CS 278 Lab 5

2.2.2 (b) Theorem:

For every integer n such that $0 \le n \le 4$, $2^{(n+2)} > 3^n$

Proof:

If $n \in \{-1, 0, 1\}$

$n = -1$; $2^{(-1+2)} > 3^{-1}$; $\frac{1}{2} > \frac{1}{3}$

$n = 0$; $2^{(0+2)} > 3^0$; $4 > 1$

$n = 1$; $2^{(1+2)} > 3^1$; $8 > 3$

(c) Theorem:

For all positive integers $n \le 4$ $(n+1)^3 \ge 3^n$

Proof:

$n \in \{1, 2, 3, 4\}$

$n = 1$; $(1+1)^3 \ge 3^1$; $8 \ge 3$

$n = 2$; $(2+1)^3 \ge 3^2$; $27 \ge 9$

$n = 3$; $(3+1)^3 \ge 3^3$; $64 \ge 27$

$n = 4$; $(4+1)^3 \ge 3^4$; $125 \ge 81$

2.2.2 (c) For every positive integer x, $x^3 < 2^x$

Counterexample: $x = 2$; $2^3 < 2^2$ $8 \not< 4$

(d) Every positive integer can be expressed as sum of the squares of two integers

Domain: $x, y, z \in \{1, 2, \ldots, \infty\}$

$\forall z (\exists (x^2 + y^2) = z)$

Counter example: $z = 24$

2.2.4 (b) There is no largest integer

Domain: $n \in \{-\infty, \ldots, \infty\}$

$\forall n (n \leq \infty)$

(c) Every real number besides 0 has a multiplicative inverse

$\forall n ((n \neq 0) \, n \times 1/n = 1)$

2.2.5 (d)

Theorem: There are integers c and d such that $7c + 5d = 1$

Proof: $c = 14$, $d = 10$ $\frac{7}{14} + \frac{5}{10} = 1$

$1/2 + 1/2 = 1$

(e)

Theorem: There are 3 positive integers x, y, z, that satisfy $x^2 + y^2 = z^2$

Proof: $x = 3$, $y = 4$, $z = 5$, $3^2 + 4^2 = 5^2$

$9 + 16 = 25$

(f)

Theorem: There exist a negative number that is equal to its cube

Proof: -1, $-1^3 = -1$

(h)

Theorem: For every pair of real numbers, x and y, there exists a real number z such that $x - z = z - y$

Proof: $x - z = z - y$

$x - z + y = z$

$x + y = 2z$

$z = (x+y)/2$

there will always satisfy a real number z that satisfies this equation

2.2.6 (b)

Theorem: There exists an integer that is smaller than every other integer

counterexample: $\forall n \, (n \nless \{\text{all integers}\})$

Because there are infinite integers in the set, there can not be an integer that is greater than or less than every other integer in the set

(d)

Theorem: There are positive integers m and n such that $\sqrt{m+n} = \sqrt{m} + \sqrt{n}$

counterexample: $\forall m, n \in \{1, 2, \ldots\} \left(\sqrt{m+n} < \sqrt{m} + \sqrt{n}\right)$