PHYS4080 N-Body Project

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1 The N-Body Solver

In this project, we utilised the python package `pytreegrav` [1], which is a pure Python implementation of brute-force and tree-based N-body numerical solver. Using this in conjunction with a leapfrog numerical integrator, we were able to quickly and accurately simulate gravitationally-self interacting systems with a precision loss of less than 0.1%. All simulations herein were done in so-called N-body units, which reduces the computational overhead and allows for generally more precise computation. All of the code used for this project is available on the public GitHub repository, including supplementary figures and animations of gravitational systems.

2 Cold Collapse

We begin by examining the collapse of a uniform sphere of particles, where the system has $\sum M = 1$ and $R = 1$ (in N-body units, requiring $G = 1$ also). For such a system, we can estimate the velocity a typical particle would need to be in order to be in equilibrium via the virial theorem, where

$$\frac{GM}{R} = \sigma^2 \tag{1}$$

We could turn our attention to any reasonable measure of mass and radius, and for the purposes of this report we choose the half mass radius. Since we have a unit sphere of uniform density, the half mass radius is $R_{1/2} = 0.5^{1/3}$. The reason for this power is intuitive if we consider one dimension, where the half mass 'radius' of a line is clearly at $r = 0.5$. Extending this to two dimensions we would require a slightly larger radius to account for the quadratic effect of area, meaning $r = 0.5^{1/2}$, and so the half mass radius for a unit sphere naturally has a cubic root power. Hence, in N-body units, we have

$$\begin{aligned} \sigma^2 &= \frac{0.5}{0.5^{1/3}} \\ \Rightarrow \sigma &= \sqrt{0.5^{2/3}} \\ \sigma &= (0.5)^{1/3} \end{aligned} \tag{2}$$

And so for the unit sphere of particles, we'd expect that each particle should have $|v| \sim \sigma$ in any random direction in order for the entire system to be in rough equilibrium. In order for them to be in true equilibrium, we might require them to have that same velocity in a direction perpendicular to the normal vector of the sphere at each particle orbital radius, but such precision won't be considered herein. As a proof of concept, we'd simulated a system with these properties, and it is available to view at this link in the repository.

Conveniently this value of σ obviously doesn't depend on the number of particles in the system, considering that we'd taken the half mass as our quantity of interest. Since all particles have the same mass (namely $M = 1/N$) and we're examining the unit sphere, this half mass radius is constant with particle number.

To assess the characteristic timescale of collapse, we utilise the so-called free fall timescale, or t_{ff}. This timescale (for a system of radius R) is calculated by considering the period of a circular orbit at $R/2$, and dividing it by two. This is because a mass falling from $x = R$ to $x = -R$ (with the center of mass of the system at $x = 0$) takes the same length of time as a circular orbit of radius $R/2$. So, the time for a particle at $x = R$ to fall to $x = 0$ is half that time. Mathematically,

$$t_{ff} = \frac{t_{\text{orbit}}}{2} = \frac{\pi}{2}\frac{R^{3/2}}{\sqrt{2GM}} \tag{3}$$

Therefore, in N-body units with $G = M = R = 1$, we have the characteristic free-fall time of $t_{ff} = \pi/2\sqrt{2}$.

With all of these properties of the system calculated, we then constructed a system of $N = 4096$ particles of identical mass in the unit sphere and gave them 5% of the 'equilibrium' velocity, v_E, in a random direction. This was simulated up to $T = 5$ in time steps of $\Delta t = 0.02$ yielding a change in energy (as a result of smoothing) of less than 1%. Our smoothing parameter was set to 0.2 which was fine tuned to minimise the change in energy of the system over the simulation. An animation of this simulation is included in the repository. In that animation, particles are coloured according to a gradient red to yellow according to their *initial* distance from the center of mass. We can see that particles that were initially near to the center tend to star near to the center over time.

