MATH2100 Assignment 4

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Question 1

An example random walk of 500 time steps (each with a distance step of 1), across 3 examples, is shown in Figure 1:

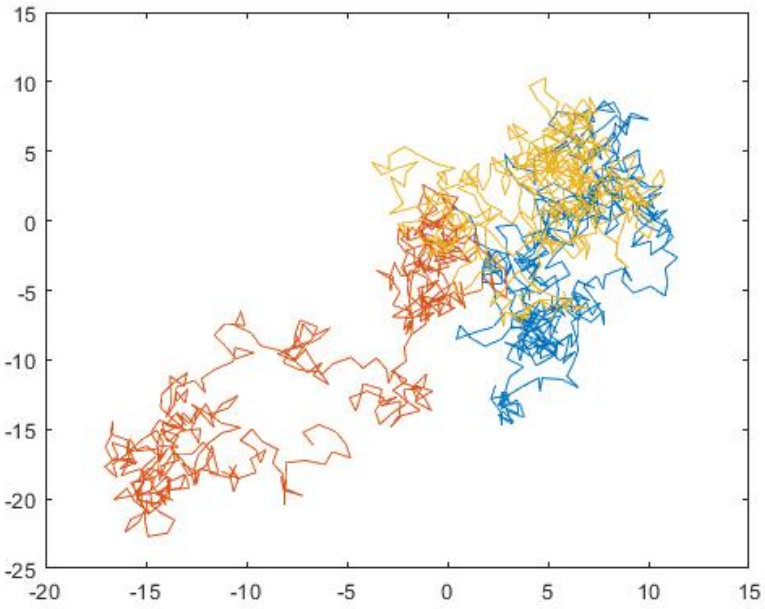

Figure 1: 3 Random Walks Beginning at the Origin

In Figure 1, each of the three examples are their own colour.

Question 2

For a group of 1000 random walkers, their positions at $t = 100$, $t = 200$, and $t = 300$ are shown in Figure 2.

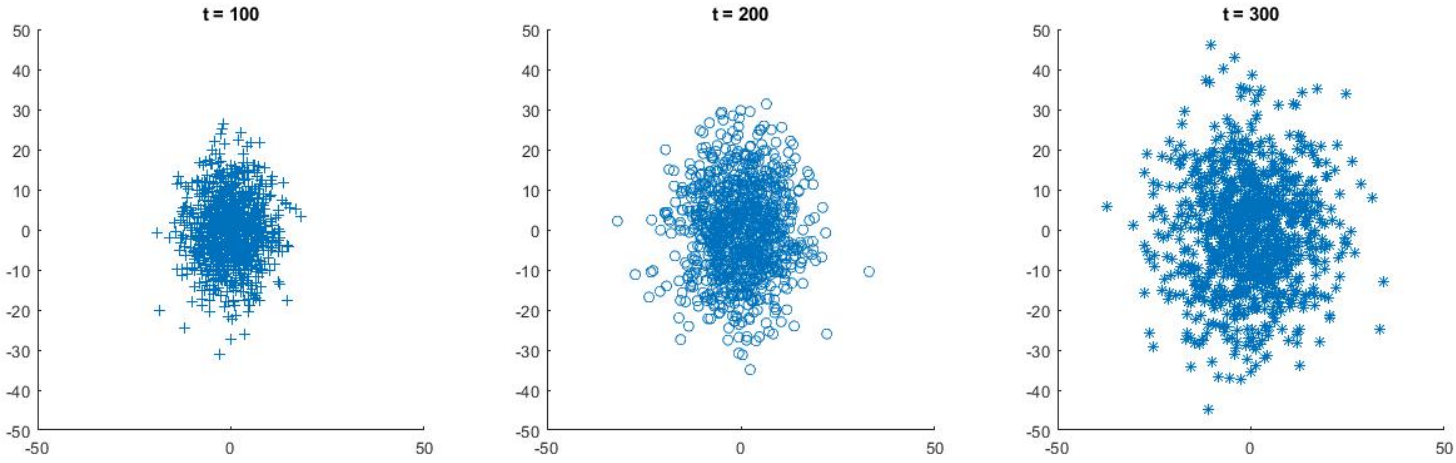

Figure 2: Distribution of $n = 1000$ Walkers at Varying Timepoints

Although the direction of each individual step is random, it can clearly be seen that for larger time samples, the group of walkers tend to disperse.

