State-Aware TrueSkill For Tennis Prediction

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Imperial College London

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GOALS

- Comprehensive Overview Of TrueSkill

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- Use TrueSkill To Model Tennis

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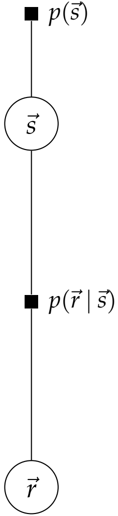

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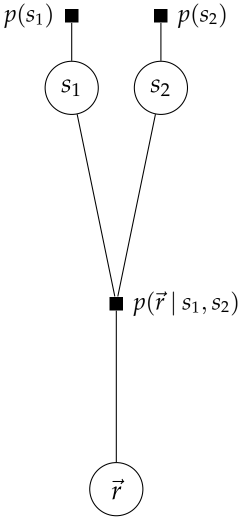

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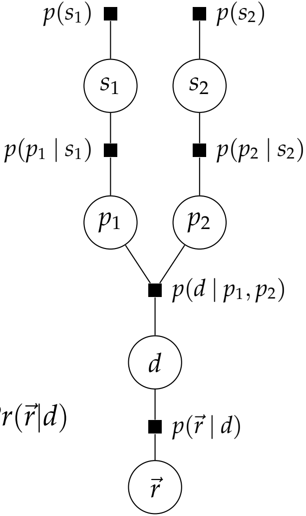
- Factorising Likelihood

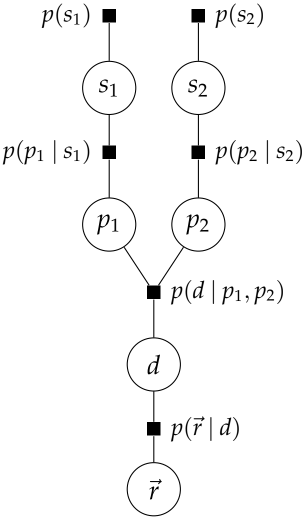
$$Pr(\vec{r}|s_1, s_2) \triangleq Pr(p_1|s_1)Pr(p_2|s_2)Pr(d|p_1, p_2)Pr(\vec{r}|d)$$



Specification Of Factors In TrueSkill

- Gaussian Skill Priors

$$p(s_i) = \mathcal{N}(s_i \mid \mu_i, \sigma_i^2 + \tau^2)$$



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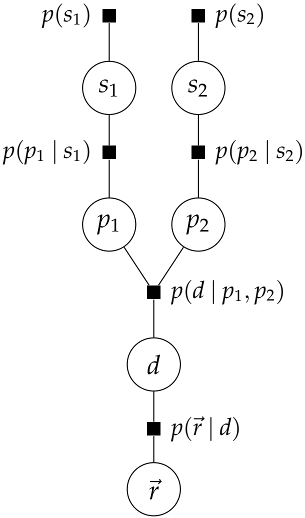
$$p(p_i \mid s_i) = \mathcal{N}(p_i \mid s_i, \beta^2)$$



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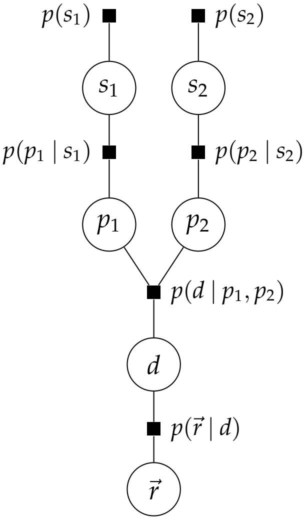

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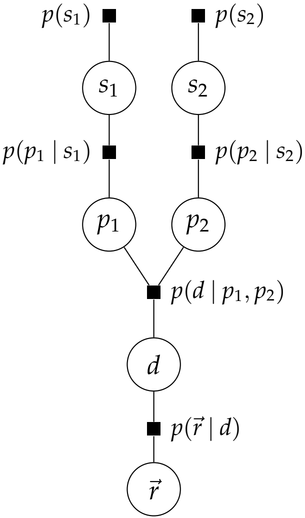
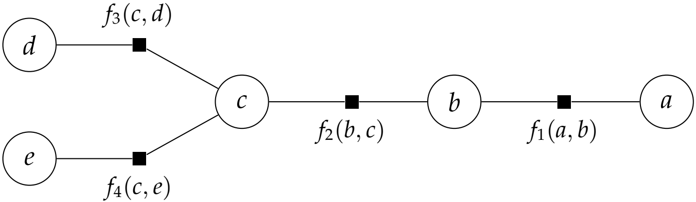
- Outcome-Truncation Factor

