Concepts in Machine Learning Winter Institute in Data Science

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2024 - 01 - 08

Building Models

Modeling Helper Functions

Example: mtcars

Example: Social Pressure Experiment (recipes)

Regularization Methods: LASSO, ridge regression, elastic nets

Building Models

What are our goals?

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- 1. Generative modeling
- 2. Predictive modeling

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- 2. Predictive modeling

Breiman (2001)

► Theory (novel theory, prior theory, prior findings)

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- Raw data ("data look nonlinear, so ... + $\beta x^2 + ...$ ")
- Specification searching (repeat modeling with same data)
- Training and testing (repeat modeling, different data)

All the important ones (No omitted variable bias)

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- No irrelevant ones (No included variable bias)

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Helpful?

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Helpful?

- ► Affect outcome
- ► Confounders
- ▶ Pre-treatment only
- ► Avoid post-treatment
- ► "In-horizon"
- ► Test something "out-of-horizon"

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- ▶ Pre-treatment only
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- ► "In-horizon"
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(Sometimes it will depend on goals.)

What to include, when thousands of predictors?

What to include, when thousands of predictors? "Machine learning"

What to include, when thousands of predictors? "Machine learning"

(but "machine learning" can mean different things.)



I finally found it in real life: the consultant who runs OLS in Excel and calls it machine learning

9:17 AM \cdot Jan 31, 2019 \cdot Twitter for iPhone

54 Retweets	7 Quote Tweets	511 Likes		
\Diamond	17	\bigcirc	\uparrow	

Figure 1: Don't do this.



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Figure 1: Don't do this.

If you can't describe the procedure's "learning", it may not be "machine learning".



. . .

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Figure 1: Don't do this.

If you can't describe the procedure's "learning", it may not be "machine learning".

There should probably be some testing/training, regularization, $\,$

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Modeling Helper Functions

```
data(sim1)
lm out \leftarrow lm(y \sim x, data = sim1)
tidy(lm out)
## # A tibble: 2 x 5
## term estimate std.error statistic p.value
## <chr> <dbl>
                         <dbl>
                                  <dbl> <dbl>
## 1 (Intercept) 4.22 0.869 4.86 4.09e- 5
                  2.05 0.140 14.7 1.17e-14
## 2 x
```

2 x

##

```
data(sim1)
lm_out \leftarrow lm(y \sim x, data = sim1)
tidy(lm out)
## # A tibble: 2 x 5
## term estimate std.error statistic p.value
## <chr> <dbl> <dbl> <dbl> <dbl>
## 1 (Intercept) 4.22 0.869 4.86 4.09e- 5
```

glance(lm out) ## # A tibble: 1 x 12 r.squared adj.r.squared sigma statistic p.value d: ## <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <

1 0.885 0.880 2.20 215. 1.17e-14_{28/103}

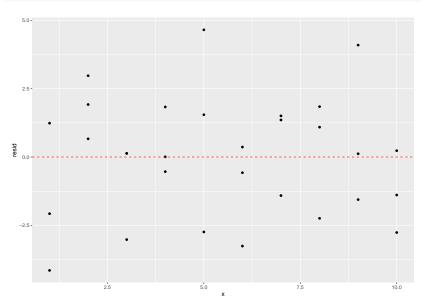
2.05 0.140 14.7 1.17e-14

Special mutate() functions:

Special mutate() functions:

```
(sim1 <- sim1 |> add_residuals(lm out))
## # A tibble: 30 \times 3
##
                       resid
           Х
                 У
      <int> <dbl>
                       <dbl>
##
##
              4.20 - 2.07
           1 7.51
                    1.24
##
           1 \quad 2.13 \quad -4.15
##
    3
           2 8.99
##
                    0.665
##
    5
           2 10.2 1.92
##
           2 11.3 2.97
           3 \quad 7.36 \quad -3.02
##
##
    8
           3 10.5
                    0.130
           3 10.5 0.136
##
           4 12.4
                    0.00763
##
   10
       20
          more rows
```

```
ggplot(sim1, aes(x, resid)) + geom_point() +
  geom_hline(yintercept = 0, linetype = 2, color = "re
```

