PHY 329 Homework 1

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Problem 5.7

We are tasked with determining the roots of $f(x) = -12 - 21x + 18x^2 - 2.75x^3$ using three methods.

Part a) Graphically

Using MATLAB, we can graph the function and $y = 0$. After zooming in several times, we can approximate the roots x_r through visual inspection. Then, we can evaluate $f(x_r)$ for each root to find our deviation from the theoretical value of $f(x_r) = 0$.

Using this method, I found roots $x_r = -0.415, 2.220, 4.740$ with corresponding values $f(x_r) = -0.0005$, $-0.0003, 2.352e-5$.

Part b) Bisection

We are then asked to find the first root using bisection with initial guesses $x_l = -1, x_u = 0$ and a stopping criterion of 1%. I implemented a function to perform this using MATLAB, producing a root of $x = -0.4147$.

Part c) False Position

Using the same initial guesses and stopping criterion, we are asked to perform the calculation using False Position, which I again implemented in MATLAB, again producing a root of $x = -0.4147$.

Problem 5.8

We must determine the first nontrivial solution to $sin(x) = x^2$ using graphical inspection and bisection with $x_l = 0.5, x_u = 1$ and stopping criterion of 2%. This, of course, requires that we rearrange the function such that it reads $0 = AE$ for AE some arbitrary expression. Graphically, I was able to estimate a root of $x = 0.877$ rad while bisection produced a root of $x = 0.8768$ rad.

Problem 5.9

We must determine the positive real root of $\ln(x^2) = 0.7$.

Part a) Graphically

Using the aforementioned graphical method, I estimated a root of $x = 1.4197$.

Part b) Bisection

Using the given parameters $x_l = 0.5, x_u = 2$ and a maximum number of iterations maxit $= 3$, I estimated a root $x = 1.4141$.

Part c) False Position

Using the same parameters, we are asked to perform the calculation using false position. As I hadn't coded a maximum number of iterations into my formula, I decided to perform the calculations manually with the assistance of MATLAB. This resulted in a root $x = 1.448.$ I concluded that false position may need more iterations to work on this type of function.

Problem 5.19

We are asked to solve for the distance at which a point charge would exhibit an electrostatic force $F = 1.25$N in the presence of a ring of radius $a = 0.85$m. Both the point and ring have charge $q = 2e - 5$C. The force can be calculated according to:

$$F = \frac{1}{4\pi e_0} \frac{q^2 x}{(x^2 + a^2)^{3/2}} \tag{1}$$

Using the given parameters and rearranging the formula, we can treat this as a root finding problem. I graphed the function to estimate the roots graphically, resulting in two roots $x- = 0.2, 1.0$ m. Then, using the $fzero$ function built into MATLAB, I found roots of $x = 0.2310, 1.2913$ m. My hypothesis for the existence of two roots is due to the exponent on x in the denominator.

Problem 5.15

A uniform beam subject to a linearly increasing distributed load results in an elastic curve described by:

$$y = \frac{\omega_0}{120EIL}(-x^5 + 2L^2x^3 - L^4x) \tag{2}$$

We are asked to use bisection to determine the point of maximum deflection, defined as the point where $dy/dx = 0$. We are given parameters $L = 600\text{cm}, E = 50000\text{kN/cm}^2, I = 30000\text{cm}^4, \omega_0 = 2.5\text{kN/cm}$.

First, I graphed the derivative, which I calculated analytically on scratch paper as:

$$\frac{dy}{dx} = \frac{\omega_0}{120EIL}(-5x^4 + 6L^2x^2 - L^4) \tag{3}$$

Then, I graphed the function to estimate the root(s), resulting in estimates of $x = 270$ and $x = 600$ cm. Understanding that the latter would be nonphysical as it occurs at the end of the rod, I only continued with the first guess.

I first tried bisection, but was receiving errors. After doing some research, I realized that, since the fourth-order polynomial merely "reflects" off the y-axis at a point (the true root) instead of crossing it, bisection would not work as it will never be able to approach the root from the left x_l and the right x_r such that $f(x_l) * f(x_r) < 0$.

Trying another method, I entered my estimate into MATLAB's $fzero$ function along with the derivative and received an estimated root of $x = 268.328$ cm. Then, I evaluated the original function at this value to estimate a maximum deflection of -0.5152 cm.

Problem 5.24

Archimedes's principle states that the buoyancy force is equal to the weight of the fluid displaced by a submerged body. Given a partially submerged sphere of radius $r = 1$ m and density $\rho_s = 200\text{kg/m}^3$ in water of density $\rho_w = 1000\text{kg/m}^3$, we are asked to use bisection to find the height of the non-submerged portion of the sphere.

To solve this problem, I assumed the ball to be in static equilibrium such that the force of gravity acting downward equals the buoyancy force acting upwards.

$$\frac{4\pi}{3}r^3\rho_s g = (\frac{4\pi}{3}r^3 - \frac{\pi h^2}{3}(3r-h))\rho_w g \tag{4}$$

After entering our given parameters and rearranging into a root finding problem, I graphed it to find suitable bounds. Then, using $x_l = 1, x_u = 1.5$ with my bisection code, I estimated a root of $x = 1.4257$ m, the height of the non-submerged portion of the sphere.

Problem 6.3

We are tasked with determining the highest real root of $f(x) = x^3 - 6x^2 + 11x - 6.1$.

