Regression with Randomized Experiments

Ryan T. Moore

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The Linear Regression Estimator

Estimating the ATE

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```

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seats <- read_csv("http://j.mp/2YfZdgv")
```

```r
head(seats)
```

```
## # A tibble: 6 x 6
##      GP village reserved female irrigation water
##   <dbl>   <dbl>    <dbl>  <dbl>      <dbl> <dbl>
## 1     1       2        1      1          0    10
## 2     1       1        1      1          5     0
## 3     2       2        1      1          2     2
## 4     2       1        1      1          4    31
## 5     3       2        0      0          0     0
## 6     3       1        0      0          0     0
```

Estimating the ATE

Explore assignment:

```r
table(seats$reserved)
```

```
##
##   0   1
## 214 108
```

Estimating the ATE

Estimate treatment effect with difference in means:

```r
mean(seats$water[seats$reserved == 1]) -
  mean(seats$water[seats$reserved == 0])
```

```
## [1] 9.252423
```

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In areas with reservations, about 9.3 more drinking water projects were undertaken.

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Estimate coefficients

```r
lm_out <- lm(water ~ reserved, data = seats)
lm_out
```

```
##
## Call:
## lm(formula = water ~ reserved, data = seats)
##
## Coefficients:
## (Intercept)     reserved
##      14.738        9.252
```

Estimated model

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Causal interpretation of coefficient comes from the science of the policy experiment, **not** from regression.

Social Pressure Mail Experiment

Gerber, Green, and Larimer (2008): Social pressure mailer experiment; households randomly receive one of four messages

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Dear Registered Voter:

DO YOUR CIVIC DUTY AND VOTE!

Why do so many people fail to vote? We've been talking about this problem for years, but it only seems to get worse.

The whole point of democracy is that citizens are active participants in government; that we have a voice in government. Your voice starts with your vote. On August 8, remember your rights and responsibilities as a citizen. Remember to vote.

DO YOUR CIVIC DUTY — VOTE!

Social Pressure Mail Experiment

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Dear Registered Voter:

YOU ARE BEING STUDIED!

Why do so many people fail to vote? We've been talking about this problem for years, but it only seems to get worse.

This year, we're trying to figure out why people do or do not vote. We'll be studying voter turnout in the August 8 primary election.

Our analysis will be based on public records, so you will not be contacted again or disturbed in any way. Anything we learn about your voting or not voting will remain confidential and will not be disclosed to anyone else.

DO YOUR CIVIC DUTY — VOTE!

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Dear Registered Voter:

WHAT IF YOUR NEIGHBORS KNEW WHETHER YOU VOTED?

Why do so many people fail to vote? We've been talking about the problem for years, but it only seems to get worse. This year, we're taking a new approach. We're sending this mailing to you and your neighbors to publicize who does and does not vote.

The chart shows the names of some of your neighbors, showing which have voted in the past. After the August 8 election, we intend to mail an updated chart. You and your neighbors will all know who voted and who did not.

DO YOUR CIVIC DUTY — VOTE!

MAPLE DR	Aug 04	Nov 04	Aug 06
9995 JOSEPH JAMES SMITH	Voted	Voted	
9995 JENNIFER KAY SMITH		Voted	
9997 RICHARD B JACKSON		Voted	
9999 KATHY MARIE JACKSON		Voted	
9999 BRIAN JOSEPH JACKSON		Voted	
9991 JENNIFER KAY THOMPSON		Voted	
9991 BOB R THOMPSON		Voted	
9993 BILL S SMITH			
9989 WILLIAM LUKE CASPER		Voted	
9989 JENNIFER SUE CASPER		Voted	
9987 MARIA S JOHNSON	Voted	Voted	
9987 TOM JACK JOHNSON	Voted	Voted	
9987 RICHARD TOM JOHNSON		Voted	

Multiple Regression: Many Predictors

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Model:

$$\begin{aligned} \text{Turnout 2006}_i &= \beta_0 + \beta_1 \cdot \text{Control}_i + \\ &\quad \beta_2 \cdot \text{Hawthorne}_i + \beta_3 \cdot \text{Neighbors}_i + \epsilon_i \end{aligned}$$

```r
social <- read_csv("http://j.mp/2YenEuU")

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```

```
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)            0.315      0.002 132.909    0.000
## messagesControl       -0.018      0.003  -6.905    0.000
## messagesHawthorne      0.008      0.003   2.341    0.019
## messagesNeighbors      0.063      0.003  18.944    0.000
```

Estimated model

$$\begin{aligned}\widehat{\text{Turnout 2006}}_i &= 0.31 + -0.02\,(\text{Control}_i) + 0.01\,(\text{Hawthorne}_i) + \\ &\quad 0.06\,(\text{Neighbors}_i)\end{aligned}$$

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$$\widehat{\text{Turnout 2006}}_i = 0.31 + -0.02\,(\text{Control}_i) + 0.01\,(\text{Hawthorne}_i) + 0.06\,(\text{Neighbors}_i)$$

1. What is the predicted turnout probability under `Control`?
2. What is the predicted turnout probability under `Hawthorne`?
3. What is the predicted turnout probability under `Neighbors`?
4. What is the predicted turnout probability under `Civic Duty`?

