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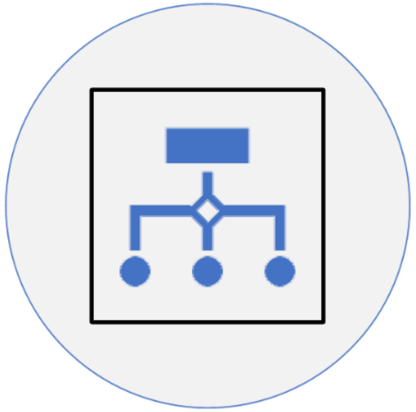
General Additive Models(GAMs)

Overview

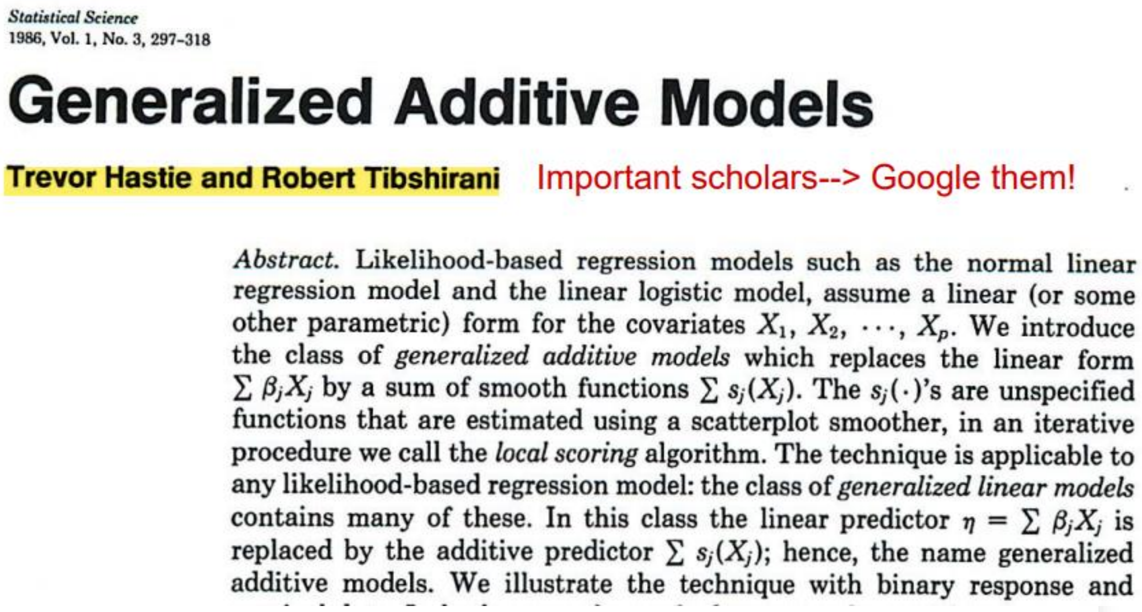
WHAT IS GAM?

WHY GAM?



HOW GAM?

What is GAM?



Statistical Science
1986, Vol. 1, No. 3, 297–318

Generalized Additive Models

Trevor Hastie and Robert Tibshirani

Important scholars--> Google them!

Abstract. Likelihood-based regression models such as the normal linear regression model and the linear logistic model, assume a linear (or some other parametric) form for the covariates $X_1, X_2, \cdots, X_p$. We introduce the class of *generalized additive models* which replaces the linear form $\sum \beta_j X_j$ by a sum of smooth functions $\sum s_j(X_j)$. The $s_j(\cdot)$'s are unspecified functions that are estimated using a scatterplot smoother, in an iterative procedure we call the *local scoring* algorithm. The technique is applicable to any likelihood-based regression model: the class of *generalized linear models* contains many of these. In this class the linear predictor $\eta = \sum \beta_j X_j$ is replaced by the additive predictor $\sum s_j(X_j)$; hence, the name generalized additive models. We illustrate the technique with binary response and

-invented by **Trevor Hastie and Robert Tibshirani in 1986 →**

- Extension of linear models that allow for

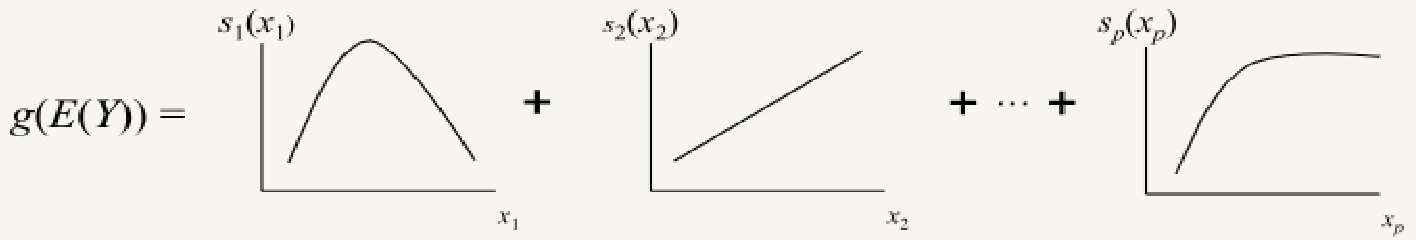
non-linear relationships between the predictors and response

- Use smoothing functions to model non-linearities

-Math: Relationships btw Y and each Si can follow smooth pattern which can be linear or non-linear →

S(Xi) instead of X(i)

S(Xi) Follows data



We can write the GAM structure as:

