PHY 329 Homework 5

Ryan J Schlimme
EID: rjs4499

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Problem 22.1

We are asked to solve the initial value problem $\frac{dy}{dt} = yt^2 - 1.1y, y(0) = 1$ using various methods starting from $t = 0$/ Since, it did not specify an ending time, I set the end of the interval to be $t = 2$ for all methods.

Part a) Analytically

$$\begin{aligned}
\frac{dy}{y} &= (t^2 - 1.1)dt \\
\ln y &= \frac{1}{3}t^3 - 1.1t + A \\
y(t) &= Be^{\frac{1}{3}t^3 - 1.1t} \\
y(0) &= B = 1 \\
y(t) &= e^{\frac{1}{3}t^3 - 1.1t}
\end{aligned}$$

This exact solution is denoted in Figure 1 by the blue line.

Part b) Euler's Method

Using Euler's method with $h = 0.5$ and $h = 0.25$, we find marginally accurate results depicted by the orange line $h = 0.5$ and the yellow line for $h = 0.25$ in 1.

Part c) Midpoint Method

As a correction to Euler's method, we implement the Midpoint method with $h = 0.5$ as shown in Figure 1 as a purple line. Note how it seems on the same order of accuracy as regular Euler at $h = 0.25$.

Part d) Fourth-Order Runge Kutta

To approximate even more correctly, we implement the Fourth-Order Runge Kutta approximation as shown by the green line in Figure 1. This seems the most accurate of the three methods we tested.

Problem 22.3

We are asked to solve the IVP $\frac{dy}{dt} = -y + t^2, y(0) = 1$ over the interval $t = 0$ to $t = 3$, using a step size of $h = 0.5$ for the following methods.

Part a) Non-Iterative Heun's Method

The blue trace on Figure 2 represents the results.

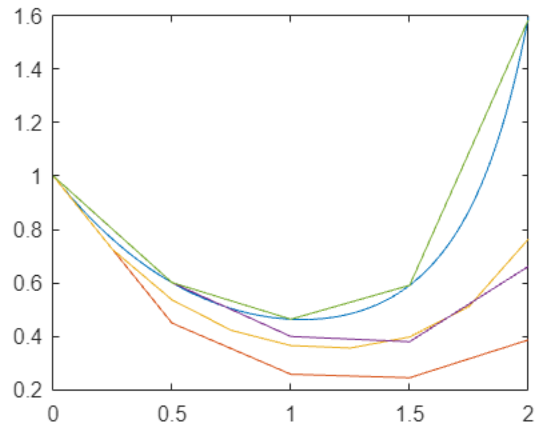

Figure 1: Graph of y vs t for the differential equation in Problem 22.1, comparing numerical techniques to analytic answer

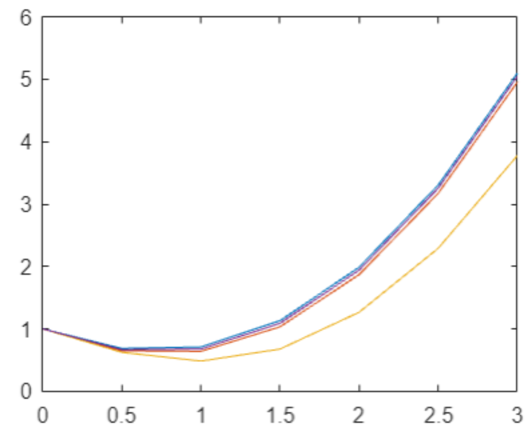

Figure 2: Graph of y vs t for the differential equation in Problem 22.3, comparing numerical techniques

Part b) Heun's Method

The orange trace on Figure 2 represents the results.

Part c) Midpoint Method

The yellow trace on Figure 2 represents the results. This is the only obviously-deviating method not agreeing with the other three.

Part d) Ralston's Method

The purple trace on Figure 2 represents the results.

Problem 22.6

We are asked to use Euler's method to solve for the maximum height of a projectile launched from the surface of the Earth at an initial velocity $v(0) = 1500$ m/s modeled by the equation $\frac{dv}{dt} = -g(0)\frac{R^2}{(R+x)^2}$. $g(0) = 9.81$ m/s^2, $R = 6.37e6$ m. Additionally, we must recognize that $dx/dt = v$.

