Use cases for integrals:

Net change theorem

$$\int_a^b f'(x) \, dx = f(b) - f(a)$$

The integral of the rate of change ($f'(a)$) is the net change ($f(b) - f(a)$).

Ex: Suppose you have a bird's eye view of a car and know it's x and y velocity in meters per sec.

$$V_x(t) = 1 \qquad V_y(t) = 1 + \cos(2t)$$

What is its displacement and distance traveled after 5 sec?

$$\Delta x = \int_0^5 1 \, dt = t \Big|_0^5 = 5$$

$$\Delta y = \int_0^5 1 + \cos(2t) \, dt = t + \tfrac{1}{2} \sin(2t) \Big|_0^5 = 5 + \tfrac{1}{2} \sin(10)$$

Displacement from origin: $\sqrt{(5)^2 + \left(5 + \tfrac{1}{2}\sin(10)\right)^2} = 6.88$ meters

Total distance traveled:

$$Speed(t) = \sqrt{V_x^2 + V_y^2}$$

$$\int_0^5 Speed(t) \, dt = \int_0^5 \sqrt{25 + \left(1 + \cos(2t)\right)^2} \, dt \approx 7.42 \text{ meters}$$

Total distance traveled on the $x = \int_0^5 |V_x| \, dt = 5$ meters

Total distance traveled on the $y = \int_0^5 |V_y| \, dt = 4.73$ meters