



# Theory and Applications of Sparsity for Radar Sensing of Ionospheric Plasma

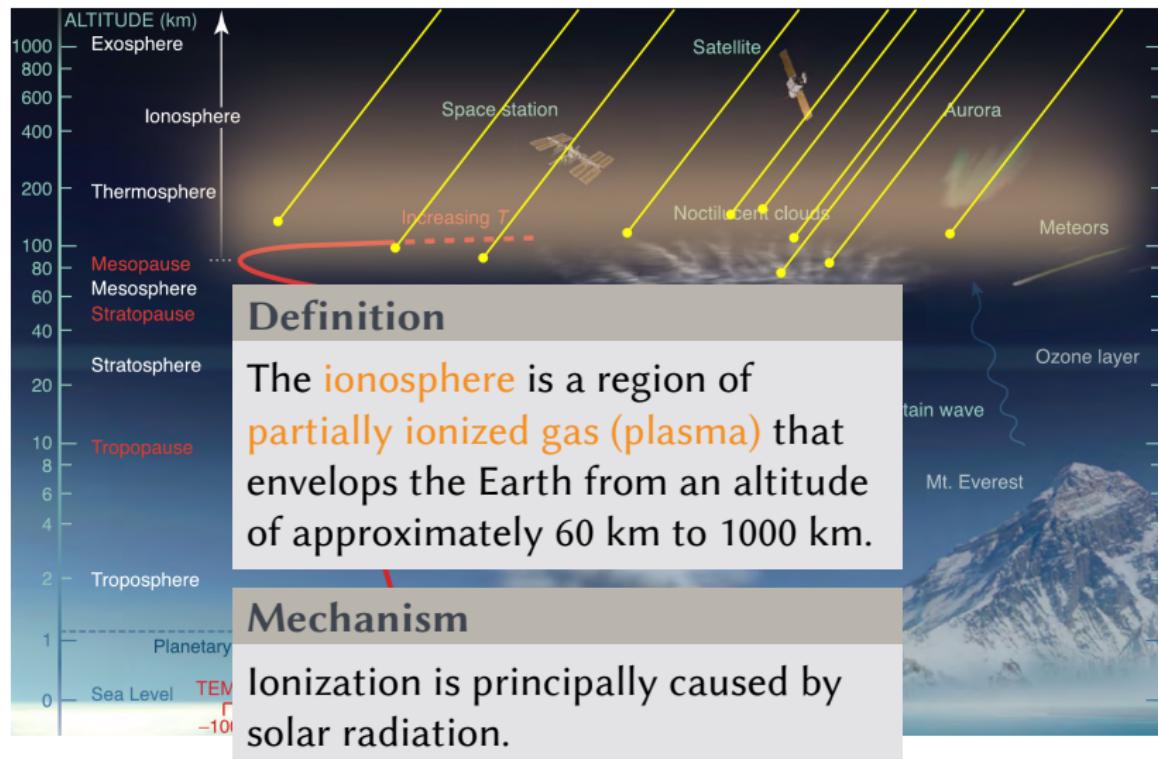
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Stanford University

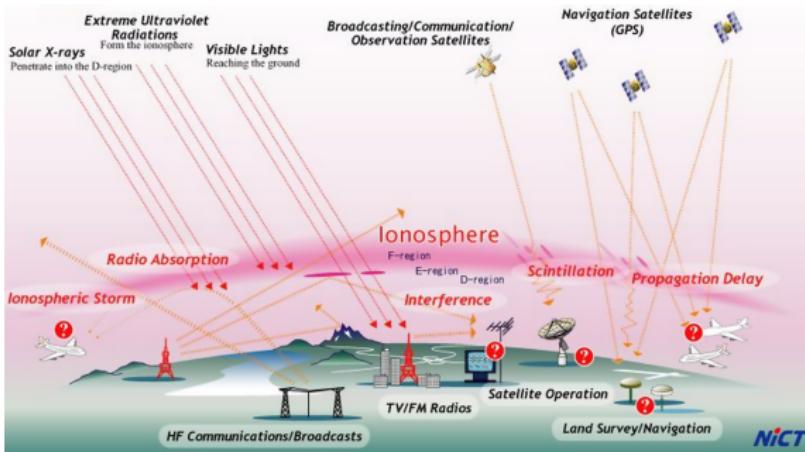
March 19, 2014

Advisor: Sigrid Close

# The ionosphere



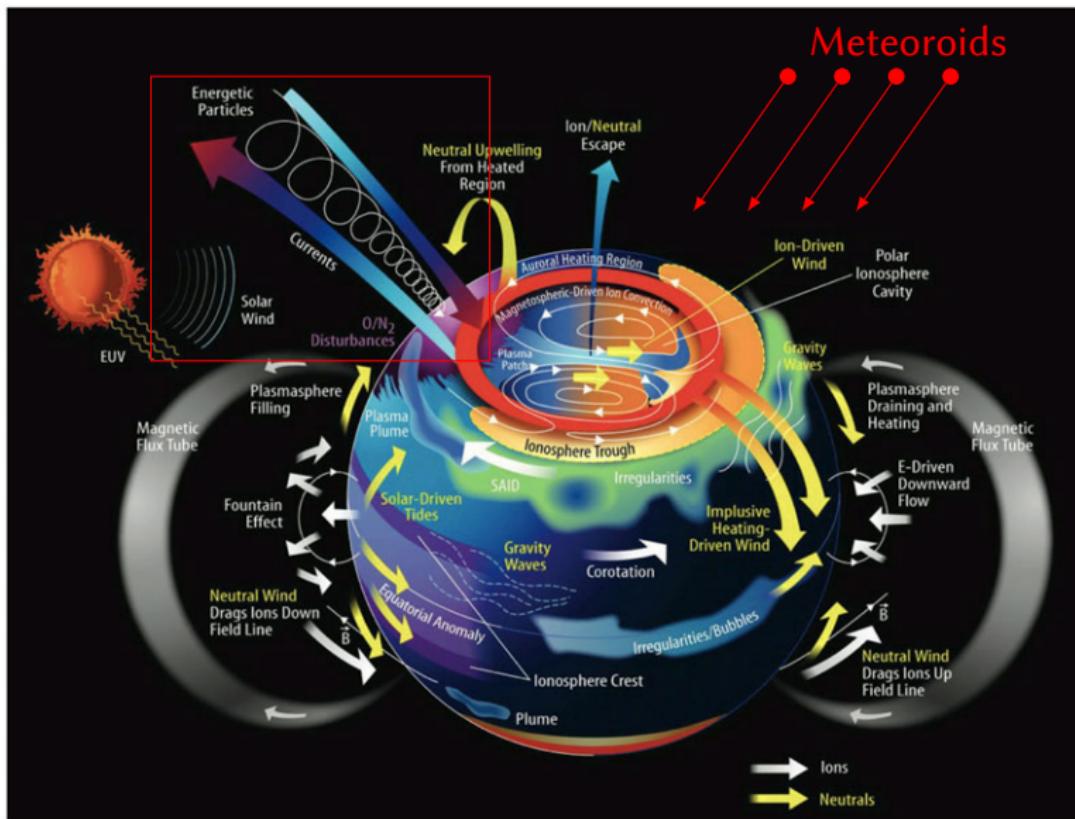
# Importance of the ionosphere Communications



- ▶ Radio waves of lower frequency (less than  $\approx 10$  MHz) reflect off ionosphere.
- ▶ Microwave frequencies (e.g. GPS) can pass through, but experience frequency-dependent refraction and delay.
- ▶ Plasma irregularities additionally perturb signals.

# Importance of the ionosphere

## Science of the geospace system



# Space weather: meteoroids and meteors

## Definition (Meteoroid)

- ▶ Solid body in space
- ▶ Small (< 1 m in diameter)
- ▶ Fast (from 11 to 72 km/s)
- ▶ Exponentially more common as size decreases

Micro-meteorites (landed on Earth)



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## Definition (Meteor)

- ▶ Plasma created by meteoroid entering the atmosphere
- ▶ Typically form at altitudes between 80 km and 120 km



2011/10/01 00:37:01.079 (LT) 0157 00076 V00395+216 UF0CapY2 MSFC ALaM0

NASA (2011)

# Importance of meteoroids

Impact  
damage to  
Space Shuttle  
Atlantis



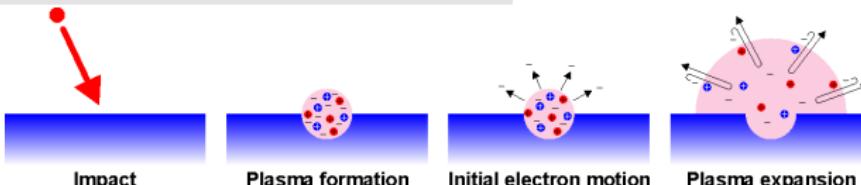
## High speed impacts

Threat to satellites and spacecraft from

- ▶ mechanical damage
- ▶ **electrical** damage

## Required observations

- ▶ Rate/count
- ▶ Size
- ▶ Speed
- ▶ Composition



# Measuring the ionosphere

## Definition

Incoherent Scatter Radars (ISRs, locations shown below) are radars sensitive enough to measure scattering from electrons in the background ionosphere.

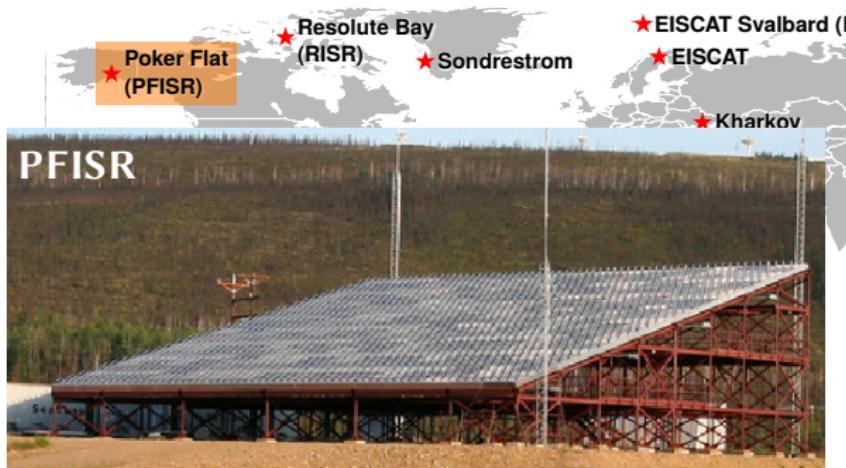


Range of ionospheric regions: polar, mid-latitude, equatorial.

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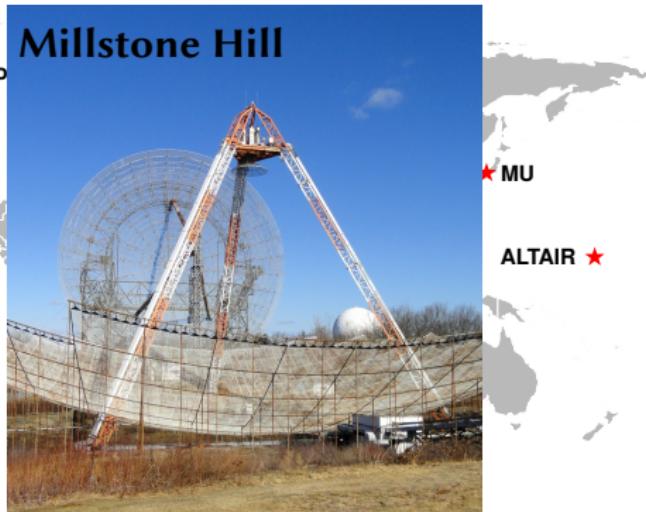
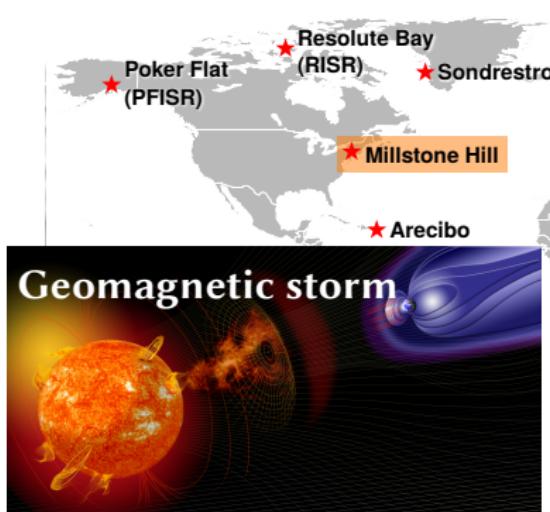


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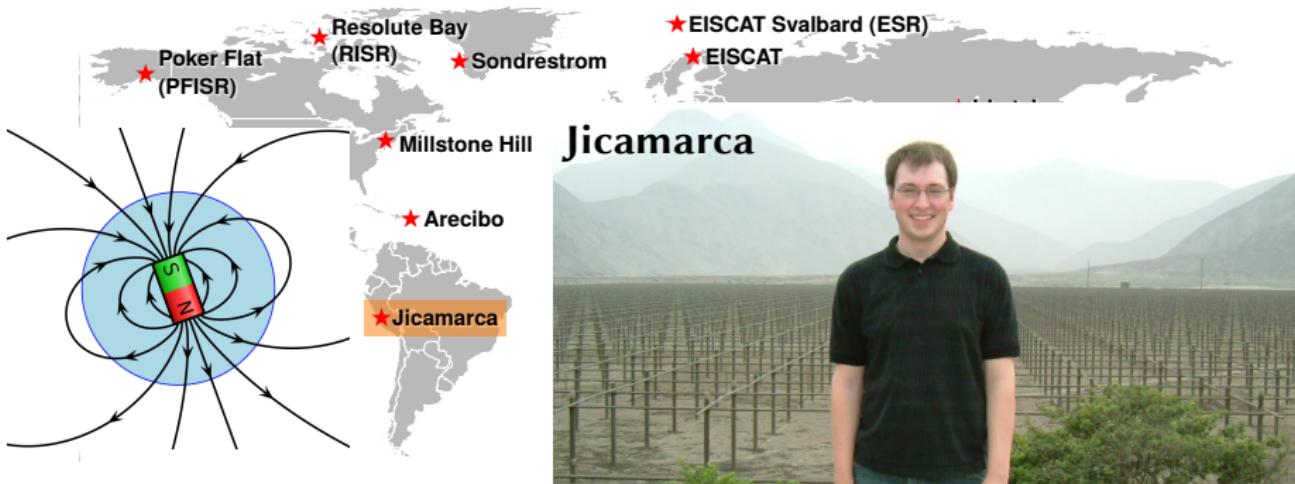


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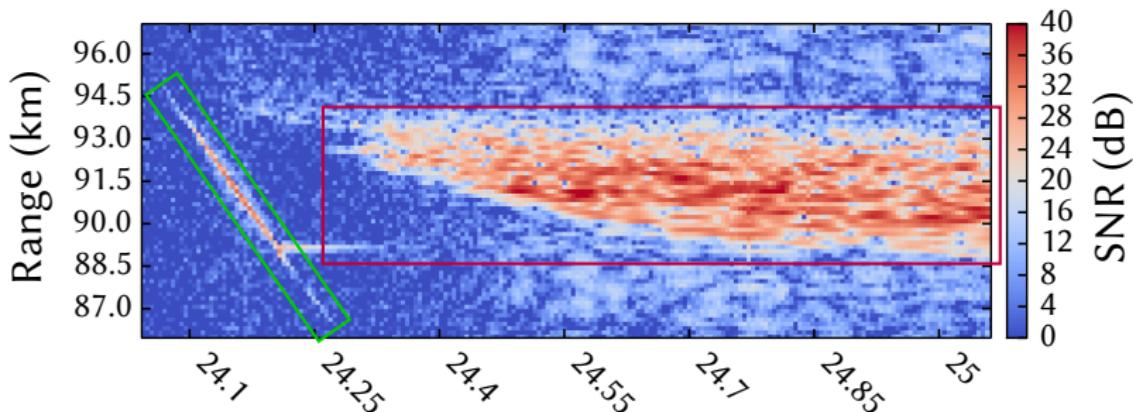


Range of ionospheric regions: polar, mid-latitude, equatorial.

