PHYS3080 Final Exam

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1 Question 1

a. i. The math is shown below

Q1 a. $v = 200$ km/s

$R_I = 4$ kpc $R_o = 12$ kpc

$$\omega = \frac{v}{R}$$

$$\Rightarrow \omega_I = \frac{v}{R_I} = \frac{200 \text{ km/s}}{4 \text{ kpc}}$$

$$\simeq 1.618 \times 10^{-15} \text{ s}^{-1}$$

$$\simeq 0.05097 \text{ Myr}^{-1}$$

$$\omega_o = \frac{v}{R_o} = \frac{200 \text{ km/s}}{12 \text{ kpc}} = \frac{1}{3}\omega_I$$

$$\simeq 0.017 \text{ Myr}^{-1}$$

The inner edge will have gained 3 orbits on the outer edge when the diff is 6π

$$\Rightarrow 6\pi = (\omega_I - \omega_o)T$$

$$T = \frac{6\pi}{\left(\frac{2}{3}\omega_I\right)}$$

$$= \frac{6\pi}{\left(\frac{2}{3} \times 0.05097 \text{ Myr}^{-1}\right)}$$

$$\simeq 554.7 \text{ Myr}$$

ii. Given that spiral arms were naively calculated to exist for ≤ 554.7 Myr, and spiral galaxies (with well defined spirals) are observed after billions of years, there must be some other physical process at work.

iii. The Lin-Shu density wave theory solves the winding problem by proposing that the observed spiral arms are not coherent structures on their own (which would be subject to the calculations above) but that they are quasi-static density waves of matter that rotates about the galactic center. Stars and gas move through the galaxy independent of the spiral arms, often passing through the spiral arms. This means that, due to the elliptical nature of stellar orbits, the spiral arms represent an overdensity of matter in the galaxy. As stars and gas move in and out of the spiral structure throughout their orbits, the spiral arms remain with well-defined shape over astronomical timescales.

b. i. Dwarf galaxies may fall below the $S_{0.5}$ relation since they are characterised by a much smaller mass, and hence the constituent stars will have a lower rotational velocity about the galactic center. Since the $S_{0.5}$ relation relies on rotational velocity, this is a source of discrepancy.

ii. Math below:

Q1 b. ii. $M_* = 10^6 M_\odot$, $S_{0.5} = 10^{1.2}$

$$\log M_* = 1.15 - 0.4 M_i \Rightarrow M_i = \frac{1.15 - \log M_0}{0.4}$$

The actual magnitude is then

$$M_a = \frac{1.15 - \log(10^6)}{0.4} \simeq -12.1$$

The magnitude predicted by the $S_{0.5}$ mass is

$$M_s = \frac{1.15 - \log(10^{8.3})}{0.4} \simeq -17.9$$

$$\Rightarrow \Delta M = |M_a - M_s| = |-12.1 - (-17.9)|$$
$$= 5.8$$

as required.

iii. Math below:

iii. $$m - M = 5 \log_{10}\left(\frac{d}{10\text{pc}}\right)$$

$$\Rightarrow d = 10 \cdot 10^{\left(\frac{m-M}{5}\right)} \text{ pc}$$

with $m - M = \Delta M = 5.8$

$$d = 10 \cdot 10^{\left(\frac{5.8}{5}\right)} \text{ pc} \simeq 145 \text{ pc}$$

2 Question 2

a. Math and sketch below:

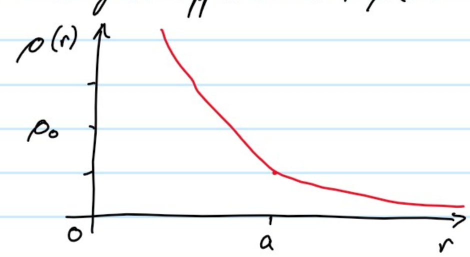
Q2 a. $$\rho(r) = \frac{\rho_0}{\frac{r}{a}\left(1 + \frac{r^2}{a^2}\right)}$$

when $r = a$, $\rho(r) = \frac{\rho_0}{2}$

when $r << a$, $\rho(r) \gg \rho_0$

$r >> a$, $\rho(r) \ll \rho_0$

To a first approximation, $\rho(r) \propto r^{-3}$



b. Math below:

b. $\rho = \frac{V}{M} \Rightarrow M = \frac{V}{\rho}$

$V \simeq \frac{4}{3}\pi r^3$ (volume of sphere).