Alternatively, estimate

$$\begin{aligned}\text{Turnout } 06_i &= \beta_0 \cdot \text{CivicDuty}_i + \beta_1 \cdot \text{Contrl}_i + \beta_2 \cdot \text{Hawthorne}_i + \\ &\quad \beta_3 \cdot \text{Neighbors}_i + \epsilon_i\end{aligned}$$

```r
lm_out <- lm(primary2006 ~ -1 + messages, data = social)
summary(lm_out)$coefficients
```

```
##                      Estimate  Std. Error  t value Pr(>|t
## messagesCivic Duty 0.3145377 0.002366570 132.9087
## messagesControl    0.2966383 0.001057939 280.3927
## messagesHawthorne  0.3223746 0.002367004 136.1952
## messagesNeighbors  0.3779482 0.002367097 159.6674
```

Estimated model:

$$\begin{aligned}\widehat{\text{Turnout 2006}}_i &= 0.31\,(\text{Civic Duty}_i) + 0.3\,(\text{Control}_i) + \\ &\quad 0.32\,(\text{Hawthorne}_i) + 0.38\,(\text{Neighbors}_i)\end{aligned}$$

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Models with Covariates

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We consider adding covariates to our linear model to

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- (not to reduce bias in estimator, if simple design)

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- "Included variable bias" (see https://t.ly/ttYpD)

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- E.g., make better turnout predictions by including registrant's age and prior turnout, in addition to whether received `Neighbors` postcard
- Age and prior turnout are *causally prior* to treatment
- Let's just compare `Neighbors` to `Control`.

Models with Covariates

```r
# Subset to two treatment conditions:
social.neighbor <- subset(social,
                          (messages == "Control") |
                            (messages == "Neighbors"))

# Calculate age:
social.neighbor <- social.neighbor |>
  mutate(age = 2006 - yearofbirth)
```

```r
# Multiple regression:
lm_out <- lm(primary2006 ~ age + primary2004 +
               messages, data = social.neighbor)

summary(lm_out)$coefficients |> round(3)
```

```
##                    Estimate Std. Error t value Pr(>|t
## (Intercept)           0.046      0.003  13.274
## age                   0.004      0.000  58.873
## primary2004           0.147      0.002  76.464
## messagesNeighbors     0.080      0.003  31.670
```

Now, make different prediction depending on registrant's age and prior turnout, not just treatment status.

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Estimated model:

$$\begin{aligned}\widehat{\text{Turnout 2006}}_i \quad &= \quad 0.05 + 0.004\,(\text{Age}_i) + 0.15\,(\text{Turnout 2004}_i) + \\ &\quad\; 0.08\,(\text{Neighbors}_i)\end{aligned}$$

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Coefficients: how much 1-unit difference in predictor affects my prediction for turnout in 2006, *assuming the other predictors do not change.* (The *ceteris paribus* assumption.) Sometimes a valid assumption, sometimes not.

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&= 0.004 \times 10
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Not causal.

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- Reserved seats for women may only produce more water projects in *larger* villages, e.g.
- Reserved seats may produce *fewer* water projects in small villages

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RI test for heterogeneity:

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- Create reference dist'n of 1000 diff-in-variances

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RI test for heterogeneity:

- Calculate observed diff-in-variances
- Assume $\tau_i = \widehat{\tau}$
- Impute missing potential outcomes
- Create reference dist'n of 1000 diff-in-variances
- p-value = prop of ref dist'n as extreme or more extreme than observed diff-in-variances.

Testing for Heterogeneity

```
obs_diff_in_vars <- var(seats$water[seats$reserved == 1])
  var(seats$water[seats$reserved == 0])
```

Testing for Heterogeneity

```
obs_diff_in_vars <- var(seats$water[seats$reserved == 1]) -
  var(seats$water[seats$reserved == 0])
```

```
obs_diff_in_vars
```

```
## [1] 2266.654
```

Testing for Heterogeneity

```
n_perms <- 2000                     # how many permutations
store_dvs <- rep(NA, n_perms) # storage
base_assg <- c(rep(0, 214), rep(1, 108))
```

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n_perms <- 2000                     # how many permutations
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```r
for(i in 1:n_perms){
  perm_tr <- sample(base_assg)
  diff_in_var <- var(seats$water[perm_tr == 1]) -
    var(seats$water[perm_tr == 0])
  store_dvs[i] <- diff_in_var
}
```

Testing for Heterogeneity

```r
n_perms <- 2000                   # how many permutations
store_dvs <- rep(NA, n_perms) # storage
base_assg <- c(rep(0, 214), rep(1, 108))
```

```r
for(i in 1:n_perms){
  perm_tr <- sample(base_assg)
  diff_in_var <- var(seats$water[perm_tr == 1]) -
    var(seats$water[perm_tr == 0])
  store_dvs[i] <- diff_in_var
}
```

```r
p_val <- mean(abs(store_dvs) >= abs(obs_diff_in_vars))

p_val
```

```
## [1] 0.005
```

Testing for Heterogeneity

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n_perms <- 2000                    # how many permutations
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}
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```r
p_val <- mean(abs(store_dvs) >= abs(obs_diff_in_vars))

p_val
```

```
## [1] 0.005
```

Since $p < \alpha = 0.05$, reject null of "no heterogeneity".

Heterogeneous Effects

If evidence or theory for heterogeneous effects, estimate them!