# Radar meteors

## Types of scattering

- ▶ **Head:** ball of plasma traveling with the meteoroid
- ▶ **Trail:** plasma left in wake of meteoroid

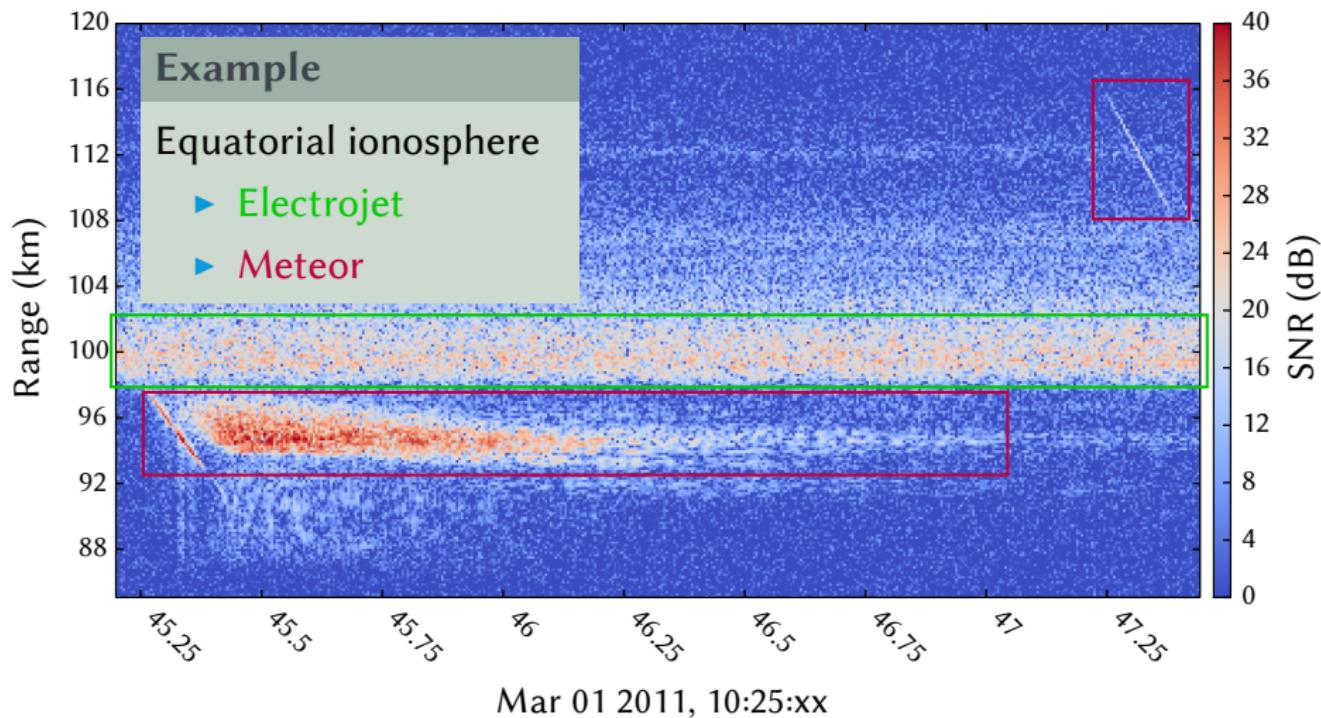


Mar 01 2011, 10:04:xx

Time of Radar Pulse

# Variety of ionospheric plasma

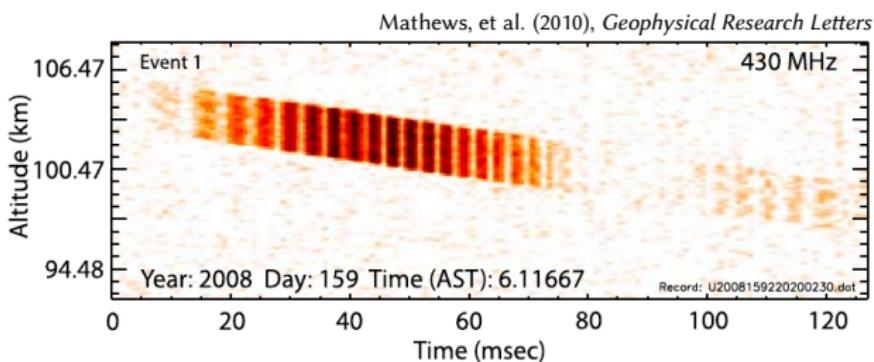
There are many different plasma phenomena, each presenting its own challenges for observation.



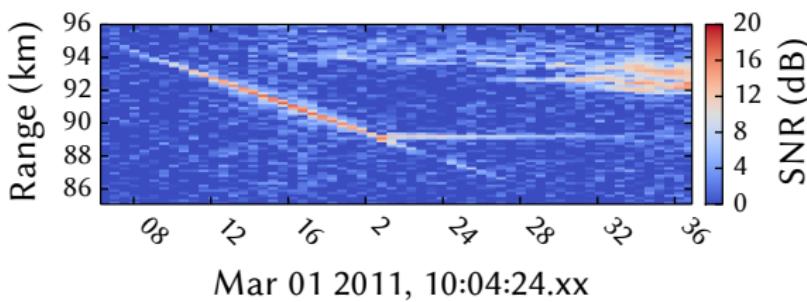
# At the limit: meteors

Scientific progress is limited by current signal processing techniques!

Fragmentation

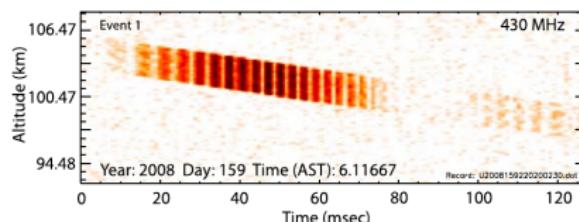
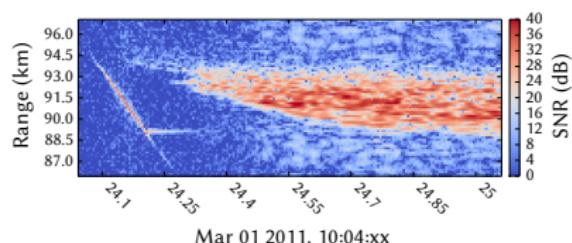
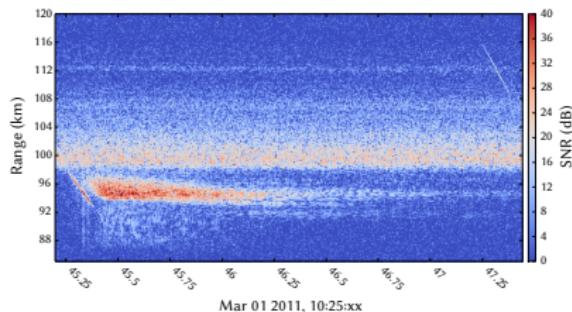


Flares and terminal events



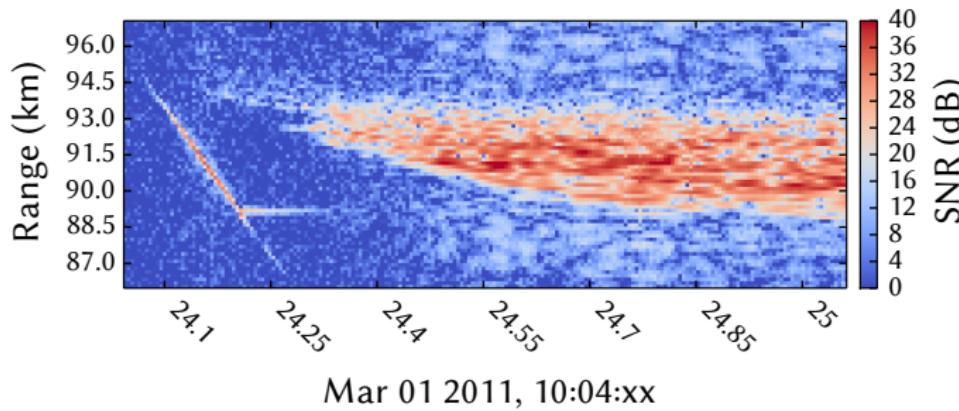
# Measurement challenges

1. **Differentiation** in a crowded and variable environment  
(e.g. meteors and electrojet)
2. **Self-interference** of range-spread targets  
(e.g. meteor trails)
3. **High resolution** to observe small-scale processes  
(e.g. meteoroid fragmentation and flares)
4. **Flexibility** to achieve all of the above at once



# A way forward: sparsity

Radar targets are typically sparse in range and frequency!



Can use sparsity to systematically improve the quality of radar measurements.

# The world is sparse

## Definition

- ▶ A vector is **sparse** if only a few of its elements are nonzero.
- ▶ Additionally, the term is often applied loosely to when this is approximately true (i.e. most of the elements close to zero).

Many natural phenomena have sparse representations:

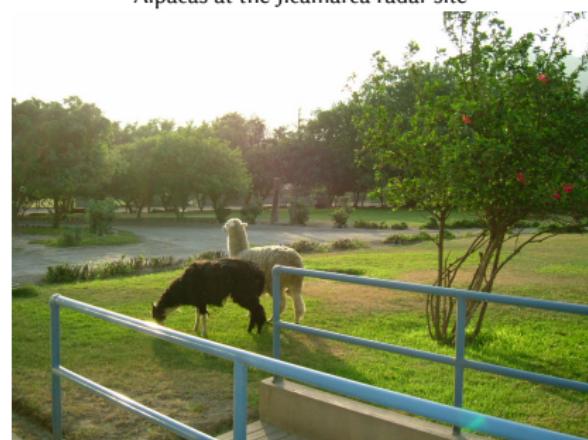
- ▶ Music and speech (mp3)
- ▶ Images (jpeg)

## Alpacas (right): 8.5x savings

Uncompressed: 3600 KB

Compressed: 420 KB

Alpacas at the Jicamarca radar site



# Sparsity revolution

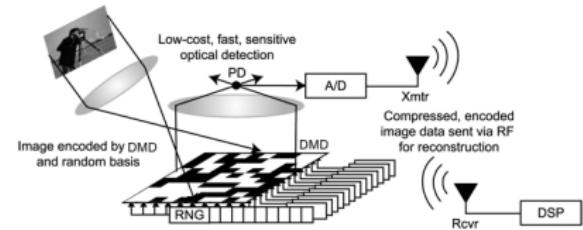
Exploitation of inherent signal sparsity through compressed sensing is leading to advancements in many subjects.

## Single pixel camera (1)

Smaller, cheaper, more flexible

[dsp.rice.edu/cscamera](http://dsp.rice.edu/cscamera)

(1)

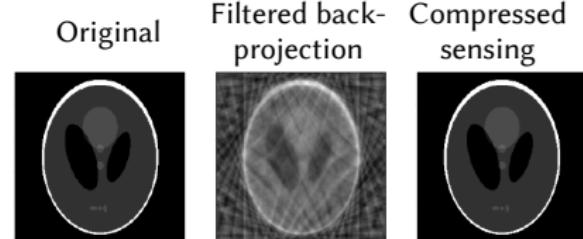


## MRI (2)

Faster scans, higher resolution

[eecs.berkeley.edu/mlustig/CS.html](http://eecs.berkeley.edu/mlustig/CS.html)

(2)



## Genomics

High-throughput screening

[erlichlab.wi.mit.edu](http://erlichlab.wi.mit.edu)

# Contributions

In order to enable **flexible high-resolution measurements** of ionospheric plasma phenomena, including interfering and range-spread targets, I made the following contributions:

1. Derived a discrete **radar model** that captures signal sparsity in delay-frequency space.
2. Uncovered the adjoint relationship between the model and existing matched filter techniques, giving an **intuitive interpretation** of waveform inversion as iterative filtering.
3. **Analyzed the model** to show how it exactly represents distributed scatterers and preserves their sparsity.
4. Created and tuned an **efficient implementation** of waveform inversion using modern convex optimization techniques.
5. **Demonstrated the flexibility and effectiveness** of the inversion technique by removing filtering artifacts from meteor observations made with a variety of radar waveforms.

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# Additional contributions

In addition, I also made the following contributions that will not be covered in this talk:

1. Created a range-time-frequency clustering algorithm for detection and classification of ionospheric radar signals.
2. Simulated the statistical accuracy of interpolated matched filter estimation for range and range-rate super-resolution of meteor head echoes.

# Outline

Introduction and Motivation

Radar Background

My Radar Model

Sparsity Background

Waveform Inversion

Conclusion

# Outline

## Introduction and Motivation

## Radar Background

Pulse Encoding/Decoding

Measurement Ambiguity

Current Techniques

## My Radar Model

## Sparsity Background

## Waveform Inversion

## Conclusion

# Radar pulse length

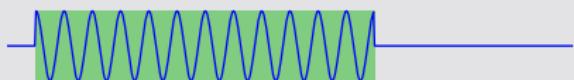
Some applications favor a shorter radar pulse, while others favor a longer radar pulse.

