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MAT-230

Module One – 1-5 Problem Set

Problem one:

(a) Every patient was given the medication or the placebo or both.

Logical Expression: $\forall x\ (D(x) \lor P(x))$
Negation: $\neg\forall x\ (D(x) \lor P(x))$
Applying De Morgan's law: $\exists x\ \neg(D(x) \lor P(x))$
Further Application: $\exists x\ (\neg D(x) \land \neg P(x))$
English Translation: There is at least one patient who was given neither the medication nor the placebo.

(b) Every patient who took the placebo had migraines

Logical Expression: $\forall x\ (P(x) \rightarrow M(x))$
Conditional Identity Applied: $\forall x\ (\neg P(x) \lor M(x))$
Negation: $\neg\forall x\ (\neg P(x) \lor M(x))$
Applying De Morgan's law: $\exists x\ \neg(\neg P(x) \lor M(x))$
Further Application: $\exists x\ (P(x) \land \neg M(x))$
English Translation: There is at least one patient who took the placebo and did not have migraines.

(c) There is a patient who had migraines and was given the placebo.

Logical Expression: $\exists x\ (M(x) \land P(x))$
Negation: $\neg\exists x\ (M(x) \land P(x))$
Applying De Morgan's law: $\forall x\ \neg(M(x) \land P(x))$
Further Application: $\forall x\ (\neg M(x) \lor \neg P(x))$
English Translation: Every patient either did not have migraines or was not given the placebo or both.

Problem Two:

(a) $\neg\forall x\ (P(x) \land \neg Q(x)) \equiv \exists x\ (\neg P(x) \lor Q(x))$

Beginning with $\neg\forall x\ (P(x) \land \neg Q(x))$. The negation of a universal quantifier is equivalent to the existence quantifier with a negated statement: $\neg\forall x\ (P(x) \land \neg Q(x)) \equiv \exists x\ \neg(P(x) \land \neg Q(x))$

Then I applied De Morgan's law to move the negation inside the parentheses: $\exists x\ \neg(P(x) \land \neg Q(x)) \equiv \exists x\ (\neg P(x) \lor \neg\neg Q(x))$

Then I simplified by removing the double negation: $\exists x\ (\neg P(x) \lor \neg\neg Q(x)) \equiv \exists x\ (\neg P(x) \lor Q(x))$

Thus, the equivalence is proven.

(b) $\neg\forall x\ (\neg P(x) \to Q(x)) \equiv \exists x\ (\neg P(x) \land \neg Q(x))$

Beginning this problem with $\neg\forall x\ (\neg P(x) \to Q(x))$. The implication can be rewritten using the identity ($p \to q \equiv \neg p \lor q$): $\neg\forall x\ (\neg P(x) \to Q(x)) \equiv \neg\forall x\ (P(x) \lor Q(x))$

The negation of a universal quantifier is the existence quantifier with the negated statement: $\neg\forall x\ (P(x) \lor Q(x)) \equiv \exists x\ \neg(P(x) \lor Q(x))$

Then, applying De Morgan's law to move the negation inside: $\exists x\ \neg(P(x) \lor Q(x)) \equiv \exists x\ (\neg P(x) \land \neg Q(x))$

Thus, the equivalence is proven.

(c) $\neg\exists x\ (\neg P(x) \lor (Q(x) \land \neg R(x))) \equiv \forall x\ (P(x) \land (\neg Q(x) \lor R(x)))$

Start with $\neg\exists x\ (\neg P(x) \lor (Q(x) \land \neg R(x)))$. The negation of an existential quantifier is a universal quantifier with a negated statement: $\neg\exists x\ (\neg P(x) \lor (Q(x) \land \neg R(x))) \equiv \forall x\ \neg(\neg P(x) \lor (Q(x) \land \neg R(x)))$

Then, applying De Morgan's law to the negation of the disjunction and the conjunction inside: $\forall x\ \neg(\neg P(x) \lor (Q(x) \land \neg R(x))) \equiv \forall x\ (P(x) \land \neg(Q(x) \land \neg R(x)))$

Again, I Applied De Morgan's law again to the negation of the conjunction: $\forall x\ (P(x) \land \neg(Q(x) \land \neg R(x))) \equiv \forall x\ (P(x) \land (\neg Q(x) \lor \neg\neg R(x)))$

Then I simplified by removing the double negation: $\forall x\ (P(x) \land (\neg Q(x) \lor \neg\neg R(x))) \equiv \forall x\ (P(x) \land (\neg Q(x) \lor R(x)))$

Thus, the equivalence is proven.

Problem Three:

(a) $\forall x \forall y\ (x \neq y) \rightarrow M(x, y)$
Since M(2,3) is false, the statement doesn't hold for all pairs x and y where x is not equal to y. Thus the statement is false.

(b) $\forall x \exists y\ \neg M(x, y)$
Since this statement must hold for all x, and there is no person y to who person 1 who has not sent an email, thus the statement is also false.

(c) $\exists x \forall y\ M(x, y)$
Since there is at least one x ($x = 1$) for which M(x,y) is true for all y; thus the statement is true.

Problem Four:

(a) The reciprocal of every positive number less than one is greater than one.
This can be expressed as a universal statement. The predicate in this involves a number being positive, less than one, and the reciprocal being > 1.
Logical expression: $\forall x\ ((0 < x < 1) \rightarrow (1/x > 1))$

(b) There is no smallest number.
To say there is no smallest number means that for every number, there is another number that is smaller than it.
Logical expression: $\forall x\ \exists y\ (y < x)$

(c) Every number other than 0 has a multiplicative inverse.
This is applied for every number except zero; there exists another number which when multiplied by it gives one.
Logical expression: $\forall x\ (x \neq 0 \rightarrow \exists y\ (x * y = 1))$

Problem Five:

(a) Write an element from the set A x B x C.

An element would be a 3-tuple where the first element comes from set A, the second from set B, and the third would come from set C.
Example element: tall, foam, non-fat

(b) Write an element from the set B x A x C.

This would also be a 3-tuple where the first element comes from set B, the second from set A, and the third from set C.
Example element: no-foam, grande, whole

(c) Write the set B x C using roster notation.

B x C would consist of all possible ordered pairs where the first element is from set B and the second is from set C.
B x C in roster notation: {(foam,non−fat),(foam,whole),(no−foam,non−fat),(no−foam,whole)}