$$g(E(Y)) = \alpha + s_1(x_1) + \cdots + s_p(x_p),$$

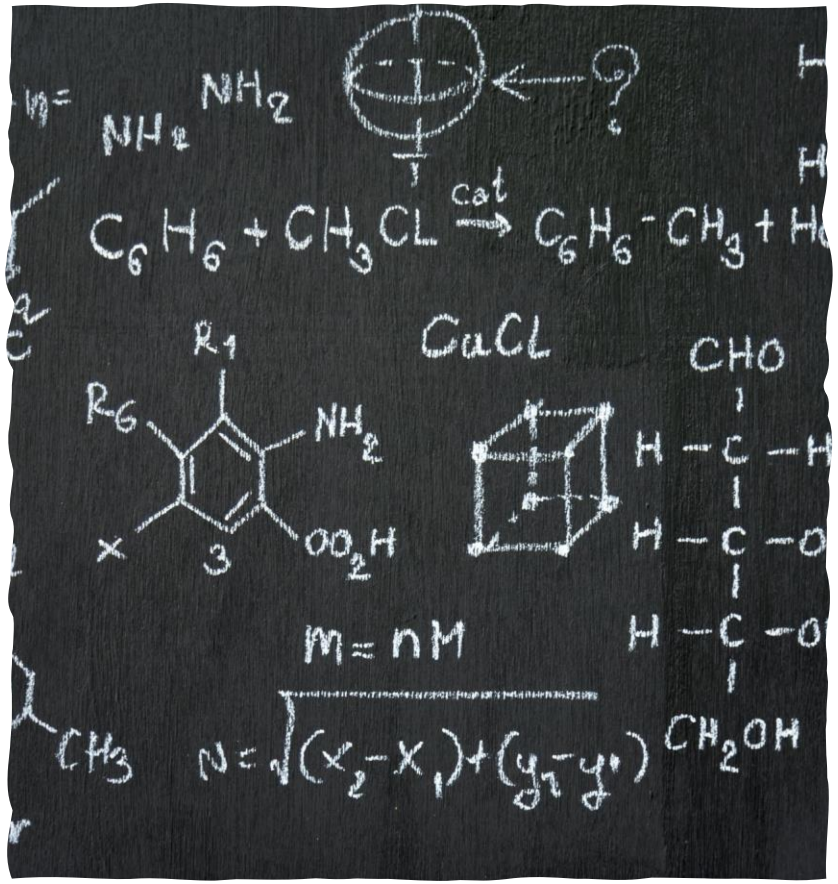
Why GAM?

- Powerful and yet simple technique → Easy to interpret
- Many real-world relationships are non-linear → uncover hidden trends
- Flexibility to model non-linear effects→ Nature is not [always]linear!

Ordinary Least Square

- What is OLS? Great invention because of beautiful interpretation
- Finds coefficients that minimize the sum of squared residuals
- Limitations of OLS? Assumes linear relationship between predictors and response
- OLS may not be suitable for data with non-linear relationships
- GAMs provide more flexibility to overcome this limitation

Math

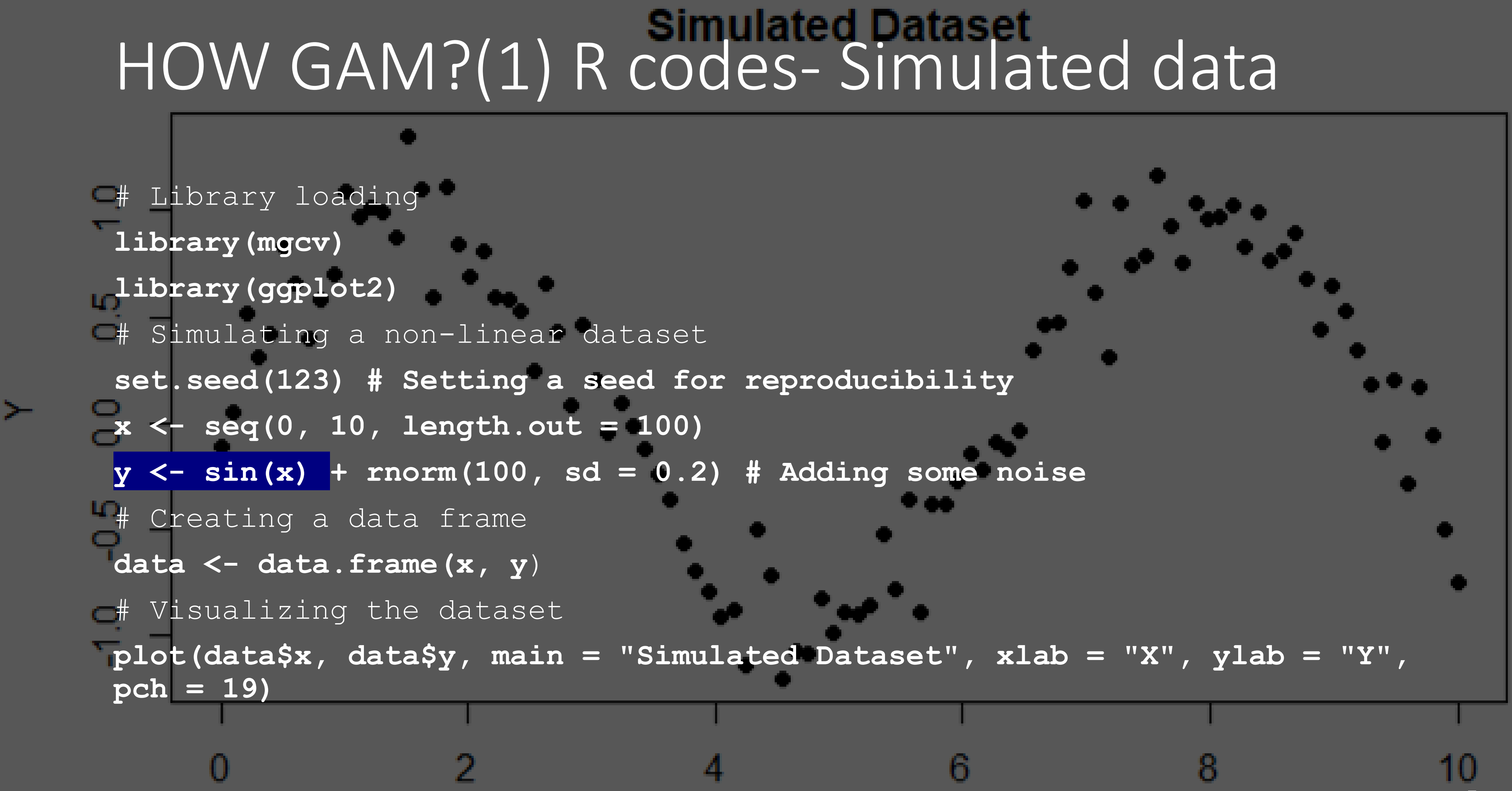


- **OLS regression model** $y = \beta_0 + \beta_1 x_1 + \ldots + \beta_p x_p + \varepsilon$

- **GAM model** $y = \beta_0 + S_1(x_1) + \ldots + S_p(x_p) + \varepsilon$
- **Key differences:**
  -Sj(xj)are smoothing functions, not linear terms
  - Allow for non-linear relationships between predictors and response

- Properties of smoothing functions:
  - Flexible
  - can take on variety of shapes
  - avoid overfitting data
  - Estimated in data-driven way from the data
  -GAMs estimate the smoothing functions to capture non-linearities

