THE UNIVERSITY OF QUEENSLAND AUSTRALIA | CREATE CHANGE

Information	
Course code and title	MATH3403, Partial Differential Equations
Semester	Semester 2, 2021
Type	Online, non-invigilated assignment, under ‘take home exam’ conditions.
Technology	File upload to Blackboard Assignment
Date and time	Your assignment will begin at the time specified by your course coordinator. You have a fixed 12-hour window from this time in which it must be completed. You can access and submit your paper at any time within the 12 hours. Even though you have the entire 12 hours to complete and submit this assessment, the expectation is that it will take students around 2 hours to complete. Note that you must leave sufficient time to submit and upload your answers.
Permitted materials	This assignment is open book.
Recommended materials	Ensure the following materials are available during the available time: bilingual dictionary; phone/camera/scanner
Instructions	You will need to download the question paper included within the Blackboard Test. Once you have completed the assignment, upload a single pdf file with your answers to the Blackboard assignment submission link. You may submit multiple times, but only the last uploaded file will be graded. You can print the question paper and write on that paper or write your answers on blank paper (clearly label your solutions so that it is clear which problem it is a solution to) or annotate an electronic file on a suitable device.
Who to contact	Given the nature of this assessment, responding to student queries and/or relaying corrections during the allowed time may not be feasible. If you have any concerns or queries about a particular question or need to make any assumptions to answer the question, state these at the start of your solution to that question. You may also include queries you may have made with respect to a particular question, should you have been able to ‘raise your hand’ in an examination-type setting. If you experience any interruptions during the allowed time, please collect evidence of the interruption (e.g. photographs, screenshots or emails). If you experience any technical difficulties during the exam, contact the Course Coordinator <d.oelz@uq.edu.au>. Note that this is for technical difficulties only.
Late or incomplete submissions	In the event of a **late submission**, you will be required to submit evidence that you completed the assessment in the time allowed. This will also apply if there is an **error in your submission** (e.g. corrupt file, missing pages, poor quality scan). We **strongly recommend** you use a phone camera to take time-stamped photos (or a video) of every page of your paper during the time allowed (even if you submit on time). If you submit your paper after the due time, then you should send details to SMP Exams (exams.smp@uq.edu.au) as soon as possible after the end of the time

	allowed. Include an explanation of why you submitted late (with any evidence of technical issues) AND time-stamped images of every page of your paper (eg screen shot from your phone showing both the image and the time at which it was taken).
Further important information	Academic integrity is a core value of the UQ community and as such the highest standards of academic integrity apply to assessment, whether undertaken in-person or online. This means: • You are permitted to refer to the allowed resources for this assignment, and you must not use any instances of work that has been submitted previously elsewhere. • You are not permitted to consult any other person – whether directly, online, or through any other means – about any aspect of this assignment during the period that it is available. • If it is found that you have given or sought outside assistance with this assignment, then that will be deemed to be cheating. If you submit your answers after the end of allowed time, the following penalties will be applied to the total mark available for the assessment: • Less than 5 minutes – 5% penalty • From 5 minutes to less than 15 minutes – 20% penalty • More than 15 minutes – 100% penalty These penalties will be applied unless there is sufficient evidence of problems with the system and/or process that were beyond your control. Undertaking this online assignment deems your commitment to UQ's academic integrity pledge as summarised in the following declaration: *"I certify that I have completed this assignment in an honest, fair and trustworthy manner, that my submitted answers are entirely my own work, and that I have neither given nor received any unauthorised assistance on this assignment".*

Total marks: 100

Q1 (17 marks) Find the solution for $t > 0$ of the following problem,

$$\begin{cases} \partial_t u - x\partial_x u = x^2\,\partial_{xx} u\,, \quad 1 < x < e\,, \quad t > 0\,, \\ u(t, x = 1) = u(t, x = e) = 0\,, \quad t > 0\,, \\ u(t = 0, x) = 1\,, \quad 1 < x < e\,. \end{cases}$$

Show your working justifying all the steps.

Q2 (16 marks)

(a) Use the method of characteristics to find the general solution of the equation

$$-2u_{xx} + u_{xy} + yu_x = 0\,.$$

(b) Use the result in a) to find the general solution of the equation

$$-2u_{xx} + u_{xy} + yu_x = y\,.$$

Q3 (17 marks) Consider the following system of equations

$$\begin{cases} u_x + (2 - x)u_y = -u \quad \text{on} \quad \{y > 0\,,\ x \in \mathbb{R}\}\,, \\ u(x, y = 0) = \exp(x) \quad \text{for} \quad x < x_0\,. \end{cases}$$

Take x_0 as the largest value such that this system has a solution and find a closed-form solution.

Q4 (17 marks) Using the fundamental solution, solve the following IVBP explicitly. (The solution should **not** involve an integral.)

$$u_t - u_{xx} = -au, \quad x \in \mathbb{R}^+\,,$$
$$u(x, 0) = \delta(x - b),\ u_x(0, t) = 0\,,$$

where $a, b > 0$.

Q5 (16 marks) For a given function $f : (0, L) \to \mathbb{R}$ consider the Poisson equation on $(0, L)$ with homogeneous boundary conditions,

$$-u_{xx} = f(x)\,,$$
$$u(0) = 0, \quad u(L) = 0\,.$$

Solve this problem by first finding the associated Green's function and then using the Representation Formula. (No marks will be assigned if the solution is computed without making use of the Green's Function!)

Q6 (17 marks)

(a) Let $U \subset \mathbb{R}^3$ be open, bounded and connected, and let $f, g \in \mathcal{C}(\bar{U})$. Use the energy method to show that the solution $u : \overline{U} \to \mathbb{R}$ to

$$\Delta u = f \quad \text{on } U\,,$$
$$u = g \quad \text{on } \partial U\,,$$

is unique. (Hint: Use the Divergence Theorem.)

(b) Find **all** solutions to $u_{xx} + u_{yy} = 5$ on the disk defined by $x^2 + y^2 < a^2$ where $a > 0$ with $u = 0$ on the boundary ($x^2 + y^2 = a^2$).

END OF ASSESSMENT