Randomization (Design-based) Inference for Experiments

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An Exercise

A volunteer?

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The task: select the 2 folders with messages

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Select!

Randomization Inference

The possible choices:

■■□□□	□■□■□
■□■□□	□■□□■
■□□■□	□□■■□
■□□□■	□□■□■
□■■□□	□□□■■

Randomization Inference

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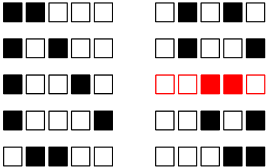

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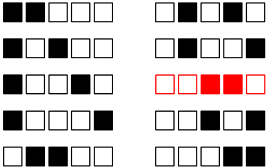

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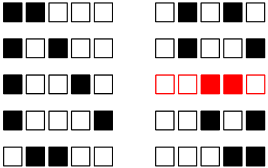

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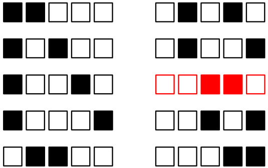

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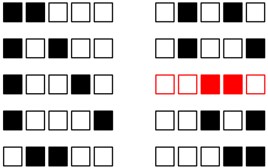

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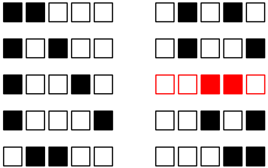

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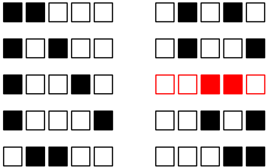

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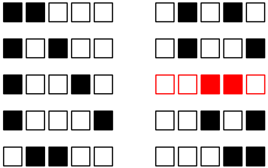

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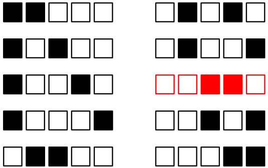

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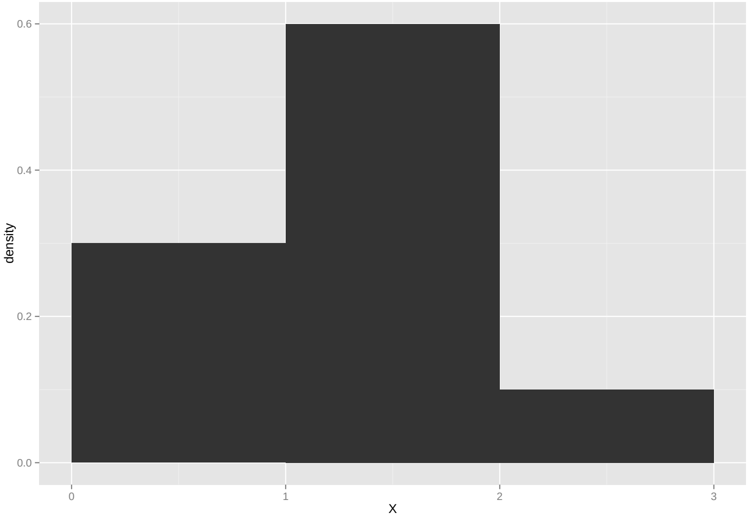
The Randomization Distribution of X



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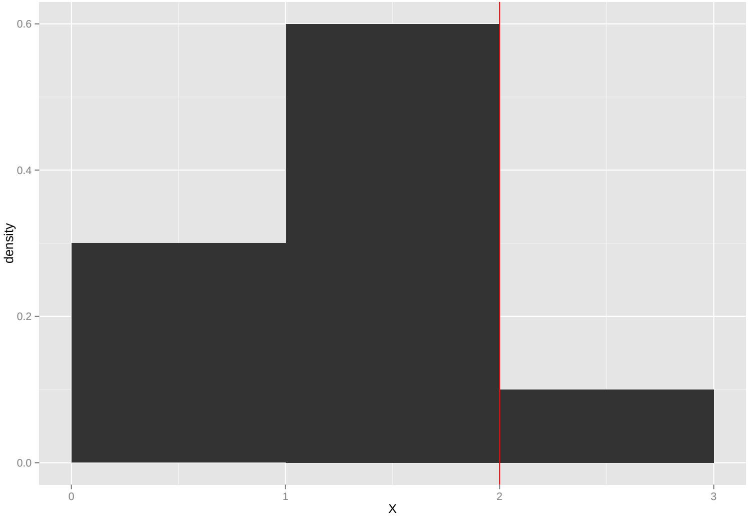