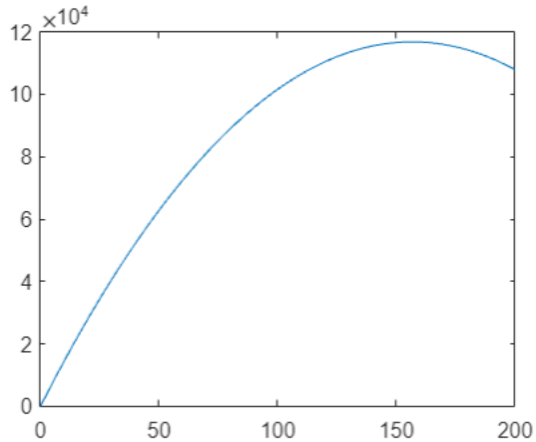

Figure 3: Graph of x vs t for the system of differential equations in Problem 22.6

Using this information, we can solve for a system of first order ordinary differential equations using a modified version of the Euler ODE script. Using a common step size $h = 0.5$ over the time interval $t = 0, 200$ produces the following graphical result shown in 3.

We can find the maximum of the functional variable x as the maximum height by using MATLAB's max function on the vector x generated by our ODE solver, resulting in a maximum height of 116.78 km.

Problem 22.7

We are asked to solve the following system of ODE's over the interval $t = 0$ to $t = 0.4$ using a step size $h = 0.1$ given initial conditions $y(0) = 2, z(0) = 4$ using two methods.

$$\frac{dy}{dt} = -2y + 4e^{-t}$$
$$\frac{dz}{dt} = -\frac{yz^2}{3}$$

Part a) Euler's Method

The blue trace on Figure 4 represents the results for the first equation. The blue trace on Figure 5 represents the results for the second equation.

Part b) Fourth-Order Runge Kutta

The orange trace on Figure 4 represents the results for the first equation. The orange trace on Figure 5 represents the results for the second equation.

Problem 22.13

We are asked to develop an M-file to solve a system of ODE's using Euler's Method and plot the results. The following code block illustrates my solution as a function.

```
function [t,x,y]=eulersys(e, f, tspan, x0, y0, h, varargin)
if nargin<4, error("at least 4 inputs required"), end
ti = tspan(1);tf = tspan(2);
if ~(tf>ti), error('upper limit must be greater than lower'), end
t = (ti:h:tf)'; n = length(t);
if t(n)<tf
```

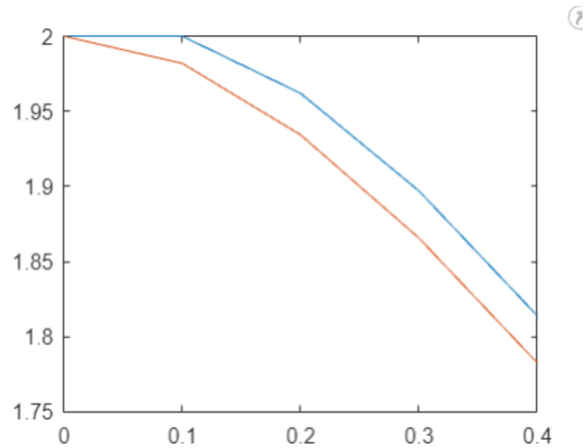

Figure 4: Graph of y vs t for the system of differential equations in Problem 22.7

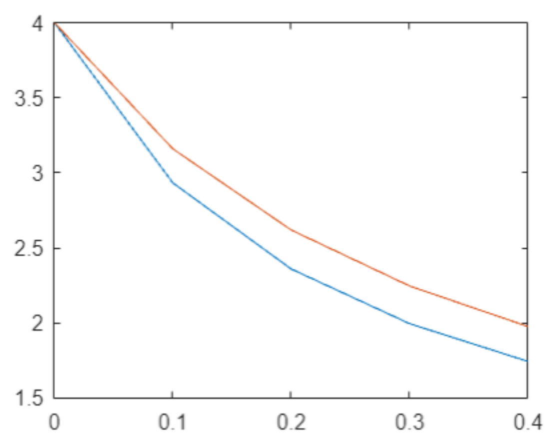

Figure 5: Graph of z vs t for the system of differential equations in Problem 22.7

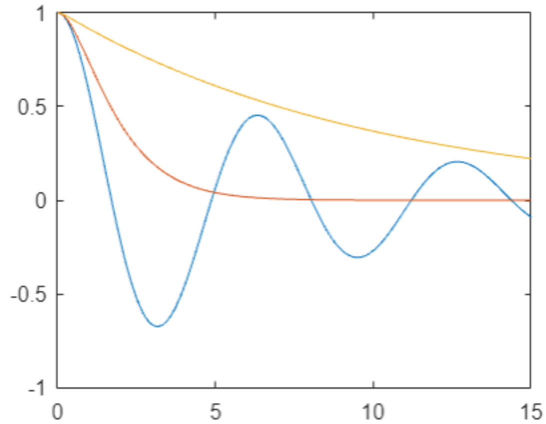

Figure 6: Graph of x vs t for the system of differential equations in Problem 22.15

```
    t(n+1) = tf;
    n = n + 1;
end
y = y0*ones(n,1);
x = x0*ones(n,1);
for i = 1:n-1
    x(i+1) = x(i) + e(t(i), x(i), y(i))*h;
    y(i+1) = y(i) + f(t(i), x(i), y(i))*h;
end
figure(1);plot(t,x)
figure(2);plot(t,y)
end
```