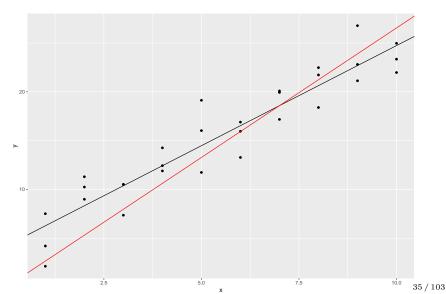


Special mutate() functions:

```
(sim1 <- sim1 |> add_predictions(lm out))
## # A tibble: 30 x 4
##
                  resid
         Х
              У
                         pred
     <int> <dbl> <dbl> <dbl>
##
           4.20 -2.07
                         6.27
##
         1
##
         1 7.51 1.24 6.27
##
   3
         1 2.13 -4.15 6.27
         2 8.99 0.665
##
                         8.32
##
   5
         2 10.2 1.92
                         8.32
         2 11.3 2.97
                         8.32
##
         3 7.36 -3.02
##
                        10.4
   8
         3 10.5 0.130
                        10.4
##
         3 10.5 0.136
                        10.4
##
         4 12.4
                 0.00763 12.4
## 10
      20
         more rows
```

```
lm_out2 \leftarrow lm(y \sim x - 1, data = sim1)
```

```
ggplot(sim1, aes(x, y)) + geom_point() +
geom_abline(intercept = coef(lm_out)[1], slope = coef(lm_geom_abline(intercept = 0, slope = coef(lm_out2)["x"], coef(lm_out2)["x"]
```



```
glance(lm_out)
## # A tibble: 1 x 12
   r.squared adj.r.squared sigma statistic p.value d:
##
      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <
##
      0.885 0.880 2.20 215. 1.17e-14
## 1
## # i 3 more variables: deviance <dbl>, df.residual <int>
glance(lm out2)
## # A tibble: 1 x 12
##
   r.squared adj.r.squared sigma statistic p.value df
       ##
## 1
      0.970 0.969 2.94 NA
                                      NA
                                           NA
```

i 3 more variables: deviance <dbl>, df.residual <int>

```
## # A tibble: 1 x 12
## r.squared adj.r.squared sigma statistic p.value df
## <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> = 1.17e-14
## i 3 more variables: deviance <dbl>, df.residual <int>
```

glance(lm_out2)

 $(R^2 \text{ and predictive quality are not the same thing } \dots)$

modelr Helper Functions

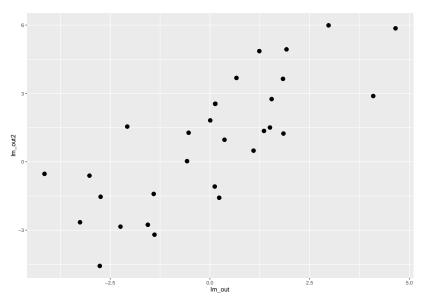
```
( sim1 <- sim1 |> spread residuals(lm out, lm out2) )
## # A tibble: 30 x 6
##
                  resid pred lm out lm out2
              V
         Х
##
     <int> <dbl> <dbl> <dbl>
                                <dbl>
                                       <dbl>
##
           4.20 -2.07
                         6.27 - 2.07
                                       1.55
##
  2
         1 7.51
                1.24
                         6.27 1.24
                                       4.86
   3
         1 2.13 -4.15
##
                        6.27 - 4.15
                                      -0.529
##
         2 8.99
                 0.665
                         8.32 0.665
                                       3.68
   5
         2 10.2 1.92
                         8.32 1.92
                                       4.93
##
         2 11.3 2.97
                         8.32 2.97
##
                                       5.99
         3 7.36 -3.02
                        10.4 -3.02
                                      -0.607
##
##
   8
         3 10.5 0.130
                        10.4 0.130
                                       2.54
         3 10.5 0.136
                        10.4 0.136
##
                                       2.55
         4 12.4
                 0.00763 12.4 0.00763
                                       1.82
##
  10
## # i 20 more rows
```

```
ggplot(sim1, aes(x, lm_out2)) + geom_point() +
 geom_hline(yintercept = 0, linetype = 2, color = "red")
Im_out2
```

