



# Theory and Applications of Sparsity for Radar Sensing of Ionospheric Plasma

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# The ionosphere

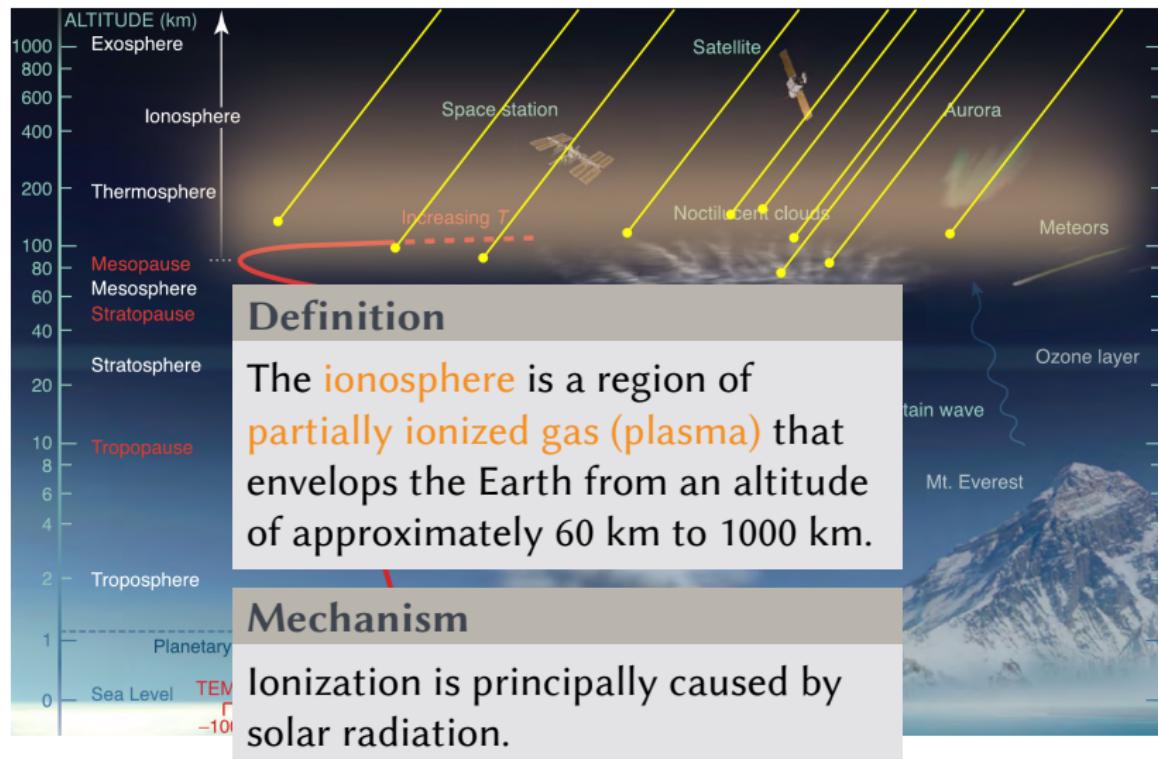
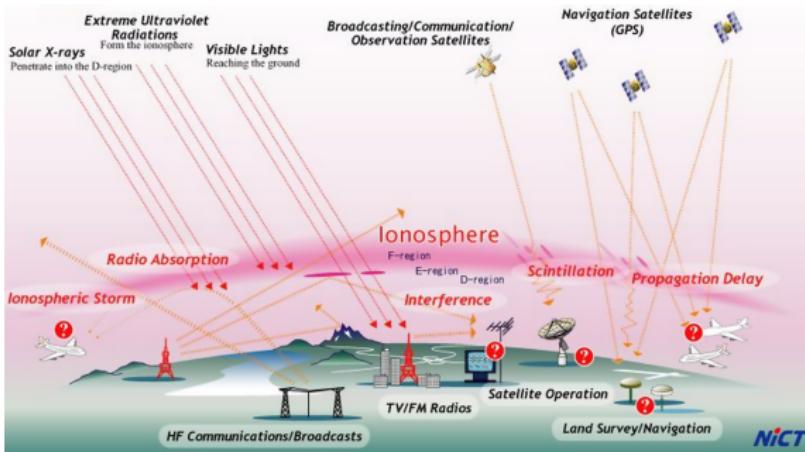


Image from NSF (2011), CEDAR: The New Dimension

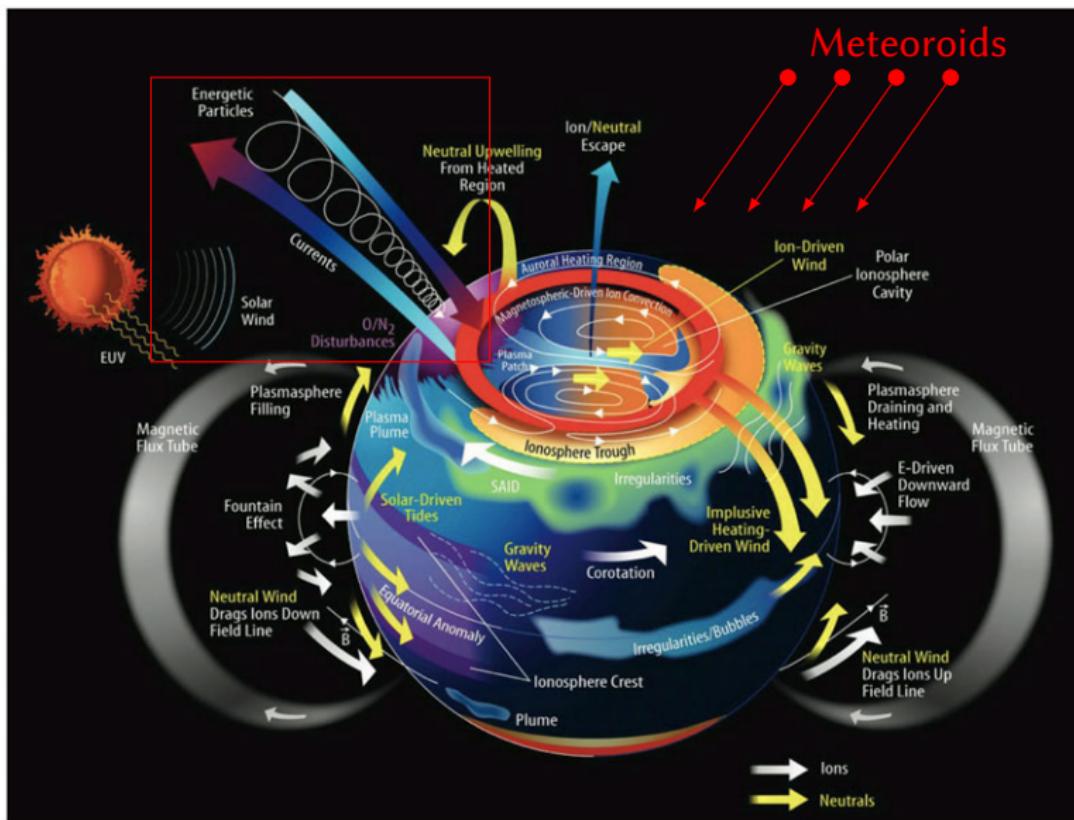
# Importance of the ionosphere Communications



- ▶ Radio waves of lower frequency (less than  $\approx 10$  MHz) reflect off ionosphere.
- ▶ Microwave frequencies (e.g. GPS) can pass through, but experience frequency-dependent refraction and delay.
- ▶ Plasma irregularities additionally perturb signals.

# Importance of the ionosphere

## Science of the geospace system



# Space weather: meteoroids and meteors

## Definition (Meteoroid)

- ▶ Solid body in space
- ▶ Small (< 1 m in diameter)
- ▶ Fast (from 11 to 72 km/s)
- ▶ Exponentially more common as size decreases

Micro-meteorites (landed on Earth)



© James K. Bowden 2013

## Definition (Meteor)

- ▶ Plasma created by meteoroid entering the atmosphere
- ▶ Typically form at altitudes between 80 km and 120 km

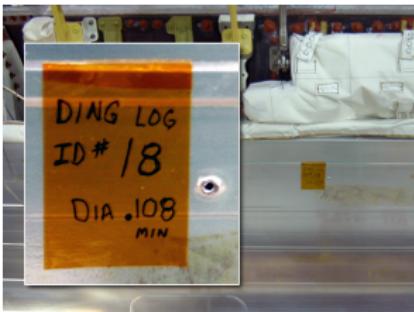


2011/10/01 00:37:01.079 (LT) 0157 00076 V00395+218 UF0CapY2 MSFC ALaM0

NASA (2011)

# Importance of meteoroids

Impact  
damage to  
Space Shuttle  
Atlantis



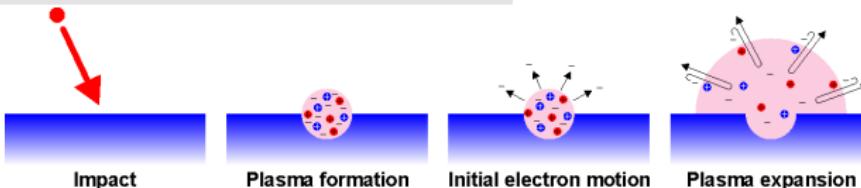
## High speed impacts

Threat to satellites and spacecraft from

- ▶ mechanical damage
- ▶ **electrical** damage

## Required observations

- ▶ Rate/count
- ▶ Size
- ▶ Speed
- ▶ Composition



# Measuring the ionosphere

## Definition

Incoherent Scatter Radars (ISRs, locations shown below) are radars sensitive enough to measure scattering from electrons in the background ionosphere.

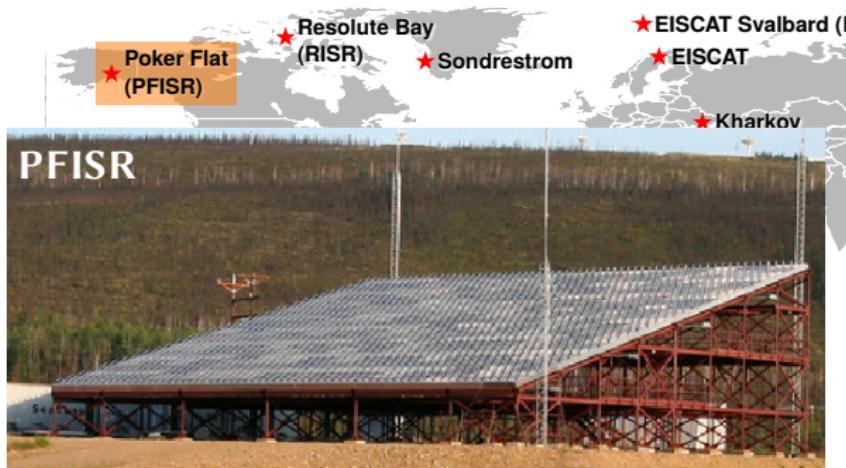


Range of ionospheric regions: polar, mid-latitude, equatorial.

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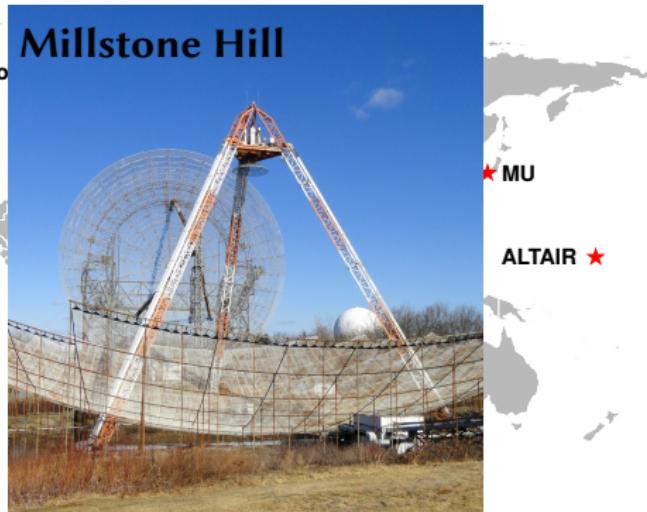
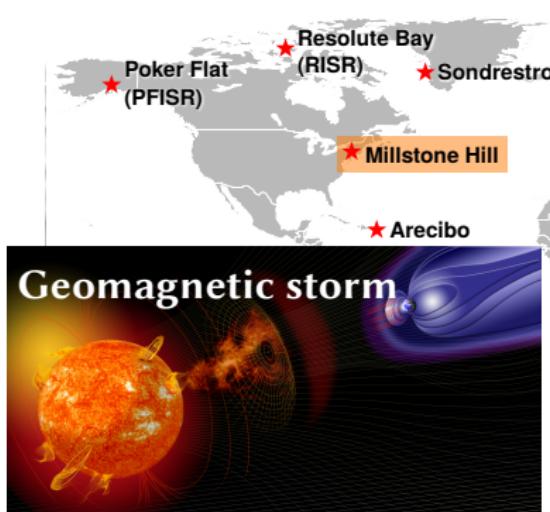


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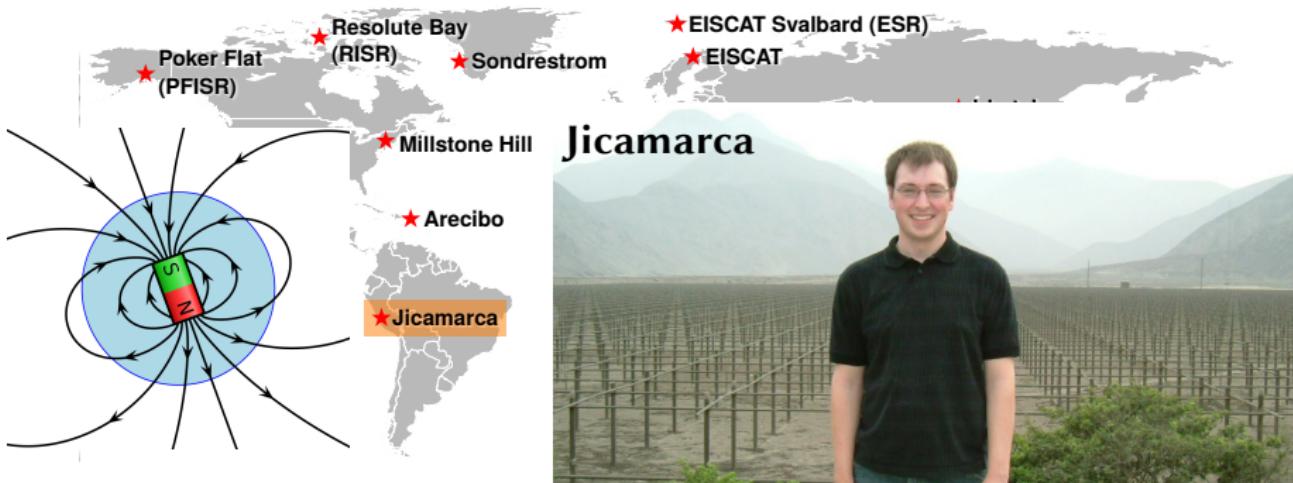


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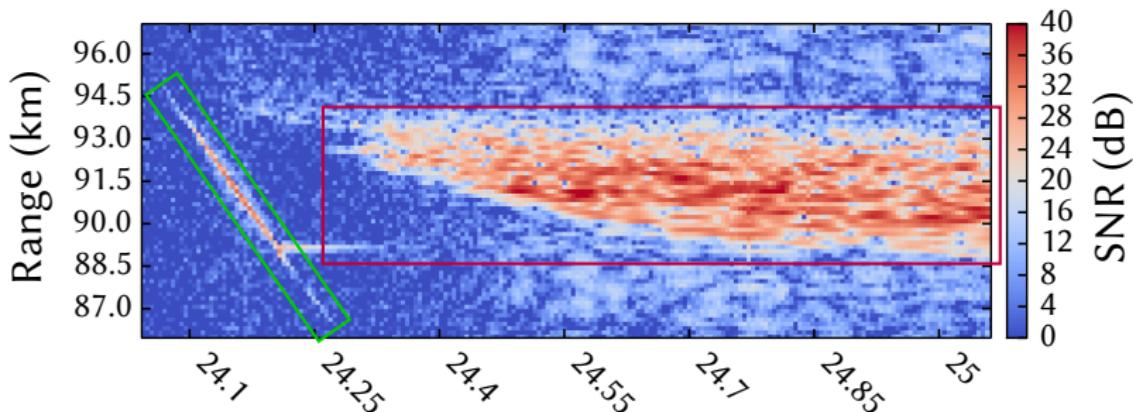


Range of ionospheric regions: polar, mid-latitude, equatorial.

