Ryan Tennant
Homework 2
BI 471/571
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1. 4.1
   (a)

$$\frac{dN}{dt} = rN[1 - \left(\frac{N}{K}\right)^{\theta}]$$

   Equilibrium is:

$$\frac{dN}{dt} = 0$$

   Two equilibria:

N = 0 and $N = K$

   Solve for N:

$$0 = rN[1 - \left(\frac{N}{K}\right)^{\theta}]$$

$$0/\left[1 - \left(\frac{N}{K}\right)^{\theta}\right] = rN[1 - \left(\frac{N}{K}\right)^{\theta}]/[1 - \left(\frac{N}{K}\right)^{\theta}]$$

$$0/r = rN/r$$

N = 0

&

$$0/rN = rN\left[1 - \left(\frac{N}{K}\right)^{\theta}\right]/rN$$

$$0 - 1 = 1 - \left(\frac{N}{K}\right)^{\theta} - 1$$

$$-(-1) = -(-\left(\frac{N}{K}\right)^{\theta})$$

$$ln(1) = ln\left(\frac{N}{K}\right)^{\theta}$$

$$\frac{0}{\theta} = \frac{\theta ln\left(\frac{N}{K}\right)}{\theta}$$

$$e^{0} = e^{ln\left(\frac{N}{K}\right)}$$

$$(K)1 = \frac{N}{K}(K)$$

$$N = K$$

   Is it stable though?

$$F(N) = rN\left(1-\left(\frac{N}{K}\right)^{\theta}\right)$$

$$= rN - rN\left(\frac{N}{K}\right)^{\theta}$$

$$\frac{\partial F}{\partial N} = r - r\left(\frac{N}{K}\right)^{\theta}$$

Since... ...And $\hat{N} = 0$

$$\frac{\partial n}{\partial t} \approx n \frac{\partial F}{\partial N}\bigg|_{N=\hat{N}}$$

Then... $\frac{\partial n}{\partial t} = n\left(r - r\left(\frac{N}{K}\right)^{\theta}\right)\bigg|_{N=0}$

Also $\hat{N} = K$

$$\frac{\partial n}{\partial t} = \left(r - r\left(\frac{N}{K}\right)^{\theta}\bigg|_{N=K}\right)n \quad \text{, where } \left(\frac{N}{K}\right) = 1$$

$$= (r - r^{\theta})\,n$$

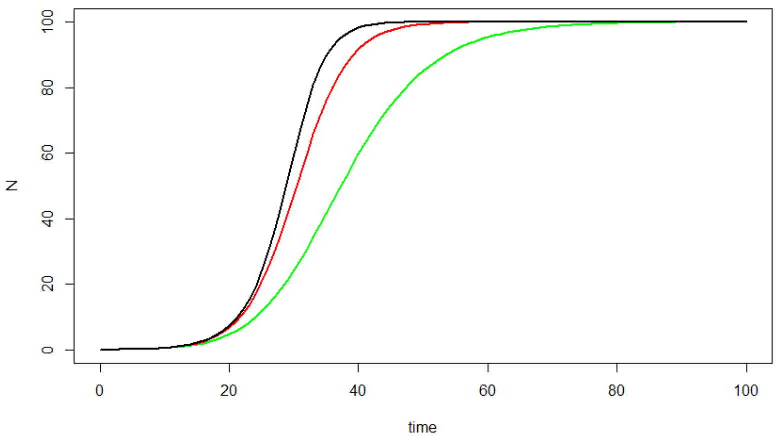
Ends up being

$$n(t) = n_0 e^{(r-r^{\theta})t}$$

$$n(t) = n(0)e^{(r-r^{\theta})t}$$

So as long as $\theta > 1$ the system goes towards the negative and is unstable, however if $\theta < 1$ the equilibrium is stable.