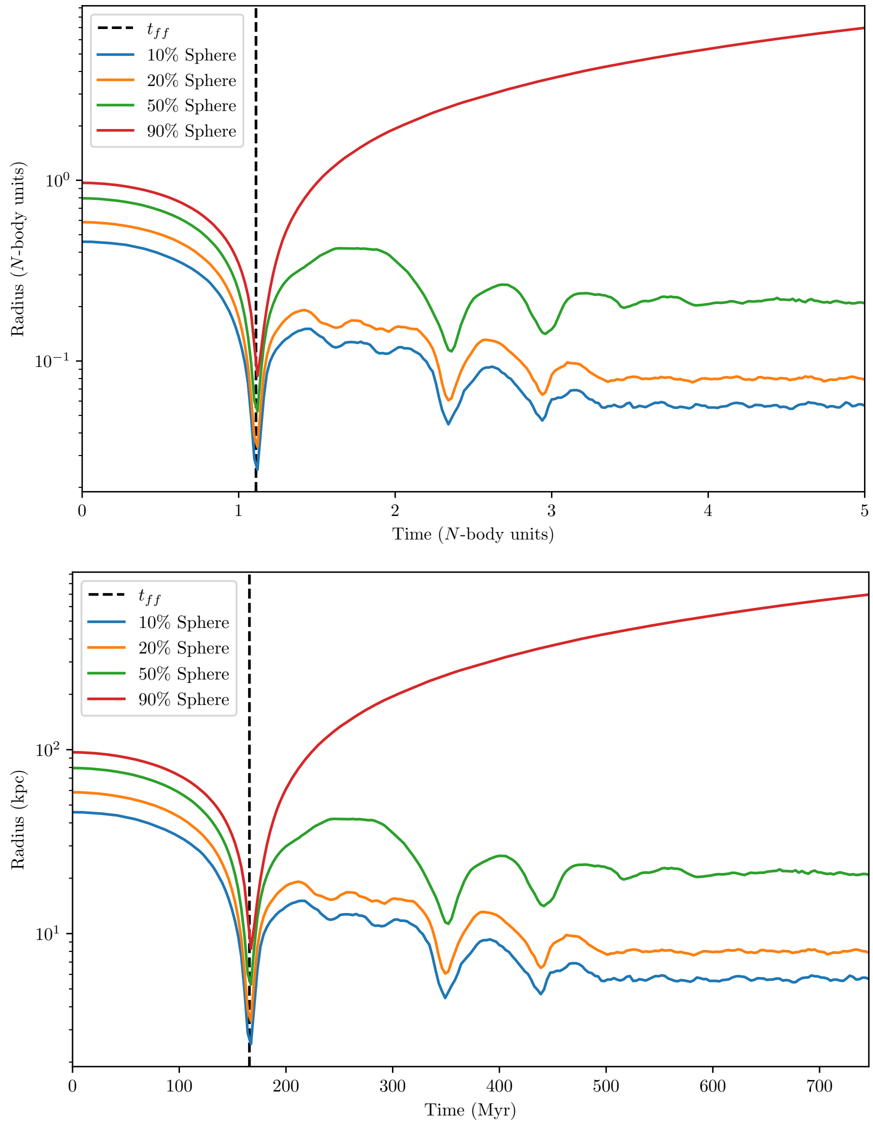

Figure. 1 *Top:* For a 'cold' collapse scenario of $N = 4096$ identical particles of mass $M = 1/N$ beginning in a uniformly distributed unit sphere with $v = 0.05v_E$, we see a characteristic collapse time of effectively the free-fall time. We plotted the 10, 20, 50, and 90% particle radii spheres over time, which shows that we have a small proportion of particles effectively escaping the system (90% line), where most particles sink deeper into the potential well where they reach an equilibrium after $T \sim 4$.
Bottom: Although computed in N-body units, we're able to convert the output into physically meaningful units for a Milky Way-type halo with $M = 10^{12}M_\odot$ and $R = 100$kpc. We see that the time to reach equilibrium is on the order of 600Myr, consistent with what we expect from cosmological models.

The exact same simulation could be converted from N-body units to 'real' units by specifying the mass and length scales in the system. For example, converting between masses and lengths is as simple as multiplying by the desired quantities. To convert from N-body time to real time units (i.e. seconds), we use the dimensional relation

$$[t] = \left[\sqrt{\frac{R^3}{MG}}\right] = \text{seconds} \qquad (4)$$

where R, M, and G are in their SI variants.
We present the particle distributions over the duration of the collapse for an N-body case and an analogous Milky-Way type halo system in Figure 1.

3 Galaxy Collisions

We now turn to the case of gravitationally interacting galaxies. Using the provided file `discgal.txt` (which contains initial positions and velocity vectors in 3D - the rotation of which is evident in this animation), we initiated two identical galaxies with center of mass (CoM) separations of ~ 30.4 N-body units, specifically with $\Delta x = 30$ and $\Delta y = 5$, keeping $\Delta z = 0$. For four separate simulations, we initiated a first simulation with the galaxies having a relative velocity of 4 times the escape velocity, $4v_e$, in the x-direction only (animation here), a second with $\Delta v = 2.25v_e$ (animation here), a third with $\Delta v = 1.25v_e$ (animation here), and a final simulation with $\Delta v = 0.5v_e$ (animation here).