$p(r \mid d) = \mathbb{I}(d > 0)$ if player 1 won

$p(r \mid d) = \mathbb{I}(d < 0)$ if player 2 won



FACTOR GRAPH EXAMPLE



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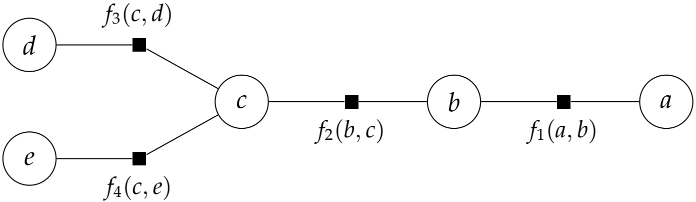

$$p(b) = \sum_a \sum_c \sum_d \sum_e f_1(a, b) f_2(b, c) f_3(c, d) f_4(c, e)$$

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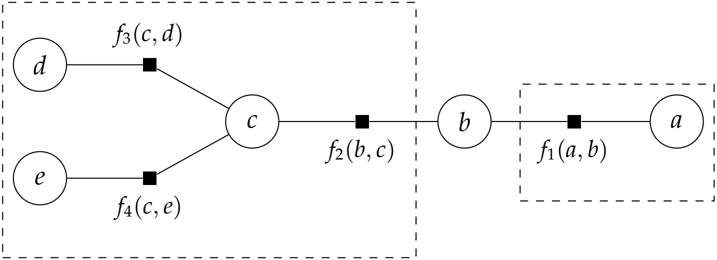

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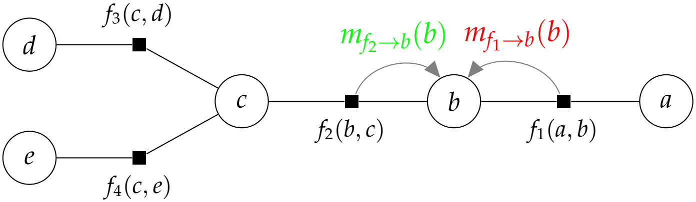

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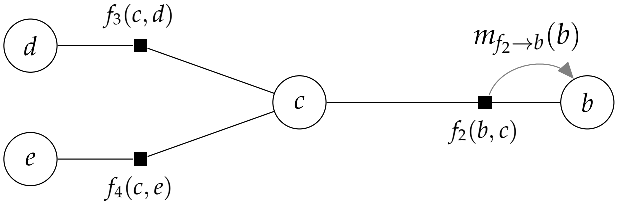

$$m_{f_2\to b}(b) = \sum_c \sum_d \sum_e f_2(b,c) f_3(c,d) f_4(c,e)$$

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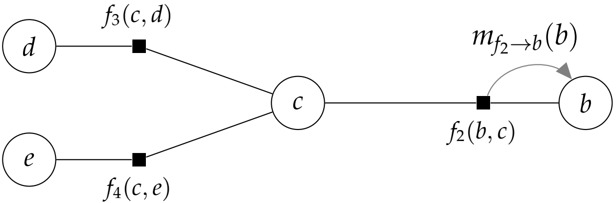

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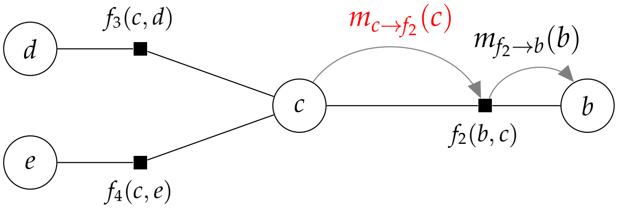