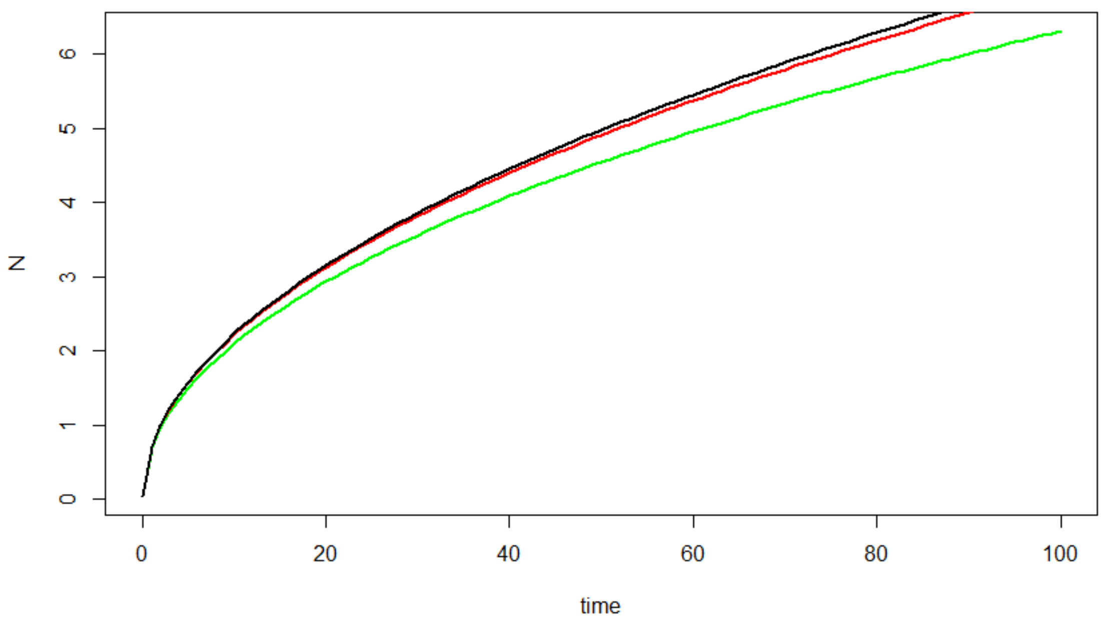
(b)



Above: $dN/dt = rN[1-(N/K)^\wedge\theta\,]$

Below: $\frac{\frac{dN}{dt}}{N} = r[1-\left(\frac{N}{K}\right)^{\theta}]$

Where, Green: θ= 0.5, Red: θ = 1.0, Black: θ = 1.5



It appears that the increase in the value of θ increases the per capita growth rate most notably as long as its above 1. It seems to have the greatest affect if its below 1.

```
## 4.1 (b)
```

```
log.growth <- function(t, y, p) {
  N <- y[1]
  with(as.list(p), {
    dN.dt <- r * (1-(N/K)^theta)
    return(list(dN.dt/N))
  })
}

p.5<- c('r'=0.25, 'K'=100, 'theta'=.5)
p.6<- c('r'=0.25, 'K'=100, 'theta'=1)
p.7<- c('r'=0.25, 'K'=100, 'theta'=1.5)
y0<-c('N'=.05)
t<-0:100

sim.5<-ode(y=y0, times=t, func=log.growth, parm=p.5,
method='lsoda')
sim.5<-as.data.frame(sim.5)
sim.6<-ode(y=y0, times=t, func=log.growth, parm=p.6,
method='lsoda')
sim.6<-as.data.frame(sim.6)
sim.7<-ode(y=y0, times=t, func=log.growth, parm=p.7,
method='lsoda')
sim.7<-as.data.frame(sim.7)

plot(N~time,dat=sim.5,type='l',col='green',lwd=2)
points(N~time,dat=sim.6,type='l',col='red',lwd=2)
points(N~time,dat=sim.7,type='l',col='black',lwd=2)
```

(c) It appears that this model is a little more useful than the normal logistic growth model since it allows you to tweak the relationship between N/K without completely changing the model.

4.3

$$\frac{dN}{dt} = rN(N-a)[1-(\frac{N}{K})]$$

(a)

2.

(a) Write a function to simulate continuous time logistic growth in R. Using the values r = 0.25 and K = 100, simulate and plot this model over the time range t = 0 to t = 100, using `runif()` to randomly draw the initial conditions 0.01 <N0 < 0.1 from a uniform distribution. Include any code and figures.

