Randomized Experiments and Covariates

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Motivating Example

Blocking: Randomization Improved

Applications: Seguro Popular, Perry Preschool, Ignatieff Campaign

Two Issues: Outliers and Interference

Blocking for Sequential Experiments

Motivating Example

Random Assignment Mechanisms

- Simple/complete randomization
  (Bernoulli trial, prob π)
- Complete randomization / random allocation
  (fixed proportion to tr)
- Blocked randomizations
  (fixed proportion to tr, w/in group)
- Cluster randomizations
  (assignment at higher level)

Motivation: A Causal Inference Question

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Suppose we observationally measure

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		Diff in Means: (Yes − No)	40

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```
x <- sample(0:1, size = 10, replace = TRUE)
```

```
x
```

```
##  [1] 1 1 1 1 1 1 1 1 1 1
```

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- If $\text{Var}(Y_i(0)) \neq \text{Var}(Y_i(1))$, allocate $\rightarrow$ higher-Variance condition

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$\widehat{ATE}_{+\text{cov}} = -7.5, 0, 7.5$
$\widehat{ATE}_{-\text{cov}} = 2.5, 0, -2.5$ (less variance!)

Example: Canvassing and Enrollment

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What *would have* happened to "No" precincts if "Yes"?

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What do we really want to know?

Does canvassing actually *change* enrollment in precinct?
(Or, just Party $\rightarrow$ Enrollment?)

What *would have* happened to "No" precincts if "Yes"?

What would have happened under *other* conditions?

Example: Canvassing and Enrollment

Suppose we can know both *potential outcomes* . . .

Precinct	Party	Canvass?	Enroll % if No Canvass	Enroll % if Canvass
1	Dem	_	20	60
2	Dem	_	30	70
3	Rep	_	20	30
4	Rep	_	30	40
		Means:	25	50

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$$\text{ATE} = 50 - 25 = 25$$

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$\text{ATE} = 50 - 25 = 25$

(True or an estimate?)

Example: Canvassing and Enrollment

Another way to think about same information:

Precinct	Party	Canvass?	Enroll % if No Canvass	Enroll % if Canvass	True Precinct Effect
1	Dem	_	20	60	40
2	Dem	_	30	70	40
3	Rep	_	20	30	10
4	Rep	_	30	40	10
		Means:	25	50	25

$$\text{ATE} = (40 + 40 + 10 + 10)/4 = 25$$

The Fundamental Problem of Causal Inference

We can't observe both "Canvassed" and "Not Canvassed" for a precinct.

We can't observe both *potential outcomes* (*counterfactuals*).

So, how can we get a good causal estimate?

Example: Canvassing and Enrollment

Suppose we observe ...

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Estimated ATE $= 65 - 25 = 40$

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Estimated ATE $= 65 - 25 = 40$ ☹ (too big)

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Estimated ATE $= 45 - 30 = 15$

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In practice, we don't know all potential outcomes.

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Let's ensure X does not predict T.

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Creating pre-treatmnt groups that look same on *predictors*.

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Under random allocation, 5 possible estimates: 40, 15, 25, 35, 10.

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- Blocked pairs randomization: $2^{\frac{n}{2}} = 2^{\frac{4}{2}} = 4$

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- **Can** observe X's, predictors of Y
- Check whether $T \overset{?}{\Rightarrow} X$

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  $\rightsquigarrow$ different actors interested in different effects
- Guidelines for limited/uncertain resources

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Define *population average treatment effect*, PATE:

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$$D \equiv \frac{1}{n/2} \sum_{i \in \{I_i=1, T_i=1\}} Y_i - \frac{1}{n/2} \sum_{i \in \{I_i=1, T_i=0\}} Y_i$$

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then, the total *estimation error* is

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Blocking controls Δ_{T_X}, and, to the degree correlated, Δ_{T_U}.

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- More covariates: no exact comparable units (5 covars, 3 levels: $3^5 = 243$)

Usual Blocking

- Exact on one or two discrete covariates (best predictor)
- More covariates: no exact comparable units (5 covars, 3 levels: $3^5 = 243$)
- More covariates often done informally

Multivariate, Continuous Blocking

Moore (2012)

- Like matching on PS, MD, start with dimension reduction

Multivariate, Continuous Blocking

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- Like matching, select an algorithm

Multivariate, Continuous Blocking

Moore (2012)

- Like matching on PS, MD, start with dimension reduction
- Like matching, select an algorithm
- Like matching, select other attributes (caliper, etc.)

Multivariate, Continuous Blocking

- Collect the predictors $\mathbf{X}$

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- Calculate how different units are from each other on $\mathbf{X}$'s

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- Create blocks of similar units
- Randomize within blocks

A Matrix of Distances

(First 5 rows of `x100`, variables `b1` and `b2`)

