MATH3202 Assignment 1

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Section A

The mathematical formulation for our model pertaining to the *Pacific Paradise Gas* client is listed below. We identify the relevant data to the optimisation problem, as well as the variables (and their constraints) that provide the optimal solution to the cost minimisation problem.

Sets

N	Nodes
S	Supplying Nodes, $S \subset N$
E	Edges (pipes that link nodes)
T	Time (day)

Data

l_e	Length of edge $e \in E$ (km)
d_{nt}	Gas demand at node $n \in N$ on day $t \in T$ (MJ)
c_s	Cost per unit gas from supplier $s \in S$ (\$/MJ)
SC_s	Maximum supply (supplying capacity) of supplier $s \in S$ over one day (MJ)
f_e, t_e	"From" and "to" nodes of each edge $e \in E$
$\text{Max}S$	Maximum supply from an individual supplier over the entire time duration (MJ)
$\text{Max}F$	Maximum flow of gas within any one pipeline over one day (MJ)
$\text{dist}C$	Extra cost per distance of pipeline gas flow (\$/MJ/km)
$\text{im}C$	Cost per unit imbalanced gas within one edge (pipeline between nodes) (\$)

Variables

y_{nt}	Gas supplied by suppl $n \in N$ on day $t \in T$
x_{et}	Net gas transmission along edge $e \in E$ on day $t \in T$
I_{et}	Change in total gas imbalance for edge $e \in E$ on day $t \in T$
AI_{et}	Absolute value of change in total gas imbalance for edge $e \in E$ on day $t \in T$

Objective

$$\min \sum_{t \in T} \left(\left[\sum_{s \in S} c_s y_{st} \right] + \left[\sum_{e \in E} \text{dist}C \cdot x_{et} \cdot l_e + \text{im}C \cdot AI_{et} \right] \right)$$

Constraints

$$y_{st} \leq SC_s \qquad \forall s \in S,\ \forall t \in T \tag{1}$$

$$y_{nt} \leq 0 \qquad \forall n \in N \setminus S,\ \forall t \in T \tag{2}$$

$$y_{nt} + \sum_{\substack{e \in E \\ s.t.\ t_e = n}} (x_{et} + I_{et}) = \sum_{\substack{e \in E \\ s.t.\ f_e = n}} x_{et} + d_{nt} \qquad \forall n \in N,\ \forall t \in T \tag{3}$$

$$x_{et} \leq \text{Max}F \qquad \forall e \in E,\ \forall t \in T \tag{4}$$

$$\sum_{t \in T} y_{nt} \leq \text{Max}S \qquad \forall n \in N \tag{5}$$

$$\sum_{t \in T} I_{et} = 0 \qquad \forall e \in E \tag{6}$$

$$AI_{et} \geq I_{et};\ AI_{et} \geq -I_{et};\ AI_{et} \geq 0 \qquad \forall e \in E,\ \forall t \in T \tag{7}$$

Constraints (1) and (2) ensure that no supplying node is supplying more than its capacity, and no non-supplying node is supplying any gas into the pipeline system. Constraint (3) ensures that the pipeline gas flow between nodes is balanced, up to a specified imbalance (given by I_{et}) that allows for a surplus/deficit of gas within each pipe $e \in E$. Constraint (4) ensures that the maximum flow across any one pipe during each day is not more than the maximum specified flow. Constraint (5) requires that any one supplying node cannot supply more than a restricted amount across the full time duration. Finally, constraint (6) ensures that there is no *net* imbalance in any pipe across the time duration and constraint (7) mathematically requires that the variable AI_{et} is the absolute value of I_{et} (i.e. $AI_{et} = |I_{et}|$).

Gurobi Implementation

The model, programmed utilising `gurobipy`, is available in the file attached with this document.

Section B

Responses to Communications

Communication 1

The initial communication of the optimisation problem provided a rich foundation on which the entirety of the project was based on. Given the positions of the nodes (and the pipelines connecting them), we produced a linear optimisation model which provided the optimal supply values for each supplier in the pipeline system. This problem was solved for given demands across the system for *one* given day, and we arrived at an optimal value of \$156314 to meet the demand across the Pacific Paradise nation.
This solution was subject to the objective modification that gas transmission expended \$0.01 per MJ gas per kilometer pipeline, and was supplied by the four specified suppliers at various locations in the system. This transmission cost raised the optimal solution by a fair amount, especially at high demand nodes far from suppliers. This constraint also impacted the demand for supplying nodes, giving less cost efficient suppliers more use for nearby nodes.

Communication 2

The second communication saw us implement a maximum single-pipeline transmission of 489 MJ over one day. As any further constraint makes the system less efficient, this saw our optimal cost rise \$18 to \$156332 to supply Pacific Paradise for one day. In conjunction with the pipeline transmission cost, this constraint saw greater use of less cost-efficient suppliers, hence the increase in optimal cost.

Communication 3

The third communication required us to account for variability in the daily demand for each node across two weeks in Pacific Paradise. In effect, our one dimensional variables which we were optimising over now had another dimension of time, where the total supply cost of one day wasn't necessarily the same as the next or previous day. Of course, the cost over two weeks was significantly more than the single day cost and so the optimal solution for the two-week supply was \$2197205.

Communication 4

As in communications 1 and 2, communication 4 required us to further utilise cost-inefficient suppliers due to a constraint on the total gas contribution of any one supplier over the two weeks (of 11265 MJ). This distributed demand on supply somewhat away from the inexpensive suppliers at nodes 27 and 36, and towards the lower-capacity suppliers at nodes 20 and 45. Across the two weeks, this raised the optimal cost solution to approximately \$2206548.

Communication 5

With the added fact of gas compressibility within pipes, we added an imbalance variable within our model to account for daily flow imbalances within pipes, such that some extra gas could be bought on cheaper days and distributed at a later date. With a \$0.10 cost per MJ of imbalance each day for each pipe, we were able to *reduce* our optimal cost (relative to the cost at communication 4) down to approximately \$2203275 – a net reduction of \$3273. This imbalance variable was subject to a constraint that the *net* imbalance over the two weeks was 0 MJ (such that the imbalance prior to the first day

and after the last day were equal to 0 MJ).
For each day over the two week time period, we observed several pipelines have an absolute net imbalance change of over 100 MJ – a value which we deem a large imbalance. Across the two weeks, the only large imbalances observed were deficits in the pipe flow, and hence didn't impact on the maximum flow constraint within pipes; even with accounting for the imbalance variable within the pipeline flow constraint, the optimal cost solution remained the same. These deficit imbalances are given in the table below.

Day	Node 1	Node 2	Imbalance (MJ)
6	3	14	−105
6	37	28	−136
6	19	4	−223
12	19	44	−122
13	17	26	−116
13	9	21	−145

Table 1: Large Imbalances in Pipelines over the Two Week Period

As noted, the net savings after allowing for imbalances was on the order of a few thousand dollars over the two weeks. Considering that the total cost is on the order of millions of dollars over the two weeks, even a total restriction on imbalances (i.e. not allowing an imbalance) wouldn't contribute significantly to the objective value (relatively speaking). What savings we do see is largely due to a reduction in the distance-transmission cost, where pipelines can store extra gas (or have a deficit) near particular nodes on particular days where *other* pipelines have reached their flow capacity (and so gas would otherwise need to take a larger route around the 'full' pipes to reach said particular node). Since the distance-transmission cost is typically one or two orders of magnitude larger than the imbalance cost, we see a net savings in reducing the former cost.