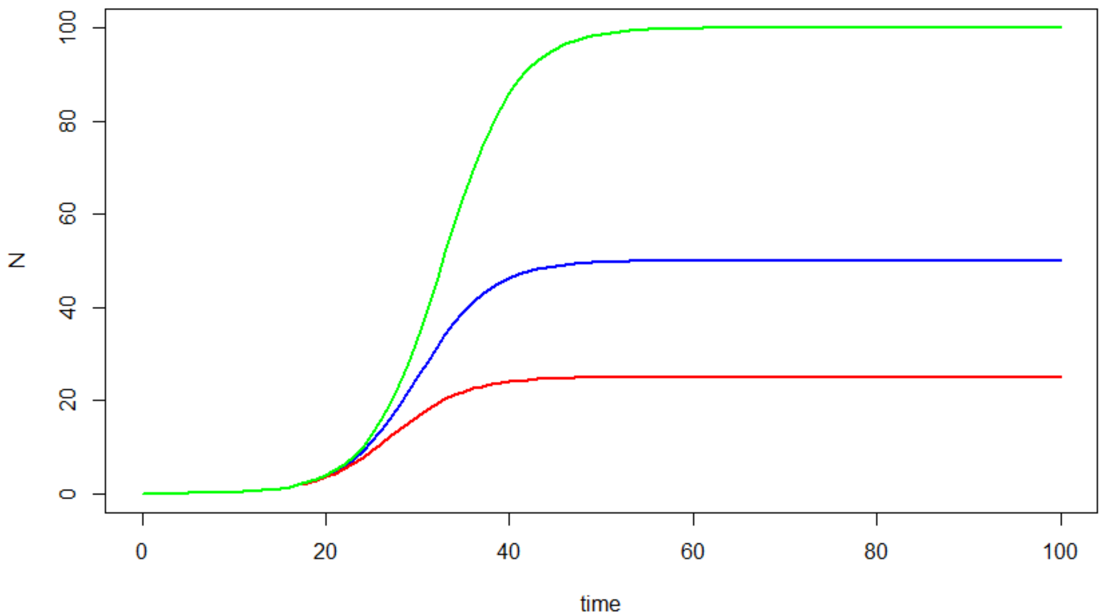
```
install.packages('deSolve')
library('deSolve')
log.growth <- function(t, y, p) {
N <- y[1]
with(as.list(p), {
dN.dt <- r * N * (1-(N/K)^theta)
return(list(dN.dt))
})
}
```

```
p<- c('r'=0.25, 'K'=100, 'theta'=1)

y0<-c('N'=runif(1, min=0.01,max=.1))
t<-0:100
sim<-ode(y=y0, times=t, func=log.growth, parm=p, method='lsoda')
sim<-as.data.frame(sim)
```

(b) Simulate this model two additional times for the same parameter set, but with K = 50 and K = 25. Plot the population level growth rate (hint: ?diff) vs. population abundance for all three simulations (on the same plot). Include any code and figures

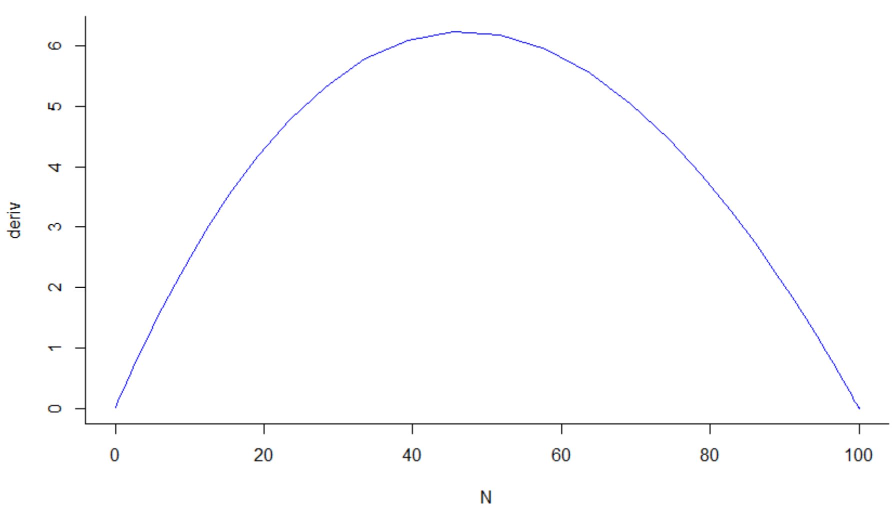
```
plot(N~time,dat=sim,type='l',col='green')

p.2<-c('r'=0.25,'K'=50,'theta'=1)
sim.2<-ode(y=y0, times=t, func=log.growth, parm=p.2, method='lsoda')
sim.2<-as.data.frame(sim.2)

plot(N~time,dat=sim.2,type='l',col='blue',ylim=c(0,100))
points(N~time,dat=sim,type='l',col='blue')

p.3<-c('r'=.25,'K'=25,'theta'=1)
sim.3<-ode(y=y0, times=t, func=log.growth, parm=p.3, method='lsoda')
sim.3<-as.data.frame(sim.3)
points(N~time,dat=sim.3,type='l',col='red')
?plot
plot(N~time,dat=sim.2,type='l',col='blue',ylim=c(0,200))
points(N~time,dat=sim.2,type='l',col='blue',lwd='2')
points(N~time,dat=sim.3,type='l',col='red',lwd='2')
points(N~time,dat=sim,type='l',col='green',lwd='2')
```