HOW GAM?(1) R codes- Simulated data

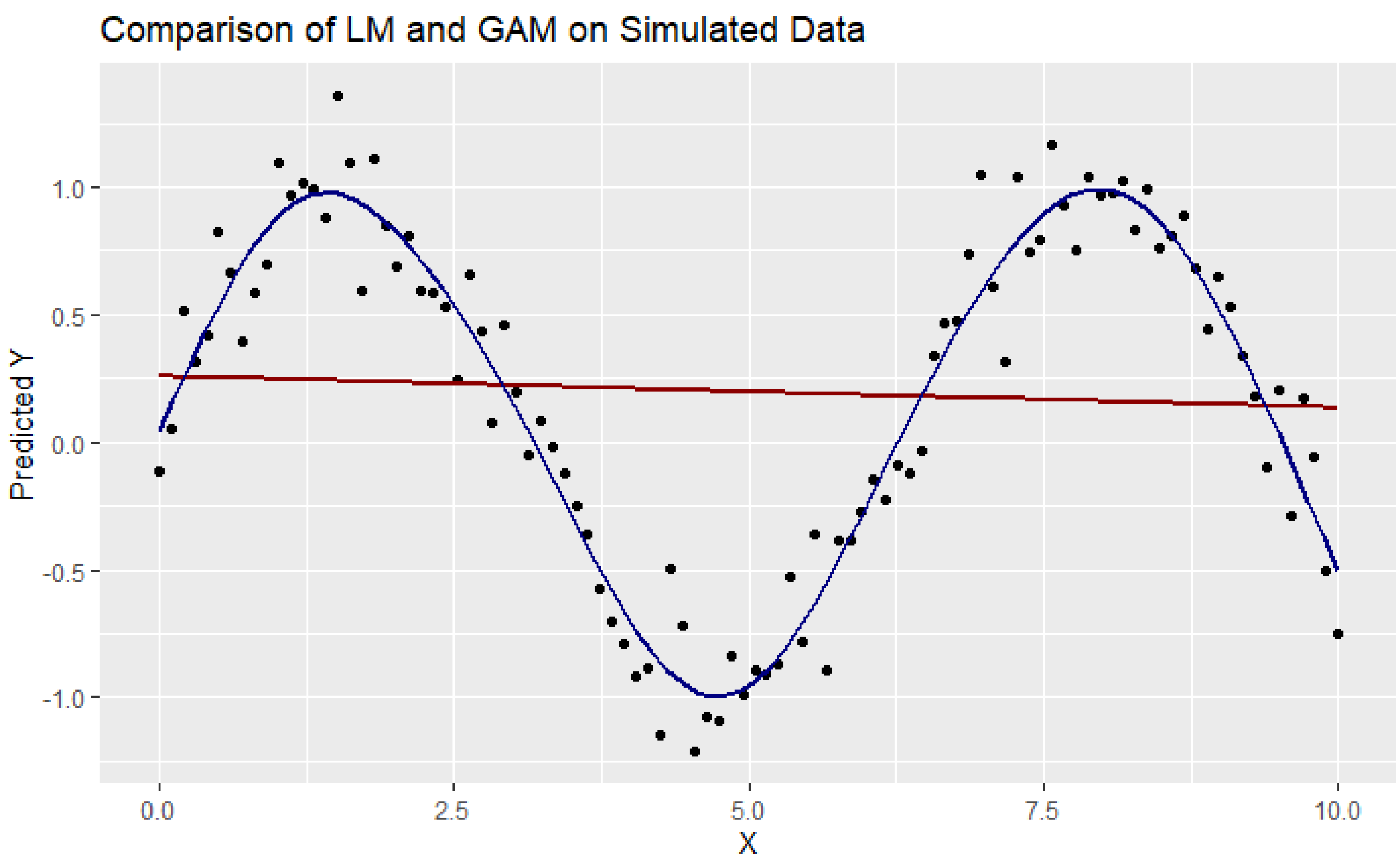



```
# Library loading
library(mgcv)
library(ggplot2)
# Simulating a non-linear dataset
set.seed(123) # Setting a seed for reproducibility
x <- seq(0, 10, length.out = 100)
y <- sin(x) + rnorm(100, sd = 0.2) # Adding some noise
# Creating a data frame
data <- data.frame(x, y)
# Visualizing the dataset
plot(data$x, data$y, main = "Simulated Dataset", xlab = "X", ylab = "Y",
pch = 19)
```

HOW GAM?(1) run GAM vs OLS Model

```
# Fit a linear model
lm_model_simulated <- lm(y ~ x, data = data)
# Fit a GAM model
gam_model_simulated <- gam(y ~ s(x), data = data)
# Create a plot with both fits
ggplot(data, aes(x = x, y = y)) +
  geom_point() +
  geom_smooth(method = "lm", colour = "darkred", se = FALSE) +
  geom_smooth(method = "gam", formula = y ~ s(x), colour = "navyblue", se = FALSE)
+
  labs(title = "Comparison of LM and GAM on Simulated Data",
       x = "X",
       y = "Predicted Y")
```

How GAM(1) Result; GAM vs OLS Simulated data




```
lm(y ~ x, data = data)→ Model Driven
vs
gam(y ~ s(x), data = data)→ Data Driven
```

Model	R-squared	RMSE	p-value	AIC	Signif. smooth terms
LM	0.0028	0.6806	0.5986	210.8	NA
GAM	0.927	0.175	<2e-16	-44.97	s(x) p<2e-16

$$R^2 = 1 - \frac{SS_{RES}}{SS_{TOT}} = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2}$$

$$AIC = 2k - 2\ln(\hat{L})$$

AIC = Akaike information criterion

k = number of estimated parameters in the model

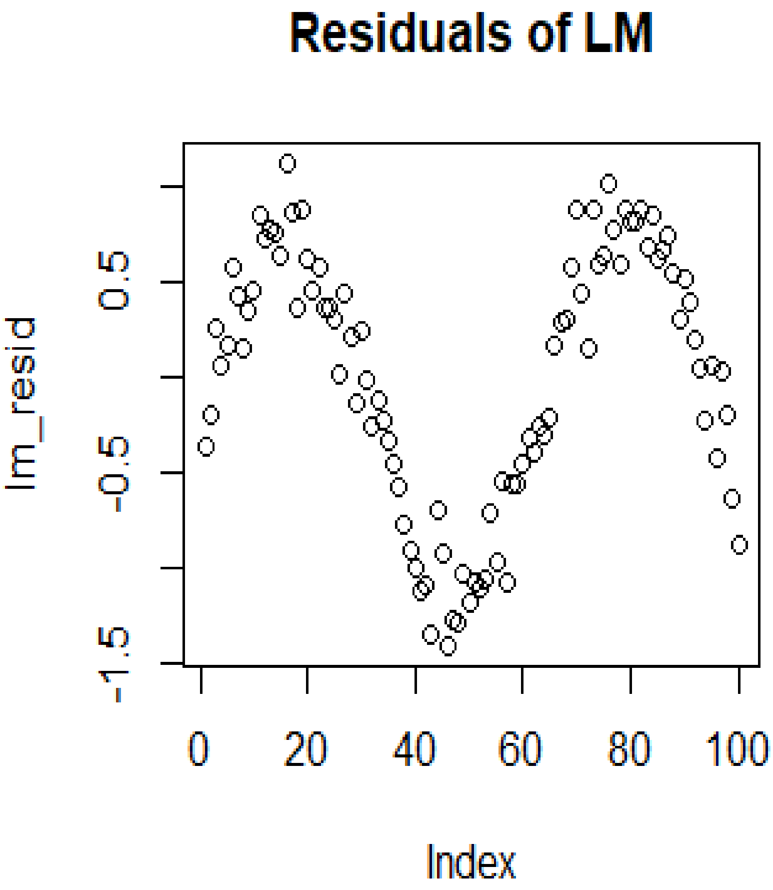
$\hat{L}$ = maximum value of the likelihood function for the model

