# Concepts in Machine Learning Winter Institute in Data Science

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2025 - 01 - 08

**Building Models** 

Modeling Helper Functions

Example: mtcars

Example: Social Pressure Experiment (recipes)

Regularization Methods: LASSO, ridge regression, elastic nets

# **Building Models**

What are our goals?

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- 1. Generative modeling
- 2. Predictive modeling

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- 2. Predictive modeling

Breiman (2001)

► Theory (novel theory, prior theory, prior findings)

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- Raw data ("data look nonlinear, so ... +  $\beta x^2 + ...$ ")
- Specification searching (repeat modeling with same data)
- Training and testing (repeat modeling, different data)

All the important ones (No omitted variable bias)

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- No irrelevant ones (No included variable bias)

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Helpful?

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- No irrelevant ones (No included variable bias)

## Helpful?

- ► Affect outcome
- ► Confounders
- ▶ Pre-treatment only
- ► Avoid post-treatment
- ► "In-horizon"
- ► Test something "out-of-horizon"

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- ► "In-horizon"
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(Sometimes it will depend on goals.)

What to include, when thousands of predictors?

What to include, when thousands of predictors? "Machine learning"

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(but "machine learning" can mean different things.)



# I finally found it in real life: the consultant who runs OLS in Excel and calls it machine learning

9:17 AM  $\cdot$  Jan 31, 2019  $\cdot$  Twitter for iPhone

<b>54</b> Retweets	7 Quote Tweets	<b>511</b> Likes		
$\Diamond$	17	$\bigcirc$	$\uparrow$	

Figure 1: Don't do this.



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Figure 1: Don't do this.

If you can't describe the procedure's "learning", it may not be "machine learning".



. . .

# I finally found it in real life: the consultant who runs OLS in Excel and calls it machine learning

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Figure 1: Don't do this.

If you can't describe the procedure's "learning", it may not be "machine learning".

There should probably be some testing/training, regularization,  $\,$ 

25 / 103

# Modeling Helper Functions

```
data(sim1)
lm_out \leftarrow lm(y \sim x, data = sim1)
tidy(lm_out)
## # A tibble: 2 x 5
## term estimate std.error statistic p.value
## <chr> <dbl>
                          <dbl>
                                  <dbl> <dbl>
## 1 (Intercept) 4.22 0.869 4.86 4.09e- 5
## 2 x
                  2.05 0.140 14.7 1.17e-14
```

```
glance(lm_out) |> select(1:5)
## # A tibble: 1 x 5
    r.squared adj.r.squared sigma statistic p.value
##
       <dbl> <dbl> <dbl> <dbl> <dbl> <
##
              0.880 2.20 215. 1.17e-14
       0.885
## 1
glance(lm out) |> select(6:12)
## # A tibble: 1 x 7
      df logLik AIC BIC deviance df.residual nobs
##
## <dbl> <dbl> <dbl> <dbl> <int> <int>
                            136.
## 1 1 -65.2 136. 141.
                                        28
                                             30
```

Special mutate() functions:

Special mutate() functions:

```
(sim1 <- sim1 |> add_residuals(lm out))
## # A tibble: 30 \times 3
##
                       resid
           Х
                 У
      <int> <dbl>
                       <dbl>
##
##
              4.20 - 2.07
           1 7.51
                    1.24
##
           1 \quad 2.13 \quad -4.15
##
    3
           2 8.99
##
                    0.665
##
    5
           2 10.2 1.92
##
           2 11.3 2.97
           3 \quad 7.36 \quad -3.02
##
##
    8
           3 10.5
                    0.130
           3 10.5 0.136
##
           4 12.4
                    0.00763
##
   10
       20
          more rows
```

```
ggplot(sim1, aes(x, resid)) + geom_point() +
  geom_hline(yintercept = 0, linetype = 2, color = "re
 5.0 -
 2.5 -
resid
 -2.5 -
```