Question 3

For $n = 5000$ walkers, their distances from the origin at various time points were recorded and displayed in histograms, as can be seen in Figure 3.

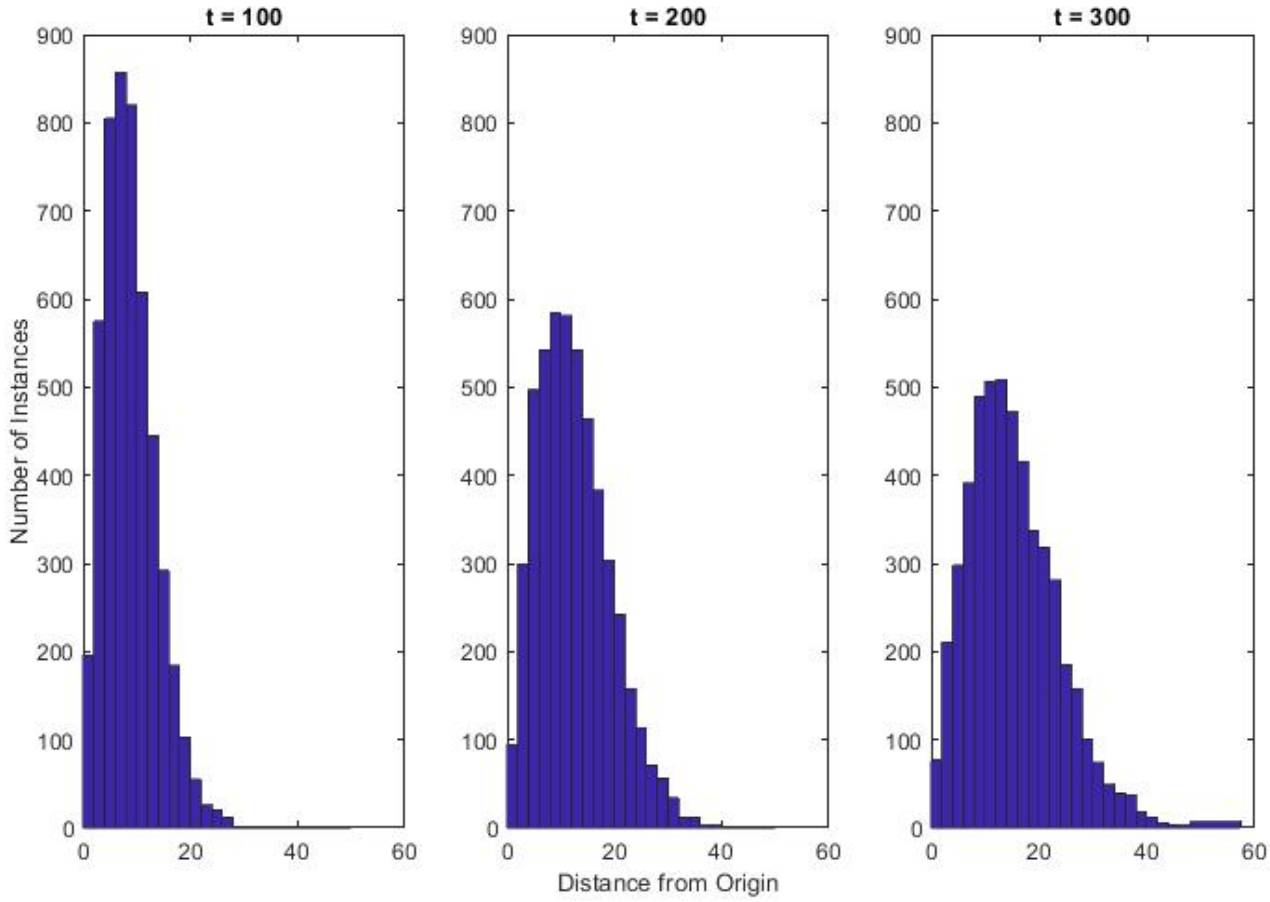

Figure 3: Distance from Origin at Various Time Points

As the time point increases, it is clear that the spread of the walkers' final positions broadens.