```
##      1001 1002 1003 1004 1005
## 1001 0.00 2.68 2.51 2.46 1.36
## 1002 2.68 0.00 2.80 1.77 1.71
## 1003 2.51 2.80 0.00 1.09 1.53
## 1004 2.46 1.77 1.09 0.00 1.11
## 1005 1.36 1.71 1.53 1.11 0.00
```

Classes of Blocking Algorithms

- Optimal: consider all blockings; pick best.
  (High-order problem)

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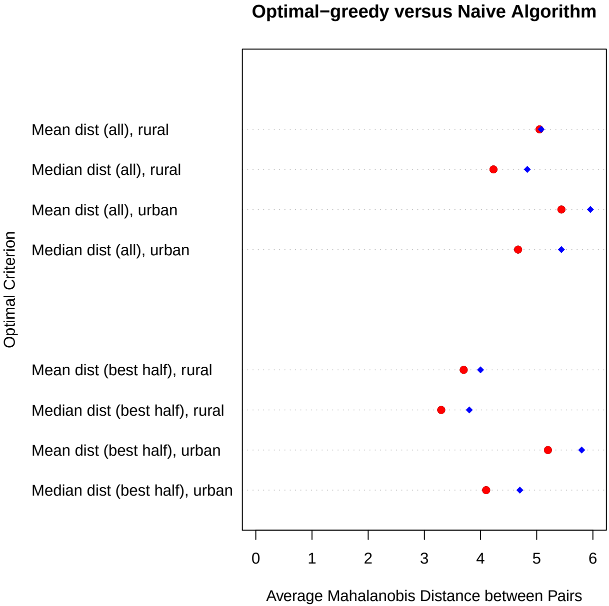
- Optimal: consider all blockings; pick best.
  (High-order problem)
- Optimal-greedy: consider all distances, pick best.
- Naive greedy: Get best block for this unit now.

Which Multivariate Blocking Algorithm? (optimal greedy)



Deploying Limited Resources: opt-greedy algorithm

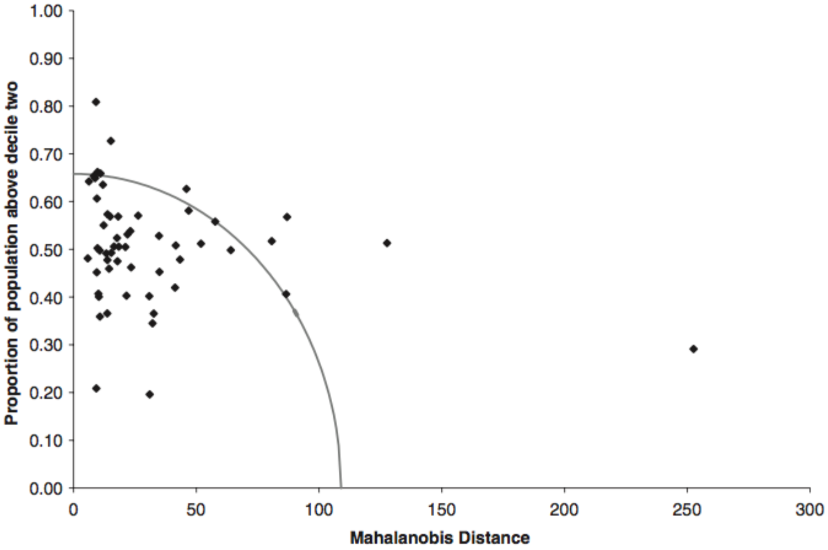


Figure 4. Choosing a subset of pairs for survey. We conducted our survey in 100 of

Why Block: Balance

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Simulation study: 100 units, $X_1 \sim N(0,1)$, $X_2 \sim \mathrm{Unif}(0,1)$, $X_3 \sim \chi^2_2$; 1000 such experiments. Assg treatmnt in 3 ways. p-value from `xBalance`.

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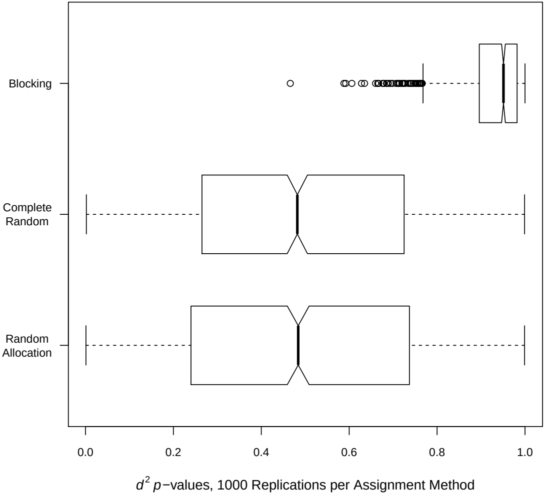

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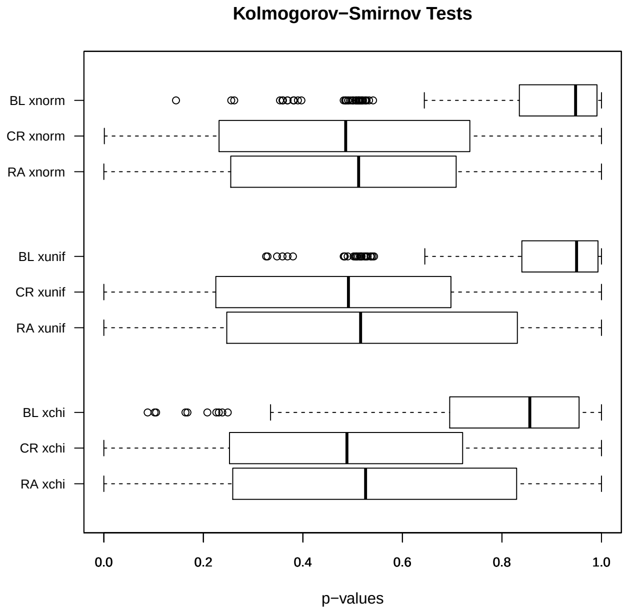
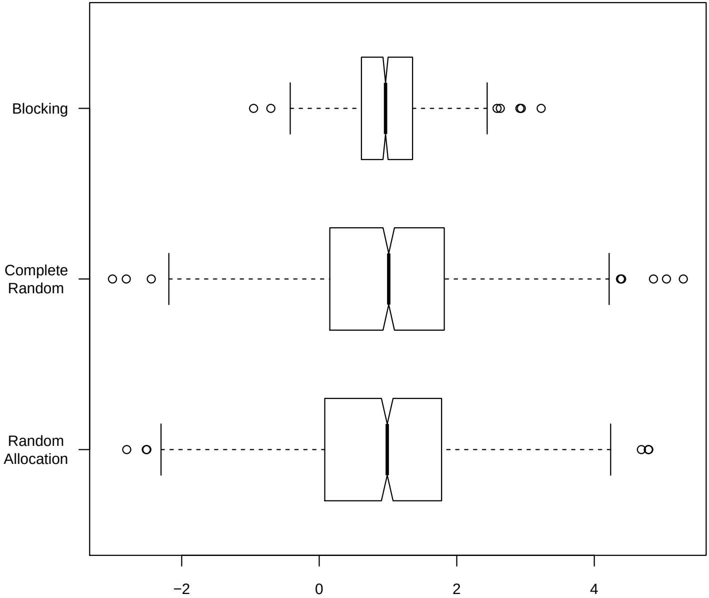
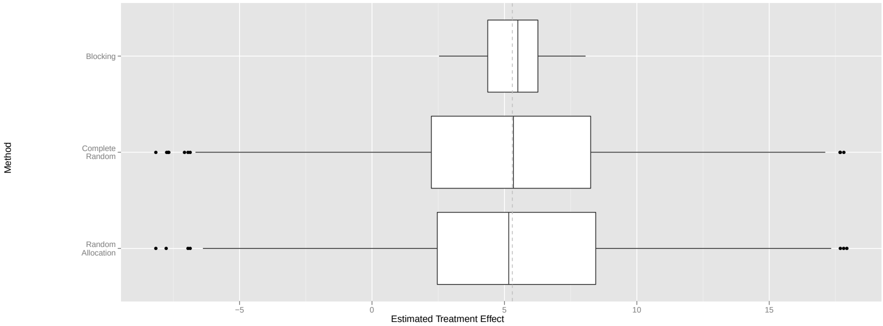


Why Block: Efficiency



Why Block: Efficiency under TE Heterogeneity



Applications: Seguro Popular, Perry Preschool, Ignatieff Campaign