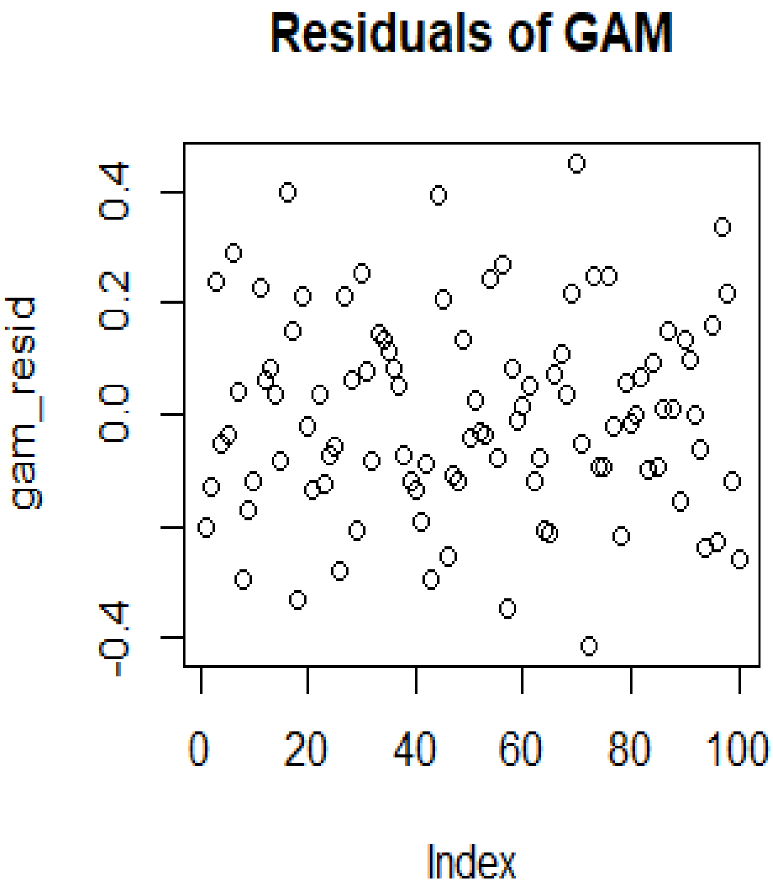
$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (Predicted_i - Actual_i)^2}{N}}$$

Model caparison
key model performance metrics

Linear vs. GAM Model Comparison
Residuals

- The sin wave pattern in the LM residuals is → the linear model cannot fit the true nonlinear (sinusoidal) relationship well → There is still nonlinearity that the model misses.
- The random scatter of the GAM residuals → flexibly fitting the nonlinearity in the data-→ There is no structure left that the model has not captured.



