Concepts in Machine Learning Winter Institute in Data Science

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2024 - 01 - 08

Building Models

Modeling Helper Functions

Example: mtcars

Example: Social Pressure Experiment (recipes)

Regularization Methods: LASSO, ridge regression, elastic nets

Building Models

What are our goals?

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- 1. Generative modeling
- 2. Predictive modeling

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- 1. Generative modeling
- 2. Predictive modeling

Breiman (2001)

► Theory (novel theory, prior theory, prior findings)

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- Raw data ("data look nonlinear, so ... + $\beta x^2 + ...$ ")
- Specification searching (repeat modeling with same data)
- Training and testing (repeat modeling, different data)

All the important ones (No omitted variable bias)

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- No irrelevant ones (No included variable bias)

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Helpful?

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Helpful?

- ► Affect outcome
- ► Confounders
- ▶ Pre-treatment only
- ► Avoid post-treatment
- ► "In-horizon"
- ► Test something "out-of-horizon"

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- ▶ Pre-treatment only
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- ► "In-horizon"
- ► Test something "out-of-horizon"

(Sometimes it will depend on goals.)

What to include, when thousands of predictors?

What to include, when thousands of predictors? "Machine learning"

What to include, when thousands of predictors? "Machine learning"

(but "machine learning" can mean different things.)



I finally found it in real life: the consultant who runs OLS in Excel and calls it machine learning

9:17 AM \cdot Jan 31, 2019 \cdot Twitter for iPhone

54 Retweets	7 Quote Tweets	511 Likes		
\Diamond	17	\bigcirc	\uparrow	

Figure 1: Don't do this.



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Figure 1: Don't do this.

If you can't describe the procedure's "learning", it may not be "machine learning".



. . .

I finally found it in real life: the consultant who runs OLS in Excel and calls it machine learning

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Figure 1: Don't do this.

If you can't describe the procedure's "learning", it may not be "machine learning".

There should probably be some testing/training, regularization, $\,$

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Modeling Helper Functions

```
data(sim1)
lm out \leftarrow lm(y \sim x, data = sim1)
tidy(lm out)
## # A tibble: 2 x 5
## term estimate std.error statistic p.value
## <chr> <dbl>
                         <dbl>
                                  <dbl> <dbl>
## 1 (Intercept) 4.22 0.869 4.86 4.09e- 5
                  2.05 0.140 14.7 1.17e-14
## 2 x
```

2 x

##

```
data(sim1)
lm_out \leftarrow lm(y \sim x, data = sim1)
tidy(lm out)
## # A tibble: 2 x 5
## term estimate std.error statistic p.value
## <chr> <dbl> <dbl> <dbl> <dbl>
## 1 (Intercept) 4.22 0.869 4.86 4.09e- 5
```

glance(lm out) ## # A tibble: 1 x 12 r.squared adj.r.squared sigma statistic p.value d: ## <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <

1 0.885 0.880 2.20 215. 1.17e-14_{28/103}

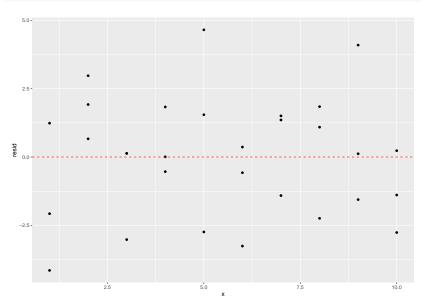
2.05 0.140 14.7 1.17e-14

Special mutate() functions:

Special mutate() functions:

```
(sim1 <- sim1 |> add_residuals(lm out))
## # A tibble: 30 \times 3
##
                       resid
           Х
                 У
      <int> <dbl>
                       <dbl>
##
##
              4.20 - 2.07
           1 7.51
                    1.24
##
           1 \quad 2.13 \quad -4.15
##
    3
           2 8.99
##
                    0.665
##
    5
           2 10.2 1.92
##
           2 11.3 2.97
           3 \quad 7.36 \quad -3.02
##
##
    8
           3 10.5
                    0.130
           3 10.5 0.136
##
           4 12.4
                    0.00763
##
   10
       20
          more rows
```

```
ggplot(sim1, aes(x, resid)) + geom_point() +
  geom_hline(yintercept = 0, linetype = 2, color = "re
```