The results of these simulations are shown in those above animations, and in Figures 2 and 3.

Based on these plots, we conclude that the merger result of two galaxies tends to be larger (by more than a factor of two) than each of the individual galaxies. Although we might expect that the merger would result

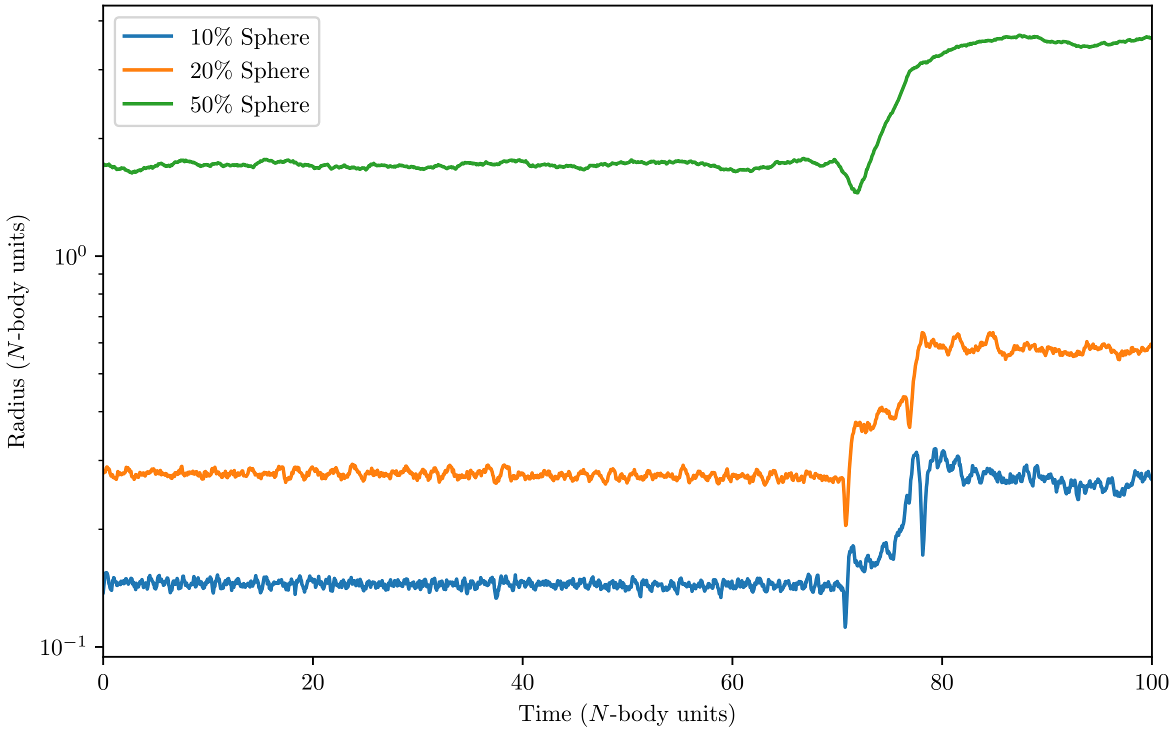

Figure. 2 For a galaxy collision between two equal mass galaxies with initial relative velocity of $\Delta v = 0.5v_e$, we see a clear 'collision' time at the region where their innermost particle sphere radii of *one* galaxy suddenly increases, here at about $T \sim 70$. After some time (roughly $\Delta T \sim 15$ for these initial conditions), the cores of these galaxies settle into a new equilibrium, while some particles far from the core escape the system as evident in the animations. We see that the 10 and 20% spheres are more quickly impacted by the merger than the 50% sphere due to their close proximity to the center of mass; a small change in the total center of mass (both in absolute mass and location) is relatively much larger for particles close to it than further away.

in a galaxy exactly twice as big, we rationalise that the extra size is a result of the injected energy into the individual system on the basis of their initial relative velocities. Since the merger result has essentially 0 net velocity, this energy has to go somewhere and naturally ends up going into increasing the radii of particle orbits.

