PHY 329 Homework 4

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Problem 18.1

Given the data shown in Figure 1, we are asked to fit the data with various cubic splines:

Part a) Natural End Conditions

Figure 2 compares the spline to the data.

Part b) Not-a-Knot End Conditions

Figure 3 compares the spline to the data.

Part c) Piecewise Hermite Interpolation

Figure 4 compares the spline to the data.

Problem 18.7

The data shown in Figure 5 was generated by $f(x) = 0.0185x^5 - 0.444x^4 + 3.9125x^3 - 15.456x^2 + 27.069x - 14.1$.
We are asked to fit a cubic spline with the following end conditions:

Part a) Not-a-Knot End Conditions

Figure 6 compares the spline to the function and data.

Part b) Clamped End Conditions

Figure 7 compares the spline to the function and data.

Problem 18.11

Given 5 equidistantly spaced values of the function $f(x) = 1/(1 + 25x^2)$ over interval $[-1, 1]$, we are asked to perform three tasks.

x	1	2	2.5	3	4	5
$f(x)$	1	5	7	8	2	1

Figure 1: Data to fit cubic splines

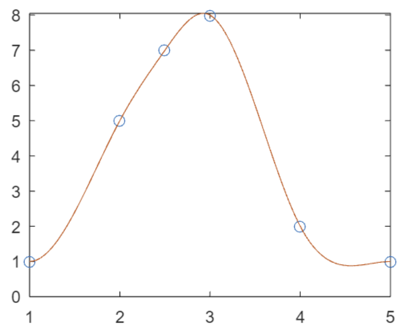

Figure 2: Comparing Natural cubic spline (orange) with data which generated the spline (circles)

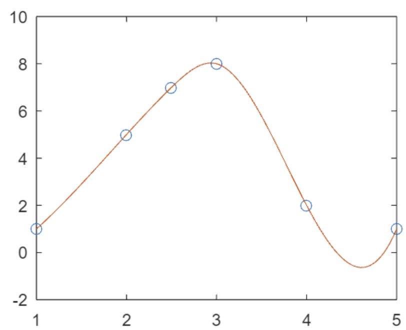

Figure 3: Comparing Not-a-Knot cubic spline (orange) with data which generated the spline (circles)

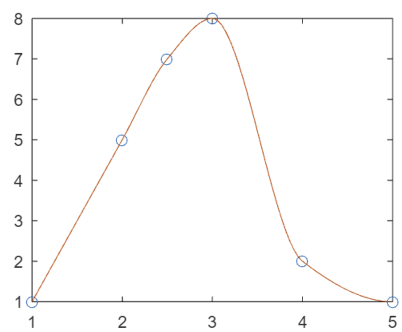

Figure 4: Comparing Piecewise Hermite Interpolation cubic spline (orange) with data which generated the spline (circles)

x	1	3	5	6	7	9
$f(x)$	1.000	2.172	4.220	5.430	4.912	9.120

Figure 5: Data generated by the function

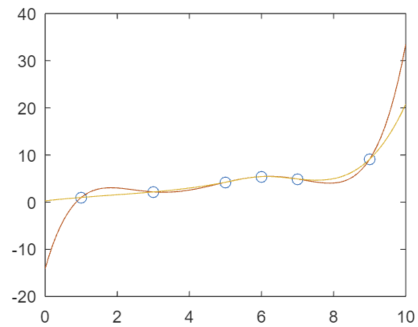

Figure 6: Comparing Not-a-Knot cubic spline (yellow) with original function (orange) along with the subset of data which generated the spline (circles)

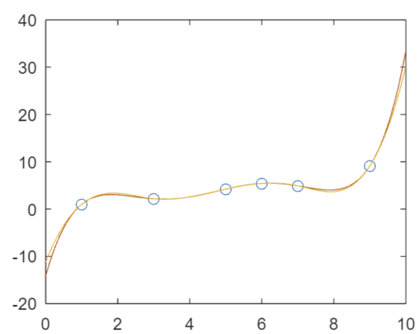

Figure 7: Comparing Clamped cubic spline (yellow) with original function (orange) along with the subset of data which generated the spline (circles)

