

PHYS2041 Short Lab Report

Atomic Spectroscopy of Sodium for Visible Spectra

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Abstract

Through the use of a prism atomic spectrometer and a low-pressure sodium lamp, the emission spectrum, both wavelength and energies, of sodium was studied and quantified. Linear regression analysis allowed for the formulation of energies with respect to shell and principal quantum number within reasonable accuracy as a modification/derivation of the Bohr Model.

1 Introduction

The spectra characteristics of near-hydrogen elements are especially important to quantify, especially in areas of science such as Stellar Astronomy and Astrophysics, Optics, or Chemistry. Specifically, understanding the emission spectra of Sodium provides crucial understanding in the lifecycle of main sequence stars, specifically by determining the sodium content (and in part the metallicity) and correlating to stage in the stellar cycle [5]. Using an atomic spectrometer, together with low-pressure sodium and mercury lamps, the emission wavelengths of atomic Sodium can be determined via refraction in a glass prism.

2 Theory

The first postulate of Bohr's theory states that an electron of an atom is subject to the Coulomb attraction, F_C , given by

$$F_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2} \quad (1)$$

This force constantly acts upon the electron, with the resultant equation of motion being found by equating F_C to $m_e a$, where $a = v^2/r$ (where m_e is the mass of the electron, v is its speed, and r its orbital radius)

$$m_e \frac{v^2}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2} \quad (2)$$

$$\Rightarrow m_e v^2 r = \frac{1}{4\pi\epsilon_0} \cdot e^2 \quad (3)$$

However, the second postulate of Bohr's theory states that only certain orbits are allowed, specifically those with an angular momentum that is an integral multiple of the reduced Planck's constant \hbar

$$L = m_e v r_n = n\hbar = \frac{nh}{2\pi} \quad (4)$$

Squaring equation (4) allows for a form that can be applied to equation (3)

$$\begin{aligned}
L^2 &= m_e^2 v^2 r_n^2 = n^2 \hbar^2 = \frac{n^2 \hbar^2}{4\pi^2} \\
m_e v^2 r_n &= \frac{n^2 \hbar^2}{4m_e r_n \pi^2} \\
\Rightarrow \frac{n^2 \hbar^2}{4m_e r_n \pi^2} &= \frac{1}{4\pi\epsilon_0} \cdot e^2 \\
\Rightarrow r_n &= \frac{n^2 \hbar^2 \epsilon_0}{e^2 m_e \pi} \tag{5}
\end{aligned}$$

Now, [1] gives the potential energy of an electron in some orbit r_n as

$$E_{pot_n} = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_n} \tag{6}$$

and, given that the kinetic energy of a particle is $E_{kin} = \frac{1}{2}mv^2$, the kinetic energy of an electron in orbit r_n is calculated using equation (2)

$$E_{kin_n} = \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2r_n} \tag{7}$$

Combining equations (6) and (7) gives the total energy, E_{tot}

$$\begin{aligned}
E_{tot_n} &= E_n = E_{pot_n} + E_{kin_n} \\
&= \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2r_n} - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_n} \\
E_n &= -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2r_n} \tag{8}
\end{aligned}$$

Substituting equation (5) into equation (8) gives the total energy of an electron with respect to its principle quantum number

$$E_n = -\frac{e^4 m_e}{8h^2 \epsilon_0^2} \cdot \frac{1}{n^2} = -R \cdot \frac{1}{n^2} \tag{9}$$

The electron angular momentum states are qualitatively described by a consecutive series of letters; s , p , d , f , and g , corresponding to 'sharp', 'principal', 'diffuse', and 'fundamental', where g follows on alphabetically from fundamental [4]. Each letter simultaneously corresponds to consecutively increasing azimuthal quantum numbers, l , where the sharp angular momentum state represents the $l = 0$ state of the system. For each of these angular momentum states, linear regression of a plot of $1/\sqrt{-E}$ vs the principal quantum number n , allows for an equation representing the energy of the valence electron in terms of its quantum number. With this, all possible states of the electron are available to be calculated.

3 Experiment

Figure 1 shows the rough setup and design of a prism spectrometer. In it, light produced from an elemental lamp is passed through a slit where it travels through a collimator, where it then leaves with the rays parallel. The light then refracts through a prism, where it is deflected at a measured angle, and beamed into a focuser, where the separated wavelengths of light are visible on a flat surface behind the focuser.