Experiment: Randomized Health Infrastructure & Insurance

King et al. (2009)

- **Intervention**: resources for medical services, preventive care, pharmaceuticals, access, and financial health protection
- **Beneficiaries**: 50M Mexicans (half the pop) w/o access to health care
- **Cost** 2005 full +1% GDP new money annually
- One of **largest** health reforms of any country in 2 decades
- Most **visible** accomplishment of Fox administration
- Major **issue** 2006 pres. campaign (vote choice effect: 7-11%)
- **Randomized** 74/148 health clusters to first roll-out, infrastructure spending, encouragement to enroll
- But – do **better** than pure coin flip

Designing the *Seguro Popular* Experiment

How can we learn **most** from 148 cluster experiment?

- Ensure causes of HH expenditures evenly *balanced* betwn Tr and Co

  (otherwise, Tr might be poorer, or Co more insured)
- Make sure effect estimates as *precise* as possible

  (prefer estimate of "9 to 11%" to "−10 to 30%")

Designing the *Seguro Popular* Experiment

- Data on clusters:

cluster	population	educ years	doctors	nurses
MCSSA000364	3130	4.00	1	1
MCSSA000504	6492	5.11	1	1
MCSSA008221	5096	4.26	2	2

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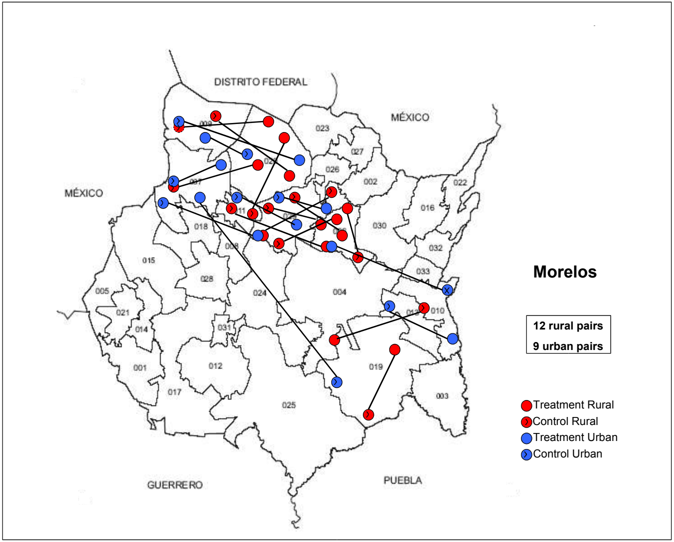
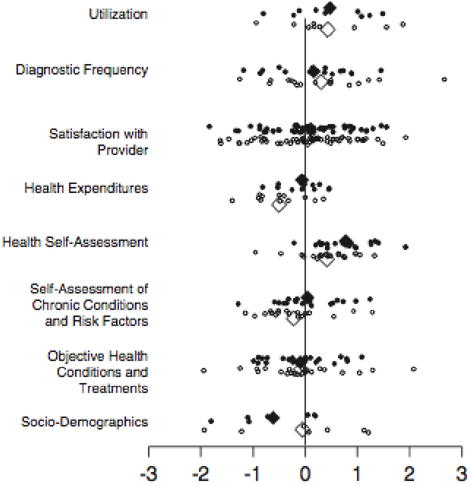
- Calculate how different clusters are from each other
- Mahalanobis distance between every pair of units:
  $MD_{ij} = \sqrt{(\mathbf{x}_i - \mathbf{x}_j)'\hat{\Sigma}^{-1}(\mathbf{x}_i - \mathbf{x}_j)}$

Blocked Pairs, Morelos



Balance in Baseline Outcomes, *Seguro Popular*

King et al. (2007)



Balance in Baseline Outcomes, *Seguro Popular*

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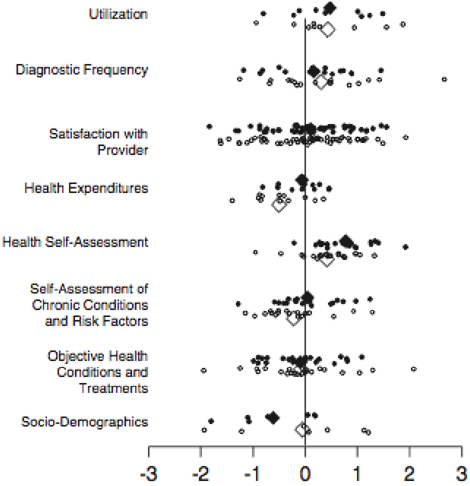
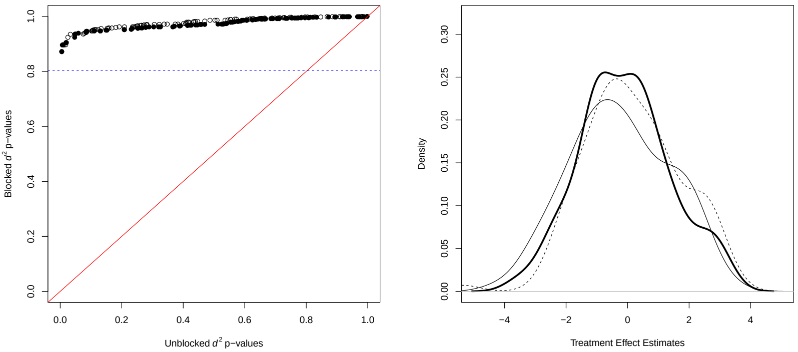


(Plus, SE's would have been 2× to 6× larger!)

Blocking in Applications: Balance and Efficiency

Moore (2012): Perry Preschool Experiment

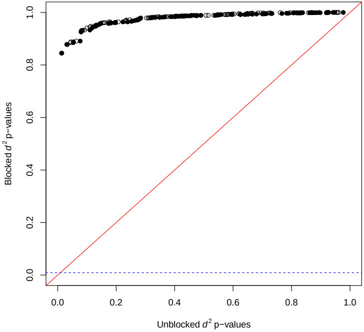
Left: QQ plot of balance (100 blocked vs. unblocked)
Right: Est TE under sharp null (100 blocked vs. unblocked)

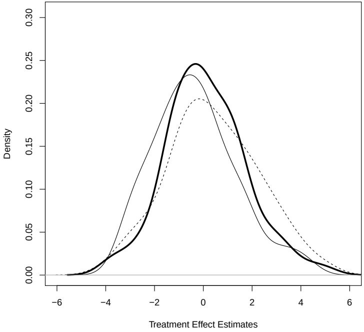


(SES, sex, IQ)

Balance in Applications: Balance and Efficiency

Considering more variables . . .





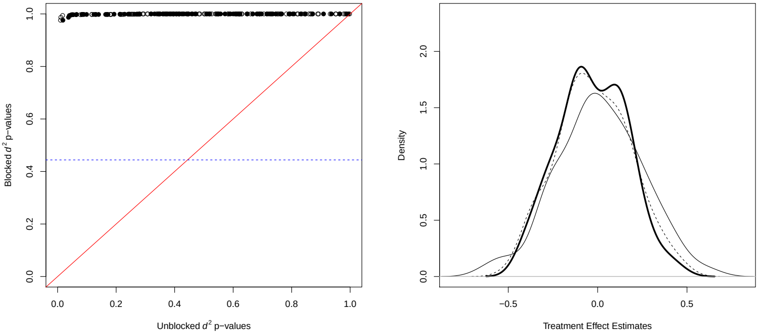
(+ siblings, AFDC, mom empl, educ, father, . . .)

Balance in Applications

Loewen and Rubenson (2011): persuade party delegates to support Ignatieff

Left: QQ plot of balance (100 blocked vs. unblocked)

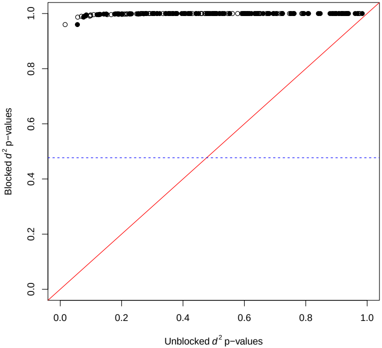
Right: Est TE under sharp null (100 blocked vs. unblocked)

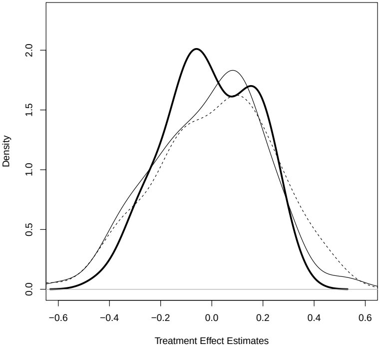


(province, pledged, special constituency)

Balance in Applications

Considering more variables . . .





(+ attention, interest)

Multivariate Continuous Blocking with `blockTools`

Moore and Schnakenberg (2023)

In R,

