PHY 329 Homework 3

Ryan J Schlimme
EID: rjs4499

October 19, 2023

Problem 7.7

Find the maximum of the function $f(x) = 4x - 1.8x^2 + 1.2x^3 - 0.3x^4$.

Part a

Use Golden-Section Search with $x_l = -2, x_u = 4$ and 1% error.

Using an M file that finds the minimum value of a function using Golden-Section search, I can find the minimum of the vertical reflection of the function, which will produce the x value corresponding to the maximum of f and the value of the maximum multiplied by -1.

Using this method, I find maximum value $f = 5.8853$ at location $x = 2.3282$.

Part b

Use Parabolic Interpolation with $x_1 = 1.75, x_2 = 2, x_3 = 2.5$ and 5 iterations.

I developed a function to do this using MATLAB. Using my function, I was able to find a maximum $f = 5.8846$ at location $x = 2.3112$ which agrees with Part a relatively well.

Problem 7.11

We are tasked with investigating the function $f(x) = \sin x + \sin 2x/3$ over the interval [2,20].

Part a

We are asked to graph the function over the interval which is shown in Figure 1.

Part b

We are then asked to use the built-in MATLAB function, fminbnd with initial guesses of $x_l = 4, x_u = 8$ to find one of the extrema.

This yields a minimum $f = -1.2160$ at $x = 5.3622$.

Part c

We are then asked to find the extrema using Golden-Section by hand and stop once we have converged the first 3 significant figures.

First, we define the golden distance $d = 0.61803(x_u - x_l)$. Then, we define two interior points $x_1 = x_l + d, x_2 = x_u - d$. Finally, we compute $f(x_1)$ and $f(x_2)$. Since we know we are looking for a minima, we look for the lesser of the two. We take that to be our minima and use the information about it's location to determine new bounds.

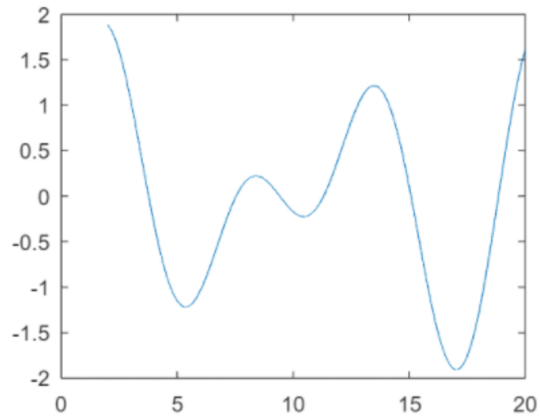

Figure 1: Graph of the function over the interval of interest

$$d = 0.61803(8-4) = 2.4721$$
$$x_1 = 4 + d = 6.4721$$
$$x_2 = 8 - d = 5.5279$$
$$f(x_1) = sin(6.4721) + sin(6.4721(3/2)) = -0.7342$$
$$f(x_2) = sin(5.5279) + sin(5.5279(3/2)) = -1.2028$$

Therefore, $x_2 = 5.5279$ is the location of the minima with minimum value $f(x_2) = -1.2028$, and the true minima lies on interval [4, x_1].

$$d = 0.61803(6.4721-4) = 1.5278$$
$$x_1 = 4 + d = 5.5278$$
$$x_2 = 8 - d = 4.0000$$
$$f(x_1) = sin(5.5278) + sin(5.5278(3/2)) = -1.2028$$
$$f(x_2) = sin(4.0000) + sin(4.0000(3/2)) = -0.2996$$

Therefore, $x_1 = 5.5278$ is the location of the minima with minimum value $f(x_1) = -1.2028$. Since we have converged to 3 significant figures, we end the computation and take

Problem 7.25

We are asked to find the minimum of $f(x, y) = 2y^2 - 2.25xy - 1.75y + 1.5x^2$ using fminsearch. This produces a minimum at $x = 0.5676, y = 0.7568$. This corresponds to a minimum value $f = -0.6622$.