Graph 1: Logistical model of the three parameters (Yellow: K=100, Blue: K=50, Red: K=25):




```
sim$deriv <- diff(sim$N)
sim$deriv <- c(diff(sim$N), NA)
plot(deriv~N, data = sim, type='l',col='blue', bty='l')
max(sim$deriv,na.rm=TRUE)
which(sim$deriv ==max(sim$deriv,na.rm=TRUE))
33
sim$N[which(sim$deriv ==max(sim$deriv,na.rm=TRUE))]
45.43845
```

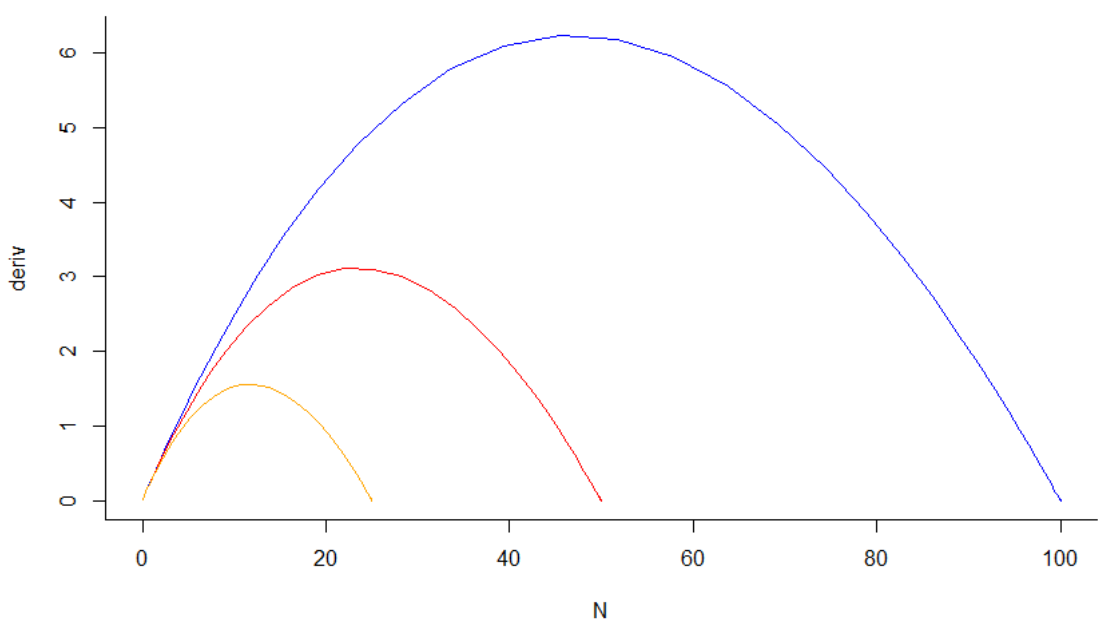
Graph 2: Derivative of logistical growth simulated with K=100 (Blue)




```
## sim.2
```

```
sim.2$deriv <- diff(sim.2$N)
sim.2$deriv <- c(diff(sim.2$N), NA)
points(deriv~N, data = sim.2, type='l',col='red', bty='l')
max(sim.2$deriv,na.rm=TRUE)
which(sim.2$deriv ==max(sim.2$deriv,na.rm=TRUE))
30
sim.2$N[which(sim.2$deriv ==max(sim.2$deriv,na.rm=TRUE))]
22.01992

## sim.3
sim.3$deriv <- diff(sim.3$N)
sim.3$deriv <- c(diff(sim.3$N), NA)
points(deriv~N, data = sim.3, type='l',col='orange', bty='l')
max(sim.3$deriv,na.rm=TRUE)
which(sim.3$deriv ==max(sim.3$deriv,na.rm=TRUE))
28
sim.3$N[which(sim.3$deriv ==max(sim.3$deriv,na.rm=TRUE))]
12.21354
```