$\Rightarrow M = \frac{4}{3}\pi r^3 \times \frac{\frac{r}{a}(1+\frac{r^2}{a^2})}{\rho_0}$

$= \frac{4\pi \frac{r^4}{a}(1+\frac{r^2}{a^2})}{3\rho_0}$

since ρ_0 is a constant, as $r \to \infty$, so does the mass (since the radius is on the numerator). Hence, the total mass is divergent.

So, the true dark matter density in galaxies must fall off for large radii. Perhaps introducing a maximum radius (dependent on scale-radius) could fix this.

c. Math below:

c. $V = \sqrt{\frac{G\,M(<R)}{R}}$

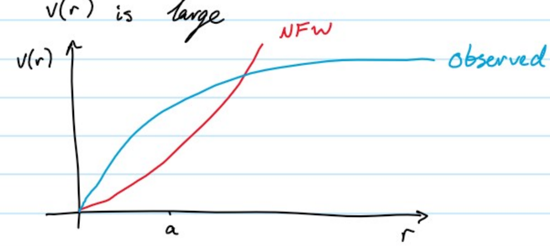
$v(r) = \sqrt{\frac{G}{r} \cdot \frac{4\pi r^4(1+\frac{r^2}{a^2})}{3a\rho_0}}$

$= \sqrt{\frac{4\pi G r^3(1+\frac{r^2}{a^2})}{3a\rho_0}}$ ①

$\therefore v(r) \sim r^{3/2}$

for $r \ll a$, the denominator dominates in ① and so $v(r)$ is small

for $r \gg a$, the numerator dominates, and $v(r)$ is large



The red-line shows the NFW profile, and the blue-line shows a rough approximation of observed profiles with dark matter halos.

d. Math below:

d. $\rho(r) = \dfrac{\rho_0}{\frac{r}{a}\left(1+\frac{r^2}{a^2}\right)}$

$a = 5\,\text{kpc}$, $\rho_0 = 1.5\ M_\odot/\text{pc}^3$, $r = 8.2\,\text{kpc}$

$$\Rightarrow \rho(8.2\,\text{kpc}) = \frac{1.5}{\frac{8.2}{5}\left(1+\left(\frac{8.2}{5}\right)^2\right)}\ M_\odot/\text{pc}^3$$

$$\simeq 0.25\ M_\odot/\text{pc}^3$$

Since $M_\odot = 1.98 \times 10^{30}$ kg
$\text{pc} = 3.09 \times 10^{16}$ m

$$\rho(8.2\,\text{kpc}) = 0.25 \times \frac{(1.98\times10^{30}\ \text{kg}/M_\odot)\,M_\odot}{(3.09\times10^{16}\ \text{m}/\text{pc})^3\ \text{pc}^3}$$

$$\simeq 1.67 \times 10^{-20}\ \text{kg}/\text{m}^3$$

$$\simeq 1.7 \times 10^{-20}\ \text{kg}/\text{m}^3 \quad \text{(2 sig fig)}$$

3 Question 3

Galaxies are characterised by their stellar populations, which are directly influenced by the merger histories of the host galaxy. There are a multitude of types of galaxy mergers which are characterised by the progenitor galaxies, their chemical composition, masses, velocities and the list goes on. Due to the immense mass of a galaxy, all galaxies in the vicinity will be subject to a significant gravitational attraction which is the underlying cause of such mergers.