2.5

10.0

ggplot(sim1, aes(lm_out, lm_out2)) + geom_point(size = 3)

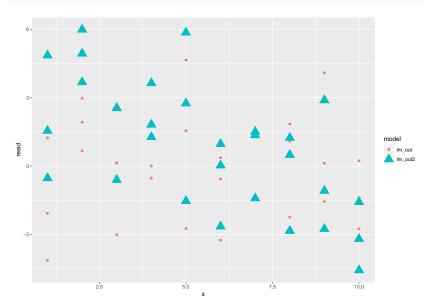


modelr Helper Functions

```
data(sim1)
( sim1 <- sim1 |> gather residuals(lm out, lm out2) )
## # A tibble: 60 \times 4
                 y resid
##
    model x
## <chr> <int> <dbl> <dbl>
## 1 lm out 1 4.20 -2.07
## 2 lm out 1 7.51 1.24
## 3 lm out 1 2.13 -4.15
   4 lm out 2 8.99 0.665
##
## 5 lm out 2 10.2 1.92
  6 lm out 2 11.3 2.97
##
## 7 lm out 3 7.36 -3.02
   8 lm out 3 10.5 0.130
##
   9 lm out 3 10.5 0.136
##
## 10 lm out 4 12.4 0.00763
## # i 50 more rows
```

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```
ggplot(sim1, aes(x, resid)) +
  geom_point(aes(color = model, size = model, shape = model)
```



modelr Helper Functions

- add_residuals()
- spread_residuals()
- gather_residuals()
- add_predictions()
- spread_predictions()
- gather_predictions()

Other Helpers for Many Models: tidy()

```
11 <- list(lm_out, lm_out2)</pre>
11 |> map_df(tidy)
## # A tibble: 3 \times 5
##
    term estimate std.error statistic p.value
##
    <chr>
                 <dbl>
                           <dbl>
                                    <dbl>
                                            <dbl>
## 1 (Intercept)
                   4.22
                          0.869 4.86 4.09e- 5
## 2 x
                   2.05
                          0.140
                                    14.7 1.17e-14
                   2.65
## 3 x
                          0.0865 30.7 1.15e-23
```

Many Models: glance()

0.885

0.970

1

2

11 |> map df(glance) |> select(1:6)

0.969 2.94

0.880 2.20 215. 1.17e-14

NA NA

Many Models: glance()

```
11 |> map df(glance) |> select(1:6)
## # A tibble: 2 x 6
    r.squared adj.r.squared sigma statistic p.value
##
       <dbl> <dbl> <dbl> <dbl>
##
                                       <dbl> <dbl
## 1 0.885 0.880 2.20 215. 1.17e-14
## 2 0.970
               0.969 2.94 NA NA
11 |> map_df(glance) |> select(7:12)
## # A tibble: 2 x 6
##
    logLik AIC BIC deviance df.residual nobs
## <dbl> <dbl> <dbl> <int> <int>
## 1 -65.2 136. 141. 136.
                                 28 30
## 2 -74.4 153. 156. 250.
                                 29
                                      30
```

Example: mtcars

1. Feature engineering

1. Feature engineering: collect/create the data

- 1. Feature engineering: collect/create the data
- 2. Data splitting

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data

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- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)

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- 2. Data splitting: split the data
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- ► Testing. (20%?)
- 3. Feature selection

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
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- 3. Feature selection: algorithms decide predictors to include

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- 2. Data splitting: split the data
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- ► Testing. (20%?)
- 3. Feature selection: algorithms decide predictors to include
- 4. Model estimation

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)
- 3. Feature selection: algorithms decide predictors to include
- 4. Model estimation: find the slopes (e.g.)

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)
- 3. Feature selection: algorithms decide predictors to include
- 4. Model estimation: find the slopes (e.g.)
- 5. Validation + testing

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)
- 3. Feature selection: algorithms decide predictors to include
- 4. Model estimation: find the slopes (e.g.)
- 5. Validation + testing: evaluate preds from trained models using new data

