Theory and Applications of Sparsity for Radar Sensing of Ionospheric Plasma

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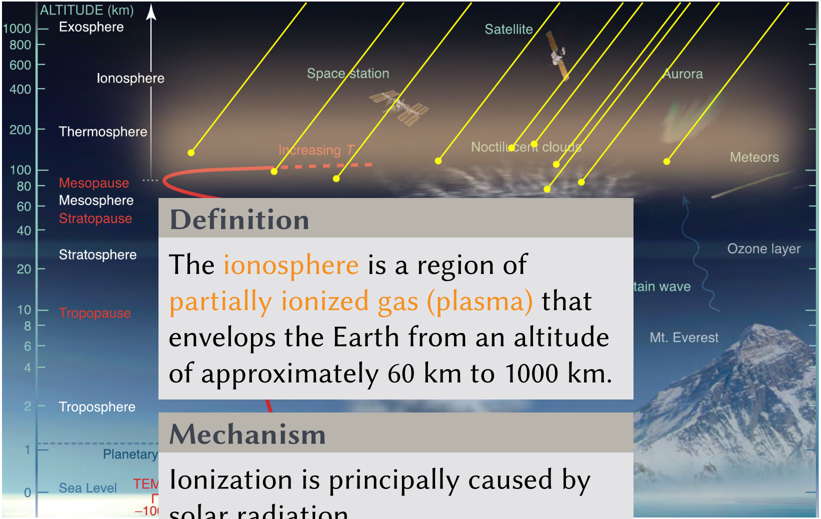
The ionosphere



Definition

The ionosphere is a region of partially ionized gas (plasma) that envelops the Earth from an altitude of approximately 60 km to 1000 km.

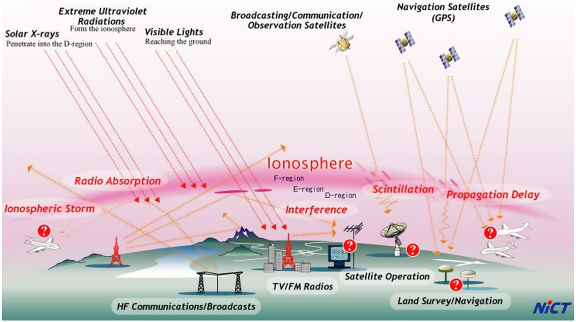
Mechanism

Ionization is principally caused by solar radiation.

Image from NSF (2011), CEDAR: The New Dimension

Importance of the ionosphere

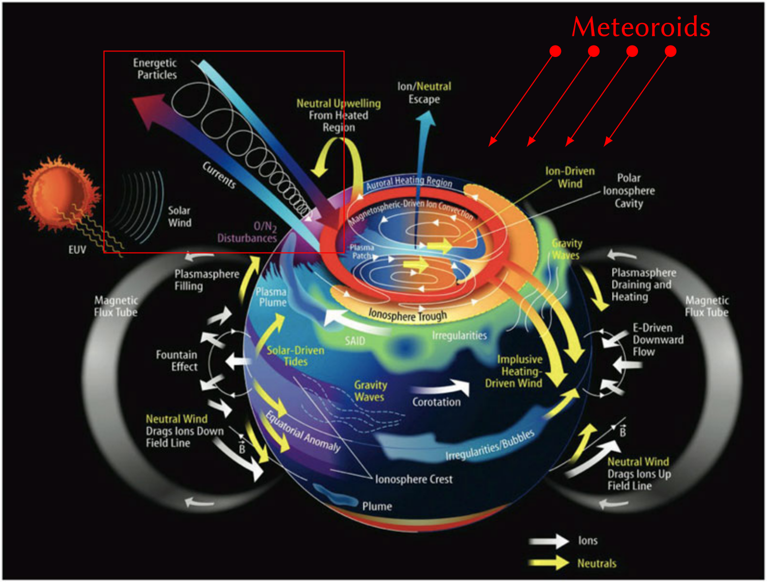
Communications



- Radio waves of lower frequency (less than ≈ 10 MHz) reflect off ionosphere.
- Microwave frequencies (e.g. GPS) can pass through, but experience frequency-dependent refraction and delay.
- Plasma irregularities additionally perturb signals.

Importance of the ionosphere

Science of the geospace system



Space weather: meteoroids and meteors

Definition (Meteoroid)

- Solid body in space
- Small (< 1 m in diameter)
- Fast (from 11 to 72 km/s)
- Exponentially more common as size decreases

Micro-meteorites (landed on Earth)

Definition (Meteor)

- Plasma created by meteoroid entering the atmosphere
- Typically form at altitudes between 80 km and 120 km

NASA (2011)

Importance of meteoroids

Impact damage to Space Shuttle Atlantis

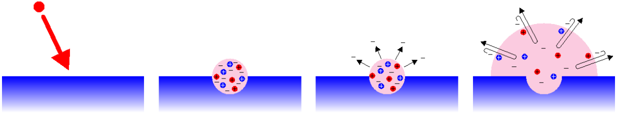
High speed impacts

Threat to satellites and spacecraft from

- mechanical damage
- electrical damage

Required observations

- Rate/count
- Size
- Speed
- Composition



Measuring the ionosphere

Definition

Incoherent Scatter Radars (ISRs, locations shown below) are radars sensitive enough to measure scattering from electrons in the background ionosphere.

Range of ionospheric regions: polar, mid-latitude, equatorial.

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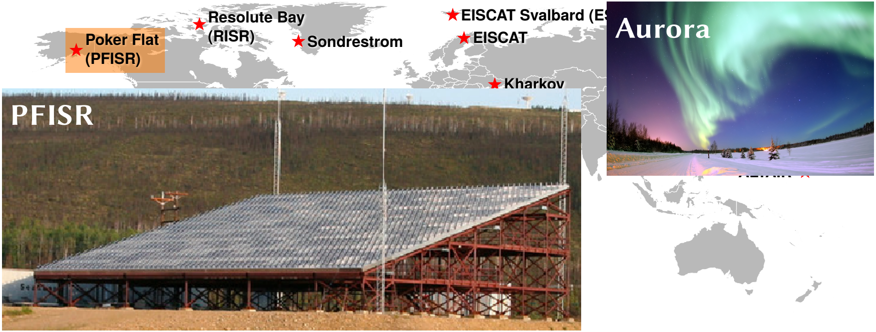

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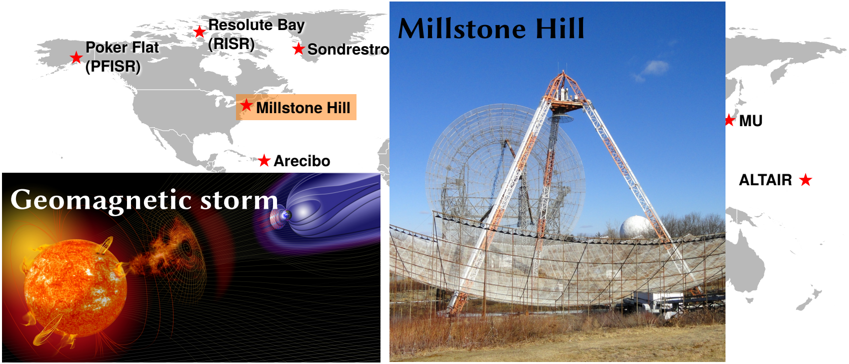

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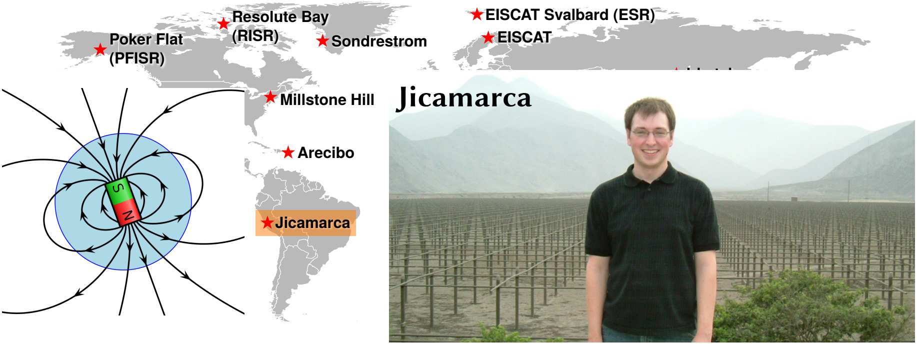
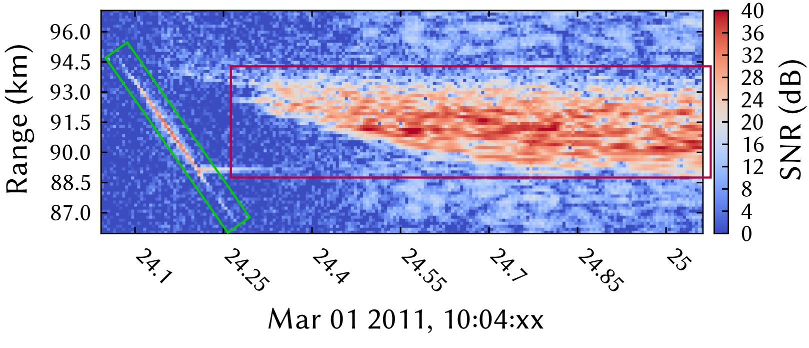


Range of ionospheric regions: polar, mid-latitude, equatorial.

