Determining the Fundamental Electric Charge Using Millikan's Oil Drop Method

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(Dated: April 27, 2024)

To measured the fundamental electric charge, q_e, using Millikan's oil drop method, we sprayed small drops of mineral oil into a capacitor and observed their behavior under only gravity and under both gravity and the electromagnetic force by varying the voltage across the capacitor plates. Using Newtonian mechanics, we calculated the charge of each oil drop from these observations. Repeating these observations and calculations for many oil droplets, we generated a histogram of charges and noticed multi-modality of roughly equally spaced peaks. We then hypothesized that each mode is a quantized energy characterized by a local mean and standard deviation. The average difference in pairwise local means is the fundamental electric charge. Using this method, we determined $q_e = (1.598 \pm 0.008) \times 10^{-19}$ Coulombs.

I. INTRODUCTION

In 1913, Robert Andrews Millikan, published the results of an experiment to determine the fundamental quantization of electric charge using what he referred to as the "oil-drop method." By observing the speed of descent of small oil drops under gravity and the speed of ascent under both gravity and an electric field, he proposed a mechanism for estimating the charge of each droplet [1]. He sprayed small oil drops into a capacitor, measured their terminal velocity under gravity, applied a voltage across the plates of the capacitor, and then measured the terminal velocity upwards of the oil drops. Then, he used these velocities to calculate the charge of each droplet, q. We seek to verify his findings as follows.

To estimate the charge, q, of each mineral oil drop, we use Newtonian mechanics. The forces acting on the oil droplet we approximate as only in the vertical direction, $\hat{y}$. When the droplet falls under the force of gravity through air, we write the sum of the forces as $m\ddot{y} = F_g - F_f$, where $F_g = mg$ is the force of gravity under Earth's gravitational acceleration, $g = 9.80665$ m/s^2, and F_f is the frictional force. As the droplets are small, they quickly reach terminal velocity, v_f, the velocity at which $\ddot{y} = 0$ such that $F_g = F_f$ [2].

We approximate the frictional force as $F_f = 6\pi\eta a v_f$ where η is the viscosity of air, a is the radius of the oil droplet, and v_f is the terminal velocity of the oil droplet [2]. We use Sutherland's model for the viscosity of air as a function of temperature, $\eta(T) = \eta_0 \left(\frac{T}{T_0}\right)^{3/2} \left(\frac{T_0+S}{T+S}\right)$ where $\eta_0 = 1.716 \times 10^{-5}$ kg/m·s, $T_0 = 273.15$ K, and $S = 110.4$ K parameters for the mixture in Earth's atmosphere [3]. There are other, more-accurate models for atmospheric viscosity, but they are lower order corrections with temperature as the main correction factor from constant atmospheric viscosity.

When we apply an external voltage, V, to the oil drops of charge, q, inside the capacitor of plate separation, d, an additional external force, $F_e = \frac{qV}{d}$ acts upon the droplet. When the signs of q and V differ, F_e opposes F_g; this is the configuration we desire. We apply sufficient voltage such that the oil droplets begin to move upwards at some terminal velocity, v_r. One must recall that the force of friction always opposes the direction of the velocity. Therefore, we now write the sum of forces as $0 = mg - \frac{qV}{d} - 6\pi\eta a v_r$. We can use this equation to determine the charge, q, of the oil droplets given the upward terminal velocity, v_r, and the mass, m [2]. Figure 1 illustrates the balancing of forces acting upon each droplet using free body diagrams.

