Module 1: Field Basics

Problem Set 1

Semester 1, 2020

Due 5pm on Friday 20 March.

Submit your problem set (hand written or typed) *either* through the PHYS2055 Tutorial slot on level 3 of the Physics Annex, *or* via Blackboard.

Answers should include detailed comments and reasoning. A grade up to 5 can be obtained by answering only questions in Part A. Higher grades require all questions to be answered (with clear, well reasoned arguments showing originality and insight).

Part A

Problem 1.1

The height of an area of land relative to some reference point is given by the equation

$$h(x,y) = 10\exp(-\frac{x^2}{10}) + \frac{y^2}{10} .$$

For each of the following, assuming just two-dimensions, make a sketch of the field - this may be hand-drawn or drawn on a computer using your own script. Include a copy of your script(s), which can be based on the on-line templates, if done on a computer. Briefly describe the important aspects of the field in each case.

(a) The height h.

(b) The gradient of the height.

(c) The divergence of the gradient.

Problem 1.2

A 200 mm long rod made of type 304 stainless steel with a diameter of 10 mm is insulated so that heat can only flow in and out of the rod at each end. The ends of the rod are maintained at 20°C while the initial temperature in the rod takes the form of part of a sine wave with a maximum temperature of 80°C.

(a) Show that a temperature profile of the form

$$T(x,t) = T_A \sin(\frac{2\pi x}{\lambda})e^{-t/\tau} + T_0$$

is a solution of the temperature diffusion equation when the diffusion constant, D, is a function of one or more of the constants T_A, T_0, λ and τ. Determine this relationship.

(b) Using the numbers provided above, and the constant-temperature boundary conditions, find numerical values for T_A, T_0 and λ.

(c) The diffusion constant of a material can be obtained from

$$D = \frac{k}{c\rho}$$

where k is the thermal conductivity of the material, c is the specific heat, and ρ is the density. Using literature values of these constants for type 304 stainless steel (quote your source), and your expression for D, calculate a numerical value for τ. What does τ mean, and does your value seem reasonable for this material? (Type 304 stainless steel is commonly used for BBQ plates)

Continued over ...

Part B - Advanced

Problem 1.3

In this problem you will develop a model for the gravitational field inside a globular cluster. A globular cluster is a spherical collection of stars that are gravitationally bound. A simple model of the density function of a globular cluster is

$$\rho(r) = \begin{cases} A\left(\frac{1}{r} - \frac{1}{r_0}\right)^2 & \text{if } r \leq r_0 \\ 0, & \text{otherwise} \end{cases}$$

where r_0 is the radius of the cluster and A is a constant.

You will need to use the divergence relation for acceleration to do your calculations, assuming spherical symmetry. This can be written as

$$\nabla \cdot \mathbf{a} = \frac{1}{r^2}\frac{\partial\left(r^2 a\right)}{\partial r} = -4\pi G\rho$$

where r is the distance from the centre of the body, a is the radial acceleration, G is the gravitational constant, and ρ is the density.

(a) Find an expression for the radial acceleration inside the globular cluster.

(b) Using the mass and radius of a real globular cluster (you will need to look up some literature to find this, quote your source), determine the constant A, and hence find a numerical value for the acceleration inside the globular cluster at $r = r_0/2$. Compare this number with the acceleration at the edge of the globular cluster ($r = r_0$).