Question 5:
Consider the evaluation of "perfect trees" using P processors. A "perfect tree" of height m is a binary tree with ~~2m-1~~ nodes. It thus has as many nodes as any binary tree of height m can have. Assume that the nodes are operators which take unit time to perform. Suppose we have P=~~2m-1/2k~~ processors, where 0 ≤ _k ≤ _m-1.

Amdahl's Law $\qquad 2^{m-1}\ 2^{k} \qquad 2^{m}-1$

A) What is the time TP in which a perfect tree can be evaluated with P processors?

B) What are the speedup SP and efficiency EP with this number of processors?

C) How does the efficiency vary as k changes?

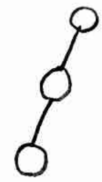
Perfect Tree

$m=2 \quad N=3$

$0 \le K \le m-1$

$0 \le K \le 1$

$0,1$

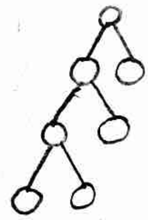


$m=3 \quad N=7$

$0 \le K \le m-1$

$0 \le K \le 2$

$0,1,2$

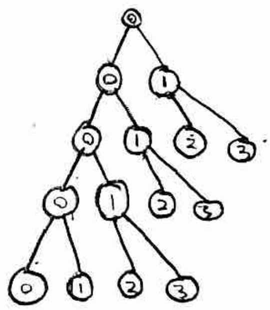


$m=4 \quad N=15$

$0 \le K \le m-1$

$0 \le K \le 3$

$0,1,2,3$



$$P = \frac{2^{m-1}}{2k}$$

N = node or operations

M = height or unit time

P = Processors

K =

$0 \le K \le m-1$

A) $TP = S + \frac{Q}{P}$ $\quad$ //sequential $=(n-1)=S$

// Parallel $= (N) = Q$

// Processors $= \frac{2^{m-1}}{2^{K}} = P$

$$TP = (n-1) + \frac{N(2^{m-1})}{2^{K}}$$

B) $SP = \frac{T(1)}{T(P)} = \frac{(n-1)}{T(P)}$

$$EP = \frac{S(P)}{P} = \frac{S(P)}{\frac{2^{m-1}}{2^{K}}} = \frac{S(P)2^{K}}{2^{m-1}}$$

C) As K increase, efficiency increases | exponentially by power of 2

As K decreases, efficiency decreases |