## Short pulse



- ▶ Low range ambiguity (interference region for multiple scatterers)
- ▶ Simple to interpret

## Long pulse

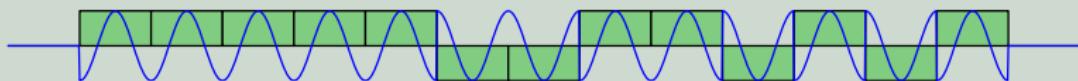


- ▶ Fine frequency resolution (observe target as it evolves in time)
- ▶ High total power

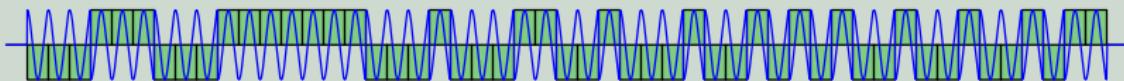
# Coded pulses

- ▶ Encoding allows long pulses to have the low range ambiguity of short pulses.
- ▶ One method: divide pulse into **bauds** of constant phase.

## Barker-13 code



## Minimum peak sidelobe code



## Linear frequency modulation (LFM chirp)



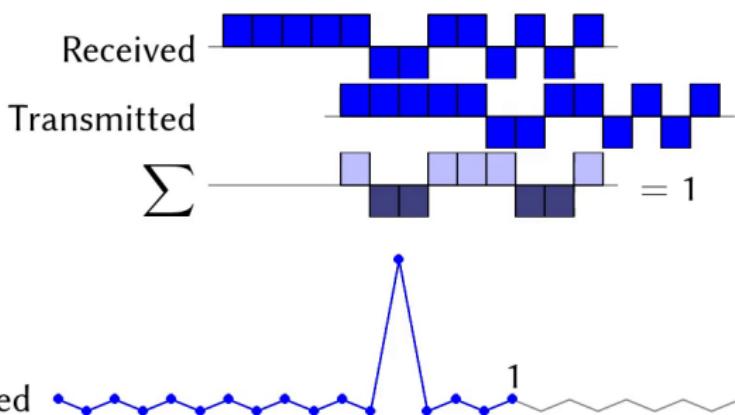
# The matched filter

## Definition

The **matched filter** correlates every segment of the received signal with the transmitted pulse.

It produces a peak at the target's range (delay), achieving a rough form of decoding with ambiguous **sidelobes**.

## Example (Barker-13 code, Minimum sidelobe code)



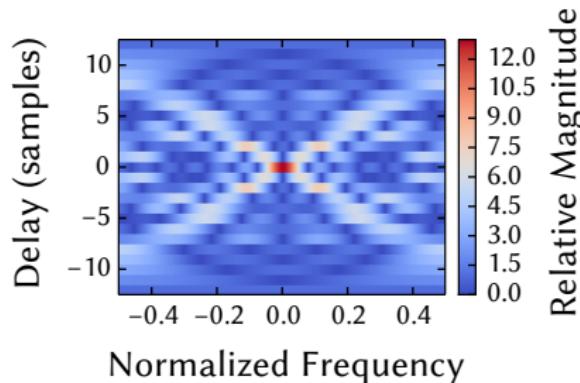
# Frequency filter banks

## Doppler frequency shift

If the target is moving **toward** or **away from** the radar, the reflected signal is frequency shifted **up** or **down**.

- ▶ Matched filter needs to be similarly frequency shifted
- ▶ True frequency shift is not known ahead of time
- ▶ Process signal with a bank of differently-shifted filters

Result of filter bank for centered “point” target, using Barker-13 code



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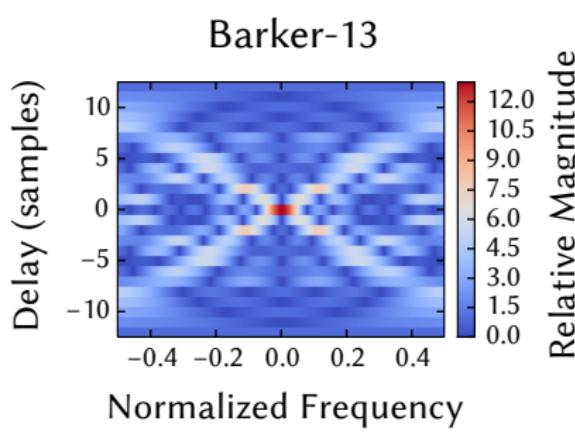
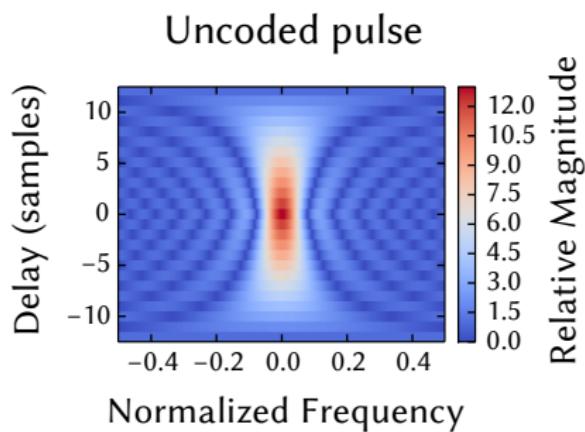
## Waveform Inversion

## Conclusion

# Ambiguity functions

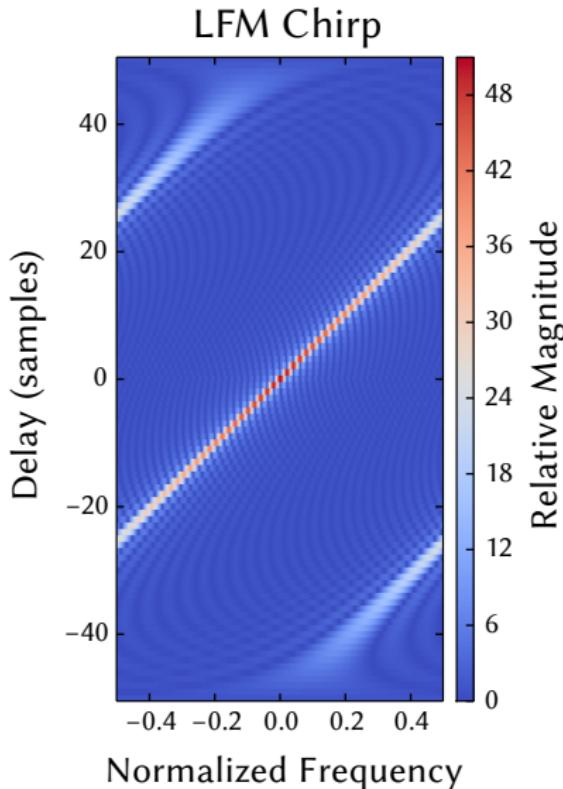
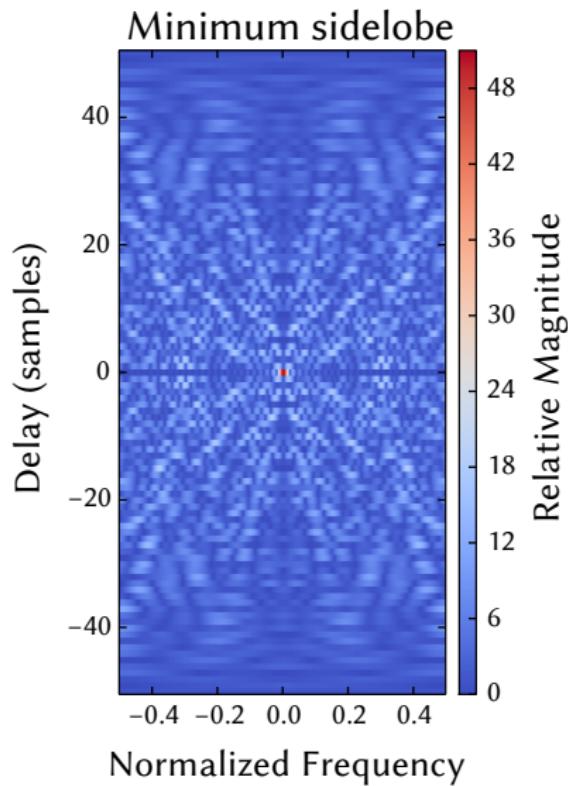
## Definition

A delay-frequency **ambiguity function** is produced when a signal is matched against itself using a filter bank.



The ambiguity function describes the delay-frequency sidelobes of a code.

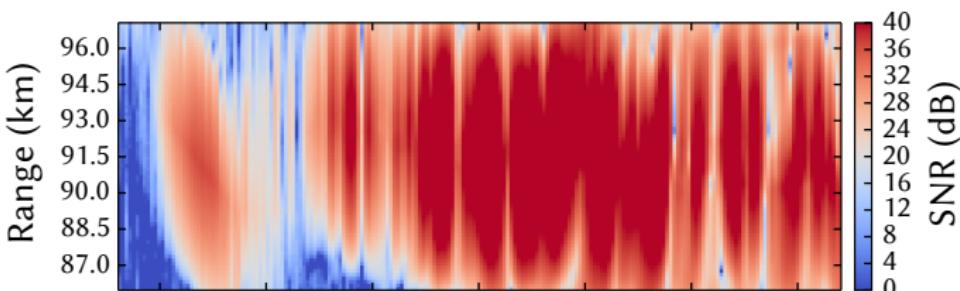
# More example ambiguity functions



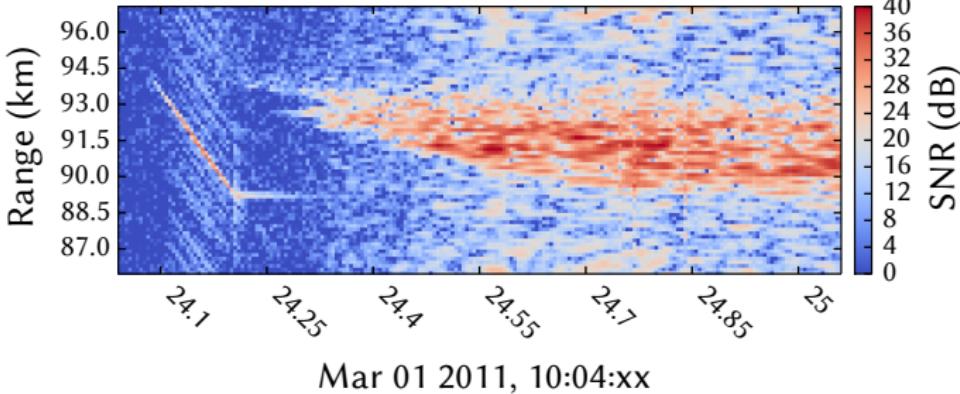
# Range sidelobes in action

Range sidelobes dominate the output of uncoded measurements, while coded pulses are better but still noticeably corrupted.

Uncoded



Minimum  
sidelobe  
code

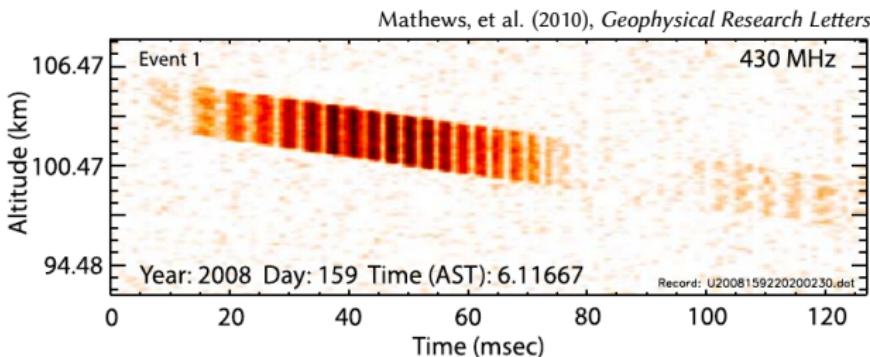


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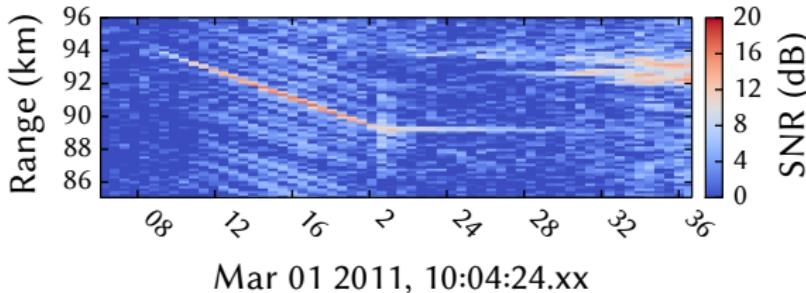
# Effect of sidelobes

Sidelobes (delay-frequency ambiguity) are the primary reason that outstanding ionospheric science questions cannot be satisfactorily answered.

Fragmentation

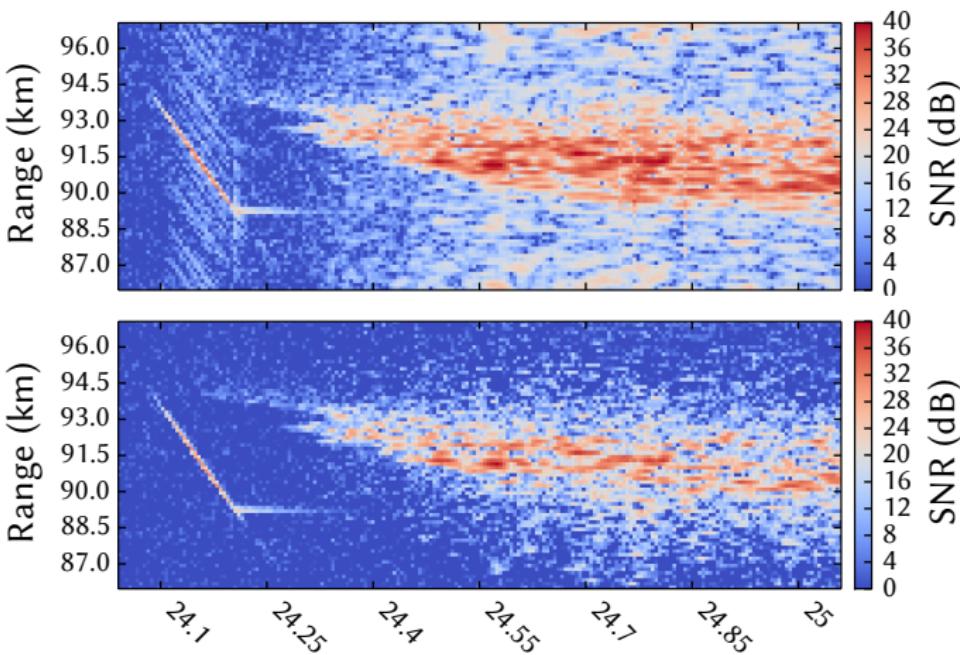


Flares and terminal events



# Goal: sidelobe removal

Minimum  
sidelobe  
code



Mar 01 2011, 10:04:xx

Infinite number of ways to decode and get a target scene that reproduces the measurements!

