Interference

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2024-08-23

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Rubin (1978)

SUTVA 2: No interference between units.

Does A's assignment affect B's potential outcomes?

Interference

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- Does assg of child A to vaccine affect health of B?

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- Does assg of MTurk survey respondent A to conjoint profile X affect response of MTurker B?
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- Does assg of child A to vaccine affect health of B?
- Does assg of A to GOTV door knock affect turnout of neighbor?

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- Does assg of develpmnt £ to village A affect trust in B?

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- Does assg of MTurk survey respondent A to conjoint profile X affect response of MTurker B?
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- Does assg of child A to vaccine affect health of B?
- Does assg of A to GOTV door knock affect turnout of neighbor?
- Does assg of develpmnt £ to village A affect trust in B?
- Other common examples?

Examples of Interference in Field Experiments

Gerber and Green (2012)

- Contagion

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- Displacement

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- Displacement
- Communication
- Knowledge of others (disappointment/pride)
- Deterrence
- Persistence
- Memory

Bias under Interference

Agent	Y if MaryH	Y if PeterH	Y if LimorH	Y if NobodyH
Mary	100	50	70	70
Peter	50	50	50	50
Limor	90	50	90	90
Mean	80	50	70	70

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- $\overline{Y}_{\text{None}} = 70$.

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Agent	Y if MaryH	Y if PeterH	Y if LimorH	Y if NobodyH
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- $Y_i(0)$ *unstable*, since depends on who assigned Tr
- $\overline{Y}_{\text{None}} = 70$.
- Calc TEs relative to $\overline{Y}_{\text{None}}$ ("uniformity")

Bias under Interference: Uniformity Trial

Agent	Y if MaryH	Y if PeterH	Y if LimorH	Y if NobodyH
Mary	100	50	70	70
Peter	50	50	50	50
Limor	90	50	90	90
Mean	80	50	70	70

- True uniformity ATE: $\frac{(100-70)+(50-50)+(90-90)}{3} = 10$

Bias under Interference: Uniformity Trial

Agent	Y if MaryH	Y if PeterH	Y if LimorH	Y if NobodyH
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  - If Peter, $\frac{50+50+50}{3} - 70 = 50 - 70 = -20$

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- So, $E(\text{Diff in Means}) = \frac{10-20+0}{3} = -\frac{10}{3}$

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- So, $E(\text{Diff in Means}) = \frac{10-20+0}{3} = -\frac{10}{3}$
- So, $E(\text{Diff in Means}) \neq ATE$
- Diff in means is *biased* for uniformity ATE!

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- What about empirical difference in means, $\widehat{\text{Diff in Means}}$?

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$E(\widehat{ATE}) \neq ATE$, so, estimator *biased* under randomization.

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- True "Limor" ATE: $\frac{(100-70)+(50-50)}{2} = 15$

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  - $E(\text{Diff in Means}) = \frac{10-20}{2} = -5$

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With interference, diff-in-means under randomization biased.

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Notation

$$Y_i(z_i, \mathbf{z}) \quad \overset{?}{=} \quad Y_i(z_i, \mathbf{z}')$$

Notation

$$
\begin{aligned}
Y_i(z_i, \mathbf{z}) &\stackrel{?}{=} Y_i(z_i, \mathbf{z}') \\
Y_1\,(1, (1, 0, 0, 1, \ldots)) &\stackrel{?}{=} Y_1\,(1, (1, 1, 0, 0, \ldots)) \\
Y_0\,(0, (0, 0, 0, 1, \ldots)) &\stackrel{?}{=} Y_0\,(0, (0, 1, 0, 0, \ldots))
\end{aligned}
$$

Estimation in Presence of Interference

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- Let $T_i' =$ whether my friend is treated
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- Then, $Y_{T_i' T_i}$ is my potential outcome using both assignments

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$$\frac{\bar{Y}_{11} + \bar{Y}_{01}}{2} - \frac{\bar{Y}_{10} + \bar{Y}_{00}}{2} = \bar{Y}_{01} - \bar{Y}_{00}$$

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$$\frac{\bar{Y}_{11} + \bar{Y}_{01}}{2} - \frac{\bar{Y}_{10} + \bar{Y}_{00}}{2} = \bar{Y}_{01} - \bar{Y}_{00}$$

which is *unbiased* for ATE.