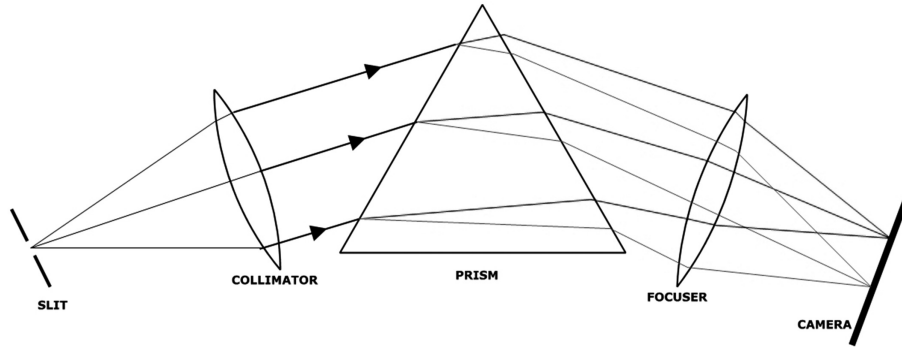


Figure 1: Typical Prism Spectrometer Design
Source: Walker, R, Trypsteen, M 2017

Once the beam of parallel (roughly white) light is refracted through the prism, discrete spectral colours are visible on the screen behind the focuser due to the process of dispersion. Knight details that the refractive index is larger for shorter wavelengths of light (the process of dispersion), and so “*violet light refracts more than red light.*” Due to this difference in refractive index, the individual wavelengths are deflected at varying degrees, and so discrete bands of spectra are seen.

3.1 Method

In order to calibrate the apparatus, the low-pressure mercury lamp was switched on and allowed to warm up for a few minutes. The entrance slit of the spectrometer was identified and opened to a width of several millimeters. Following this, the mercury lamp was placed next to the slit so that light could pass through the slit opening. A small sheet of paper was then first held at the end of the entrance telescope, where a uniform disk of light was observed. The same sheet was then positioned between the prism and the focuser, where the prism was then rotated until the refracted light could be observed on the paper. The paper was then positioned behind the focuser, prism rotated until the brightest visible band was seen on the sheet. This angle was noted, the slit closed fully, and then finally opened only very slightly. While using the eyepiece, the prism was rotated slowly until focused, bright bands of light were seen, with the corresponding angles noted. In the event that a band was out of focus, the focus knobs for the collimating lens and/or the viewing telescope were rotated. The methodology in its entirety was repeated for the low-pressure sodium lamp, and once again for the high-pressure sodium lamp, without angle measurements being taken for the latter instance.

3.2 Uncertainties

Due to the limited range of measurements taken in the spectroscopy experiment, there were minimal uncertainties encountered in the data gathered, with the only uncertainty being that of the measured deflection angle. Due to the increments in the angle scale being divided into two separate measurements, the resultant uncertainty was deemed a sum of the uncertainties of each: $\Delta\delta = \Delta\delta_1 + \Delta\delta_2$. For the larger scale (increments of approximately 1 degree), the uncertainty was decided to be half an increment, corresponding to $\Delta\delta_1 = \pm 0.5^\circ$. For the smaller scale (increments of one arc minute), the uncertainty was decided to be one whole increment, corresponding to $\Delta\delta_2 = \pm 1/60^\circ$. As a whole, the uncertainty was then $\Delta\delta = \pm 31/60^\circ$.

4 Results

The calibration of the spectrometer using the low-pressure mercury lamp (raw data in Appendix 7.2), together with known values (Appendix 7.1), yielded the linear regression shown in Figure 2