# Radar meteors

## Types of scattering

- ▶ **Head**: ball of plasma traveling with the meteoroid
- ▶ **Trail**: plasma left in wake of meteoroid

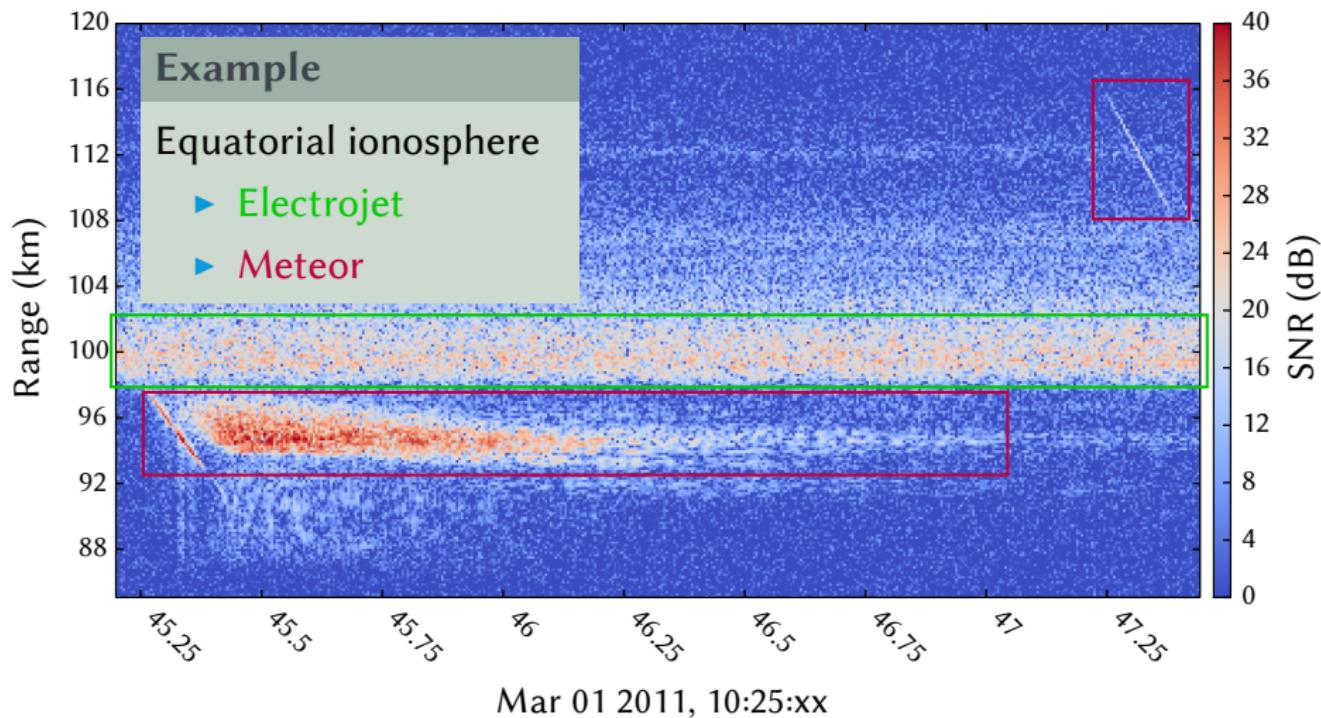


Mar 01 2011, 10:04:xx

Time of Radar Pulse

# Variety of ionospheric plasma

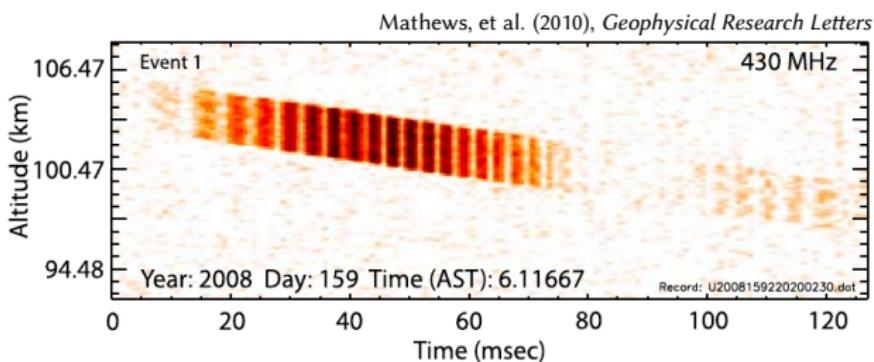
There are many different plasma phenomena, each presenting its own challenges for observation.



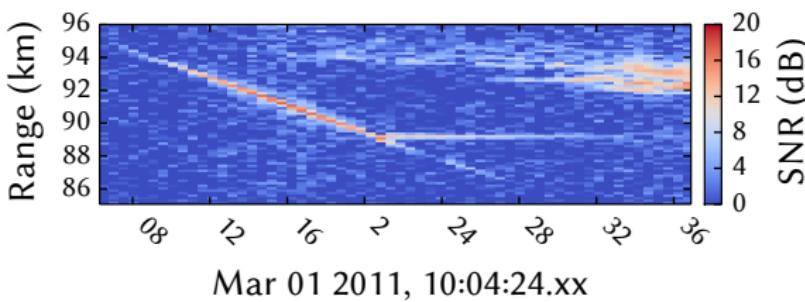
# At the limit: meteors

Scientific progress is limited by current signal processing techniques!

Fragmentation

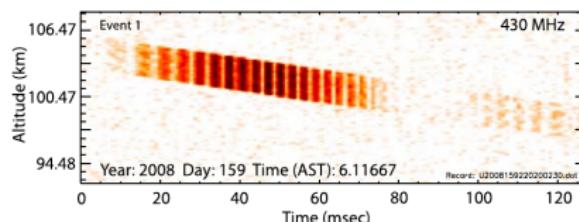
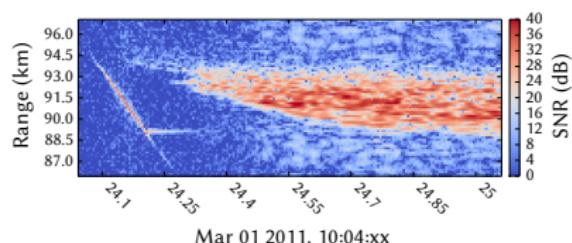
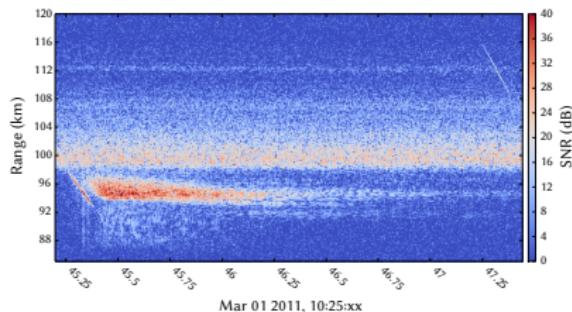


Flares and terminal events



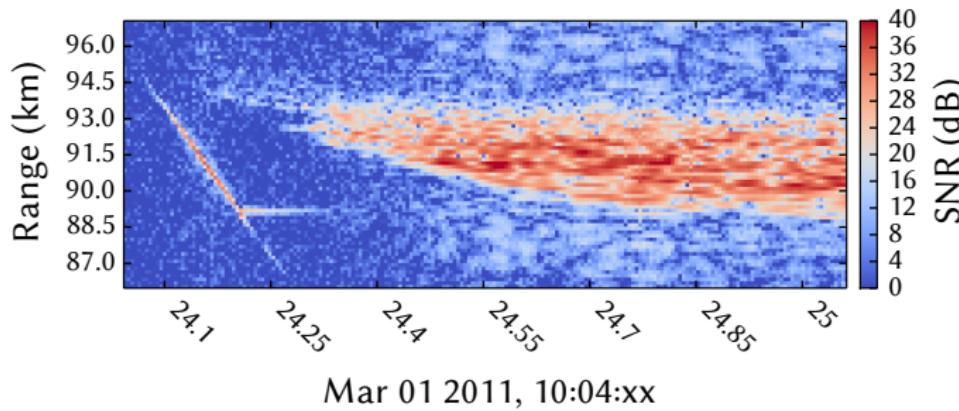
# Measurement challenges

1. **Differentiation** in a crowded and variable environment  
(e.g. meteors and electrojet)
2. **Self-interference** of range-spread targets  
(e.g. meteor trails)
3. **High resolution** to observe small-scale processes  
(e.g. meteoroid fragmentation and flares)
4. **Flexibility** to achieve all of the above at once



# A way forward: sparsity

Radar targets are typically sparse in range and frequency!



Can use sparsity to systematically improve the quality of radar measurements.

# The world is sparse

## Definition

- ▶ A vector is **sparse** if only a few of its elements are nonzero.
- ▶ Additionally, the term is often applied loosely to when this is approximately true (i.e. most of the elements close to zero).

Many natural phenomena have sparse representations:

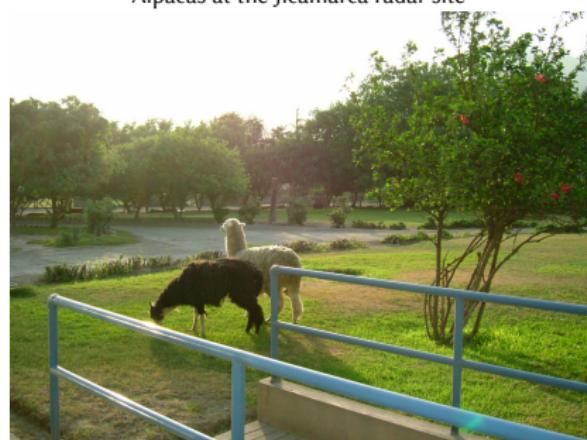
- ▶ Music and speech (mp3)
- ▶ Images (jpeg)

## Alpacas (right): 8.5x savings

Uncompressed: 3600 KB

Compressed: 420 KB

Alpacas at the Jicamarca radar site



# Sparsity revolution

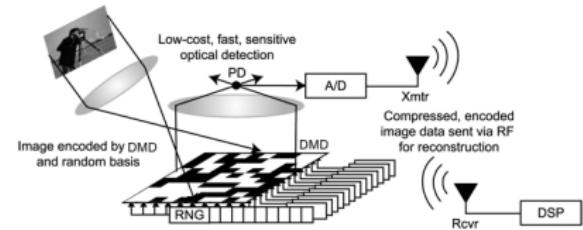
Exploitation of inherent signal sparsity through compressed sensing is leading to advancements in many subjects.

## Single pixel camera (1)

Smaller, cheaper, more flexible

[dsp.rice.edu/cscamera](http://dsp.rice.edu/cscamera)

(1)

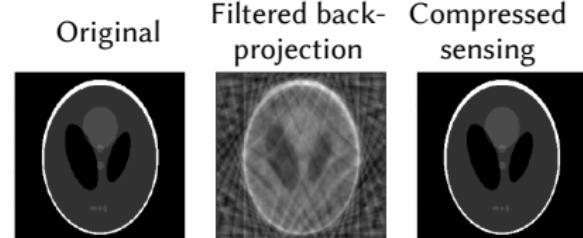


## MRI (2)

Faster scans, higher resolution

[eecs.berkeley.edu/~mlustig/CS.html](http://eecs.berkeley.edu/~mlustig/CS.html)

(2)



## Genomics

High-throughput screening

[erlichlab.wi.mit.edu](http://erlichlab.wi.mit.edu)

# Contributions

In order to enable **flexible high-resolution measurements** of ionospheric plasma phenomena, including interfering and range-spread targets, I made the following contributions:

1. Derived a discrete **radar model** that captures signal sparsity in delay-frequency space.
2. Uncovered the adjoint relationship between the model and existing matched filter techniques, giving an **intuitive interpretation** of waveform inversion as iterative filtering.
3. **Analyzed the model** to show how it exactly represents distributed scatterers and preserves their sparsity.
4. Created and tuned an **efficient implementation** of waveform inversion using modern convex optimization techniques.
5. **Demonstrated the flexibility and effectiveness** of the inversion technique by removing filtering artifacts from meteor observations made with a variety of radar waveforms.

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# Additional contributions

In addition, I also made the following contributions that will not be covered in this talk:

1. Created a range-time-frequency clustering algorithm for detection and classification of ionospheric radar signals.
2. Simulated the statistical accuracy of interpolated matched filter estimation for range and range-rate super-resolution of meteor head echoes.

# Outline

Introduction and Motivation

Radar Background

My Radar Model

Sparsity Background

Waveform Inversion

Conclusion

# Outline

## Introduction and Motivation

## Radar Background

Pulse Encoding/Decoding

Measurement Ambiguity

Current Techniques

## My Radar Model

## Sparsity Background

## Waveform Inversion

## Conclusion

# Radar pulse length

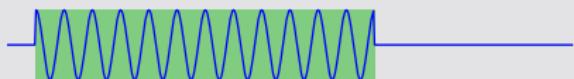
Some applications favor a shorter radar pulse, while others favor a longer radar pulse.