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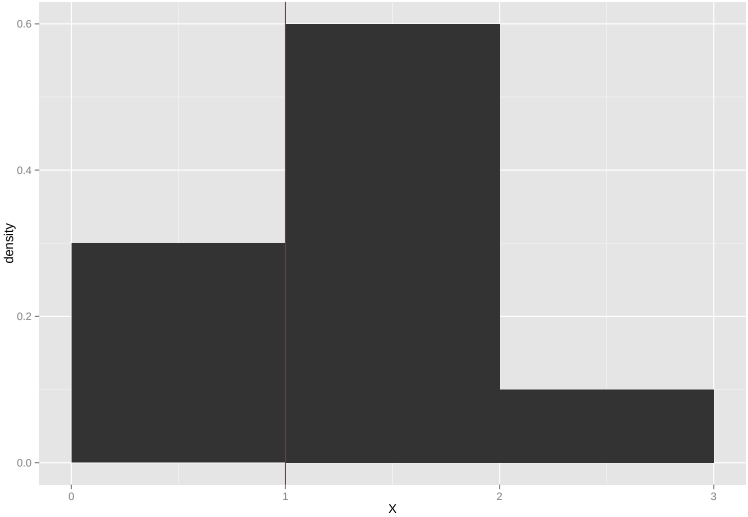

Parametric Null Hypothesis Significance Testing

- Specify and assume H_0
- Define H_A
- Examine reference dist'n (t, χ^2, ...) under H_0
- Calculate p-value
- Compare to some α; reject H_0 if $p < \alpha$

Overview of Randomization Inference

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- CA ballot ordering effects (JASA 2006)

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How many randomizations are there?

Counting Principles

Combinations: Counting selected sets

How many ways to **select** k things from a set of n things?

$$ {}_nC_k = \binom{n}{k} = \frac{{}_nP_k}{k!} = \frac{n!}{k!(n-k)!} $$

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- If only want 3 of 5? Divide by $2! = (n-k)!$ (removing permutations from last 2 slots)
- What if order doesn't matter? Divide by $3! = k!$ (6 permutations for ABC, but only one combination)

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$$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 252$$

Common Assumptions, Null Hypotheses

- Constant effect:

$$\tau_i = Y_{i1} - Y_{i0} = \tau \quad \forall i$$

- Null hypothesis of no average effect:

$$ATE = \overline{\tau} = 0$$

- Sharp null hypothesis of no effect:

$$\tau_i = 0$$

Examples

An Assignment Mechanism: Perfect Doctor

Calculate RI p-value for Perfect Doctor, under sharp null.

Patient	Y(0)	Y(1)	τ	T
1	(1)	6	(5)	1
2	(3)	12	(9)	1
3	9	(8)	(-1)	0
4	11	(10)	(-1)	0
Mean	10	9	(3)	

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(See `01-ri-perfect-dr.R`)

RI versus the t-test

Perfect Doctor:

- RI: $p = 1$
- `t.test()`: $p \approx 0.8$
- "If no tr effect, then this result typical"

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(Odd logic of NHST: "assume false thing, how strange is data?")

Randomization Inference, Example 2

Assumed table of potential outcomes:

Village	T	% if Female Head, $Y(1)$	% if Male Head, $Y(0)$	τ_i
1	_	15	10	5
2	_	15	15	0
3	_	30	20	10
4	_	15	20	-5
5	_	20	10	10
6	_	15	15	0
7	_	30	15	15
Average		20	15	5

Randomization Inference, Example 2

Suppose we randomly select 2 villages to have female-headed councils, and observe

Village	T	% if Female Head, $Y(1)$	% if Male Head, $Y(0)$	τ_i
1	F	15		
2	M		15	
3	M		20	
4	M		20	
5	M		10	
6	M		15	
7	F	30		
Average		22.5	16	6.5

Randomization Inference, Example 2

We assume the *sharp null* hypothesis (assumed values in red):

Village	T	% if Female Head, $Y(1)$	% if Male Head, $Y(0)$	τ_i
1	F	15	15	0
2	M	15	15	0
3	M	20	20	0
4	M	20	20	0
5	M	10	10	0
6	M	15	15	0
7	F	30	30	0
Average				0

Randomization Inference, Example 2

Then we estimate what the observed ATE would be for all the possible random assignments.