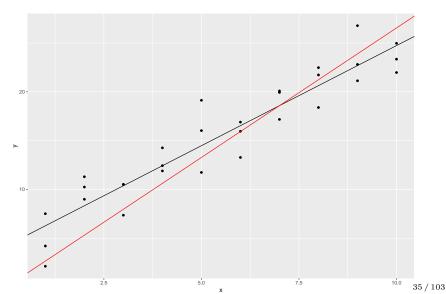


Special mutate() functions:

```
(sim1 <- sim1 |> add_predictions(lm out))
## # A tibble: 30 x 4
##
                  resid
         Х
              У
                         pred
     <int> <dbl> <dbl> <dbl>
##
           4.20 -2.07
                         6.27
##
         1
##
         1 7.51 1.24 6.27
##
   3
         1 2.13 -4.15 6.27
         2 8.99 0.665
##
                         8.32
##
   5
         2 10.2 1.92
                         8.32
         2 11.3 2.97
                         8.32
##
         3 7.36 -3.02
##
                        10.4
   8
         3 10.5 0.130
                        10.4
##
         3 10.5 0.136
                        10.4
##
         4 12.4
                 0.00763 12.4
## 10
      20
         more rows
```

```
lm_out2 \leftarrow lm(y \sim x - 1, data = sim1)
```

```
ggplot(sim1, aes(x, y)) + geom_point() +
geom_abline(intercept = coef(lm_out)[1], slope = coef(lm_geom_abline(intercept = 0, slope = coef(lm_out2)["x"], coef(lm_out2)["x"]
```



```
glance(lm_out)
## # A tibble: 1 x 12
   r.squared adj.r.squared sigma statistic p.value d:
##
      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <
##
      0.885 0.880 2.20 215. 1.17e-14
## 1
## # i 3 more variables: deviance <dbl>, df.residual <int>
glance(lm out2)
## # A tibble: 1 x 12
##
   r.squared adj.r.squared sigma statistic p.value df
       ##
## 1
      0.970 0.969 2.94 NA
                                      NA
                                           NA
```

i 3 more variables: deviance <dbl>, df.residual <int>

```
## # A tibble: 1 x 12
## r.squared adj.r.squared sigma statistic p.value df
## <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> = 1.17e-14
## i 3 more variables: deviance <dbl>, df.residual <int>
```

glance(lm_out2)

 $(R^2 \text{ and predictive quality are not the same thing } \dots)$

modelr Helper Functions

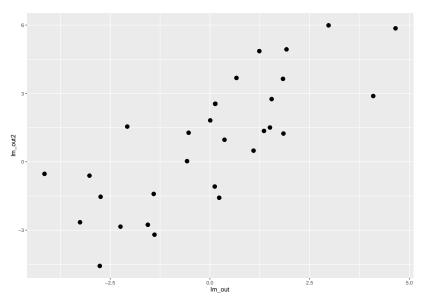
```
( sim1 <- sim1 |> spread residuals(lm out, lm out2) )
## # A tibble: 30 x 6
##
                  resid pred lm out lm out2
              V
         Х
##
     <int> <dbl> <dbl> <dbl>
                                <dbl>
                                       <dbl>
##
           4.20 -2.07
                         6.27 - 2.07
                                       1.55
##
  2
         1 7.51
                1.24
                         6.27 1.24
                                       4.86
   3
         1 2.13 -4.15
##
                        6.27 - 4.15
                                      -0.529
##
         2 8.99
                 0.665
                         8.32 0.665
                                       3.68
   5
         2 10.2 1.92
                         8.32 1.92
                                       4.93
##
         2 11.3 2.97
                         8.32 2.97
##
                                       5.99
         3 7.36 -3.02
                        10.4 -3.02
                                      -0.607
##
##
   8
         3 10.5 0.130
                        10.4 0.130
                                       2.54
         3 10.5 0.136
                        10.4 0.136
##
                                       2.55
         4 12.4
                 0.00763 12.4 0.00763
                                       1.82
##
  10
## # i 20 more rows
```

```
ggplot(sim1, aes(x, lm_out2)) + geom_point() +
 geom_hline(yintercept = 0, linetype = 2, color = "red")
Im_out2
```