Problem 7.26

We are asked to find the maximum of $f(x, y) = 4x + 2y + x^2 - 2x^4 + 2xy - 3y^2$ using fminsearch. After vertically reflecting the function, we find a maximum at $x = 0.9676, y = 0.6559$ corresponding to maximum value $f = 4.3440$.

Problem 7.37

We are tasked with minimizing the function which describes the system shown in Figure 2.

The equation which represents the potential energy can be written $p(x_1, x_2) = 0.5k_a x_1^2 + 0.5k_b(x_2 - x_1)^2 - Fx_2$. For values $k_a = 20N/m, k_b = 15N/m, F = 100N$, we find a minimum at $x_1 = 5.0001, x_2 = 11.6667$, corresponding to minimum potential energy $p = -583.3333$ J.

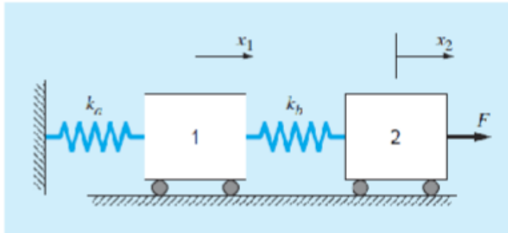

Figure 2: Two frictionless masses connected to a wall by a pair of lienar elastic springs

Problem 14.7

We are tasked with estimating the gas constant R from data relating temperature and pressure of 1 kg N_2 gas occupying $10m^3$.

Our values of T are given in degrees C so we convert to Kelvin by adding 273. Additionally, we rearrange the ideal gas law so that it reads $P = nRT/V$. This allows us to fit a linear equation such that the slope is nR/V. We can solve for n using the molar mass of $N_2 = 0.02802$ kg/mol. Therefore, we have 35.6888 moles.

Using polyfit, we find slope 29.6071 and intercept 32.4881 which we will ignore as it does not match our model. We can convert our slope into R by multiplying by V and dividing by n, yielding $R = 8.2959$ J/molK which is very close to the standard accepted value $R = 8.3145$ J/molK.

Problem 14.12

We are tasked with fitting a power law equation relating a human's weight and surface area using data on individuals 180 cm tall. By performing a log transform of our dependent and independent variable, then exponentiating the intercept, we find a model $A = 0.4149W^0.3799$. Using this model, we predict that a 95 kg, 180 cm individual would have a surface area of 2.3403 m^2.

Problem 15.3

We are asked with fitting a third order polynomial to some data for y and x. Additionally, we are asked to find the r^2 and residual standard error.

Using polyfit, I find a model of $y = 0.0467x^3 - 1.0412x^2 + 7.1438x - 11.4887$. By using the output of polyfit as the input to polyval, I can find the estimated y values for any x input. Then, I can sum over the square differences and square residuals. Then, I can compute the r squared value as 1 - sum of square differences / sum of square residuals = 0.829.

The standard error can similarly be calculated as the square root of the sum of square residuals divided by the difference between the number of data points and variables + 1. This yields a residual standard error of 0.5700 units.

Problem 15.28

I used nlinfit to perform a nonlinear regression of some data using a supplied model $p(t) = A\exp -1.5t + B\exp -0.3t + C\exp -0.05t$. This produced fit coefficients $A = 4.0046, B = 2.9213, C = 1.5647$.

Problem 16.2

Similarly, I used nlinfit to perform a nonlinear regression of temperature data onto a model sine function $T(t) = Asin(Z(t - B)) + C$ where $Z = 2\pi/365$ is the supplied period of the sine function.