```
library(tidymodels)
data_split <- initial_split(mtcars, prop = 2/3)

df_train <- training(data_split)
df_test <- testing(data_split)</pre>
```

```
library(tidymodels)
data_split <- initial_split(mtcars, prop = 2/3)</pre>
df train <- training(data split)</pre>
df_test <- testing(data_split)</pre>
dim(df train)
## [1] 21 11
dim(df test)
## [1] 11 11
```

```
lm_fit <- linear_reg() |> fit(mpg ~ ., data = df_train)
lm fit
## parsnip model object
##
##
## Call:
## stats::lm(formula = mpg ~ ., data = data)
##
## Coefficients:
## (Intercept)
                      cyl
                                 disp
                                               hp
## -52.788434 0.971688
                             0.012064 0.003002
##
         qsec
                       VS
                                   am
                                             gear
## 1.969451 -4.052556 -0.030317 8.247851
```

```
##
                              lm
                     mpg
## Duster 360
                   14.3 11.10105
## Merc 230
                  22.8 31.10956
               17.3 15.27357
## Merc 450SL
## Lincoln Continental 10.4 12.09299
## Honda Civic
                 30.4 28.05338
## Toyota Corolla 33.9 30.39759
## Toyota Corona
               21.5 20.00855
               13.3 11.64362
## Camaro 728
## Porsche 914-2 26.0 34.60792
## Ford Pantera L
                15.8 29.41662
## Ferrari Dino
                 19.7 19.64479
```

Next, predict with random forest algorithm.

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▶ Boosting: models build on prior models

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▶ Boosting: models build on prior models

Next, predict with random forest algorithm.

Ensemble learning algorithms:

- ▶ Boosting: models build on prior models ↔ pick feature, predict, upweight mispredicted data, Do several times and combine.
- ▶ Bagging: (random select units, model) \rightarrow many times. No building.

Next, predict with random forest algorithm.

Ensemble learning algorithms:

- ▶ Boosting: models build on prior models ↔ pick feature, predict, upweight mispredicted data, Do several times and combine.
- ▶ Bagging: (random select units, model) \rightarrow many times. No building.

Next, predict with random forest algorithm.

Ensemble learning algorithms:

- ▶ Boosting: models build on prior models ↔ pick feature, predict, upweight mispredicted data, Do several times and combine.
- ▶ Bagging: (random select units, model) \rightarrow many times. No building.

Random Forests are bagging algorithms.

Ranger result

Number of trees:

Target node size:

Variable importance mode:

Number of independent variables: 10

Sample size:

##

##

Call:

Type:

Mtry:

```
rf_fit <- rand_forest(mode = "regression") |>
  fit(mpg ~ ., data = df_train)
rf_fit

## parsnip model object
##
```

ranger::ranger(x = maybe_data_frame(x), y = y, num.thre

Regression

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500

21

3

5

none

parsnip::rand_forest() uses ranger engine

parsnip::rand_forest() uses ranger engine
There is also "Spark".

```
##
                                lm
                                      rf
                       mpg
## Duster 360
                    14.3 11.10105 15.65025
## Merc 230
                   22.8 31.10956 22.87099
## Merc 450SL
                  17.3 15.27357 15.83648
## Lincoln Continental 10.4 12.09299 13.16725
## Honda Civic
                      30.4 28.05338 28.26391
## Toyota Corolla
                      33.9 30.39759 28.68535
                     21.5 20.00855 23.31111
## Toyota Corona
## Camaro 728
                     13.3 11.64362 16.12771
## Porsche 914-2 26.0 34.60792 25.01991
## Ford Pantera I.
                 15.8 29.41662 18.44409
               19.7 19.64479 20.67973
## Ferrari Dino
```

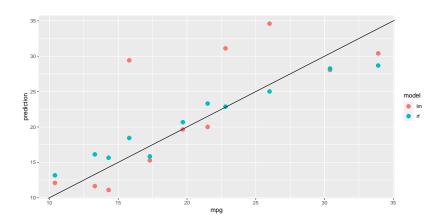
tidymodels Example