```r
library(blockTools)
data(x100)
head(x100)
```

```
##      id id2  b1  b2 g  ig
## 1 1001 101 156 795 b 729
## 2 1002 102 813 469 a 627
## 3 1003 103 950 978 a 959
## 4 1004 104 991 781 a 661
## 5 1005 105 613 759 a 819
## 6 1006 106 654 838 b 643
```

Multivariate Continuous Blocking with `blockTools`

Block:

```
block.out <- block(data = x100, id.vars = "id",
                   block.vars = c("b1", "b2"))
```

Multivariate Continuous Blocking with `blockTools`

Block:

```r
block.out <- block(data = x100, id.vars = "id",
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```

```
##   Unit 1 Unit 2   Distance
## 1   1043   1040 0.01240000
## 2   1100   1020 0.02259275
## 3   1065   1027 0.02912651
## 4   1085   1081 0.03498815
## 5   1088   1061 0.04789253
```

Multivariate Continuous Blocking with blockTools

Assign:

```
assg.out <- assignment(block.out, seed = 157)
```

Multivariate Continuous Blocking with `blockTools`

Assign:

```r
assg.out <- assignment(block.out, seed = 157)
```

```
##   Treatment 1 Treatment 2   Distance
## 1        1043        1040 0.01240000
## 2        1020        1100 0.02259275
## 3        1027        1065 0.02912651
## 4        1085        1081 0.03498815
## 5        1061        1088 0.04789253
```

Multivariate Continuous Blocking with blockTools

Diagnose:

```
diagnose(assg.out, data = x100, id.vars = "id",
         suspect.var = "b1", suspect.range = c(0,5))
```

Multivariate Continuous Blocking with `blockTools`

Diagnose:

```
diagnose(assg.out, data = x100, id.vars = "id",
          suspect.var = "b1", suspect.range = c(0,5))
```

Get block IDs

```
createBlockIDs(assg.out, data = x100, id.var = "id")
```

Multivariate Continuous Blocking with blockTools

Diagnose:

```
diagnose(assg.out, data = x100, id.vars = "id",
         suspect.var = "b1", suspect.range = c(0,5))
```

Get block IDs

```
createBlockIDs(assg.out, data = x100, id.var = "id")
```

Get balance:

```
assg2xBalance(assg.out, x100, id.var = "id",
              bal.vars = c("b1", "b2"))
```

Multivariate Continuous Blocking with blockTools

Extract conditions:

```
extract_conditions(assg.out, x100, id.var = "id")
```

```
##   [1] 2 1 1 2 2 2 2 2 1 2 2 1 1 2 1 2 2 2 2 1 2 1 1 1 2
##  [38] 1 1 2 2 1 1 1 2 2 2 1 2 2 2 1 2 1 1 2 1 2 1 2 1 1
##  [75] 1 2 1 2 2 2 2 1 1 1 1 2 1 2 2 2 1 1 1 2 1 1 1 2 1
```

Multivariate Continuous Blocking with blockTools

Extract conditions:

```
extract_conditions(assg.out, x100, id.var = "id")
```

```
##   [1] 2 1 1 2 2 2 2 2 1 2 2 1 1 2 1 2 2 2 2 1 2 1 1 1 2
##  [38] 1 1 2 2 1 1 1 2 2 2 1 2 2 2 1 2 1 1 2 1 2 1 2 1 1
##  [75] 1 2 1 2 2 2 2 1 1 1 1 2 1 2 2 2 1 1 1 2 1 1 1 2 1
```

```
x100 |> mutate(condition = extract_conditions(assg.out, x10
```

```
##        id id2  b1   b2 g  ig condition
## 1    1001 101 156  795 b 729         2
## 2    1002 102 813  469 a 627         1
## 3    1003 103 950  978 a 959         1
## 4    1004 104 991  781 a 661         2
## 5    1005 105 613  759 a 819         2
## 6    1006 106 654  838 b 643         2
## 7    1007 107 640  645 c  12         2
## 8    1008 108 681  404 a 221         2
```

Two Issues: Outliers and Interference

Covariate Weightings: Resistant Scaling

- $MD_{ij} = \sqrt{(\mathbf{x}_i - \mathbf{x}_j)'\Sigma^{-1}(\mathbf{x}_i - \mathbf{x}_j)}$

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  - regular covariance

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  - regular covariance
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- Resistant covariance estimators

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  - MCD (Minimum Covariance Determinant)
    1 constraint: include h points in interior

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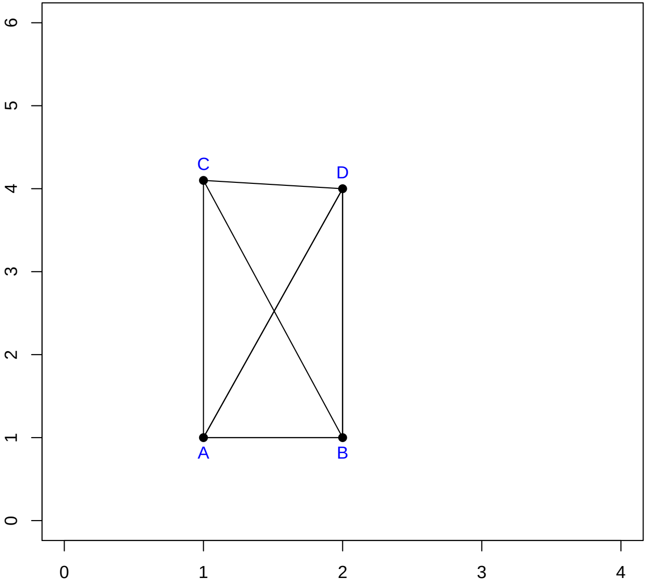
  - regular covariance
  - manual weighting
  - identity matrix (Euclidean dist)
- Resistant covariance estimators
  - MCD (Minimum Covariance Determinant)
    1 constraint: include h points in interior
  - MVE (Minimum Volume Ellipsoid)
    2 constraints: include h points, $p+1$ points on boundary