First,

Village	T	% if Female Head, $Y(1)$	% if Male Head, $Y(0)$	τ_i
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2	M		15	
3	M		20	
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5	M		10	
6	F	15		
7	M		30	
Average		15	19	-4

Randomization Inference, Example 2

Second,

Village	T	% if Female Head, $Y(1)$	% if Male Head, $Y(0)$	τ_i
1	F	15		
2	M		15	
3	M		20	
4	M		20	
5	F	10		
6	M		15	
7	M		30	
Average		12.5	20	-7.5

Randomization Inference, Example 2

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..., and all the others. The full set of $\frac{7!}{2!5!} = 21$ differences in means:

	Estimate	Frequency
	-7.5	3
	-4	5
	-0.5	6
	3	2
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Total		21

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One-sided ("women's % $>$ men's"): $p = \frac{5}{21} \approx 0.24$

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  - Assigning 500 of 1000 respondents to treatment? Randomly generate 10,000 assignments from 2.7×10^{299} possible ...
- can be computationally intensive

Randomization Inference

- Resume audit study, Bertrand and Mullainathan (2004)

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```
# A tibble: 2 x 2
  race  call_rate
  <chr>     <dbl>
1 black    0.0645
2 white    0.0965
```

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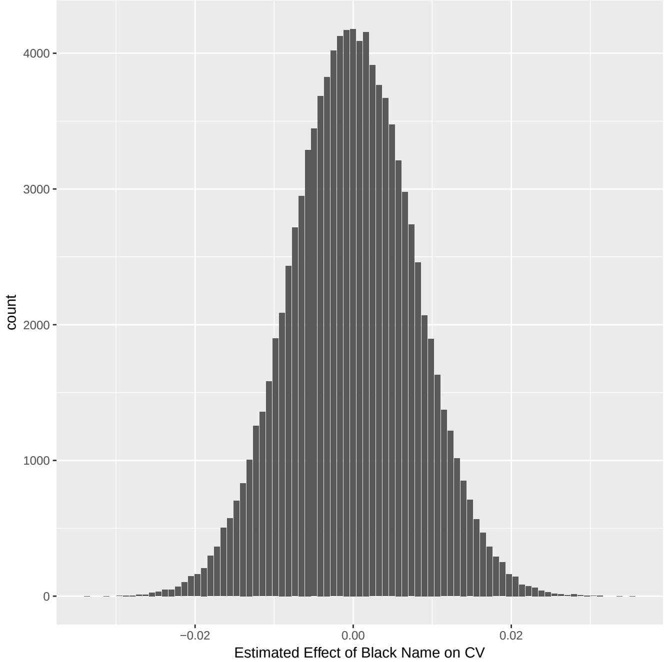