Radar meteors

Types of scattering

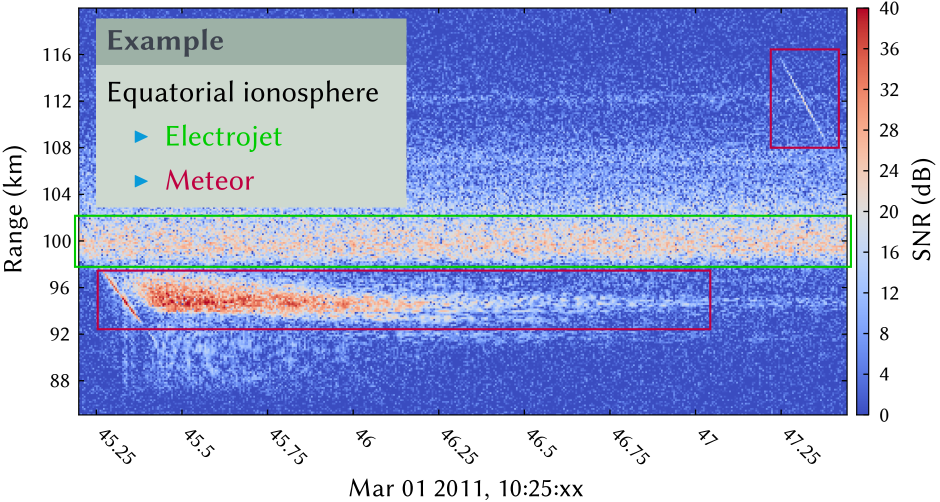
- Head: ball of plasma traveling with the meteoroid
- Trail: plasma left in wake of meteoroid



Time of Radar Pulse

Variety of ionospheric plasma

There are many different plasma phenomena, each presenting its own challenges for observation.



Example

Equatorial ionosphere

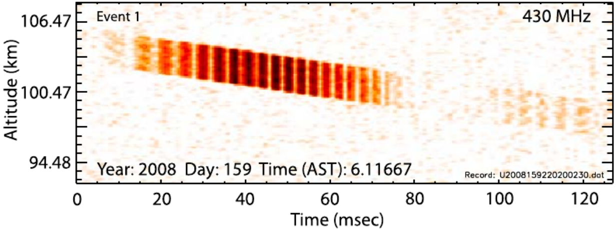
- Electrojet
- Meteor

At the limit: meteors

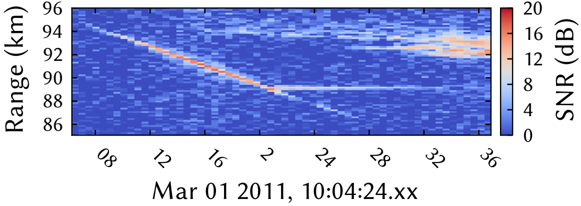
Scientific progress is limited by current signal processing techniques!

Mathews, et al. (2010), *Geophysical Research Letters*

Fragmentation

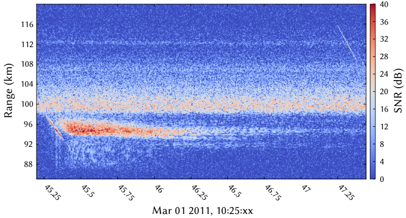


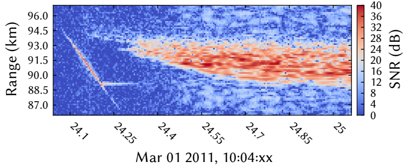
Flares and terminal events

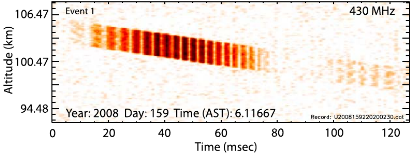


Measurement challenges

1. Differentiation in a crowded and variable environment (e.g. meteors and electrojet)
2. Self-interference of range-spread targets (e.g. meteor trails)
3. High resolution to observe small-scale processes (e.g. meteoroid fragmentation and flares)
4. Flexibility to achieve all of the above at once

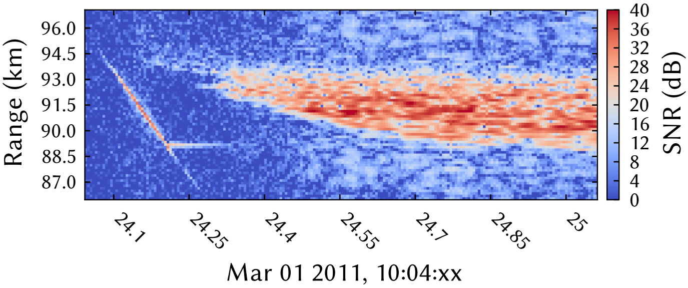






A way forward: sparsity

Radar targets are typically sparse in range and frequency!



Can use sparsity to systematically improve the quality of radar measurements.

The world is sparse

Definition

- A vector is sparse if only a few of its elements are nonzero.
- Additionally, the term is often applied loosely to when this is approximately true (i.e. most of the elements close to zero).

Many natural phenomena have sparse representations:

- Music and speech (mp3)
- Images (jpeg)

Alpacas (right): 8.5x savings

Uncompressed:	3600 KB
Compressed:	420 KB

Alpacas at the Jicamarca radar site

Sparsity revolution

Exploitation of inherent signal sparsity through compressed sensing is leading to advancements in many subjects.

Single pixel camera (1)

Smaller, cheaper, more flexible

dsp.rice.edu/cscamera

MRI (2)

Faster scans, higher resolution

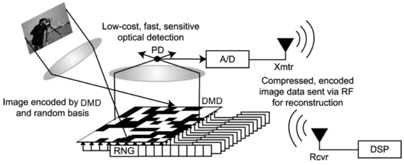
eecs.berkeley.edu/ mlustig/CS.html

Genomics

High-throughput screening

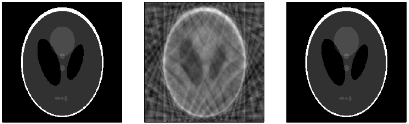
erlichlab.wi.mit.edu

(1)



(2)

Original | Filtered back-projection | Compressed sensing



Contributions

In order to enable flexible high-resolution measurements of ionospheric plasma phenomena, including interfering and range-spread targets, I made the following contributions:

1. Derived a discrete radar model that captures signal sparsity in delay-frequency space.
2. Uncovered the adjoint relationship between the model and existing matched filter techniques, giving an intuitive interpretation of waveform inversion as iterative filtering.
3. Analyzed the model to show how it exactly represents distributed scatterers and preserves their sparsity.
4. Created and tuned an efficient implementation of waveform inversion using modern convex optimization techniques.
5. Demonstrated the flexibility and effectiveness of the inversion technique by removing filtering artifacts from meteor observations made with a variety of radar waveforms.

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Additional contributions

In addition, I also made the following contributions that will not be covered in this talk:

1. Created a range-time-frequency clustering algorithm for detection and classification of ionospheric radar signals.
2. Simulated the statistical accuracy of interpolated matched filter estimation for range and range-rate super-resolution of meteor head echoes.

Outline

Radar Background

My Radar Model

Sparsity Background

Waveform Inversion

Conclusion

Outline

Radar Background
- Pulse Encoding/Decoding
- Measurement Ambiguity
- Current Techniques

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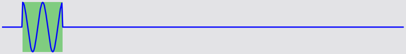
Radar pulse length

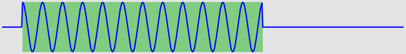
Some applications favor a shorter radar pulse, while others favor a longer radar pulse.

Short pulse



- Low range ambiguity (interference region for multiple scatterers)
- Simple to interpret

Long pulse

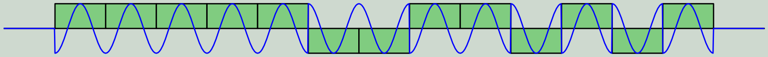


- Fine frequency resolution (observe target as it evolves in time)
- High total power

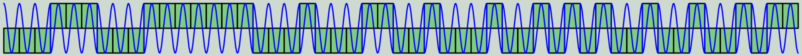
Coded pulses

- Encoding allows long pulses to have the low range ambiguity of short pulses.
- One method: divide pulse into bauds of constant phase.

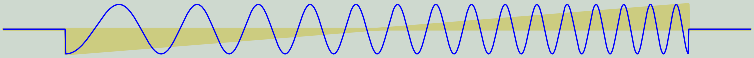
Barker-13 code



Minimum peak sidelobe code



Linear frequency modulation (LFM chirp)



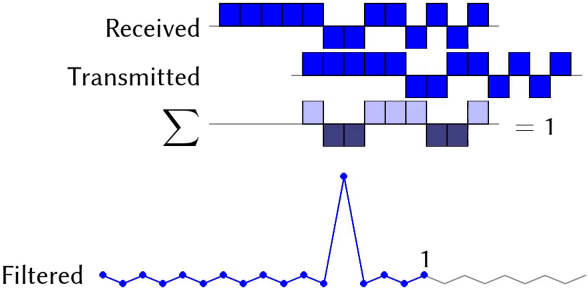
The matched filter

Definition

The matched filter correlates every segment of the received signal with the transmitted pulse.

