PHYS3051 Problem Sheet

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Question 1

i. For the strong interaction to occur in a particle decay, we need conservation of strangeness (as opposed to decays mediated by the weak interaction) among the usual conservation of charge, baryon number and mass (after accounting for kinetic energy in the products). Since the Ξ^{*-} particle has strangeness $s = -2$, charge $q = -1$ and baryon number $B = 1$, the decay products need to sum to these values too. Further, the heavy Ξ^{*-} has a mass of 1535 and so the decay products must be less than or equal to this value to be kinematically valid. With these constraints, we note the valid decay products are

$$\Xi^{*-} \longrightarrow \Xi^{-} + \pi^{0} \tag{1}$$
$$\Xi^{*-} \longrightarrow \Xi^{0} + \pi^{-} \tag{2}$$

While it should be clear that the strangeness, baryon number and charge are conserved in each strong decay mode, we have a total mass of 1456 and 1455 for the products in equations (1) and (2) respectively (which is less than the initial mass of 1535 in each case).
All other possible decay modes within the baryon decuplet violate at least one of the conservation principles and hence there can only be two strong decay modes with the information given.

ii. The quark flow diagrams for each process are shown below.

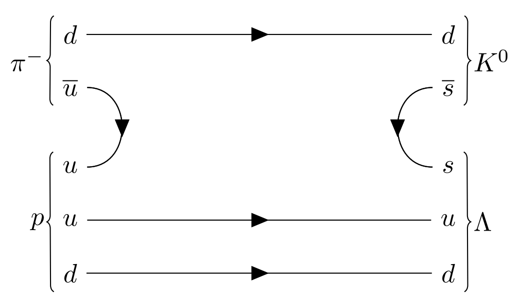

Figure 1: $\pi^- + p \longrightarrow K^0 + \Lambda$ interaction, $\Delta m = 536$ MeV$/c^2$

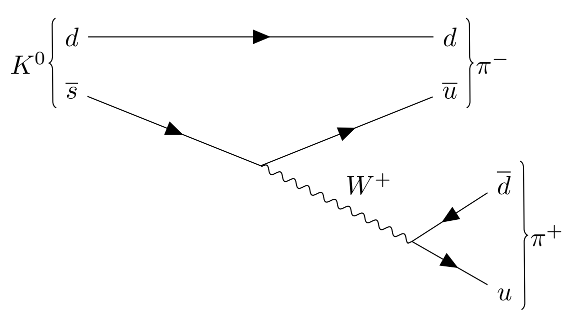

Figure 2: $K^0 \longrightarrow \pi^- + \pi^+$ interaction, $\Delta m = -218$ MeV$/c^2$

We would expect the top interaction to occur quicker, as it can proceed entirely via the strong interaction. The bottom interaction relies on the weak force, which has a characteristic time scale of several fewer orders of magnitude than the strong interaction.

Question 2

The cited article by Aaltonen *et al* details the revised mass of the W boson using recent techniques on existing data. The authors claim a mass of $M_W = 80,433.5 \pm 9.4$ MeV/c^2 as opposed to the standard model estimate of $M_W = 80,357 \pm 6$ MeV/c^2 and previous experimental best value of $M_W = 80,385 \pm 15$ MeV/c^2. This most recent mass that the authors devised used data from the now retired Fermilab Tevatron particle collider with data collected from 2002-2011, where the developments in computational techniques and physics since that time allowed for a more nuanced calculation.

This result of $M_W = 80,433.5 \pm 9.4$ MeV/c^2 is suggesting very real deviation in experimental results from the established theory of the standard model. The quoted uncertainty is more than 7σ outside of the accepted theory and suggests that the W boson is significantly heavier than the standard model predicts. If this result holds true, existing tweaks to the standard model may be supported to explain this discrepancy from theory.

So far, multiple extensions to the standard model have been proposed which *could* explain the apparent massive behaviour of the W boson. The linked Nature article suggests two favoured existing explanations which could fill this niche. One is the hypothesised supersymmetry in which every SM particle has a more massive twin, where these particles could spontaneously appear and annihilate in the region surrounding the W particle, affecting its apparent mass at the time of observation. Another favoured explanation is the Higgs particle – which effectively gives matter mass – bearing different properties to those theorised. Given that the discovery of the Higgs is only recent, there stands to be much more analysis done in the coming years with more sophisticated experiments that could shed light on its properties.