Graph 3: Model of population growth rate vs population abundance (Blue: K=100, Red: K=50, Orange: K=25)

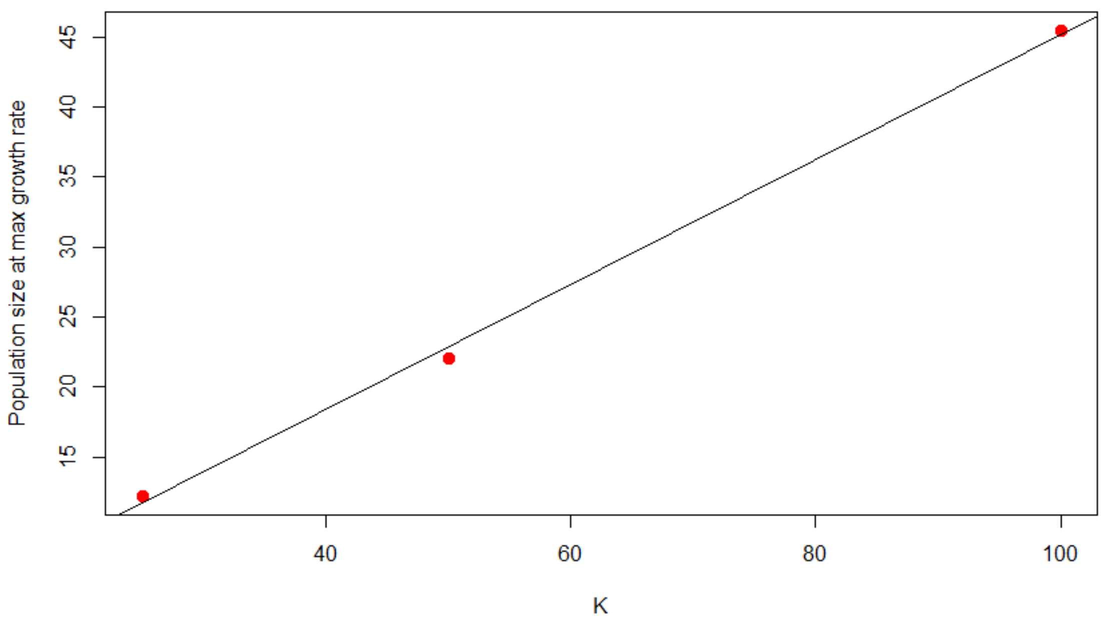


(c) Find the population abundance that yields the maximum population growth rate for each of the above three simulations. Visualize the effect of carrying capacity on population size at maximum growth rate by plotting these values against their corresponding K parameter. Include any code and figures.

`Sim`:
Population size at Max Growth Rate = 45.43845
K= 100
`Sim.2`:
Population size at Max Growth Rate = 22.01992
K= 50
`Sim.3`:
Population size at Max Growth Rate = 12.21354
K= 25

```
x<-c(100,50,25)
y<-c(45.43845, 22.01992, 12.21354)
plot(x, y,xlab="K",ylab="Population size at max growth
rate",col='red',pch=20,lwd='6')
```

Graph 4: The points for each model showing the increasing nature of logarithmic growth where as K increases the total population size at maximum growth rate increases. Regression line added to show that the numbers used in K and N were not pulled directly from the model.