It produces a peak at the target's range (delay), achieving a rough form of decoding with ambiguous sidelobes.

Example (Barker-13 code, Minimum sidelobe code)



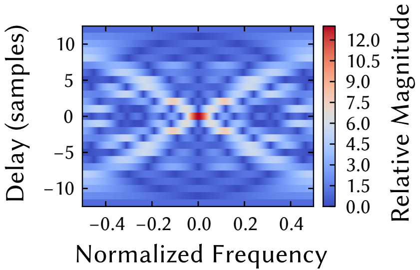
Frequency filter banks

Doppler frequency shift

If the target is moving toward or away from the radar, the reflected signal is frequency shifted up or down.

- Matched filter needs to be similarly frequency shifted
- True frequency shift is not known ahead of time
- Process signal with a bank of differently-shifted filters

Result of filter bank for centered "point" target, using Barker-13 code



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- Pulse Encoding/Decoding
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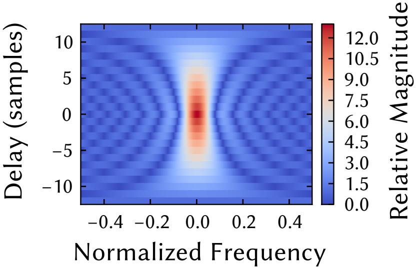
Ambiguity functions

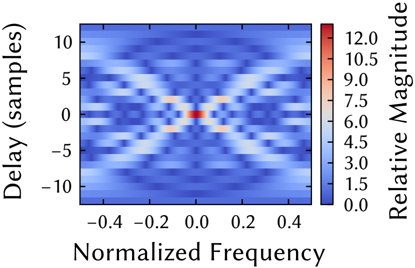
Definition

A delay-frequency ambiguity function is produced when a signal is matched against itself using a filter bank.

Uncoded pulse

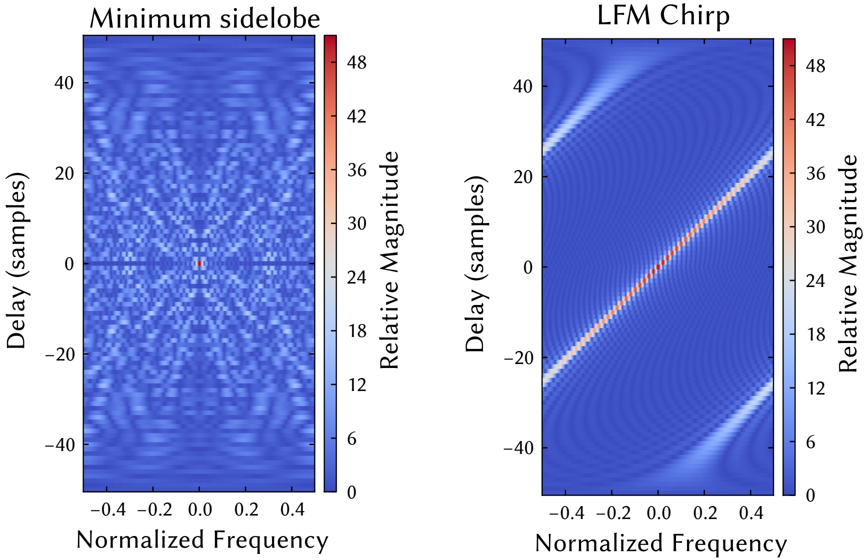


Barker-13



The ambiguity function describes the delay-frequency sidelobes of a code.

More example ambiguity functions

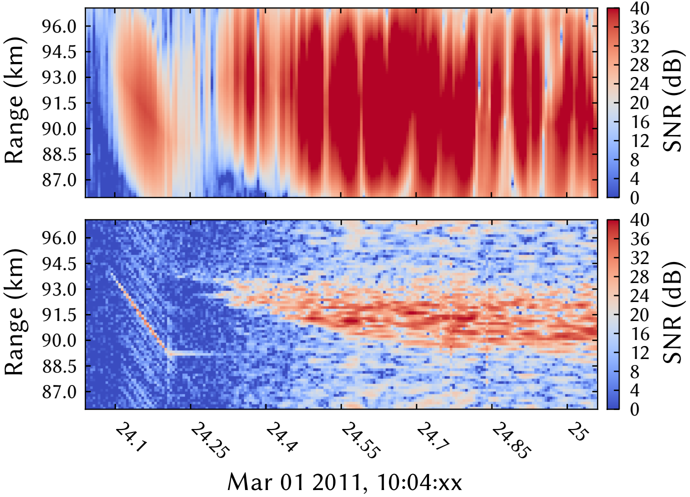


Range sidelobes in action

Range sidelobes dominate the output of uncoded measurements, while coded pulses are better but still noticeably corrupted.

Uncoded

Minimum sidelobe code

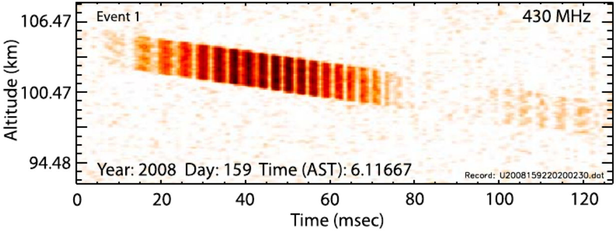


Effect of sidelobes

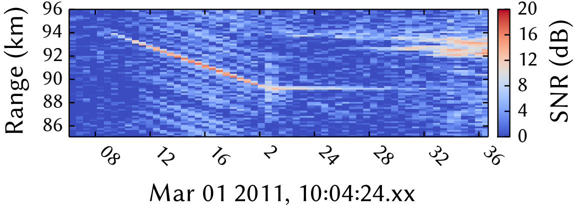
Sidelobes (delay-frequency ambiguity) are the primary reason that outstanding ionospheric science questions cannot be satisfactorily answered.

Mathews, et al. (2010), *Geophysical Research Letters*

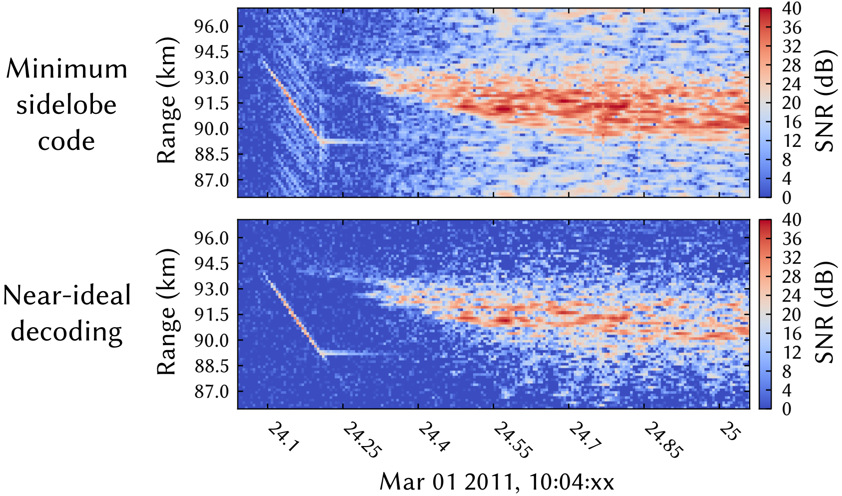
Fragmentation



Flares and terminal events



Goal: sidelobe removal



Infinite number of ways to decode and get a target scene that reproduces the measurements!

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Delay-frequency sidelobe mitigation in use

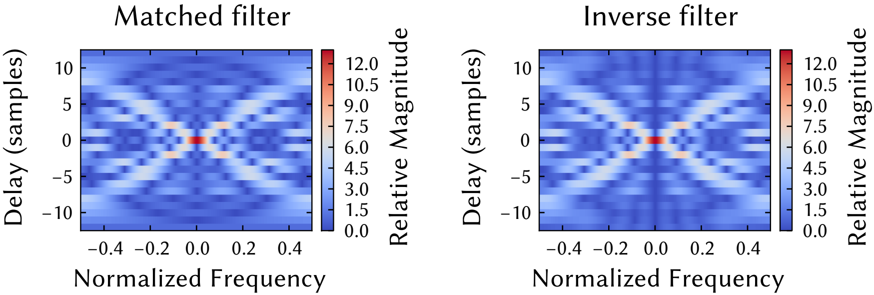
1. Codes with good sidelobe properties
   - e.g. Barker-13, minimum peak sidelobe, LFM chirp
   - Most prevalent strategy
   - Only effective when it is simple to ignore remaining sidelobes
2. Alternating codes
   - Set of codes over multiple pulses
   - When pulses summed together, sidelobes cancel
   - Target properties must remain stationary
3. Inverse filters
   - Alternative to matched filter
   - Loss of SNR depending on code
   - Produces no range sidelobes at target's frequency shift

These strategies severely constrain the flexibility and accuracy of ionospheric radar measurements!

Inverse filter ambiguity function

Example

Ambiguity function comparison for Barker-13 code.



No advantage if Doppler shift is unknown or multiple frequencies must be decoded!

