PHYS2041 Final Exam

Ryan White
s4499039

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Question 1 (10 marks):

a. The normalization condition gives a dimensionless quantity of 1. For 2 spatial dimensions, this is

$$\int_{-\infty}^{\infty} |\Psi(x,y,t)|^2 \, dx\,dy = 1$$

Since this is an integration of two variables (an area), this $dx\,dy$ has a unit of length squared (m^2). However, the wave function is squared, and the total function yields no units, so

$$\text{length}^2 (\Psi)^2 = 1$$

and Ψ must have units m^{-2} to cancel the m^2.

b. The expression (1) is the correct expectation value for kinetic energy. This is because

$$T = \frac{1}{2m} p^2$$

and since $\hat{p} = -i\hbar \,\partial/\partial x \Rightarrow \hat{p}^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$

$$\Rightarrow T = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$$

and so

$$\langle \hat{T} \rangle = -\frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \psi^* \frac{\partial^2 \psi}{\partial x^2} \, dx$$

c. Once the position is known more precisely, less so is the momentum known (according to the uncertainty principle). Therefore, the probability density will be more spread out for the momentum value, resembling graph (c)

d. Given that $[A,B] = AB - BA$ and $\hat{p} = -i\hbar \,\partial/\partial x$, the commutator of $\hat{p}$ and some $f(x)$ may be found by introducing some $g(x)$:

$$[\hat{p}, f(x)]g(x) = -i\hbar \,\partial/\partial x \,(f(x)g(x)) - (-i\hbar f(x) \,\partial/\partial x \, g(x))$$

Using the product rule, this becomes:

$$[\hat{p}, f(x)]g(x) = -i\hbar g(x) \tfrac{\partial}{\partial x} f(x) - i\hbar f(x) \tfrac{\partial}{\partial x} g(x) + i\hbar f(x) \tfrac{\partial}{\partial x} g(x)$$

$$= -i\hbar g(x) \tfrac{\partial}{\partial x} f(x)$$

dividing both sides by $g(x)$ gives

$$[\hat{p}, f(x)] = -i\hbar \tfrac{\partial}{\partial x} f(x)$$

QED

Question 2 (10 marks):

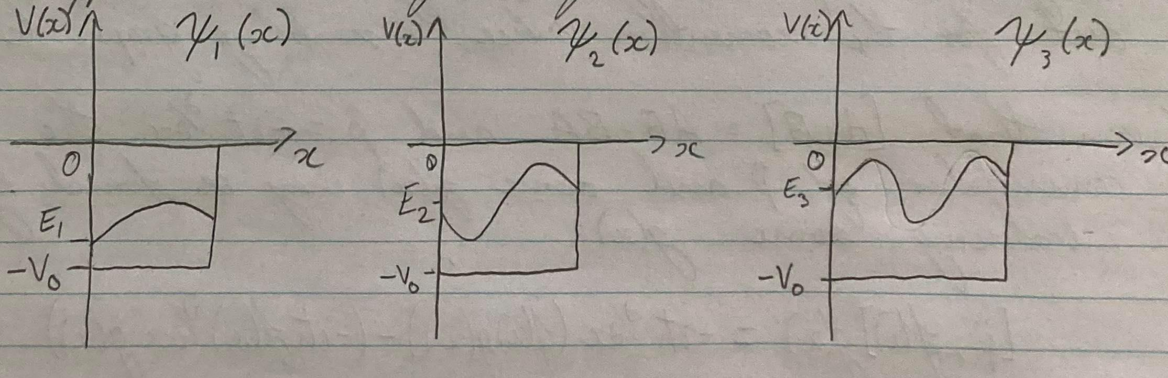
a. ~~Given that the probability density across the entire domain must be equal to 1, the tick on the vertical axis must be equal to or greater than 1 as~~

Each of the numerical values listed could make physical sense, as it depends entirely on the value of L. As long as

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1$$

then the graph makes physical sense. If L is very large, you would expect $? \geq 1$. If $L \leq 1$, you would expect $? < 1$.

b. To the best of my knowledge, there are n stationary points of a wave function $\psi_n(x)$ in a trapping potential, where there are nodes at the boundaries for an infinite well, and approximate nodes for a finite well. Thus



c. Assuming that the well is wide and deep, the energy of each state will be

$$E_n \geq \frac{n^2\pi^2\hbar^2}{2m(2L)^2} - V_0 \qquad (1)$$

The finite square well has the above equation as an approximate equality rather than $E_n \geq \ldots$

The infinite square well has

$$E_n = \frac{n^2\pi^2\hbar^2}{2m(2L)^2}$$

which is larger than the suggested energy equation (1).

As the situation is a combination of the finite and infinite square wells, I expect the energy eigenvalues to be between the analogies, but closer to that of the finite square well.

Question 3 (10 marks):

a. The new wave function would be analogous to that of the $n=1$ state, rather than the ground state. That is:

$$\psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2}} H_1(\xi) e^{-\xi^2/2}$$

where $H_1(\xi) = 2\xi$

and $\xi = \sqrt{\frac{m\omega}{\hbar}}\, x$

so therefore,

$$\psi_1(x) = \left(\frac{m^3\omega^3}{\pi\hbar^3}\right)^{1/4} \frac{2x}{\sqrt{2}} e^{-m\omega x^2/2\hbar}$$

b. As the new frequency is 2ω, the possible energies are

$$E_n = \left(n+\tfrac{1}{2}\right)\hbar(2\omega)$$

$$= (2n+1)\hbar\omega \quad \text{for } n \in \mathbb{Z}^+$$