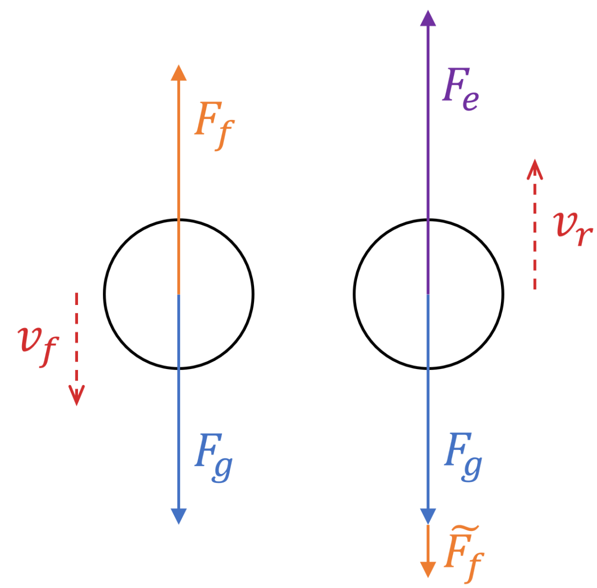

FIG. 1: Free body diagram illustrating the forces acting upon the same droplet without the applied voltage (left) and with the applied voltage (right). Notice that in both cases, the forces are balanced, indicating the droplets have no net acceleration and are at constant (terminal) velocities. The force of gravity, F_g, denoted in blue is constant for the same droplet as the mass is constant. The force of friction, F_f, denoted in orange is variable in magnitude and direction depending on the charge and velocity of the droplet. The electromagnetic force, F_e, denoted in purple opposes the force of gravity. Dashed red arrows illustrate the constant terminal velocities of the droplet in both cases.

To further charge the droplets inserted into the capacitor, we irradiate them with Thorium-232 (half life of 1.41×10^{10} years). Thorium-232 decays via alpha emission to Lead through a decay chain involving many unstable isotopes, most of which decay via alpha decay. When these positively-charged alpha particles encounter the oil droplets, they pull negative charges from the droplets. This results in an oil droplet that is more positively charged than one would expect, allowing us to observe oil droplets of many different charges which proves useful in our analysis [4].

II. EXPERIMENTAL SETUP

A. Apparatus

To determine the fundamental electric charge, we used a modified apparatus akin to Millikan's original experiment with technological improvements. Our apparatus consists of a platform with adjustable feet and leveling device such that we maintain the platform at approximately level plane. On top of this level platform, we have a cylindrical droplet viewing chamber consisting of a parallel plate capacitor, the top plate of which has a pinhole to allow drops to enter the capacitor. Above the top plate is an empty cylindrical chamber which we cap with a cover plate. The cover plate has a hole large enough to fit the tip of an atomizer which we use to spray small oil droplets into the chamber. Our first modification to Millikan's apparatus is a "tuning knob" built into the cylinder to either close the pinhole to the capacitor (isolating it from external perturbations), open the pinhole (to allow for droplets to enter), or expose the capacitor chamber to a small amount of Thorium-232 (a source of ionizing radiation to charge the oil droplets).

We shine a variable-intensity LED — which we can control through an adjustment knob — into the chamber through a small plastic screen to illuminate the volume inside the capacitor. Then, we connect the two plates of the capacitor to a well-regulated 500 V DC power supply such that we can apply a specific voltage across the plates. Our next modification to Millikan's experiment is a "plate charging switch" to either disable the external voltage, enable it in one polarity, or enable it in the opposite polarity. This allows us to quickly disable or enable the electric field as well as to quickly reverse the direction of the electric field.

Our final modification to Millikan's experiment is exchanging the viewing scope for a camera to record the droplets as they fall. The camera lens has two focus adjustment knobs to allow for a higher quality image for analysis.

Throughout our experiment, we noticed a small fissure in the rubber bulb of our atomizer, the device we used to spray oil droplets into the capacitor. Without intervention, the loss of pressure through the fissure resulted in no observable droplets. We hypothesize that the droplets produced from the damaged atomizer were too large to enter through the pinhole into the capacitor. We applied vacuum grease to the fissure as a temporary solution, but ideally the bulb needs replacing. Additionally, we noticed that should the capacitor and chamber saturate with oil, it becomes difficult to observe oil droplets. We believe any newly introduced droplets adhere to oil already in the chamber, preventing them from entering the capacitor. Therefore, we cleaned the cylindrical viewing chamber and capacitor regularly.

B. Data Collection

To determine the terminal velocities of the charged oil droplets, we use the following procedure. First, we ensure that our camera is in focus and recording by adjusting the two focusing dials until a pin inserted into the capacitor pinhole is in focus. We only refocus upon the start of a new day of data collection. Then, we open the capacitor using the tuning knob so that droplets can enter the capacitor. We set the external voltage to around 500 V but deactivate it using the plate charging switch; this ensures that no external voltage is applied but we can quickly apply it when needed.

Now, we spray droplets into the cylindrical viewing chamber using the atomizer. The method we found to work best is one quick, hard pulse of the bulb to spray the droplets followed by a softer, slower pulse to distribute the droplets throughout the chamber. Quickly, we turn the tuning knob to irradiate the droplets for an arbitrary time; we typically irradiated for 5-10 seconds, but since we want a variety of charges, it is not important to irradiate for a consistent time period.