Outline

Radar Background

My Radar Model
Defining the Model
Representation of Targets

Sparsity Background

Waveform Inversion

Conclusion

Matched filtering as imaging

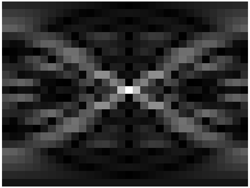
Imaging analogy

Matched filter “image” is blurred by the ambiguity function:

$$h[n,p] * \chi[n,p] = x[n,p]$$

Target reflectivity (source image) | Ambiguity function (point-spread func.) | Matched filter result (measured image)

$*$



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Radar model from matched filter

Ambiguity equation

$$x[n,p] = \chi[n,p] * h[n,p]$$

$$A^*\left(y[m]\right) = A^*\left(A\left(h[n,p]\right)\right)$$

- Matched filtering is A^*
- Ambiguity function is A^*A
- Indices:

m	sample
n	frequency
p	delay

Radar model

Simplify by removing excess matched filtering operation:

$$y[m] = A\left(h[n,p]\right)$$

Measured signal | Radar model | Target reflectivity

- Radar model is adjoint (conjugate transpose) of matched filter
- Non-invertible because under-determined
- Sparsity of target reflectivity is key to solution

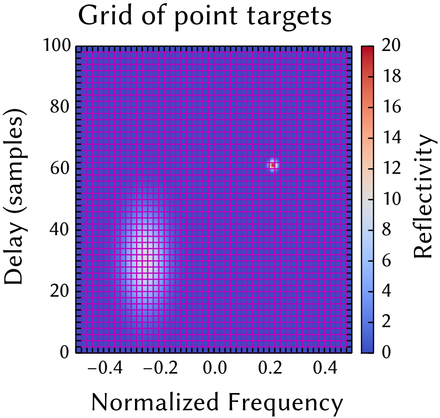
Alternative interpretation

Matched filter (A^*)

Correlates received signal with expected return from point targets with different delays and frequency shifts.

Radar model (A)

Simulates received signal as sum of returns from point targets with different delays and frequency shifts.



$$y = A(h[n, p])$$

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Model discretization from radar signal equation

Narrow-band radar equation for received signal

$$y(t) = \int_0^T \int_{-\infty}^{\infty} s(t-\lambda) e^{2\pi i f(t-\lambda)} h(f,\lambda)\, df\, d\lambda$$

Measured signal | Transmitted signal | Frequency shift | Target reflectivity

t	Sample time	$y(m\tau) \to y[m]$	$m = 0, 1, \ldots, M-1$
f	Frequency	$h(n\phi, :) \rightsquigarrow h[n, :]$	$n = 0, 1, \ldots, N-1$
λ	Delay	$h(:, p\tau) \rightsquigarrow h[:, p]$	$p = 0, 1, \ldots, P-1$

Discretized radar model

$$y[m] = \sum_{p=0}^{P-1} \sum_{n=0}^{N-1} \frac{1}{\sqrt{N}} s[m-p+L-1] e^{2\pi i nm/N} h[n,p] = A(h)$$

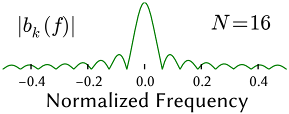
Reflectivity coefficients from function

Can write the "point target" reflectivity coefficients in terms of the original reflectivity function:

$$h[n,p] = \int_{p\tau}^{(p+1)\tau} \left[h(f,\lambda) e^{2\pi i f \lambda} * b_{p+1}(f) \right] (n\phi)\, d\lambda$$

1. Blur in frequency by convolving with wrapped sinc function:

$$b_k(f) = \frac{1}{N} e^{-\pi i (2k+N-1)\tau f} \frac{\sin(N\pi\tau f)}{\sin(\pi\tau f)}$$

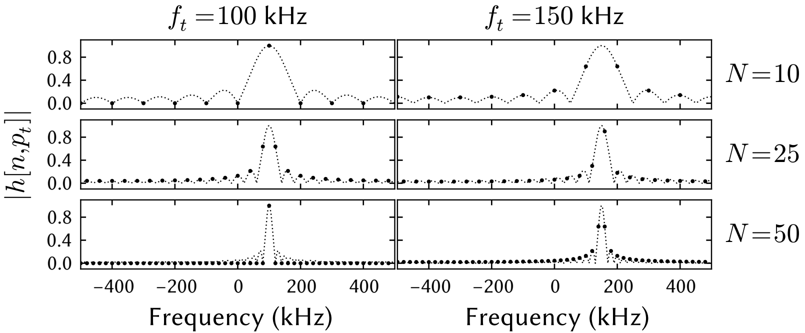


2. Sample at discretized frequency point $n\phi$
3. Integrate over delay window of discretized delay point $p\tau$

Representing an off-grid point target

Off-grid point targets are still relatively sparse!

- All coefficients outside delay window p_t are zero
- For frequency index $n = 0, 1, \ldots, N-1$, behavior dictated by wrapped sinc:



Relative sparsity improves with increasing N
(same number significant coefficients, many more zeros)

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- Compressed Sensing
- Convex Optimization

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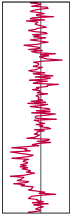
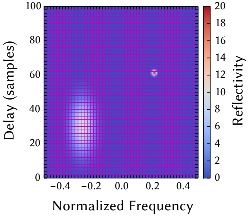
Under-determined systems of equations

Not enough measurements to constrain unknown values (e.g. the radar model):



$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} \quad & A & \quad \end{bmatrix} \begin{bmatrix} h \end{bmatrix}$$



(vectorized as h)

Measurement Model Unknown

- Infinite number of solutions
- Often know that the true solution should be sparse
- Finding the sparsest solution is hard in general

Theory of compressed sensing

Definition

Compressed sensing is a theory to guarantee solution of an under-determined set of equations.

Approximate guidelines for application

- Solution known to be sparse
- Measurements are “incoherent” (global with low correlation)
- Minimum number of measurements on the order of the solution sparsity (number of nonzeros)

Benefit

Can solve easy convex optimization problem instead of hard combinatorial problem.

Equivalent convex optimization problem

Sparsest solution to noisy measurements

$$\begin{array}{ll} \text{Find sparsest} & h \\ \text{subject to} & \|y - A(h)\|_2 < \eta \end{array} \qquad \|h\|_2^2 = \sum_k |h_k|^2$$

l_1-regularized least-squares (convex)

$$\underset{h}{\text{minimize}} \quad \frac{1}{2}\|y - A(h)\|_2^2 + \lambda \|h\|_1 \qquad \|h\|_1 = \sum_k |h_k|$$

The l_1-norm promotes sparsity!

Outline

Radar Background

My Radar Model

Sparsity Background

- Compressed Sensing
- Convex Optimization

Waveform Inversion

Conclusion

First-order methods

We want to efficiently solve

$$\underset{h}{\text{minimize}} \quad \frac{1}{2}\|y - A(h)\|_2^2 + \lambda\|h\|_1$$

but for systems A that are too large for matrix methods.

First-order methods

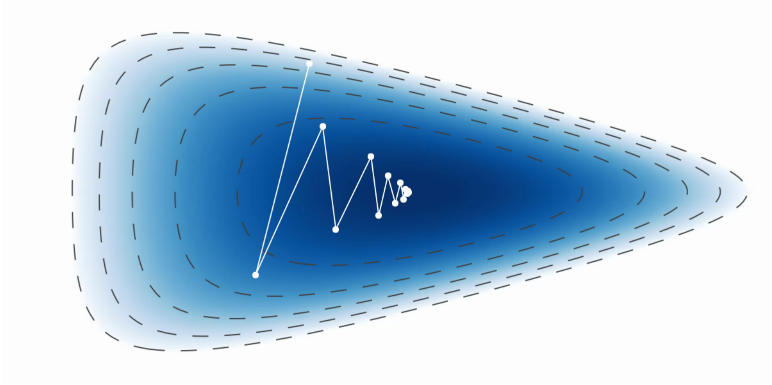
- Explicit matrix for A not needed
- Only need to be able to compute $A(\cdot\,)$ and $A^*(\cdot\,)$

l_1-regularized least-squares is non-smooth because of the l_1 norm term, so minimizing it requires a special approach.

First-order step

Smooth

If the function being minimized is differentiable, the most natural first-order step is a gradient step.



First-order step

Non-smooth

For a non-differentiable function, an attractive alternative is the proximal (or prox) step.

$$x^{k+1} := \arg\min_x \left(G(x) + \frac{1}{2\mu}\|x - x^k\|_2^2 \right)$$

Non-smooth function   Smoothing around current iterate

- This is surprisingly easy to solve and can be evaluated much like a gradient!
- Note that as x^k approaches the minimum x^*, the smoothing term goes to zero.

The prox operator is defined for non-smooth $G(x)$ as

$$\mathbf{prox}_{\mu G}(v) = \arg\min_x \left(G(x) + \frac{1}{2\mu}\|x - v\|_2^2 \right).$$

Proximal gradient method

The proximal gradient method combines gradient and prox steps to solve

$$\underset{x}{\text{minimize}} \quad F(x) + G(x)$$

for smooth $F(x)$ and non-smooth $G(x)$.