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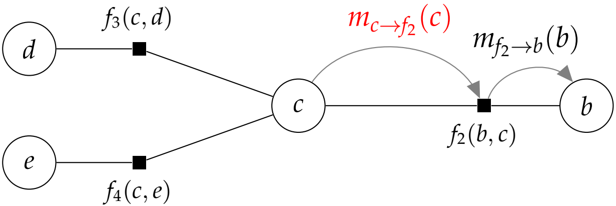

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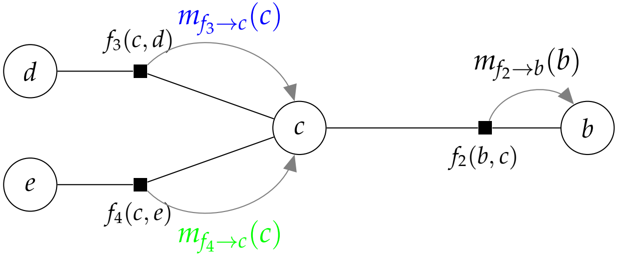

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Sum-Product Algorithm

- Variable Node To Factor Node

$$m_{x_m \to f_s}(x_m) = \prod_{l \in ne(x_m) \setminus f_s} \left(m_{f_l \to x_m}(x_m) \right)$$

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- Factor Node To Variable Node

$$m_{f_s \to x}(x) = \sum_{x_1} \cdots \sum_{x_M} \left(f_s(x, x_1, \ldots, x_M) \prod_{i \in ne(f_s) \backslash x} \left(m_{x_i \to f_s}(x_i) \right) \right)$$

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$$p(x) = \prod_{f_i \in ne(x)} m_{f_i \to x}(x)$$

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$$\Longrightarrow \; p(x) = m_{f \to x}(x) \prod_{f_i \in ne(x) \setminus f} m_{f_i \to x}(x) \quad \forall f \in ne(x)$$

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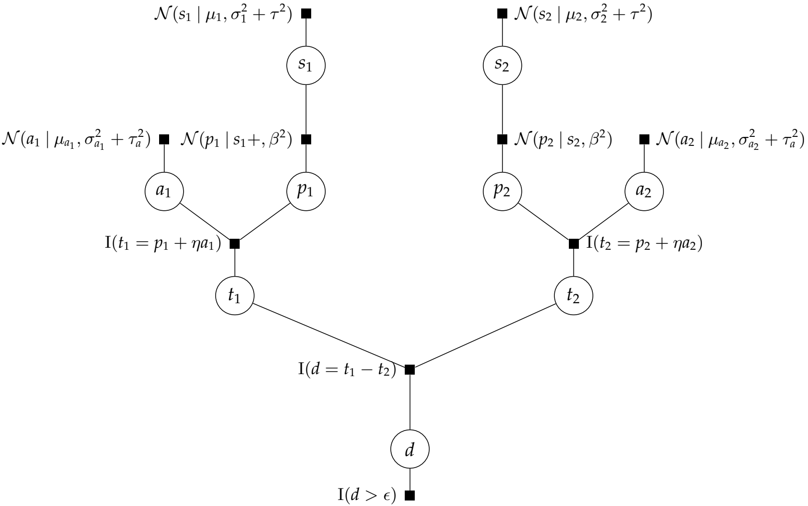
Results On Seperate Test Set

Data Granularity	Selection Based On	Brier Score	Error Rate
Match	Brier	0.199784	0.312693
Match	Error	0.202965	0.319917
Point	Brier	0.249268	0.477656
Point	Error	0.249284	0.477803

Table: Performance On A Separate Test Set Of Naïve Models

FACTOR GRAPH REPRESENTATION



Point Level Results On ATP Dataset

- Selection Of $\beta : 21 \rightarrow 10$

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- Selection Of β : $21 \rightarrow 10$
- Brier Score : $0.249268 \rightarrow 0.225575$

Point Level Results On ATP Dataset

- Selection Of β : $21 \rightarrow 10$
- Brier Score : $0.249268 \rightarrow 0.225575$
- Error Rate: $0.477656 \rightarrow 0.349461$

FUTURE WORK

- Extending The Model To Cover Multiplayer Games

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- Elegantly Deal With Continuous Features

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