

## Use cases for derivatives

### Find a tangent line at a point

1. Find the slope at the point

$$m = f'(a)$$

2. Point slope form:  $y - f(a) = m(x - a)$

3. Slope intercept form:  $y = mx + b$  ←  $y$ -intercept

Normal line: Perpendicular to the tangent line.

- Slope is the opposite reciprocal.

Ex:  $m$  of tangent:  $\frac{3}{2}$      $m$  of normal:  $-\frac{2}{3}$

Related rates: Converting the rate of one variable into another through a common equation.

Ex: Air is being pumped into a spherical balloon so that its volume increases at a rate of  $100 \text{ cm}^3/\text{s}$ . How fast is the radius of the balloon increasing when the diameter is  $50 \text{ cm}$ ?

$$\frac{dV}{dt} = 100 \frac{\text{cm}^3}{\text{s}} \quad V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{dV}{dt} \cdot \frac{1}{4\pi r^2} \quad d = 2r \quad 50 = 2r \quad r = 25$$

$$\frac{dr(r=25)}{dt} = (100) \cdot \frac{1}{4\pi(25)^2} = \frac{1}{2\pi} \text{ cm/s}$$

Find the absolute max and min values of  $f(x)$  in an interval:

0. Graph if possible

1. Check if  $f(x)$  is continuous on the interval.

2. Find  $f'(x)$

3. Find  $x$  values where  $f'(x) = 0$  or DNE.

4. Check if those  $x$  values are in the interval.

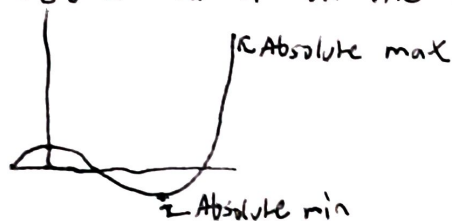
5. Find  $f(x)$  for those  $x$  values.

6. Find  $f(x)$  for the endpoints.

7. Absolute max is the largest  $f(x)$ .

Absolute min is the lowest  $f(x)$ .

Ex:  $f(x) = x^3 - 3x^2 + 1$  on the interval  $[-\frac{1}{2}, 4]$ .



$f(x)$ 's domain is  $(-\infty, \infty)$  so it's continuous on the interval.

$$f'(x) = 3x^2 - 6x$$

$$0 = 3x^2 - 6x = (3x)(x - 2)$$

$x = 0, 2$  which are both in the interval.

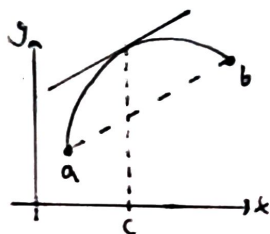
$$f(0) = 1 \quad f(2) = -3$$

$$\text{Endpoints: } f(-\frac{1}{2}) = \frac{1}{8} \quad f(4) = 17$$

$$\text{Absolute max: } (4, 17) \quad \text{Absolute min: } (2, -3)$$

## Mean value theorem

If  $f(x)$  is continuous and differentiable on an interval, then there exists a  $c$  in the interval such that  $f'(c)$  = the slope of a and b.



$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Ex: Find all  $c$ s that satisfy MVT for  $f(x) = x^3 - x$  on  $(0, 2)$ .

$f(x)$  is continuous on the interval

$$f'(x) = 3x^2 - 1$$

$$f(0) = 0 \quad f(2) = 6$$

$$f'(c) = \frac{f(0) - f(2)}{0 - 2} = \frac{-6}{-2} = 3$$

$$3 = 3c^2 - 1$$

$$4 = 3c^2$$

$$c^2 = \frac{4}{3}$$

$$c = \pm \sqrt{\frac{4}{3}}$$

$-\sqrt{\frac{4}{3}}$  isn't in the interval.  
so  $c = \sqrt{\frac{4}{3}}$