## Short pulse



- ▶ Low range ambiguity (interference region for multiple scatterers)
- ▶ Simple to interpret

## Long pulse

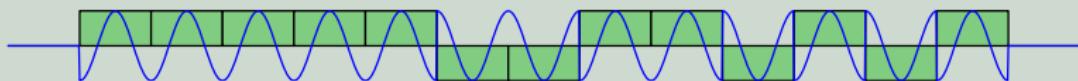


- ▶ Fine frequency resolution (observe target as it evolves in time)
- ▶ High total power

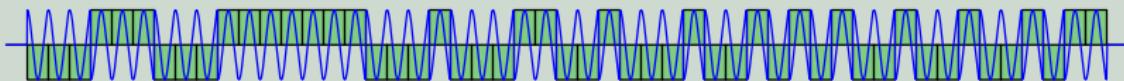
# Coded pulses

- ▶ Encoding allows long pulses to have the low range ambiguity of short pulses.
- ▶ One method: divide pulse into **bauds** of constant phase.

## Barker-13 code



## Minimum peak sidelobe code



## Linear frequency modulation (LFM chirp)



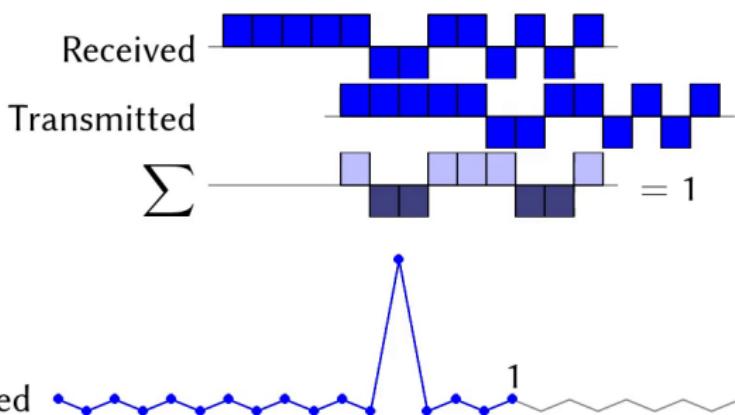
# The matched filter

## Definition

The **matched filter** correlates every segment of the received signal with the transmitted pulse.

It produces a peak at the target's range (delay), achieving a rough form of decoding with ambiguous **sidelobes**.

## Example (Barker-13 code, Minimum sidelobe code)



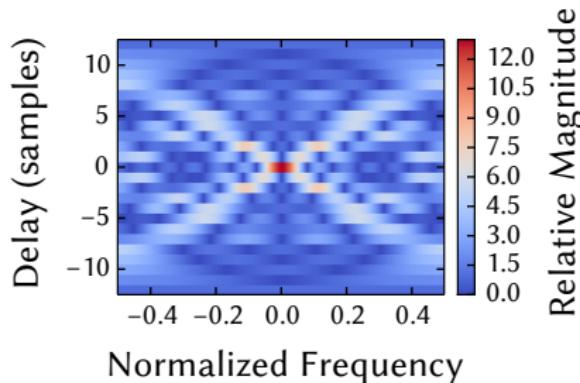
# Frequency filter banks

## Doppler frequency shift

If the target is moving **toward** or **away from** the radar, the reflected signal is frequency shifted **up** or **down**.

- ▶ Matched filter needs to be similarly frequency shifted
- ▶ True frequency shift is not known ahead of time
- ▶ Process signal with a bank of differently-shifted filters

Result of filter bank for centered “point” target, using Barker-13 code



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## Sparsity Background

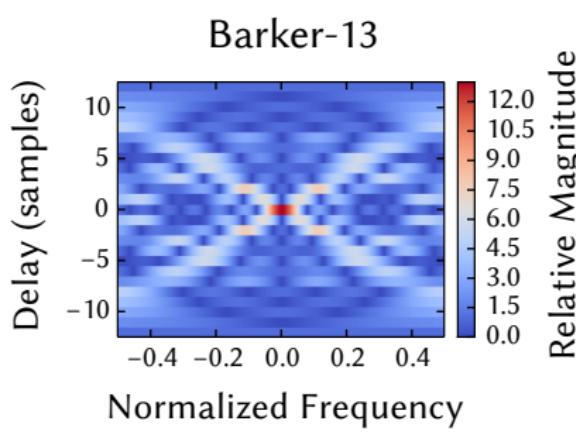
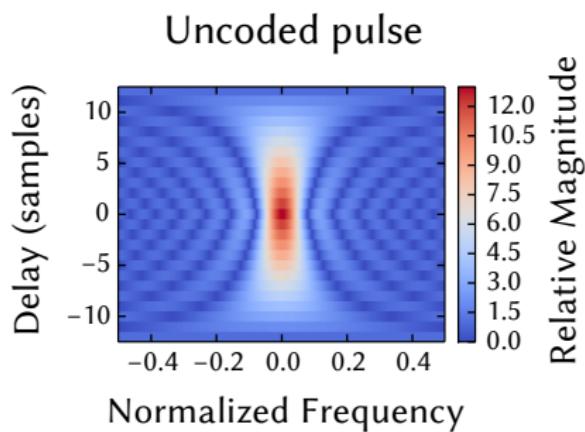
## Waveform Inversion

## Conclusion

# Ambiguity functions

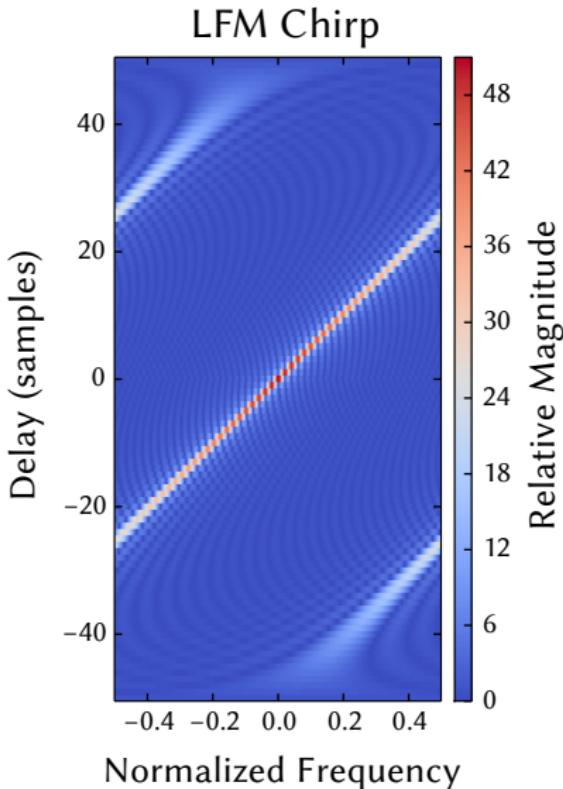
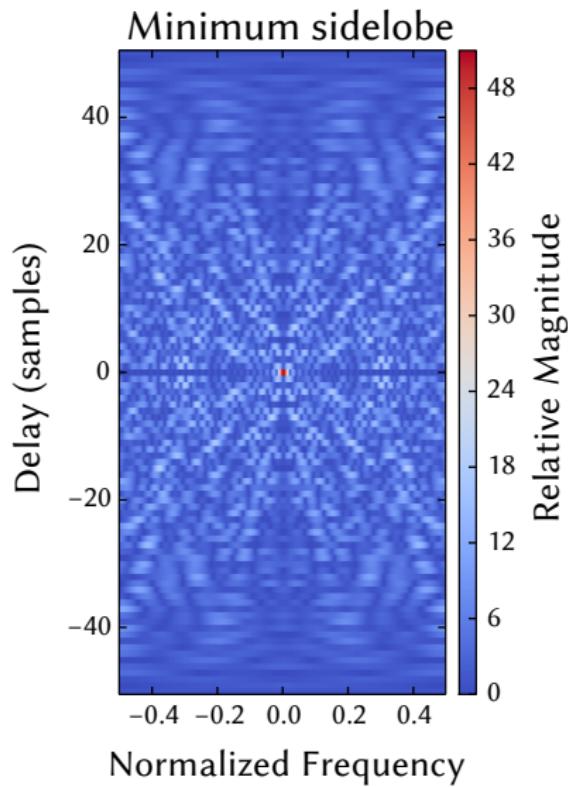
## Definition

A delay-frequency **ambiguity function** is produced when a signal is matched against itself using a filter bank.



The ambiguity function describes the delay-frequency sidelobes of a code.

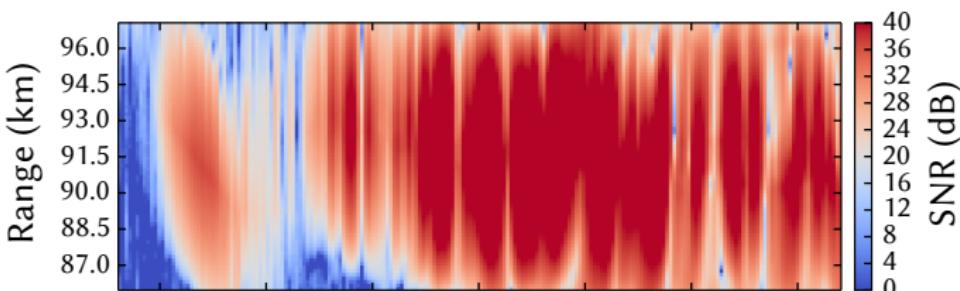
# More example ambiguity functions



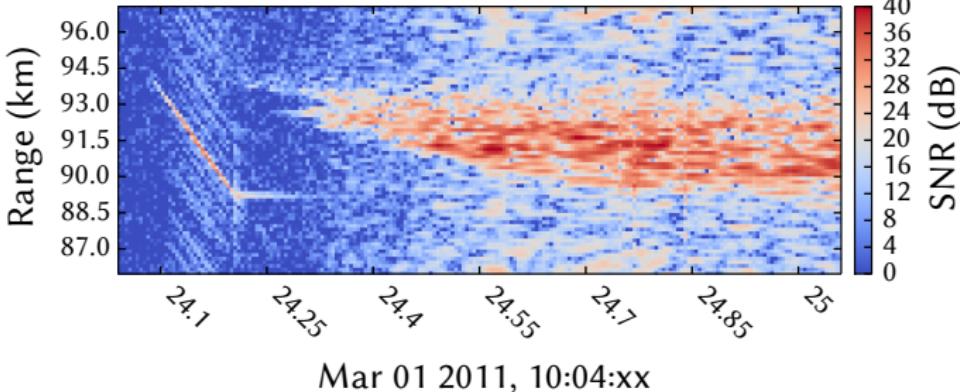
# Range sidelobes in action

Range sidelobes dominate the output of uncoded measurements, while coded pulses are better but still noticeably corrupted.

Uncoded



Minimum  
sidelobe  
code

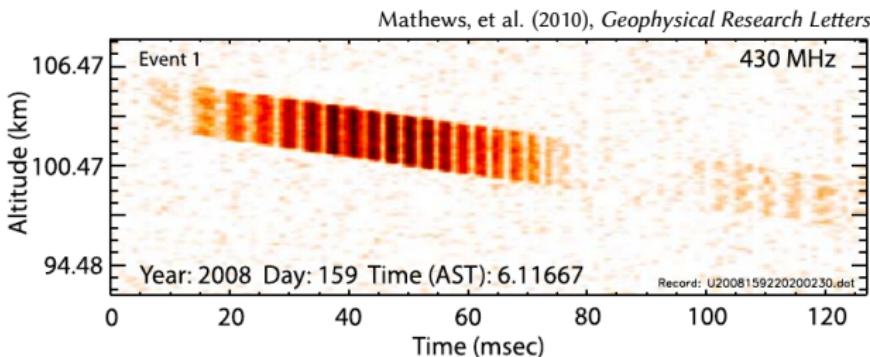


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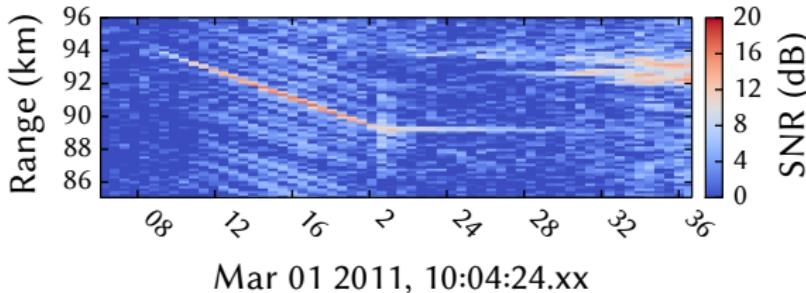
# Effect of sidelobes

Sidelobes (delay-frequency ambiguity) are the primary reason that outstanding ionospheric science questions cannot be satisfactorily answered.

Fragmentation

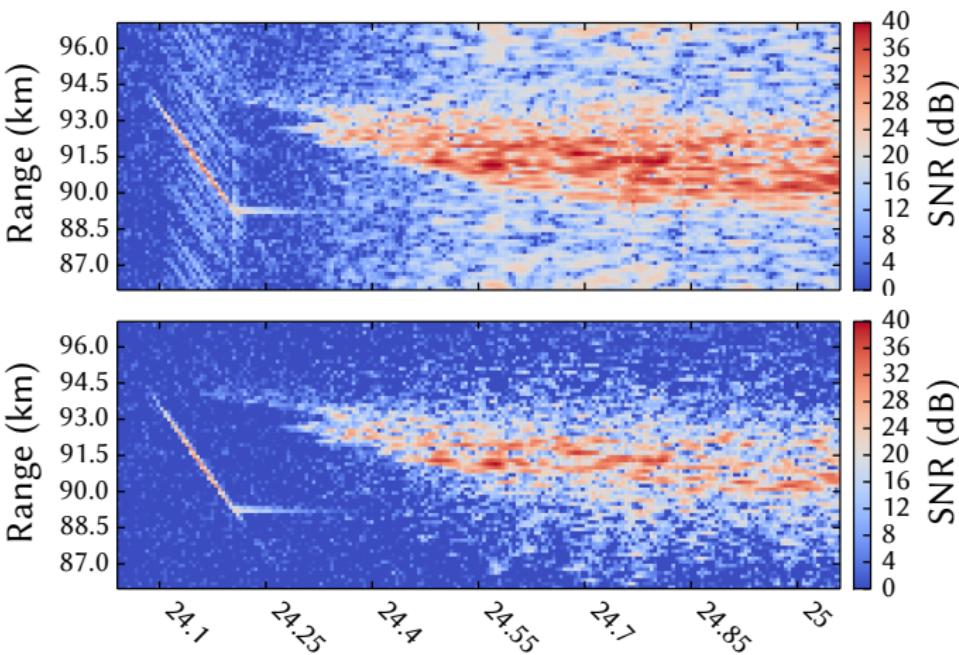


Flares and terminal events



# Goal: sidelobe removal

Minimum  
sidelobe  
code



Near-ideal  
decoding

Mar 01 2011, 10:04:xx

Infinite number of ways to decode and get a target scene that reproduces the measurements!