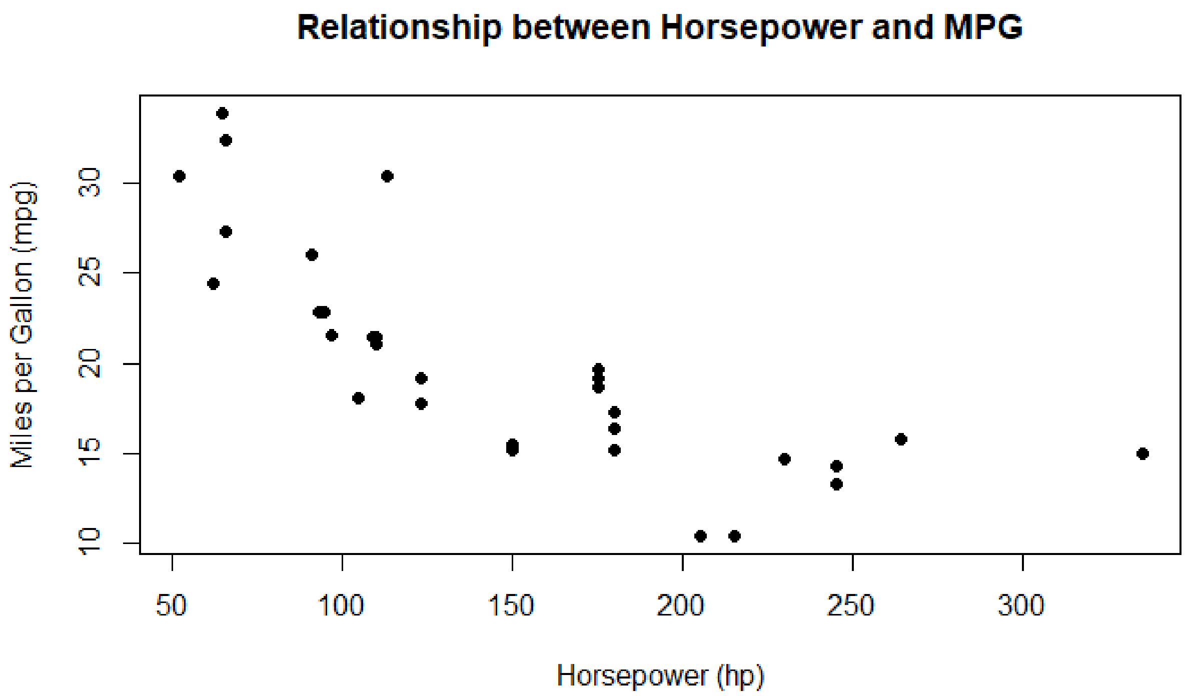


Linear vs. GAM Model Comparison
Anova test

```
anova(lm_model_simulated, gam_model_simulated, test="F")
Analysis of Variance Table

Model 1: y ~ x
Model 2: y ~ s(x)
  Res.Df    RSS     Df Sum of Sq      F    Pr(>F)
1 98.000 45.401
2 91.204  3.070 6.7964    42.331 185.04 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

HOW GAM?(2)R codes &datasets(mtcars)

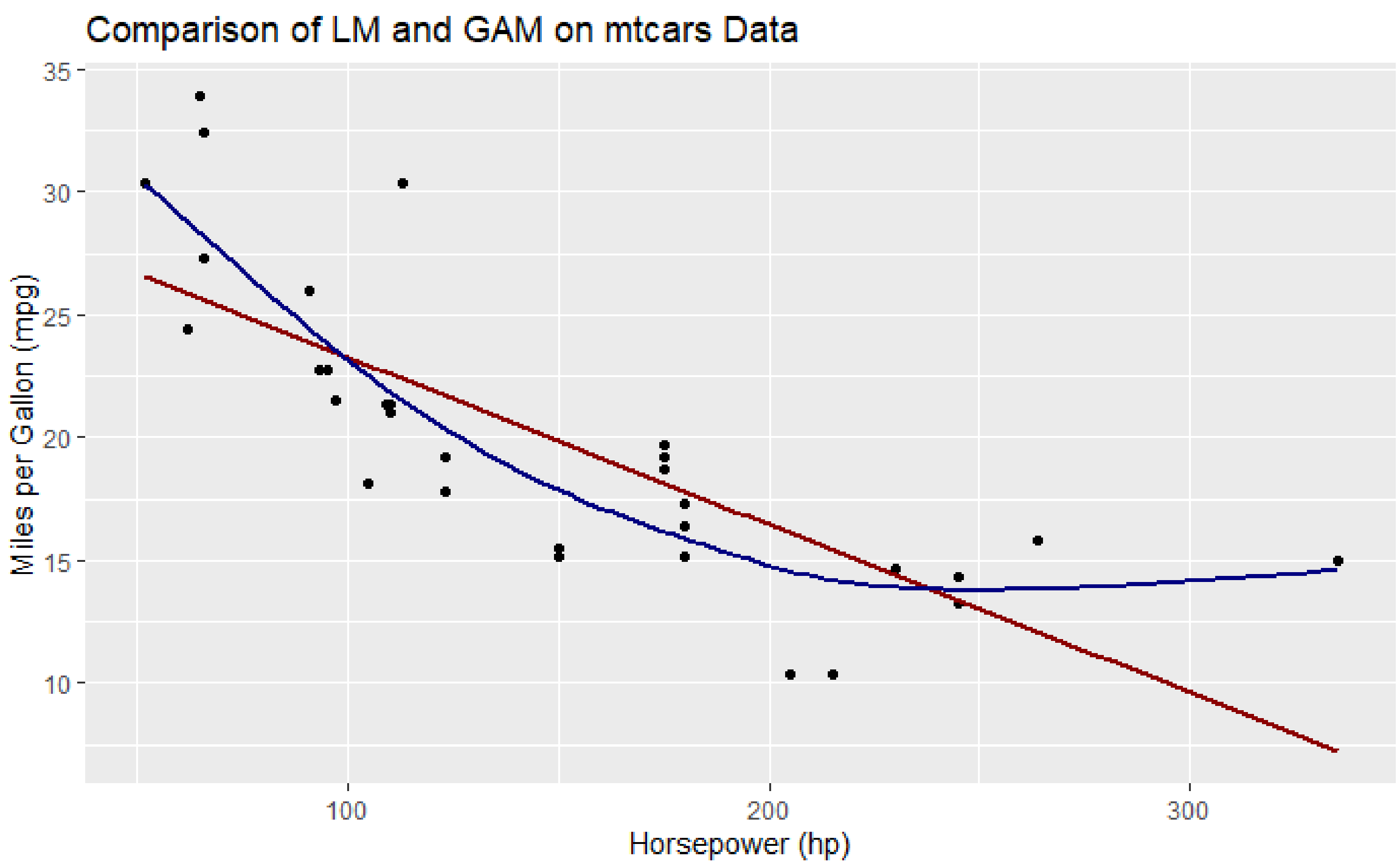
```
# Loading the mtcars dataset

data(mtcars) #The data was extracted from the 1974 Motor Trend US magazine, and comprises fuel consumption and 10 aspects of automobile design and performance for 32 automobiles

# variables of interest,mpg (Miles/(US) gallon) and hp (Gross horsepower)

# Exploring the relationship between 'mpg' and 'hp'

plot(mtcars$hp, mtcars$mpg, main = "Relationship between Horsepower and MPG", xlab = "Horsepower (hp)", ylab = "Miles per Gallon (mpg)", pch = 19)
```



HOW GAM?(2) run GAM vs OLS Model R data sets(mtcars)