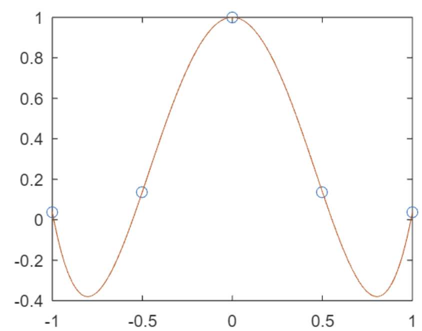

Figure 8: Runge's function fit with a fourth order polynomial from 5 equidistant points on interval $[-1, 1]$

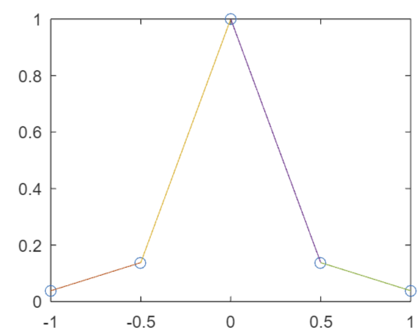

Figure 9: Runge's function fit with a linear spline

Part a) Quartic Polynomial Fit

Using a fourth order polynomial, we find $f(x) = 3.3156x^4 - 4.2772x^2 + 1.0000$. A graph of the function fit and data is shown in Figure 8.

Part b) Linear Spline

Using a linear spline, I fit the data as shown in Figure 9.

Part c) Cubic Spline

Using MATLAB's spline function, I fit the data as shown in Figure 10.

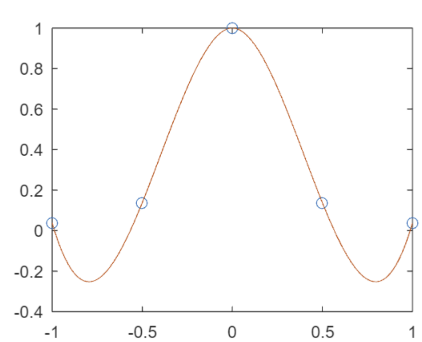

Figure 10: Runge's function fit with a cubic spline

x	0	0.1	0.3	0.5	0.7	0.95	1.2
$f(x)$	1	0.9048	0.7408	0.6065	0.4966	0.3867	0.3012

Table 1: Unevenly spaced data generated from $f(x) = e^{-x}$

Problem 19.4

We are asked to evaluate the integral $I = \int_{-2}^{4} 1 - x - 4x^3 + 2x^5 \, dx$ using various methods.

Part a) Analytically

Polynomials can be integrated easily. $I = [x - x^2/2 - x^4 + 2x^6/6\Big|_{-2}^{4} = 1104.$

Part b) Single Trapezoidal Rule

Using a single trapezoid, $I = 5280$ for an error of 378.2609%.

Part c) Composite Trapezoidal Rule

For $n = 2$, $I = 2634$ with error 138.5870%. For $n = 4$, $I = 1516.9$ with error 37.3981%.

Part d) Single Simpson's 1/3 Rule

Using Simpson's 1/3 rule with $n = 2$, $I = 1752$ for error 58.6957%. With $n = 4$, $I = 1144.5$ for error 3.6685%.

Part e) Single Simpson's 3/8 Rule

Using Simpson's 3/8 rule with $n = 3$, $I = 1392$ for error 26.0870%. With $n = 6$, $I = 1122$ for error 1.6304%.

Part f) Boole's Rule

Using Boole's rule with $n = 2$, $I = 1104$ for error 0%.

Problem 19.5

The following data shown in Table 1 was generated from the function $f(x) = e^{-x}$.

We are asked to evaluate the integral from 0 to 1.2 by various means.

Part a) Analytically

Exponential functions integrate easily. $I = \int_0^{1.2} e^{-x} \, dx = -e^{-x}\Big|_0^{1.2} \approx 0.698806.$

Part b) Trapezoidal Rule

Using the trapezoidal rule, I found $I = 0.7012$ for a true percent relative error of 0.3492%.