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# Delay-frequency sidelobe mitigation in use

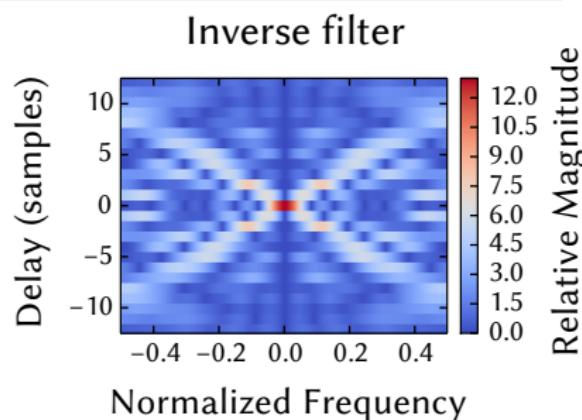
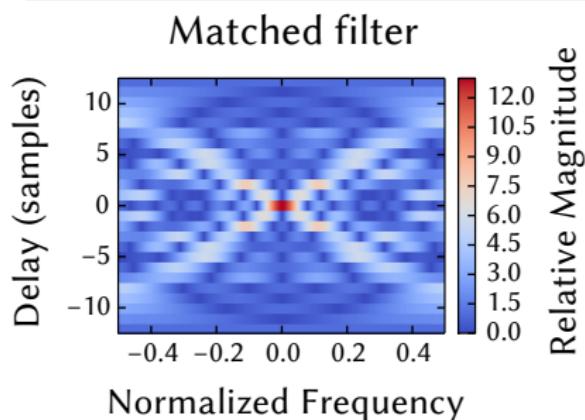
1. Codes with good sidelobe properties
  - ▶ e.g. Barker-13, minimum peak sidelobe, LFM chirp
  - ▶ Most prevalent strategy
  - ▶ Only effective when it is simple to ignore remaining sidelobes
2. Alternating codes
  - ▶ Set of codes over multiple pulses
  - ▶ When pulses summed together, sidelobes cancel
  - ▶ Target properties must remain stationary
3. Inverse filters
  - ▶ Alternative to matched filter
  - ▶ Loss of SNR depending on code
  - ▶ Produces no range sidelobes at target's frequency shift

These strategies severely constrain the flexibility and accuracy of ionospheric radar measurements!

# Inverse filter ambiguity function

## Example

Ambiguity function comparison for Barker-13 code.



No advantage if Doppler shift is unknown or multiple frequencies must be decoded!

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Defining the Model

Representation of Targets

Sparsity Background

Waveform Inversion

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# Matched filtering as imaging

## Imaging analogy

Matched filter “image” is blurred by the ambiguity function:

$$h[n, p] * \chi[n, p] = x[n, p]$$

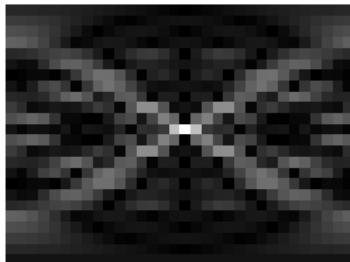
Target reflectivity  
(source image)

Ambiguity function  
(point-spread func.)

Matched filter result  
(measured image)



\*



=



# Radar model from matched filter

## Ambiguity equation

$$x[n, p] = \chi[n, p] * h[n, p]$$

$$A^*(y[m]) = A^*(A(h[n, p]))$$

- ▶ Matched filtering is  $A^*$
- ▶ Ambiguity function is  $A^*A$ 
  - $m$  sample
- ▶ Indices:
  - $n$  frequency
  - $p$  delay

## Radar model

Simplify by removing excess matched filtering operation:

$$y[m] = A(h[n, p])$$

Measured signal      Radar model      Target reflectivity

- ▶ Radar model is **adjoint** (conjugate transpose) of matched filter
- ▶ Non-invertible because under-determined
- ▶ Sparsity of target reflectivity is key to solution

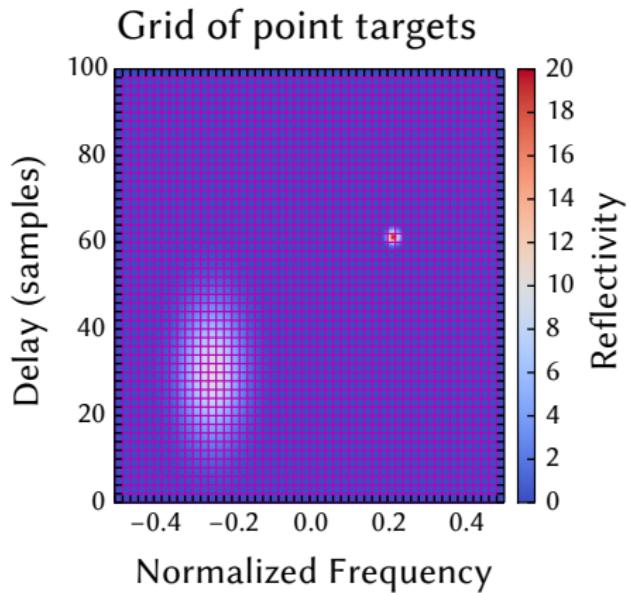
# Alternative interpretation

## Matched filter ( $A^*$ )

Correlates received signal with expected return from point targets with different delays and frequency shifts.

## Radar model ( $A$ )

Simulates received signal as **sum of returns from point targets** with different delays and frequency shifts.



$$y = A(\mathbf{h}[n, p])$$

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# Model discretization from radar signal equation

## Narrow-band radar equation for received signal

$$y(t) = \int_0^T \int_{-\infty}^{\infty} s(t - \lambda) e^{2\pi i f(t-\lambda)} h(f, \lambda) df d\lambda$$

Measured signal	Transmitted signal	Frequency shift	Target reflectivity
-----------------	--------------------	-----------------	---------------------

$$t \quad \text{Sample time} \quad y(m\tau) \rightarrow y[m] \quad m = 0, 1, \dots, M-1$$

$$f \quad \text{Frequency} \quad h(n\phi, :) \rightsquigarrow h[n, :] \quad n = 0, 1, \dots, N-1$$

$$\lambda \quad \text{Delay} \quad h(:, p\tau) \rightsquigarrow h[:, p] \quad p = 0, 1, \dots, P-1$$

## Discretized radar model

$$y[m] = \sum_{p=0}^{P-1} \sum_{n=0}^{N-1} \frac{1}{\sqrt{N}} s[m - p + L - 1] e^{2\pi i n m / N} h[n, p] = A(h)$$

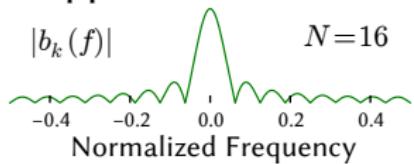
# Reflectivity coefficients from function

Can write the “point target” reflectivity coefficients in terms of the original reflectivity function:

$$h[n, p] = \int_{p\tau}^{(p+1)\tau} [h(f, \lambda) e^{2\pi i f \lambda} * b_{p+1}(f)] (n\phi) d\lambda$$

- Blur in frequency by convolving with wrapped sinc function:

$$b_k(f) = \frac{1}{N} e^{-\pi i (2k+N-1)\tau f} \frac{\sin(N\pi\tau f)}{\sin(\pi\tau f)}$$

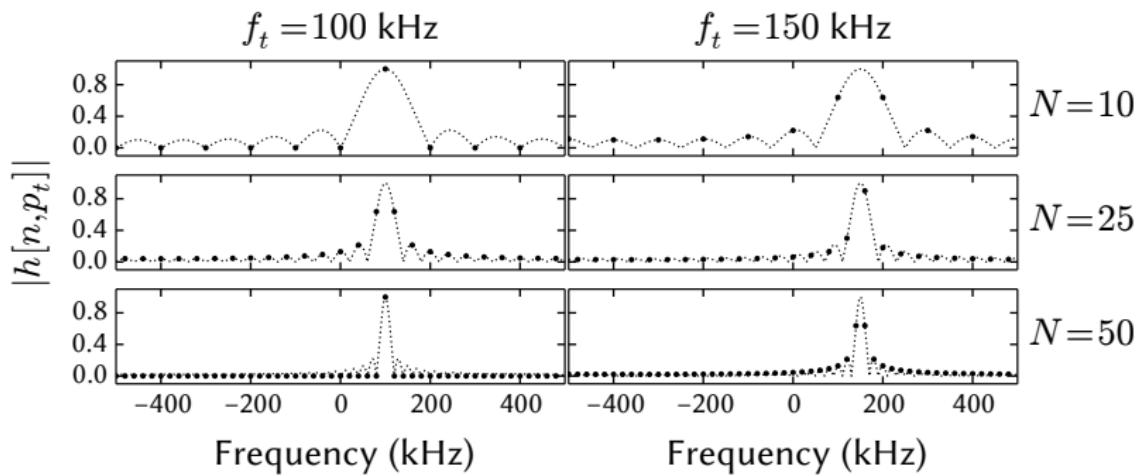


- Sample at discretized frequency point  $n\phi$
- Integrate over delay window of discretized delay point  $p\tau$

# Representing an off-grid point target

Off-grid point targets are still relatively sparse!

- ▶ All coefficients outside delay window  $p_t$  are zero
- ▶ For frequency index  $n = 0, 1, \dots, N - 1$ , behavior dictated by wrapped sinc:



Relative sparsity improves with increasing  $N$   
(same number significant coefficients, many more zeros)

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**Sparsity Background**

Compressed Sensing

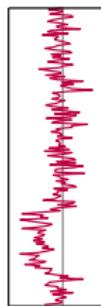
Convex Optimization

Waveform Inversion

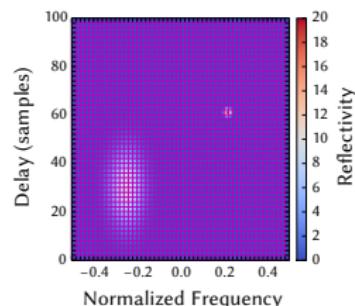
Conclusion

# Under-determined systems of equations

Not enough measurements to constrain unknown values  
(e.g. the radar model):



$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} & & \\ A & & \\ & & \end{bmatrix} \begin{bmatrix} h \end{bmatrix}$$



(vectorized as  $h$ )

Measurement    Model    Unknown

- ▶ Infinite number of solutions
- ▶ Often know that the true solution should be sparse
- ▶ Finding the sparsest solution is hard in general

# Theory of compressed sensing

## Definition

Compressed sensing is a theory to guarantee solution of an under-determined set of equations.

## Approximate guidelines for application

- ▶ Solution known to be sparse
- ▶ Measurements are “incoherent” (global with low correlation)
- ▶ Minimum number of measurements on the order of the solution sparsity (number of nonzeros)

## Benefit

Can solve easy convex optimization problem instead of hard combinatorial problem.

# Equivalent convex optimization problem

## Sparsest solution to noisy measurements

Find sparsest  $h$

subject to  $\|y - A(h)\|_2 < \eta$   $\|h\|_2^2 = \sum_k |h_k|^2$

## $l_1$ -regularized least-squares (convex)

$$\underset{h}{\text{minimize}} \quad \frac{1}{2} \|y - A(h)\|_2^2 + \lambda \|h\|_1 \quad \|h\|_1 = \sum_k |h_k|$$

The  $l_1$ -norm promotes sparsity!

# Outline

Introduction and Motivation

Radar Background

My Radar Model

**Sparsity Background**

Compressed Sensing

Convex Optimization

Waveform Inversion

Conclusion

# First-order methods

We want to efficiently solve

$$\underset{h}{\text{minimize}} \quad \frac{1}{2} \|y - A(h)\|_2^2 + \lambda \|h\|_1$$

but for systems  $A$  that are too large for matrix methods.

## First-order methods

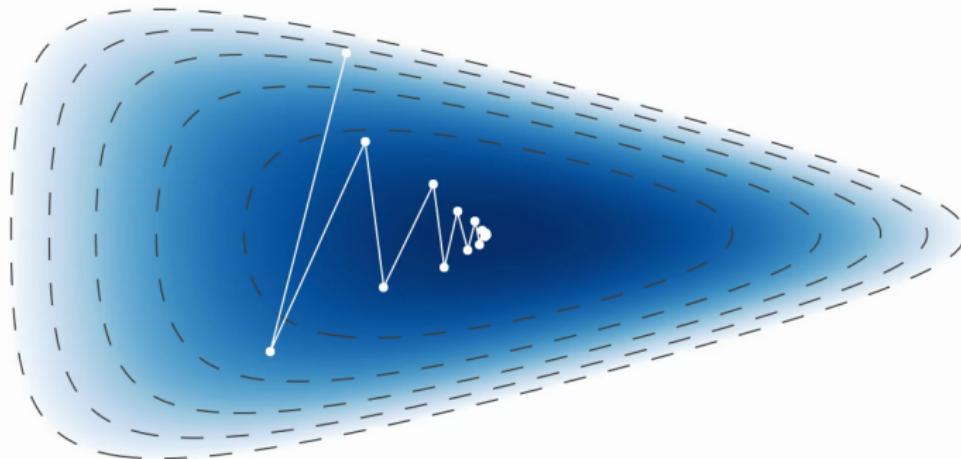
- ▶ Explicit matrix for  $A$  not needed
- ▶ Only need to be able to compute  $A(\cdot)$  and  $A^*(\cdot)$

$l_1$ -regularized least-squares is non-smooth because of the  $l_1$  norm term, so minimizing it requires a special approach.

# First-order step

## Smooth

If the function being minimized is differentiable, the most natural first-order step is a gradient step.



# First-order step

## Non-smooth

For a non-differentiable function, an attractive alternative is the proximal (or prox) step.

$$x^{k+1} := \arg \min_x \left( G(x) + \frac{1}{2\mu} \|x - x^k\|_2^2 \right)$$

Non-smooth function
Smoothing around  
current iterate

- ▶ This is surprisingly easy to solve and can be evaluated much like a gradient!
- ▶ Note that as  $x^k$  approaches the minimum  $x^*$ , the smoothing term goes to zero.