Problem 22.15

We are asked to solve the ODE $m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$ representing the displacement from equilibrium x of a mass spring system as a function of time t with a mass $m = 20$ kg, a spring constant $k = 20$ N/m, and a damping coefficient c which can take values $5, 40, 200$ Ns/m representing underdamped, critically damped, and overdamped systems. The initial displacement $x(0) = 1$ m and the initial velocity $v(0) = 0$ m/s^2. We are instructed to use a numerical method over time interval $0 \leq t \leq 15$ s.

We can break this system into a system of first order differential equations by using $dx/dt = v$. Substituting this into the original equation, we yield:

$$m\frac{dv}{dt} + cv + kx = 0$$
$$\frac{dv}{dt} = \frac{-cv - kx}{m}$$

Using Fourth-Order RK on the system, we can produce traces of $x(t)$ for each value of c as shown in Figure 6. The blue trace represents $c = 5$ Ns/m. Orange represents $c = 40$ Ns/m. Yellow represents $c = 200$ Ns/m.

Problem 22.19

We are asked to solve for the concentration as a function of time $C(t)$ described by the following system of ODEs:

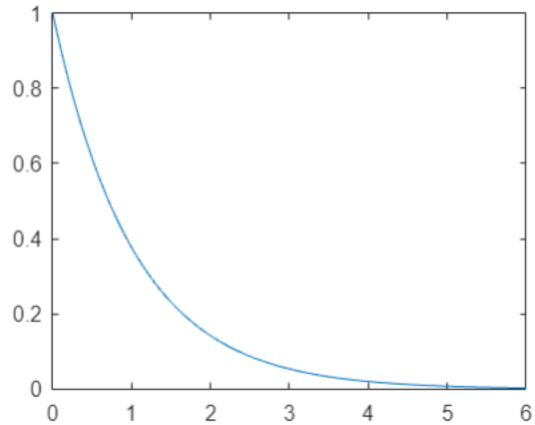

Figure 7: Graph of C vs t for the system of differential equations in Problem 22.19

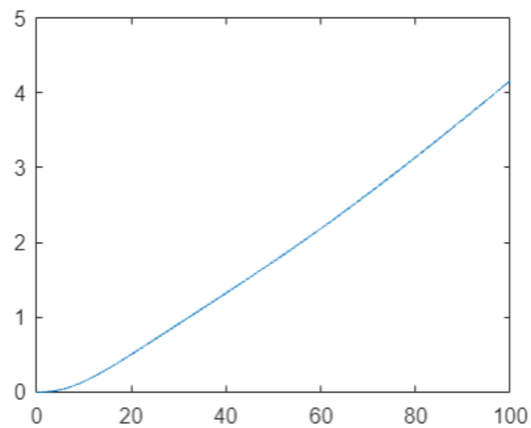

Figure 8: Graph of y vs t for the differential equation in Problem 22.20

$$\frac{dC}{dt} = -e^{-10/(T+273)}C$$
$$\frac{dT}{dt} = 1000e^{-10/(T+273)}C - 10(T-20)$$

We are given initial values of temperature $T(0) = 15$ degrees C and concentration $C(0) = 1.0$ gmol/L. Using Euler's method over the time interval $0 \le t \le 6$ s and step size 0.01, we find the results shown in figure 7.

Problem 22.20

The following equation models the deflection y of a sailboat mast subject to a wind force $f(z)$ as a function of time t, $\frac{d^2y}{dz^2} = \frac{\frac{200z}{5+z}e^{-2z/30}}{2EI}(L-z)^2$. Initially ($z = 0$), $y = 0, dy/dz = 0$. Using $L = 30, E = 1.25e8, I = 0.05$, we can solve the equation by converting it into a system of first order ODEs and solving with Fourth-Order RK.

Defining a new variable $w = dy/dz$, we can rewrite the original equation as $\frac{dw}{dz} = \frac{\frac{200z}{5+z}e^{-2z/30}}{2EI}(L-z)^2$. The results are illustrated in Figure 8.