After the irradiation, we switch the tuning knob to the closed position and flip the plate charging switch to apply the external voltage. We observe many droplets start accelerating upwards. Sometimes, they accelerate downwards as they are still negatively charged; in these cases, we flip the polarity of the capacitor using the plate charging switch. Once they rise for sufficient time (2-3 seconds) to reach terminal velocity, v_r, we disable the external voltage and allow them to fall until they reach terminal velocity, v_f. Some droplets that left the frame such that we cannot secure either one or both terminal velocities are discarded from analysis. We repeat this method for several hundred droplets.

In order to determine the terminal velocities of the oil droplets, we use Fiji Imagej to track the oil droplets and estimate their velocities as a difference in vertical pixel number $p_2 - p_1$ divided by the time interval $t_2 - t_1$. However, this results in units of pixels/frame. We correct units by applying known calibration factors of 175,081.3 pixels/m and 15 frames/s as shown below:

$$v_f = \frac{p_2 - p_1}{175,081.3} * \frac{15}{t_2 - t_1}$$
$$v_r = \frac{\tilde{p_2} - \tilde{p_1}}{175,081.3} * \frac{15}{\tilde{t_2} - \tilde{t_1}}$$

With the terminal velocities recorded, we continue to estimate the radius of each droplet. We approximate the oil drops as perfect spheres of unknown radius, a, and known density, $\rho = 886$ kg/m^3. Using the downward terminal velocity, v_f, we calculate the radius of the oil drops as follows:

$$mg = 6\pi\eta a v_f$$
$$\frac{4}{3}\pi a^3 \rho g = 6\pi\eta a v_f$$
$$a = \left(\frac{9}{2}\frac{\eta v_f}{\rho g}\right)^{1/2}$$

This allows us to estimate the mass of the spherical oil droplets as $m = \frac{4}{3}\pi a^3 \rho$.

Next, we apply an external voltage of magnitude V to the oil droplets and observe their terminal velocity upwards, v_r, we can determine their respective charge magnitude, q. We balance the forces acting upon the droplet and insert Sutherland's parameters for atmosphere to write an equation for charge as follows:

$$0 = mg - \frac{qV}{d} + 6\pi\eta a v_r$$
$$\frac{qV}{d} = mg - 6\pi\eta a |v_r|$$
$$q = \frac{d}{V}\left(\left(\frac{4}{3}\pi a^3 \rho\right) g - 6\pi\eta a |v_r|\right)$$
$$q = \frac{d}{V}\left(\frac{4}{3}\pi\left(\frac{9}{2}\frac{\eta v_f}{\rho g}\right)^{3/2} \rho g - 6\pi\eta\left(\frac{9}{2}\frac{\eta v_f}{\rho g}\right)^{1/2} |v_r|\right)$$

which yields Equation 1:

$$q = 2.4 \times 10^{-14} * \frac{d}{V}\left[\left(\frac{T^{3/2} v_f}{T + 110.4}\right)^{3/2} - \frac{T^{3/2} v_r}{T + 110.4}\left(\frac{T^{3/2} v_f}{T + 110.4}\right)^{1/2}\right] \quad (1)$$

for charge magnitude as a function of temperature (T), which we record prior to each iteration of the experiment, the terminal speed downwards ($|v_f|$), the terminal speed upwards ($|v_r|$), the plate separation of the capacitor (d), and the applied voltage magnitude (V) [3]. Note that we only take interest in the magnitude of the charge and not the sign in order to decrease variation in charges. Any charges equal in magnitude but opposite in sign now map to the same location, further increasing the number of points to reduce our statistical uncertainty.

Repeating this calculation of charge for many oil droplets — ideally of various terminal velocities — we can create a distribution of charges for analysis.

C. Data Analysis

Figure 2 illustrates the results of our experiment. Notice the multi-modality with equally spaced means. We exclude all data above 9.2×10^{-19} C as we want to generate local means using multiple data points. The oil drops above this cutoff, labeled "5th Cutoff", are too infrequent to generate sufficiently accurate means. We denote this data as "Excluded Data" in red. The "Included Data" in blue we further subdivide using Cutoffs 1-4 (of respective values 3.0×10^{-19}, 4.4×10^{-19}, 5.9×10^{-19}, 7.4×10^{-19} C), by visually locating regions of low counts separating the modes.

After using the cutoffs to divide the data, we calculate means and standard deviations for each of the 5 sections, corresponding to 1 through 5 elementary charges. Then, we calculate the difference in means $\mu_{i+1} - \mu_i$ to generate 4 differences. Figure 3 illustrates this processing visually. We overlay red dashed lines to represent the Local Means along with black arrows to represent the distances between the means as calculated by the difference in means. By calculating the average of these differences, we estimate the fundamental electric charge, $q_e = 1.598 \times 10^{-19}$ C.