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# Delay-frequency sidelobe mitigation in use

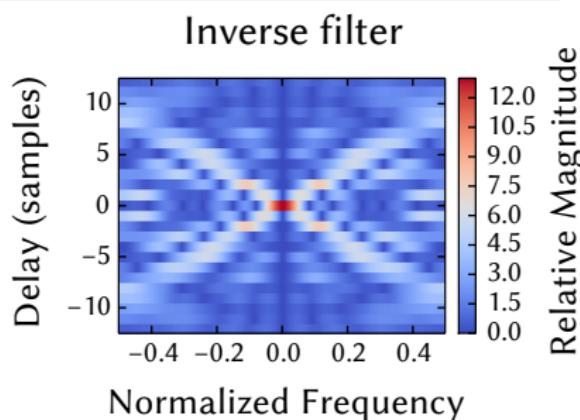
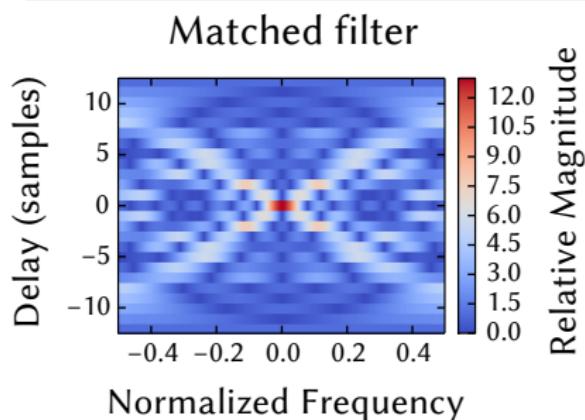
1. Codes with good sidelobe properties
  - ▶ e.g. Barker-13, minimum peak sidelobe, LFM chirp
  - ▶ Most prevalent strategy
  - ▶ Only effective when it is simple to ignore remaining sidelobes
2. Alternating codes
  - ▶ Set of codes over multiple pulses
  - ▶ When pulses summed together, sidelobes cancel
  - ▶ Target properties must remain stationary
3. Inverse filters
  - ▶ Alternative to matched filter
  - ▶ Loss of SNR depending on code
  - ▶ Produces no range sidelobes at target's frequency shift

These strategies severely constrain the flexibility and accuracy of ionospheric radar measurements!

# Inverse filter ambiguity function

## Example

Ambiguity function comparison for Barker-13 code.



No advantage if Doppler shift is unknown or multiple frequencies must be decoded!

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**My Radar Model**

Defining the Model

Representation of Targets

Sparsity Background

Waveform Inversion

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# Matched filtering as imaging

## Imaging analogy

Matched filter “image” is blurred by the ambiguity function:

$$h[n, p] * \chi[n, p] = x[n, p]$$

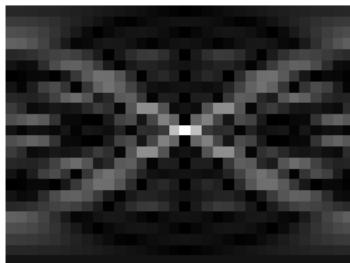
Target reflectivity  
(source image)

Ambiguity function  
(point-spread func.)

Matched filter result  
(measured image)



\*



=



# Radar model from matched filter

## Ambiguity equation

$$x[n, p] = \chi[n, p] * h[n, p]$$

$$A^*(y[m]) = A^*(A(h[n, p]))$$

- ▶ Matched filtering is  $A^*$
- ▶ Ambiguity function is  $A^*A$ 
  - $m$  sample
- ▶ Indices:
  - $n$  frequency
  - $p$  delay

## Radar model

Simplify by removing excess matched filtering operation:

$$y[m] = A(h[n, p])$$

Measured signal      Radar model      Target reflectivity

- ▶ Radar model is **adjoint** (conjugate transpose) of matched filter
- ▶ Non-invertible because under-determined
- ▶ Sparsity of target reflectivity is key to solution

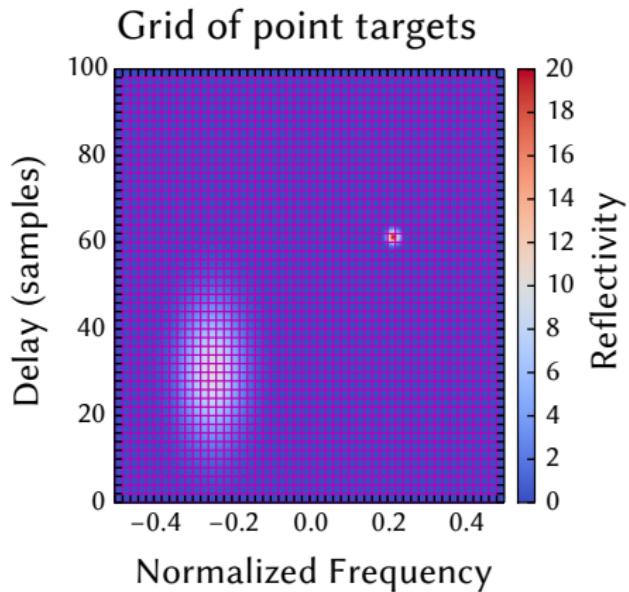
# Alternative interpretation

## Matched filter ( $A^*$ )

Correlates received signal with expected return from point targets with different delays and frequency shifts.

## Radar model ( $A$ )

Simulates received signal as **sum of returns from point targets** with different delays and frequency shifts.



$$y = A(\mathbf{h}[n, p])$$

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# Model discretization from radar signal equation

## Narrow-band radar equation for received signal

$$y(t) = \int_0^T \int_{-\infty}^{\infty} s(t - \lambda) e^{2\pi i f(t-\lambda)} h(f, \lambda) df d\lambda$$

Measured signal	Transmitted signal	Frequency shift	Target reflectivity
-----------------	--------------------	-----------------	---------------------

$t$	Sample time	$y(m\tau) \rightarrow y[m]$	$m = 0, 1, \dots, M - 1$
$f$	Frequency	$h(n\phi, :) \rightsquigarrow h[n, :]$	$n = 0, 1, \dots, N - 1$
$\lambda$	Delay	$h(:, p\tau) \rightsquigarrow h[:, p]$	$p = 0, 1, \dots, P - 1$

## Discretized radar model

$$y[m] = \sum_{p=0}^{P-1} \sum_{n=0}^{N-1} \frac{1}{\sqrt{N}} s[m - p + L - 1] e^{2\pi i n m / N} h[n, p] = A(h)$$

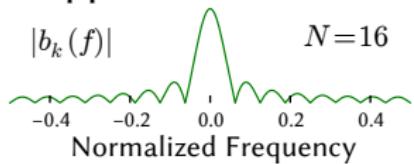
# Reflectivity coefficients from function

Can write the “point target” reflectivity coefficients in terms of the original reflectivity function:

$$h[n, p] = \int_{p\tau}^{(p+1)\tau} [h(f, \lambda) e^{2\pi i f \lambda} * b_{p+1}(f)] (n\phi) d\lambda$$

- Blur in frequency by convolving with wrapped sinc function:

$$b_k(f) = \frac{1}{N} e^{-\pi i (2k+N-1)\tau f} \frac{\sin(N\pi\tau f)}{\sin(\pi\tau f)}$$

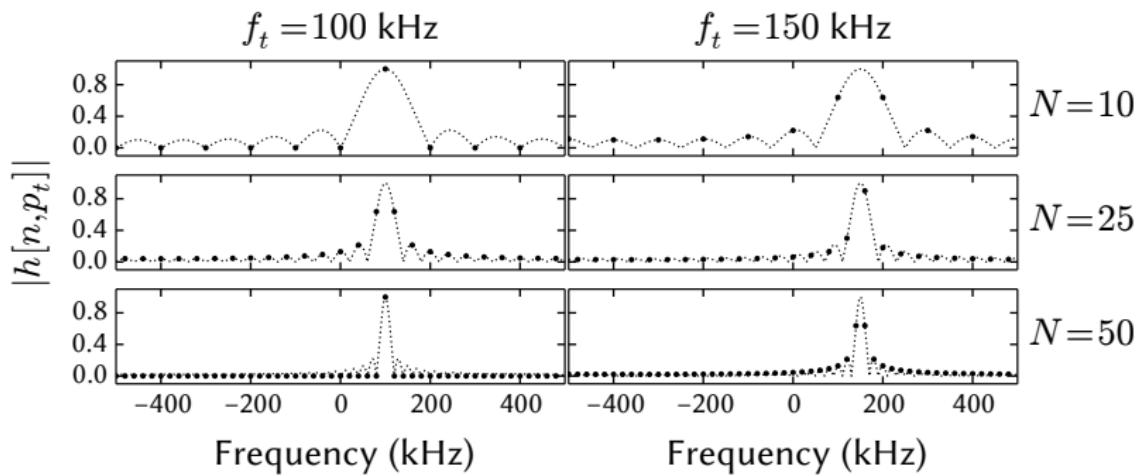


- Sample at discretized frequency point  $n\phi$
- Integrate over delay window of discretized delay point  $p\tau$

# Representing an off-grid point target

Off-grid point targets are still relatively sparse!

- ▶ All coefficients outside delay window  $p_t$  are zero
- ▶ For frequency index  $n = 0, 1, \dots, N - 1$ , behavior dictated by wrapped sinc:



Relative sparsity improves with increasing  $N$   
(same number significant coefficients, many more zeros)

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**Sparsity Background**

Compressed Sensing

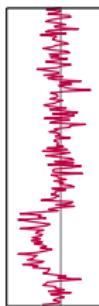
Convex Optimization

Waveform Inversion

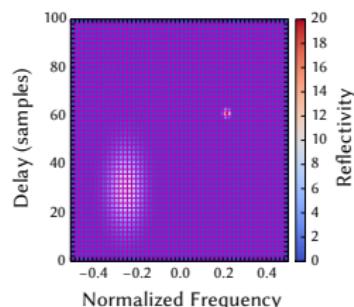
Conclusion

# Under-determined systems of equations

Not enough measurements to constrain unknown values  
(e.g. the radar model):



$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} & & \\ A & & \\ & & \end{bmatrix} \begin{bmatrix} h \end{bmatrix}$$



(vectorized as  $h$ )

Measurement    Model    Unknown

- ▶ Infinite number of solutions
- ▶ Often know that the true solution should be sparse
- ▶ Finding the sparsest solution is hard in general

# Theory of compressed sensing

## Definition

Compressed sensing is a theory to guarantee solution of an under-determined set of equations.

## Approximate guidelines for application

- ▶ Solution known to be sparse
- ▶ Measurements are “incoherent” (global with low correlation)
- ▶ Minimum number of measurements on the order of the solution sparsity (number of nonzeros)

## Benefit

Can solve easy convex optimization problem instead of hard combinatorial problem.

# Equivalent convex optimization problem

## Sparsest solution to noisy measurements

Find sparsest  $h$

subject to  $\|y - A(h)\|_2 < \eta$   $\|h\|_2^2 = \sum_k |h_k|^2$

## $l_1$ -regularized least-squares (convex)

$$\underset{h}{\text{minimize}} \quad \frac{1}{2} \|y - A(h)\|_2^2 + \lambda \|h\|_1 \quad \|h\|_1 = \sum_k |h_k|$$

The  $l_1$ -norm promotes sparsity!

# Outline

Introduction and Motivation

Radar Background

My Radar Model

**Sparsity Background**

Compressed Sensing

Convex Optimization

Waveform Inversion

Conclusion

# First-order methods

We want to efficiently solve

$$\underset{h}{\text{minimize}} \quad \frac{1}{2} \|y - A(h)\|_2^2 + \lambda \|h\|_1$$

but for systems  $A$  that are too large for matrix methods.

## First-order methods

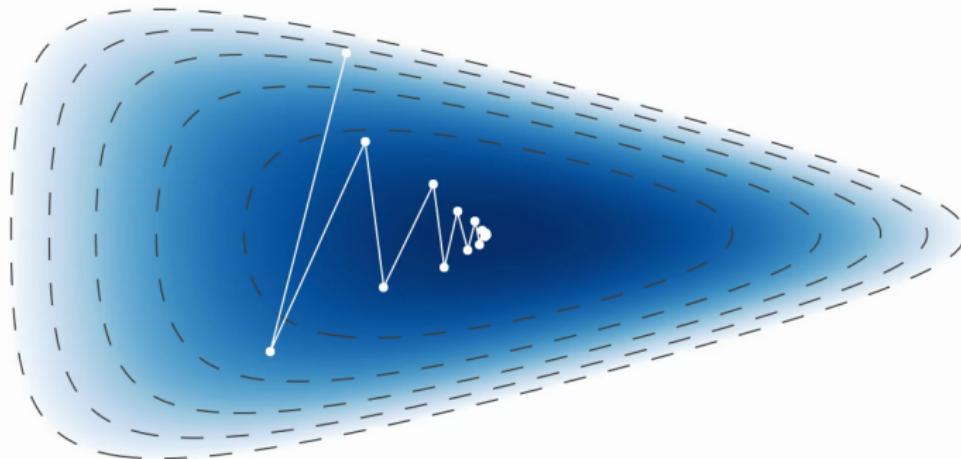
- ▶ Explicit matrix for  $A$  not needed
- ▶ Only need to be able to compute  $A(\cdot)$  and  $A^*(\cdot)$

$l_1$ -regularized least-squares is non-smooth because of the  $l_1$  norm term, so minimizing it requires a special approach.

# First-order step

## Smooth

If the function being minimized is differentiable, the most natural first-order step is a gradient step.



# First-order step

## Non-smooth

For a non-differentiable function, an attractive alternative is the proximal (or prox) step.

$$x^{k+1} := \arg \min_x \left( G(x) + \frac{1}{2\mu} \|x - x^k\|_2^2 \right)$$

Non-smooth function
Smoothing around  
current iterate

- ▶ This is surprisingly easy to solve and can be evaluated much like a gradient!
- ▶ Note that as  $x^k$  approaches the minimum  $x^*$ , the smoothing term goes to zero.

