CS 278 HW 6

2.3.1 (b) T: If m and n are integers such that $m \mid n$, then $m \mid (5n^3-2n^2+3n)$

P: ① Let m and n be integers such that $m \mid n$.

② We will prove $m \mid (5n^3-2n^2+3n)$

③ By fundamental theorem of division, there exist an integer k such that $n=km$

④ Plugging in km for n, $5n^3-2n^2+3n = 5(km)^3-2(km)^2+3km$

⑤ Factor out m gives $(5k^3m^2-2k^2m+3k)m$

~~⑥ Because k and m are both integers, [illegible] is an integer.~~

~~⑦ Sn~~

⑥ Because m is an integer, $n = (\text{some integer})(5k^3m^2-2k^2m+3k)$

⑦ Therefore ~~[illegible]~~ $m \mid 5n^3-2n^2+3n$ ■

(d) T: The sum of the squares of any two consecutive integers is odd.

P: ① Let x and y be integers that are consecutive

② We will prove x^2+y^2 is odd

③ $y = x+1$

④ plugging y in for x^2+y^2 gives $x^2+(x+1)^2$

$$= x^2+x^2+2x+1$$
$$= 2x^2+2x+1$$
$$= 2(x^2+x)+1$$

⑤ x^2 and x are both integers so x^2+x will also be an integer. Let $k = x^2+x$

⑥ $x^2+y^2 = 2k+1$. By definition x^2+y^2 will be odd ■

2.3.2 ⓒ They didn't show algebraically the change from $xz = (kw)(jy)$ to
$xz = (kj)(wy)$
and then kj is some integer m, $xz = m(wy)$

ⓓ This proof looks correct, except they should add let $m = kj$, $xz = m(wy)$. for clarity

2.3.3 ⓑ They should set $2k^2 + 2k + 2j^2 + 2j + 1$ to some integer i for clarity,
So $n^2 + m^2 = 2i$, even ■

ⓓ The equation needs to be simplified further and can't just be assumed.

$$n^2 + m^2 = (2k+1)^2 + (2j+1)^2$$
$$= 4k^2 + 4k + 1 + 4j^2 + 4j + 1$$
$$= 2(2k^2 + 2k + 2j^2 + 2j + 1)$$
$$= 2i \text{, even } ■$$

← conclude is integer, let it be i

2.4.1 ⓑ T: The sum of two odd integers is an even integer

P: ① Let x and y be two odd integers
② We will prove that $x + y$ is even
③ Since x is odd there exists integer k that $x = 2k+1$
Since y is odd there exist integer j that $y = 2j+1$
④ Plug in for $x + y = (2k+1) + (2j+1) = 2k + 2j + 2 = 2(k+j+1)$
⑤ k and j are integers, so $k+j+1$ is and integer. Let i be $k+j+1$
⑥ $x + y = 2i$, even ■

(d) T: The product of two odd integers is an odd integer.

P: ① Let x and y be two odd integers

② We will prove $x \cdot y$ is odd

③ Since x is odd there exists an integer k that $x = 2k+1$

④ Since y is odd there exists an integer j that $y = 2j+1$

⑤ Plug in for $x \cdot y = (2k+1) \cdot (2j+1)$

$$= 4kj + 2k + 2j + 1$$

$$= 2(2kj + k + j) + 1$$

⑥ Because k and j are integers, $2kj + k + j$ is an integer

⑦ Let $i = 2kj + k + j$, $x \cdot y = 2i + 1$, odd ■

(f) T: If x is an even integer and y is an odd integer then $3x + 2y$ is even

P: ① Let x and y be integers such that x is even and y is odd

② We will prove $3x + 2y$ is even

③ Since x is even there exists an integer k such that $x = 2k$

④ Since y is odd there exists an integer j such that $y = 2j+1$

⑤ Plug in for $3x + 2y = 3(2k) + 2(2j+1)$

$$= 6k + 4j + 2$$

$$= 2(3k + 2j + 1)$$

⑥ Because j and k are integers, $3k + 2j + 1$ is an integer

⑦ Let $i = 3k + 2j + 1$, $3x + 2y = 2i$, even

(g) T: If x is an even integer and y is an odd integer then $2x+3y$ is odd

P: ① Let x and y be integers such that x is even and y is odd. We will prove $2x+3y$ is odd.