Part c) Trapezoidal and Simpson's

Using a combination of trapezoidal and Simpson's rules, I found $I = 0.6988$ for a true relative percent error of -0.0072%.

t	1	2	3.25	4.5	6	7	8	8.5	9	10
v	5	6	5.5	7	8.5	8	6	7	7	5

Table 2: Time-varying velocity data

x	0	4	6	8	12	16	20
ρ	4.00	3.95	3.89	3.80	3.60	3.41	3.30
A	100	103	106	110	120	133	150

Table 3: Data from a variable density rod for calculation of total mass

Problem 19.8

We are asked to determine the distance traveled given the data shown in Table 2.

Part a) Trapezoidal Rule

We are asked to find the distance traveled by integrating using the Trapezoidal rule. Additionally, we are asked to determine the average velocity (which can be computed by dividing the integral by the change in time).

Using this method, I found $x = 60.1250$ units and average $v = 6.6806$ units/s.

Part b) Cubic Polynomial

We are asked to fit a cubic polynomial and integrate the fitted equation to determine the distance.

Using polyfit, my cubic polynomial of best fit was $v(t) = -0.0180t^3 + 0.1753t^2 + 0.0603t + 4.8507$, and using the integral function, I found $x = 60.0206$ units.

Problem 19.13

The total mass of a variable density rod is given by $m = \int_0^L \rho(x)A(x)\,dx$. For a 20 m rod, the following data was measured as depicted in Table 3. We are asked to solve for the total mass.

Using trapezoidal rule, I found $m = 8.6314e3$ grams.

Problem 19.10

Given $f(z) = 200(\frac{z}{5+z})e^{-2z/h}$, we are asked to compute $F = \int_0^{30} f(z)\,dz$ and $d = \frac{\int_0^{30} zf(z)\,dz}{\int_0^{30} f(z)\,dz}$ by two methods.

Part a) Composite Trapezoidal Rule

Using n = 6 and a script to calculate integrals using composite trapezoidal rule, I found answers $F = 1.4027e3, d = 13.7199$.

Part b) Composite Simpson 1/3 Rule

Using Composite Simpson's 1/3 rule, I found $F = 1.2308e3, d = 13.0971$.

Problem 20.1

Using Romberg integration with an error tolerance of 0.5%, we are asked to evaluate $I = \int_1^2 (x + \frac{1}{x})^2\,dx$ and compare against the analytical answer.

Analytically, the result is simple. We expand the integrand into the "polynomial" $x^2 + 2 + x^{-2}$. Then, we perform the integration to yield an integral $[x^3/3 + 2x - x^{-3}/3]\Big|_1^2 = 29/6 \approx 4.8333$.

Using Romberg integration, we find $I = 4.8335$ for an error of 0.003%.

Problem 20.2

We are tasked with solving the integral:

$$I = \int_0^8 -0.055x^4 + 0.86x^3 - 4.2x^2 + 6.3x + 2\,dx$$

Part a) Analytically

Integrating a polynomial is trivial:

$$I = [-0.055/5x^5 + 0.86/4x^4 - 4.2/3x^3 + 6.3/2x^2 + 2x]\Big|_0^8 I = 20.992$$

Part b) Romberg Integration

Using an error tolerance of 0.5%, we find an integral of 20.992.

Part c) Three-Point Gauss Quadrature

We find an integral of 20.992.

Part d) MATLAB integral Function

We find an integral of 20.992.

Problem 20.7

The heat ΔH required to induce a temperature change ΔT is given by $\Delta H = mC(T)\Delta T$. Given 1000 g of mass and $C(T) = 0.132 + 1.56\text{e-}4T + 2.64\text{e-}7T^2$, we are tasked with producing a plot of ΔH versus ΔT.

I performed this by taking an integral from $T = -100$ to T_{max} for $T_{max} \in [-100, 200]$. The result is shown in Figure 11.

Problem 20.14

The voltage across a capacitor as a function of time is given by $V(t) = \frac{1}{C}\int_0^t i(t)\,dt$. Given a 10^{-5} F capacitor and the data shown in Figure 12, we generate a plot of voltage versus time shown in Figure 13.

