CS 278 Lab 6

2.4.4 (b)

T: If $x+y$ is an even integer, then x and y are both even integers

(false) Set $x+y$ to 10, then set $x=5, y=5$.
Both are odd integers

(d) T: If x and x are real numbers and $x<y$, then $x^2<y^2$

(false) Set $x=-2$, $y=1$

$-2<1$

$-2^2 \not< 1^2$

$4 < 1$

(g) T: The average of two even numbers is even.

(false) Set $x=6$, $y=8$. $\frac{6+8}{2}=\frac{14}{2}=7$ odd

(h) T: The average of two odd integers is an integer

P: ① Let x and y be odd integers. We shall prove that $(x+y)/2$ is an integer.

② Since x is odd there is an integer k such that $x=2k+1$
Since y is odd there is an integer j such that $y=2j+1$

③ Plug x, y into $(x+y)/2$: $(2k+1+2j+1)/2$

median

$(2k+2j+2)/2$

$\frac{\cancel{2}(k+j+1)}{\cancel{2}}$

④ Median is integer k + integer j + 1. Addition of integers is an integer.

⑤ Therefore, the average of two odd integers is an integer ∎

(i) T: IF x and y are integers such that xy is a perfect square, then x and y are also perfect squares.

(false) set $x=5$, $y=5$. $xy = 5 \cdot 5 = 25$
Neither x or y is a perfect square

(l) T: If x, y, and z are integers and $x \mid yz$, then $x \mid y$ or $x \mid z$

P: ① Let x, y, z be integers such that $x \mid yz$

② We will prove that $x \mid y$ or $x \mid z$

③ From fundemental theorem of division for some integer k $\frac{x}{yz} = k$

④ $\frac{x}{z} = ky$. k and y are both integers so $(k \cdot y)$ is an integer.

⑤ Therefore $x \mid z$ by Fundemental theorem of division $\frac{x}{z} = (\text{integer})$

⑥ $\frac{x}{y} = kz$, k and z are both integers so $(k \cdot z)$ is an integer

⑦ Therefore $x \mid y$ by Fundemental theorem of division $\frac{x}{y} = (\text{integer})$ ∎

(M) T: If x, y, and z are integers and $x \mid (y+z)$, then $x \mid y$ or $x \mid z$

(false) set $x = 8$, $y = 3$, $z = 5$; $8 \mid (3+5)$ true
$8 \nmid 3$, $8 \nmid 5$

(n) T: If x, y, and z are integers such that $x \mid (y+z)$ and $x \mid y$, then $x \mid z$

(False) Set $x = 10$, $y = 2$, $z = 3$

$10 \mid (2+3)$
and $10 \mid 2$
but $10 \nmid 3$

(o) T: If x and y are integers and $x \mid y^2$, then $x \mid y$

P: (1) Let x, y be integers such that $x \mid y^2$

(2) We will prove $x \mid y$

(3) By fundemental theorem of division, if $x \mid y^2$, there exist a value k such that

$$\frac{x}{y^2} = k$$

(4) Therefore, $\frac{x}{y} = ky$. k and y are both integers So $(k \cdot y)$ is also an integer

(5) Let integer $j = (k \cdot y)$, therefore $\frac{x}{y} = j$ (integer)

(6) By fundemental theorem of division $x \mid y$ ■