2.5

10.0

ggplot(sim1, aes(lm_out, lm_out2)) + geom_point(size = 3)

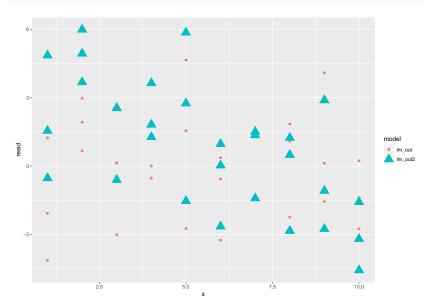


modelr Helper Functions

```
data(sim1)
( sim1 <- sim1 |> gather residuals(lm out, lm out2) )
## # A tibble: 60 \times 4
                 y resid
##
    model x
## <chr> <int> <dbl> <dbl>
## 1 lm out 1 4.20 -2.07
## 2 lm out 1 7.51 1.24
## 3 lm out 1 2.13 -4.15
   4 lm out 2 8.99 0.665
##
## 5 lm out 2 10.2 1.92
  6 lm out 2 11.3 2.97
##
## 7 lm out 3 7.36 -3.02
   8 lm out 3 10.5 0.130
##
   9 lm out 3 10.5 0.136
##
## 10 lm out 4 12.4 0.00763
## # i 50 more rows
```

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```
ggplot(sim1, aes(x, resid)) +
  geom_point(aes(color = model, size = model, shape = model)
```



modelr Helper Functions

- add_residuals()
- spread_residuals()
- gather_residuals()
- add_predictions()
- spread_predictions()
- gather_predictions()

Other Helpers for Many Models: tidy()

```
11 <- list(lm_out, lm_out2)</pre>
11 |> map_df(tidy)
## # A tibble: 3 \times 5
##
    term estimate std.error statistic p.value
##
    <chr>
                 <dbl>
                           <dbl>
                                    <dbl>
                                            <dbl>
## 1 (Intercept)
                   4.22
                          0.869 4.86 4.09e- 5
## 2 x
                   2.05
                          0.140
                                    14.7 1.17e-14
                   2.65
## 3 x
                          0.0865 30.7 1.15e-23
```

Many Models: glance()

0.885

0.970

1

2

11 |> map df(glance) |> select(1:6)

0.969 2.94

0.880 2.20 215. 1.17e-14

NA NA

Many Models: glance()

```
11 |> map df(glance) |> select(1:6)
## # A tibble: 2 x 6
    r.squared adj.r.squared sigma statistic p.value
##
       <dbl> <dbl> <dbl> <dbl>
##
                                       <dbl> <dbl
## 1 0.885 0.880 2.20 215. 1.17e-14
## 2 0.970
               0.969 2.94 NA NA
11 |> map_df(glance) |> select(7:12)
## # A tibble: 2 x 6
##
    logLik AIC BIC deviance df.residual nobs
## <dbl> <dbl> <dbl> <int> <int>
## 1 -65.2 136. 141. 136.
                                 28 30
## 2 -74.4 153. 156. 250.
                                 29
                                      30
```

Example: mtcars

1. Feature engineering

1. Feature engineering: collect/create the data

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- 2. Data splitting

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data

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- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
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- ► Testing. (20%?)
- 3. Feature selection

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
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- 3. Feature selection: algorithms decide predictors to include

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- 2. Data splitting: split the data
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- ► Testing. (20%?)
- 3. Feature selection: algorithms decide predictors to include
- 4. Model estimation

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)
- 3. Feature selection: algorithms decide predictors to include
- 4. Model estimation: find the slopes (e.g.)

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)
- 3. Feature selection: algorithms decide predictors to include
- 4. Model estimation: find the slopes (e.g.)
- 5. Validation + testing

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)
- 3. Feature selection: algorithms decide predictors to include
- 4. Model estimation: find the slopes (e.g.)
- 5. Validation + testing: evaluate preds from trained models using new data

