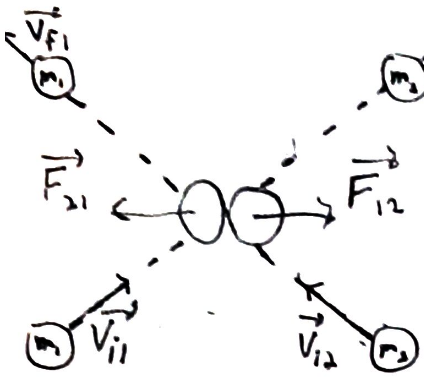
Collisions



$\vec{F}_{21} = -\vec{F}_{12}$ Newton's 3rd Law

$\vec{F}_{21} + \vec{F}_{12} = 0$

$m_1\vec{a}_1 + m_2\vec{a}_2 = 0$ Newton's 2nd Law

$$m_1 \frac{d\vec{V}_1}{dt} + m_2 \frac{d\vec{V}_2}{dt} = 0$$

$$\frac{d}{dt}\underbrace{\left(m_1\vec{V}_1 + m_2\vec{V}_2\right)} = 0 \qquad \boxed{\vec{P} = m\vec{V}} \qquad \boxed{\Delta\vec{P} = 0}$$

The total momentum stays constant w/ respect to time.

$$\vec{F}_{net} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V}) = \frac{d\vec{P}}{dt}$$

when mass is constant

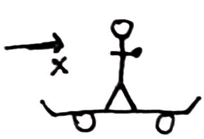
$$\boxed{\vec{F}_{net} = \frac{d\vec{P}}{dt}}$$

Ex: You throw a ball on a frictionless skateboard.

$V_i = 0 \quad V_b = 10\frac{m}{s}$

m_b = Mass of ball = 3 kg After you throw, what v do you move back?

m_p = Mass of person = 50 kg



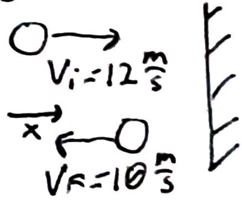
$\Delta P = 0$

$P_F = P_i$

$m_p V_p + m_b V_b = (m_p + m_b) V_i$

$$V_p = \frac{(m_p + m_b)V_i - m_b V_b}{m_p} = \frac{0 - (3)(10)}{50} = -\frac{3}{5}\frac{m}{s}$$

Ex: A ball is throw at a wall. Suppose it was in contact for 0.0001 s. What average force gave it that acceleration?



$m = 0.2$ kg

$$\vec{F}_{avg} = \frac{\Delta P}{\Delta t} = \frac{P_F - P_i}{t} = \frac{mV_F - mV_i}{t} = 0.2\frac{-10-12}{0.0001}$$

$$= -44{,}000\text{N}$$

Elastic - Objects bounce off. $\Delta P = 0$ $\Delta K = 0$

Inelastic - Objects bounce off, but some kenetic energy is lost.

$\Delta P = 0$ $\Delta K = 0$

Perfectly inelastic - Objects stick together after collision.

Before:

$O\rightarrow \qquad \leftarrow O$

$m_1 \qquad m_2$

$\vec{V}_{i1} \qquad \vec{V}_{i2}$

After:

Elastic: $\vec{V}_{f1} \leftarrow O \qquad O \rightarrow \vec{V}_{f2}$

$$\Delta P = 0$$

$$P_i = P_f$$

$$m_1\vec{V}_{i1} + m_2\vec{V}_{i2} = m_1\vec{V}_{f1} + m_2\vec{V}_{f2}$$

$$m_1\vec{V}_{i1} - m_1\vec{V}_{f1} = m_2\vec{V}_{f2} - m_2\vec{V}_{i2}$$

$$① \quad m_1(\vec{V}_{i1} - \vec{V}_{f1}) = m_2(\vec{V}_{f2} - \vec{V}_{i2})$$

$$\Delta K = 0$$

$$K_i = K_f$$

$$\frac{1}{2}m_1\vec{V}_{i1}^2 + \frac{1}{2}m_2\vec{V}_{i2}^2 = \frac{1}{2}m_1\vec{V}_{f1}^2 + \frac{1}{2}m_2\vec{V}_{f2}^2$$

$$m_1\vec{V}_{i1}^2 + m_2\vec{V}_{i2}^2 = m_1\vec{V}_{f1}^2 + m_2\vec{V}_{f2}^2$$

$$m_1\vec{V}_{i1}^2 - m_1\vec{V}_{f1}^2 = m_2\vec{V}_{f2}^2 - m_2\vec{V}_{i2}^2$$

$$m_1(\vec{V}_{i1}^2 - \vec{V}_{f1}^2) = m_2(\vec{V}_{f2}^2 - \vec{V}_{i2}^2)$$

$$m_1(\vec{V}_{i1} - \vec{V}_{f1})(\vec{V}_{i1} + \vec{V}_{f1}) = m_2(\vec{V}_{f2} - \vec{V}_{i2})(\vec{V}_{f2} + \vec{V}_{i2})$$

$$② \quad \vec{V}_{i1} + \vec{V}_{f1} = \vec{V}_{f2} + \vec{V}_{i2}$$

Use equations ① and ② to solve for:

$$\vec{V}_{f1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{V}_{i1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{V}_{i2}$$

$$\vec{V}_{f2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{V}_{i2} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{V}_{i2}$$