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Estimand: Conditional Average Treatment Effect (CATE)

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Estimand: Conditional Average Treatment Effect (CATE)

$$\begin{aligned} CATE &= E(Y_1 - Y_0 | X = x) \\ &= E(Y_1 | X = x) - E(Y_0 | X = x) \end{aligned}$$

Estimating Heterogeneous Treatment Effects

- Subset on covariate X, estimate ATE within subsets

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- Subset on covariate X, estimate ATE within subsets
- Estimate with interaction term in single model

Interaction Terms for Heterogeneous Effects

- Add predictor variable representing subgroups in multiple regression equation

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```
lm_out <- lm(primary2006 ~ primary2004 +
               messages + primary2004:messages,
             data = social.neighbor)
summary(lm_out)$coef |> round(3)
```

```
##                                Estimate Std. Error t valu
## (Intercept)                       0.237      0.001 176.32
## primary2004                       0.149      0.002  69.96
## messagesNeighbors                 0.069      0.003  20.93
## primary2004:messagesNeighbors     0.027      0.005   5.23
```

Estimated model:

$$\widehat{\text{Turnout 2006}}_i = 0.24 + 0.15\,(\text{Turnout 2004}_i) + 0.07\,(\text{Neighbors}_i) + 0.03\,(\text{Turnout 2004}_i)\,(\text{Neighbors}_i)$$

Estimated model:

$$
\begin{aligned}
\widehat{\text{Turnout 2006}}_i &= 0.24 + 0.15\,(\text{Turnout 2004}_i) + 0.07\,(\text{Neighbors}_i) + \\
&\quad 0.03\,(\text{Turnout 2004}_i)\,(\text{Neighbors}_i)
\end{aligned}
$$

Calculate

1. Predicted turnout prob under `Control`, if no 2004 vote?

Estimated model:

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Calculate

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2. Predicted turnout prob under `Neighbors`, if no 2004 vote?

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Calculate

1. Predicted turnout prob under `Control`, if no 2004 vote?
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3. Treatment effect for those who didn't vote in 2004?

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$$\widehat{\text{Turnout 2006}}_i = 0.24 + 0.15\,(\text{Turnout 2004}_i) + 0.07\,(\text{Neighbors}_i) + 0.03\,(\text{Turnout 2004}_i)\,(\text{Neighbors}_i)$$

Calculate

1. Predicted turnout prob under `Control`, if no 2004 vote?
2. Predicted turnout prob under `Neighbors`, if no 2004 vote?
3. Treatment effect for those who didn't vote in 2004?
4. Predicted turnout prob under `Control`, if voted in 2004?
5. Predicted turnout prob under `Neighbors`, if voted in 2004?
6. Treatment effect for those who voted in 2004?

Interaction Terms for Heterogeneous Effects

$$\widehat{\text{Turnout 2006}}_i \quad = \quad \beta_0 + \beta_1 X_i + \beta_2 T_i + \beta_3 X_i T_i$$

Interaction Terms for Heterogeneous Effects

$$\begin{aligned}
\widehat{\text{Turnout 2006}}_i &= \beta_0 + \beta_1 X_i + \beta_2 T_i + \beta_3 X_i T_i \\
&= \underbrace{\beta_0}_{\bar{Y} \text{ if } (0,0)} + \underbrace{\beta_1}_{\bar{Y} \text{ if } X_i=1} X_i + \underbrace{\beta_2}_{\text{ATE if } X_i=0} T_i + \underbrace{\beta_3}_{\text{HTE}} X_i T_i
\end{aligned}$$

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\widehat{\text{Turnout 2006}}_i &= \beta_0 + \beta_1 X_i + \beta_2 T_i + \beta_3 X_i T_i \\
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\widehat{\text{Turnout 2006}}_i &= \underbrace{0.24}_{\bar{Y}\text{ for (0,0), Prior Nonvoters, Control}} + \\
&\quad \underbrace{0.15}_{\text{Additional }\bar{Y}\text{ for Prior Voters}} (\text{Turnout 2004}_i) + \\
&\quad \underbrace{0.07}_{\text{ATE for Prior Nonvoters; part of ATE for Prior Voters}} (\text{Neigh} \\
&\quad \underbrace{0.03}_{\text{Additional ATE for Prior Voters}} (\text{Turnout 2004}_i)(\text{Neighb}
\end{aligned}$$

Exercise:

Consider the data on UK MP candidates at
http://j.mp/32PHfFd

1. Read the data into R and name it `mps`
2. Create a new variable called `winner`. Set it equal to `1` if `margin` is greater than 0; set it to `0` if `margin` is less than zero.
3. Consider `winner` to be a randomly assigned treatment. Estimate the causal effect of `winner` on `ln.net`, net worth at death of these candidates.
4. Investigate whether the effect of `winner` varies by `party`.

Quantile Average Treatment Effects

A *quantile* is a cut-point, or a position, in a statistical distribution.

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- half the values below it?
- $\frac{3}{4}$ of the values below it?
- $\frac{1}{5}$ of the values below it?
- $\dots$

These are *quantiles*.

Quantile Average Treatment Effects (data)