Algorithm

(Step size μ) Iterate:

Gradient step $\quad z^{k+1} := x^k - \mu \nabla F\left(x^k\right)$

Prox step $\quad x^{k+1} := \mathbf{prox}_{\mu G}\left(z^{k+1}\right)$

Proximal gradient method applied

Proximal gradient for l_1-regularized least-squares:

$$\underset{h}{\text{minimize}} \quad \frac{1}{2}\|y - A(h)\|_2^2 + \lambda\|h\|_1$$

$$F(h) = \frac{1}{2}\|y - A(h)\|_2^2 \quad \Longrightarrow \quad \nabla F(h) = -A^* \left(y - A(h)\right)$$

$$G(h) = \lambda\|h\|_1 \quad \Longrightarrow \quad \mathbf{prox}_{\mu G}(v) = \mathbf{soft}_{\lambda\mu}(v) = \begin{cases} v - \lambda\mu, & v > \lambda\mu \\ 0, & |v| \leq \lambda\mu \\ v + \lambda\mu, & v < -\lambda\mu \end{cases}$$

Iterative Soft Thresholding

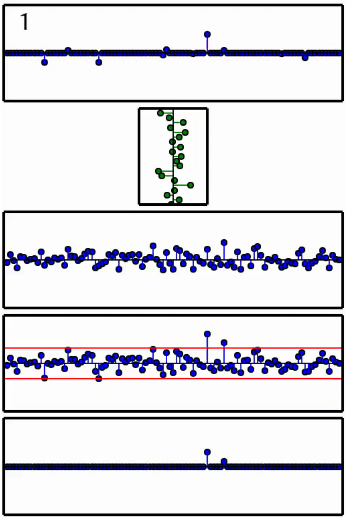
Guess (e.g. reflectivity): h

Calculate error: $z = y - A(h)$

Matched filter the error: $A^*(z)$

Add previous guess: $h + A^*(z)$

Threshold to form new guess: $h = \textbf{soft}(h + A^*(z))$



Interpretation: iterative matched filtering with thresholding!

Outline

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Waveform Inversion

- Implementation
- Experimental Results

Conclusion

Combined algorithmic advances

First order prox algorithms are an active research area, so I tested and combined multiple proposed enhancements:

1. Accelerated proximal gradient method
   - Adds simple "momentum" term
   - Better theoretical convergence
2. Adaptive restart
   - Reset acceleration when it opposes prior step
3. Adaptive step size
   - Increase step size every iteration
   - Decrease as necessary to ensure convergence

Convergence comparison

Method	Iterations
Prox gradient	4463
+ Acceleration	2911
+ Adaptive restart	397
+ Adaptive step	105

Testing similar algorithms

Other prox-based algorithms are worthy of consideration:

- Linearized alternating direction method of multipliers (ADMM)
- Primal-dual hybrid gradient (PDHG)

Closely related, these both solve

$$\underset{x}{\text{minimize}} \quad F(x) + G(x)$$

when both $F(x)$ and $G(x)$ can be non-smooth.

Convergence comparison

Method	Iterations
Accelerated proximal gradient	105
Linearized ADMM (novel adaptive step)	154
PDHG (standard fixed step)	4464

Code

Principles

1. Easy to develop
2. Fast when necessary
3. Freely available for collaboration and community use

Answers

1. Python with NumPy
2. Radar model is bottleneck: implement with Cython or Numba
3. Available on github: github.com/ryanvolz

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The Jicamarca incoherent scatter radar

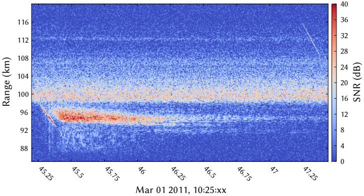
Specifications:

- Located outside Lima, Peru
- VHF (50 MHz)
- Phased array of 96×96 crossed half-wave dipoles

- 1 MHz bandwidth (TX/RX)

Reasons:

- Equatorial ionosphere is challenging

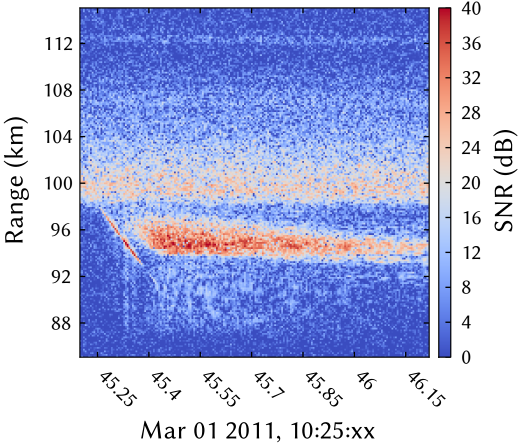


- Detection rate of meteors is better at lower frequencies
- Interferometry and/or dual polarization receive

Jicamarca experiment goals

Goals

- Test sparsity-based waveform inversion in crowded environment
- Directly compare effectiveness of different waveforms



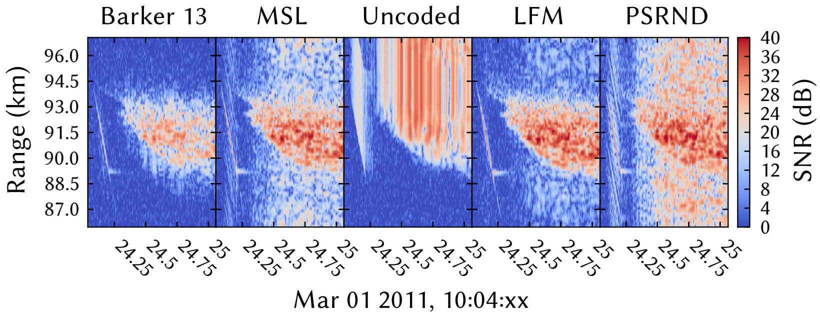
Jicamarca meteor experiment

Description

Alternating sequence of 5 common waveforms for observing meteor region (80-140 km altitude).

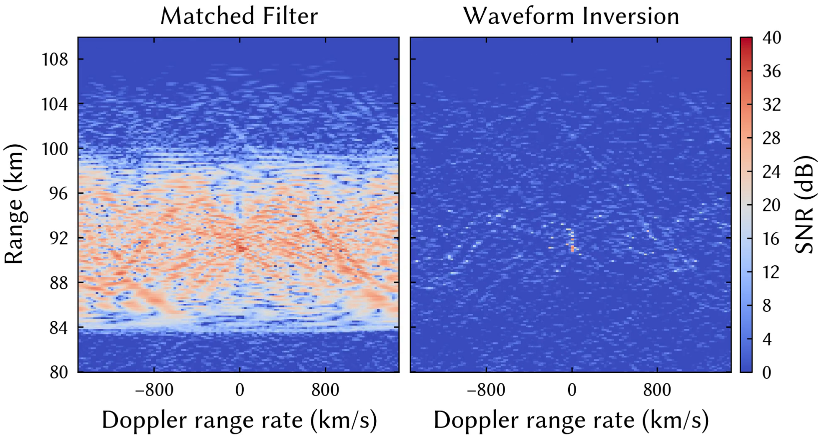
Parameters

- Pulse interval of 1 ms
- Sample time of 1 μs (150 m range resolution)



Movie of meteor sidelobe removal

Decoding comparison for minimum sidelobe code:

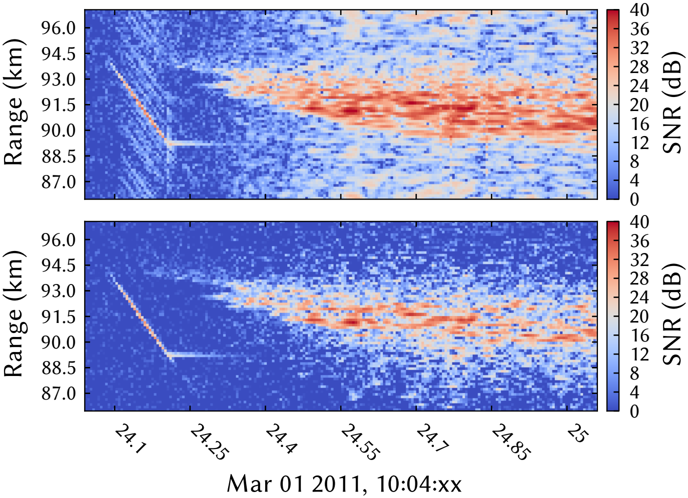


(Total elapsed time of 1 second)