D. Error Analysis

Since we estimate the fundamental electric charge as an average of the differences in means, we calculate the statistical error using the following derivation. For each difference in means, we calculate its respective variance, σ_j^2, as the sum of the variance of each sample, σ_i^2, divided by the number of data points in each sample, n_i. Then, we average the variances of each difference in means and take the square root to determine the average standard deviation of our estimated fundamental electric charge, σ_q.

$$\sigma_j^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$$\sigma_q = \left(\frac{1}{N}\sum_j^N \sigma_j^2\right)^{1/2} \quad (2)$$

Using this equation, we estimated our statistical error as 1.5×10^{-40}.

However, there exists error intrinsic to the uncertainty in our measurements which we must now account for. We propagate error as the uncertainty in each of our independent variables as follows:

$$\sigma_q = \left(\left(\frac{\partial q}{\partial d}\right)^2 \sigma_d^2 + \left(\frac{\partial q}{\partial v_f}\right)^2 \sigma_{v_f}^2 + \left(\frac{\partial q}{\partial v_r}\right)^2 \sigma_{v_r}^2 + \left(\frac{\partial q}{\partial T}\right)^2 \sigma_T^2 + \left(\frac{\partial q}{\partial V}\right)^2 \sigma_V^2\right)^{1/2} \quad (3)$$

By calculating the partial derivatives of Equation 1, imputing our experimental input values, and using our measurement

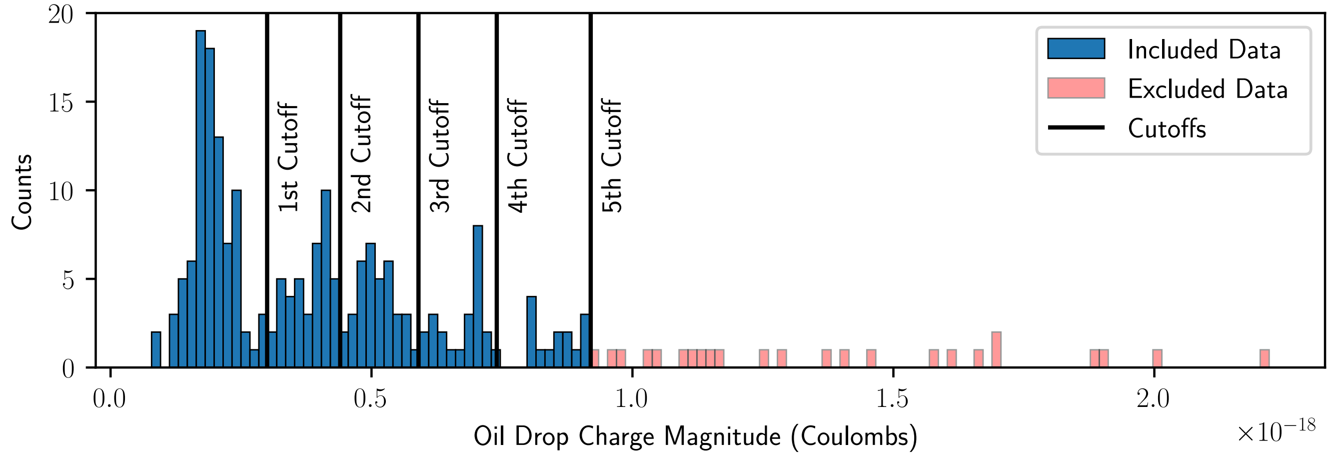

FIG. 2: Histogram of the charges of all observed oil droplets. In blue, we denote the data we save for further processing (Included Data), and in red we denote the data we discard due to insufficient quantity of charges of similar magnitudes (Excluded Data). Black vertical lines denote our visually-determined cutoff values to further divide the Included Data into regions of approximately similar charge.