```r
# Loading necessary libraries
library(mgcv)
library(ggplot2)
# Fit a linear model
lm_model_mtcars <- lm(mpg ~ hp, data = mtcars)
# Fit a GAM model
gam_model_mtcars <- gam(mpg ~ s(hp) , data = mtcars)
# Create a plot with both fits
ggplot(mtcars, aes(x = hp, y = mpg)) +
  geom_point() +
  geom_smooth(method = "lm", colour = "darkred", se = FALSE) +
  geom_smooth(method = "gam", formula = y ~ s(x), colour = "navyblue", se = FALSE) +
  labs(title = "Comparison of LM and GAM on mtcars Data",
       x = "Horsepower (hp)",
       y = "Miles per Gallon (mpg)")
```

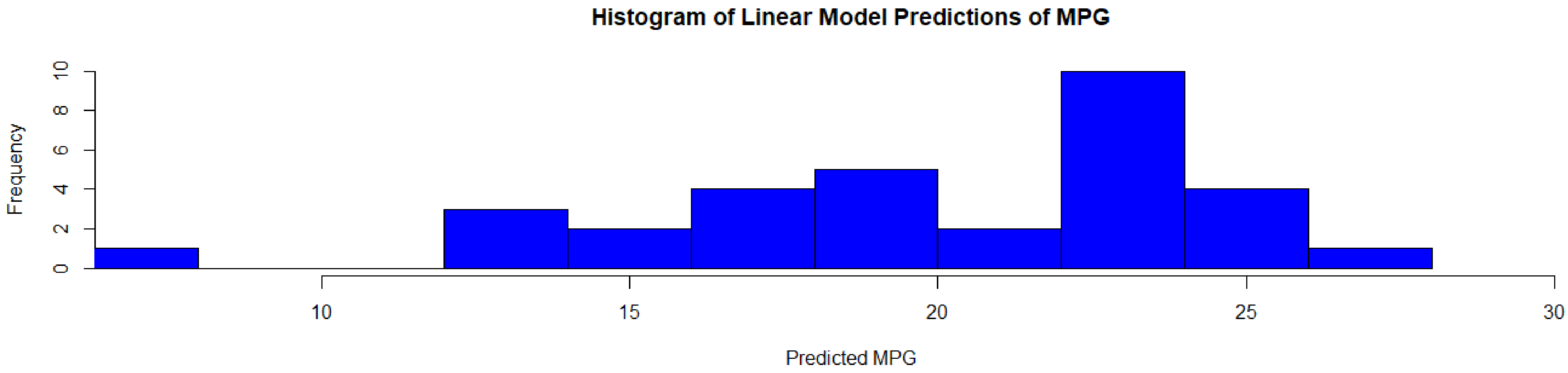
How GAM(2) Result; GAM vs OLS mtcars data

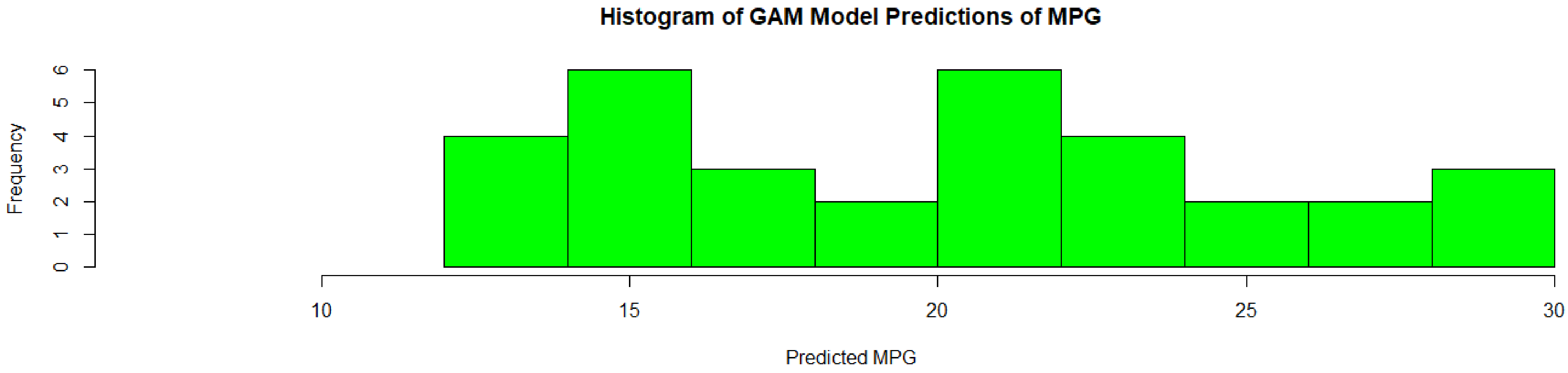


lm(mpg ~ hp, data = mtcars)→ Model Driven

vs

gam(mpg ~ s(hp), data = mtcars)→ Data Driven


Histogram of Linear Model Predictions of MPG
Frequency
0
2
4
6
8
10
10
15
20
25
30
Predicted MPG


Histogram of GAM Model Predictions of MPG
Frequency
0
1
2
3
4
5
6
10
15
20
25
30
Predicted MPG

Real World application: *model*→

$$T_{\text{anom}} \sim s(\text{TCF}) + s(\text{IMP}) + ti(\text{TCF}, \text{IMP}) + s(\text{ELEV}) + s(\text{ST}-\text{WS}, \text{by} = \text{ST}-\text{WD}) + s(\text{LON}, \text{LAT})$$

2.3. Analysis using generalized additive models (GAMs)

The relationship between some biophysical variables—most notably tree canopy cover—and air or LST can be nonlinear in nature (Ziter *et al* 2019, Logan *et al* 2020). GAMs are a nonparametric technique that can fit smooth curves between predictor and response variables using penalized regression splines (Pedersen *et al* 2019). In this study, we used the *gam* function in the R package 'mgcv' (version 1.8.31) and fit the models using fast restricted maximum likelihood.