Question 3

We have that $U^{\mu\nu}$ is symmetric and $V^{\mu\nu}$ is anti-symmetric.

i. We want to show that the symmetry properties of $U^{\mu\nu}$ and $V^{\mu\nu}$ apply to $U_{\mu\nu}$ and $V_{\mu\nu}$. Starting with U, we have that

$$U_{\mu\nu} = g_{\mu\alpha}g_{\nu\beta}U^{\alpha\beta}$$

But $U^{\alpha\beta} = U^{\beta\alpha}$, so

$$\begin{aligned} U_{\mu\nu} &= g_{\mu\alpha}g_{\nu\beta}U^{\beta\alpha} \\ &= g_{\nu\beta}g_{\mu\alpha}U^{\beta\alpha} \\ &= U_{\nu\mu} \end{aligned}$$

And so $U_{\mu\nu}$ is symmetric as expected.
As above,

$$V_{\mu\nu} = g_{\mu\alpha}g_{\nu\beta}V^{\alpha\beta}$$

but $V^{\alpha\beta} = -V^{\beta\alpha}$, so

$$\begin{aligned} V_{\mu\nu} &= g_{\mu\alpha}g_{\nu\beta}(-V^{\beta\alpha}) \\ &= -g_{\nu\beta}g_{\mu\alpha}V^{\beta\alpha} \\ &= -V_{\nu\mu} \end{aligned}$$

And so $V_{\mu\nu}$ is anti-symmetric as expected.

ii. We want to show that $U^{\mu\nu}V_{\mu\nu} = 0$. Due to the anti-symmetry of $V_{\mu\nu}$, we have that

$$U^{\mu\nu}V_{\mu\nu} = -U^{\mu\nu}V_{\nu\mu}$$

but we also have symmetry in $U^{\mu\nu}$, so

$$U^{\mu\nu}V_{\mu\nu} = -U^{\nu\mu}V_{\nu\mu}$$

Since this is a contraction, we can relabel the right hand side $\nu \leftrightarrow \mu$, giving

$$U^{\mu\nu}V_{\mu\nu} = -U^{\mu\nu}V_{\mu\nu}$$

where the equality is only true if $U^{\mu\nu}V_{\mu\nu} = 0$ as desired.

iii. We have the second-rank arbitrary tensor

$$T^{\mu\nu} = U^{\mu\nu} + V^{\mu\nu}$$

Relabelling $\nu \leftrightarrow \mu$, we get

$$T^{\nu\mu} = U^{\nu\mu} + V^{\nu\mu}$$

Due to the symmetry properties of $U^{\mu\nu}$ and $V^{\mu\nu}$, we also have that

$$T^{\nu\mu} = U^{\mu\nu} - V^{\mu\nu}$$

Hence

$$\begin{aligned} T^{\mu\nu} + T^{\nu\mu} &= U^{\mu\nu} + V^{\mu\nu} + U^{\mu\nu} - V^{\mu\nu} \\ &= 2U^{\mu\nu} \\ \implies U^{\mu\nu} &= \frac{1}{2}\left(T^{\mu\nu} + T^{\nu\mu}\right) \end{aligned}$$

which is the form of $U^{\mu\nu}$ purely in terms of $T^{\mu\nu}$. If we subtract the two tensors instead of adding, we get

$$\begin{aligned} T^{\mu\nu} - T^{\nu\mu} &= U^{\mu\nu} + V^{\mu\nu} - U^{\mu\nu} + V^{\mu\nu} \\ &= 2V^{\mu\nu} \\ \implies V^{\mu\nu} &= \frac{1}{2}\left(T^{\mu\nu} - T^{\nu\mu}\right) \end{aligned}$$

which is the form of $V^{\mu\nu}$ purely in terms of $T^{\mu\nu}$.