5.0

7.5

2.5

10.0

Special mutate() functions:

```
(sim1 <- sim1 |> add_predictions(lm out))
## # A tibble: 30 x 4
##
                  resid
         Х
              У
                         pred
     <int> <dbl> <dbl> <dbl>
##
           4.20 -2.07
                         6.27
##
         1
##
         1 7.51 1.24 6.27
##
   3
         1 2.13 -4.15 6.27
         2 8.99 0.665
##
                         8.32
##
   5
         2 10.2 1.92
                         8.32
         2 11.3 2.97
                         8.32
##
         3 7.36 -3.02
##
                        10.4
   8
         3 10.5 0.130
                        10.4
##
         3 10.5 0.136
                        10.4
##
         4 12.4
                 0.00763 12.4
## 10
      20
         more rows
```

```
lm_out2 \leftarrow lm(y \sim x - 1, data = sim1)
```

```
ggplot(sim1, aes(x, y)) + geom_point() +
  geom_abline(intercept = coef(lm_out)[1], slope = coef(lm_out)
  geom_abline(intercept = 0, slope = coef(lm_out2)["x"], co
 20 -
 10-
```

```
glance(lm_out)
## # A tibble: 1 x 12
   r.squared adj.r.squared sigma statistic p.value d:
##
      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <
##
      0.885 0.880 2.20 215. 1.17e-14
## 1
## # i 3 more variables: deviance <dbl>, df.residual <int>
glance(lm out2)
## # A tibble: 1 x 12
##
   r.squared adj.r.squared sigma statistic p.value df
       ##
## 1
      0.970 0.969 2.94 NA
                                      NA
                                           NA
```

## # i 3 more variables: deviance <dbl>, df.residual <int>

```
## # A tibble: 1 x 12
## r.squared adj.r.squared sigma statistic p.value df
## <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> = 1.17e-14
## i 3 more variables: deviance <dbl>, df.residual <int>
```

#### glance(lm\_out2)

 $(R^2 \text{ and predictive quality are not the same thing } \dots)$ 

#### modelr Helper Functions

```
( sim1 <- sim1 |> spread residuals(lm out, lm out2) )
## # A tibble: 30 x 6
##
                  resid pred lm out lm out2
              V
         Х
##
     <int> <dbl> <dbl> <dbl>
                                <dbl>
                                       <dbl>
##
           4.20 -2.07
                         6.27 - 2.07
                                       1.55
##
  2
         1 7.51
                1.24
                         6.27 1.24
                                       4.86
   3
         1 2.13 -4.15
##
                        6.27 - 4.15
                                      -0.529
##
         2 8.99
                 0.665
                         8.32 0.665
                                       3.68
   5
         2 10.2 1.92
                         8.32 1.92
                                       4.93
##
         2 11.3 2.97
                         8.32 2.97
##
                                       5.99
         3 7.36 -3.02
                        10.4 -3.02
                                      -0.607
##
##
   8
         3 10.5 0.130
                        10.4 0.130
                                       2.54
         3 10.5 0.136
                        10.4 0.136
##
                                       2.55
         4 12.4
                 0.00763 12.4 0.00763
                                       1.82
##
  10
## # i 20 more rows
```

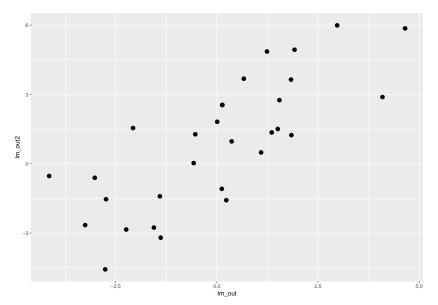
```
ggplot(sim1, aes(x, lm_out2)) + geom_point() +
  geom_hline(yintercept = 0, linetype = 2, color = "red")
lm_out2
 -3-
```