Table 1. Data descriptions.

Variable name	Short name	Description
Vegetation		
Tree canopy	TCF	1 m tree canopy map derived from 2018 City of DC lidar data
Soft canopy	SCF	Tree canopy that does not overhang impervious surface
Hard canopy	HCF	Tree canopy that overhangs impervious surface
Canopy patches	PATCH	Soft canopy patches large enough to have cores (MSPA)
Distributed canopy	DISTRB	Soft canopy, connected or unconnected, no core (MSPA)
Pervious-open	PV-O	Area that is neither soft canopy nor impervious surface
Built environment		
Impervious surface	IMP	Impervious surface from City of DC planimetric data
Building height (sum)	BH	Building heights summed in area (DC building footprints and lidar data)
Building height (IMP norm)	BH-norm	Building heights as above but normalized by IMP to decorrelate
Skyview factor	SVF	Skyview factor calculated using DC lidar data in SAGA GIS
Physiographic		
Elevation	ELEV	City of DC lidar Digital Terrain Model (2018)
Quantile elevation	Q-ELEV	Quantile (local) elevation within 300 m radius
Distance from water	DIST-W	Euclidean distance from Potomac and Anacostia rivers
Car data		
Spatial coordinates	LON, LAT	Temperature measurement locations geographic coordinates
Mobile temperature	MBL-T	Temperature measurements (celsius)
Miles per hour	MPH	Car travel speed
Station data		