```
library(tidymodels)
data_split <- initial_split(mtcars, prop = 2/3)

df_train <- training(data_split)
df_test <- testing(data_split)</pre>
```

```
library(tidymodels)
data_split <- initial_split(mtcars, prop = 2/3)</pre>
df train <- training(data split)</pre>
df_test <- testing(data_split)</pre>
dim(df train)
## [1] 21 11
dim(df test)
## [1] 11 11
```

```
lm_fit <- linear_reg() |> fit(mpg ~ ., data = df_train)
lm fit
## parsnip model object
##
##
## Call:
## stats::lm(formula = mpg ~ ., data = data)
##
## Coefficients:
## (Intercept)
                       cyl
                                  disp
                                                hp
## -127.13656 3.24904
                              0.01493
                                            0.06491
##
         qsec
                       VS
                                    am
                                               gear
## 4.21793 -3.09021
                                    NΑ
                                           10.66257
```

```
##
                            lm
                   mpg
## Mazda RX4
            21.0 17.68862
## Mazda RX4 Wag 21.0 18.87465
## Hornet 4 Drive 21.4 18.42586
## Hornet Sportabout 18.7 19.98952
## Merc 240D
                24.4 21.63137
## Merc 230
           22.8 37.74679
## Merc 280C 17.8 22.20879
## Toyota Corona 21.5 19.32277
## Porsche 914-2 26.0 33.08862
## Ford Pantera L 15.8 39.18207
## Maserati Bora 15.0 24.66780
```

Next, predict with random forest algorithm.

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▶ Boosting: models build on prior models

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Next, predict with random forest algorithm.

Ensemble learning algorithms:

- ▶ Boosting: models build on prior models ↔ pick feature, predict, upweight mispredicted data, Do several times and combine.
- ▶ Bagging: (random select units, model) \rightarrow many times. No building.

Next, predict with random forest algorithm.

Ensemble learning algorithms:

- ▶ Boosting: models build on prior models ↔ pick feature, predict, upweight mispredicted data, Do several times and combine.
- ▶ Bagging: (random select units, model) \rightarrow many times. No building.

Next, predict with random forest algorithm.

Ensemble learning algorithms:

- ▶ Boosting: models build on prior models ↔ pick feature, predict, upweight mispredicted data, Do several times and combine.
- ▶ Bagging: (random select units, model) \rightarrow many times. No building.

Random Forests are bagging algorithms.

Ranger result

Number of trees:

Target node size:

Variable importance mode:

Number of independent variables: 10

Sample size:

##

##

Call:

Type:

Mtry:

```
rf_fit <- rand_forest(mode = "regression") |>
  fit(mpg ~ ., data = df_train)
rf_fit

## parsnip model object
##
```

ranger::ranger(x = maybe_data_frame(x), y = y, num.thre

Regression

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500

21

3

5

none

parsnip::rand_forest() uses ranger engine

parsnip::rand_forest() uses ranger engine
There is also "Spark".

```
##
                              lm
                                    rf
                    mpg
## Mazda RX4
               21.0 17.68862 20.45133
## Mazda RX4 Wag 21.0 18.87465 20.36097
## Hornet 4 Drive 21.4 18.42586 18.54591
## Hornet Sportabout 18.7 19.98952 16.77318
## Merc 240D
                   24.4 21.63137 21.83636
## Merc 230
                   22.8 37.74679 21.47683
## Merc 280C 17.8 22.20879 19.54339
## Toyota Corona
                   21.5 19.32277 22.28518
## Porsche 914-2 26.0 33.08862 25.05484
## Ford Pantera I. 15.8 39.18207 17.33194
## Maserati Bora 15.0 24.66780 16.16053
```

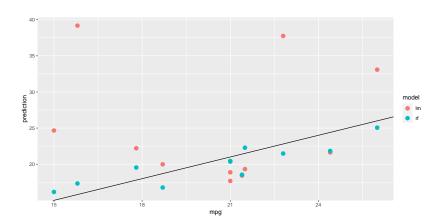
tidymodels Example

```
out_preds_long <- out_preds |>
 pivot_longer(cols = c(lm, rf),
              names to = "model",
              values to = "prediction")
out preds long
## # A tibble: 22 x 3
##
       mpg model prediction
     <dbl> <chr>
##
                 <dbl>
## 1 21 lm
                       17.7
##
   2 21 rf
                       20.5
   3 21 lm
##
                       18.9
   4 21 rf
##
                       20.4
##
   5 21.4 lm
                      18.4
   6 21.4 rf
##
                       18.5
   7 18.7 lm
                       20.0
##
##
   8
      18.7 rf
                       16.8
```

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tidymodels Example