```
set.seed(326370675)
n <- 1000
Y1 <- Y0 <- runif(n)
# If low baseline, negative TE:
Y1[Y0 <.5] <- Y0[Y0 <.5] - rnorm(length(Y0[Y0 <.5]))
# If high baseline, positive TE:
Y1[Y0 >.5] <- Y0[Y0 >.5] + rnorm(length(Y0[Y0 >.5]))
D <- sample((1:n) %% 2)  # Assign 0/1
Y <- D * Y1 + (1 - D) * Y0  # Y_obs
samp <- tibble(D, Y)
library(quantreg)
ate <- coef(lm(Y ~ D, data = samp))[2]
qtes <- rq(Y ~ D,
           tau = seq(.05, .95, length.out = 10), # At what
           data = samp, method = "fn")
```

Quantile Average Treatment Effects (data)

```r
set.seed(326370675)
n <- 1000
Y1 <- Y0 <- runif(n)
# If low baseline, negative TE:
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qtes <- rq(Y ~ D,
           tau = seq(.05, .95, length.out = 10), # At what p
           data = samp, method = "fn")
```
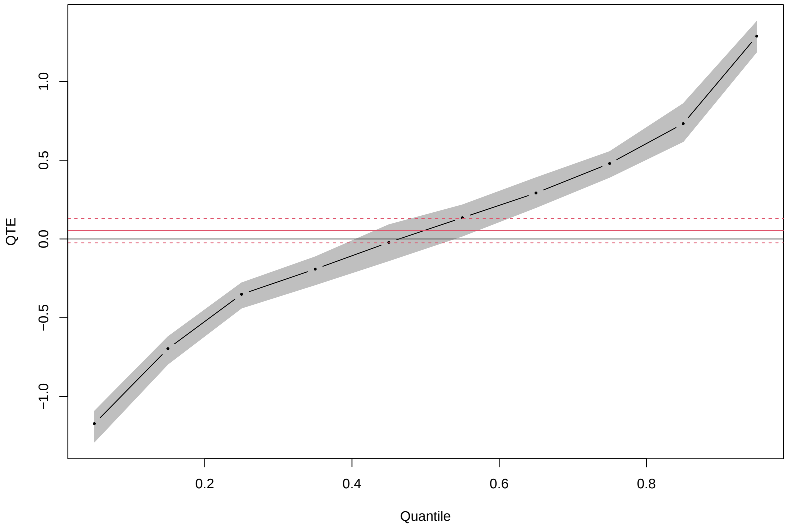

(See https://bit.ly/3dnNhGP)



QTE
1.0
0.5
0.0
−0.5
−1.0
0.2
0.4
0.6
0.8
Quantile

Quantile Average Treatment Effects (new data)

```r
set.seed(21578100)
n <- 1000
Y1 <- Y0 <- runif(n)
# If low baseline, zero TE:
Y1[Y0 <.5] <- Y0[Y0 <.5]
# If high baseline, positive TE:
Y1[Y0 >.5] <- Y0[Y0 >.5] + rnorm(length(Y0[Y0 >.5]))
D <- sample((1:n) %% 2)  # Assign 0/1
Y <- D * Y1 + (1 - D) * Y0  # Y_obs
samp <- tibble(D, Y)

ate <- coef(lm(Y ~ D, data = samp))[2]
qtes <- rq(Y ~ D,
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```

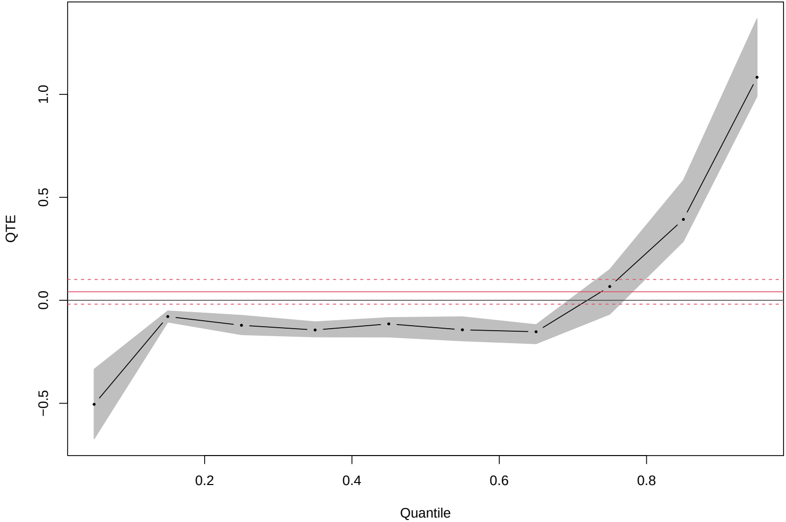

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Inference for Experiments using Regression Adjustment

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Lin (2013) compares several estimates of SE for linear models with experimental data, adjusting for covariates x.

Tailored for fixed-sample, variation-from-assignment situations.

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“Sandwich” estimators of variance:

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Tailored for fixed-sample, variation-from-assignment situations.

"Sandwich" estimators of variance:

$$Var\left(\widehat{\beta_{\text{OLS}}}\right) = (X'X)^{-1}(X'\text{diag}(\hat{\epsilon}_1, \ldots, \hat{\epsilon}_n)X)(X'X)^{-1}$$

Inference for Experiments using Regression Adjustment

Model:

$$Y_i = \beta_0 + \beta_1 T_i + \beta_2 (x_i - \bar{x}) + \beta_3 T_i (x_i - \bar{x}) + \epsilon_i$$

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"Full set of treatment by (demeaned) covariate interactions"

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Then, use "sandwich" HC2 standard errors.