Estimation in Presence of Interference

- But, if

$$\underbrace{\bar{Y}_{11} - \bar{Y}_{01}}_{\text{spillover eff on Tr}} > \underbrace{\bar{Y}_{10} - \bar{Y}_{00}}_{\text{spillover eff on Co}}$$

then tend to overest $\bar{Y}_{01} - \bar{Y}_{00}$

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$$\underbrace{\bar{Y}_{11} - \bar{Y}_{01}}_{\text{spillover eff on Tr}} \quad > \quad \underbrace{\bar{Y}_{10} - \bar{Y}_{00}}_{\text{spillover eff on Co}}$$
then tend to overest $\bar{Y}_{01} - \bar{Y}_{00}$
- If
$$\underbrace{\bar{Y}_{11} - \bar{Y}_{01}}_{\text{spillover eff on Tr}} \quad < \quad \underbrace{\bar{Y}_{10} - \bar{Y}_{00}}_{\text{spillover eff on Co}}$$
then tend to underestimate $\bar{Y}_{01} - \bar{Y}_{00}$.

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  - Randomly assign treatment to some individuals in communities

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  - Given individuals within communities
  - Randomly assign whether community gets 20% or 80% treatment
  - Randomly assign treatment to some individuals in communities
- "Detecting Spillover Effects: Design and Analysis of Multilevel Experiments", Sinclair, McConnell, and Green (2012)

Design for Estimating Interference

OES project with IRS (US tax authority):

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- assign preparers to "some client letters", "no client letters"

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OES project with IRS (US tax authority):

- assign preparers to “some client letters”, “no client letters”
- assign clients in “some client letters” preparers to letters/no letters

Diagnosis of Potential Interference

1. Block:

```r
data(x100)
block.out <- block(data = x100, groups = "g",
                   id.vars = c("id"),
                   block.vars = c("b1", "b2"))
```

2. Assign:

```r
assg.out <- assignment(block.out, seed = 157)
```

Diagnosis of Potential Interference

Diagnose interference after assgnmnt (1D, Linday et al. (2001))

3. Diagnose:

```
diagnose(assg.out, data = x100, id.vars = "id",
    suspect.var = "b1", suspect.range = c(0, 5))
```

```
##
## Units differing by at least 0 and no
## more than 5 on b1:
##
## Group: a
##    Unit 1  Unit 2  Difference
## 1  1073    1098        0
## 2  1002    1036        1
## 3  1016    1060        1
## 4  1039    1076        3
##
## Group: b
```

Further Examination of Design

4. Get block IDs:

```
createBlockIDs(assg.out, data = x100, id.var = "id")
```

```
##   [1] 29 17 14  5 17 33 35 10 21 41 39 45 32 49 36 12 18
##  [26]  6 37 31  4 11 20 16 47 28 48 12 23 18  2 19 48 14
##  [51] 32 11 40 15 29  8 23  1  9 13  3 24 26 28  3 50  5
##  [76]  2 25 25 26 43 16 46 35  1 44 45 50 37  7 30 10 38
```

Further Examination of Design

5. Get balance:

```
assg2xBalance(assg.out, x100, id.var = "id",
    bal.vars = c("b1", "b2"))
```

```
## $Group1
##      strata():   unstrat
##      stat      Treatment Control adj.diff std.diff    z
## vars
## b1                 -23.7     0.0    -23.7    -0.08   -0.
## b2                 28.6     0.0     28.6     0.10    0.
## ---Overall Test---
##         chisquare df p.value
## unstrat     0.161  2   0.922
## ---
## Signif. codes:  0 '***' 0.001 '** ' 0.01 '*  ' 0.05 '.
##
## $Group2
##      strata():   unstrat
```

Avoiding Potential Interference due to Proximity

Are units too near each other?