The **prox operator** is defined for non-smooth  $G(x)$  as

$$\mathbf{prox}_{\mu G}(v) = \arg \min_x \left( G(x) + \frac{1}{2\mu} \|x - v\|_2^2 \right).$$

# Proximal gradient method

The **proximal gradient method** combines gradient and prox steps to solve

$$\underset{x}{\text{minimize}} \quad F(x) + G(x)$$

for smooth  $F(x)$  and non-smooth  $G(x)$ .

## Algorithm

(Step size  $\mu$ ) Iterate:

Gradient step       $z^{k+1} := x^k - \mu \nabla F(x^k)$

Prox step       $x^{k+1} := \mathbf{prox}_{\mu G}(z^{k+1})$

# Proximal gradient method applied

Proximal gradient for  $l_1$ -regularized least-squares:

$$\underset{h}{\text{minimize}} \quad \frac{1}{2} \|y - A(h)\|_2^2 + \lambda \|h\|_1$$

$$F(h) = \frac{1}{2} \|y - A(h)\|_2^2 \quad \Rightarrow \quad \nabla F(h) = -A^*(y - A(h))$$

$$G(h) = \lambda \|h\|_1 \quad \Rightarrow \quad \mathbf{prox}_{\mu G}(v) = \mathbf{soft}_{\lambda\mu}(v) = \begin{cases} v - \lambda\mu, & v > \lambda\mu \\ 0, & |v| \leq \lambda\mu \\ v + \lambda\mu, & v < -\lambda\mu \end{cases}$$

# Iterative Soft Thresholding

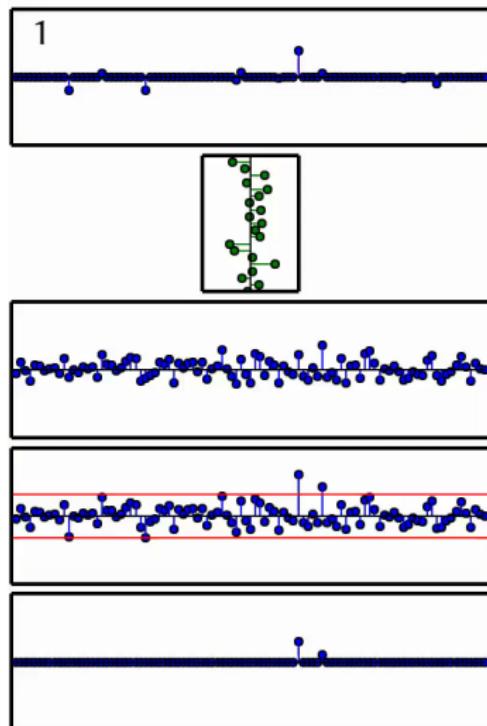
Guess  
(e.g. reflectivity):  $h$

Calculate error:  $z = y - A(h)$

Matched filter  
the error:  $A^*(z)$

Add previous  
guess:  $h + A^*(z)$

Threshold to  
form new guess:  
 $h = \text{soft}(h + A^*(z))$



**Interpretation:** iterative matched filtering with thresholding!

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Waveform Inversion

Implementation

Experimental Results

Conclusion

# Combined algorithmic advances

First order prox algorithms are an active research area, so I tested and combined multiple proposed enhancements:

1. Accelerated proximal gradient method
  - ▶ Adds simple “momentum” term
  - ▶ Better theoretical convergence
2. Adaptive restart
  - ▶ Reset acceleration when it opposes prior step
3. Adaptive step size
  - ▶ Increase step size every iteration
  - ▶ Decrease as necessary to ensure convergence

Convergence comparison	
Method	Iterations
Prox gradient	4463
+ Acceleration	2911
+ Adaptive restart	397
+ Adaptive step	105

# Testing similar algorithms

Other prox-based algorithms are worthy of consideration:

- ▶ Linearized alternating direction method of multipliers (ADMM)
- ▶ Primal-dual hybrid gradient (PDHG)

Closely related, these both solve

$$\underset{x}{\text{minimize}} \quad F(x) + G(x)$$

when both  $F(x)$  and  $G(x)$  can be non-smooth.

## Convergence comparison

Method	Iterations
Accelerated proximal gradient	105
Linearized ADMM (novel adaptive step)	154
PDHG (standard fixed step)	4464

# Code

## Principles

1. Easy to develop
2. Fast when necessary
3. Freely available for collaboration and community use

## Answers

1. Python with NumPy
2. Radar model is bottleneck: implement with Cython or Numba
3. Available on github:  
[github.com/ryanvolz](https://github.com/ryanvolz)



python™



NumPy



Cython



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# The Jicamarca incoherent scatter radar

## Specifications:

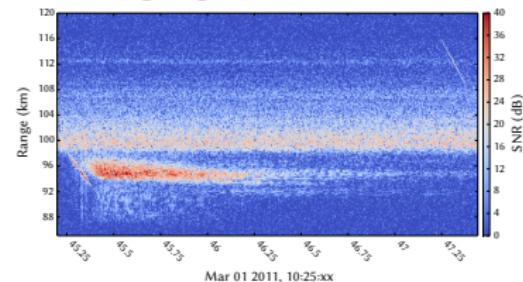
- ▶ Located outside Lima, Peru
- ▶ VHF (50 MHz)
- ▶ Phased array of  $96 \times 96$  crossed half-wave dipoles



- ▶ 1 MHz bandwidth (TX/RX)

## Reasons:

- ▶ Equatorial ionosphere is challenging

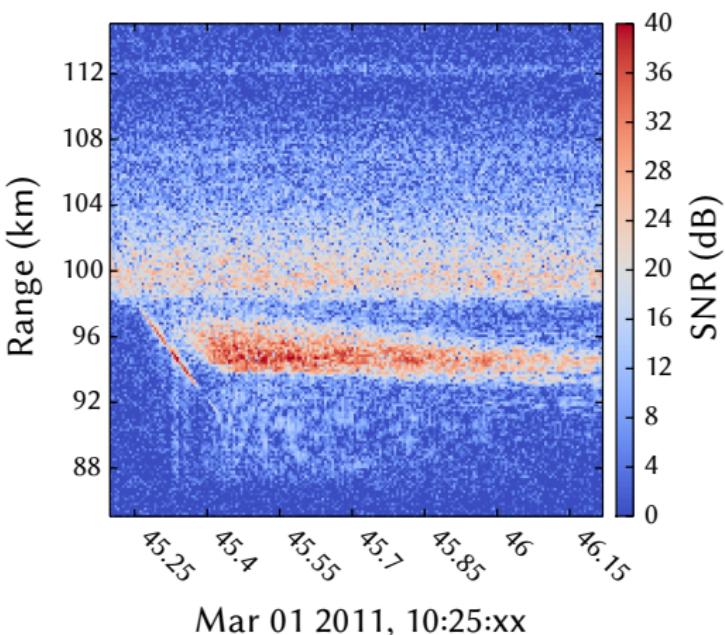


- ▶ Detection rate of meteors is better at lower frequencies
- ▶ Interferometry and/or dual polarization receive

# Jicamarca experiment goals

## Goals

- ▶ Test sparsity-based waveform inversion in crowded environment
- ▶ Directly compare effectiveness of different waveforms



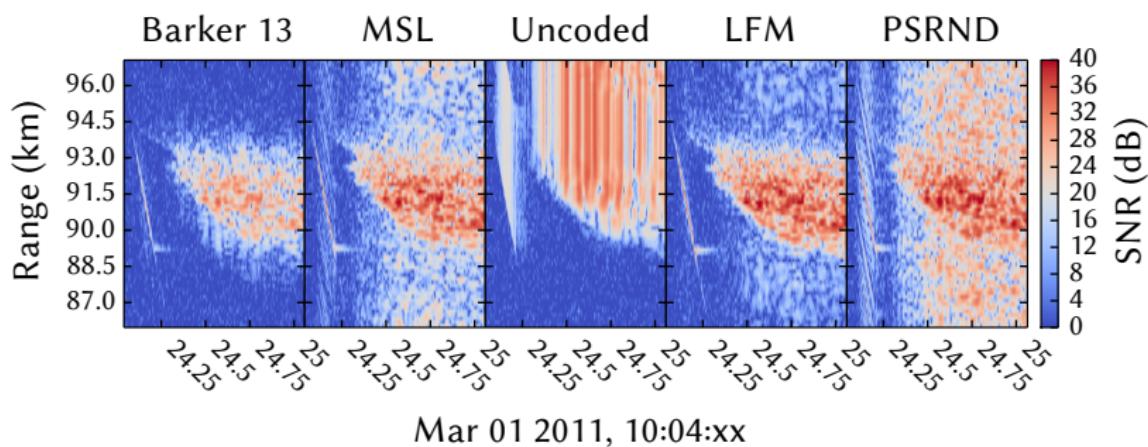
# Jicamarca meteor experiment

## Description

Alternating sequence of 5 common waveforms for observing meteor region (80-140 km altitude).

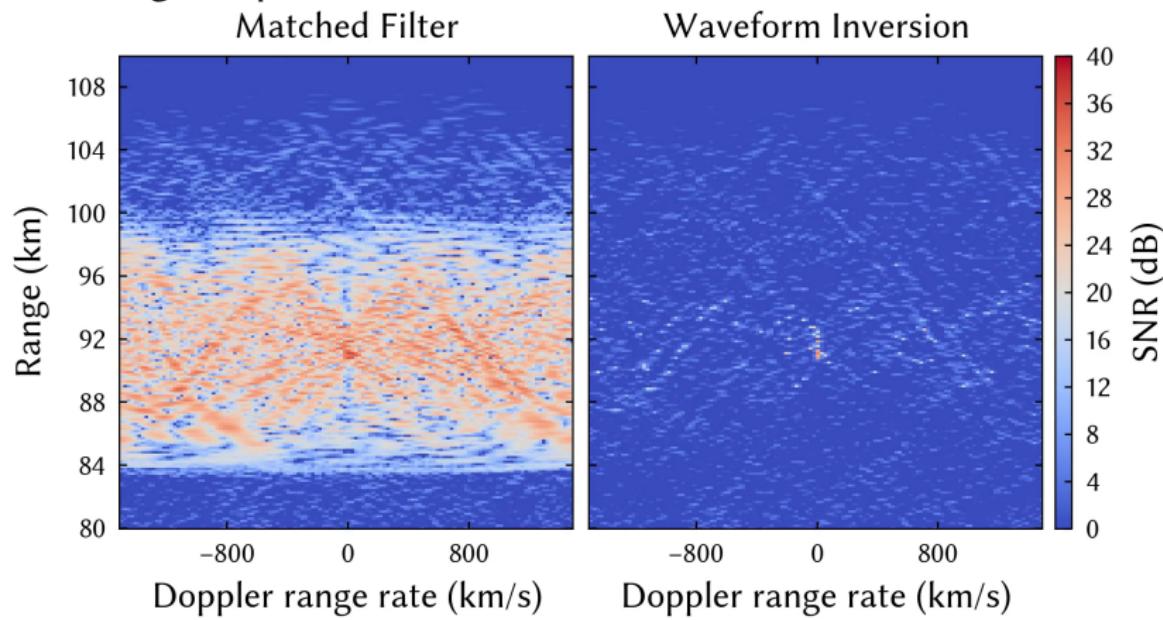
## Parameters

- ▶ Pulse interval of 1 ms
- ▶ Sample time of  $1 \mu\text{s}$   
(150 m range resolution)



# Movie of meteor sidelobe removal

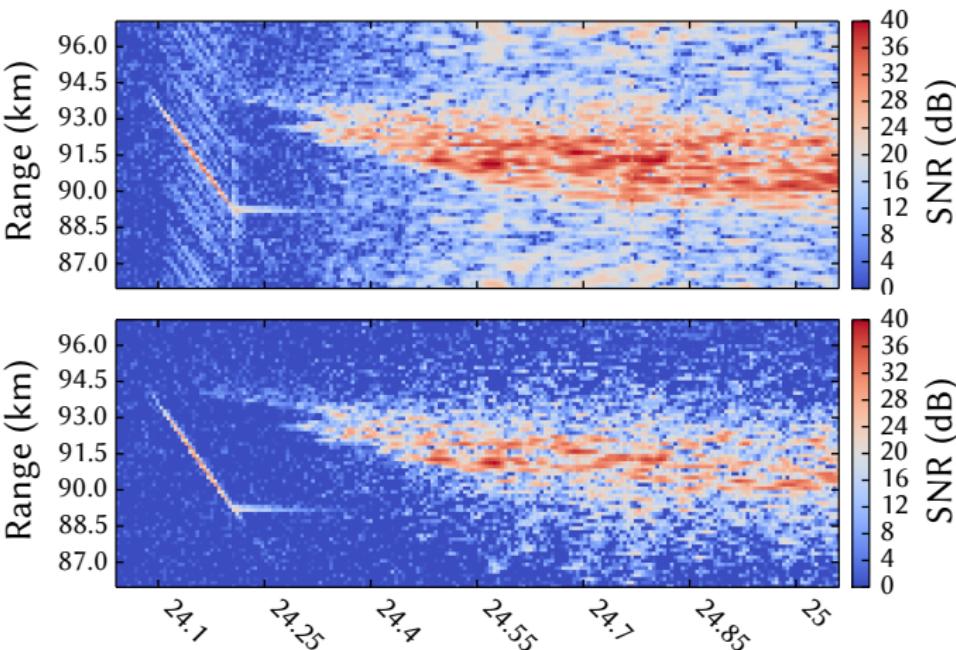
Decoding comparison for minimum sidelobe code:



(Total elapsed time of 1 second)