Part a) Graphically

Using the aforementioned graphical method, I found a result of $x = 3.04665.$

Part b) Newton-Raphson

To estimate the root with Newton-Raphson, I analytically calculated the derivative of the polynomial as $f'(x) = 3x^2 - 12x + 11$. Using a maximum of 3 iterations and an initial guess $x_0 = 3.5$, I estimated a root of $x = 3.0473.$

Part c) Secant

Using a maximum of 3 iterations and initial guess parameters $x_0 = 2.5, x_1 = 3.5$, I estimated a root of $x = 3.0479.$

Part d) Modified Secant

Using a maximum of 3 iterations, an initial guess $x_0 = 3.5$, and perturbation fraction $\delta = 0.01$, I estimated a root of $x = 3.049$.

Part e) All Roots

Since this function is a polynomial, I can use the built in MATLAB function *roots*. This produced a vector of roots, $x = 3.047, 1.899, 1.054.$

Problem 6.4

We are tasked with finding the lowest positive root of $f(x) = 7\sin(x)e^{-x} - 1$.

Part a) Graphically

Using the aforementioned graphical method, I found a lowest positive root of $x = 0.17018$.

Part b) Wegstein

Using Wegstein with initial parameters $x = -0.5, x = 0.3$, we get an inaccurate result of $x = 0.2182.$ Despite testing other initial parameters, the output did not deviate. I am not sure why this method failed when the others succeeded.

Part c) Newton-Raphson

After analytically calculating the derivative $f'(x) = 7e^{-x}(\cos x - \sin x)$, I used Newton-Raphson with initial parameter $x_0 = 0.3$ and 3 iterations maximum to produce an estimate of $x = 0.1702.$

Part d) Modified Secant

Using a maximum of 5 iterations and initial parameters $x_0 = 0.3, \delta = 0.01$, the modified secant method produced an estimate of $x = 0.1702$.

Problem 6.19

The following expression for impedance, Z, in terms of resistance, R, capacitance, C, inductance, L, and angular frequency, ω, solves a specific LRC circuit.

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + (\omega C - \frac{1}{\omega L})^2} \tag{5}$$

To find the angular frequency that results in an impedance of 100 Ω, given the parameters $R = 225\Omega, C = 0.6e-6$ F, and $L = 0.5$ H, we can rearrange the problem into a root finding problem, $0 = (\frac{1}{R^2} + (\omega C - \frac{1}{\omega L}))^{-1/2} - 100$.

Graphing the function, I estimated a root at $\omega = 0.6$ rad. Using MATLAB's $fzero$ function, I found a root of $\omega = 0.663$ rad.

Problem 6.20

Using conservation of energy, one can find the following expression for the deflection of nonlinear springs:

$$0 = \frac{2k_2 d^{5/2}}{5} + 0.5k_1 d^2 - mgd - mgh \tag{6}$$

where m is the mass of a block, h is the distance the block is released above the spring, d is the compression of the spring, and k_1 and k_2 are spring constants.

Given the parameters $k_1 = 40000\text{g/s}^2, k_2 = 40\text{g/(s}^2\text{m}^{0.5}, m = 95\text{g}, g = 9.81\text{m/s}^2$, and $h = 0.43$ m, we can solve for d using MATLAB root finding.

After graphing the function and estimating a root at $d = 0.2$ m, I used MATLAB to estimate the root at $d = 0.1667$ m.

Problem 6.21

One can model trajectory using the function:

$$y = (\tan(\theta))x - \frac{g}{2v^2 \cos^2\theta}x^2 + y_0 \tag{7}$$

Given $v = 30$ m/s and $x = 90$ m, $y_0 = 1.8$ m, and $y = 1$ m, we can use root finding to find θ.

When graphing the function, I estimated a root at $\theta = 0.6$ rad. However, there also existed a solution around $\theta = 0.9$ rad and these two roots continued in both directions cyclically. The infinite repeating roots is expected from a trigonometric function, but the existence of the root at $\theta = 0.9$ rad I cannot explain. I used MATLAB and confirmed a root at $\theta = 0.663$ rad. However, I was also able to confirm the root at $\theta = 0.899$ rad.

Problem 6.38

Two cables initially of lengths L suspend a scoreboard which deform the cables to lengths L' at a distance d below the unperturbed system. The scoreboard weighs 9000 N. Since each cable obeys Hooke's law such that the axial elongation is represented by $L' - L = FL/(A_c E)$, where F is the tension in the stretched cables, A_c is the cable's cross-sectional area, and E is the modulus of elasticity.

Given the parameters $L = 45\text{m}, A_c = 6.362e-4\text{m}^2, E = 1.5e11$ N/m^2, one can solve for the vertical deformation distance d and the elongation $L' - L$.

Each cable has a vertical force component equal to half the total weight, or 4500 N each. Using trigonometry, this means that $F \sin\theta = 4500$. Additionally, $\sin\theta = d/L'$. Therefore, $F = 4500L'/d$. Using Pythagoras's theorem, $L'^2 = L^2 + d^2$, along with rearranging Hooke's law, $L' = L + F\alpha$, where $\alpha = L/A_cE$, we can write $(L + F\alpha)^2 = L^2 + d^2$. Using $d = 4500L'/F$ and Hooke's law $L' = F\alpha + L$ can write this as $L^2 + 2FL\alpha + (F\alpha)^2 = L^2 + (\frac{4500}{F}(F\alpha + L))^2$. Simplifying into a root finding problem, we yield $2F\alpha + (F\alpha)^2 - (\frac{4500}{F}(F\alpha + L))^2 = 0$. Using MATLAB, we find $F = 98913$ N. Then, using $d = \frac{4500}{F}(F\alpha + L)$, we find vertical deflection of $d = 2.0494$ m downward. Then, using $L' - L = F\alpha$, we find the elongation in each cable $L' - L = 0.0466$ m.