```
ggplot(out_preds_long, aes(mpg, prediction)) +
  geom_point(aes(color = model), size = 3) +
  geom_abline(slope = 1, intercept = 0)
```



tidymodels Example

Evaluate:

```
out preds |> metrics(truth = mpg, estimate = lm) |>
 rename(lm = .estimate) |>
 left join(out preds |>
             metrics(truth = mpg, estimate = rf) |>
             rename(rf = .estimate))
## Joining with 'by = join by(.metric, .estimator)'
## # A tibble: 3 \times 4
## .metric .estimator
                           lm rf
## <chr> <chr> <dbl> <dbl>
## 1 rmse standard 9.40 1.63
## 2 rsq standard 0.000476 0.783
## 3 mae standard 6.74 1.46
```

Example: Social Pressure Experiment (recipes)

Data Splitting

```
social <- read_csv("https://raw.githubusercontent.com/"
soc_split <- initial_split(social)
soc_train <- training(soc_split)
soc_test <- testing(soc_split)</pre>
```

Data Splitting

```
social <- read csv("https://raw.githubusercontent.com/)</pre>
soc split <- initial_split(social)</pre>
soc train <- training(soc split)</pre>
soc test <- testing(soc split)</pre>
dim(soc train)
## [1] 229399
dim(soc test)
## [1] 76467
```

```
social_recip <- recipe(primary2006 ~ ., data = soc_train)
social_recip</pre>
```

summary(social_recip)

```
social_recip <- social_recip |>
step_mutate(age = 2006 - yearofbirth) |>
step_dummy(all_nominal(), -all_outcomes())
```

```
social_recip
##
## -- Recipe ------
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
##
## -- Operations
## * Variable mutation for: 2006 - yearofbirth
## * Dummy variables from: all_nominal(), -all_outcomes()
```

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```
social_recip <- social_recip |>
step_zv(all_predictors())
```

```
##
## -- Recipe ------
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
##
## -- Operations
## * Variable mutation for: 2006 - yearofbirth
## * Dummy variables from: all_nominal(), -all_outcomes()
## * Zero variance filter on: all_predictors()
```

social_recip

```
social_recip <- social_recip |>
step_center(all_predictors(), -primary2004)
```

```
social_recip
##
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
##
## -- Operations
## * Variable mutation for: 2006 - yearofbirth
## * Dummy variables from: all_nominal(), -all_outcomes()
## * Zero variance filter on: all_predictors()
## * Centering for: all_predictors(), -primary2004
```

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Recipe complete. Time to prep and bake.

Recipe complete. Time to prep and bake.

```
social recip |>
  prep()
##
## -- Recipe
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
```

```
soc_train_processed <- social_recip |>
  prep() |>
  bake(new data = NULL)
soc train processed
## # A tibble: 229,399 x 22
##
      yearofbirth primary2004 hhsize primary2006
                                                        age se
##
            <dbl>
                         <dbl> <dbl>
                                              <dbl>
                                                      <dbl>
## 1
          -19.2
                              0 - 0.184
                                                  0 19.2
##
           -8.21
                              1 - 0.184
                                                      8.21
    3
##
           19.8
                              1 - 0.184
                                                  1 - 19.8
          -6.21
##
    4
                             0 - 0.184
                                                      6.21
##
    5
          3.79
                             0 - 0.184
                                                  1 - 3.79
           11.8
                             0 - 1.18
                                                  0 - 11.8
##
    6
            0.793
##
                              0 - 0.184
                                                  0 - 0.793
##
    8
            4.79
                              1 0.816
                                                  1 - 4.79
            7.79
                              0 - 0.184
                                                  1 - 7.79
##
                              1 - 0.184
                                                  0
                                                      1.21
## 10
           -1.21
                                                        90 / 103
```