The **prox operator** is defined for non-smooth  $G(x)$  as

$$\mathbf{prox}_{\mu G}(v) = \arg \min_x \left( G(x) + \frac{1}{2\mu} \|x - v\|_2^2 \right).$$

# Proximal gradient method

The **proximal gradient method** combines gradient and prox steps to solve

$$\underset{x}{\text{minimize}} \quad F(x) + G(x)$$

for smooth  $F(x)$  and non-smooth  $G(x)$ .

## Algorithm

(Step size  $\mu$ ) Iterate:

Gradient step       $z^{k+1} := x^k - \mu \nabla F(x^k)$

Prox step       $x^{k+1} := \mathbf{prox}_{\mu G}(z^{k+1})$

# Proximal gradient method applied

Proximal gradient for  $l_1$ -regularized least-squares:

$$\underset{h}{\text{minimize}} \quad \frac{1}{2} \|y - A(h)\|_2^2 + \lambda \|h\|_1$$

$$F(h) = \frac{1}{2} \|y - A(h)\|_2^2 \quad \Rightarrow \quad \nabla F(h) = -A^*(y - A(h))$$

$$G(h) = \lambda \|h\|_1 \quad \Rightarrow \quad \text{prox}_{\mu G}(v) = \text{soft}_{\lambda\mu}(v) = \begin{cases} v - \lambda\mu, & v > \lambda\mu \\ 0, & |v| \leq \lambda\mu \\ v + \lambda\mu, & v < -\lambda\mu \end{cases}$$

# Iterative Soft Thresholding

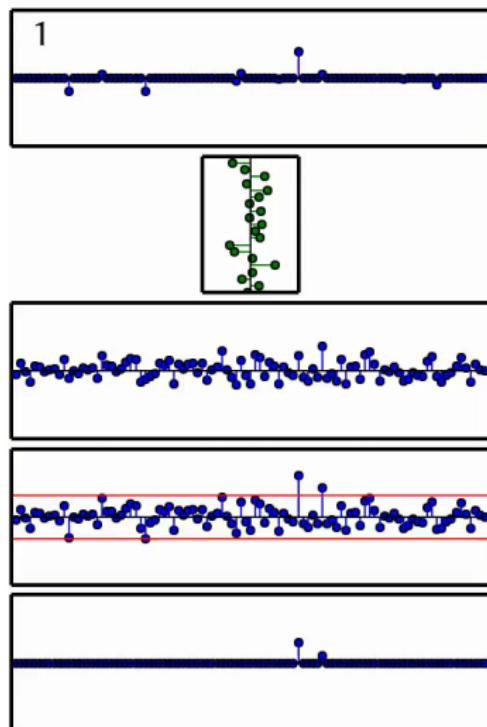
Guess  
(e.g. reflectivity):  $h$

Calculate error:  $z = y - A(h)$

Matched filter  
the error:  $A^*(z)$

Add previous  
guess:  $h + A^*(z)$

Threshold to  
form new guess:  
 $h = \text{soft}(h + A^*(z))$



**Interpretation:** iterative matched filtering with thresholding!

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Waveform Inversion

Implementation

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Conclusion

# Combined algorithmic advances

First order prox algorithms are an active research area, so I tested and combined multiple proposed enhancements:

1. Accelerated proximal gradient method
  - ▶ Adds simple “momentum” term
  - ▶ Better theoretical convergence
2. Adaptive restart
  - ▶ Reset acceleration when it opposes prior step
3. Adaptive step size
  - ▶ Increase step size every iteration
  - ▶ Decrease as necessary to ensure convergence

Convergence comparison	
Method	Iterations
Prox gradient	4463
+ Acceleration	2911
+ Adaptive restart	397
+ Adaptive step	105

# Testing similar algorithms

Other prox-based algorithms are worthy of consideration:

- ▶ Linearized alternating direction method of multipliers (ADMM)
- ▶ Primal-dual hybrid gradient (PDHG)

Closely related, these both solve

$$\underset{x}{\text{minimize}} \quad F(x) + G(x)$$

when both  $F(x)$  and  $G(x)$  can be non-smooth.

## Convergence comparison

Method	Iterations
Accelerated proximal gradient	105
Linearized ADMM (novel adaptive step)	154
PDHG (standard fixed step)	4464

# Code

## Principles

1. Easy to develop
2. Fast when necessary
3. Freely available for collaboration and community use

## Answers

1. Python with NumPy
2. Radar model is bottleneck: implement with Cython or Numba
3. Available on github:  
[github.com/ryanvolz](https://github.com/ryanvolz)



python™



NumPy



Cython



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# The Jicamarca incoherent scatter radar

## Specifications:

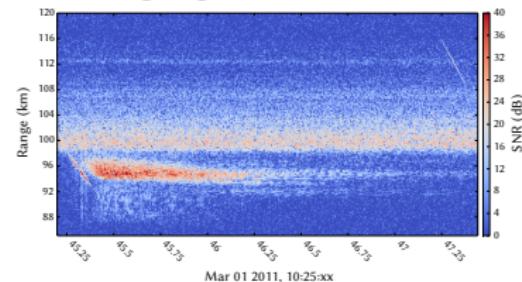
- ▶ Located outside Lima, Peru
- ▶ VHF (50 MHz)
- ▶ Phased array of  $96 \times 96$  crossed half-wave dipoles



- ▶ 1 MHz bandwidth (TX/RX)

## Reasons:

- ▶ Equatorial ionosphere is challenging

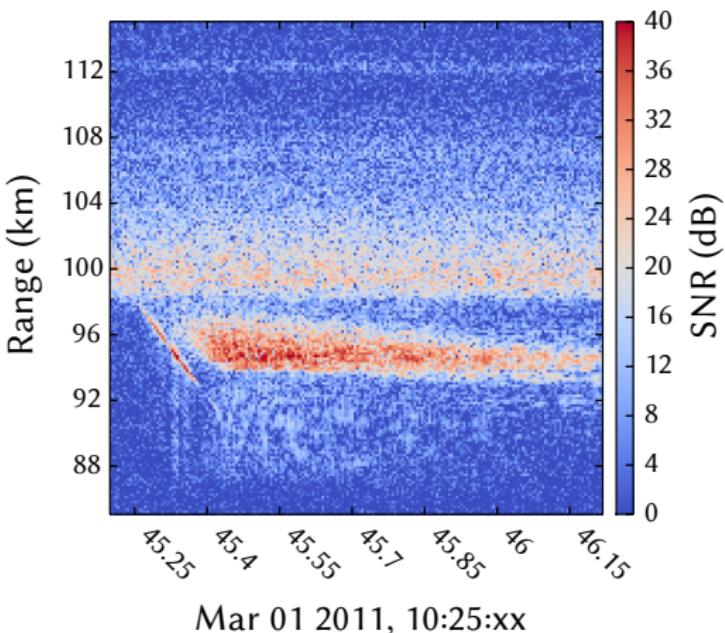


- ▶ Detection rate of meteors is better at lower frequencies
- ▶ Interferometry and/or dual polarization receive

# Jicamarca experiment goals

## Goals

- ▶ Test sparsity-based waveform inversion in crowded environment
- ▶ Directly compare effectiveness of different waveforms



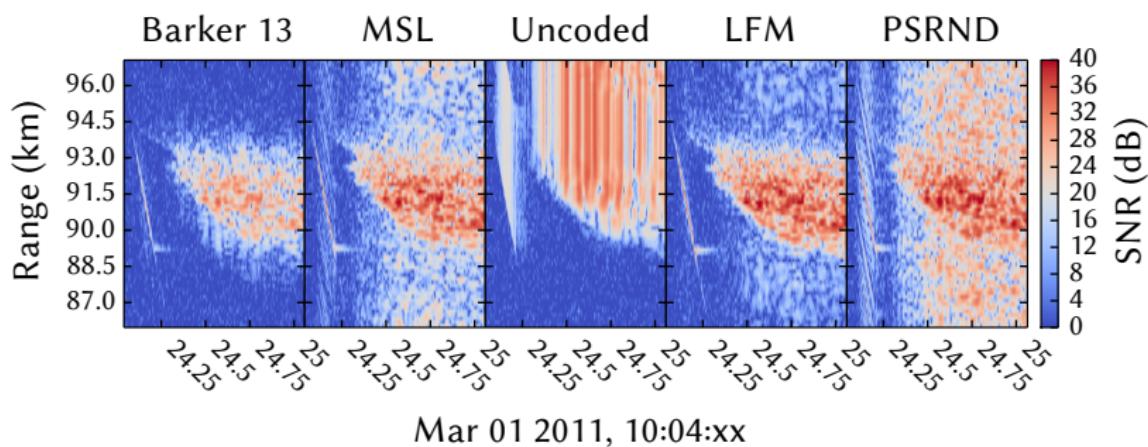
# Jicamarca meteor experiment

## Description

Alternating sequence of 5 common waveforms for observing meteor region (80-140 km altitude).

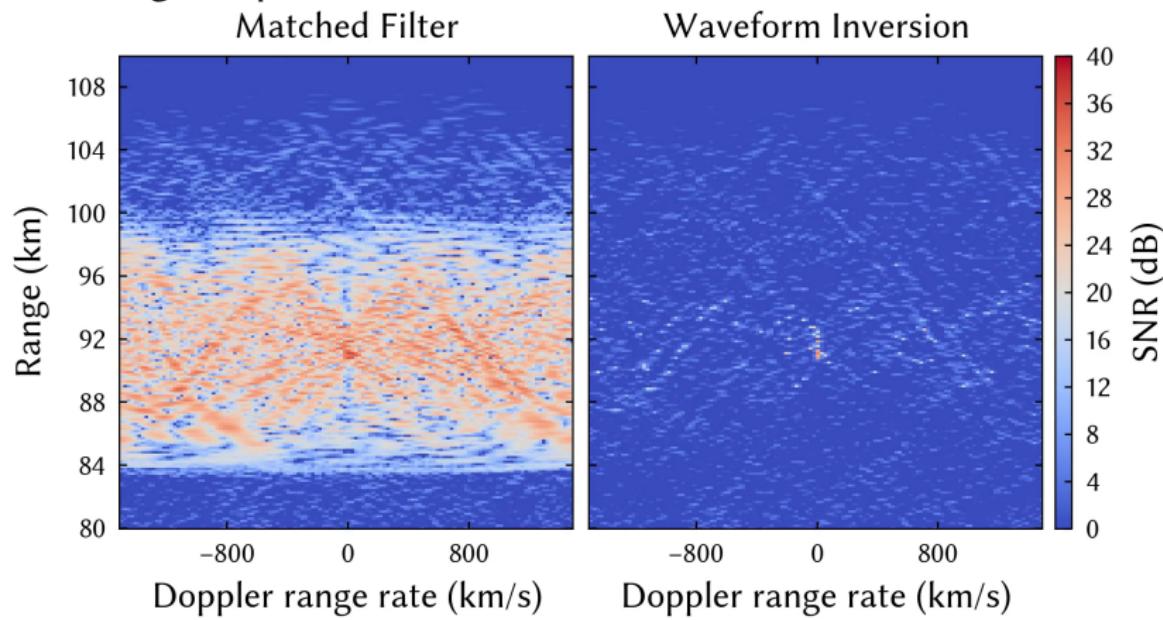
## Parameters

- ▶ Pulse interval of 1 ms
- ▶ Sample time of  $1 \mu\text{s}$   
(150 m range resolution)



# Movie of meteor sidelobe removal

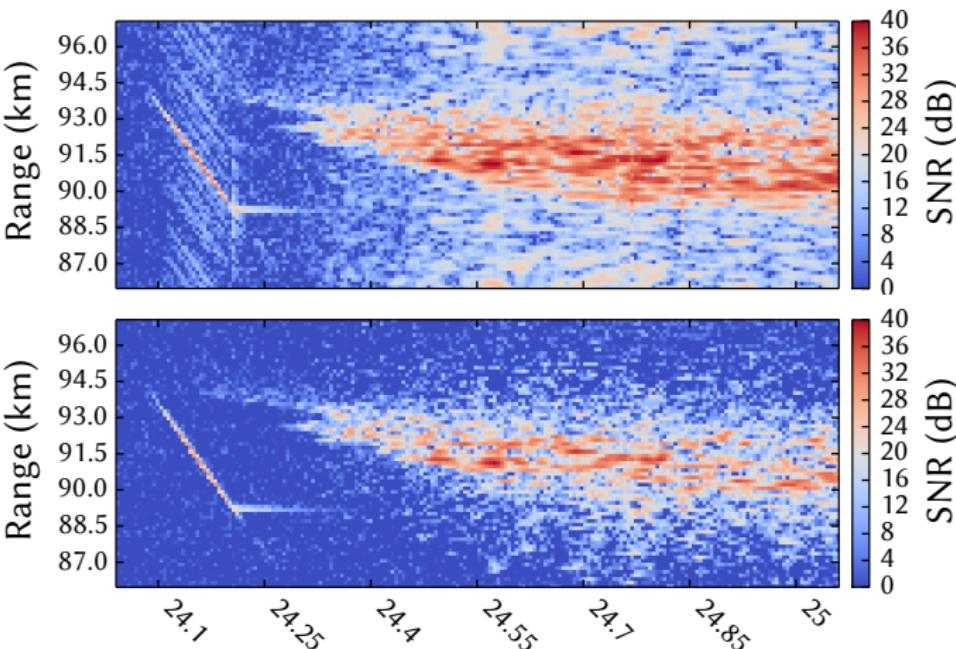
Decoding comparison for minimum sidelobe code:



(Total elapsed time of 1 second)

