Data Science Methods in Causal Inference

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2024-08-23

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Data Science in Causal Inference

Approaches of Prediction and Causal Inference

Two Cultures, (Breiman 2001)

- *Data Models*: our "social science modeling"

Approaches of Prediction and Causal Inference

Two Cultures, (Breiman 2001)

- *Data Models*: our "social science modeling"
- *Algorithmic Models*: our "data science algorithms"

Methods for Prediction and Causal Inference

- Cross-validation
- Regression/Decision trees
- Random forests

James et al. (2021)

Cross-validation

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k-fold cross-validation to select method

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$$CV_{(k)} = \frac{1}{k}\sum_{i=1}^{k}\text{MSE}_i$$

- Select model that minimises $CV_{(k)}$

CV for Linear Model

```r
## Make data

mk_data <- function(n = 90, n_folds = 10){

  df <- tibble(
    x1 = rnorm(n),
    x2 = rnorm(n),
    x3 = rnorm(n),
    y = 0.1 * x1 + 0.2 * x2 + 0.5 * x3 + rnorm(n),
    cv_fold = sample(rep(1:n_folds, (n / n_folds)))
  )

}

df <- mk_data()
```

CV for Linear Model

```
head(df)
```

```
# A tibble: 6 x 5
       x1      x2      x3       y cv_fold
    <dbl>   <dbl>   <dbl>   <dbl>   <int>
1  1.34   -0.617   0.0877 -0.358       10
2 -0.333  -0.0812  0.817  -0.112        6
3 -0.267   0.262  -0.402   0.875        9
4 -1.15   -0.657   0.182   0.168        4
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```

```
table(df$cv_fold)
```

```

 1  2  3  4  5  6  7  8  9 10
 9  9  9  9  9  9  9  9  9  9
```

CV for Linear Model

```
cv_lm <- function(data, fmla){

  n_folds <- max(data$cv_fold)
  store_mses <- vector("numeric", length = n_folds)

  for(idx in 1:n_folds){

    df_train <- data |> filter(cv_fold != idx)
    df_test <- data |> filter(cv_fold == idx)

    lm_out <- lm(fmla, data = df_train)

    predictions <- predict(lm_out, newdata = df_test)

    store_mses[idx] <- mean((df_test$y - predictions)^2)}

  test_error_cv_k <- mean(store_mses)
  return(test_error_cv_k)
```

CV for Linear Model

```
cv_lm(data = df, fmla = y ~ x1 + x2)
```

```
[1] 1.274226
```

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```
df <- mk_data()
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```

```
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```

CV for Linear Model

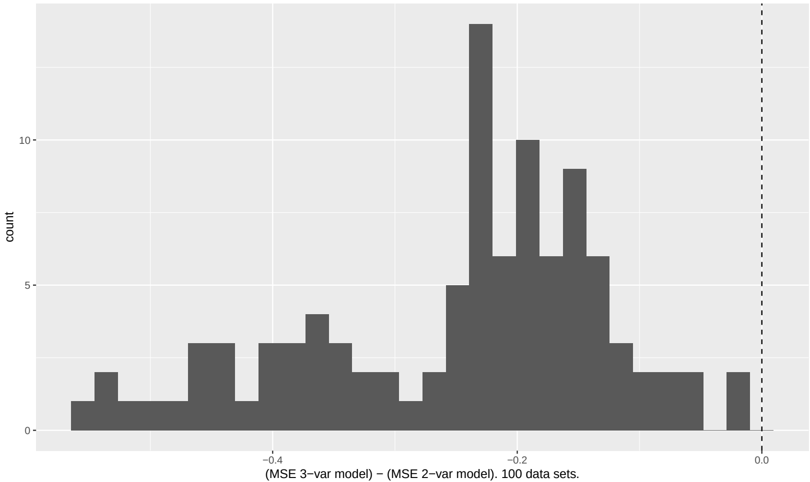

Figure 1: MSE always less (better) for 3-variable model.

Regression Trees

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$$\sum_{j=1}^{J} \sum_{i \in R_j} \left(y_i - \hat{y}_{R_j} \right)$$

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$$\sum_{i:x\in R_1(j,s)} \left(y_i - \hat{y}_{R_1(j,s)}\right)^2 + \sum_{i:x\in R_2(j,s)} \left(y_i - \hat{y}_{R_2(j,s)}\right)^2$$

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$$\sum_{m=1}^{|T|} \sum_{i: x_i \in R_m} \left(y_i - \hat{y}_{R_m}\right)^2 + \alpha |T|$$

Sum squared pred. error (plus penalty that grows with tree size) across units in region, then regions.