Example: Minimum sidelobe code

Matched Filter

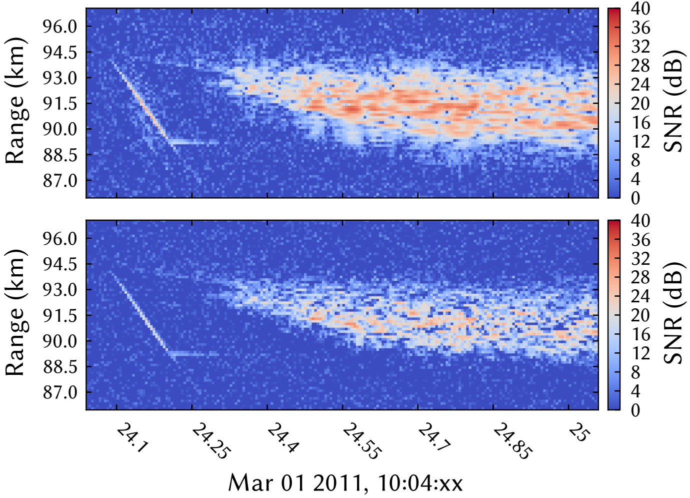
Waveform Inversion



Example: Barker-13 code

Matched Filter

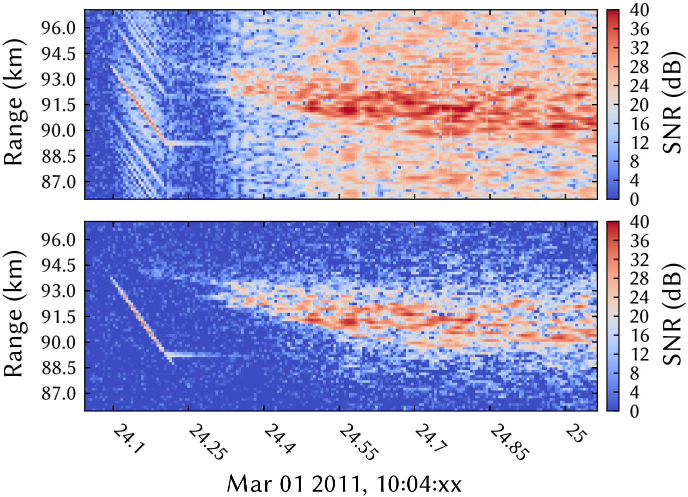
Waveform Inversion



Example: Pseudorandom code

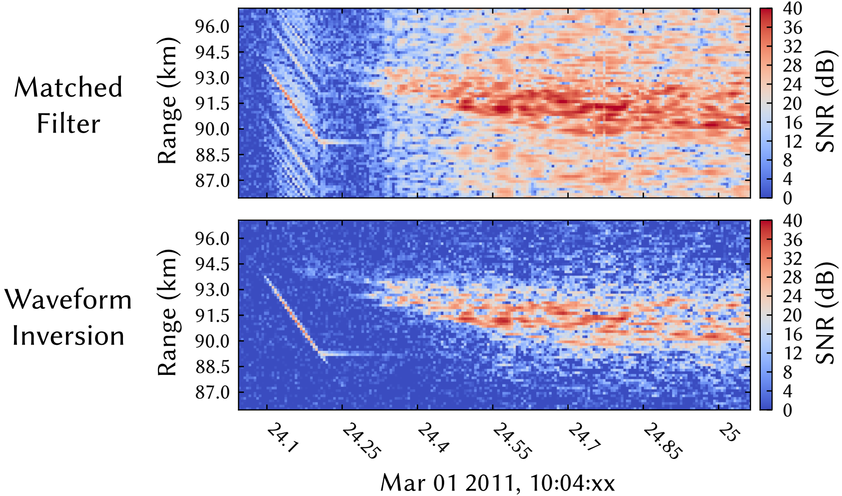
Matched Filter

Waveform Inversion



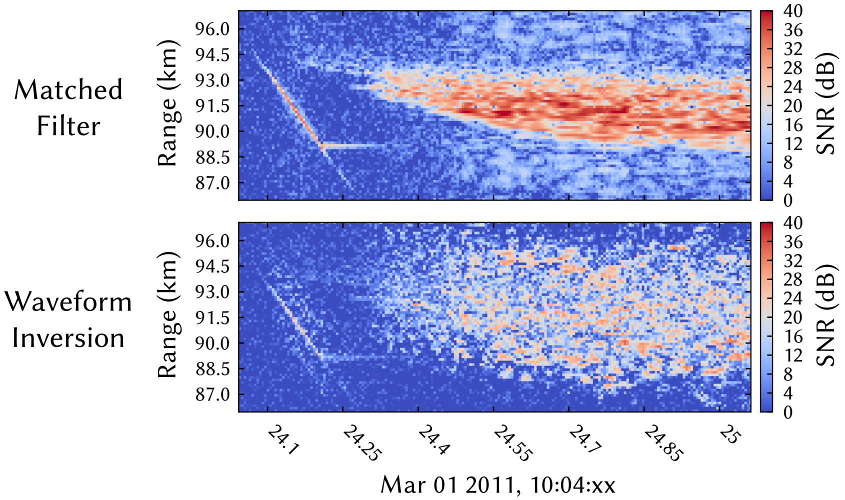
Works with variety of codes

Example: Pseudorandom code



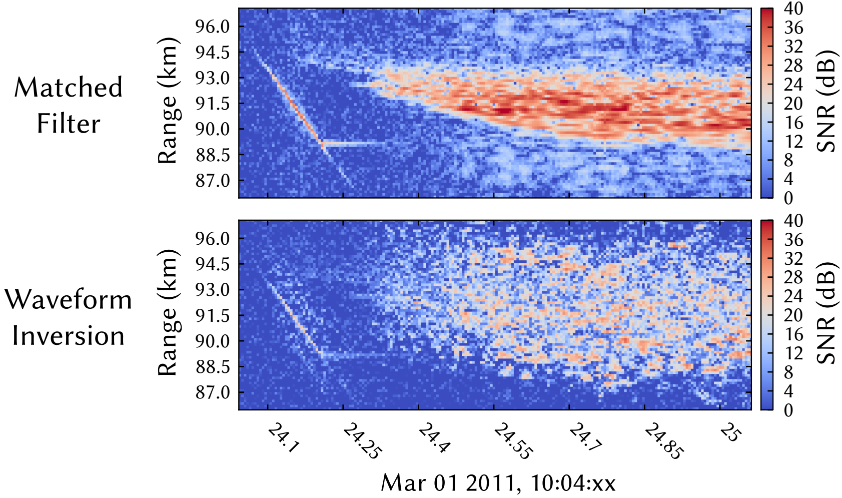
Works with variety of codes

Example: LFM chirp



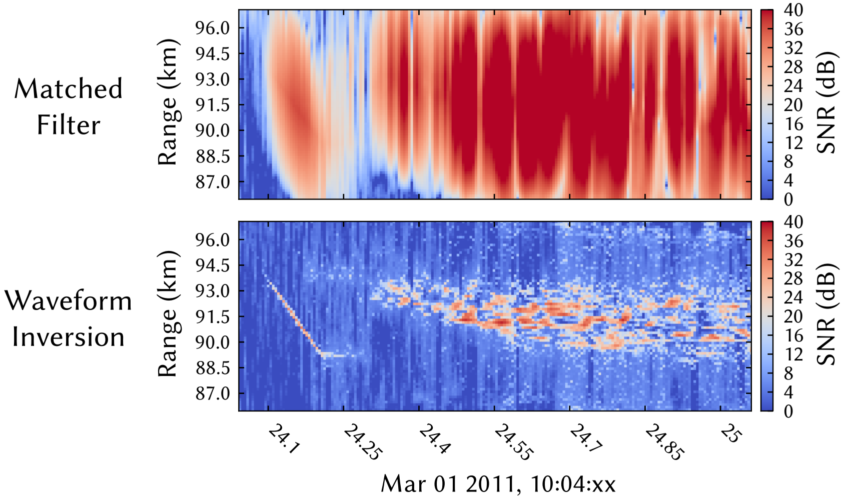
Some codes are troublesome

Example: LFM chirp



Some codes are troublesome

Example: Uncoded



Works even with uncoded pulses!

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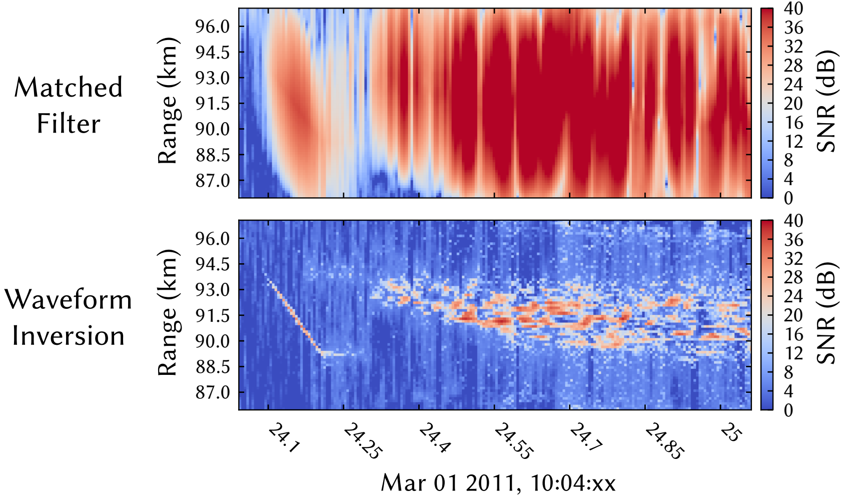

