Use cases for derivatives

Find a tangent line at a point

1. Find the slope at the point
   $m = f'(a)$
2. Point slope form: $y - f(a) = m(x-a)$
3. Slope intercept form: $y = mx + b$ ← y-intercept

Normal line: Perpendicular to the tangent line.
- Slope is the opposite reciprical.

Ex: m of tangent: $\frac{3}{2}$ m of normal: $-\frac{2}{3}$

Related rates: Converting the rate of one variable into another through a common equation.

Ex: Air is being pumped into a spherical balloon so that its volume increases at a rate of $100\ cm^3/s$. How fast is the radius of the balloon increasing when the diameter is 50cm?

$\frac{dV}{dt} = 100\ \frac{cm^3}{s}$ $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

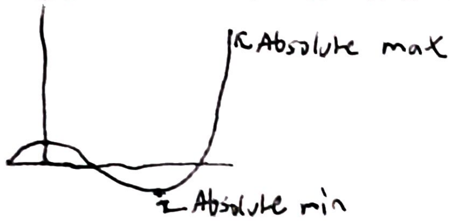
$\frac{dr}{dt} = \frac{dV}{dt} \cdot \frac{1}{4\pi r^2}$ $d = 2r$ $50 = 2r$ $r = 25$

$\frac{dr(r=25)}{dt} = (100) \cdot \frac{1}{4\pi (25)^2} = \frac{1}{25\pi}$ cm/s

Find the absolute max and min values of $f(x)$ in an interval:

0. Graph if possible
1. Check if $f(x)$ is continious on the interval.
2. Find $f'(x)$
3. Find x values where $f'(x)=0$ or DNE.
4. Check if those x values are in the interval.
5. Find $f(x)$ for those x values.
6. Find $f(x)$ for the endpoints.
7. Absolute max is the lagest $f(x)$.
   Absolute min is the lowest $f(x)$.

Ex: $f(x)=x^3-3x^2+1$ on the interval $[-\frac{1}{2},4]$.



$f(x)$'s domain is $(-\infty,\infty)$ so it's continious on the interval.

$f'(x)=3x^2-6x$ $\qquad$ $0=3x^2-6x=(3x)(x-2)$

$x=0,2$ which are both in the interval.

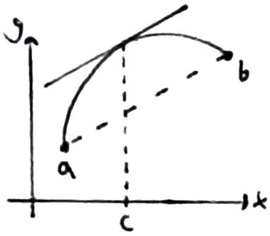
$f(0)=1 \quad f(2)=-3$

Endpoints: $f(-\frac{1}{2})=\frac{1}{8} \quad f(4)=17$

Absolute max: $(4,17)$ $\qquad$ Absolute min: $(2,-3)$

Mean value theorem

If $f(x)$ is continious on $[a,b]$ and differentiable on (a,b), then there exists a C in the interval such that $f'(c)=$ the slope of a and b.



$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

Ex: Find all Cs that satisfy MVT for $f(x)=x^3-x$ on $(0,2)$.

$f(x)$ is continious on the interval

$f'(x)=3x^2-1$

$f(0)=0 \quad f(2)=6$

$$f'(c)=\frac{f(0)-f(2)}{0-2}=\frac{-6}{-2}=3$$

$3=3c^2-1 \qquad 4=3c^2 \qquad c^2=\frac{4}{3} \qquad c=\pm\sqrt{\frac{4}{3}}$

$-\sqrt{\frac{4}{3}}$ isn't in the interval.

so $c=\sqrt{\frac{4}{3}}$

Graphing a function

Finding local minimums and maximums

1. Find $f'(x)$
2. Find x values that cause $f'(x)=0$ or DNE
3. Draw number line with x values
4. Pick nums between to see if $f'(x)$ is pos or neg
5. Local min: $f'(x)=0$ and $f'(x)$ changes from neg to pos.
   Local max: $f'(x)=0$ and $f'(x)$ changes from pos to neg.

Finding inflection points

Same steps as local min/max, but with $f''(x)$.

Concave up: $f''(x)$ pos Concave down: $f''(x)$ neg

Ex: $f(x)=-4x^3-12x^2+5$

$f'(x)=-12x^2-24x=(-12x)(x+2)$

$x=0,-2$