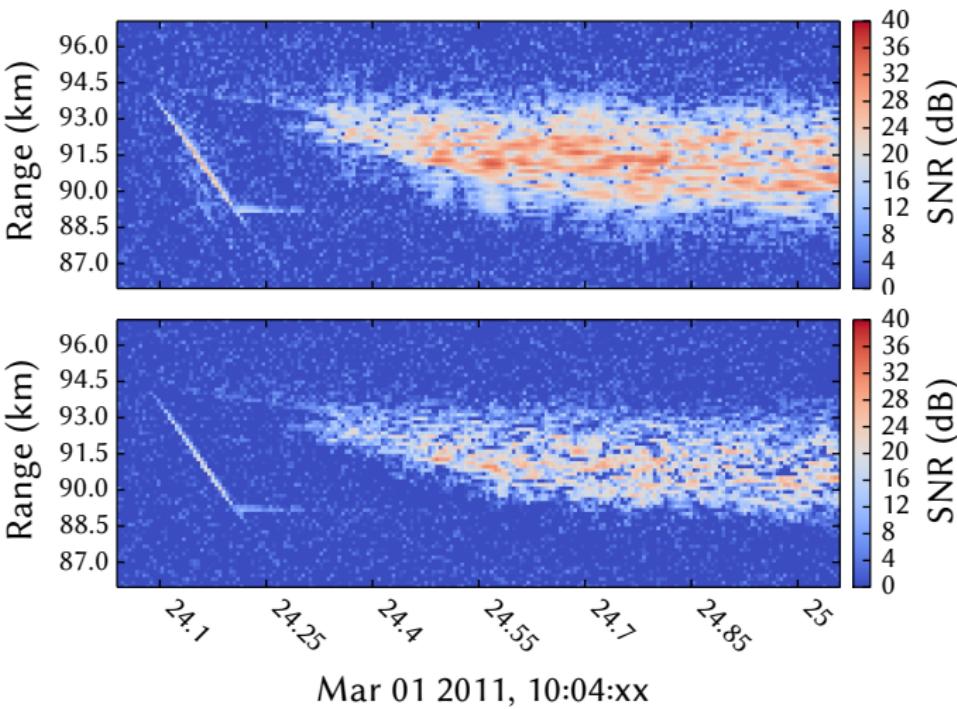
# Example: Minimum sidelobe code

Matched  
Filter



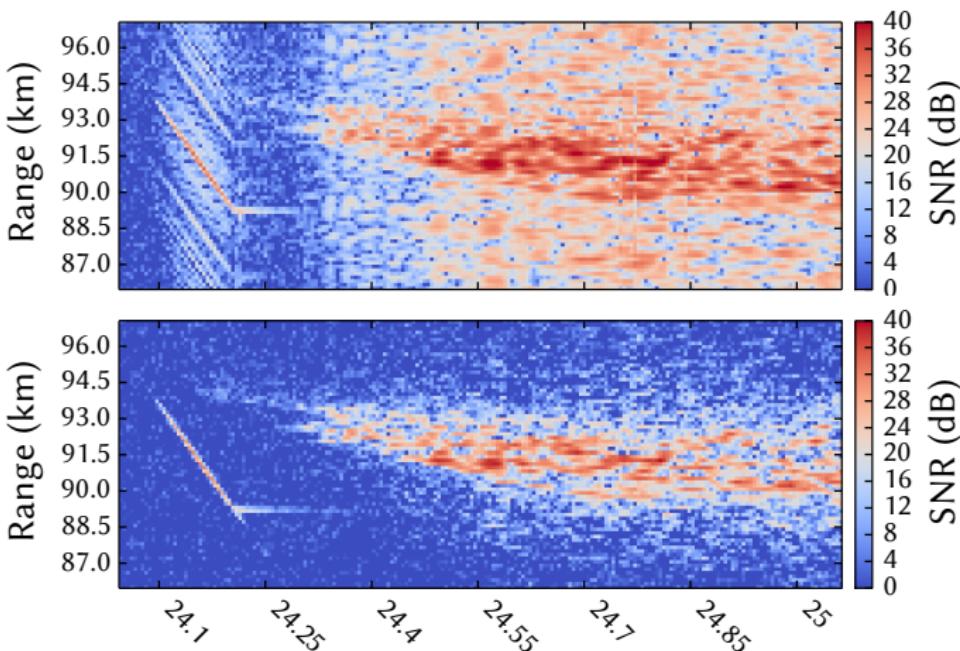
# Example: Barker-13 code

Matched  
Filter



# Example: Pseudorandom code

Matched  
Filter

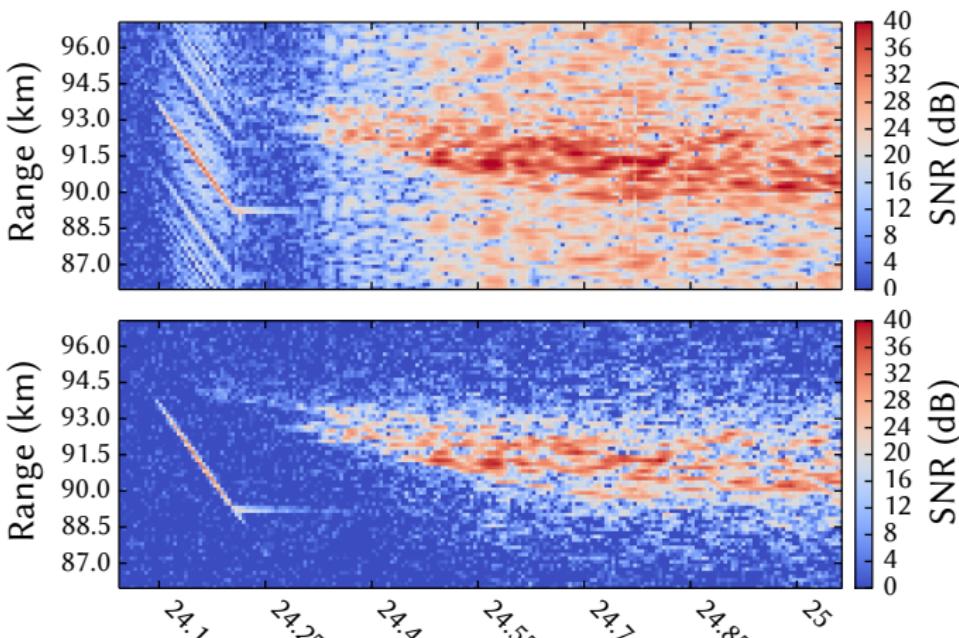


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Works with variety of codes

# Example: Pseudorandom code

Matched  
Filter



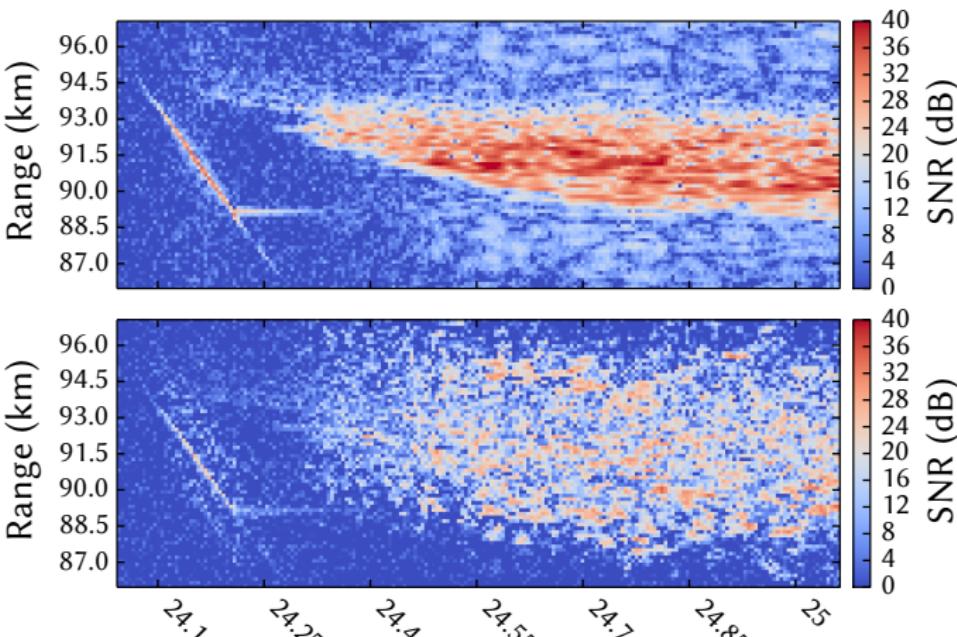
Waveform  
Inversion

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Works with variety of codes

# Example: LFM chirp

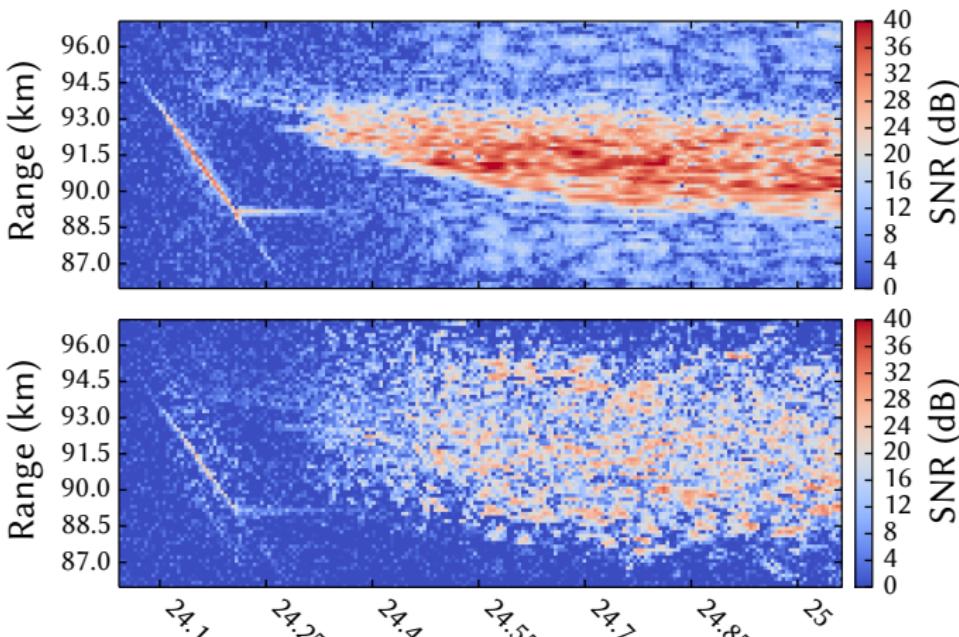
Matched  
Filter



Some codes are troublesome

# Example: LFM chirp

Matched  
Filter

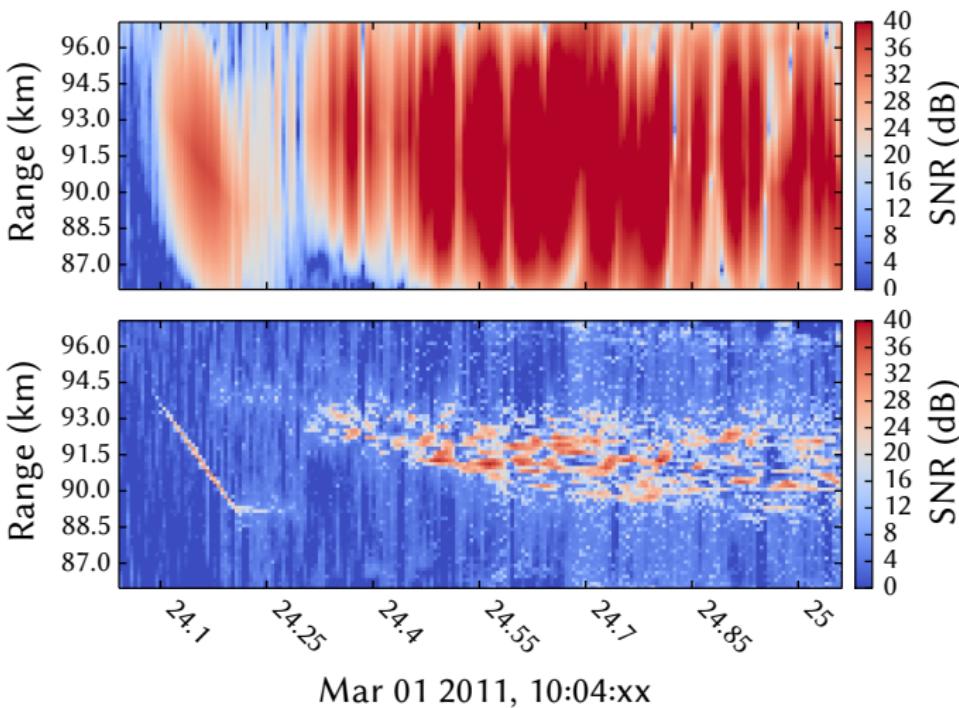


Waveform  
Inversion

Some codes are troublesome

# Example: Uncoded

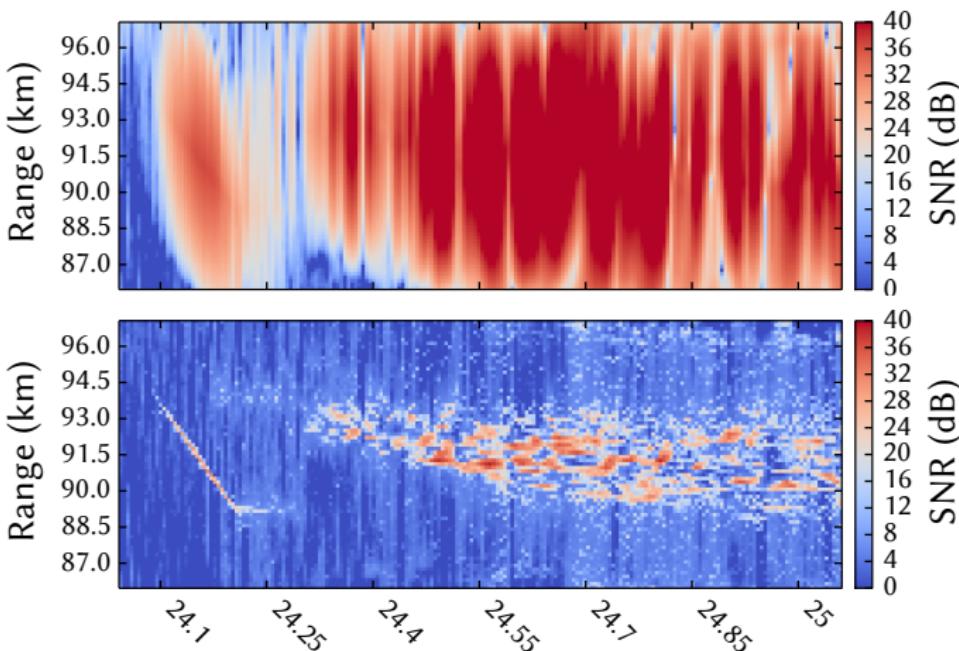
Matched  
Filter



Works even with uncoded pulses!