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4. Using that α, select best subtree from Step 2

Example: Regression Tree

Effect of office-holding on wealth
(Eggers and Hainmueller 2009):

```
library(qss)
library(rsample)
library(tree)

data("MPs")
mps <- MPs |> mutate(age = yod - yob,
                     is_labour = if_else(party == "labour",
                     is_london = if_else(region == "Greate
                     is_winner = if_else(margin > 0, 1, 0))
  select(ln.net, age, is_labour, is_london, is_winner) |>
  na.omit()
```

Example: Regression Tree

```
set.seed(765076184)

mp_split <- initial_split(mps, prop = 0.7)

mp_train <- training(mp_split)
mp_test <- testing(mp_split)
```

Example: Regression Tree

```
tree_mp <- tree(ln.net ~ ., data = mp_train)
plot(tree_mp)
text(tree_mp)
```

Figure 2: The regression tree (for training data)

Example: Regression Tree

Would pruning help?

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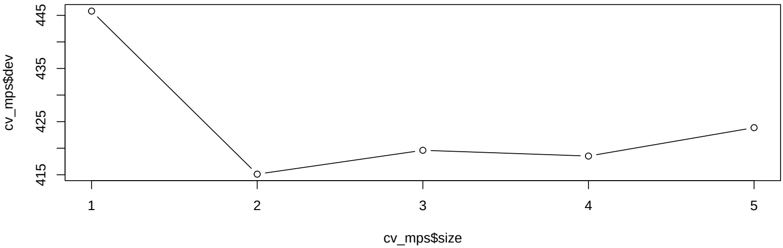
```
cv_mps <- cv.tree(tree_mp, K = 10)

plot(cv_mps$size, cv_mps$dev, type = "b")
```



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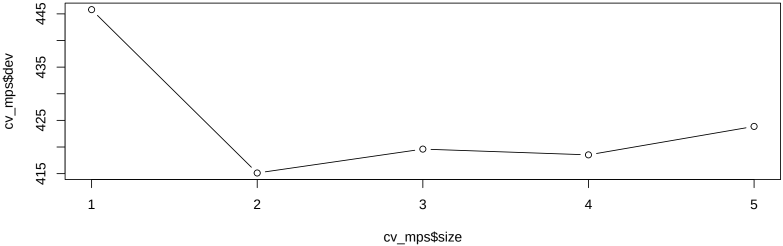

Figure 3: Subtree size 2 minimises SSR

Example: Regression Tree

```r
prune_mps <- prune.tree(tree_mp, best = 2)

plot(prune_mps)
text(prune_mps)
```

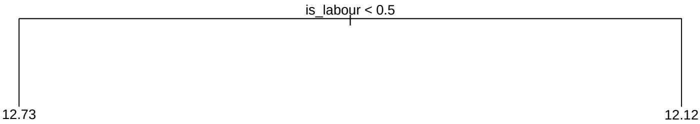

Figure 4: The pruned tree

Example: Regression Tree

Predict for test set:

- MSE for pruned: 1.922
- MSE for full: 1.945

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(Pretty good for 1 split!?)

Random Forests

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Next: random forest algorithm

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Bagging: *b*ootstrap *agg*regation

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- (Linear regression: lower variance)

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- (Often choose $m \approx \sqrt{p}$)
- So, different splits consider different predictors
- So, trees will look very different to each other

Example: Random Forests

```r
library(randomForest)

# Full bag:
bag_mps <- randomForest(ln.net ~ ., data = mp_train,
                        ntree = 500, mtry = 4,
                        importance = TRUE)

# Decorrelate:
rf_mps <- randomForest(ln.net ~ ., data = mp_train,
                       ntree = 500, mtry = 2,
                       importance = TRUE)
```

Example: Random Forests

Predict:

```
preds_bag <- predict(bag_mps, newdata = mp_test)

preds_rf <- predict(rf_mps, newdata = mp_test)
```

- MSE for RF: 1.995
- MSE for full bag: 2.536

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Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \epsilon$$

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \epsilon$$

```r
lm_out <- lm(ln.net ~ is_winner, data = mps)
lm_out
```

```

Call:
lm(formula = ln.net ~ is_winner, data = mps)

Coefficients:
(Intercept)    is_winner
    12.2464       0.5176
```