Question 4

Figure 4 compares simulated random walks (shown by the purple bars) against the theoretical frequency distribution (shown by the red curve).

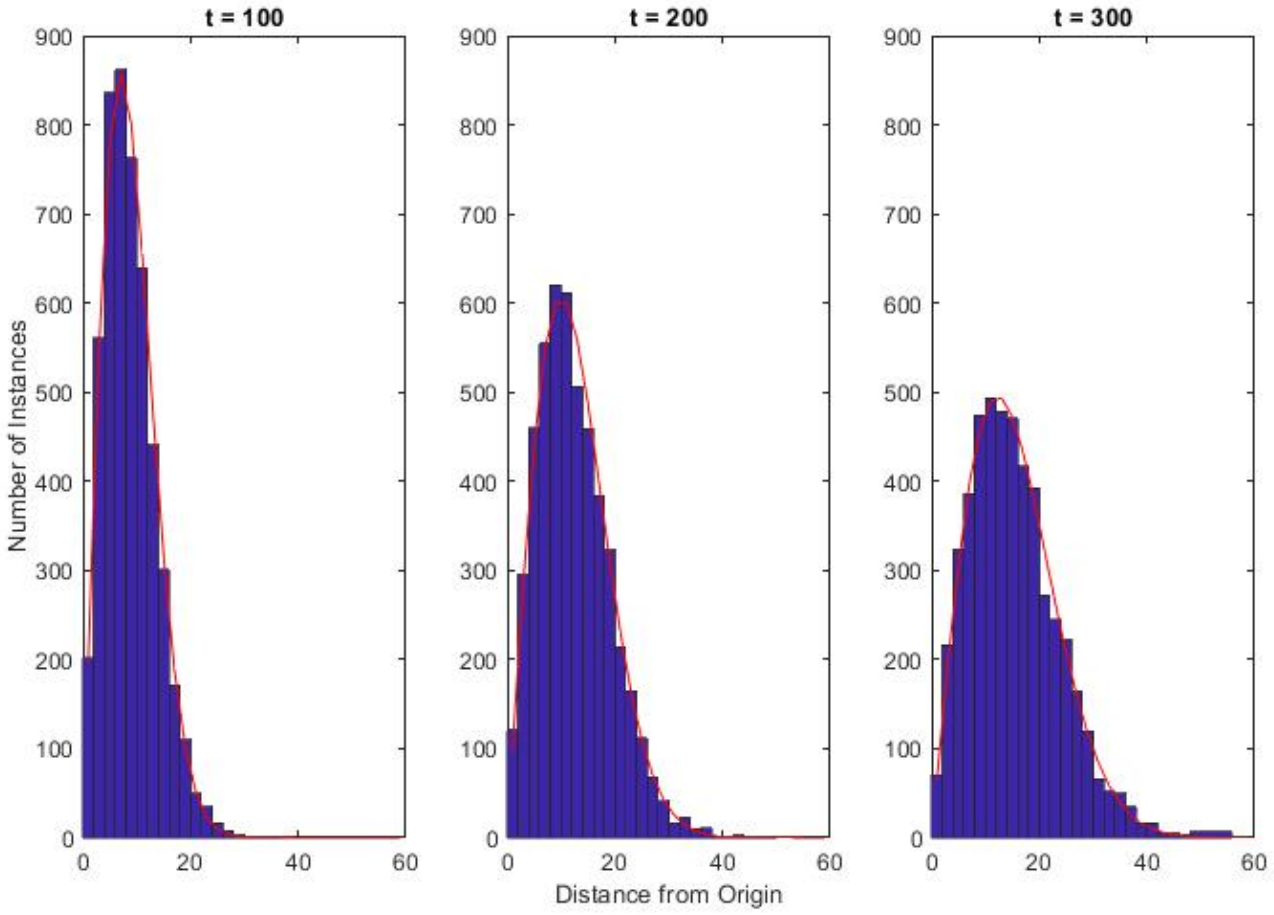

Figure 4: Distance from Origin Comparison of Theoretical and Simulated

As can be seen, the simulated random walks align consistently with the theoretical predictions from the formula