Avoiding Potential Interference due to Proximity

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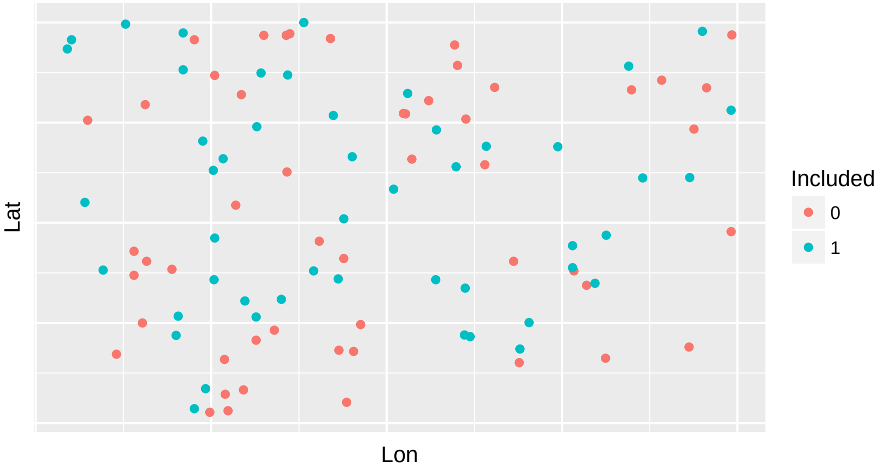
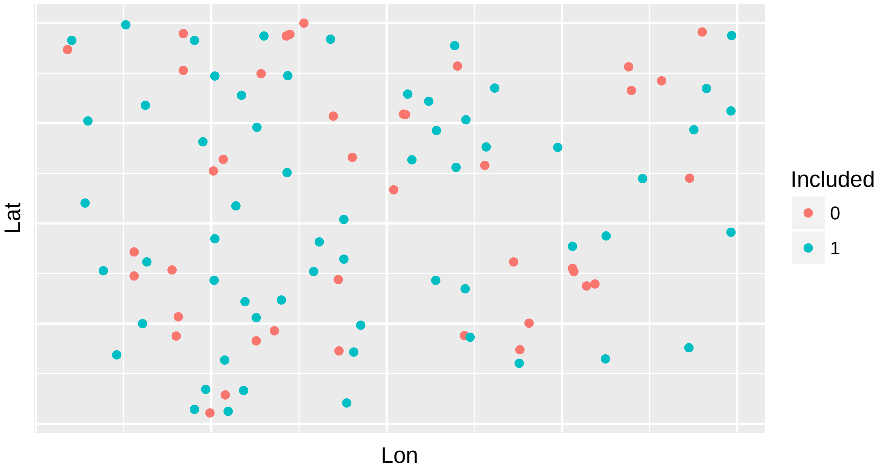


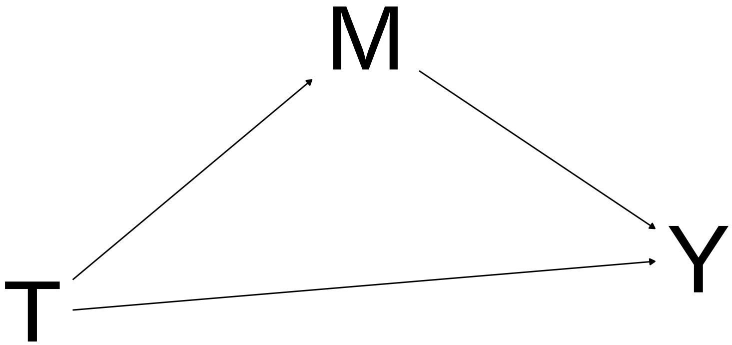
Avoiding Potential Interference due to Proximity

3000 iterations, max min distance:



Estimating Direct and Indirect Effects

- Different to mediation *direct*, *indirect* effects



Estimating Direct and Indirect Effects

Hudgens and Halloran (2008) (on Ali et al. (2005))

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Think of this as different problem

- (Though, "effect of treatment through others" works?)

