PHYS3080 Problem Set 1

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Question 1

a. The absolute magnitude can be found from the apparent magnitude and the distance to Leo I by

$$m - M = 5\log_{10}\left(\frac{d}{10\text{pc}}\right)$$

Substituting in the given values gives:

$$\begin{aligned}\Rightarrow M &= m - 5\log_{10}\left(\frac{d}{10\text{pc}}\right)\\ &= 11.2 - 5\log_{10}\left(\frac{250\times 10^3\text{pc}}{10\text{pc}}\right)\\ &\simeq -10.79 \text{ mag}\end{aligned}$$

The luminosity (in solar units) can then be calculated by:

$$\begin{aligned}L &= 10^{-0.4(M-M_\odot)}L_\odot\\ &= 10^{-0.4(-10.79-4.83)}L_\odot\\ &\simeq 1.77\times 10^6 L_\odot\end{aligned}$$

And so Leo I has an absolute magnitude of -10.79 with an absolute luminosity of 1.77 million solar luminosities.

b. The absolute half-life radius (as opposed to the angular radius) of Leo I can be found by:

$$\begin{aligned}\tan(r_h(\theta)) &= \frac{r_h}{d}\\ \Rightarrow r_h &= d\tan(r_h(\theta))\\ &= 250\times 10^3 \times \tan\left(4\text{arcmin}\times\frac{\pi}{60\times 180}\right)\text{ pc}\\ &\simeq 290.9\text{pc}\end{aligned}$$

Then, the approximate mass of the dwarf galaxy can be found by

$$M = \frac{3\pi r_h \sigma^2}{2G} \tag{1}$$

Substituting in known values into equation 1 then gives

$$\begin{aligned}M &= \frac{3\pi\times(290.9\text{pc})\times(8.8\text{km/s})^2}{2\times 0.0043\left(\text{pc km}^2\text{ sec}^{-2}\ M_\odot^{-1}\right)}\\ &\simeq 2.469\times 10^7 M_\odot\end{aligned}$$

And so the calculated mass of Leo I is approximately $2.469\times 10^7 M_\odot$.

c. The mass-to-light ratio for Leo I is

$$\frac{M}{L} = \frac{2.469 \times 10^7 M_\odot}{1.77 \times 10^6 L_\odot} \simeq 13.95 \frac{M_\odot}{L_\odot}$$

This implies that this dwarf galaxy *could* be comprised entirely of stars for a couple of key reasons, although it is unlikely.

Firstly, the luminosity of stars increases exponentially with linearly increasing mass, and so a mass-to-light ratio above 1 suggests that the matter in Leo 1 is, at least on average, less luminous than the Sun. Assuming that this mass is totally stellar mass, this would imply that the bulk of the dwarf galaxy's stars must be less massive (and consequently much less luminous) than the Sun to account for this luminosity deficiency.

Now, notice that the mass-to-light ratio of Leo I (13.95 in solar units) is significantly higher than that of both the bulge and disk of the Milky Way (approx. 5 and 3 solar units respectively). A key distinction is to be made, however, in that Leo I is not a spiral galaxy and would tend to have a higher mass-to-light ratio being an elliptical galaxy (even though a dwarf). Schneider quotes dSph galaxies (to which Leo I belongs) as having mass to light ratios in the range of 5-100, and so Leo I being more luminous than one might expect for a dSph indicates that it very well could be predominately stellar matter that makes up it's mass profile.

Question 2

a. i. In the visible spectrum, the dust lanes and the brightest (and most massive) OB stars dominate the appearance of the spiral arms. This is mainly due to the interstellar extinction with the former, which hides the light behind the gas. These gas/dust lanes also are primarily responsible for star formation in the disk of the galaxy. Since these gas/dust lanes are at the forefront of the spiral arms, they appear to be the leading component of each arm and (when viewing the sum of all of the components of the spiral arms) can indicate the direction of rotational motion of the galactic disk.

ii. Ultraviolet imaging of spiral arms reveals the massive, newly formed stars that were created as a result of the spiral arm local gas overdensity that lead to the Jean's collapse of gas clouds. These massive stars are by far the most prominent feature of the spiral arms in the ultraviolet spectrum, and they trail directly behind the gas and dust clouds that are prominent in the visible spectrum. This distance between the bright HII regions (and the massive OB stars) to the dust lanes is due to the time it takes for the stars to form, since it's not an instantaneous process.

iii. Infra-red imaging immediately reveals the older stars formed by the overdensities of the leading gas/dust lanes, which have survived longer than the massive OB stars. Since these stars are older than their more massive counterparts, they appear further away from the leading dust lanes since they have experienced more time since formation and have trailed off from the spiral arm density wave.