5.0

7.5

10.0

#### ggplot(sim1, aes(lm\_out, lm\_out2)) + geom\_point(size = 3)

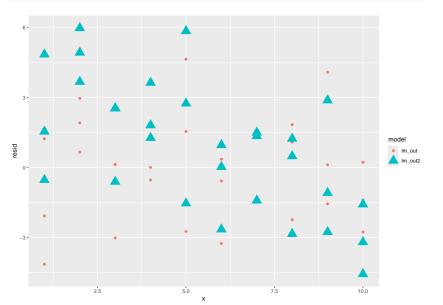


#### modelr Helper Functions

```
data(sim1)
( sim1 <- sim1 |> gather residuals(lm out, lm out2) )
## # A tibble: 60 \times 4
                 y resid
##
    model x
## <chr> <int> <dbl> <dbl>
## 1 lm out 1 4.20 -2.07
## 2 lm out 1 7.51 1.24
## 3 lm out 1 2.13 -4.15
   4 lm out 2 8.99 0.665
##
## 5 lm out 2 10.2 1.92
  6 lm out 2 11.3 2.97
##
## 7 lm out 3 7.36 -3.02
   8 lm out 3 10.5 0.130
##
   9 lm out 3 10.5 0.136
##
## 10 lm out 4 12.4 0.00763
## # i 50 more rows
```

41 / 103

```
ggplot(sim1, aes(x, resid)) +
  geom_point(aes(color = model, size = model, shape = model)
```



#### modelr Helper Functions

- add\_residuals()
- spread\_residuals()
- gather\_residuals()
- add\_predictions()
- spread\_predictions()
- gather\_predictions()

#### Other Helpers for Many Models: tidy()

```
11 <- list(lm_out, lm_out2)</pre>
11 |> map_df(tidy)
## # A tibble: 3 \times 5
##
    term estimate std.error statistic p.value
##
    <chr>
                 <dbl>
                           <dbl>
                                    <dbl>
                                            <dbl>
## 1 (Intercept)
                   4.22
                          0.869 4.86 4.09e- 5
## 2 x
                   2.05
                          0.140
                                    14.7 1.17e-14
                   2.65
## 3 x
                          0.0865 30.7 1.15e-23
```

#### Many Models: glance()

0.885

0.970

## 1

## 2

11 |> map df(glance) |> select(1:6)

0.969 2.94

0.880 2.20 215. 1.17e-14

NA NA

#### Many Models: glance()

```
11 |> map df(glance) |> select(1:6)
## # A tibble: 2 x 6
    r.squared adj.r.squared sigma statistic p.value
##
       <dbl> <dbl> <dbl> <dbl>
##
                                       <dbl> <dbl
## 1 0.885 0.880 2.20 215. 1.17e-14
## 2 0.970
               0.969 2.94 NA NA
11 |> map_df(glance) |> select(7:12)
## # A tibble: 2 x 6
##
    logLik AIC BIC deviance df.residual nobs
## <dbl> <dbl> <dbl> <int> <int>
## 1 -65.2 136. 141. 136.
                                 28 30
## 2 -74.4 153. 156. 250.
                                 29
                                      30
```

Example: mtcars

1. Feature engineering

1. Feature engineering: collect/create the data

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- 2. Data splitting

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- 2. Data splitting: split the data

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- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)

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- 2. Data splitting: split the data
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- ightharpoonup Testing. (20%?)
- 3. Feature selection

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- 4. Model estimation

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)
- 3. Feature selection: algorithms decide predictors to include
- 4. Model estimation: find the slopes (e.g.)

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)
- 3. Feature selection: algorithms decide predictors to include
- 4. Model estimation: find the slopes (e.g.)
- 5. Validation + testing

- 1. Feature engineering: collect/create the data
- 2. Data splitting: split the data
- ► Training. (80%? further split ("cross-validation")?)
- ▶ Validation. (for hyperparams; can be small (?))
- ► Testing. (20%?)
- 3. Feature selection: algorithms decide predictors to include
- 4. Model estimation: find the slopes (e.g.)
- 5. Validation + testing: evaluate preds from trained models using new data

```
library(tidymodels)
data_split <- initial_split(mtcars, prop = 2/3)

df_train <- training(data_split)
df_test <- testing(data_split)</pre>
```

```
library(tidymodels)
data_split <- initial_split(mtcars, prop = 2/3)</pre>
df train <- training(data split)</pre>
df_test <- testing(data_split)</pre>
dim(df train)
## [1] 21 11
dim(df test)
## [1] 11 11
```

```
lm_fit <- linear_reg() |> fit(mpg ~ ., data = df_train)
lm fit
## parsnip model object
##
##
## Call:
## stats::lm(formula = mpg ~ ., data = data)
##
## Coefficients:
## (Intercept)
                      cyl
                                 disp
                                               hp
## -22.67228 -0.49328
                              0.01129
                                           0.01901
##
         qsec
                       VS
                                   am
                                             gear
      3.20799 -4.19044 -1.84671
                                           1.58796
##
```

```
##
                             ٦m
                   mpg
## Datsun 710
                22.8 24.692631
          22.8 35.122259
## Merc 230
## Merc 280C 17.8 21.218543
## Merc 450SLC 15.2 18.318645
## Chrysler Imperial 14.7 8.185006
## Fiat 128
                  32.4 27.711266
## Toyota Corona 21.5 28.429983
## AMC Javelin 15.2 17.863348
## Pontiac Firebird 19.2 15.456183
## Ferrari Dino 19.7 21.071413
## Maserati Bora 15.0 17.150351
```