```
out_preds_long <- out_preds |>
 pivot_longer(cols = c(lm, rf),
              names to = "model",
              values to = "prediction")
out preds long
## # A tibble: 22 x 3
##
       mpg model prediction
     <dbl> <dbl> <dbl>
##
## 1 14.3 lm
                    11.1
##
   2 14.3 rf
                      15.7
##
   3 22.8 lm
                      31.1
##
   4 22.8 rf
                      22.9
##
   5 17.3 lm
                      15.3
   6 17.3 rf
##
                      15.8
   7 10.4 lm
##
                      12.1
##
   8
     10.4 rf
                      13.2
```

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tidymodels Example

```
ggplot(out_preds_long, aes(mpg, prediction)) +
  geom_point(aes(color = model), size = 3) +
  geom_abline(slope = 1, intercept = 0)
```



tidymodels Example

Evaluate:

```
out preds |> metrics(truth = mpg, estimate = lm) |>
 rename(lm = .estimate) |>
 left join(out preds |>
             metrics(truth = mpg, estimate = rf) |>
             rename(rf = .estimate))
## Joining with 'by = join by(.metric, .estimator)'
## # A tibble: 3 \times 4
## .metric .estimator
                      lm rf
## <chr> <chr> <dbl> <dbl>
## 1 rmse standard 5.79 2.40
## 2 rsq standard 0.572 0.945
## 3 mae standard 4.23 2.02
```

Example: Social Pressure Experiment (recipes)

Data Splitting

```
social <- read_csv("https://raw.githubusercontent.com/"
soc_split <- initial_split(social)
soc_train <- training(soc_split)
soc_test <- testing(soc_split)</pre>
```

Data Splitting

```
social <- read csv("https://raw.githubusercontent.com/)</pre>
soc split <- initial_split(social)</pre>
soc train <- training(soc split)</pre>
soc test <- testing(soc split)</pre>
dim(soc train)
## [1] 229399
dim(soc test)
## [1] 76467
```

```
social_recip <- recipe(primary2006 ~ ., data = soc_train)
social_recip</pre>
```

summary(social_recip)

```
social_recip <- social_recip |>
step_mutate(age = 2006 - yearofbirth) |>
step_dummy(all_nominal(), -all_outcomes())
```

```
social_recip
##
## -- Recipe ------
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
##
## -- Operations
## * Variable mutation for: 2006 - yearofbirth
## * Dummy variables from: all_nominal(), -all_outcomes()
```

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```
social_recip <- social_recip |>
step_zv(all_predictors())
```

```
##
## -- Recipe ------
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
##
## -- Operations
## * Variable mutation for: 2006 - yearofbirth
## * Dummy variables from: all_nominal(), -all_outcomes()
## * Zero variance filter on: all_predictors()
```

social_recip

```
social_recip <- social_recip |>
step_center(all_predictors(), -primary2004)
```

```
social_recip
##
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
##
## -- Operations
## * Variable mutation for: 2006 - yearofbirth
## * Dummy variables from: all_nominal(), -all_outcomes()
## * Zero variance filter on: all_predictors()
## * Centering for: all_predictors(), -primary2004
```

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Recipe complete. Time to prep and bake.

Recipe complete. Time to prep and bake.

```
social recip |>
  prep()
##
## -- Recipe
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
```

```
soc_train_processed <- social_recip |>
  prep() |>
  bake(new data = NULL)
soc train processed
## # A tibble: 229,399 x 22
      yearofbirth primary2004 hhsize primary2006
##
                                                     age se
##
            <dbl>
                        <dbl> <dbl>
                                            <dbl> <dbl>
             2.78
                                                0 - 2.78
## 1
                            1 0.815
## 2
          -10.2
                            0 -0.185
                                                0 10.2
    3
##
           27.8
                            1 3.81
                                                1 - 27.8
          -3.22
##
    4
                            0 1.81
                                                0
                                                  3.22
##
    5
           -15.2
                            0 - 0.185
                                                0 15.2
           -11.2
                            0 - 1.19
                                                1 11.2
##
    6
##
            7.78
                            1 - 0.185
                                                1 - 7.78
##
    8
            22.8
                            1 - 1.19
                                                0 - 22.8
                                                1 17.2
           -17.2
                            0 - 0.185
##
                            0 - 0.185
                                                0
                                                   14.2
## 10
           -14.2
```