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Then, use "sandwich" HC2 standard errors.

Causal estimate:

$$\widehat{ATE}_{\text{interact}} = \hat{\beta}_1$$

Inference for Experiments using Regression Adjustment

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Inference for Experiments using Regression Adjustment

Result: SE's that are

- Consistent under heteroskedasticity
- At least as efficient as usual $\widehat{ATE}_{\text{OLS}}$
- More efficient if $n_T \neq n_C$
- Doesn't *hurt* precision
  ($\widehat{ATE}_{\text{OLS}}$ does if x varies more with τ_i than Y_1, Y_0, e.g.)

Inference for Experiments using Regression Adjustment

Recovering the ATE

Suppose categorical G with 2 levels. Let G_i be indicator for first level. Expect heterogeneous treatment effects.

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$$Y_i = \beta_0 + \beta_1 T_i + \beta_2 G_i + \beta_3 T_i G_i + \epsilon_i \tag{1}$$

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What's $\widehat{ATE}$? Not $\hat{\beta}_1$. Not $\hat{\beta}_1 + \hat{\beta}_3$.

(Need to weight by how many of each G type we have.)

Inference for Experiments using Regression Adjustment

Recovering the ATE

$$Y_i = \beta_0 + \beta_1 T_i + \beta_2 (G_i - \bar{G}) + \beta_3 T_i (G_i - \bar{G}) + \epsilon_i$$

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$$\widehat{ATE}_{G1} \quad = \quad \hat{\beta}_1 + \hat{\beta}_3 (1 - \bar{G})$$

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What's $\widehat{ATE}$ for $G_i = 1$ group?

$$\begin{aligned} \widehat{ATE}_{G1} &= \hat{\beta}_1 + \hat{\beta}_3 (1 - \bar{G}) \\ &= \hat{\beta}_1 + \hat{\beta}_3 \times (\text{prop. of } G = 0) \end{aligned}$$

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$$Y_i = \beta_0 + \beta_1 T_i + \beta_2 (G_i - \bar{G}) + \beta_3 T_i (G_i - \bar{G}) + \epsilon_i$$

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What's $\widehat{ATE}$ for $G_i = 0$ group?

$$\widehat{ATE}_{G0} \quad = \quad \hat{\beta}_1 + \hat{\beta}_3 (0 - \bar{G})$$

Inference for Experiments using Regression Adjustment

Recovering the ATE

$$Y_i = \beta_0 + \beta_1 T_i + \beta_2 (G_i - \bar{G}) + \beta_3 T_i (G_i - \bar{G}) + \epsilon_i$$

What's $\widehat{ATE}$ for $G_i = 0$ group?

$$\begin{aligned} \widehat{ATE}_{G0} &= \hat{\beta}_1 + \hat{\beta}_3 (0 - \bar{G}) \\ &= \hat{\beta}_1 - \hat{\beta}_3 \bar{G} \end{aligned}$$

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\end{aligned}$$

Inference for Experiments using Regression Adjustment

So, to adjust for covariates in experimental data,

1. Estimate

$$Y_i = \beta_0 + \beta_1 T_i + \beta_2(x_i - \bar{x}) + \beta_3 T_i(x_i - \bar{x}) + \epsilon_i$$

2. Use HC2 standard errors (or analogous CR2 for clustered designs)

```r
library(estimatr)
lm_robust(y ~ t + I(x - mean(x)) + t * I(x - mean(x)),
          data = df)
```

Inference for Experiments using Regression Adjustment

So, to adjust for covariates x in experimental data,

```
lm_lin(y ~ t, covariates = ~ x, data = df)
```

Inference for Experiments using Regression Adjustment

With two levels of treatment,

$$Y_i = \beta_0 + \beta_1 T1_i + \beta_2 T2_i + \beta_3 (x_i - \bar{x}) + \beta_4 T1_i (x_i - \bar{x}) + \beta_5 T2_i (x_i - \bar{x}) + \epsilon_i$$

Inference for Experiments using Regression Adjustment

With two levels of treatment,

$$Y_i = \beta_0 + \beta_1 T1_i + \beta_2 T2_i + \beta_3 (x_i - \bar{x}) + \beta_4 T1_i (x_i - \bar{x}) + \beta_5 T2_i (x_i - \bar{x}) + \epsilon_i$$

etc.

“Controlling for Blocks”

Beware LSDV: "controlling for blocks"

A first-thought for estimating ATE for blocked design ...

Beware LSDV: "controlling for blocks"

A first-thought for estimating ATE for blocked design ...