Covariate Weightings: Resistant Scaling



Covariate Weightings: Resistant Scaling

Given outliers, use resistant estimates of Σ and blocks will stay *same*:

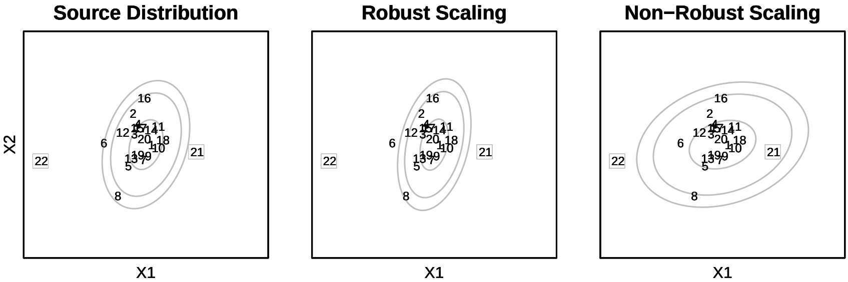
Covariate Weightings: Resistant Scaling

Given outliers, use resistant estimates of Σ and blocks will stay *same*:

		Distance from A		
Scale Matx	Points	B	C	D
Cov	A-D	**1.73**	1.76	2.45
Cov	A-$D + (1, 100)$	2.00	**0.08**	2.03
Cov	A-$D + (20, 100)$	1.72	1.01	**0.74**
MCD	A-D	**1.73**	1.76	2.45
MCD	A-$D + (1, 100)$	**1.73**	1.76	2.45
MCD	A-$D + (20, 100)$	**1.73**	1.76	2.45
MVE	A-D	**1.73**	1.76	2.45
MVE	A-$D + (1, 100)$	**1.73**	1.76	2.45
MVE	A-$D + (20, 100)$	**1.73**	1.76	2.45

(**Bold** = closest to A)

Covariate Weightings: Resistant Scaling



Covariate Weightings

Baseline: Exclude outliers, regular MD

	Unit 1	Unit 2	Distance
1	15	4	0.18
2	9	7	0.37
3	13	5	0.37
4	18	10	0.43
5	17	3	0.44
6	14	11	0.50
7	20	1	0.65
8	16	2	0.95
9	12	6	1.32
10	19	8	2.12

Covariate Weightings

Include outliers, resistant MD: Same blocks! ☺

	Unit 1	Unit 2	Distance
1	15	4	0.18
2	13	5	0.37
3	9	7	0.42
4	18	10	0.46
5	17	3	0.50
6	14	11	0.59
7	20	1	0.76
8	16	2	1.03
9	12	6	1.56
10	19	8	2.18
11	22	21	13.43

Covariate Weightings

Include outliers, non-resistant MD: Blocks shuffled by outliers ☹

	Unit 1	Unit 2	Distance
1	17	15	0.12
2	9	7	0.24
3	19	13	0.25
4	14	11	0.28
5	10	1	0.33
6	12	3	0.45
7	4	2	0.58
8	20	18	0.69
9	8	5	1.45
10	22	6	2.24
11	21	16	3.64

Interference

- We worry that effects may spillover: *interference*
- But nearby units are similar!
- Restrict blocks to units in a range

Multivariate Continuous Blocking with `blockTools`

Other arguments to `block()`

- `vcov.data`
- `n.tr`
- `algorithm`
- `distance`: `mahalanobis`, `mcd`, `mve`
- `weight`
- `level.two`: block states by most similar cities
- `valid.var`, `valid.range`: Goldilocks
- `seed`: (for `mcd` and `mve`)

Blocking for Sequential Experiments

What about Sequential Experiments?

Moore and Moore (2013)

What about Sequential Experiments?

Moore and Moore (2013)

tall, male,
senior, biology

What about Sequential Experiments?

Moore and Moore (2013)

Have info!

Common Discrete Covariate-Adaptive Allocations

Biased coins

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Biased coins

Define unique covariate profile $\mathbf{p}$

Common Discrete Covariate-Adaptive Allocations

Biased coins

Define unique covariate profile $\mathbf{p}$

use

$$Pr(t = T) = \begin{cases} \frac{2}{3} & \text{if } n_{\mathbf{p}T} < n_{\mathbf{p}C} \\ \frac{1}{3} & \text{if } n_{\mathbf{p}T} > n_{\mathbf{p}C} \end{cases}$$

Common Discrete Covariate-Adaptive Allocations

Minimization

Common Discrete Covariate-Adaptive Allocations

Minimization

Given $\mathbf{p}$, score each variable s_j, $\left(\frac{n_{\mathbf{p}C}-n_{\mathbf{p}T}}{n_{\mathbf{p}C}+n_{\mathbf{p}T}+1}\right)$
combine (sum), rank conditions, assign π's

Common Discrete Covariate-Adaptive Allocations

Minimization

Given $\mathbf{p}$, score each variable s_j, $\left(\frac{n_{\mathbf{p}C}-n_{\mathbf{p}T}}{n_{\mathbf{p}C}+n_{\mathbf{p}T}+1}\right)$
combine (sum), rank conditions, assign π's

