Use cases for integrals:

Net change theorem

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

The integral of the rate of change ($f'(x)$) is the net change ($f(b) - f(a)$).

Ex: Suppose you have a bird's eye view of a car and know it's x and y velocity in meters per sec.

$V_x(t) = 1$ $\qquad$ $V_y(t) = 1 + \cos(2t)$

What is its displacement and distance traveled after 5 sec?

$$\Delta x = \int_0^5 1\,dt = t\Big|_0^5 = 5$$

$$\Delta y = \int_0^5 1 + \cos(2t)\,dt = t + \tfrac{1}{2}\sin(2t)\Big|_0^5 = 5 + \tfrac{1}{2}\sin(10)$$

Displacement from origin: $\sqrt{(5)^2 + \left(5 + \tfrac{1}{2}\sin(10)\right)^2} = 6.88$ meters

Total distance traveled:

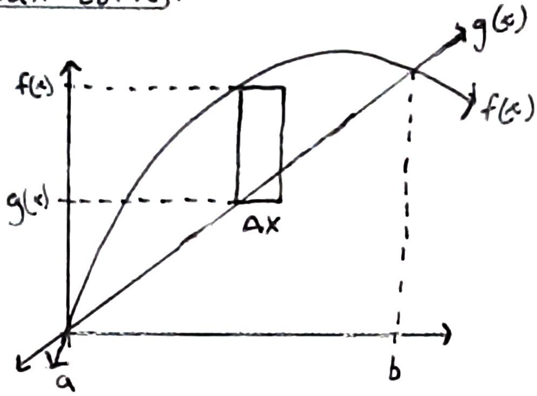
$$\text{Speed}(t) = \sqrt{V_x^2 + V_y^2}$$

$$\int_0^5 \text{Speed}(t)\,dt = \int_0^5 \sqrt{25 + (1 + \cos(2t))^2}\,dt \approx 7.42 \text{ meters}$$

Total distance traveled on the $x = \int_0^5 |V_x|\,dt = 5$ meters

Total distance traveled on the $y = \int_0^5 |V_y|\,dt = 4.73$ meters

Area between curves:



$$\text{Area} = \int_a^b f(x) - g(x)\, dx$$

$$= \left| \int_a^b g(x) - f(x)\, dx \right|$$

1. Find a and b usually by finding intersection points.
2. Find which function is ontop or use absolute value
3. Solve definite integral

Ex: Find area bounded by $y = \sin x$ and $y = \cos x$ on $[0, \frac{\pi}{2}]$

$\sin x = \cos x \qquad x = \frac{\pi}{4}$

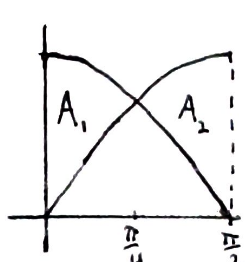
$$\text{Area} = \left| \int_0^{\pi/4} \sin x - \cos x\, dx \right| + \left| \int_{\pi/4}^{\pi/2} \sin x - \cos x\, dx \right|$$

$$= \left| -\cos x - \sin x \Big|_0^{\pi/4} \right| + \left| -\cos x - \sin x \Big|_{\pi/4}^{\pi/2} \right|$$

$$= \left| -\cos\frac{\pi}{4} - \sin\frac{\pi}{4} - \left(-\cos 0 - \sin 0\right) \right| + \left| -\cos\frac{\pi}{2} - \sin\frac{\pi}{2} - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right) \right|$$

$$= \left| -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - (-1 - 0) \right| + \left| -0 - 1 - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) \right| = \left| -\sqrt{2} + 1 \right| + \left| -1 + \sqrt{2} \right|$$

$$= \sqrt{2} - 1 + \sqrt{2} - 1 = 2(\sqrt{2} - 1) = 2\sqrt{2} - 2$$



You can also integrate with respect to y

$$\int_c^d f(x) - g(x)\, dx$$