Estimate linear model with indicators for each block:

$$Y_i = \beta_0 + \beta_1 T_i + \gamma_1 B_{i1} + \gamma_2 B_{i2} + \ldots + \epsilon_i$$

Beware LSDV: "controlling for blocks"

However, when

1. TE varies by block
2. $P(T = 1)$ varies by block

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- biased for the estimate (ATE)

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Using "block fixed effects" or "controlling for block IDs" is

- biased for the estimate (ATE)
- biased for the standard error ($SE(ATE)$)

Beware LSDV: "controlling for blocks"

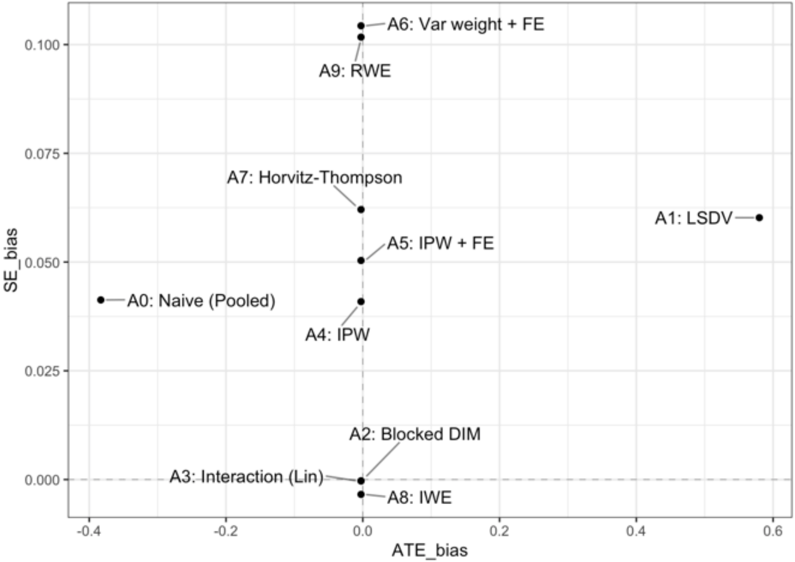
However, when

1. TE varies by block
2. $P(T = 1)$ varies by block

Using "block fixed effects" or "controlling for block IDs" is

- biased for the estimate (ATE)
- biased for the standard error ($SE(ATE)$)
- worse than most everything else!

Beware LSDV: "controlling for blocks"



Beware LSDV: "controlling for blocks"

The intuition: regression over-weights high-variance blocks

block	block_ATE	block_ATE_est	n_j	p_j	sample_weight	fe_weight
1	4	4.06	100	0.5	0.33	0.45
2	2	1.66	100	0.7	0.33	0.38
3	0	0.02	100	0.9	0.33	0.16

Beware LSDV: "controlling for blocks"

- Should I adjust for continuous x, or block IDs, where blocks come from coarsened x?

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Beware LSDV: "controlling for blocks"

- Should I adjust for continuous x, or block IDs, where blocks come from coarsened x?
- If both TE and Tr Prob vary across blocks, use **block IDs** to remove bias
  (since those determine assignment)
- In all 4 of the "probabilities vary/constant" by "treatment effects vary/constant" possibilities, using **both** the IDs and the variables themselves feels like overkill, but performs well.

(Related note: regression weights the cases)

FIGURE 1 Example of nominal and effective samples from Jensen (2003)

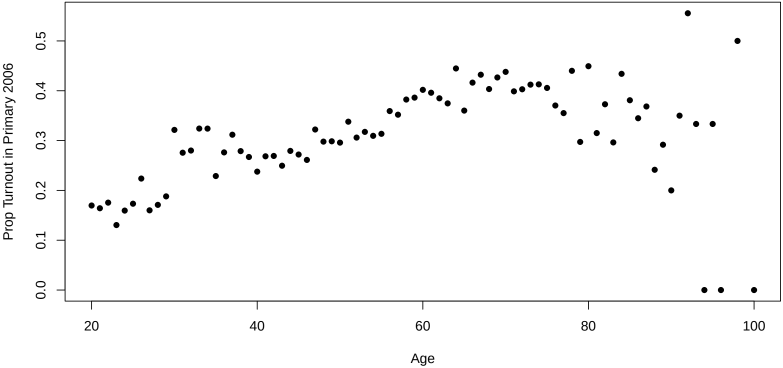
- Aronow and Samii (2016) shows how multiple regression weights its cases
- Interp: for causal inf., need causal methods, not just regression + representative sample

Nonlinear Terms

Terms to Capture Nonlinear Relationships

Let's take a step back from causal inference, and just think about trying to model the relationship between turnout probability and age.

```r
# Calculate turnout prop for every year of age:
prop_turnout <- tapply(social.neighbor$primary2006,
                       social.neighbor$age, mean)
# Plot:
plot(prop_turnout ~ names(prop_turnout), xlab = "Age",
     ylab = "Prop Turnout in Primary 2006", pch = 16)
```



This looks like a nonlinear relationship. Let's include an age-squared term to try to model this nonlinearity:

```
lm_out <- lm(primary2006 ~ age + I(age^2),
             data = social.neighbor)
lm_out
```

```
##
## Call:
## lm(formula = primary2006 ~ age + I(age^2), data = social
##
## Coefficients:
## (Intercept)          age     I(age^2)
##  -2.868e-02    9.818e-03   -5.572e-05
```

The estimated model is

$$\widehat{\text{Turnout 2006}}_i = -0.02868 + 0.00982\,(\text{Age}_i) + -6 \times 10^{-5}\left(\text{Age}_i^2\right)$$