The nature and outcome of mergers are directly related to the mass of the two galaxies merging, of which there are two *main* classes of mergers: minor mergers and major mergers. The former describes galaxy mergers where one galaxy typically has less than a third of the more massive galaxy's mass, and are comparatively peaceful in comparison to the major mergers, where the two progenitor galaxies will have a comparable mass. Main examples of the so-called minor mergers are where small dwarf galaxies (perhaps dwarf ellipticals, irregular galaxies, or large globular clusters) are subject to tidal stripping by a large elliptical or spiral galaxy. Such mergers would generally be referred to as *dry*, where the constituent gas of each galaxy has a minor role in the merger and would not be significantly affected.
In the case of major mergers, this would usually occur between spiral or elliptical galaxies or a combination thereof. Especially in the context of two merging spiral galaxies, the interacting gas between them would constitute a *wet* merger. The gas and dust lanes of each galaxy would collide, and the friction between them would effectively dynamically cool them enough so that rapid and violent star formation might form and consume a significant fraction of the galaxies' constituent gas. This may not happen immediately, though, since due to the aforementioned gravitational attraction over time, the two interacting galaxies would typically have extremely large relative velocities.
After a first collision between them, *dynamical friction* becomes significant. This is when stars and gas from one galaxy experience a significant gravitational attraction to the mass of the other galaxy and hence the relative velocity magnitudes between galaxies is reduced. After some time, this velocity reduces to such a point where the galaxies no longer are moving away from each other and move together once more. On subsequent passes/collisions, this velocity will reduce to such a point where the galaxies effectively merge into one. All the while, significant star formation takes place from the aforementioned interacting gas.

This process is the most-accepted cause of the formation of elliptical galaxies - galaxies which are characterised by an almost random and chaotic stellar rotation profile. Since the two merging galaxies almost certainly would not have aligning angular momentum vectors in their disks (due to the more-or-less random orientation of different galaxies), the galaxy's collision will result in the addition of their angular momentum vectors and hence a likely reduction in order - the stars dynamically heat up and orbit in random orientations about the galactic center (often in a spheroidal distribution). Because of this, gas will not interact steadily anymore and star formation practically ceases. What remains is a stellar population characterised by a uniform age. As time goes on, the most massive blue stars die which leaves an older, redder population of stars with non-uniform rotation.

The classic Hubble tuning fork diagram classifies galaxy types nicely, although doesn't provide an accurate depiction of the galaxy-type hierarchy. Since elliptical galaxies are the product of mergers (at least non-dwarf ellipticals are), they are better described by the ATLAS3D comb diagram which takes into account actual physical parameters of galaxies [rotation, colour, ellipticity, and gas fraction] rather than an idealised (and outdated) formation process.

4 Question 4

a. The w_i in the Friedmann equation represents the equation of state of different parameters that impact cosmological expansion. A more negative equation of state effectively corresponds to a higher contribution to the expansion of the universe. For some parameter Ω_i, the equation of state for this parameter is given by the ratio of its pressure to density contribution:

$$w_i \equiv \frac{p_i}{\rho_i c^2}$$

b. The Ω_i in Friedmann's equation correspond to the contribution of each type of component (mass, radiation, etc) on the expansion of the universe. The critical density is given such that the sum of all of the parameter densities sum to 1.

c. The three main Ω_i that appear in the Friedmann equation are:

- Radiation density: Ω_r, which dictates the contribution of relativistic energy (photons, neutrinos, etc). The equation of state for radiation is $w_r = 1/3$.
- Matter density: Ω_m, which dictates the contribution of non-relativistic matter (i.e. baryons and cold dark matter). The equation of state for matter is $w_m = 0$.
- Dark energy (or cosmological constant density): Ω_Λ, which dictates the contribution of dark energy on the expansion of the universe. The equation of state for the cosmological constant is $w_\Lambda = -1$.

d. Math below:

Q4 d. $\Omega_0 = 1$, $\Omega_m = 0.3 \times \Omega_0$
$= 0.3$

Flat universe $\Rightarrow \sum \Omega_i = 1$
$\Rightarrow \Omega_\Lambda = 0.7$

$$H^2(a) = H_0^2 \sum \Omega_i a^{-3(1+w_i)}$$
$$= H_0^2 \left(\Omega_m a^{-3} + \Omega_\Lambda\right)$$
$$= H_0^2 \left(0.3 a^{-3} + 0.7\right)$$

$$a = \frac{1}{1+z} \Rightarrow a^{-3} = (1+z)^3$$

$$\Rightarrow H^2(z) = H_0^2 \left(0.3(1+z)^3 + 0.7\right)$$

$$H(z) = H_0 \sqrt{0.3(1+z)^3 + 0.7}$$

e. Math below:

e. Flat universe $\Rightarrow \sum \Omega_i = 1$
Matter dominated $\Rightarrow$ all other parameters $\simeq 0$
$\Rightarrow \Omega_m = 1$

$$H^2(a) = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \sum \Omega_i a^{-3(1+w_i)}$$

$$\Rightarrow \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 a^{-3(1+0)} = H_0^2 a^{-3}$$

$$\Rightarrow \frac{\dot{a}}{a} = H_0 a^{-\frac{3}{2}}$$
$$\dot{a} = H_0 a^{-1/2}$$

but $\dot{a} = \frac{da}{dt}$, and so

$$\frac{da}{dt} = H_0 a^{-1/2}$$
$$\Rightarrow \int_0^a a^{1/2} da = H_0 \int_0^t dt$$

$$\frac{2}{3} \cdot a^{\frac{3}{2}} = H_0 t$$

When $t = t_0$, $a = 1$, so

$$\frac{2}{3} \cdot 1 = H_0 t_0 \Rightarrow t_0 = \frac{2}{3H_0}$$

At half of its present age, $t = \frac{t_0}{2} = \frac{1}{3H_0}$

$$\Rightarrow \frac{2}{3} a^{\frac{3}{2}} = H_0 \cdot \frac{1}{3H_0}$$

$$a^{\frac{3}{2}} = \frac{3}{2} \cdot \frac{1}{3}$$

$$\Rightarrow a = \left(\frac{1}{2}\right)^{\frac{2}{3}} \simeq 0.63$$

$\therefore$ universe was 63% of its current size at half of its age.

5 Question 5

a. Math below:

Q5 a. Since $z \sim 3600 > 1090$, we can use the CMB temperature relation:

$$T_{then} = T_{now}(1+z)$$
$$= 2.725(1+3600)\ K$$
$$\simeq 9813\ K$$

b. Math below:

b. $\epsilon = k_B T$, $T = 9813\ K$

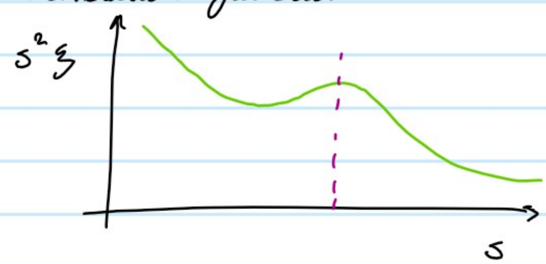
$$\Rightarrow \epsilon = 8.62 \times 10^{-5} \left(\frac{eV}{K}\right) \times 9813\ K$$
$$\simeq 0.85\ eV$$

c. The rapid expansion of the universe during inflation solves the horizon and flatness problems, since the rapid expansion essentially flattens out any initial curvature that the universe had (especially over large distance scales). The whole universe was in causal contact prior to the inflationary period, and so the rapid expansion meant that the nearly perfect isotropy of the CMB can be explained by causal processes that occured before expansion.

d. Graphs below:

d. Power spectrum:

Correlation function:



Peak gives length scale of acoustic oscillations
$\Rightarrow$ standard ruler for large scale universe structure.

e. The curvature of the early universe can be inferred from the position of each of the peaks in the CMB power spectrum. A universe with higher curvature would shift the peaks further right, and lower curvature would shift the peaks further left. Where the peaks are indicate practically 0 curvature (i.e. a flat universe).