MATH3202 Assignment 3

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Formulation

The mathematical formulation for our model pertaining to the *Pacific Paradise Gas* client is listed below. We identify the relevant data to the optimisation problem, and the stages/states/actions that provide the optimal solution to the profit maximisation problem. For the sake of this problem, we seek the optimal solution given by the value function $V_0(0, 0, 0)$ as defined below.

Data

d_t	demand for no. of gas cylinders on day t
hd_t	high demand for no. of gas cylinders on day t
LD_t	demand for no. of large gas cylinders on day t
p	probability of high demand of gas cylinders on any given day
r_n	sale price (revenue) of a normal 45kg gas cylinder
r_L	sale price (revenue) of a large 90kg gas cylinder
cap_n	capacity for overnight 45kg cylinder storage
cap_L	capacity for overnight 90kg cylinder storage
BDC	base delivery cost
NDC	normal (45kg) cylinder delivery cost
LDC	large (90kg) cylinder delivery cost
Dcap	maximum weight of cylinders available for transport on delivery truck
M_n	mass of a normal cylinder (45kg)
M_L	mass of a large cylinder (90kg)

Stages

Days $\quad t \in \{0, \ldots, 13\}$

States

s_t	number of stored 45kg cylinders on day t
L_t	number of stored 90kg cylinders on day t

Actions

n_t	number of 45kg cylinders to order/deliver on day t
N_t	number of 90kg cylinders to order/deliver on day t

Value Function

$$\begin{aligned} V_t(t, s_t, L_t) &= \text{expected maximum profit if we start day } t \text{ with } s_t \text{ 45kg and } L_t \text{ 90kg cylinders currently in storage} \\ &= \max_{\substack{0 \leq n_t \leq \max_{n_t} \\ 0 \leq N_t \leq \max_{N_t}}} \{\text{profit}(t, s_t, L_t, n_t, N_t) + pV_{t+1}\left(t+1,\ \text{high\_step}(t, s_t, L_t, n_t, N_t),\ \text{large\_step}(t, L_t, N_t)\right) + \\ &\qquad + (1-p)V_{t+1}\left(t+1,\ \text{small\_step}(t, s_t, L_t, n_t, N_t),\ \text{large\_step}(t, L_t, N_t)\right)\} \end{aligned}$$

Base Case

$$V_{14}(14, s_t, L_t) = 0$$

Supplementary Functions

$\max_{n_t}$	$\min\{\text{cap}_n - s_t + hd_t + 2 \times \text{deficit}(t, L_t, N_t),\ \text{floor}((\text{Dcap} - M_L \times N_t)/M_n)\}$
$\max_{N_t}$	$\min\{\text{cap}_L - L_t + LD_t,\ \text{floor}(\text{Dcap}/M_L)\}$
$\text{deficit}(t, L_t, N_t)$	deficit function for how many large cylinders we're missing to meet (mandatory) demand $\text{deficit}(t, L_t, N_t) = \max\{0, LD_t - L_t - N_t\}$
$DC(n_t,\ N_t)$	delivery cost function for delivering n_t 45kg and N_t 90kg cylinders: $DC(n_t, N_t) = BDC + NDC \times n_t + LDC \times N_t$
$\text{profit}(t, s_t, L_t, n_t, N_T)$	profit for day t given the current storage (s_t, L_t) and the delivery (n_t, N_t) $\text{profit}(t, s_t, L_t, n_t, N_t) = (p \times \min\{hd_t,\ s_t + n_t - 2 \times \text{deficit}(t, L_t, N_t)\}+$ $+(1-p)\min\{d_t,\ s_t + n_t - 2 \times \text{deficit}(t, L_t, N_t)\}) \times r_n +$ $+LD_t \times r_L - \min\{n_t + N_t, 1\} \times DC(n_t, N_t)$
$\text{small\_step}(t, s_t, L_t, n_t, N_t)$	calculates the next 45kg inventory amount given the current inventory and the sales $\text{small\_step}(t, s_t, L_t, n_t, N_t) = s_t + n_t - 2 \times \text{deficit}(t, L_t, N_t)-$ $-\min\{d_t, s_t + n_t - 2 \times \text{deficit}(t, L_t, N_t)\}$
$\text{high\_step}(t, s_t, L_t, n_t, N_t)$	calculates the next 45kg inventory amount given the current inventory and the high sales $\text{high\_step}(t, s_t, L_t, n_t, N_t) = s_t + n_t - 2 \times \text{deficit}(t, L_t, N_t)-$ $-\min\{hd_t, s_t + n_t - 2 \times \text{deficit}(t, L_t, N_t)\}$
$\text{large\_step}(t, L_t, N_t)$	calculates the next 90kg inventory amount given the current inventory and the sales $\text{large\_step}(t, L_t, N_t) = L_t + N_t + \text{deficit}(t, L_t, N_t) - LD_t$

Python Implementation

The model, programmed utilising a recursive value function, is available in the file attached with this document. Each (outdated) value function is given in this file, albeit without using many of the supplementary functions listed above.