Next, predict with random forest algorithm.

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▶ Boosting: models build on prior models

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▶ Boosting: models build on prior models

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Ensemble learning algorithms:

- ▶ Boosting: models build on prior models ↔ pick feature, predict, upweight mispredicted data, . . . . Do several times and combine.
- ▶ Bagging: (random select units, model)  $\rightarrow$  many times. No building.

Next, predict with random forest algorithm.

Ensemble learning algorithms:

- ▶ Boosting: models build on prior models ↔ pick feature, predict, upweight mispredicted data, . . . . Do several times and combine.
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Next, predict with random forest algorithm.

Ensemble learning algorithms:

- ▶ Boosting: models build on prior models ↔ pick feature, predict, upweight mispredicted data, . . . . Do several times and combine.
- ▶ Bagging: (random select units, model)  $\rightarrow$  many times. No building.

Random Forests are bagging algorithms.

## Ranger result

## Number of trees:

## Target node size:

## Variable importance mode:

## Number of independent variables: 10

## Sample size:

##

##

## Call:

## Type:

## Mtry:

```
rf_fit <- rand_forest(mode = "regression") |>
  fit(mpg ~ ., data = df_train)
rf_fit

## parsnip model object
##
```

ranger::ranger(x = maybe\_data\_frame(x), y = y, num.thre

Regression

69 / 103

500

21

3

5

none

parsnip::rand\_forest() uses ranger engine

parsnip::rand\_forest() uses ranger engine
There is also "Spark".

```
##
                              lm
                                    rf
                    mpg
## Datsun 710
                22.8 24.692631 26.61184
## Merc 230
          22.8 35.122259 23.42771
## Merc 280C
           17.8 21.218543 19.83090
## Merc 450SLC 15.2 18.318645 15.99974
## Chrysler Imperial 14.7 8.185006 12.63439
## Fiat 128
                   32.4 27.711266 28.17196
                   21.5 28.429983 24.29935
## Toyota Corona
## AMC Javelin
             15.2 17.863348 17.57158
## Pontiac Firebird 19.2 15.456183 16.54506
## Ferrari Dino 19.7 21.071413 20.43014
## Maserati Bora 15.0 17.150351 15.38326
```

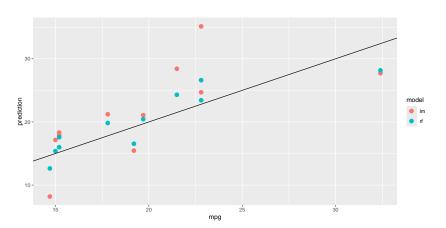
### tidymodels Example

```
out_preds_long <- out_preds |>
 pivot_longer(cols = c(lm, rf),
             names to = "model",
             values to = "prediction")
out preds long
## # A tibble: 22 x 3
##
      mpg model prediction
## <dbl> <chr> <dbl>
              24.7
## 1 22.8 lm
##
   2 22.8 rf 26.6
##
   3 22.8 lm 35.1
##
   4 22.8 rf 23.4
##
   5 17.8 lm
                21.2
   6 17.8 rf
##
                 19.8
   7 15.2 lm
##
                    18.3
##
   8
     15.2 rf
                    16.0
```