b. i. The Tully-Fisher relation relies on the assumption that the mass-to-light ratio of spiral galaxies is constant. It also assumes that the ratio of baryons to dark matter is a constant, as are the proportions of star types across all spiral galaxies. This doesn't account for the mass of the (non-luminous) gas in a galaxy.

ii. By comparing the left and right panels in Fig 3.29, one can assume that dwarf galaxies contain a significant fraction of their mass in the form of gas and dust. Since the luminosity of a galaxy is multiplied by some mass-to-luminosity ratio for spiral galaxies to obtain the (stellar) mass of said galaxy, the dwarf galaxies had their mass severely underrepresented since the M/L ratio did not account for gas mass (visualised by the sagging of the green points in the left panel).

Once the gas mass is accounted for (by the strength of the 21cm line and molecular emission), the masses of the dwarf galaxies were shifted up (seen in the right panel) which then strengthened the fit of the Baryonic Tully-Fisher relation.

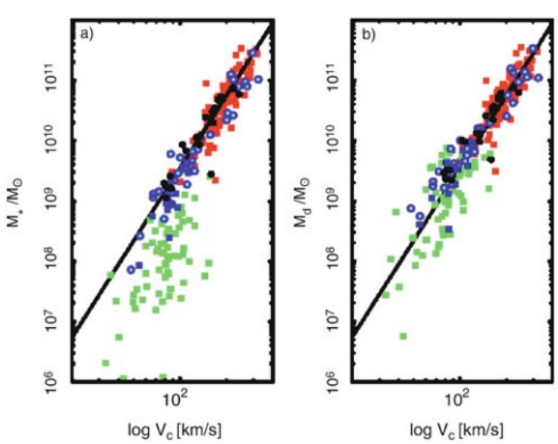

Fig. 3.29 Tully-Fisher Relation (left) vs Baryonic Tully-Fisher Relation (right)

c. i. To start with, Seyfert galaxies are defined as being spiral galaxies with extremely luminous nuclei and so they, at least in principle, could be found on the Tully-Fisher relation. In practice, matching a host-galaxy of an AGN to a scaling relation could prove difficult on account of the extreme luminosity due to the AGN.
Similarly to Seyfert Galaxies, Radio Galaxies are defined as elliptical galaxies with an active nucleus. As such, Radio Galaxies could be found on the fundamental plane or the Faber-Jackson relation.

The remaining AGN classes (namely LINERs, Quasars, Blazars, etc) are not restricted to a type of galaxy in particular and are only differentiated by their spectrum and luminosity, defined by the angle of inclination of the line-of-sight of the observer to the central SMBH in the host-galaxy. As such, they cannot be routinely placed on any one scaling-relation with certainty without more knowledge of the AGN's host galaxy.

ii. With an increasing brightness of AGN, it becomes increasingly inaccurate to estimate the distance to said AGN using the aforementioned scaling relations (at least in the visible, ultraviolet and radio spectrums). This is because the scaling relations rely *heavily* on an assumed constant mass-to-luminosity ratio for which AGNs would break on account of their extremely high luminosity. Perhaps the scaling relations for AGNs would be more accurate in wavelengths for which the AGNs aren't particularly luminous in, so that the luminosity of the galaxy as a whole could be estimated without contamination from the bright galactic nucleus.

d. First, assuming that the Virgo cluster is gravitationally bound, obeys the virial theorem, and that the velocity dispersion of its members is approximately equal to that of it's radial velocity dispersion, the total mass of the cluster is found by

$$M = \frac{3\pi R_G \sigma_v^2}{2G} \tag{2}$$

However, the value R_G must be found as an absolute distance as opposed to an apparent angular extent in the sky. To do this, employ the same method using the tan function as in Q1b:

$$R_G = d \times \tan(R_G(\theta)) = 16.5\text{Mpc} \times \tan\left(6.5 \times \frac{\pi}{180}\right) \approx 1.88\text{Mpc}$$

Substituting this, as well as all other given constants, into equation 2 gives:

$$M = \frac{3\pi R_G \sigma_v^2}{2G} = \frac{3\pi(1.88\text{Mpc})(638\text{km/s})}{2 \times 0.0043\left(\text{pc km}^2 \text{ sec}^{-2}\ M_\odot^{-1}\right)} \simeq 8.386 \times 10^{14} M_\odot$$

This value corresponds to the *total* mass of the gravitational potential of the cluster, not just the luminous matter present within the galaxies that make up the cluster.