1. What is the predicted turnout probability for a 30-year-old?
2. What is the predicted turnout probability for a 60-year-old?
3. What is the predicted turnout probability for a 80-year-old?
4. What is the predicted turnout probability for a 90-year-old?

```r
predict(lm_out, data.frame(age = 40))
```

```
##         1
## 0.2748905
```

```r
predict(lm_out, data.frame(age = 60))
```

```
##         1
## 0.3598197
```

```r
predict(lm_out, data.frame(age = 80))
```

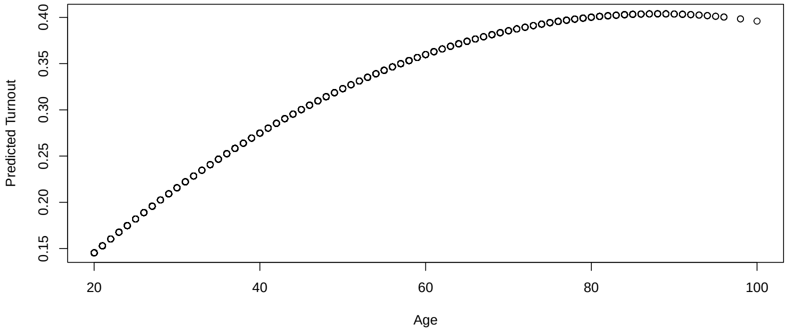
```
##         1
## 0.4001768
```

```r
predict(lm_out, data.frame(age = 100))
```

```
##         1
## 0.3959618
```

We can also look at all the predicted values:

```
plot(lm_out$fitted.values ~ social.neighbor$age,
     xlab = "Age", ylab = "Predicted Turnout")
```



How Causal Inference is Different

Data Science Approaches

Three tasks of data science:

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- Description

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Models/algorithms central to all three.

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Models/algorithms central to all three.

Hernán, Hsu, and Healy (2019)

Description

- Identifying patterns, etc.

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- E.g., clustering to discover groups

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- E.g., regression, random forests, neural networks, ...

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- Potential outcomes/counterfactual/interventionist perspective

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- E.g., experiments, observational causal designs, …

Causal Inference with Models

Consider two loaded datasets:

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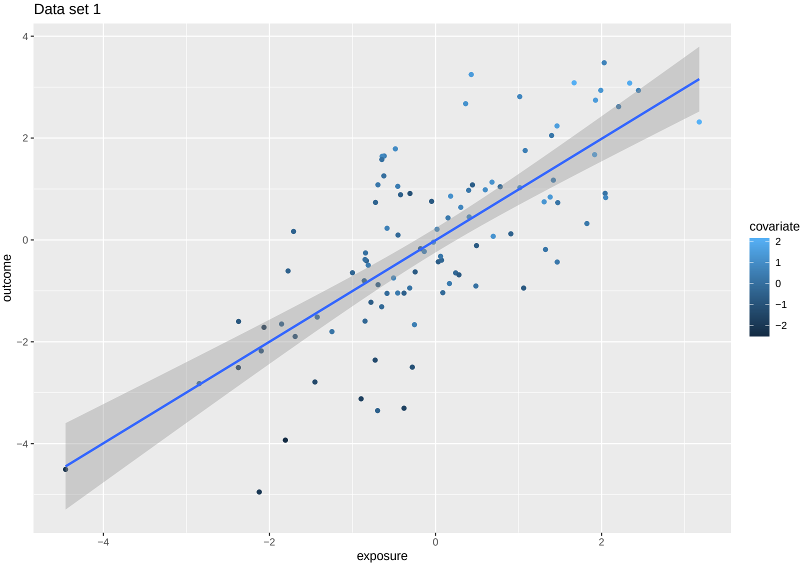
```r
str(df1)
```

```
## tibble [100 x 3] (S3: tbl_df/tbl/data.frame)
##  $ covariate: num [1:100] -0.622 1.137 -0.238 1.529 -0.1
##  $ exposure : num [1:100] 0.0332 0.3627 0.2422 1.4633 0.
##  $ outcome  : num [1:100] -0.429 2.675 -0.647 2.238 1.04
```

```r
str(df2)
```

```
## tibble [100 x 3] (S3: tbl_df/tbl/data.frame)
##  $ exposure : num [1:100] 0.4862 0.0653 -1.4021 -0.546 -
##  $ outcome  : num [1:100] 1.706 0.669 -1.597 -1.733 0.61
##  $ covariate: num [1:100] 2.24 0.924 -0.999 -2.343 0.207
```

Causal Inference with Models



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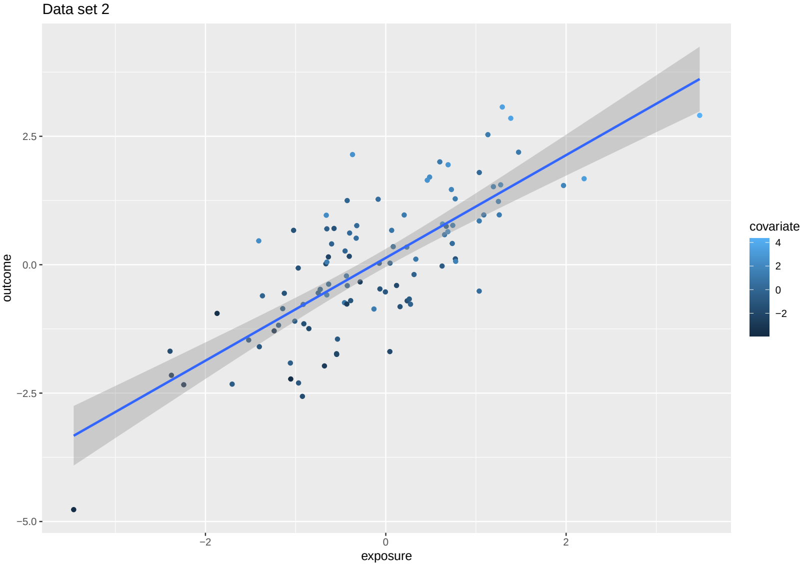