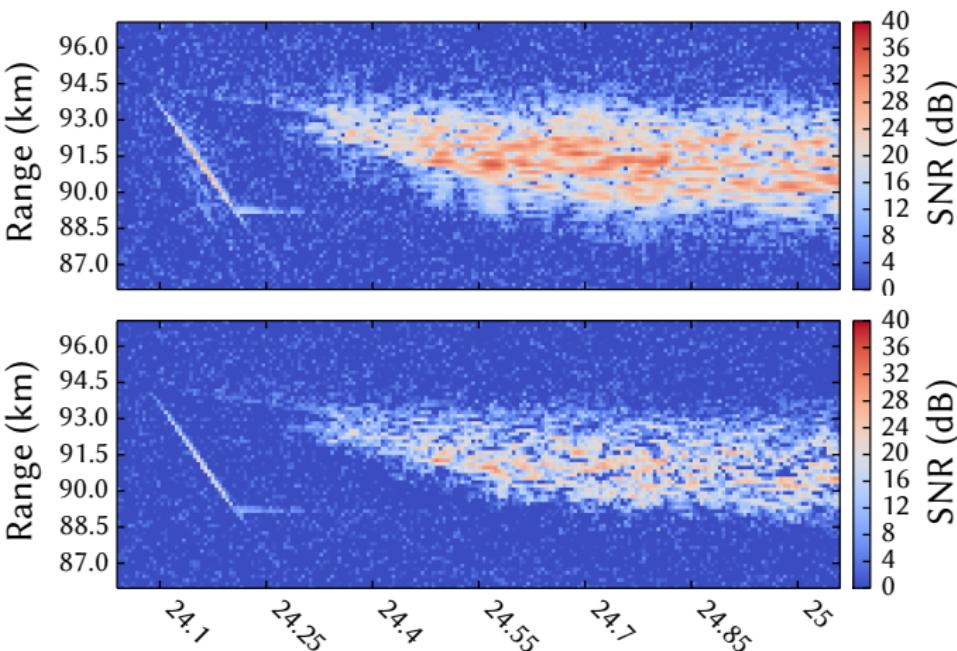
# Example: Minimum sidelobe code

Matched  
Filter



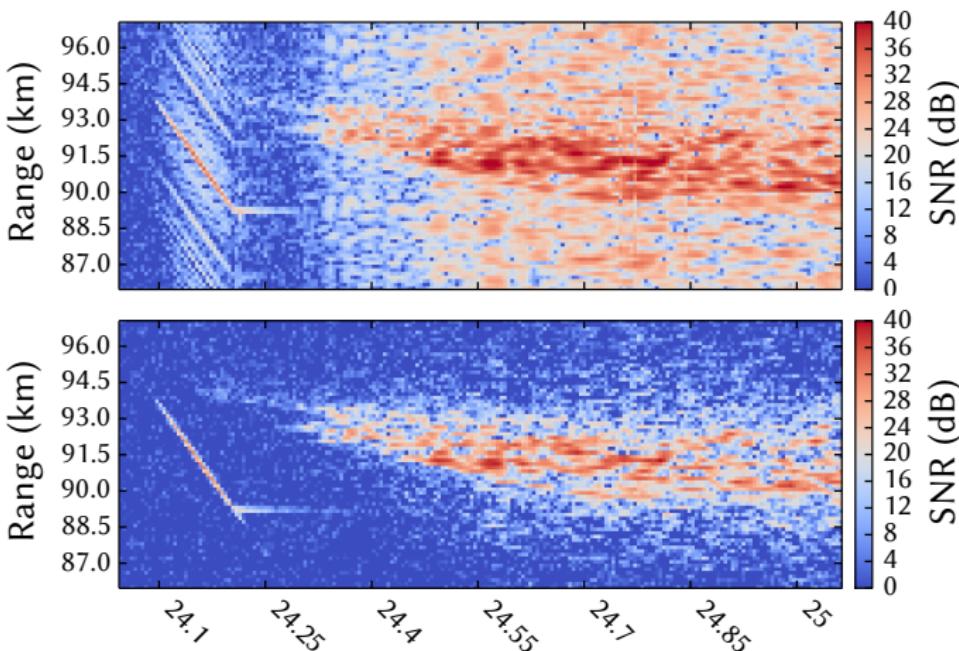
# Example: Barker-13 code

Matched  
Filter



# Example: Pseudorandom code

Matched  
Filter

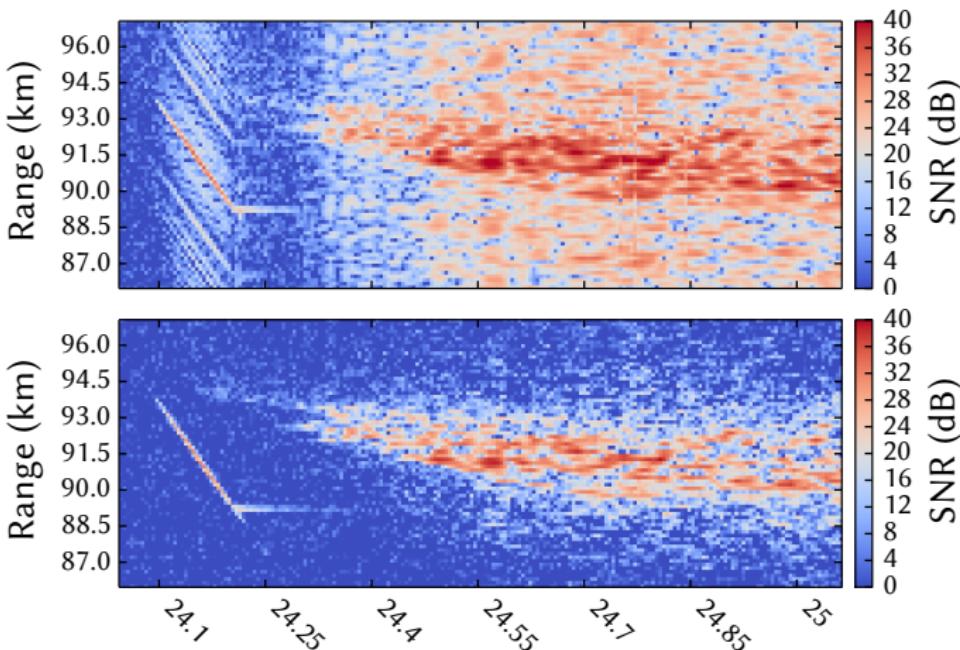


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Works with variety of codes

# Example: Pseudorandom code

Matched  
Filter



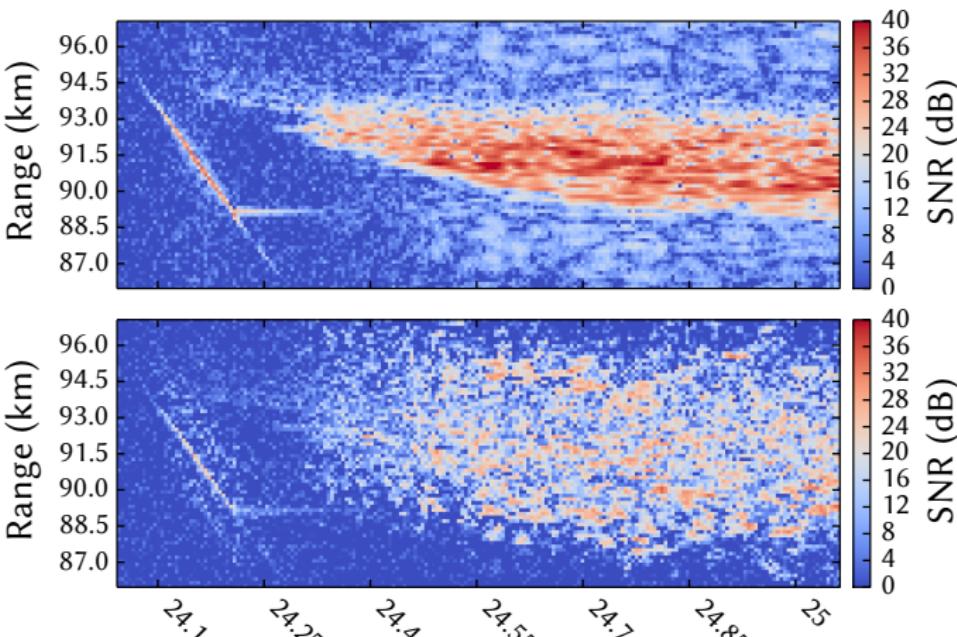
Waveform  
Inversion

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Works with variety of codes

# Example: LFM chirp

Matched  
Filter

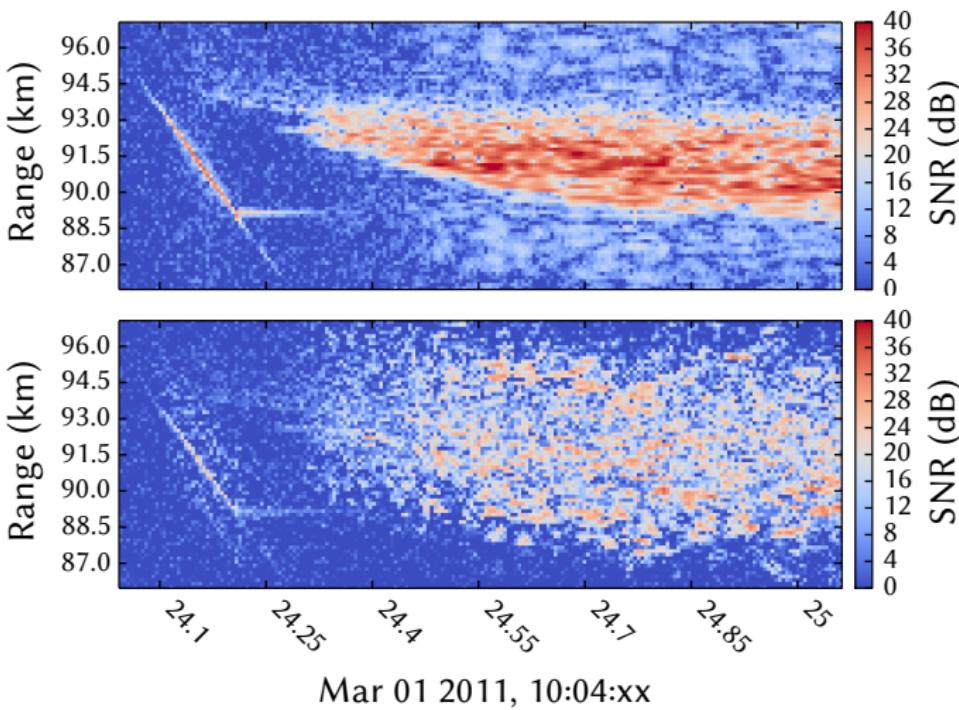


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Some codes are troublesome

# Example: LFM chirp

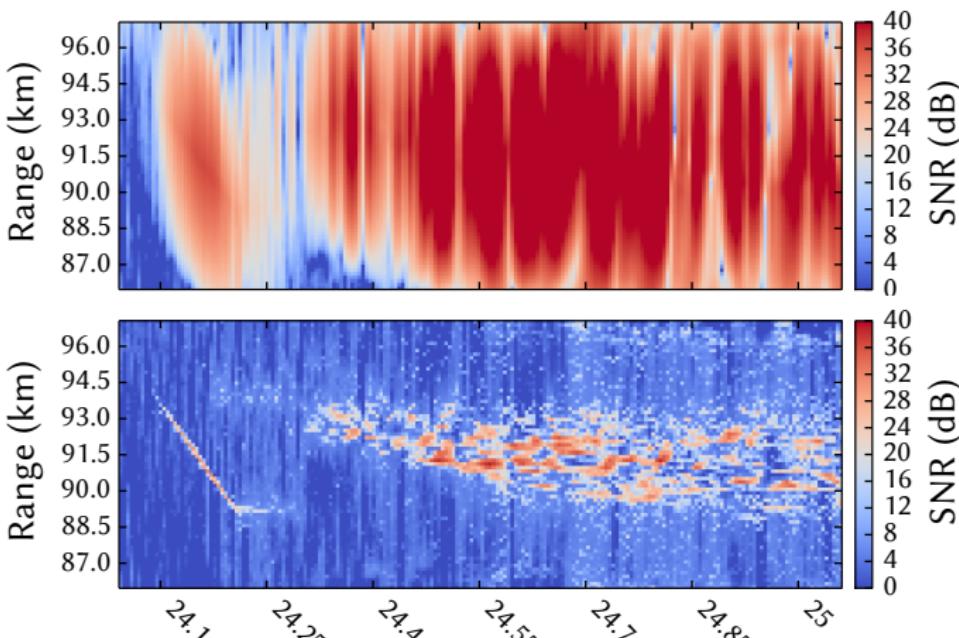
Matched  
Filter



Some codes are troublesome

# Example: Uncoded

Matched  
Filter

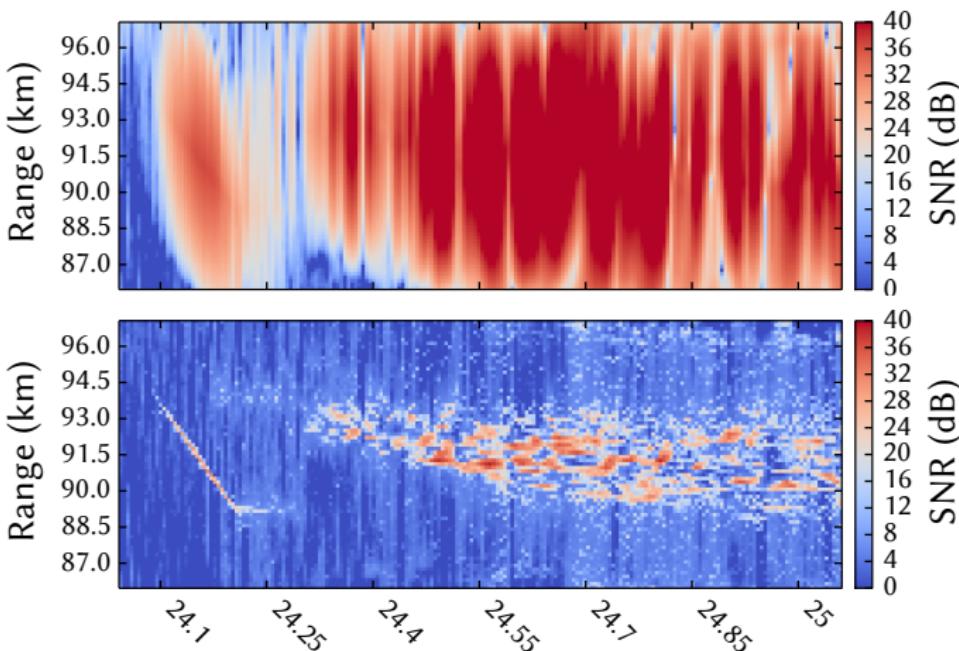


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Works even with uncoded pulses!

