Extra-Solar Planets

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Introduction

Trivia:

- First discovery of two exoplanets orbiting a pulsar (PSR 1257+12) by Aleksander Wolszczan and Dale Frail in 1992.
- Over 3500 exoplanets discovered since then.
- **The High Accuracy Radial Velocity Planet Searcher (HARPS)** has discovered over 130 exoplanets. Measures the Doppler shift of radiated light from stars to indirectly detect the gravitational tug of a planet.
- Was the most successful observatory until 2012, since surpassed by The Kepler Space Observatory.
- **Some notable discoveries; Proxima Centauri B** - closest Earth-like exoplanet; **Trappist-1 system** - Most exoplanets orbiting a single star, 3 out of the 7 planets in the habitable zone; **Kepler-438b** - Highest "Earth similarity index" of a confirmed exoplanet with ESI = 0.88; **Ross 128b** - Second place with ESI =0.86.

Motivation:

- To better understand planetary formation.
- Investigate atmospheric compositions of planets for signs of life.
- A new home in the distant future?
- Why not?!

Fitting a Linear Model to raw data

Plot:

- Oscillating relative velocity indicates presence of exoplanet.
- A well fit model will give a good approximation of the exoplanet's orbit.
- We can use the information to infer the planet's mass.

Fitting the data:

- Best fit parameters β found by solving the 'normal equations'

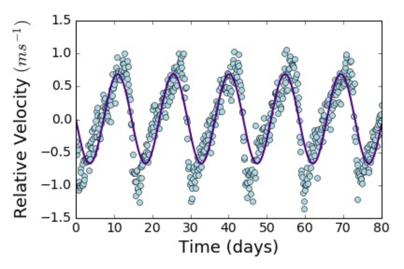
$$(\mathbf{X}^T\mathbf{X})\beta = \mathbf{X}^T\mathbf{y}. \qquad (1)$$

- Substitute β, $\omega = 2\pi/14.6$ into (2) to get (3).
    - **Linear model used:**

$$V_{||} = \beta_0 \sin(\omega t) + \beta_1 \cos(\omega t), \qquad (2)$$

$$V_{||} = -0.679 \sin(2\pi t/14.6) + 0.009 \cos(2\pi t/14.6). \qquad (3)$$

- **Light blue dots:** Data ν of the star's relative velocity in the direction of Earth.
- **Dark blue line:** The linear model (3) to fit the data.



Testing for goodness of fit

Residuals help to understand how well models represent raw data. Residuals are calculated by

$$r_i = \nu_i - V_{||i} \tag{4}$$

Testing methods:

- **RMSD** measures the average difference between the data and model. Smaller values indicate a better fit.

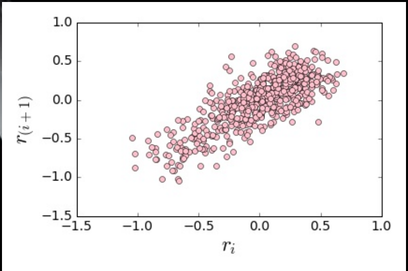
$$\sqrt{\frac{1}{N}\sum_{i=1}^{N} r_i^2} \tag{5}$$

- **Auto-correlation co-efficient ACC** Measures the strength of correlation between the residuals. Uncorrelated residuals indicate a good fit.

$$a_k \equiv \frac{\sum_{j=1}^{N}(r_j - \bar{r})(r_{j+k} - \bar{r})}{\sum_{j=1}(r_j - \bar{r})^2} \tag{6}$$

Residual plot (linear model):

- **Pink dots:** *ith* residual vs $(i+1)th$ residual.
- **RMSD** $= 0.331$. Indicates an average difference of 0.3ms^{-1} in $V_{||}$ between the data and model. $\overline{Error} \approx 15\%$.
- **ACC** $= 0.789$. Indicates a large positive correlation between the residuals, this is verified by the plot.
- The linear model is not a good fit.