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

```r
t.test(ln.net ~ is_winner, data = mps)
```

```

	Welch Two Sample t-test

data:  ln.net by is_winner
t = -3.9552, df = 287.65, p-value = 9.636e-05
alternative hypothesis: true difference in means betwee
95 percent confidence interval:
 -0.7751044 -0.2599998
sample estimates:
mean in group 0 mean in group 1
       12.24641        12.76396
```

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Homogeneous effects:

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```r
lm_out <- lm(ln.net ~ is_winner + is_labour +
               is_london + age, data = mps)
lm_out
```

```

Call:
lm(formula = ln.net ~ is_winner + is_labour + is_london + a
    data = mps)

Coefficients:
(Intercept)    is_winner    is_labour    is_london
  12.078838     0.398818    -0.477549     0.161134      0.00
```

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

```
lm_lin(ln.net ~ is_winner, covariates = ~ is_labour + is_lo
```

```
                          Estimate  Std. Error     t valu
(Intercept)            1.226687e+01 0.078894901 155.483661
is_winner              3.459885e-01 0.131207672   2.636953
is_labour_c           -1.613663e-01 0.152608515  -1.057387
is_london_c            2.427360e-01 0.250214401   0.970111
age_c                  4.740367e-03 0.007031323   0.674178
is_winner:is_labour_c -9.104022e-01 0.264395760  -3.443331
is_winner:is_london_c -8.847770e-02 0.426241818  -0.207576
is_winner:age_c       -4.778657e-05 0.012753800  -0.003746
                         CI Lower    CI Upper  DF
(Intercept)           12.111785723 12.42195044 416
is_winner              0.088075873  0.60390123 416
is_labour_c           -0.461346226  0.13861367 416
is_london_c           -0.249106208  0.73457813 416
```

CATEs: Conditional ATEs

- *Conditional average treatment effect* (CATE): avg treatment effect for subset of population

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- *Conditional average treatment effect* (CATE): avg treatment effect for subset of population
- Sometimes "CACE"
- Inference: not "evidence against TE $= 0$?", but "evidence against $\text{CATE}_1 = \text{CATE}_2$?"

Homogeneous and Heterogeneous Effects: Estimation

Heterogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

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$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

- β_1 gives TE for `Group == 0`

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- β_1 gives TE for `Group == 0`
- $\beta_1 + \beta_3$ gives TE for `Group == 1`

Homogeneous and Heterogeneous Effects: Estimation

Heterogeneous effects:

```r
lm_out <- lm(ln.net ~ is_winner * is_labour +
               is_london + age, data = mps)
coef(lm_out) |> round(3)
```

```
      (Intercept)           is_winner          is_labou
           11.959               0.780            -0.162
              age is_winner:is_labour
            0.005              -0.914
```

Causal Forests

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- Use $\hat{y}_{R_j}$ as pred value for obs in R_j
- (Tory, winner, London $\rightarrow$ 13.84)
- $$\hat{y}_{R_j} = \frac{1}{|R_j|} \sum_{i \in R_j} Y_i$$

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$$Y(0), Y(1) \perp\!\!\!\perp T | \mathbf{X}$$

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$$\hat{\hat{\tau}}_{R_j} = \frac{1}{|\{T, R_j\}|} \sum_{\{T,R_j\}} Y_i - \frac{1}{|\{C, R_j\}|} \sum_{\{C,R_j\}} Y_i$$

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☺

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  - Prediction, estimation of $\hat{\hat{\tau}}$ uses only $\mathcal{I}$
- Build a random forest (decorrelated deep trees picking from m predictors) of causal trees

Example: Causal Forests

```r
library(grf)

X <- mps |> select(age, is_labour, is_london)

W <- mps |> select(is_winner) |>
  unlist() |> as.numeric()

Y <- mps |> select(ln.net) |> unlist()

cf_out <- causal_forest(X, Y, W)
```

Example: Causal Forests

```
cf_out
```

```
GRF forest object of type causal_forest
Number of trees: 2000
Number of training samples: 424
Variable importance:
    1     2     3
0.537 0.393 0.070
```

Example: Causal Forests

```
cf_out
```

```
GRF forest object of type causal_forest
Number of trees: 2000
Number of training samples: 424
Variable importance:
    1     2     3
0.537 0.393 0.070
```

(“How frequently was i the split feature?”)

Example: Causal Forests

