**DEPARTMENT OF MATHEMATICS**

# MATH1052
# Assignment 2
# Semester 1, 2018

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*Assignment 2 is due by 5pm on Friday 23rd of March*

You may find some of these problems challenging. Attendance at weekly tutorials is assumed.

Family name:

Given names:

Student number:

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Marker's use only

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Each question marked out of 3.

- Mark of 0: You have not submitted a relevant answer, or you have no strategy present in your submission.
- Mark of 1: Your submission has some relevance, but does not demonstrate deep understanding or sound mathematical technique.
- Mark of 2: You have the right approach, but need to fine-tune some aspects of your calculations.
- Mark of 3: You have demonstrated a good understanding of the topic and techniques involved, with well-executed calculations.

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|---|---|---|---|
| Q1a: | Q1b: | Q1c: | Q2a: |
| Q2b: | Q2c: | Q2d: | Q3a: |
| Q3b: | Q3c: | | |

Total (out of 30):

**(1)** Two countries $A$ and $B$ have the same natural growth rate $k > 0$, so their populations grow according to $\frac{dP_A}{dt} = kP_A$, $\frac{dP_B}{dt} = kP_B$,respectively. A more realistic model is given by

$$\frac{dP_A}{dt} = kP_A - aP_A + bP_B,$$
$$\frac{dP_B}{dt} = kP_B - bP_B + aP_A,$$

where $a \geq 0$ denotes the emigration rate from $A$ to $B$ and $b \geq 0$ the emigration rate from $B$ to $A$.

a) Show that $P_A$ satisfies a second order ODE.

b) Assuming $a + b > 0$, obtain the general solution for $P_A(t)$.

c) Given that the initial population of $A$, denoted $N_A$, is steady (so $\frac{dP_A}{dt} = 0$, at $t = 0$), determine the population at time $t$.

**(2)** The price $P(t)$ of a company's goods is modelled by the system of ODEs

$$\frac{dP}{dt} = -k(L(t) - l),$$
$$\frac{dL}{dt} = Q(t) - S(t),$$
$$S(t) = 500 - 40P - 10\frac{dP}{dt},$$
$$Q(t) = 250 - 5P,$$

where, at time $t$, $S(t)$ is the level of (forecasted) sales, $L(t)$ is the inventory level, $Q(t)$ is the production level, $l$ is a given constant (optimal level) and $k > 0$ is a constant parameter.

a) Show by differentiation that $P(t)$ satisfies the ODE

$$\frac{d^2P}{dt^2} + 10k\frac{dP}{dt} + 35kP = 250k.$$

(b) Obtain the general solution for $P(t)$ in the case $k \geq 7/5$.

(c) Repeat part (b) for the case $k < 7/5$.

(d) Determine the steady-state solution for $P(t)$ (ie the solution as $t \to \infty$) and show it is independent of $k$.

**(3)** By completing the square, sketch by hand, the following curves, labelling the significant features.

(a) $x - y^2 + 4y - 5 = 0$

(b) $x^2 - 4y^2 - 4x + 24y - 36 = 0$

(c) $25x^2 + 4y^2 - 250x + 525 = 0$