200 mama marra

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names(soc_train_processed)

```
"primary2004
##
    [1] "yearofbirth"
                                             "primary2000
##
    [3] "hhsize"
    [5] "age"
##
                                             "sex male"
##
    [7] "messages Control"
                                             "messages Ha
    [9] "messages Neighbors"
##
                                             "age x year
   [11] "age x primary2004"
##
                                             "age x hhsi:
   [13] "age x sex male"
##
                                             "age x messa
   [15] "age_x_messages_Hawthorne"
                                             "age x messa
  [17] "yearofbirth x primary2004"
                                             "primary2004
                                             "primary2004
## [19] "primary2004 x sex male"
## [21] "primary2004_x_messages_Hawthorne"
                                             "primary2004
```

```
soc_test_processed <- social_recip |>
 prep() |>
  bake(new data = soc test)
soc_test_processed
## # A tibble: 76,467 x 22
##
     yearofbirth primary2004 hhsize primary2006
                                                     age se
##
            <dbl>
                        <dbl>
                               <dbl>
                                           <dbl>
                                                   <dbl>
## 1
           -5.22
                            0
                               0.815
                                                   5.22
##
          -6.22
                            0
                              0.815
                                                   6.22
   3
         25.8
##
                            0
                              0.815
                                               1 - 25.8
        -0.223
##
   4
                            0 0.815
                                                   0.223
##
   5
           10.8
                            0 - 0.185
                                               0 - 10.8
         -11.2
                            0 - 0.185
                                               0 11.2
##
   6
```

0 - 0.185

0 -1.19

0

0 - 0.185

1.81

7.22

1 17.2

1 - 11.8

0 - 22.8

92 / 103

##

9 ## 10

8

-7.22

-17.2

11.8

22.8

: 76 /F7 mama marra

Regularization Methods: LASSO, ridge regression, elastic nets

Feature Selection

▶ Wrappers: pick subset of covars, train on data (estimate model), test on hold-out, score predictions. Keep best-scoring subset.

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- ► Embeds: select features and estimate model at same time. Penalize using more predictors.

OLS reminder

Minimize SSR:

$$\underset{\beta}{\operatorname{arg\,min}} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\underset{\beta}{\operatorname{arg\,min}} \sum_{i=1}^{n} (\mathbf{y} - \mathbf{X}\hat{\beta})^2$$

L1 regularization: the LASSO (Least Absolute Shrinkage and Selection Operator)

$$\underset{\beta}{\operatorname{arg\,min}} \left[\sum_{i=1}^{n} \left(y_i - \mathbf{X} \hat{\beta} \right)^2 + \lambda \sum_{j=1}^{k} |\hat{\beta}_j| \right]$$

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L2 regularization: Ridge regression

$$\underset{\beta}{\operatorname{arg\,min}} \left[\sum_{i=1}^{n} \left(y_i - \mathbf{X} \hat{\beta} \right)^2 + \lambda \sum_{j=1}^{k} \hat{\beta}_j^2 \right]$$

Mix L1 and L2: Elastic net

$$\underset{\beta}{\operatorname{arg\,min}} \left(\frac{\sum_{i=1}^{n} \left(y_i - \mathbf{X} \hat{\beta} \right)^2}{2n} + \lambda \left[\alpha \sum_{j=1}^{k} |\hat{\beta}_j| + \frac{1 - \alpha}{2} \sum_{j=1}^{k} \hat{\beta}_j^2 \right] \right)$$

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Regularized trees, ...

R packages for Regularization, etc.

- ▶ glmnet
- ▶ caret

See also tidymodels, parsnip, ...

References

Breiman, Leo. 2001. "Statistical Modeling: The Two Cultures (with comments and a rejoinder by the author)." *Statistical Science* 16 (3): 199–231. https://doi.org/10.1214/ss/1009213726.