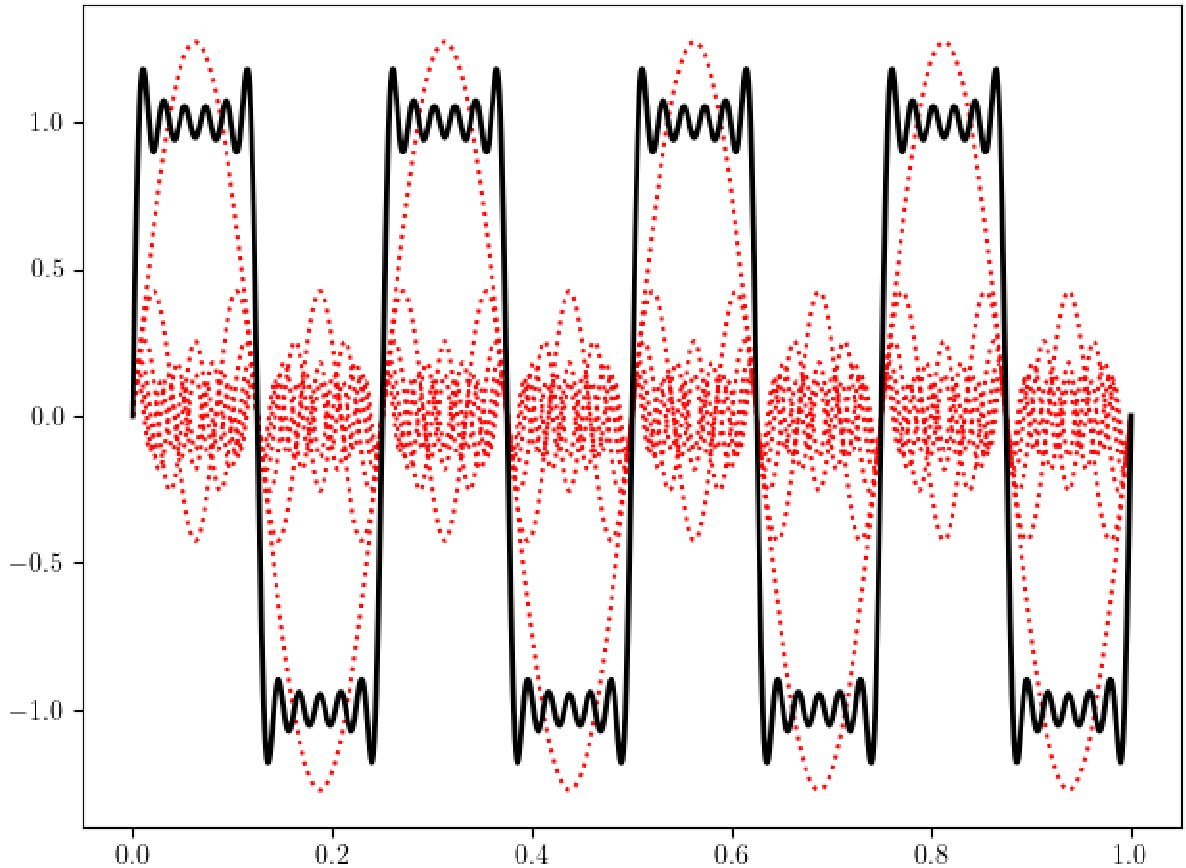

Figure 3: Plot of the first 6 terms of the Fourier series denoted by thin red dotted lines as well as the sum of the terms denoted by a thick black line

This produced a function $T(t) = 8.4418 sin(0.0172(t - 470.9652)) + 12.0341$, corresponding to an average temperature of 12.0341 degrees C, an amplitude of 8.4418 degrees C, and a hottest day of at day 197.2152 (when considering a 365 day year), or approximately June 28.

Problem 16.6

We are asked with plotting the first 6 terms of the following Fourier series of a square wave which oscillates from -1 to 1, completing one period every 0.25 s.

$$f(t) = \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \sin \frac{2\pi(2n-1)t}{0.25}$$

We are instructed to plot the first 6 terms of the series individually using thin red dotted lines and the sum of the first 6 terms using a bold, solid, black line over the interval $t = 0$ s to $t = 1$ s.

Figure 3 illustrates the requested plot.

Evidently, the first 6 terms only moderately resemble $f(t)$. We need to compute infinitely many terms to fully recover the original function. Without that, we don't have perfect jumps from +1 to -1, and toward the jump, we observe the Gibb's phenomena, the prevalence of higher frequency oscillations around jump discontinuities.