$$p(d) = \frac{2d}{t} e^{-\frac{d^2}{t}}$$

Appendices

0.1 Question 1 Matlab Code

```
[WX1, WY1] = randomwalk(500);
[WX2, WY2] = randomwalk(500);
[WX3, WY3] = randomwalk(500);

plot(WX1, WY1);
hold on
plot(WX2, WY2);
plot(WX3, WY3);

function [X, Y] = randomwalk(N)
    [X, Y] = deal(zeros(1, N+1));    %deal zero vectors to X and Y coordinates
    for n = 1:N
        dX = sign(randn) * rand(1); dY = sign(randn) * sqrt(1 - dX^2); %find value of x, 0<= x <=
            1, with random sign, and value of y with random sign that gives triangle with sides x,
            y and hypotenuse 1
        X(n + 1) = X(n) + dX; Y(n + 1) = Y(n) + dY;                  %assign stepped values to
            initial value on X and Y coords
    end
end
```

0.2 Question 2 Matlab Code

```
time = [100, 200, 300];      %choose time points
walkers = 1000;              %number of random walks
[t1xcoords, t1ycoords, t2xcoords, t2ycoords, t3xcoords, t3ycoords] = deal(zeros(1, walkers));
    %initialise vectors
for t = 1:length(time)
    for n = 1:walkers
        [x, y] = randomwalk(time(t));
        if t == 1
            t1xcoords(n) = x(time(t)+1);
            t1ycoords(n) = y(time(t)+1);
        elseif t == 2
            t2xcoords(n) = x(time(t)+1);
            t2ycoords(n) = y(time(t)+1);
        else
            t3xcoords(n) = x(time(t)+1);
            t3ycoords(n) = y(time(t)+1);
        end
    end
end

figure
subplot(1,3,1)
scatter(t1xcoords, t1ycoords, '+');
axis([-50 50 -50 50]);
title('t = 100');
hold on
subplot(1,3,2)
```

```
scatter(t2xcoords, t2ycoords, 'o');
axis([-50 50 -50 50]);
title('t = 200');
subplot(1,3,3)
scatter(t3xcoords, t3ycoords, '*');
axis([-50 50 -50 50]);
title('t = 300');


function [X, Y] = randomwalk(N)
    [X, Y] = deal(zeros(1, N+1));    %deal zero vectors to X and Y coordinates
    for n = 1:N
        dX = sign(randn) * rand(1); dY = sign(randn) * sqrt(1 - dX^2); %find value of x, 0<= x <=
            1, with random sign, and value of y with random sign that gives triangle with sides x,
            y and hypotenuse 1
        X(n + 1) = X(n) + dX; Y(n + 1) = Y(n) + dY;                  %assign stepped values to
            initial value on X and Y coords
    end
end
```