```
cf_pred_est_var <- predict(cf_out, X,
                           estimate.variance = TRUE)
```

Example: Causal Forests

```
cf_pred_est_var <- predict(cf_out, X,
                           estimate.variance = TRUE)
```

```
cf_preds <- cf_pred_est_var$predictions

df_cf <- tibble(X,
                cf_te = cf_preds,
                cf_se = sqrt(cf_pred_est_var$variance.e
                te_1se_lower = cf_te - cf_se,
                te_1se_upper = cf_te + cf_se)
```

Example: Causal Forests

Avg pred treatment effect in honest sample:

```
mean(cf_preds)
```

```
[1] 0.3805919
```

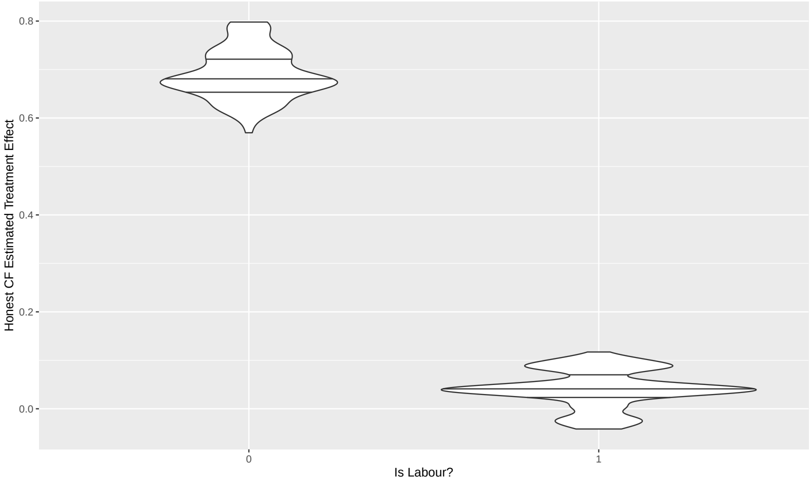
Example: Causal Forests

A doubly-robust ATE from honest sample:

```
average_treatment_effect(cf_out)
```

```
 estimate   std.err
0.3673061 0.1376409
```

Example: Causal Forests Results, Party



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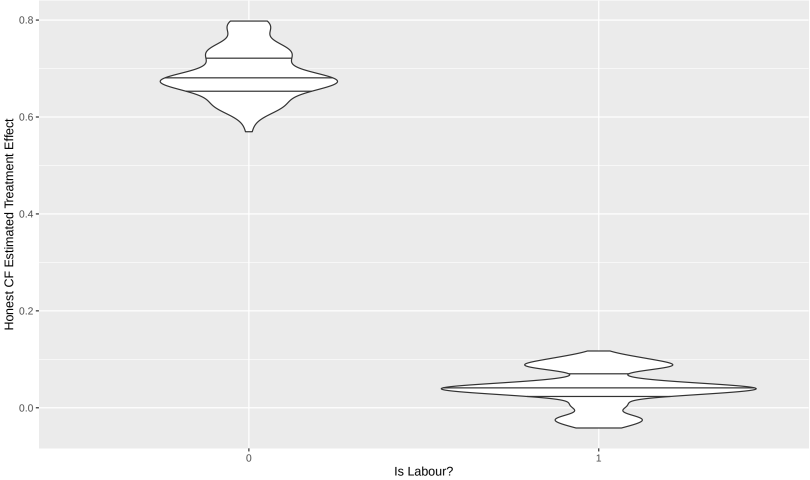

- Mean CF TE, Tory: 0.69

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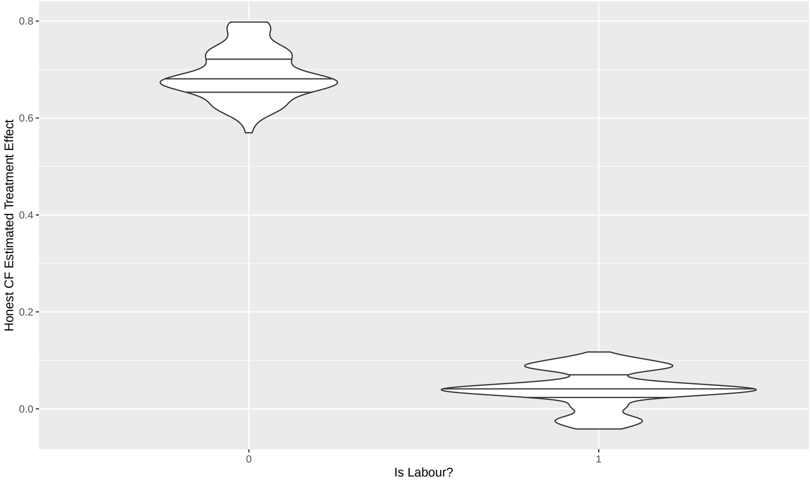

- Mean CF TE, Tory: $0.69 \rightsquigarrow £242,000$

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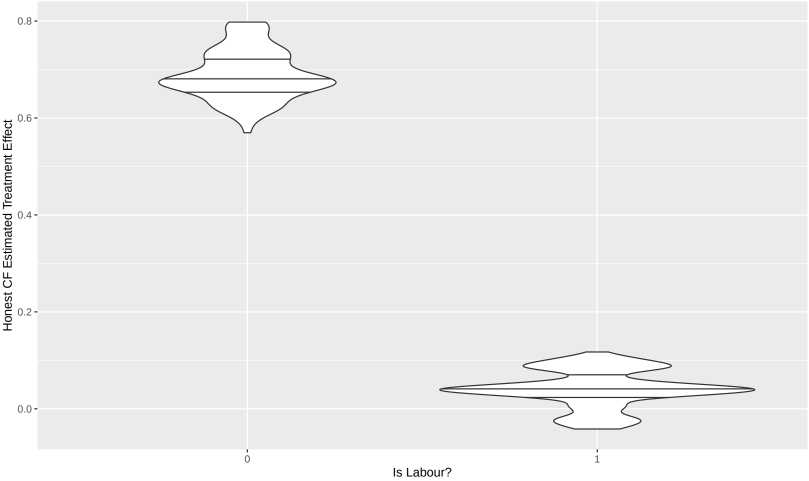
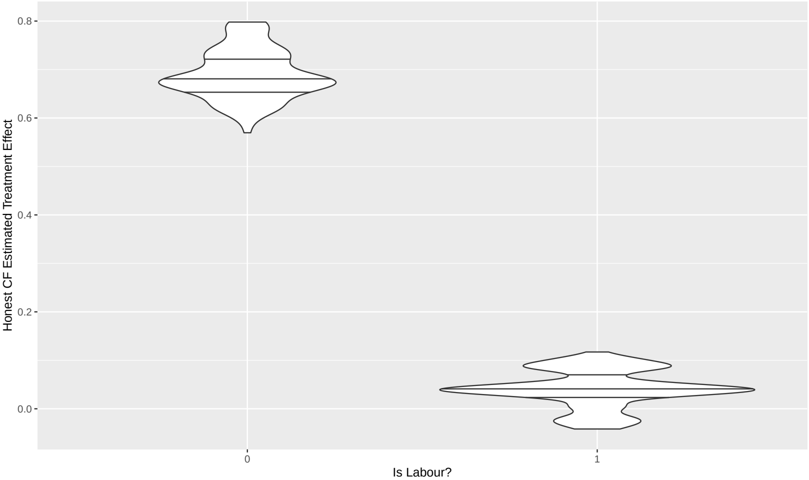


- Mean CF TE, Tory: 0.69 $\rightsquigarrow$ £242, 000
- Mean CF TE, Labour: 0.041

Example: Causal Forests Results, Party



- Mean CF TE, Tory: 0.69 $\rightsquigarrow$ £242,000
- Mean CF TE, Labour: 0.041 $\rightsquigarrow$ £10,000

(mix of medians/means here ...)

Example: Causal Forests Results, Party

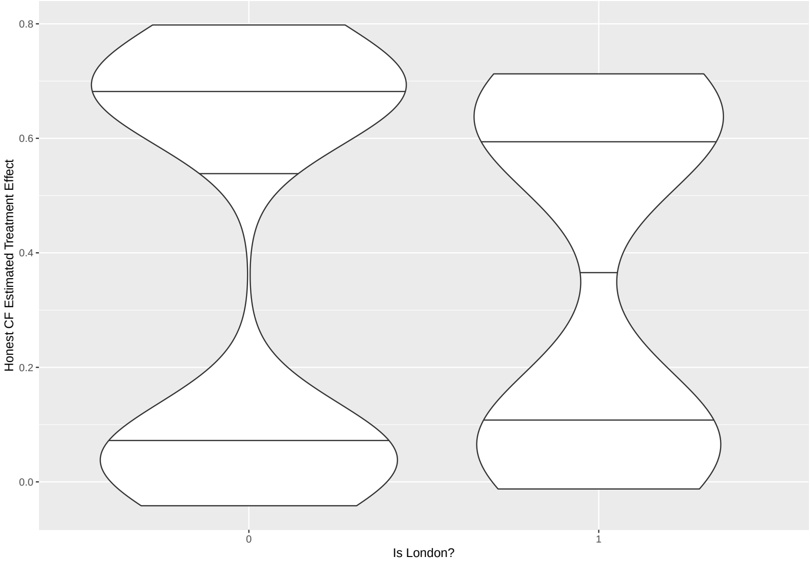
```r
average_treatment_effect(
  cf_out,
  subset = X$is_labour == 0)
```