73 / 103

#### tidymodels Example

```
ggplot(out_preds_long, aes(mpg, prediction)) +
  geom_point(aes(color = model), size = 3) +
  geom_abline(slope = 1, intercept = 0)
```



#### tidymodels Example

Evaluate:

```
out preds |> metrics(truth = mpg, estimate = lm) |>
 rename(lm = .estimate) |>
 left join(out preds |>
             metrics(truth = mpg, estimate = rf) |>
             rename(rf = .estimate))
## Joining with 'by = join by(.metric, .estimator)'
## # A tibble: 3 \times 4
## .metric .estimator
                      lm rf
## <chr> <chr> <dbl> <dbl>
## 1 rmse standard 5.37 2.39
## 2 rsq standard 0.473 0.783
## 3 mae standard 4.44 2.05
```

# Example: Social Pressure Experiment (recipes)

#### Data Splitting

```
social <- read_csv("https://raw.githubusercontent.com/"
soc_split <- initial_split(social)
soc_train <- training(soc_split)
soc_test <- testing(soc_split)</pre>
```

#### Data Splitting

```
social <- read csv("https://raw.githubusercontent.com/)</pre>
soc split <- initial_split(social)</pre>
soc train <- training(soc split)</pre>
soc test <- testing(soc split)</pre>
dim(soc train)
## [1] 229399
dim(soc test)
## [1] 76467
```

```
social_recip <- recipe(primary2006 ~ ., data = soc_train)
social_recip</pre>
```

#### summary(social\_recip)

```
social_recip <- social_recip |>
step_mutate(age = 2006 - yearofbirth) |>
step_dummy(all_nominal(), -all_outcomes())
```

```
social_recip
##
## -- Recipe ------
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
##
## -- Operations
## * Variable mutation for: 2006 - yearofbirth
## * Dummy variables from: all_nominal() and -all_outcomes
                                                 82 / 103
```

```
social_recip <- social_recip |>
step_zv(all_predictors())
```

```
##
## -- Recipe ------
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
##
## -- Operations
## * Variable mutation for: 2006 - yearofbirth
## * Dummy variables from: all_nominal() and -all_outcomes()
## * Zero variance filter on: all_predictors()
```

social\_recip

```
social_recip <- social_recip |>
step_center(all_predictors(), -primary2004)
```

```
social_recip
##
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
##
## -- Operations
## * Variable mutation for: 2006 - yearofbirth
## * Dummy variables from: all_nominal() and -all_outcomes()
## * Zero variance filter on: all_predictors()
## * Centering for: all_predictors() and -primary2004
```

86 / 103

Recipe complete. Time to prep and bake.

Recipe complete. Time to prep and bake.

```
social recip |>
  prep()
##
## -- Recipe
##
## -- Inputs
## Number of variables by role
## outcome: 1
## predictor: 5
```

```
soc_train_processed <- social_recip |>
  prep() |>
  bake(new data = NULL)
soc train processed
## # A tibble: 229,399 x 22
      yearofbirth primary2004 hhsize primary2006
##
                                                      age ser
##
            <dbl>
                        <dbl> <dbl>
                                            <dbl> <dbl>
## 1
             4.81
                             0 - 0.185
                                                 1 - 4.81
##
             6.81
                             1 - 0.185
                                                0 - 6.81
    3
##
             3.81
                             0 - 0.185
                                                   -3.81
            26.8
##
    4
                             1 1.82
                                                0 - 26.8
##
    5
             9.81
                            0 - 0.185
                                                0 -9.81
             9.81
                             0 - 0.185
                                                0 -9.81
##
    6
##
           -9.19
                             1 - 0.185
                                                0
                                                   9.19
##
    8
           -14.2
                             1 - 0.185
                                                 1 14.2
             3.81
                             0 -1.18
##
```