Using a physical model to fit the data

- It is good idea to construct a better model derived from Newton's Law of Gravitation. The new model is a 3-dimensional autonomous system given by;

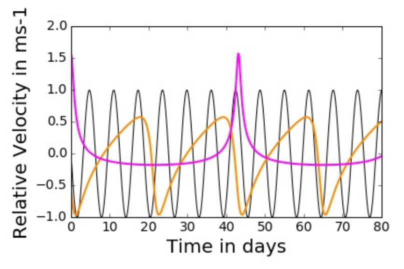
$$\dot{r} = s, \quad \dot{s} = \frac{p}{r^3} - \frac{1}{r^2}, \quad \dot{\phi} = \frac{\sqrt{p}}{r^2}.$$

- $r(0) = \frac{p}{1+e}$ where p is the size of the orbit, and e is the eccentricity.
- This system was solved numerically using a the 4th order Runga-Kutta method.

Investigating the physical model

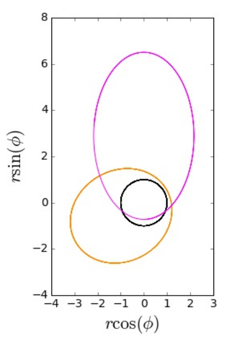
The relative velocity of the star is calculated with;

$$v_{||} = \dot{r}\cos(\phi) - r\dot{\phi}\sin(\phi).$$



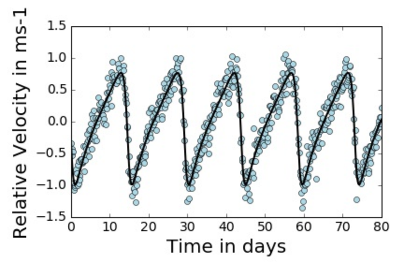
The above plot shows 3 different types of signal produced by varying the eccentricity and size of orbit of a modelled exoplanet. The signal of the circular orbit matches the real data best, so we should expect the data was produced by a circular orbiting exoplanet.

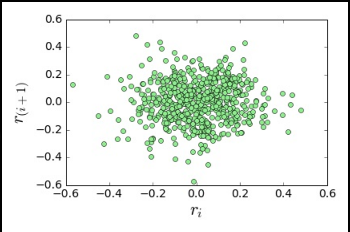
The figure below shows 3 examples of exoplanet orbits around a centre of mass at the origin obtained by varying the parameters p, e, and ϕ.



Fitting a non-linear model

Using the least squares method and the $v_{||}$ residuals to find non-linear best fit parameters, the plot below shows the new model, black line, fitting much more closely to the data than the linear model earlier.





The above plot shows no correlation between the residuals of $v_{||}$, indicating that the new model is a good fit to the data.

Inferring the mass of the exoplanet

Best fit parameters β for the non-linear model allow the mass of the planet to be inferred.

- Assumed the star m_1 shares same mass of the sun.
- Used Kepler's 3rd Law to find the semi-major axis;

$$a^3 = \frac{G(m_1 + m_2)}{\Omega^2}. \tag{7}$$

  Max distance from the star is $a \approx 0.117$au.
  Min distance $b \approx 1.005$au

- Used (8) to infer the planet's mass from it's orbital eccentricity.

$$m_2 = V_{||\,\mathrm{max}} \sqrt{\frac{m_1 a}{G} \frac{1-e}{1+e}} \tag{8}$$

  The planet's mass is $m_2 \approx 1.005 \times 10^{25}$kg.

Exoplanet's characteristics:

- Estimated to be 1.68 times more massive than Earth.
- Orbit's the host star 10, and 8.5 times closer at it's minimum and maximum distance respectively than Earth does the sun.
- Orbit's the star every 14.6 days.
- Likely rocky in nature since it's mass is similar to Earth's
- Assuming the planet is rocky with no atmosphere, average surface temperature estimated in the range 8-900 Kelvin.
- Almost certainly devoid of life.