We note that it seems as though an initial relative velocity between the two galaxies of just *over* v_e will still result in a merger. We can start to see this behaviour in the $v_i = 1.25v_e$ line in Figure 3 and in its animation, where many of the stars of one galaxy are captured by the other and the CoM motion is greatly perturbed. A logical explanation for why this is, despite at first seeming fallacious, lies in dynamical friction. In all interaction cases, we see large perturbations in the orbits of the stars, with many seemingly escaping from the system and others sinking deeper into the common potential. At low enough relative velocity (but still larger than v_e), we expect a majority of stars to be captured within the common gravitational potential of the two galaxies and eventually merge.

References

[1] Michael Y. Grudić and Alexander B. Gurvich. ""pytreegrav": A fast Python gravity solver". In: *Journal of Open Source Software* 6.68 (2021), p. 3675. DOI: `10.21105/joss.03675`. URL: `https://doi.org/10.21105/joss.03675`.

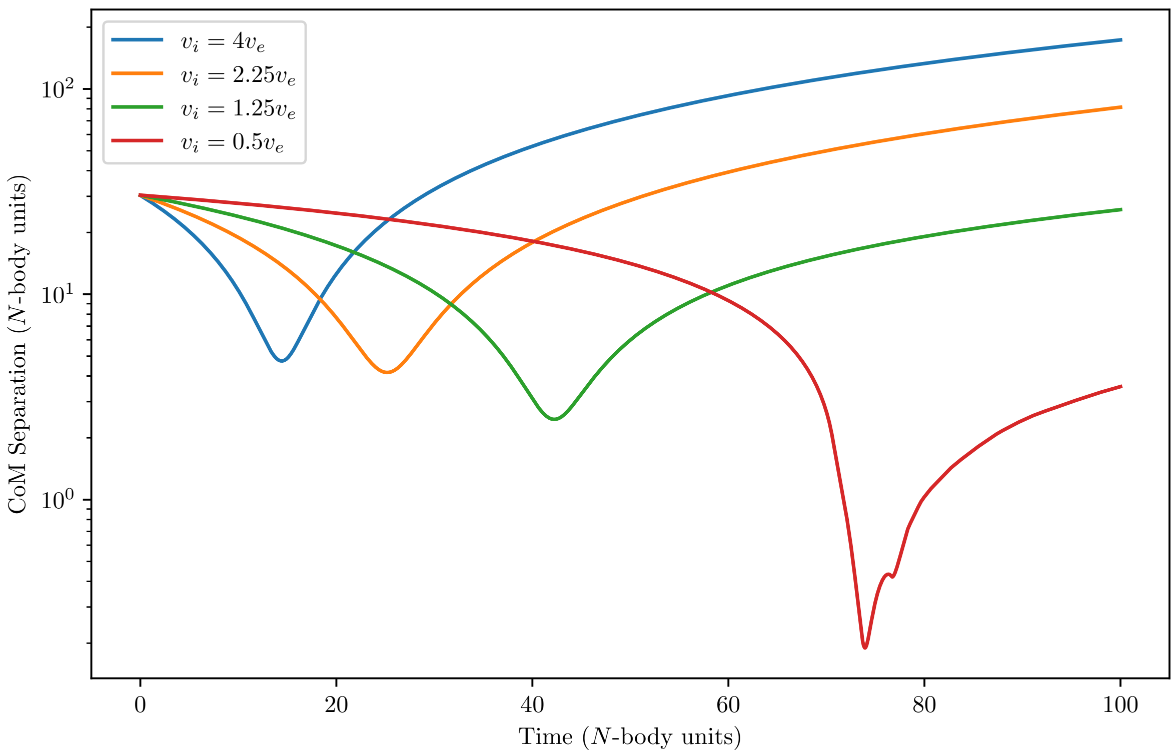

Figure. 3 We can clearly see the interaction effects of the two galaxies via their CoM separation profiles. For large initial velocities, we see a larger minimum CoM separation which steadily gets smaller for lower initial velocities. We posit that if the late-time separation is lower than the initial separation, then the two galaxies will eventually merge. On this basis, we clearly see a merger in the $\Delta v = 0.5v_e$ scenario, and a possible merger for the $1.25v_e$ relative velocity iteration. We note a steeper decline in the separation curve as the CoM gets less separated. Intuitively, most of the mass in the galaxies lie near to their respective CoM, and so a small separation in CoM propagates into a larger interaction effect as a result of the inverse square law.