Finding Integrals

1. Limit definition

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \underbrace{\sum_{i=1}^{n} f(x_i)\,\Delta x}_{\text{Riemann sum}}$$

$f(x_i) = \text{Height}$

$\Delta x = \text{Width} = \frac{b-a}{n}$

$x_i = a + i\Delta x$

- We use an infinite number of rectangles to get the area under the curve.
- If we don't use an infinite number, we can use the left, right, midpoint, smallest y value (lower sum), or largest y value (upper sum) to estimate the area under the curve.
- We can also get a rough estimate by finding the max (M) and min (m) of $f(x)$.

If $m \leq f(x) \leq M$, then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

$$\sum_{i=1}^{n} c = c\sum_{i=1}^{n} = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=1}^{n} a_i + b_i = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \qquad \sum_{i=1}^{n} a_i - b_i = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

2 Properties of integrals

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \qquad \int_a^a f(x)\,dx = \emptyset$$

$$\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx \qquad \int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int_a^b \big(f(x)+g(x)\big)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx \qquad \int_a^b \big(f(x)-g(x)\big)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$