Causal Inference with Models

Model each

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lm_df1 <- lm(outcome ~ exposure, data = df1)
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```

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## # A tibble: 4 x 4
##   data  term        estimate std.error
##   <chr> <chr>          <dbl>     <dbl>
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## 2 df1   exposure     0.996      0.0927
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- Is this good? Is it correct?
- What if we adjust for covariate?

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```r
lm_df1_adj <- lm(outcome ~ exposure + covariate, data = df1
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```
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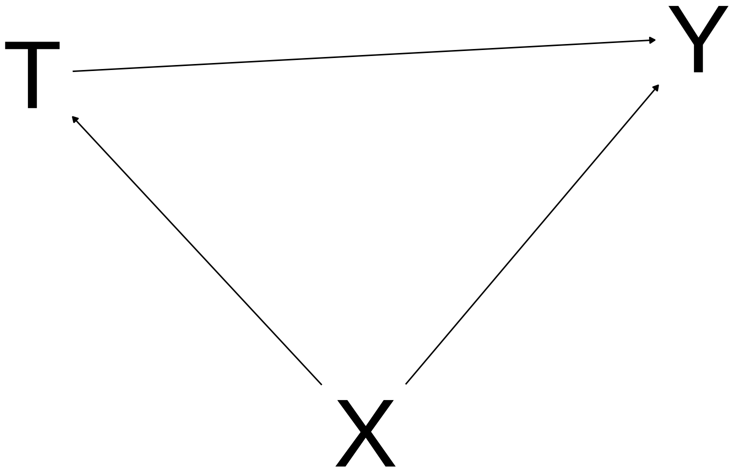

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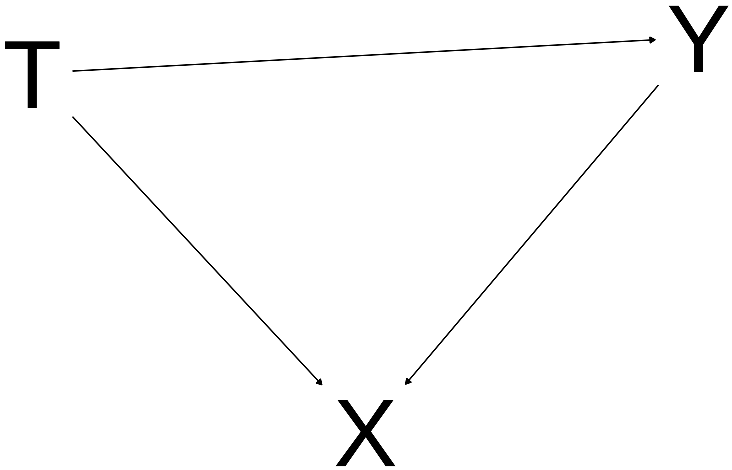

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g_coll <- dagitty("dag{ x -> y ; x -> c <- y }")
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```
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```

```
## { c }
```

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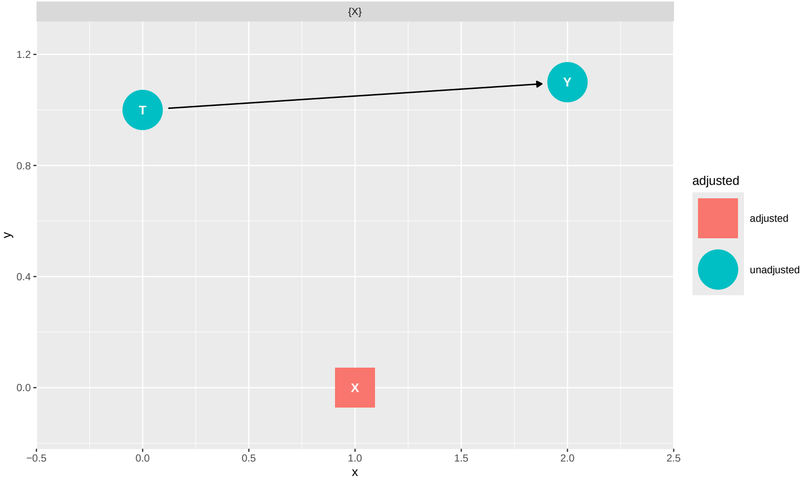

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So, correct adjustments to reveal causal effect of $T \rightsquigarrow Y$:

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`df1`, **adjust** for X, $\beta = 0.5$:

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## # A tibble: 1 x 4
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## 1 df1   exposure    0.501     0.108
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`df2`, **do not adjust** for X, $\beta = 1$:

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(Data from D'Agostino McGowan (2023))

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- Though, methods from prediction can aid causal inference (see, especially, *causal forests*)
- "Causal euphimisms" don't help (Hernán 2018)

References I

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