Estimating Direct and Indirect Effects

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- Here,
  - *direct*: from treating i
  - *indirect*: from treating $j \neq i$

Think of this as different problem

- (Though, “effect of treatment through others” works?)
- (Common concept, but not really a mediating *variable*)

Estimating Direct and Indirect Effects

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Table 1. Risk of cholera in recipients of killed oral cholera vaccines or placebo, by level of coverage of the bari during one year of follow-up, based on data from Ali et al. (2005)

Level of vaccine coverage	Target population	Vaccine recipients: Total	Vaccine recipients: Cases	Vaccine recipients: Risk per 1,000 population	Placebo recipients: Total	Placebo recipients: Cases	Placebo recipients: Risk per 1,000 population
>50%	22,394	12,541	16	1.27	6,082	9	1.47
41–50%	24,159	11,513	26	2.26	5,801	27	4.65
36–40%	24,583	10,772	17	1.58	5,503	26	4.72
28–35%	25,059	8,883	22	2.48	4,429	26	5.87
<28%	24,954	5,627	15	2.66	2,852	20	7.01

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- Total effect: $7.01 - 1.27 = 5.74$
- Overall effect: $35/8479 - 25/18623 = 2.79/1000$

Estimating Direct and Indirect Effects

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Designing the randomized experiment:

Table 2. Illustrative example of a two-stage randomized placebo-controlled vaccine trial based on data from Ali et al. (2005)

		Vaccine recipients ($Z_{ij} = 1$)		Placebo recipients ($Z_{ij} = 0$)	
Group i	Group assignment S_i	Total $\sum_j Z_{ij}$	Cases $\sum_j Z_{ij} Y_{ij}(\mathbf{Z}_i)$	Total $\sum_j (1 - Z_{ij})$	Cases $\sum_j (1 - Z_{ij}) Y_{ij}(\mathbf{Z}_i)$
1	1	12,541	16	12,541	18
2	1	11,513	26	11,513	54
3	0	10,772	17	25,134	119
4	0	8,883	22	20,727	122
5	0	5,627	15	13,130	92

NOTE: Group assignment $S_i = 1$ (0) corresponds to 50% (30%) vaccine coverage.

Hudgens and Halloran (2008) (on Ali 2005)

Table 3. Estimates of population average direct, indirect, total, and overall effects per 1,000 individuals per year for data in Table 2

Effect	Parameter	Estimate	Estimated variance
Direct	$\overline{CE}^D(\psi)$	1.30	.856
Direct	$\overline{CE}^D(\phi)$	3.64	.178
Indirect	$\overline{CE}^I(\phi, \psi)$	2.81	3.079
Total	$\overline{CE}^T(\phi, \psi)$	4.11	.672
Overall	$\overline{CE}^O(\phi, \psi)$	2.37	1.430

- ψ: 50% coverage
- ϕ: 30% coverage

Application: Displacing Election Irregularities

Ichino and Schündeln (2012) in Ghana

Design:

- Blocks of 3 constituencies, select 1 for Tr, 2 for Co

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- Where election observers are sent, smaller registration irregularities
- In nearby control areas, *larger* irregularities

Sobel (2006)

If interference, diff-in-means estimator (or regression coef)

- is **not** unbiased for ATE
- is difference:

(ITT for Tr group) – (indirect/spillover effect on Co group)

Rosenbaum (2007)

- Randomization inference gives valid coverage, even if interference

Null Hypotheses:

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- if exposing block gets everyone sick, then no primary effect of i getting directly exposed

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Effect, but no Primary Effect:

- if exposing block gets everyone sick, then no primary effect of i getting directly exposed
- if news raises anxiety in HH, irrelevant if I saw news

Next: Causal Forests

References I

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