	Sex		Age	
	M	F	Young	Old
Control	1	3	3	3
Treatment	2	4	3	1
s_j (Old M)	$-\frac{1}{4}$			$\frac{2}{5}$

Common Discrete Covariate-Adaptive Allocations

Minimization

Given $\mathbf{p}$, score each variable s_j, $\left(\frac{n_{\mathbf{p}C}-n_{\mathbf{p}T}}{n_{\mathbf{p}C}+n_{\mathbf{p}T}+1}\right)$
combine (sum), rank conditions, assign π's

	Sex		Age	
	M	F	Young	Old
Control	1	3	3	3
Treatment	2	4	3	1
s_j (Old M)	$-\frac{1}{4}$			$\frac{2}{5}$

(bias toward Treatment)

Common Discrete Covariate-Adaptive Allocations

1. Biased coins

   $\rightsquigarrow$ require unique, replicated covariate profiles

Common Discrete Covariate-Adaptive Allocations

1. Biased coins

   $\rightsquigarrow$ require unique, replicated covariate profiles

2. Minimization

   $\rightsquigarrow$ ignores joint distribution

Perfect balance?

	Sex		Age	
	M	F	Young	Old
Control	3	3	3	3
Treatment	3	3	3	3

Common Discrete Covariate-Adaptive Allocations

1. Biased coins

   $\rightsquigarrow$ require unique, replicated covariate profiles

2. Minimization

   $\rightsquigarrow$ ignores joint distribution

Perfect balance?

	Sex		Age	
	M	F	Young	Old
Control	3	3	3	3
Treatment	3	3	3	3

Perfect imbalance?

	Young M	Old M	Young F	Old F
Control	3	0	0	3
Treatment	0	3	3	0

My Approach

- Allow discrete covariates, restriction to exact profile $\mathbf{p}$

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- Move beyond counts in conditions

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My Approach

- Allow discrete covariates, restriction to exact profile $\mathbf{p}$
- Move beyond counts in conditions
- Incorporate ordered, continuous covariates
  - Define dissimilarity between current unit and conditions (Average pairwise Mahalanobis distance $\overline{MD}_{qt} = \frac{1}{R}\sum_{r=1}^{R} MD_{qr}$)
  - Assign higher prob to more dissimilar conditions

My Approach

- Allow discrete covariates, restriction to exact profile $\mathbf{p}$
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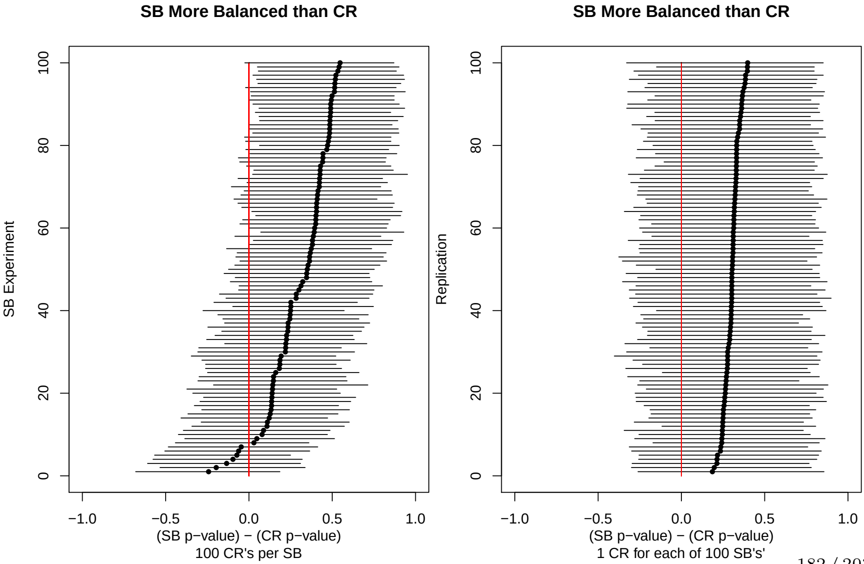
  - **Realistic data**
- Implement in actual sequential trial

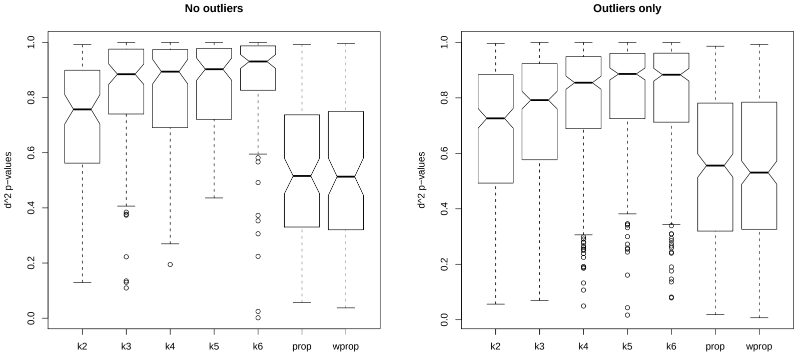
Seq. Blocking More Balanced than CR

Correlated MVN



Selecting a Method for Experiments

Simulate 100 datasets, each has
(48 units, realistic means, SDs, bivariate corrs for 10 covariates)