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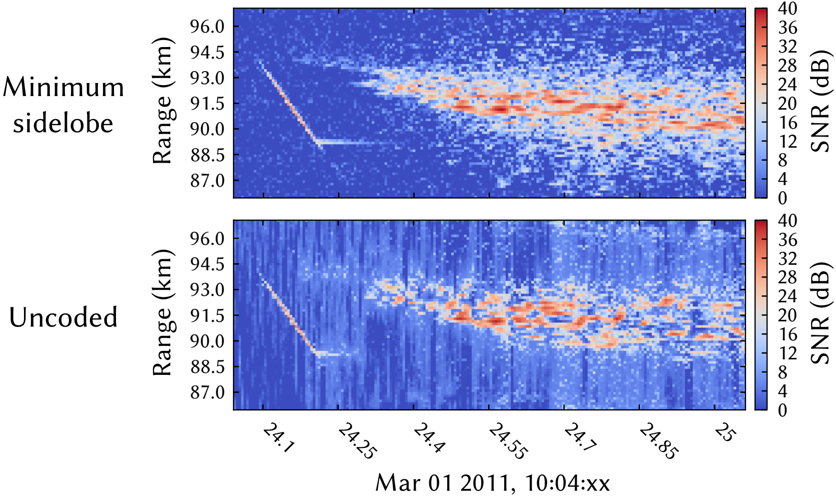
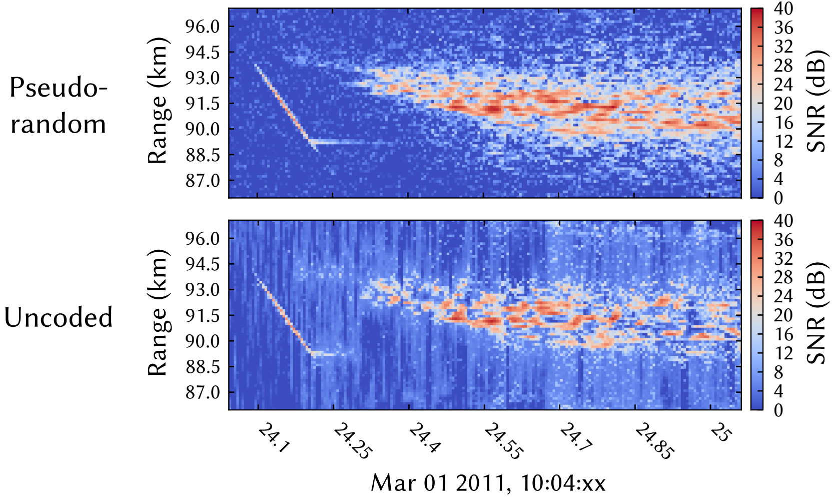
Waveform inversion code comparison



Quality of solution depends on fidelity of waveform

Waveform inversion code comparison



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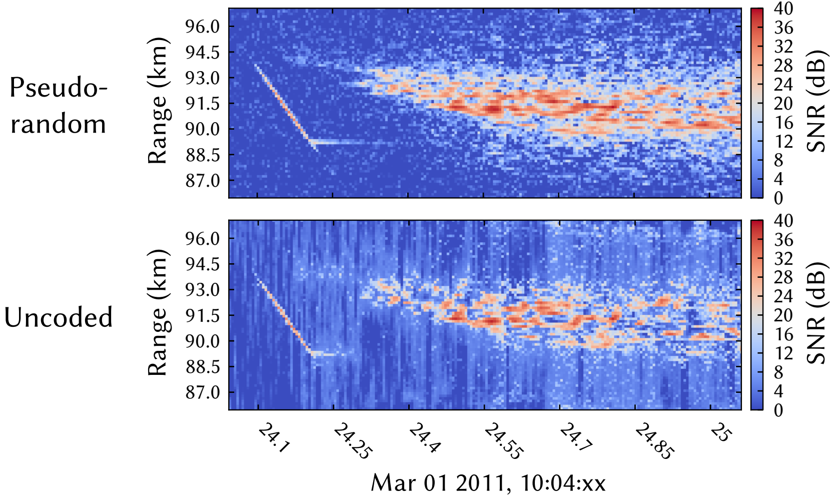
Waveform inversion code comparison



Quality of solution depends on fidelity of waveform

Outline

Radar Background

My Radar Model

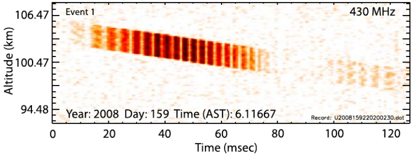
Sparsity Background

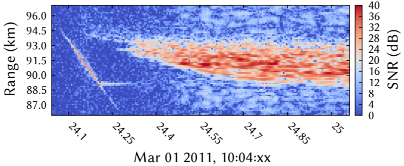
Waveform Inversion

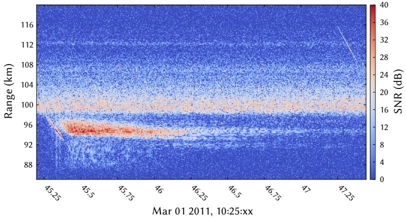
Conclusion

Benefits of waveform inversion

1. Sidelobe removal -
   Removes ambiguity for observation of multiple or range-spread targets (e.g. fragmentation and non-specular trails)
2. Full frequency decoding -
   Decodes over full frequency spectrum simultaneously, for target differentiation in crowded environments (e.g. equatorial, flares)
3. Flexibility -
   Enables use of many different waveforms







Contributions

In order to enable flexible high-resolution measurements of ionospheric plasma phenomena, including interfering and range-spread targets, I made the following contributions:

1. Derived a discrete radar model that captures signal sparsity in delay-frequency space.
2. Uncovered the adjoint relationship between the model and existing matched filter techniques, giving an intuitive interpretation of waveform inversion as iterative filtering.
3. Analyzed the model to show how it exactly represents distributed scatterers and preserves their sparsity.
4. Created and tuned an efficient implementation of waveform inversion using modern convex optimization techniques.
5. Demonstrated the flexibility and effectiveness of the inversion technique by removing filtering artifacts from meteor observations made with a variety of radar waveforms.

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Further research

Near-universal radar measurements

Submitted NSF postdoc proposal to work at MIT Haystack (Millstone Hill) and create near-universal measurement mode across radar chain using uniquely-tuned waveforms.

Self-calibration

Iteratively solve for reflectivity *and* transmitted waveform to better characterized the radar system.

Passive radar

Waveform flexibility invites use for parasitic radio science (using FM radio, digital TV signals) where the transmitted waveform cannot be controlled.

Acknowledgements

Thank you for coming!

Questions

Radar Background
- Pulse Encoding/Decoding
- Measurement Ambiguity
- Current Techniques

My Radar Model
- Defining the Model
- Representation of Targets

Sparsity Background
- Compressed Sensing
- Convex Optimization

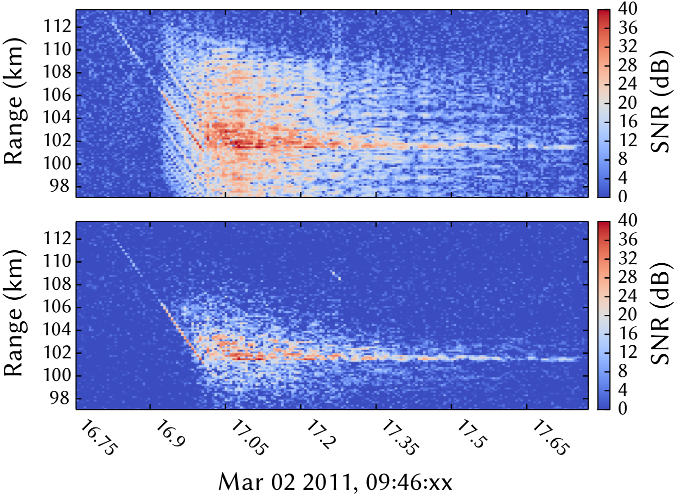
Waveform Inversion
- Implementation
- Experimental Results

Conclusion

Another example: Pseudorandom code

Matched Filter

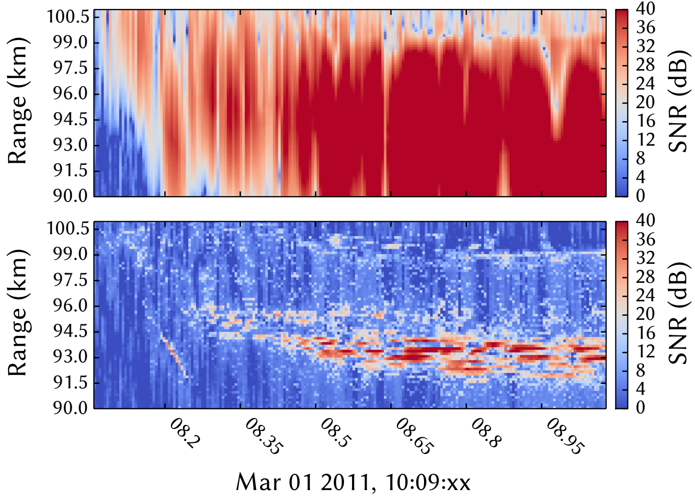
Waveform Inversion



Different example: Uncoded

Matched Filter

Waveform Inversion



Radar model derivation assumptions

- Continuous model holds
- No signal is received from outside the sampled delay window
- Transmitted waveform is piecewise-constant in equal length bauds

The prox operator

The proximal operator, or prox operator, is defined for non-smooth $G(x)$ as

$$\mathbf{prox}_{\mu G}(v) = \arg\min_x \left(G(x) + \frac{1}{2\mu} \|x - v\|_2^2 \right).$$

Projection onto a set

If $G(x)$ is the indicator function for a closed convex set $\mathcal{C}$,

$$G(x) = \begin{cases} 0 & x \in \mathcal{C} \\ \infty & x \notin \mathcal{C} \end{cases}$$

$\mathbf{prox}_G(v)$ is projection of v onto $\mathcal{C}$.