```
 estimate   std.err
0.8530553 0.1983866
```

```r
average_treatment_effect(
  cf_out,
  subset = X$is_labour == 1)
```

```
  estimate    std.err
-0.1665371  0.1828122
```

Example: Causal Forests Results, London



Example: Causal Forests Results, London

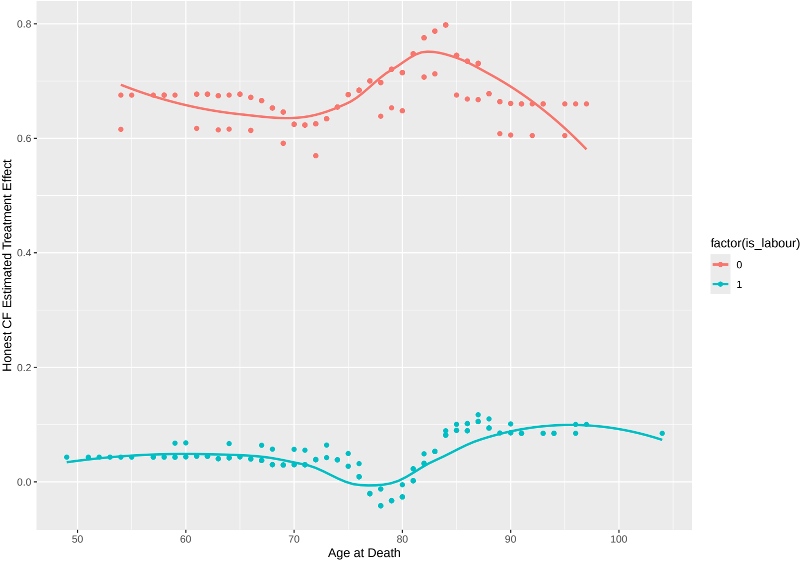
```
average_treatment_effect(
  cf_out,
  subset = X[, "is_london"] == 1)
```

```
 estimate   std.err
0.2454707 0.3964525
```

```
average_treatment_effect(
  cf_out,
  subset = X[, "is_london"] == 0)
```

```
 estimate   std.err
0.3847111 0.1469377
```

Example: Causal Forests Results, Age



Some Next Ideas ...

- Feature Selection
- Regularization/Shrinkage
  (LASSO, ridge, elastic net)
- Double LASSO for treatment effects
  (models for treatment and outcome)

Conclusions

The Big Picture

	Treatment Assignment	
Unit Selection	Randomized	Not Randomized
Randomized	Randomized Experiment (gold standard)	Survey Sampling (allows population inference)
Not Randomized	Controlled Experiment (allows causal inference)	Observational Study (large potential for bias)

Final Thought on Importance of Comparison Groups (Tufte 1974)

Final Thought on Importance of Comparison Groups (Tufte 1974)

One day when I was a junior medical student, a very important Boston surgeon visited the school and delivered a great treatise on a large number of patients who had undergone successful operations for vascular reconstruction. At the end of the lecture, a young student at the back of the room timidly asked, "Do you have any controls?" Well, the great surgeon drew himself up to his full height, hit the desk, and said, "Do you mean did I not operate on half of the patients?" The hall grew very quiet then. The voice at the back of the room very hesitantly replied, "Yes, that's what I had in mind." Then the visitor's fist really came down as he thundered, "Of course not. That would have doomed half of them to their death." God, it was quiet then, and one could scarcely hear the small voice ask, "Which half?"[4]

Thank you.

Thank you.

Your engagement, your ideas, your questions, your participation, your good nature, your stamina (800 minutes!), and your hard work have been a joy to share.

It has been a great honor to teach you this week.

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Stay in touch.

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