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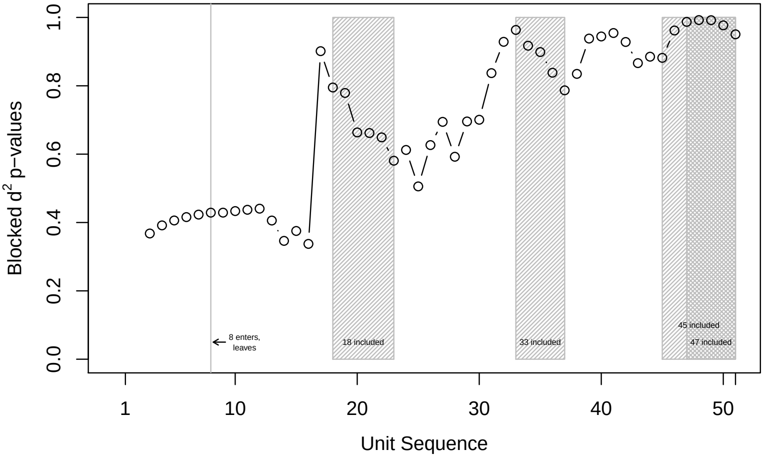
- 52 assigned total; 46 assigned Nov 09 - Jan 11; 39 follow-up.
- Two Conditions (daily practice)
  - `memory`: retrieve specific memories of life events in response to cue words
  - `anagrams`: rearrange letters of cue words to create new words (control)

Inputting Data in Real Time: Generalized Query

Unit 1

```
> seqblock1(query=T)
How many identification variables are there?
> 1
Enter the name of ID variable 1 without quotation marks.
> id
Enter the value of 'id'.
> 10624
How many exact blocking variables are there?
> 0
How many blocking variables are there?
> 2
Enter the name of blocking variable 1.
> x1
Enter the value of ‘x1’.
> 100
Should ‘x1’ be restricted to certain values? [n/y]
> no
Enter the name of blocking variable 2.
> x2
Enter the value of ‘x2’.
> 80
How many experimental/treatment conditions are there?
> 2
```

Balance: Moore & Moore 2013



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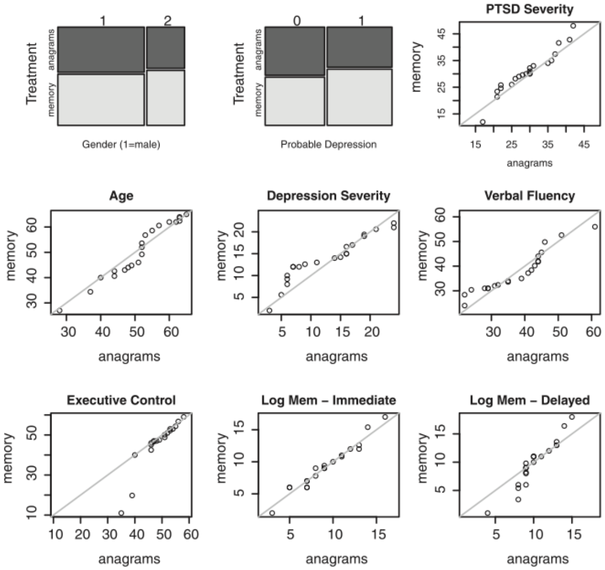

RI Confidence Intervals from `blockTools`

Get RI confidence interval from sequentially blocked data:

```
set.seed(407357912)
df <- tibble(id = 1:50,
             prov = sample(1:2, 50, replace = TRUE),
             age = sample(seq(18, 55, 1), 50, replace =
             treat = sample(0:1, 50, replace = TRUE),
             yobs = treat + sample(15:20, 50, replace =
```

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Check summary statistics:

```
df |> group_by(treat) |> summarise(m_yobs = mean(yobs)
```

```
## # A tibble: 2 x 2
##   treat m_yobs
##   <int>  <dbl>
## 1     0   17.7
## 2     1   18.3
```

RI Confidence Intervals from `blockTools`

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```
invertRIconfInt(df,
                outcome.var = "yobs", tr.var = "treat",
                tau.abs.min = -2, tau.abs.max = 3,
                tau.length = 20, n.sb.p = 200,
                id.vars = "id", id.vals = "id",
                exact.vars = c("prov", "age"), exact.vals =
```

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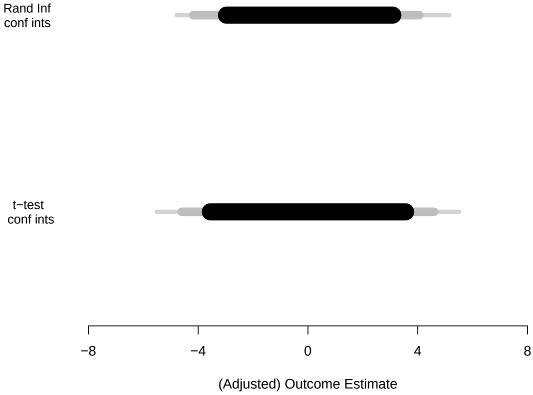
```
$ci95
[1] -0.16  0.11  0.37  0.63  0.89  1.16  1.42

$ci90
[1] -0.16  0.11  0.37  0.63  0.89  1.16

$ci80
[1] 0.11  0.37  0.63  0.89  1.16
```

RI Confidence Intervals: Moore & Moore 2013

Blocked estimates 9-15% more efficient



Analysis for Blocked Designs

If assignment probability for i varies by block j, let $p_{ij} = m_{ij}/N_j$. Weight each obs with

$$w_{ij} = \frac{d_i}{p_{ij}} + \frac{1 - d_i}{1 - p_{ij}}$$

Testing for Blocked Designs

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If blocks used to randomize, incorporate into RI.

1. Use actual blocks created, when possible
2. Reassign hypothetical treatment 1000 times
3. Obtain distribution of ATEs under (e.g.) sharp null
4. Compare observed ATE to randomisation distribution

Next:
Clustered Designs
Regression and Experiments

References I

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