# Example: Uncoded

Matched  
Filter



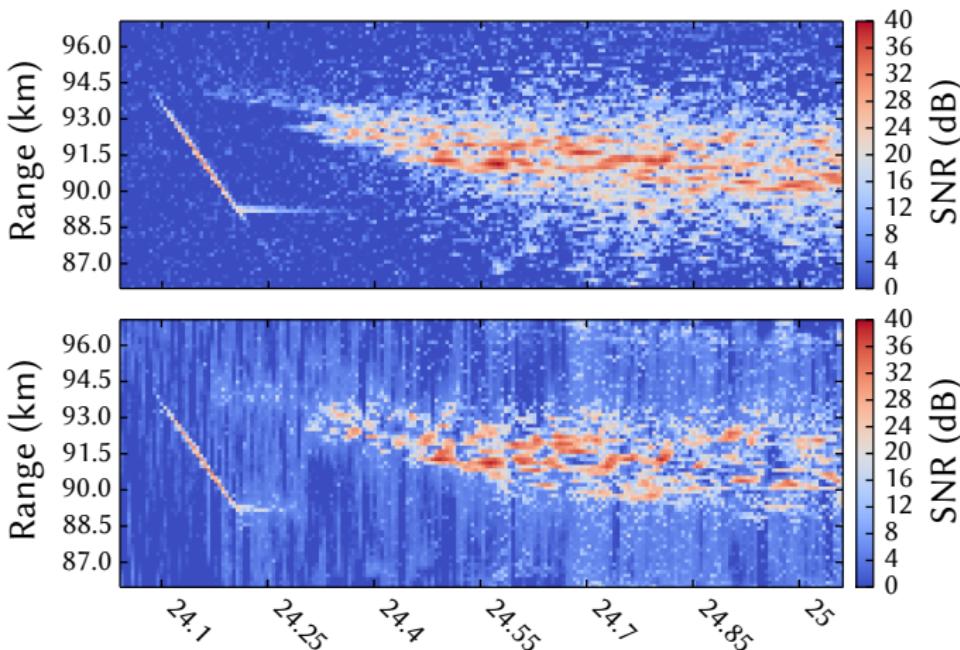
Waveform  
Inversion

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Works even with uncoded pulses!

# Waveform inversion code comparison

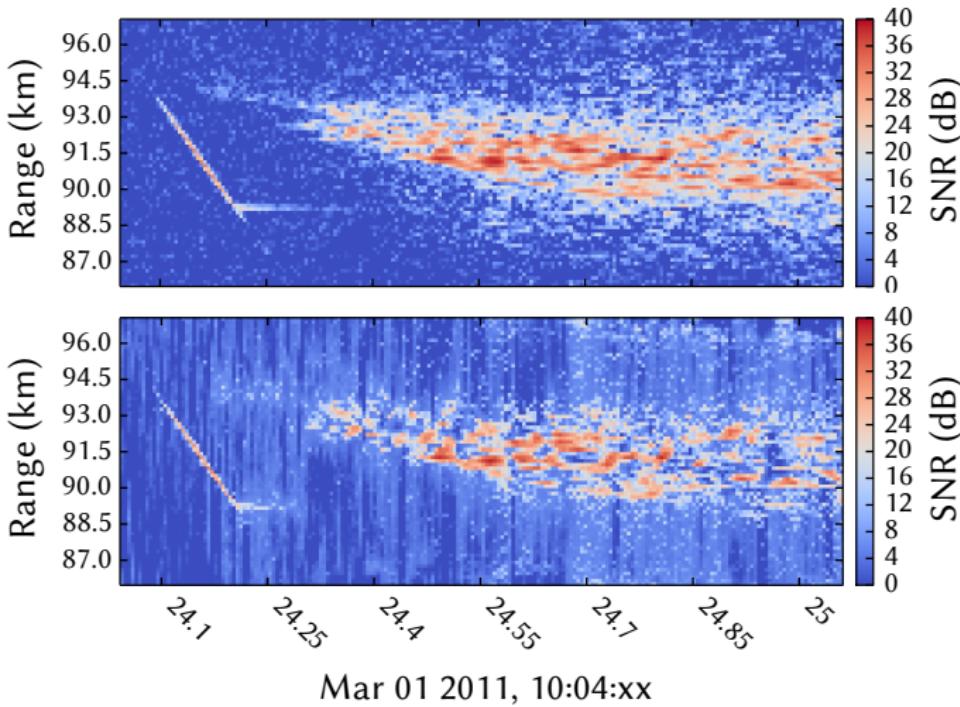
Minimum sidelobe



Quality of solution depends on fidelity of waveform

# Waveform inversion code comparison

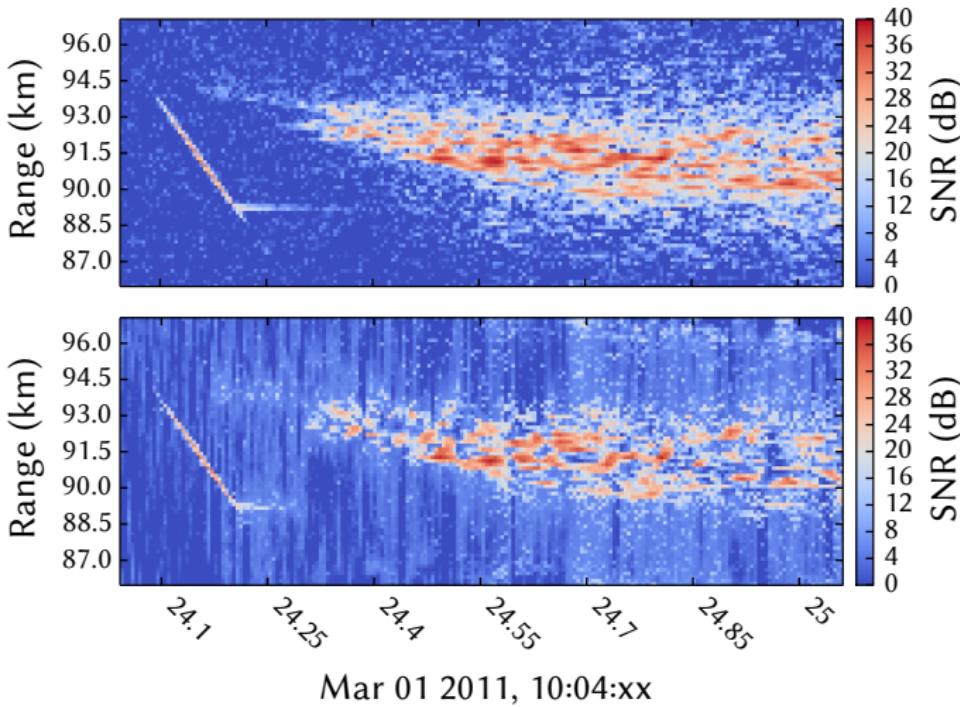
Pseudo-random



Quality of solution depends on fidelity of waveform

# Waveform inversion code comparison

Pseudo-random



Quality of solution depends on fidelity of waveform

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My Radar Model

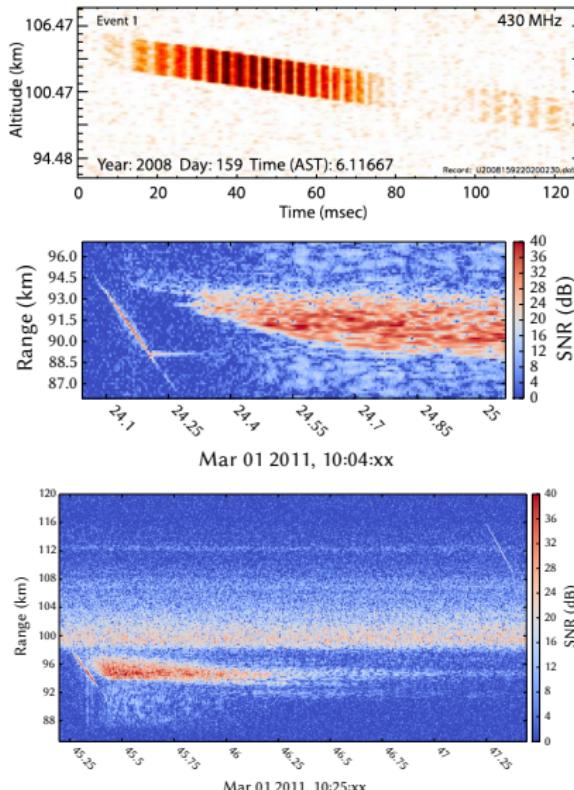
Sparsity Background

Waveform Inversion

Conclusion

## Benefits of waveform inversion

1. **Sidelobe removal** -  
Removes ambiguity for observation of multiple or range-spread targets  
(e.g. fragmentation and non-specular trails)
  2. **Full frequency decoding** -  
Decodes over full frequency spectrum simultaneously, for target differentiation in crowded environments  
(e.g. equatorial, flares)
  3. **Flexibility** -  
Enables use of many different waveforms



# Contributions

In order to enable **flexible high-resolution measurements** of ionospheric plasma phenomena, including interfering and range-spread targets, I made the following contributions:

1. Derived a discrete **radar model** that captures signal sparsity in delay-frequency space.
2. Uncovered the adjoint relationship between the model and existing matched filter techniques, giving an **intuitive interpretation** of waveform inversion as iterative filtering.
3. **Analyzed the model** to show how it exactly represents distributed scatterers and preserves their sparsity.
4. Created and tuned an **efficient implementation** of waveform inversion using modern convex optimization techniques.
5. **Demonstrated the flexibility and effectiveness** of the inversion technique by removing filtering artifacts from meteor observations made with a variety of radar waveforms.

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# Further research

## Near-universal radar measurements

Submitted NSF postdoc proposal to work at MIT Haystack (Millstone Hill) and create near-universal measurement mode across radar chain using uniquely-tuned waveforms.

## Self-calibration

Iteratively solve for reflectivity *and* transmitted waveform to better characterize the radar system.

## Passive radar

Waveform flexibility invites use for parasitic radio science (using FM radio, digital TV signals) where the transmitted waveform cannot be controlled.

# Acknowledgements

Thank you for coming!

# Questions

## Introduction and Motivation

### The Ionosphere

- 2 The ionosphere
- 3 Importance of the ionosphere: Communications
- 4 Importance of the ionosphere: Science of the geospace system
- 5 Space weather: meteoroids and meteors
- 6 Importance of meteoroids

### Radar Measurements

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- 8 Radar meteors
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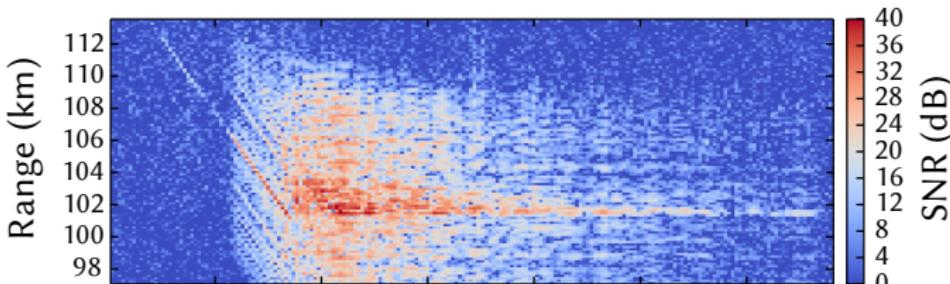
### Questions

- 71 Questions

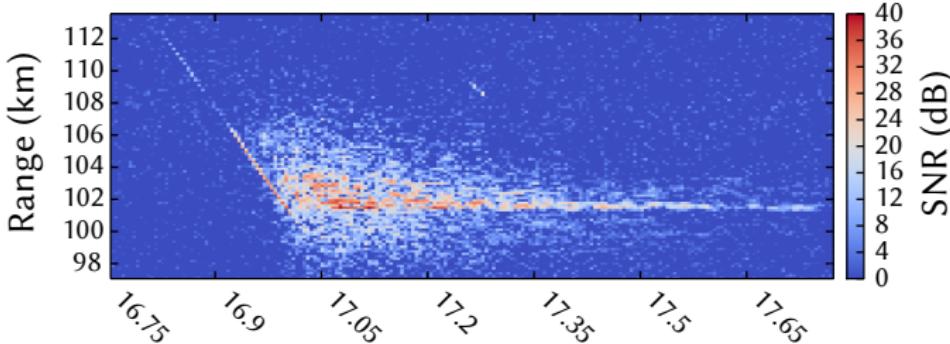
# Appendix

# Another example: Pseudorandom code

Matched  
Filter



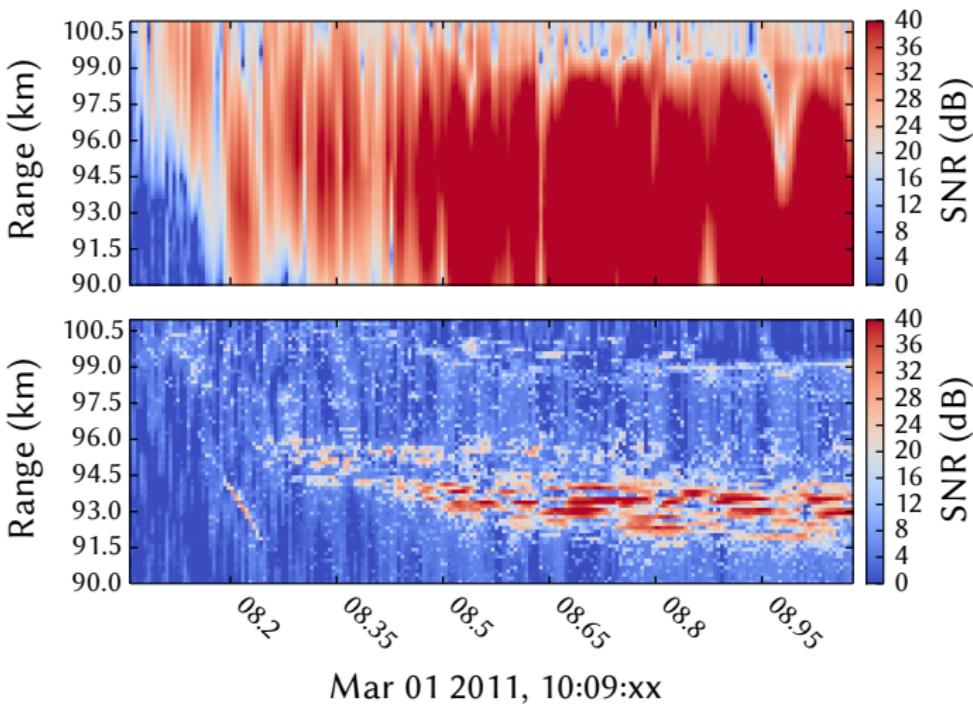
Waveform  
Inversion



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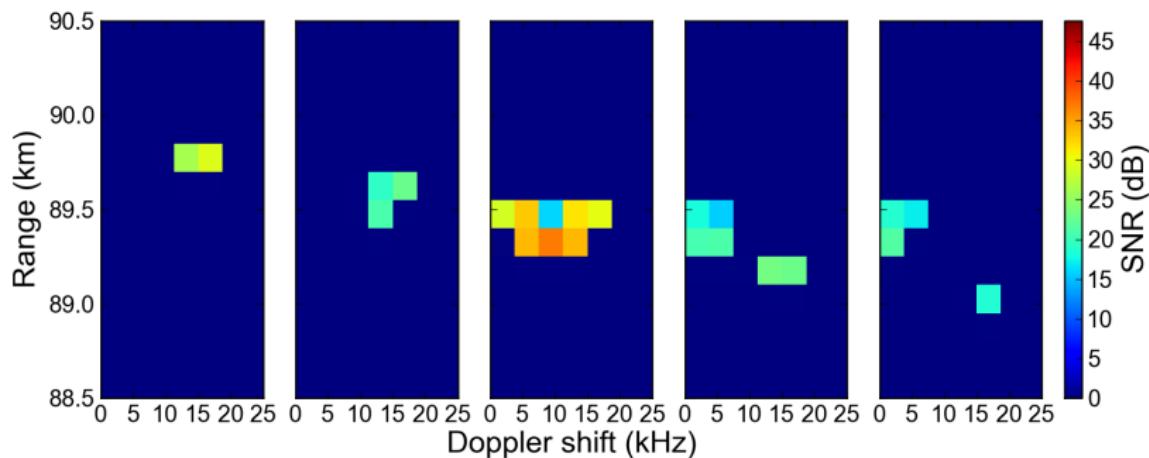
# Different example: Uncoded