Accelerated proximal gradient method

$$
\begin{aligned}
\text{Set step size} \quad & \mu^{k+1} \\
& \gamma := \mu^{k} / \mu^{k+1} \\
\text{Acceleration parameter} \quad & t^{k+1} := \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\gamma \left(t^{k}\right)^{2}} \\
& \theta := \left(t^{k} - 1\right) / t^{k+1} \\
\text{Acceleration step} \quad & w^{k+1} := x^{k} + \theta \left(x^{k} - x^{k-1}\right) \\
\text{Gradient step} \quad & z^{k+1} := w^{k+1} - \mu^{k+1} \nabla F \left(w^{k}\right) \\
\text{Prox step} \quad & x^{k+1} := \mathbf{prox}_{\mu G} \left(z^{k+1}\right)
\end{aligned}
$$

Waveform inversion equation

Rewrite matched filter using the model in terms of solution h:

$$\begin{aligned} A^*(y) &= A^*\left(A(h) + y - A(h)\right) \\ &= A^*A(h) + A^*\left(y - A(h)\right) \\ &= \left(\frac{1}{N}I + A^*A - \frac{1}{N}I\right)(h) + A^*\left(y - A(h)\right) \\ &= \frac{1}{N}h + \left(A^*A - \frac{1}{N}I\right)(h) + A^*\left(y - A(h)\right) \end{aligned}$$

Ambiguity function is A^*A, and sidelobes are $A^*A - \frac{1}{N}I$. The matched filter is usually scaled by a factor of $\sqrt{N}$, so the waveform inversion result with sidelobes removed is given by:

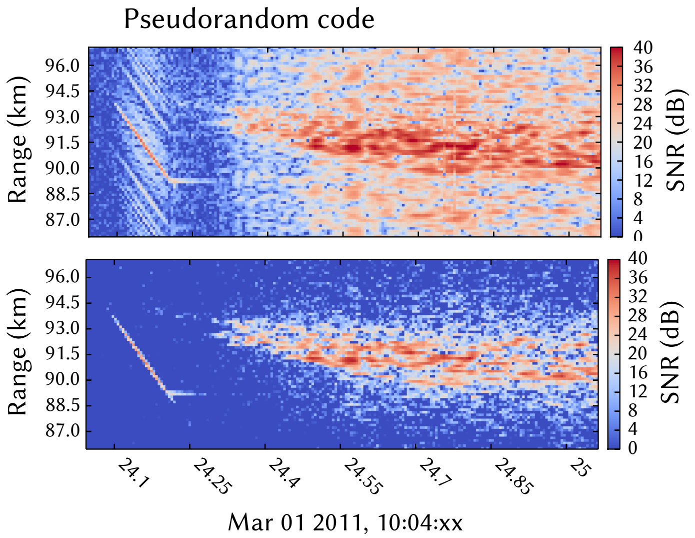
$$\frac{1}{\sqrt{N}}h + \sqrt{N}A^*\left(y - A(h)\right)$$

(sparse result plus matched filtered noise)

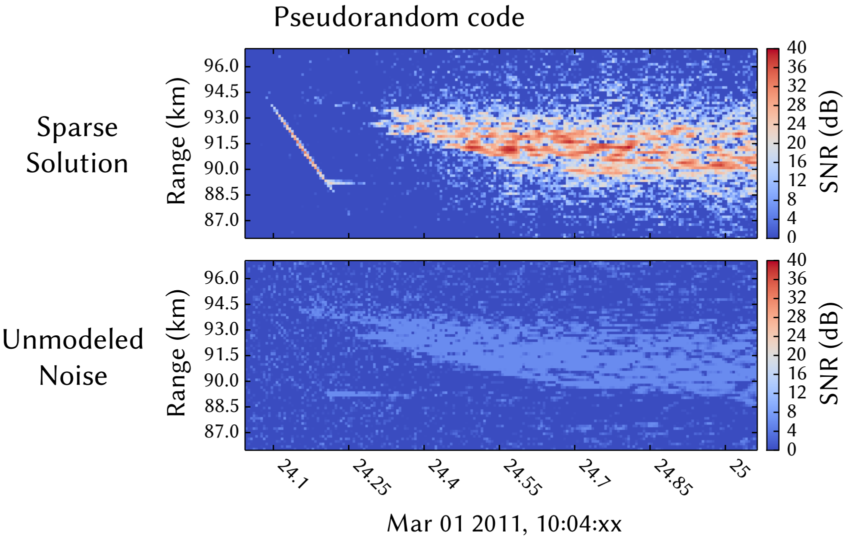
Sparse solution

Matched Filter

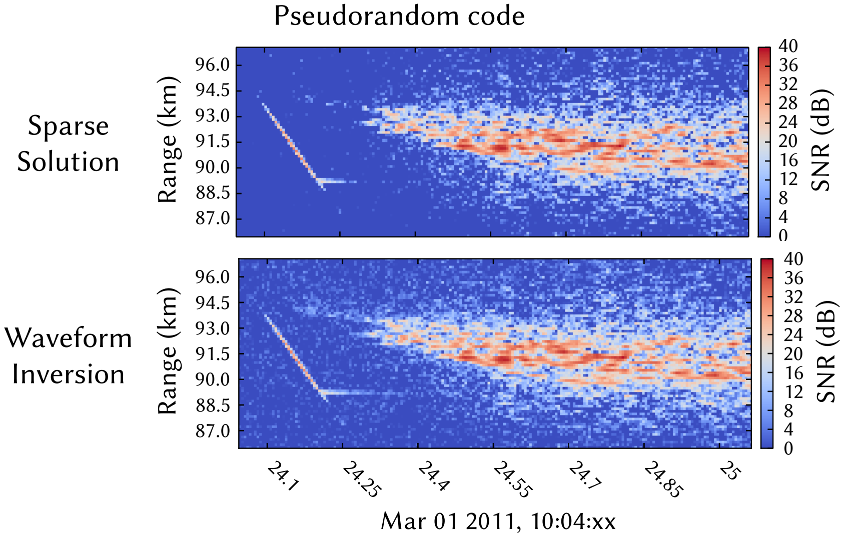
Sparse Solution



Unmodeled noise



Sidelobe removal



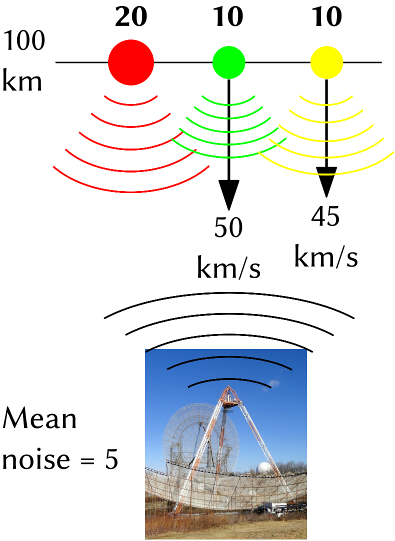
Simulation parameters

Point target simulation

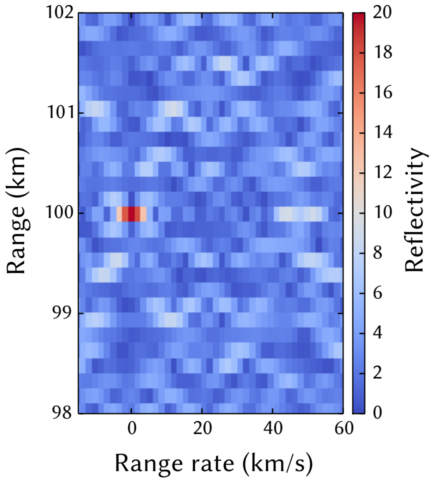
- 3 targets at 100 km range:

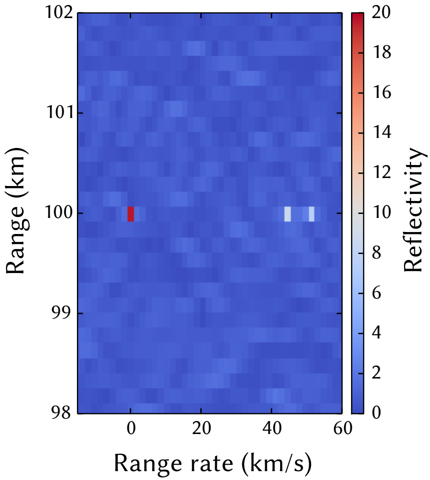
Reflectivity	Range rate
20	0 km/s
10	45 km/s
10	50 km/s

- Mean noise power of 5
- 51-baud minimum sidelobe code



Simulation results





Jicamarca flare

- Waveform inversion of a meteor head echo and flare
- Minimum sidelobe code data from Jicamarca experiment
- 5 pulses around the event