# Example: Uncoded

Matched  
Filter

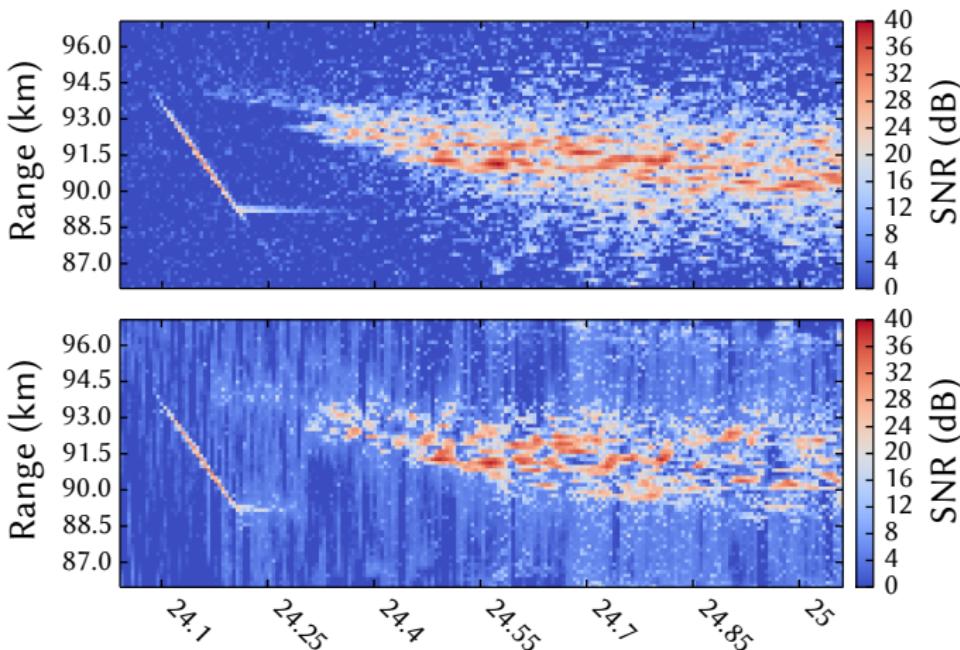


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Works even with uncoded pulses!

# Waveform inversion code comparison

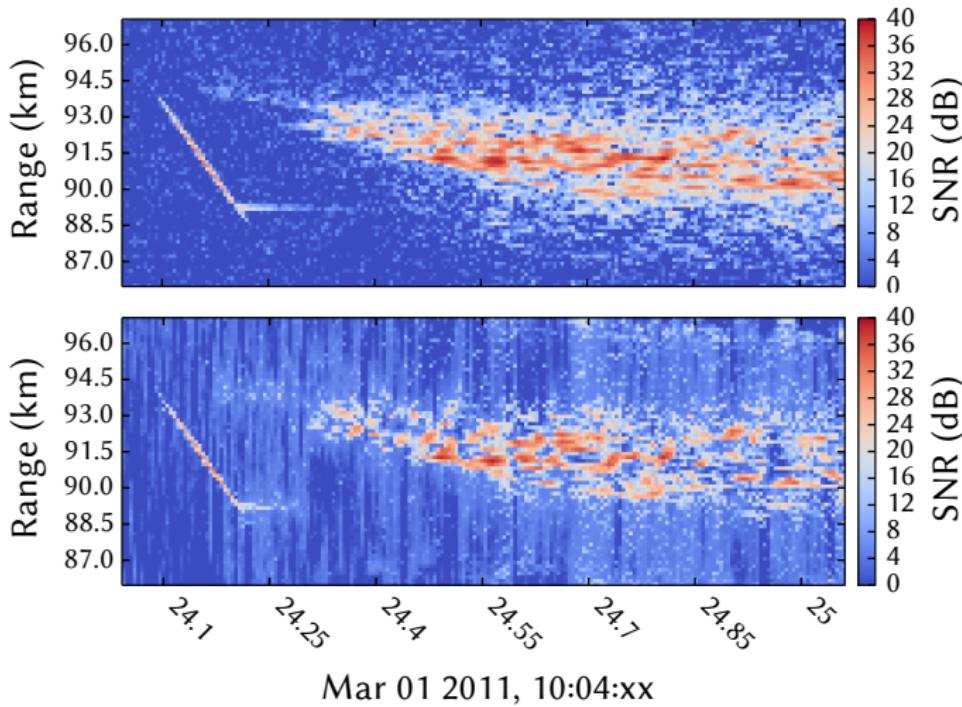
Minimum sidelobe



Quality of solution depends on fidelity of waveform

# Waveform inversion code comparison

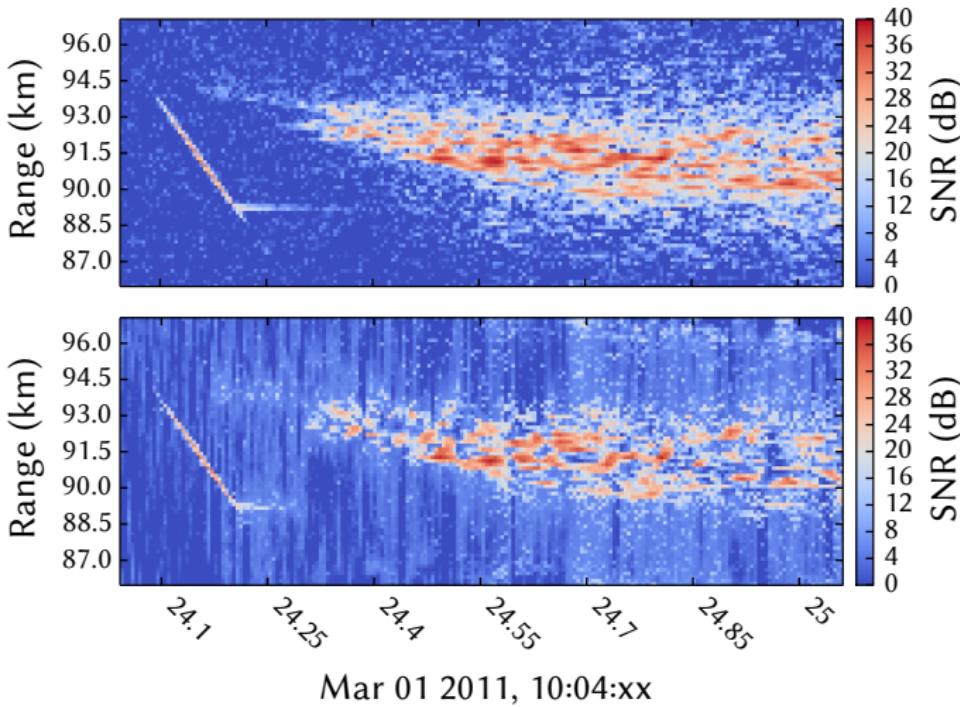
Pseudo-random



Quality of solution depends on fidelity of waveform

# Waveform inversion code comparison

Pseudo-random



Quality of solution depends on fidelity of waveform

# Outline

Introduction and Motivation

Radar Background

My Radar Model

Sparsity Background

Waveform Inversion

Conclusion

# Benefits of waveform inversion

## 1. Sidelobe removal -

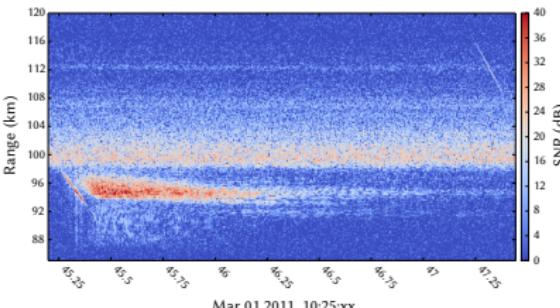
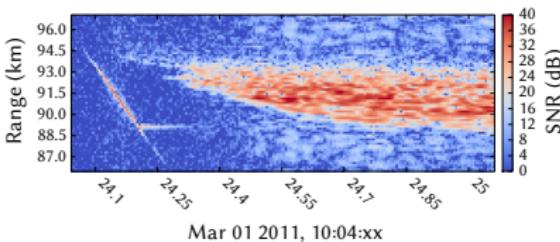
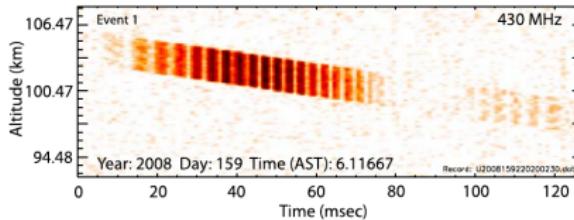
Removes ambiguity for observation of multiple or range-spread targets  
(e.g. fragmentation and non-specular trails)

## 2. Full frequency decoding -

Decodes over full frequency spectrum simultaneously, for target differentiation in crowded environments  
(e.g. equatorial, flares)

## 3. Flexibility -

Enables use of many different waveforms



# Contributions

In order to enable flexible high-resolution measurements of ionospheric plasma phenomena, including interfering and range-spread targets, I made the following contributions:

1. Derived a discrete radar model that captures signal sparsity in delay-frequency space.
2. Uncovered the adjoint relationship between the model and existing matched filter techniques, giving an intuitive interpretation of waveform inversion as iterative filtering.
3. Analyzed the model to show how it exactly represents distributed scatterers and preserves their sparsity.
4. Created and tuned an efficient implementation of waveform inversion using modern convex optimization techniques.
5. Demonstrated the flexibility and effectiveness of the inversion technique by removing filtering artifacts from meteor observations made with a variety of radar waveforms.

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# Further research

## Near-universal radar measurements

Submitted NSF postdoc proposal to work at MIT Haystack (Millstone Hill) and create near-universal measurement mode across radar chain using uniquely-tuned waveforms.

## Self-calibration

Iteratively solve for reflectivity *and* transmitted waveform to better characterize the radar system.

## Passive radar

Waveform flexibility invites use for parasitic radio science (using FM radio, digital TV signals) where the transmitted waveform cannot be controlled.

# Acknowledgements

Thank you for coming!

# Questions

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### The Ionosphere

- 2 The ionosphere
- 3 Importance of the ionosphere: Communications
- 4 Importance of the ionosphere: Science of the geospace system
- 5 Space weather: meteoroids and meteors
- 6 Importance of meteoroids

### Radar Measurements

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- 8 Radar meteors
- 9 Variety of ionospheric plasma
- 10 At the limit: meteors
- 11 Measurement challenges

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- 13 The world is sparse
- 14 Sparsity revolution

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- 16 Additional contributions

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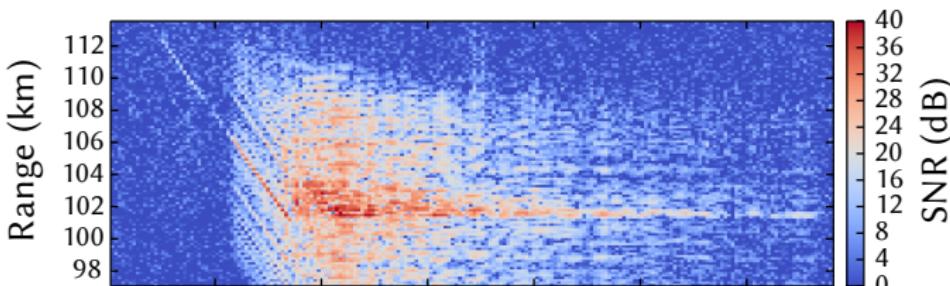
### Questions

- 71 Questions

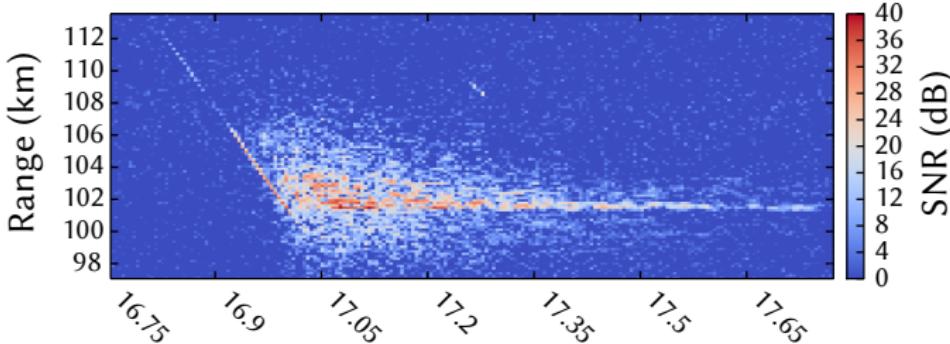
# Appendix

# Another example: Pseudorandom code

Matched  
Filter



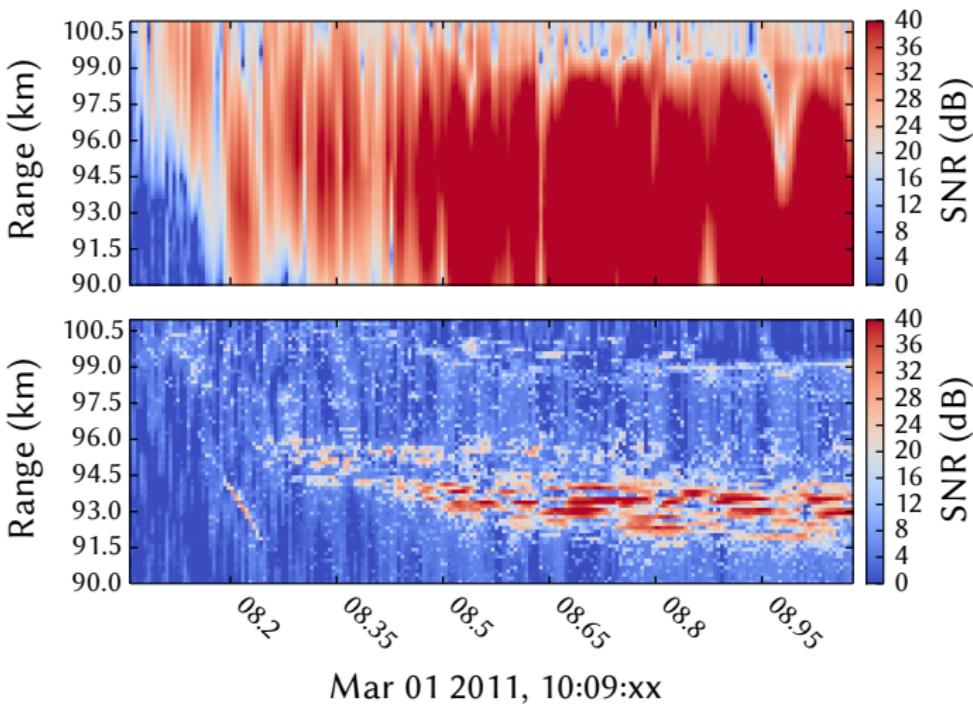
Waveform  
Inversion



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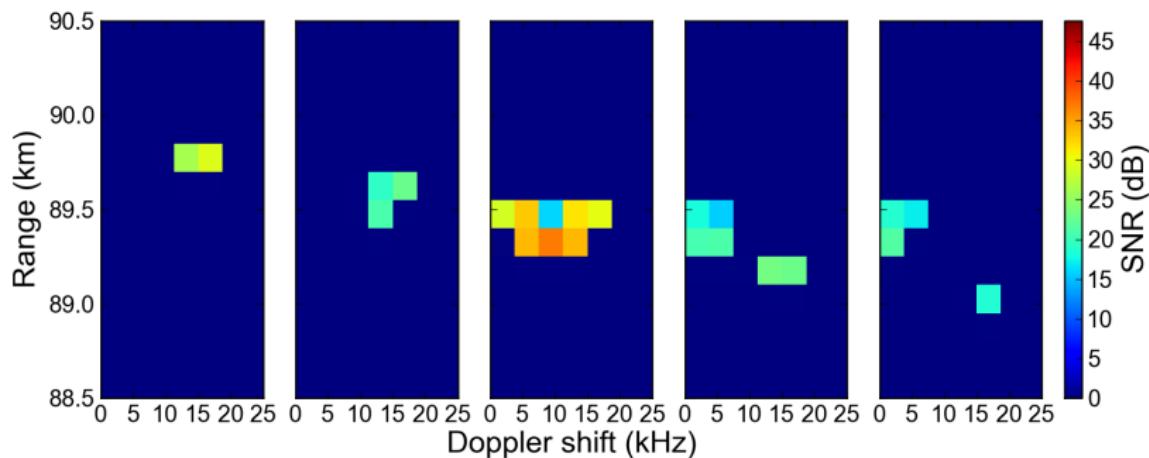
# Different example: Uncoded