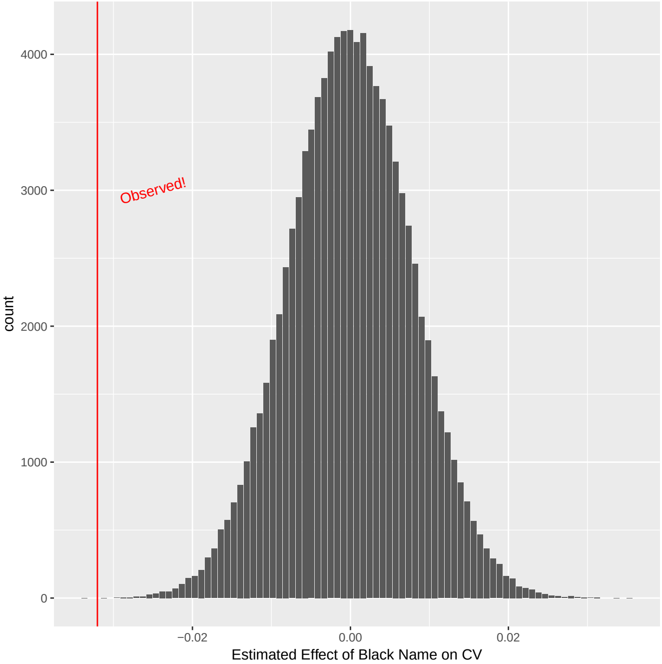
- See `01-ri-resume-donate.R`


4000
3000
2000
1000
0
count
−0.02
0.00
0.02
Estimated Effect of Black Name on CV



Observed!
count
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3000
2000
1000
0
−0.02
0.00
0.02
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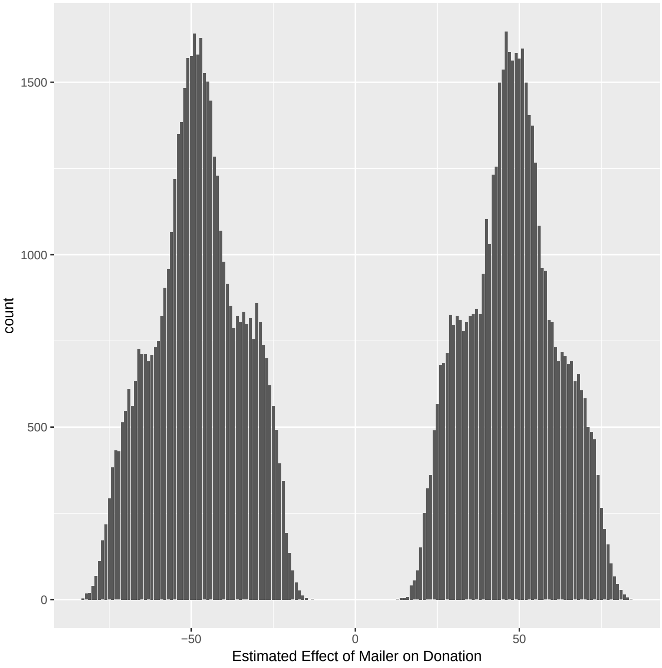
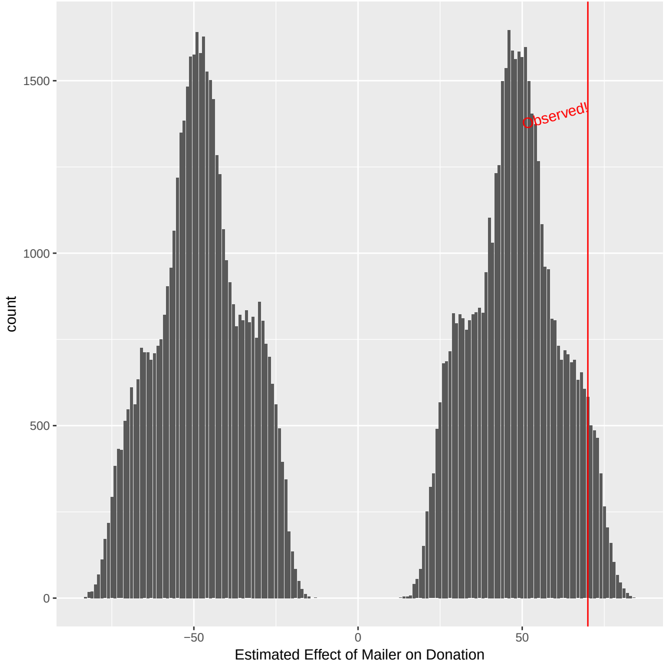
Randomization Inference

- Gerber and Green (2012) donations example, p. 65
- Possible values $\tau_i \in (-\infty, \infty)$
- Y_1, Y_0, τ likely very skewed
- See `01-ri-resume-donate.R`


1500
1000
500
0
count
−50
0
50
Estimated Effect of Mailer on Donation


Observed!
1500
1000
500
0
count
−50
0
50
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No need to derive the correct asymptotic standard error (SE).

“There is only one test.”

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– Allen Downey, posit::conf(2024)

Next:
Covariates in Experiments

Bertrand, Marianne, and Sendhil Mullainathan. 2004. "Are Emily and Greg More Employable Than Lakisha and Jamal? A Field Experiment on Labor Market Discrimination." *American Economic Review* 94 (4): 991–1013.

Gerber, Alan S., and Donald P. Green. 2012. *Field Experiments: Design, Analysis, and Interpretation.* New York, NY: WW Norton.