Station temperature	ST-T	Temperature (celsius) averaged across four downtown DC stations
Station wind speed	ST-WS	Wind speed at one representative station
Station wind direction	ST-WD	Wind direction at one representative station
Station solar radiation	ST-SR	Solar radiation at one representative station

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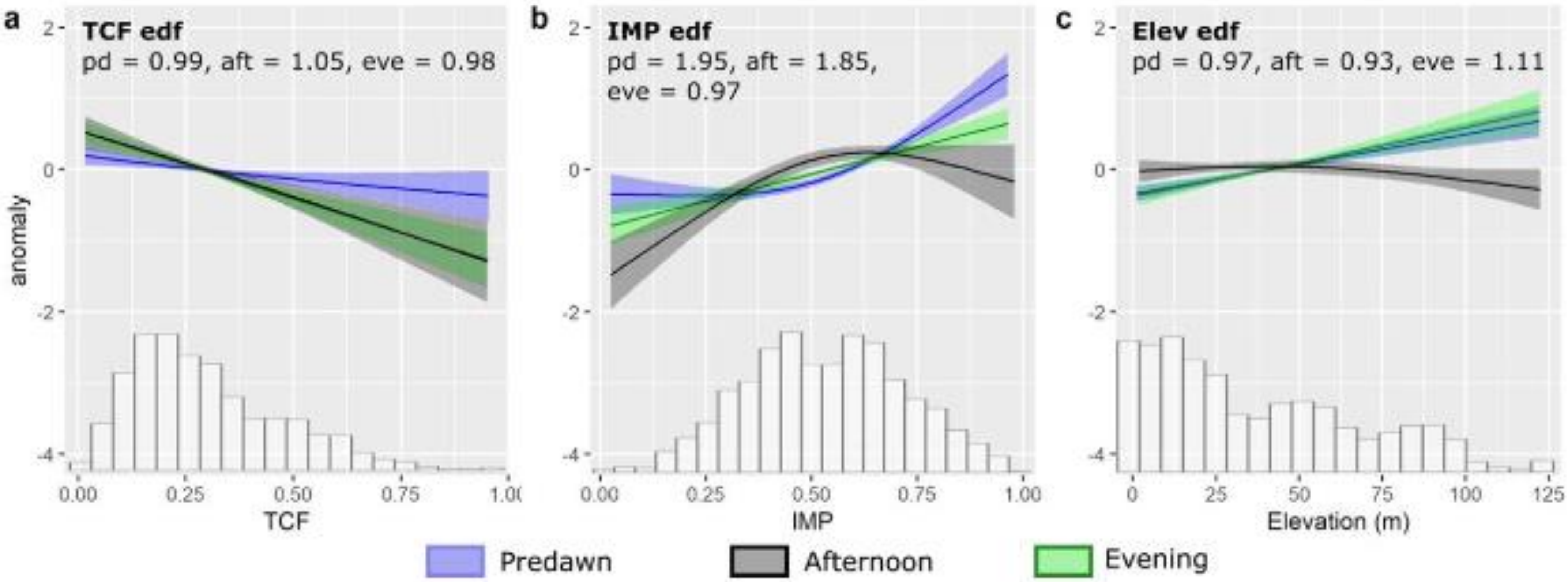

Figure 3. Model 1 results at 200 m scale by time of day where blue is predawn, black is afternoon, and green is evening. Data distributions accompany below. Estimated degrees of freedom (edf) average of 20 iterations. (a) Tree canopy fraction (TCF) cooling; (b) warming from impervious surface fraction (IMP); (c) temperature change driven by elevation.

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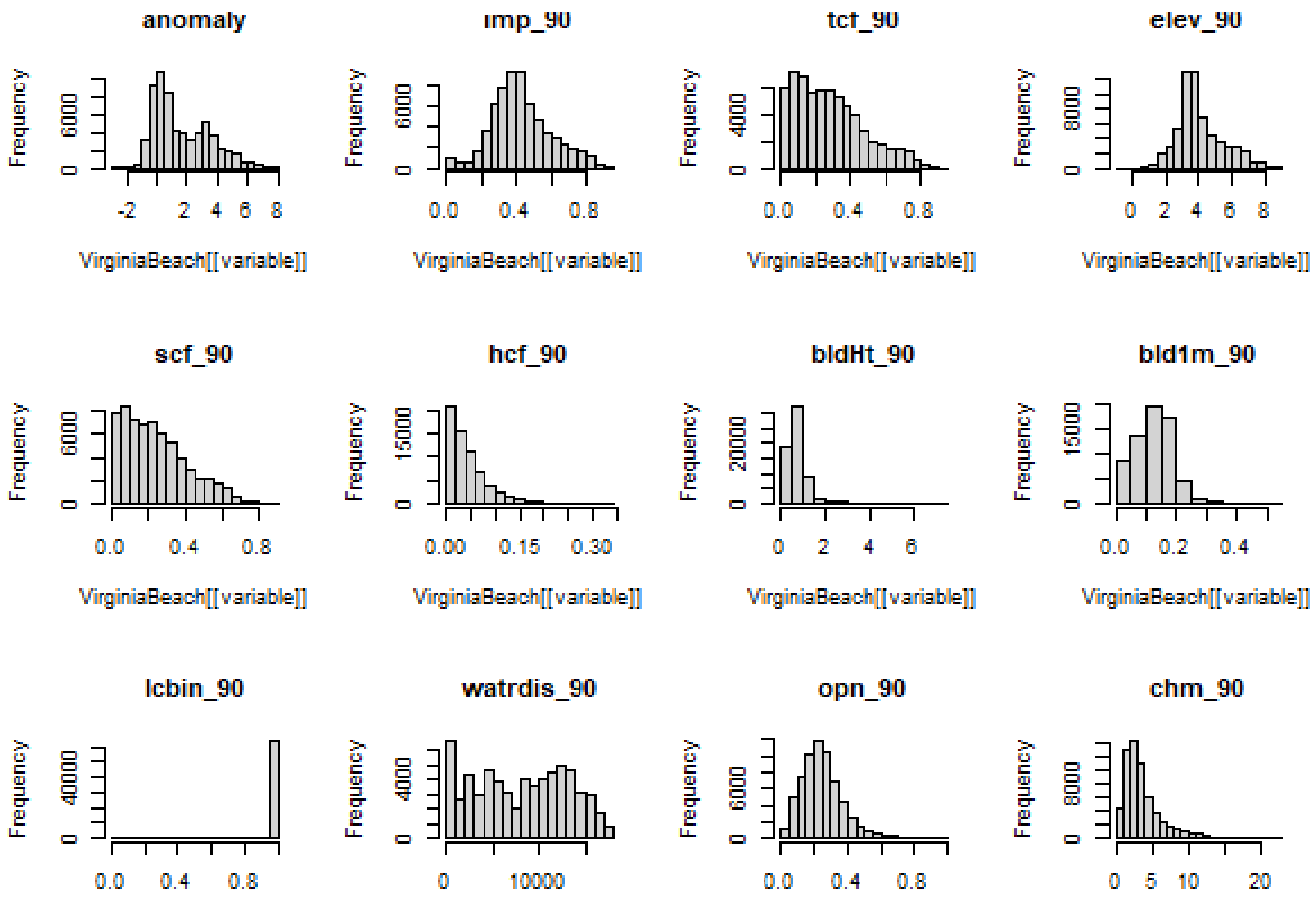
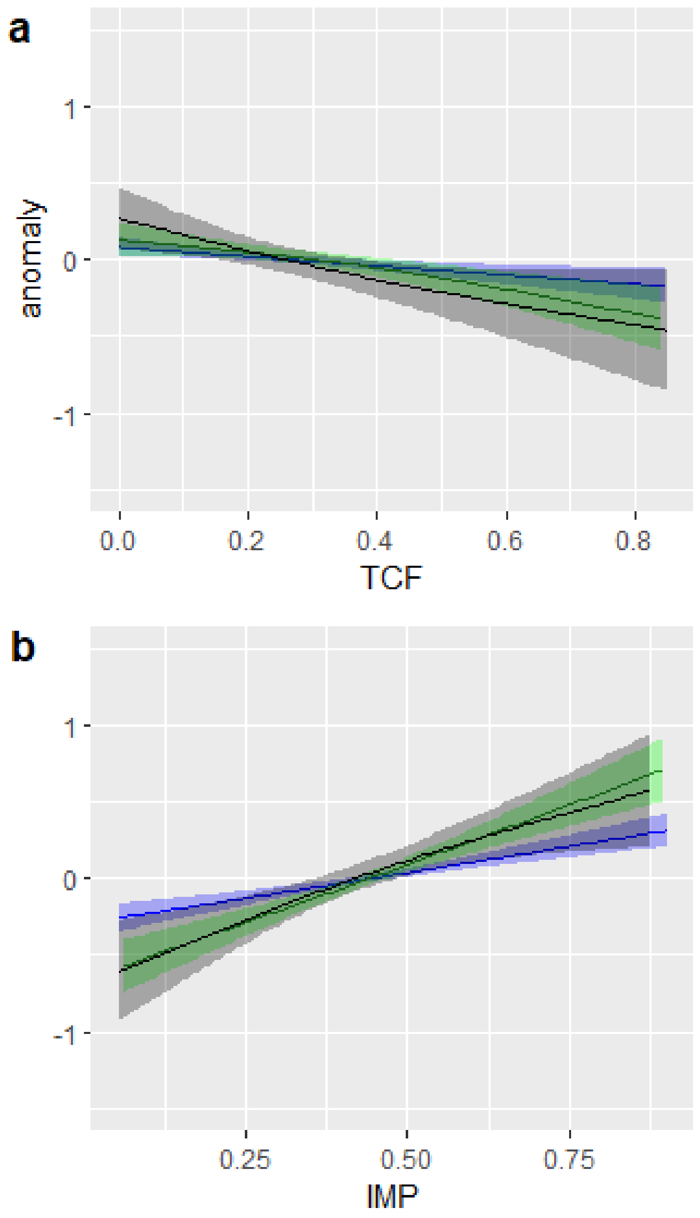
How GAM(3) Real world application; Tree Canopy and Temperature N ~ 62,000

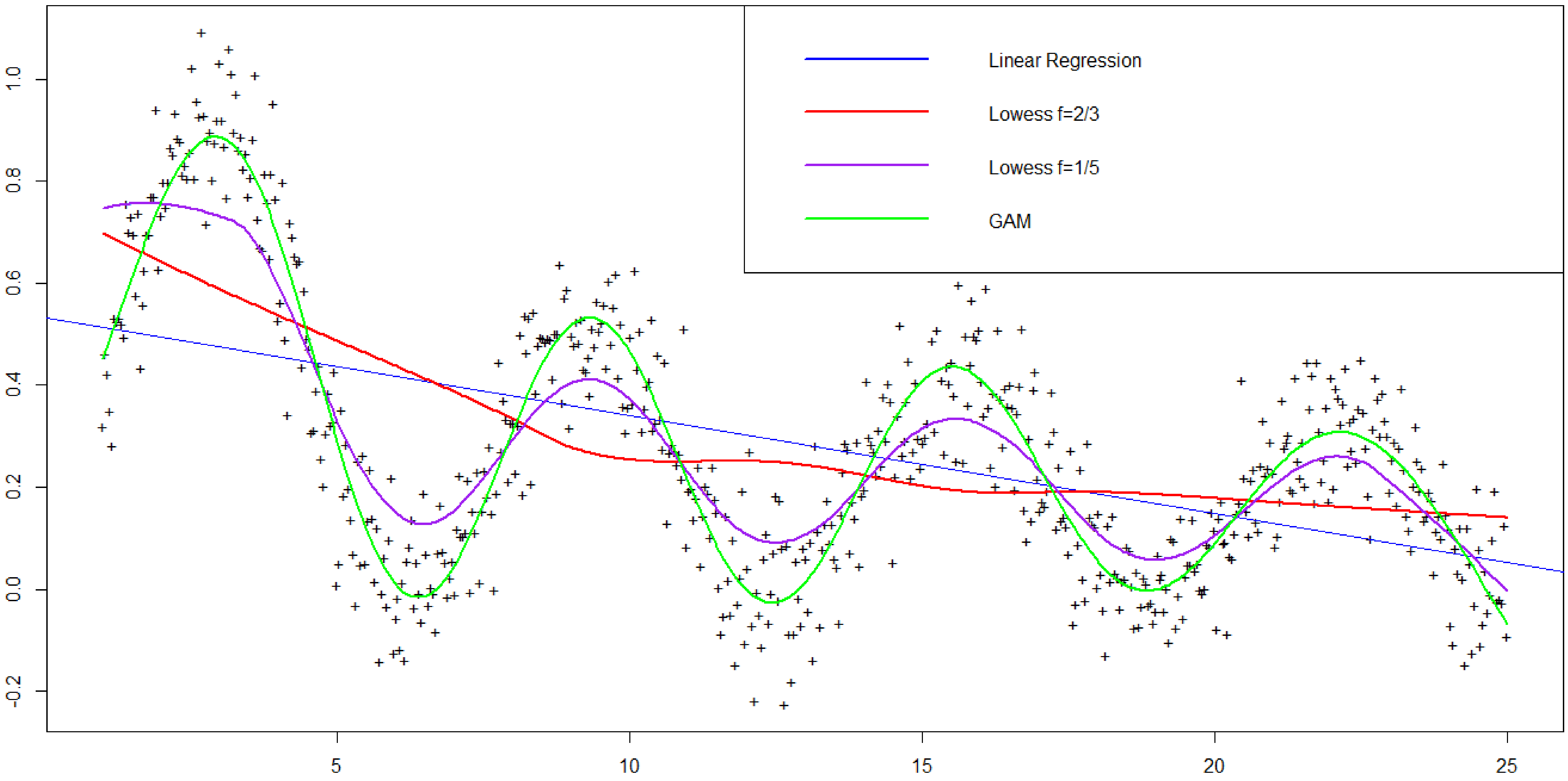
Morning= 23166
Afternoon=17396
Evening=20698

Virginia Beach, July 15th, 2021


anomaly
imp_90
tcf_90
elev_90
scf_90
hcf_90
bldHt_90
bld1m_90
lcbin_90
watrdis_90
opn_90
chm_90
Frequency
VirginiaBeach[[variable]]


a
anomaly
TCF
b
IMP

Jeff's slide 2 **- Running Lowess**



Thank You!

References

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[7] Michael Alonzo et al 2021 Environ. Res. Lett. Spatial configuration and time of day impact the magnitude of urban tree canopy cooling

[8] https://jeffgill.org/classes/

[9] Openai & ClaoudAI