Matched  
Filter



# Jicamarca flare

- ▶ Waveform inversion of a meteor head echo and flare
- ▶ Minimum sidelobe code data from Jicamarca experiment
- ▶ 5 pulses around the event:



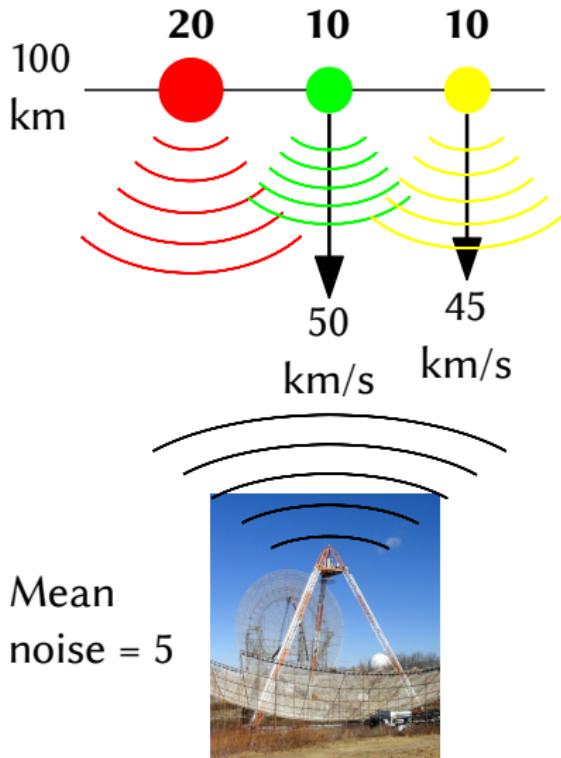
# Simulation parameters

## Point target simulation

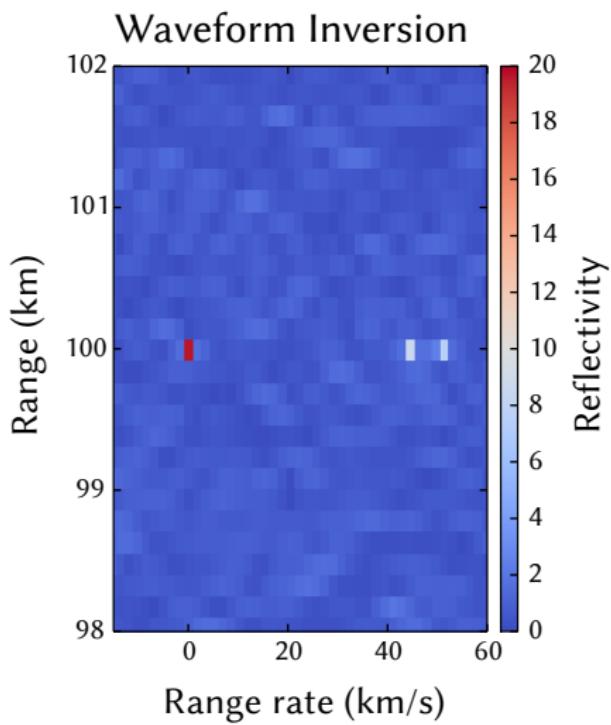
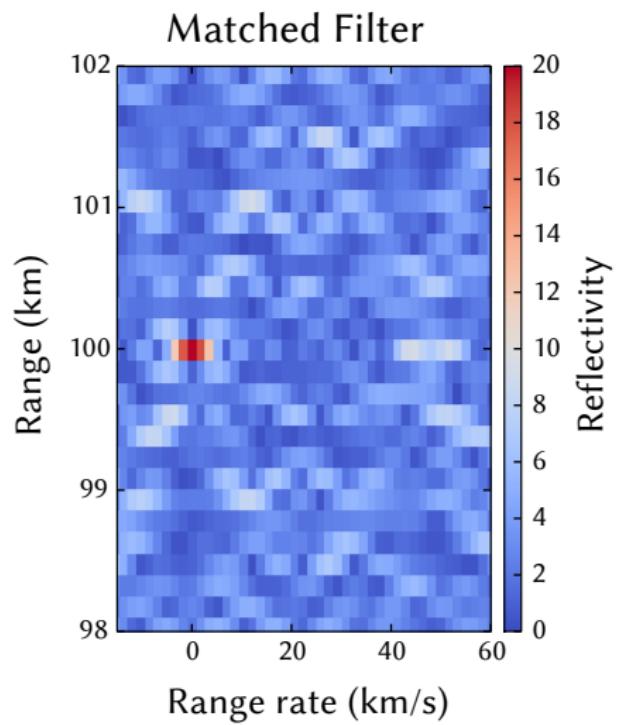
- ▶ 3 targets at 100 km range:

Reflectivity	Range rate
20	0 km/s
10	45 km/s
10	50 km/s

- ▶ Mean noise power of 5
- ▶ 51-baud minimum sidelobe code



# Simulation results



# The prox operator

The proximal operator, or **prox operator**, is defined for non-smooth  $G(x)$  as

$$\mathbf{prox}_{\mu G}(v) = \arg \min_x \left( G(x) + \frac{1}{2\mu} \|x - v\|_2^2 \right).$$

## Projection onto a set

If  $G(x)$  is the indicator function for a closed convex set  $\mathcal{C}$ ,

$$G(x) = \begin{cases} 0 & x \in \mathcal{C} \\ \infty & x \notin \mathcal{C} \end{cases}$$

$\mathbf{prox}_G(v)$  is projection of  $v$  onto  $\mathcal{C}$ .

# Accelerated proximal gradient method

Set step size       $\mu^{k+1}$

$$\gamma := \mu^k / \mu^{k+1}$$

Acceleration parameter       $t^{k+1} := \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\gamma(t^k)^2}$

$$\theta := (t^k - 1) / t^{k+1}$$

Acceleration step       $w^{k+1} := x^k + \theta(x^k - x^{k-1})$

Gradient step       $z^{k+1} := w^{k+1} - \mu^{k+1} \nabla F(w^k)$

Prox step       $x^{k+1} := \mathbf{prox}_{\mu G}(z^{k+1})$

# Sidelobe math

- ▶ The function  $A^*A$  operating on a target scene  $h$  models the matched filter result of measuring that target.
- ▶ The sidelobes are the off-diagonal results of the function, given by the operation  $A^*A - \frac{1}{N}I$ .
- ▶ So to remove sidelobes from the matched filtered data  $A^*(y)$  using the approximating sparse solution  $h$ , we must calculate:

$$A^*(y) - \left( A^*A - \frac{1}{N}I \right) (h) = A^* (y - A(h)) + \frac{1}{N}h$$

- ▶ The standard matched filter is usually scaled by a factor of  $\sqrt{N}$  relative to the model, so the equivalent waveform inversion result with sidelobes removed is given by:

$$\frac{1}{\sqrt{N}}h + \sqrt{N}A^* (y - A(h))$$

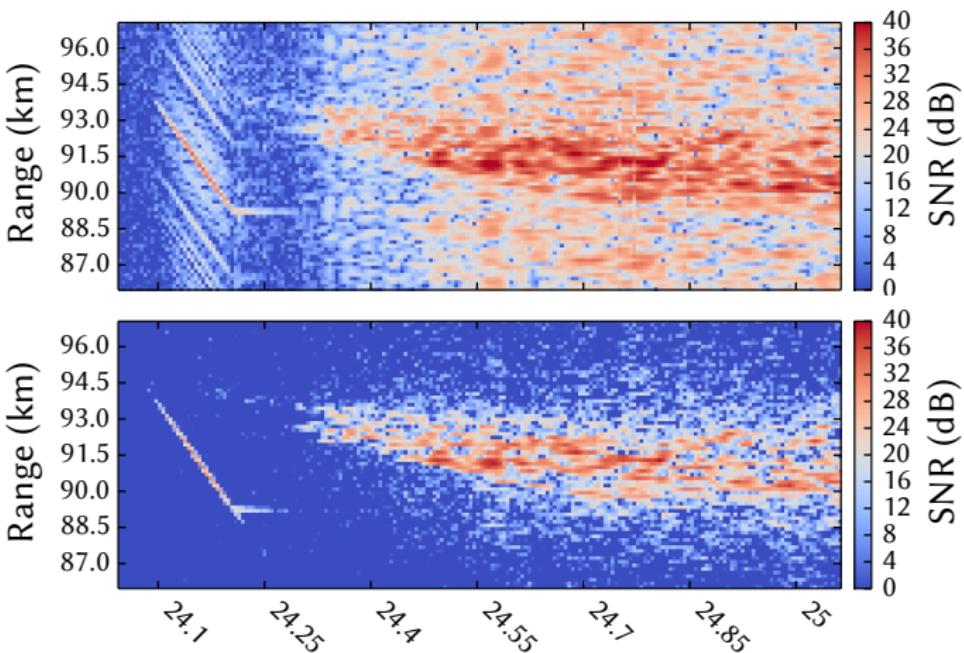
(sparse result plus matched filtered noise)

# Sparse solution

Matched  
Filter

Sparse  
Solution

Pseudorandom code



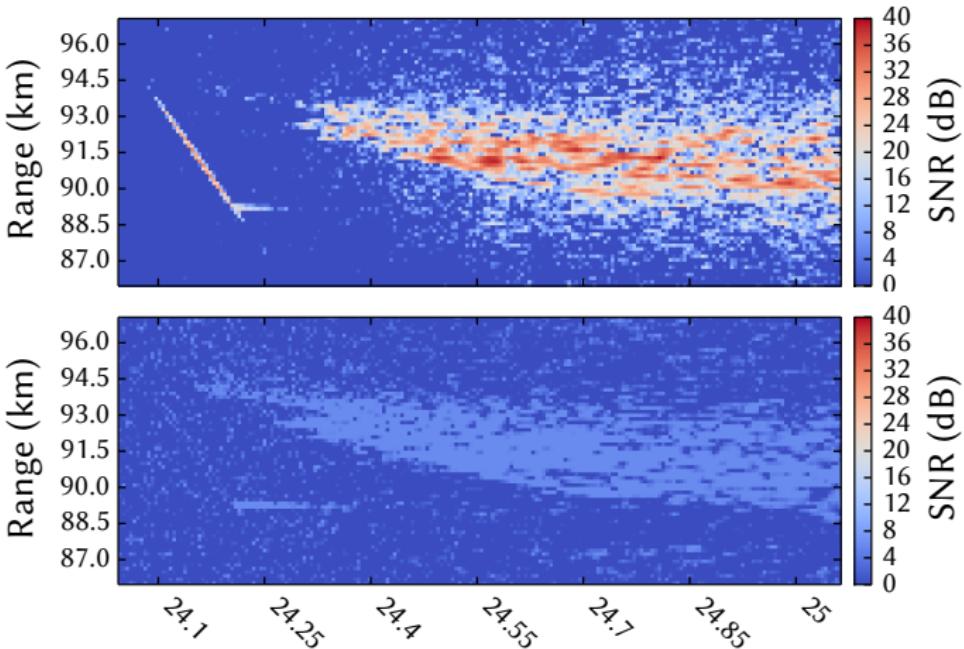
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# Unmodeled noise

Sparse Solution

Unmodeled Noise

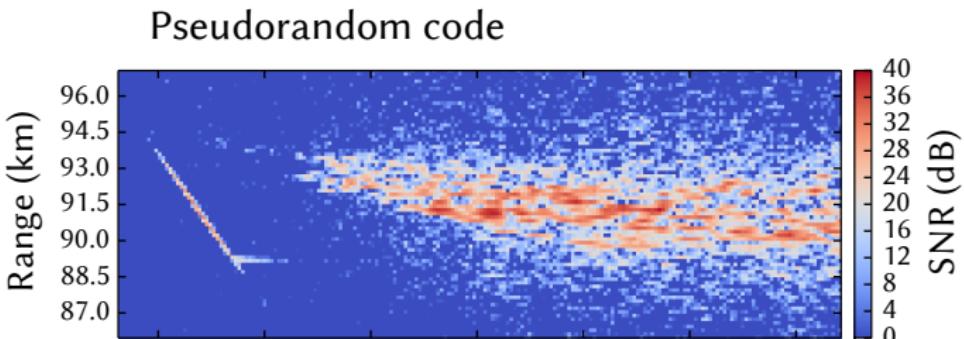
Pseudorandom code



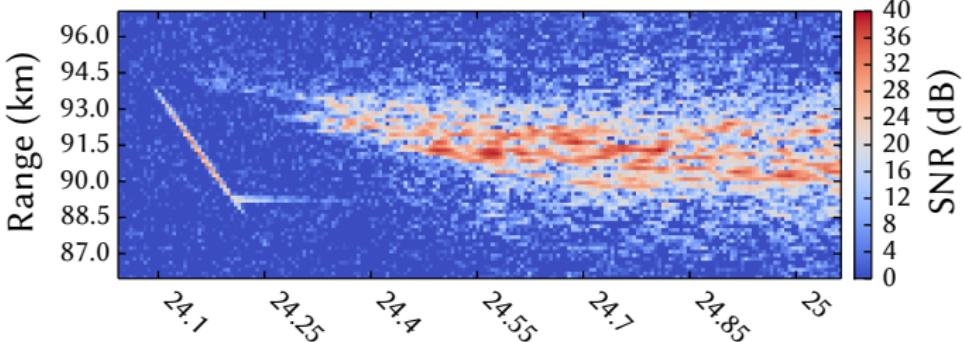
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# Sidelobe removal as sparse plus noise

Sparse Solution



Sparse plus Noise



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