② Since x is even there exist an integer k such that $x=2k$

③ Since y is odd there exists an integer j such that $y=2j+1$

④ Plug in for $2x+3y = 2(2k)+3(2j+1)$

$$= 4k+6j+3$$

$$= 2(2k+3j+1)+1$$

⑤ Because k and j are integers, $2k+3j+1$ is an integer

⑥ Let $i=2k+3j+1$, $2x+3y=2i+1$, odd ■

(j) T: If x is an odd integer then $(-1)^x=-1$

P: ① Let x be an integer that is odd

② Since x is odd there exists an integer k such that $x=2k+1$

③ ~~Plug in for (-1)~~ We will prove $(-1)^x=-1$

④ Plug in for $(-1)^{2k+1}$

$$= (-1)^{2k} \times (-1)^1$$

$$= [(-1)^2]^k \times (-1)$$

$$= 1^k \times (-1)$$

$$= 1 \times (-1)$$

$$= -1 \quad ■$$

2.4.2 (c) T: If x, y are rational numbers then $3x+2y$ is also rational

P: ① Let x and y be rational numbers

② We will prove $3x+2y$ is rational

③ Since x is rational there exists integers p, q such that $x=\frac{p}{q}$

④ Since y is rational there exists integers m, n such that $y=\frac{m}{n}$

⑤ Plug in for $3x+2y = \frac{3p}{q}+\frac{2m}{n}$

$$= \frac{3pn+2mq}{qn}$$

(6) Because p, q, m, n are integers, $3pn + 2mq$ is an integer. Let it be called a

(7) Because q, n are integers, qn is an integer. Let it be called b

(8) Simplify $\frac{(3pn+2mq)}{(qn)} = \frac{a}{b} \rightarrow$ rational number ∎

(9) $3x + 2y = \frac{a}{b}$

(d) T: If x and y are rational numbers then $3x^2+2y$ is also a rational number

P: (1) Let x and y be rational numbers, we will prove $3x^2+2y$ is rational

(2) Because x is rational there exists integers p, q such that $x = \frac{p}{q}$

(3) Because y is rational there exists integers m, n such that $y = m/n$

(4) Plug into
$$3x^2+2y = 3\left(\frac{p}{q}\right)^2 + 2\left(\frac{m}{n}\right)$$
$$= \frac{3p^2}{q^2} + \frac{2m}{n}$$
$$= \frac{3p^2n + 2mq^2}{q^2n}$$

(5) Because p, n, m, q are integers, $3p^2n+2mq^2$ is an integer, let it be a. And q^2n is an integer, let it be b

(6) Plug into $\frac{3p^2n+2mq^2}{q^2n} = \frac{a}{b} = 3x^2+2y$, rational ∎

(f) T: The average of two rational numbers is rational

P: (1) Let x and y be rational numbers

(2) We will prove $(x+y)/2$ is rational

(3) Because x is rational there exists integers p, q that $x = p/q$

(4) Because y is rational there exists integers n, m such that $y = m/n$

(5) Plug in
$$(x+y)/2 = \left(p/q + m/n\right)/2 = \frac{p}{2q} + \frac{m}{2n} = \frac{2pn + 2mq}{4nq}$$

(6) ~~That exists~~ Because p, n, m, q are integers, $2pn + 2mq$ is an integer, let it be a

(7) Because n, q are integers, $4nq$ is an integer, let it be b

(8) Plug in
$$(x+y)/2 = \frac{2pn + 2mq}{4nq} = \frac{a}{b} = \text{rational} \quad \blacksquare$$