Exam information	
Course code and name	MATH2100 Applied Mathematical Analysis
Semester	Semester 2, 2020
Exam type	Online, non-invigilated
Exam date and time	Please refer to your personalised timetable
Exam duration	You have a 12-hour window in which you must complete your exam. You can access and submit your exam at any time within the 12 hours. Even though you have the entire 12 hours to complete and submit your exam, the expectation is that it will take most students between 2 and 2.5 hours.
Reading time	Reading time has not been formally allocated for online exams, however students are encouraged to review and plan their approach for the exam before they start. The total exam time should be sufficient to do this.
Exam window	You must commence your exam during the time listed in your personalised timetable. The exam will remain open **only** for the duration of the exam.
Weighting	This exam is weighted at 52% of your total mark for this course.
Permitted materials	This is an open book exam – all materials permitted. You may use Wolfram Alpha or Mathematica for calculating integrals.
Instructions	Answer all questions You can print the exam and write in the exam paper, or write your answers on blank paper (clearly label your solutions so that it is clear which problem it is a solution to), or annotate an electronic file on a suitable device. You must submit your answers as a single electronic file through Blackboard before the end of the allowed time. You should include your **name and student number** on the first page of the file that you submit
Who to contact	If you have any concerns or queries about a particular question, or need to make any assumptions to answer the question, state these at the start of your solution to that question. You may also include queries you may have made with respect to a particular question, should you have been able to 'raise your hand' in an examination room. If you experience any **technical difficulties** during the exam, contact the Library AskUs (https://web.library.uq.edu.au/contact-us) service for advice (open 7am–10pm, 7 days a week, Brisbane time): Chat: https://support.my.uq.edu.au/app/chat/chat_launch_lib/p/45 Phone: +61 7 3506 2615 Email: examsupport@library.uq.edu.au You should also ask for an email documenting the advice provided so you can provide this on request. In the event of a **late submission**, you will be required to submit evidence that you completed the exam in the time allowed. We recommend you use a phone camera to take photos (or a video) of every page of your exam. Ensure that the photos are time-stamped. If you submit your exam after the due time then you should send details (including any evidence) to SMP Exams (exams.smp@uq.edu.au) as soon as possible after the end of the exam.

Important exam condition information	The normal academic integrity rules apply. You cannot cut-and-paste material other than your own work as answers. You are not permitted to consult any other person – whether directly, online, or through any other means – about any aspect of this assessment during the period that this assessment is available. If it is found that you have given or sought outside assistance with this assessment then that will be deemed to be cheating and will result in disciplinary action. By undertaking this online assessment, you will be deemed to have acknowledged UQ's academic integrity pledge by making the following declaration: *"I certify that my submitted answers are entirely my own work and that I have neither given nor received any unauthorised assistance on this assessment item"*.

1. (20 marks, show your workings)
Consider the following system of equations,

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -3x\,(2y+1) \\ -y+\cos(x) \end{pmatrix}$$

for $0 \leq x \leq 8$.

(a) Find the stationary points and determine their stability and nature.

(b) Compute the information you need to sketch the phase portrait close to the critical points.

(c) Sketch the phase portrait of the nonlinear system. Explicitly sketch and name trajectories which connect fixed points.

(Working Space Only)

(Working Space Only)

2. (15 marks, show your workings)
Consider the following system of equations:

$$\begin{cases} y''(t) - y(t) = t \ , \\ y(0) = 0 \ , \\ y'(0) = 0 \ . \end{cases}$$

(a) Find the transfer function and compute its inverse Laplace transform.

(b) USE THE CONVOLUTION THEOREM to find $y(t)$. Remark: No marks will be assigned if you compute $y(t)$ in a way which avoids applying the convolution theorem!

(Working Space Only)

3. (15 marks, show your workings) Consider the system of differential equations for the unknown functions $x(t)$ and $y(t)$,

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -2xy^3 \\ -y \end{pmatrix} . \tag{1}$$

(a) Find all stationary points.

(b) Find the solution curves written as $y(x)$ and parametrised by a constant of integration C.

(c) Use the information from (a) and (b) to sketch the phase portrait. Include arrowheads which denote the direction of the trajectories you sketch. Justify your answer!

(d) Consider the trajectory $(x(t), y(t))$ which satisfies (1) and starts at $(x(0), y(0)) = (-1, 1)$. Use the information collected in (a)-(c) to find $\lim_{t\to\infty}(x(t), y(t))$. Justify your answer!

(Working Space Only)

4. **(15 marks, show your workings)** Determine the Fourier series of the function $f(x) = x^2$ on the $[-1, 1]$ interval. Plot the truncated Fourier series with 1, 2 and 3 nonzero terms, together with $f(x)$ to illustrate how the truncated series provides a better approximation as more terms are included.

(Working Space Only)

5. (15 marks, show your workings) Solve the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad u(0,x) = e^{-x^2}, \ \ \frac{\partial u}{\partial t}(0,x) = \sin x$$

on an infinite domain $-\infty < x < \infty$. Plot the solution $u(t,x)$ within the $-5 < x < 5$ interval at $t = 0, \pi/2$ and π.

(Working Space Only)

6. (20 marks, show your workings) A metal bar, of length $l = 1$ meter and thermal diffusivity $D = 2m^2/s$, is taken out of a 100° C oven and then fully insulated except for one end, which is fixed to a large ice cube at 0° C.

(a) Write down an initial-boundary value problem that describes the temperature $u(t, x)$ of the bar at all subsequent times.

(b) Write a series formula for the temperature distribution $u(t, x)$ at time $t > 0$.

(c) What is the equilibrium temperature distribution in the bar, i.e., for $t \to \infty$? How fast does the solution go to equilibrium?

(d) Just before the temperature distribution reaches equilibrium, what does it look like? Sketch a picture and discuss.

(Working Space Only)

Table of Laplace Transforms

$f(t), g(t)$	$F(s), G(s)$
K	$\dfrac{K}{s}$
t^n	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
$\cos(\alpha t)$	$\dfrac{s}{s^2+\alpha^2}$
$\sin(\alpha t)$	$\dfrac{\alpha}{s^2+\alpha^2}$
$e^{at}f(t)$	$F(s-a)$
$f(t-k)u(t-k) = \begin{cases} 0 & t < k \\ f(t-k) & k \leq t \end{cases}$ ($u(.)$ denotes the Heaviside step function.)	$e^{-ks}F(s)$
$\int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \ ... \ - f^{(n-1)}(0)$
$tf(t)$	$-F'(s)$
$\dfrac{f(t)}{t}$	$\int_s^\infty F(\sigma)\, d\sigma$