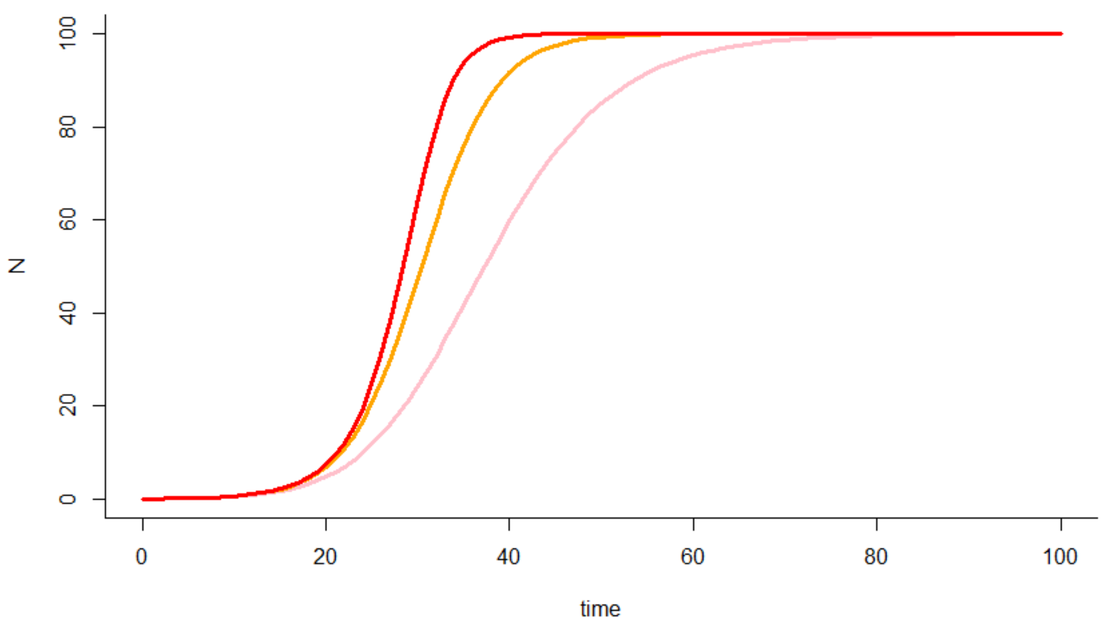
3. Suppose you manage a fishery and are tasked with maximizing the fishery's yield by managing the populations of three fish species that grow according to the theta logistic

growth model (see Hastings Ch. 4). A scientist visited the fishery and determined the theta value for each fish: 0.5 for species A, 1 for species B and 1.8 for species C. Which species will be maintained at the highest population abundance in your fishery? Include any code and figures

For this model I assumed that all other parameters for the fish species, other than theta are the same. So I used arbitrary numbers to hold the place of r, N and K to study the effects of the change of theta (r = 0.25, K = 100, and N = 0.05). The theta logistic growth model formula is as follows:

$$\frac{dN}{dt} = rN[1 - \left(\frac{N}{K}\right)^{\theta}]$$

Graph 5: Showing the theta logistic model for the three species (Pink=Species A, Orange=Species B, Red=Species C)




```
## deriv A
sim.A$deriv <- c(diff(sim.A$N), NA)

points(deriv~N, data = sim.A, type='l',col='Pink', bty='l',lwd=2)

max(sim.A$deriv,na.rm=TRUE)
3.700691
which(sim.A$deriv ==max(sim.A$deriv,na.rm=TRUE))
36
sim.A$N[which(sim.A$deriv ==max(sim.A$deriv,na.rm=TRUE))]
41.60371
```

```
## deriv B
sim.B$deriv <- c(diff(sim.B$N), NA)

points(deriv~N, data = sim.B, type='l',col='orange',
bty='l',lwd=2)

max(sim.B$deriv,na.rm=TRUE)
6.240943
which(sim.B$deriv ==max(sim.B$deriv,na.rm=TRUE))
31
sim.B$N[which(sim.B$deriv ==max(sim.B$deriv,na.rm=TRUE))]
47.49267


## deriv C
sim.C$deriv <- c(diff(sim.C$N), NA)

plot(deriv~N, data = sim.C, type='l',col='brown', bty='l',lwd=2)

max(sim.C$deriv,na.rm=TRUE)
8.98868
which(sim.C$deriv ==max(sim.C$deriv,na.rm=TRUE))
30
sim.C$N[which(sim.C$deriv ==max(sim.C$deriv,na.rm=TRUE))]
55.55655

## Plotting the peaks

xFish<-c(41.60371,47.49267,55.55655)
yFish<-c(3.700691,6.240943,8.98868)

points(x=xFish,y=yFish,col='red',pch=20,lwd='1')
```

In order to determine which fish species would be maintained at the highest population abundance I modeled the per capita growth rate against. I did this because the species with the highest growth rate at the highest N would be the one that would maintain a growth even at a large population size (or at least larger than the other species). According to this logic it appears the species C (theta = 1.8) has the greatest ability to maintain a high population.

Graph 6: Showing the derivative of the theta logistic model (the per capita growth rate) for the three species against N (Pink=Species A, Orange=Species B, Red=Species C)