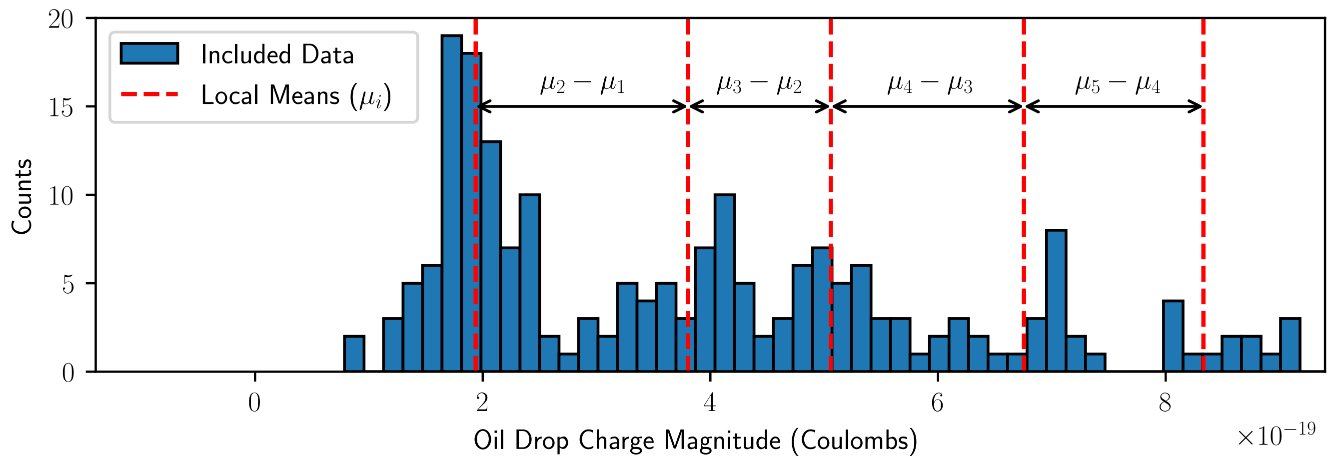

FIG. 3: Histogram of the Included Data as shown in Figure 2, overlaid with local means shown as red vertical dashed lines. The distance between means, $\mu_{i+1} - \mu_i$ is illustrated by black horizontal arrows. We use these distances to estimate the value of the fundamental electric charge, q_e.

uncertainties of:

$$\sigma_d = 0.1 \times 10^{-3} \text{ m}$$
$$\sigma_{v_f} = 8.6 \times 10^{-9} \text{ m/s}$$
$$\sigma_{v_r} = 8.6 \times 10^{-9} \text{ m/s}$$
$$\sigma_T = 1 \text{ K}$$
$$\sigma_V = 1 \text{ V}$$

we determined $\sigma_q = 8.4 \times 10^{-22}$ C or $\sigma_q = 0.008 \times 10^{-19}$ C. As this error is 10^{18} orders of magnitude larger than the statistical uncertainty shown by Equation 2, we decided not to propagate our statistical error into our final error estimate.

III. DISCUSSION

The standard accepted value for the fundamental electric charge is $q_e = 1.602176634 \times 10^{-19}$ C where Millikan observed $q_e = 1.5924 \times 10^{-19}$ C. We calculated $q_e = (1.598 \pm 0.008) \times 10^{-19}$ C. The standard accepted value lies within the 68% confidence interval we generated (mean $\pm$ sigma). We are 68% confident the fundamental electric charge lies between 1.590×10^{-19} and 1.606×10^{-19} C.

To decrease uncertainty and to improve our estimate of q_e, we can observe more charges. However, our primary sources of error result from our uncertainty in temperature and capacitor distance. Improving our measurements of these two variables using higher precision instruments could improve our estimate and reduce our error.

IV. CONCLUSIONS

We used Millikan's oil drop method to determine the fundamental electric charge. By observing the terminal velocity of small oil drops falling in an uncharged capacitor, we determined the size of the droplets. Then, by applying a voltage to the capacitor and observing the terminal velocity of the oil drops, we calculate their respective charges. Repeating this process, we generated a histogram of charges with multiple modes. We found the mean and standard deviation in each mode which we denoted as local means and standard deviations. By calculating the average difference in local means, we found the fundamental electric charge to be $(1.598 \pm 0.008) \times 10^{-19}$ Coulombs, an accurate prediction to within one standard deviation of the accepted value.

We believe that we can reduce uncertainty and improve our estimate by taking more data and investing in higher-quality temperature and distance instruments. By increasing our certainty in temperature and capacitor spacing, we can more accurately determine the charge of the oil drops.

I would like to thank my lab partner, Adrian, for his invaluable assistance during the experiment. Additionally, I would like to thank Dr Greg Sitz for assisting our efforts in and outside of the lab. I would also like to thank the creators of Python, R, Sympy, Numpy, Matplotlib, Fiji Imagej, and PowerPoint who helped make possible the analysis and figures in this paper. This paper was written and compiled in LaTeX using the Overleaf online editor.

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