Matched  
Filter



# Jicamarca flare

- ▶ Waveform inversion of a meteor head echo and flare
- ▶ Minimum sidelobe code data from Jicamarca experiment
- ▶ 5 pulses around the event:



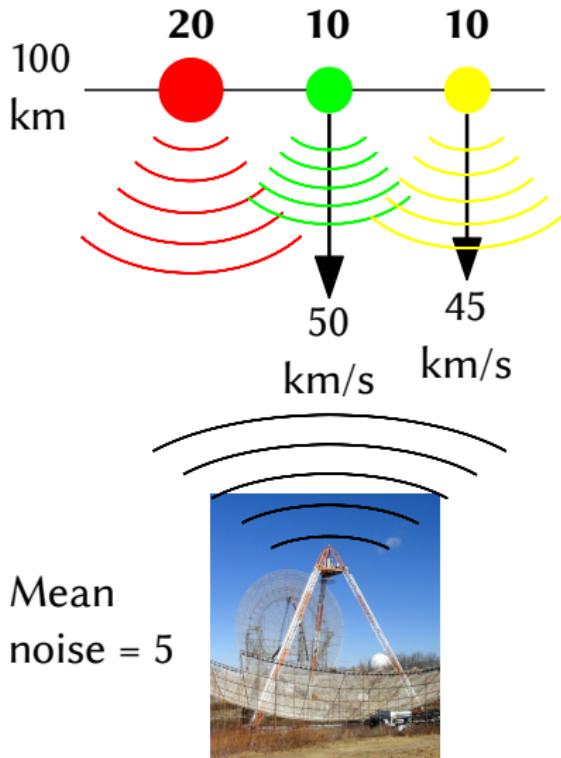
# Simulation parameters

## Point target simulation

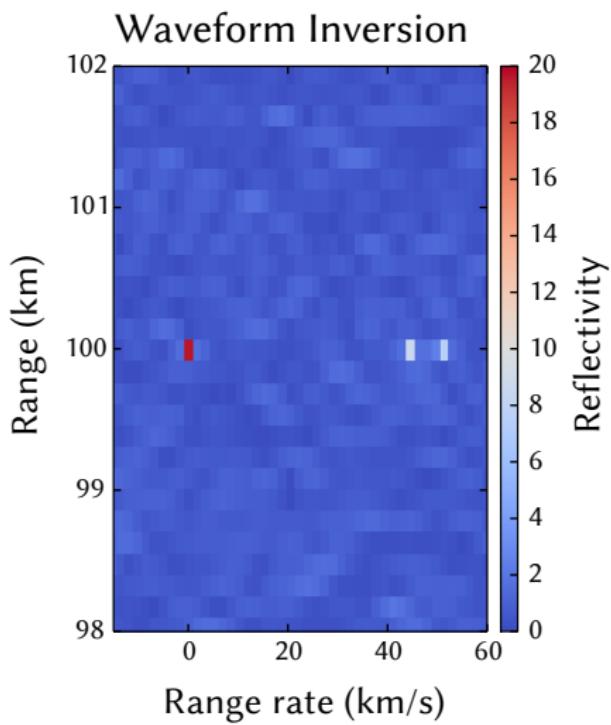
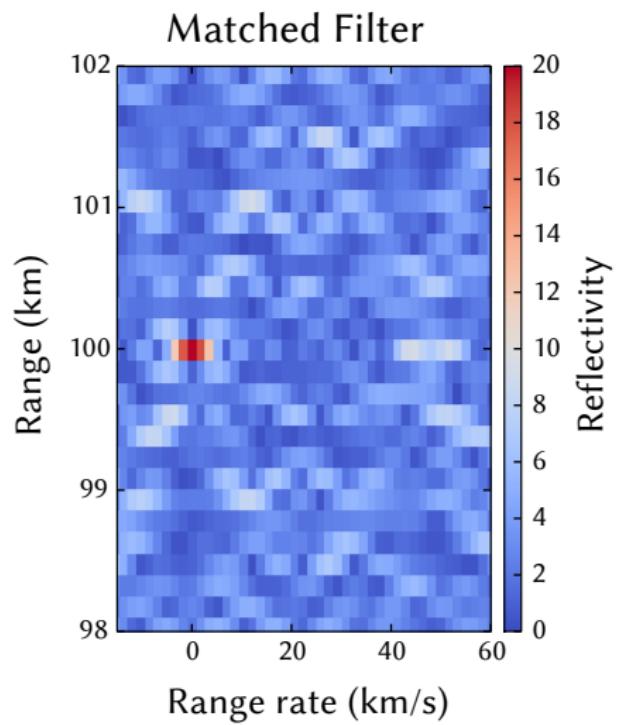
- ▶ 3 targets at 100 km range:

Reflectivity	Range rate
20	0 km/s
10	45 km/s
10	50 km/s

- ▶ Mean noise power of 5
- ▶ 51-baud minimum sidelobe code



# Simulation results



# The prox operator

The proximal operator, or **prox operator**, is defined for non-smooth  $G(x)$  as

$$\mathbf{prox}_{\mu G}(v) = \arg \min_x \left( G(x) + \frac{1}{2\mu} \|x - v\|_2^2 \right).$$

## Projection onto a set

If  $G(x)$  is the indicator function for a closed convex set  $\mathcal{C}$ ,

$$G(x) = \begin{cases} 0 & x \in \mathcal{C} \\ \infty & x \notin \mathcal{C} \end{cases}$$

$\mathbf{prox}_G(v)$  is projection of  $v$  onto  $\mathcal{C}$ .

# Accelerated proximal gradient method

Set step size       $\mu^{k+1}$

$$\gamma := \mu^k / \mu^{k+1}$$

Acceleration parameter       $t^{k+1} := \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\gamma(t^k)^2}$

$$\theta := (t^k - 1) / t^{k+1}$$

Acceleration step       $w^{k+1} := x^k + \theta(x^k - x^{k-1})$

Gradient step       $z^{k+1} := w^{k+1} - \mu^{k+1} \nabla F(w^k)$

Prox step       $x^{k+1} := \mathbf{prox}_{\mu G}(z^{k+1})$

# Sidelobe math

- ▶ The function  $A^*A$  operating on a target scene  $h$  models the matched filter result of measuring that target.
- ▶ The sidelobes are the off-diagonal results of the function, given by the operation  $A^*A - \frac{1}{N}I$ .
- ▶ So to remove sidelobes from the matched filtered data  $A^*(y)$  using the approximating sparse solution  $h$ , we must calculate:

$$A^*(y) - \left( A^*A - \frac{1}{N}I \right) (h) = A^* (y - A(h)) + \frac{1}{N}h$$

- ▶ The standard matched filter is usually scaled by a factor of  $\sqrt{N}$  relative to the model, so the equivalent waveform inversion result with sidelobes removed is given by:

$$\frac{1}{\sqrt{N}}h + \sqrt{N}A^* (y - A(h))$$

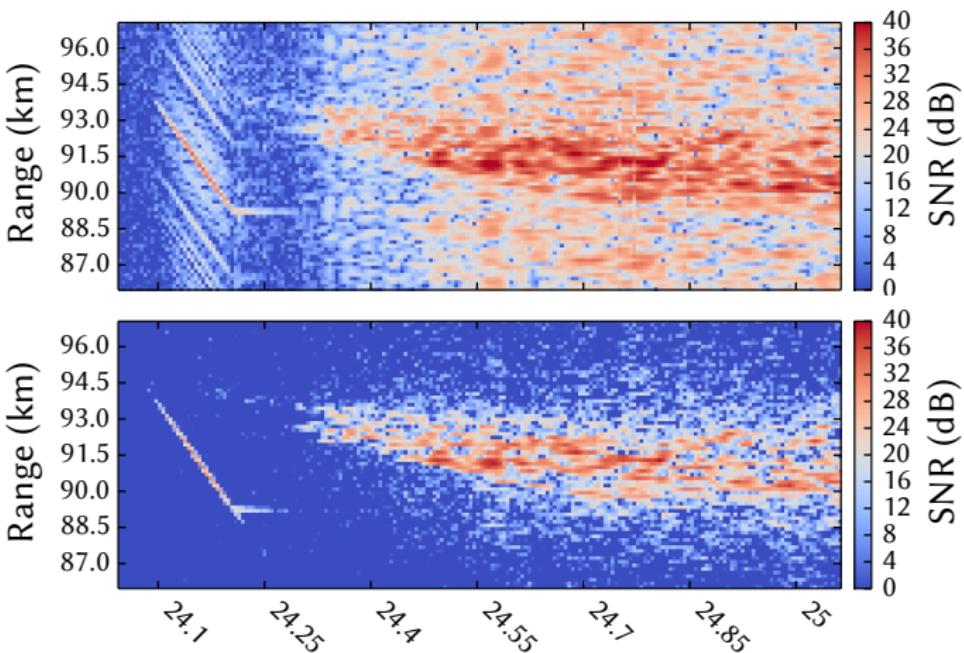
(sparse result plus matched filtered noise)

# Sparse solution

Matched  
Filter

Sparse  
Solution

Pseudorandom code



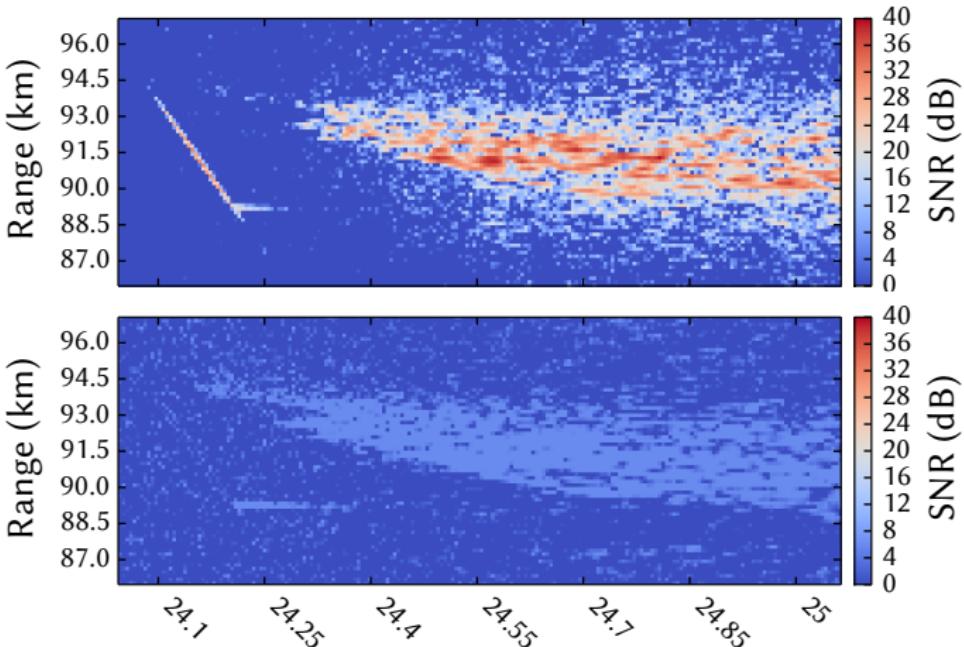
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# Unmodeled noise

Sparse  
Solution

Unmodeled  
Noise

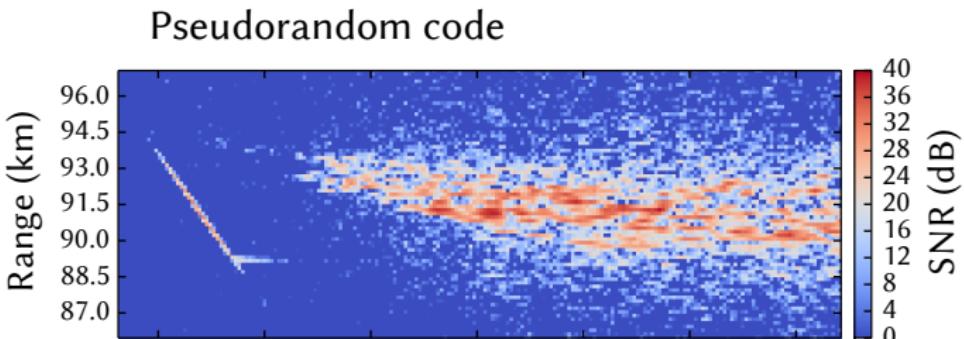
Pseudorandom code



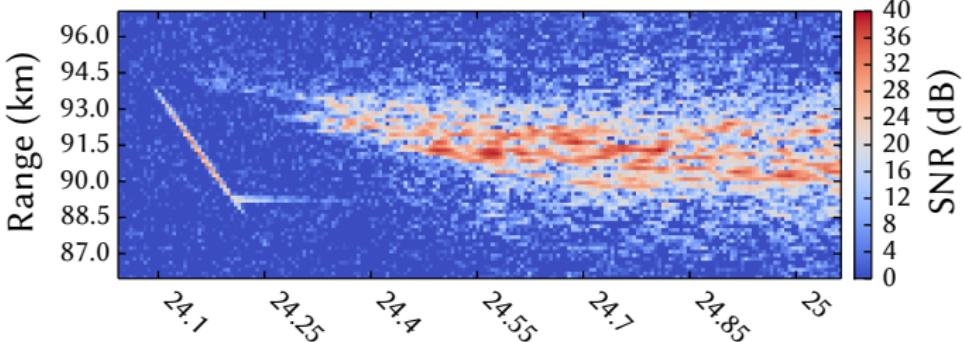
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# Sidelobe removal as sparse plus noise

Sparse Solution



Sparse plus Noise



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