Question 4

a. Tyndall scattering refers to the scattering of light by particles that are of similar size to (or are just larger than) the wavelengths in the visible spectrum. Similar to Rayleigh scattering, the intensity of the scattered light is inversely dependent on the fourth power of wavelength of the light and so blue light is scattered more readily than redder light. Typically, this scattering occurs via the interaction of light with *suspended* particles in an otherwise approximately transparent medium [1]. That is, we could expect to see Tyndall scattering via transmission of light through a glass of a water-flour mixture (where the flour particles are suspended in the water), or in an opalescent material (which usually appear blue with red light passing through).
Contrary to Tyndall scattering, Rayleigh scattering requires the scattering particles to be much smaller than the incident light, and so Tyndall scattering is typically much more intense over the same length scale than Rayleigh scattering.

b. Since the charged particle (with charge q) in question is a harmonic oscillator, we have the usual equation of motion

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

The equation of motion is subject to the electric field from the incident light, and so

$$\frac{d^2x}{dt^2} + \omega^2 x = \frac{q}{m}\mathbf{E}_{\text{inc}}$$

Since the electric dipole moment is given by

$$\mathbf{d} = q\mathbf{r}$$

and we can assume that $\mathbf{r} = x \implies \ddot{\mathbf{r}} = \ddot{x}$, we have that

$$\begin{aligned} \ddot{\mathbf{d}} &= \frac{q^2}{m}\mathbf{E}_{\text{inc}} - q\omega^2 x \\ \implies \mathbf{d} &= \frac{q^2}{m(\omega^2 - \omega_e^2)}\mathbf{E}_{\text{inc}} \end{aligned}$$

where ω is the oscillation from the harmonic motion, ω_e is the oscillation as a part of the scattering process, and $\mathbf{E}_{\text{inc}} = \hat{\mathbf{e}}_0 E_0 \exp\left(-i\omega_e t\right)$ (assuming that the field acting on the charge is the same as at the origin and that the displacement of the charge is negligible over the oscillation during vibration).
We have from the lectures that

$$\frac{d\sigma}{d\Omega} = \frac{\langle dP/d\Omega \rangle}{|\langle S_{\text{inc}} \rangle|}$$

where

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{\omega_e^4}{32\pi^4 c^3 \epsilon_0} |\mathbf{d} \times \mathbf{k}|^2$$
$$= \frac{\omega_e^4}{32\pi^4 c^3 \epsilon_0} \cdot \left| \frac{q^2}{m} \frac{E_0 e^{-i\omega_e t}}{(\omega^2 - \omega_e^2)} \right|^2 \cdot |\mathbf{e} \times \mathbf{k}|^2$$
$$= \frac{\omega_e^4 q^4 E_0^2}{32 m^2 \pi^4 c^3 \epsilon_0 (\omega^2 - \omega_e^2)^2} \cdot |\mathbf{e} \times \mathbf{k}|^2$$

and $|\langle S_{\text{inc}} \rangle| = \epsilon_0 c E_0^2/2$, so

$$\frac{d\sigma}{d\Omega} = \frac{\omega_e^4 q^4}{16 m^2 \pi^4 c^4 \epsilon_0^2 (\omega^2 - \omega_e^2)^2} \cdot |\mathbf{e} \times \mathbf{k}|^2$$

For a linearly polarised wave, $|\mathbf{e} \times \mathbf{k}|^2 = \sin^2 \theta$ where θ is the angle between the incident and scattered wave vectors. Finally we have that

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0} \right)^2 \left(\frac{q^2}{mc^2} \right)^2 \left(\frac{\omega^2}{\omega^2 - \omega_e^2} \right)^2 \sin^2 \theta$$

This assumes that the harmonic oscillations is in the same direction as the oscillation due to the incident electric field. If this weren't the case, we'd expect to have another $\sin^2 \varphi$ term attached where φ is dependent on the misalignment of the axes of oscillation.

References

[1] E. O. Kraemer and S. T. Dexter. "The Light-Scattering Capacity (Tyndall Effect) and Colloidal Behavior of Gelatine Sols and Gels". In: *The Journal of Physical Chemistry* 31.5 (May 1927), pp. 764–782. ISSN: 0092-7325. DOI: 10.1021/j150275a014. URL: https://doi.org/10.1021/j150275a014.