Math 492 Quantam Physics Problem

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1 Problem Statement

We have set out to analyze how the parameters of quantam systems govern how the state of the system changes over time.

2 Axioms and Definitions

Definition 2.1) A complex A matrix is **skew hermitian** iff its transpose conjugate is equal to $-A$. Therefore, all 2×2 skew hermitian matricies fit the form

$$\begin{pmatrix} ai & b+ci \\ -b+ci & di \end{pmatrix} \text{ or alternatively, } \begin{pmatrix} ai & \beta \\ -\bar{\beta} & di \end{pmatrix}$$

Axiom 2.2) The state of a quantam system can be described entirely by $\begin{pmatrix} x \\ y \end{pmatrix}$, where x, y are complex numbers.

Axiom 2.3) Two quantam states are considered the same if one is a multiple of another, i.e. $\begin{pmatrix} x \\ y \end{pmatrix} = \alpha \begin{pmatrix} v \\ w \end{pmatrix}$ for any $x, y, \alpha, v, w \in \mathbb{C}$, means both states are the same. Therefore any state $\begin{pmatrix} x \\ y \end{pmatrix}$ can be represented as a ratio $z = x/y$.

Axiom 2.4) The function describing the state of a quantam system at time t is the solution to the ODE

$$\dot{\begin{pmatrix} x \\ y \end{pmatrix}} = A \begin{pmatrix} x \\ y \end{pmatrix}$$

where A is a skew hermation matrix.

3 Initial Results

Combining Axiom 2.4 with the general form of a 2×2 skew hermation matrix gives

$$\dot{\begin{pmatrix} x \\ y \end{pmatrix}} = \begin{pmatrix} ai & \beta \\ -\bar{\beta} & di \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \text{ or } \begin{cases} \dot{x} = aix + \beta y \\ \dot{y} = -\bar{\beta}x + diy \end{cases}$$

So if you consider $z = y/x$, and its derivative, then we get the following new equation

$$\begin{aligned}
\dot{z} &= \frac{\dot{y}x - y\dot{x}}{x^2} \\
\dot{z} &= \frac{(-\bar{\beta}x + diy)x - y(aix + \beta y)}{x^2} \\
\dot{z} &= \frac{-\bar{\beta}x^2 + diyx - aixy - \beta y^2}{x^2} \\
\dot{z} &= -\bar{\beta} + diz - aiz - \beta z^2
\end{aligned}$$

From here we can make assumptions to make this easier to solve

4 Assume $\beta = 0, d = -a, a \neq 0$

Adding these assumptions to our initial result gives:

$$\dot{z} = -2aiz$$

Now we can define f, g as the real and complex parts of z respectively

$$\begin{aligned}
\dot{f} + i\dot{g} &= -2ai(f + ig) \\
\dot{f} + i\dot{g} &= -2aif + 2ag
\end{aligned}$$

Giving the system:

$$\begin{cases} \dot{f} = 2ag \\ \dot{g} = -2af \end{cases}$$

$$\begin{aligned}
\therefore \ddot{f} &= 2a\dot{g} \\
\ddot{f} &= -4a^2 f \\
\ddot{f} + 4a^2 f &= 0 \\
\lambda^2 + 4a^2 &= 0 \\
\lambda^2 &= -4a^2 \\
\lambda = \pm\sqrt{-4a^2} &= \pm 2ai
\end{aligned}$$

So the solution is

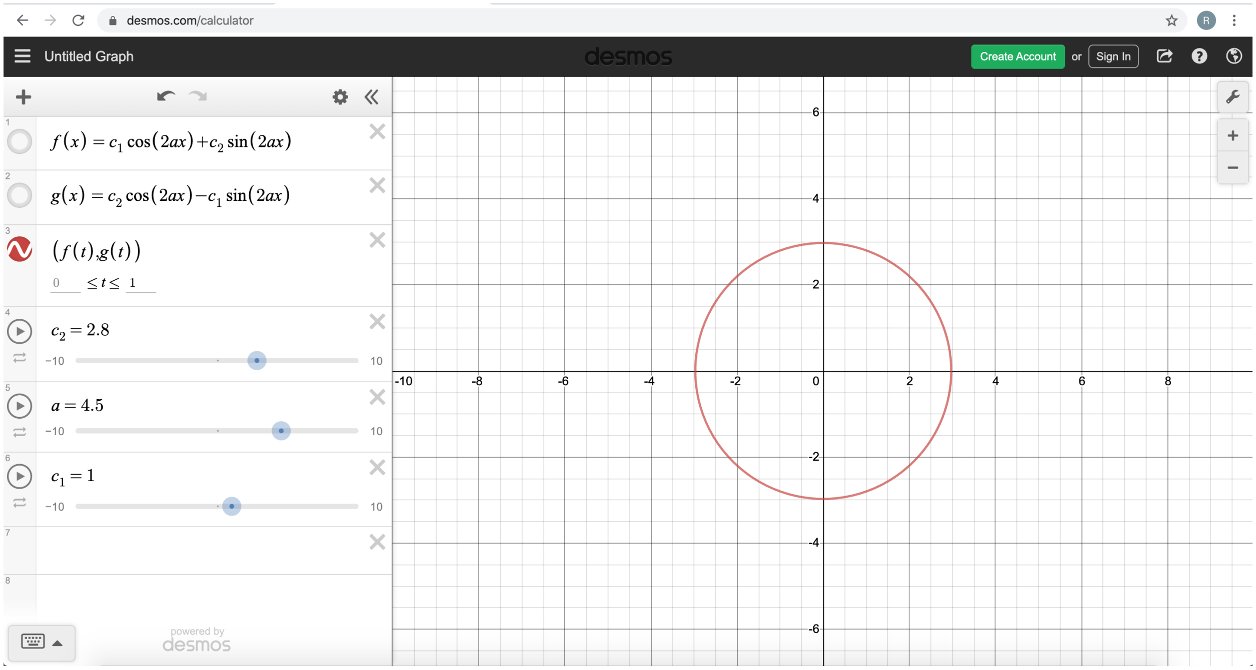
$$\begin{aligned}
f &= c_1 e^0 \cos(2at) + c_2 e^0 \sin(2at) \\
f &= c_1 \cos(2at) + c_2 \sin(2at) \\
\therefore \dot{f} &= -c_1 2a \sin(2at) + c_2 2a \cos(2at)
\end{aligned}$$

pluggint f into our equation for $\dot{f}$ gives g:

$$\begin{aligned}
\dot{f} &= 2ag \\
-c_1 2a \sin(2at) + c_2 2a \cos(2at) &= 2ag \\
-c_1 \sin(2at) + c_2 \cos(2at) &= g
\end{aligned}$$

As a vector gives:

$$\begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} c_1 \cos(2at) + c_2 \sin(2at) \\ -c_1 \sin(2at) + c_2 \cos(2at) \end{pmatrix}$$



Changing c_1 and c_2 affects the size of the circle. Changing a only affects the paramaterization