## 10

-12.2

200 mama marra

0 - 0.185

1 - 3.8112.2 90 / 103

1

#### names(soc\_train\_processed)

```
"primary2004
##
    [1] "yearofbirth"
                                             "primary2000
##
    [3] "hhsize"
    [5] "age"
##
                                             "sex male"
##
    [7] "messages Control"
                                             "messages Ha
    [9] "messages Neighbors"
##
                                             "age x year
   [11] "age x primary2004"
##
                                             "age x hhsi:
   [13] "age x sex male"
##
                                             "age x messa
   [15] "age_x_messages_Hawthorne"
                                             "age x messa
  [17] "yearofbirth x primary2004"
                                             "primary2004
                                             "primary2004
## [19] "primary2004 x sex male"
## [21] "primary2004_x_messages_Hawthorne"
                                             "primary2004
```

```
soc_test_processed <- social_recip |>
 prep() |>
  bake(new data = soc test)
soc_test_processed
## # A tibble: 76,467 x 22
     yearofbirth primary2004 hhsize primary2006
##
                                                    age se
##
            <dbl>
                        <dbl> <dbl>
                                           <dbl> <dbl>
## 1
           -5.19
                            0
                              0.815
                                                   5.19
## 2
           25.8
                            0
                              0.815
                                               1 - 25.8
```

0 0.815

0 - 0.185

0 1.82

0 - 0.185

1 - 0.185

0 - 0.185

1 0.815

0.815

0

0 - 24.8

0 - 26.8

0 -8.81

0 - 2.81

1 -8.81

92 / 103

1 - 28.8

0

7.19

7.19

## 3

9

## 4

## 5

## 6

##

## 8

##

## 10

24.8

-7.19

26.8

8.81

-7.19

2.81

8.81

28.8

## # : 76 /F7 mama marra

# Regularization Methods: LASSO, ridge regression, elastic nets

#### Feature Selection

▶ Wrappers: pick subset of covars, train on data (estimate model), test on hold-out, score predictions. Keep best-scoring subset.

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- ► Embeds: select features and estimate model at same time. Penalize using more predictors.

OLS reminder

Minimize SSR:

$$\underset{\beta}{\operatorname{arg\,min}} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\underset{\beta}{\operatorname{arg\,min}} \sum_{i=1}^{n} (\mathbf{y} - \mathbf{X}\hat{\beta})^2$$

L1 regularization: the LASSO (Least Absolute Shrinkage and Selection Operator)

$$\underset{\beta}{\operatorname{arg\,min}} \left[ \sum_{i=1}^{n} \left( y_i - \mathbf{X} \hat{\beta} \right)^2 + \lambda \sum_{j=1}^{k} |\hat{\beta}_j| \right]$$

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$$\underset{\beta}{\operatorname{arg\,min}} \left[ \sum_{i=1}^{n} \left( y_i - \mathbf{X} \hat{\beta} \right)^2 + \lambda \sum_{j=1}^{k} |\hat{\beta}_j| \right]$$

L2 regularization: Ridge regression

$$\underset{\beta}{\operatorname{arg\,min}} \left[ \sum_{i=1}^{n} \left( y_i - \mathbf{X} \hat{\beta} \right)^2 + \lambda \sum_{j=1}^{k} \hat{\beta}_j^2 \right]$$

Mix L1 and L2: Elastic net

$$\underset{\beta}{\operatorname{arg\,min}} \left( \frac{\sum_{i=1}^{n} \left( y_i - \mathbf{X} \hat{\beta} \right)^2}{2n} + \lambda \left[ \alpha \sum_{j=1}^{k} |\hat{\beta}_j| + \frac{1 - \alpha}{2} \sum_{j=1}^{k} \hat{\beta}_j^2 \right] \right)$$

Mix L1 and L2: Elastic net

$$\underset{\beta}{\operatorname{arg\,min}} \left( \frac{\sum_{i=1}^{n} \left( y_i - \mathbf{X} \hat{\beta} \right)^2}{2n} + \lambda \left[ \alpha \sum_{j=1}^{k} |\hat{\beta}_j| + \frac{1 - \alpha}{2} \sum_{j=1}^{k} \hat{\beta}_j^2 \right] \right)$$

Regularized trees, ...

# R packages for Regularization, etc.

- ▶ glmnet
- ▶ caret

See also tidymodels, parsnip, ...

#### References

Breiman, Leo. 2001. "Statistical Modeling: The Two Cultures." Statistical Science 16 (3): 199–215. http://www.jstor.org/stable/2676681.