0.3 Question 3 Matlab Code

```
time = [100, 200, 300];      %choose time points
walkers = 5000;              %number of random walks
[t1xcoords, t1ycoords, t2xcoords, t2ycoords, t3xcoords, t3ycoords] = deal(zeros(1, walkers));
    %initialise vectors
for t = 1:length(time)
    for n = 1:walkers
        [x, y] = randomwalk(time(t));
        if t == 1
            t1xcoords(n) = x(time(t)+1);
            t1ycoords(n) = y(time(t)+1);
        elseif t == 2
            t2xcoords(n) = x(time(t)+1);
            t2ycoords(n) = y(time(t)+1);
        else
            t3xcoords(n) = x(time(t)+1);
            t3ycoords(n) = y(time(t)+1);
        end
    end
end
[t1dist, t2dist, t3dist] = deal(zeros(1, walkers));
for t = 1:length(time)
    for n = 1:walkers
        t1dist(n) = sqrt(t1xcoords(n)^2 + t1ycoords(n)^2);
        t2dist(n) = sqrt(t2xcoords(n)^2 + t2ycoords(n)^2);
        t3dist(n) = sqrt(t3xcoords(n)^2 + t3ycoords(n)^2);
    end
end

figure
subplot(1,3,1)
hist(t1dist, 1:2:50);
axis([0 60 0 900]);
```

```
title('t = 100');
ylabel('Number of Instances');
hold on
subplot(1,3,2)
hist(t2dist, 1:2:50);
axis([0 60 0 900]);
xlabel('Distance from Origin');
title('t = 200');
subplot(1,3,3)
hist(t3dist, 1:2:50);
axis([0 60 0 900]);
title('t = 300');


function [X, Y] = randomwalk(N)
    [X, Y] = deal(zeros(1, N+1));    %deal zero vectors to X and Y coordinates
    for n = 1:N
        dX = sign(randn) * rand(1); dY = sign(randn) * sqrt(1 - dX^2); %find value of x, 0<= x <=
            1, with random sign, and value of y with random sign that gives triangle with sides x,
            y and hypotenuse 1
        X(n + 1) = X(n) + dX; Y(n + 1) = Y(n) + dY;                  %assign stepped values to
            initial value on X and Y coords
    end
end
```

0.4 Question 4 Matlab Code

```
time = [100, 200, 300];      %choose time points
walkers = 5000;              %number of random walks
[t1xcoords, t1ycoords, t2xcoords, t2ycoords, t3xcoords, t3ycoords] = deal(zeros(1, walkers));
    %initialise vectors
for t = 1:length(time)
    for n = 1:walkers
        [x, y] = randomwalk(time(t));
        if t == 1
            t1xcoords(n) = x(time(t)+1);
            t1ycoords(n) = y(time(t)+1);
        elseif t == 2
            t2xcoords(n) = x(time(t)+1);
            t2ycoords(n) = y(time(t)+1);
        else
            t3xcoords(n) = x(time(t)+1);
            t3ycoords(n) = y(time(t)+1);
        end
    end
end
[t1dist, t2dist, t3dist] = deal(zeros(1, walkers));
for t = 1:length(time)
    for n = 1:walkers
        t1dist(n) = sqrt(t1xcoords(n)^2 + t1ycoords(n)^2);
        t2dist(n) = sqrt(t2xcoords(n)^2 + t2ycoords(n)^2);
        t3dist(n) = sqrt(t3xcoords(n)^2 + t3ycoords(n)^2);
    end
end
```

```
X = 1:2:60;


figure
subplot(1,3,1)
hist(t1dist, 1:2:50);
axis([0 60 0 900]);
title('t = 100');
ylabel('Number of Instances');
hold on
plot(X, ((2 .* X ./ time(1)) .* 2*walkers .* exp(-1 .* X.^2 ./ time(1))), 'r');
subplot(1,3,2)
hist(t2dist, 1:2:50);
axis([0 60 0 900]);
xlabel('Distance from Origin');
title('t = 200');
hold on
plot(X, ((2 .* X ./ time(2)) .* 2 .* walkers .* exp(-1 .* X.^2 ./ time(2))), 'r');
subplot(1,3,3)
hist(t3dist, 1:2:50);
axis([0 60 0 900]);
title('t = 300');
hold on
plot(X, ((2 .* X ./ time(3)) .* 2 .* walkers .* exp(-1 .* X.^2 ./ time(3))), 'r');


function [X, Y] = randomwalk(N)
    [X, Y] = deal(zeros(1, N+1));    %deal zero vectors to X and Y coordinates
    for n = 1:N
        dX = sign(randn) * rand(1); dY = sign(randn) * sqrt(1 - dX^2); %find value of x, 0<= x <=
            1, with random sign, and value of y with random sign that gives triangle with sides x,
            y and hypotenuse 1
        X(n + 1) = X(n) + dX; Y(n + 1) = Y(n) + dY;                  %assign stepped values to
            initial value on X and Y coords
    end
end
```