200 mama marra

names(soc_train_processed)

```
"primary2004
##
    [1] "yearofbirth"
                                             "primary2000
##
    [3] "hhsize"
    [5] "age"
##
                                             "sex male"
##
    [7] "messages Control"
                                             "messages Ha
    [9] "messages Neighbors"
##
                                             "age x year
   [11] "age x primary2004"
##
                                             "age x hhsi:
   [13] "age x sex male"
##
                                             "age x messa
   [15] "age_x_messages_Hawthorne"
                                             "age x messa
  [17] "yearofbirth x primary2004"
                                             "primary2004
                                             "primary2004
## [19] "primary2004 x sex male"
## [21] "primary2004_x_messages_Hawthorne"
                                             "primary2004
```

```
soc_test_processed <- social_recip |>
 prep() |>
  bake(new data = soc test)
soc_test_processed
## # A tibble: 76,467 x 22
     yearofbirth primary2004 hhsize primary2006
##
                                                    age se
##
            <dbl>
                        <dbl> <dbl>
                                           <dbl> <dbl>
## 1
          -15.2
                            0 - 0.184
                                               0 15.2
## 2
           11.8
                            0 - 0.184
                                               0 -11.8
   3
##
           10.8
                            0 -0.184
                                               0 -10.8
          4.79
##
   4
                           1 -0.184
                                               1 - 4.79
##
   5
          -13.2
                           0 -1.18
                                               1 13.2
          -17.2
                            0 -1.18
                                               1 17.2
##
   6
##
           -2.21
                            0 1.82
                                               0 2.21
##
   8
           -7.21
                            1 - 0.184
                                               0
                                                  7.21
   9
            6.79
                            0 - 0.184
                                               0 - 6.79
##
            2.79
## 10
                            0 - 1.18
                                               0
                                                  -2.79
```

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Regularization Methods: LASSO, ridge regression, elastic nets

Feature Selection

▶ Wrappers: pick subset of covars, train on data (estimate model), test on hold-out, score predictions. Keep best-scoring subset.

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- ► Embeds: select features and estimate model at same time. Penalize using more predictors.

OLS reminder

Minimize SSR:

$$\underset{\beta}{\operatorname{arg\,min}} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\underset{\beta}{\operatorname{arg\,min}} \sum_{i=1}^{n} (y_i - \mathbf{X}\hat{\beta})^2$$

L1 regularization: the LASSO (Least Absolute Shrinkage and Selection Operator)

$$\underset{\beta}{\operatorname{arg\,min}} \left[\sum_{i=1}^{n} \left(y_i - \mathbf{X} \hat{\beta} \right)^2 + \lambda \sum_{j=1}^{k} |\hat{\beta}_j| \right]$$

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L2 regularization: Ridge regression

$$\underset{\beta}{\operatorname{arg\,min}} \left[\sum_{i=1}^{n} \left(y_i - \mathbf{X} \hat{\beta} \right)^2 + \lambda \sum_{j=1}^{k} \hat{\beta}_j^2 \right]$$

Mix L1 and L2: Elastic net

$$\underset{\beta}{\operatorname{arg\,min}} \left(\frac{\sum_{i=1}^{n} \left(y_i - \mathbf{X} \hat{\beta} \right)^2}{2n} + \lambda \left[\alpha \sum_{j=1}^{k} |\hat{\beta}_j| + \frac{1 - \alpha}{2} \sum_{j=1}^{k} \hat{\beta}_j^2 \right] \right)$$

Mix L1 and L2: Elastic net

$$\underset{\beta}{\operatorname{arg\,min}} \left(\frac{\sum_{i=1}^{n} \left(y_i - \mathbf{X} \hat{\beta} \right)^2}{2n} + \lambda \left[\alpha \sum_{j=1}^{k} |\hat{\beta}_j| + \frac{1 - \alpha}{2} \sum_{j=1}^{k} \hat{\beta}_j^2 \right] \right)$$

Regularized trees, ...

R packages for Regularization, etc.

- ▶ glmnet
- ▶ caret

See also tidymodels, parsnip, ...

References

Breiman, Leo. 2001. "Statistical Modeling: The Two Cultures (with comments and a rejoinder by the author)." *Statistical Science* 16 (3): 199–231. https://doi.org/10.1214/ss/1009213726.