Problem 21.9

For the unequally spaced data shown in Table 4, we are asked to compute the first derivative and compare with the equation $f(x) = 5\exp(-2x)x$. Actual derivatives are calculated at the midpoint of each pair of x values.

Problem 21.18

Using diff(y) we are asked to find the first and second derivative of the following data as shown in Table 5. Derivatives are placed on the right of the interval created by each pair of x data.

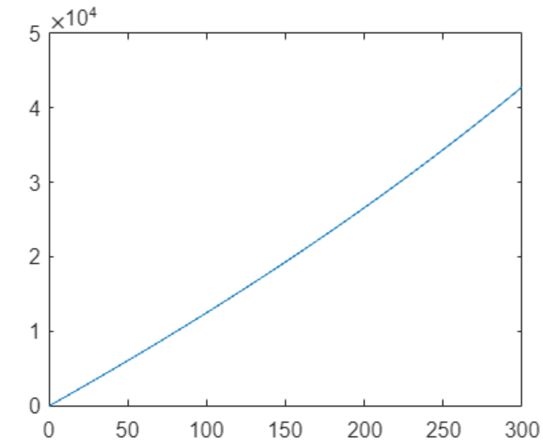

Figure 11: ΔH as a function of ΔT

t, s	0	0.2	0.4	0.6
i, 10^{-3} A	0.2	0.3683	0.3819	0.2282
t, s	0.8	1	1.2	
i, 10^{-3} A	0.0486	0.0082	0.1441	

Figure 12: Current as a function of time data

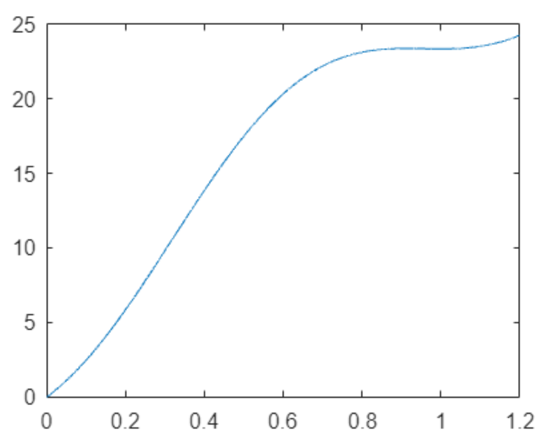

Figure 13: Voltage as a function of time given data shown in Figure 12

x	0.6	1.5	1.6	2.5	3.5
$f(x)$	0.9036	0.3734	0.3261	0.08422	0.01596
Numerical		-0.5891	-0.4730	-0.2688	-0.0683
Analytical		-0.6735	-0.4730	-0.2569	-0.0620

Table 4: Comparing numerical and analytical derivatives of the data

x	0	1	2	3	4	5	6	7	8	9	10
y	1.4	2.1	3.3	4.8	6.8	6.6	8.6	7.5	8.9	10.9	10
dy/dx		0.7000	1.2000	1.5000	2.0000	-0.2000	2.0000	-1.1000	1.4000	2.0000	-0.9000
d^2y/dx^2			0.5000	0.3000	0.5000	-2.2000	2.2000	-3.1000	2.5000	0.6000	-2.9000

Table 5: Right point numerical differentiation of the data

t	0	0.1	0.2	0.3	0.5	0.7
i	0	0.16	0.32	0.56	0.84	2.0
$V(t)$		6.4000	6.4000	9.6000	5.6000	23.2000

Table 6: Derived voltage as a function of time given current and time data

Problem 21.26

Given data for time and current as shown in Table 6, we are asked to determine the voltage for each value of time given the formula $V(t) = L\frac{di}{dt}$ for $L = 4$ H. Derivatives are placed on the right of the interval created by each pair of t data.

Problem 21.41

Using Richardson's extrapolation, we are asked to find the acceleration of a particle at time $t = 5$ s using $h = 0.5, 0.25$ given $v(t) = 2t/\sqrt{1+t^2}$. The exact solution is $a(t) = 2/(1+t^2)^{3/2}$. Therefore, $a(5) = 0.0151$ m/s^2. Using Richardson's extrapolation, we get $a(5) = 0.0158$ m/s^2 for a true percent relative error of 4.76%.