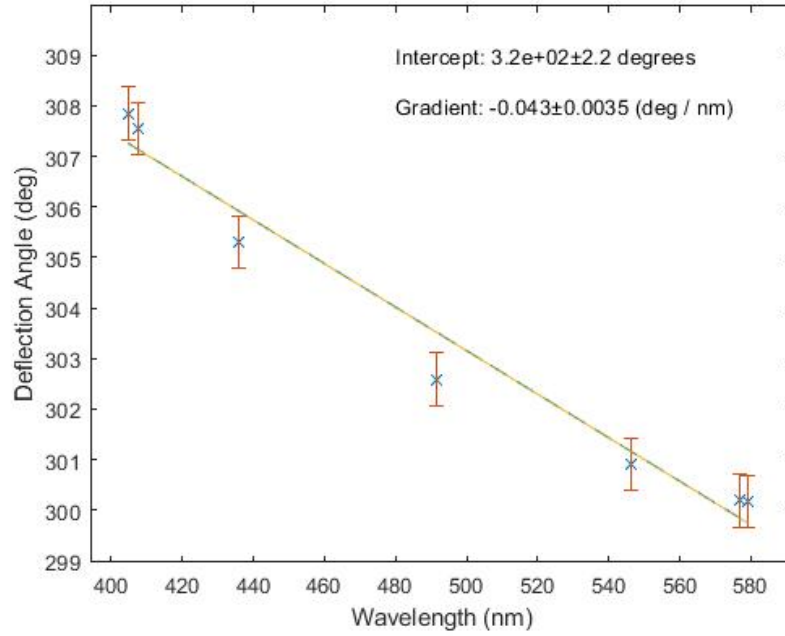


Figure 2: Deflection Angle vs Logged Mercury Emission Spectra

From this figure, with noted intercept and gradient values, a linear model was established:

$$\begin{aligned}\delta &= m\lambda + c \\ &= -0.0431\lambda + 324\end{aligned}\tag{10}$$

$$\Rightarrow \lambda = 4.31 \times 10^2 (324 - \delta)\tag{11}$$

With this model, the wavelengths of light for the Low-Pressure Sodium Lamp (LPSL) could be calculated from equation (11), with all data for this stage of the experiment shown in Table 1:

Colour	Measured Angle (Degrees)						Wavelength (nm)
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Ave	
Red	299.55	299.52	299.60	299.58	299.57	299.56	592 ± 72
Yellow	300.05	300.11	300.08	300.07	300.06	300.07	580 ± 71
Green	300.69	300.41	300.47	300.47	300.43	300.49	570 ± 71
Green	301.88	301.75	301.80	301.80	301.70	301.79	540 ± 69
Green	302.05	302.06	302.04	302.05	302.04	302.05	534 ± 69
Blue/Green	302.31	302.25	302.21	302.18	302.26	302.24	529 ± 69
Blue	303.69	303.60	303.60	303.60	303.62	303.62	497 ± 67
Violet	304.12	304.21	304.31	304.20	304.23	304.21	483 ± 67
Violet	304.54	304.54	304.57	304.52	304.50	304.53	476 ± 66
Violet	305.14	305.57	305.10	305.15	305.12	305.22	460 ± 66
Violet	305.54	305.69	305.51	305.57	305.46	305.55	452 ± 65

Table 1: Measured Deflection Angles For Emission Spectra of LPSL

Where the uncertainty in the average measured angle for each emission line was $\Delta\delta = \pm 0.23^\circ$ (Appendix 7.3.2), and the wavelength uncertainty was calculated according to Appendix 7.3.3.

5 Discussion

The Figures shown in Appendix 7.5 show the relationship between energy and the principal quantum number for each angular momentum state of the valence electron. Using the values obtained by linear regression of said data, equations for the energy multiples of each angular momentum state were formulated:

$$E_{ns} = -\frac{1}{(0.23n + 0.25)^2}; \quad E_{np} = -\frac{1}{(0.27n + 0.31)^2}; \quad E_{nd} = -\frac{1}{(0.19n + 0.66)^2} \quad (12)$$

for the s , p , and d momentum states respectively. The energy formula derived from the Bohr Model (9) was then altered to account for the azimuthal quantum number

$$E_n = -R \cdot \frac{1}{(n - \delta_l)^2} \quad (13)$$

Using the equations (12), the quantum defect numbers, δ_l , were calculated (Appendix 7.6) to be

$$\delta_s = n \left(1 - \frac{0.23}{\sqrt{R}} \right) - \frac{0.25}{\sqrt{R}} \quad \delta_p = n \left(1 - \frac{0.27}{\sqrt{R}} \right) - \frac{0.31}{\sqrt{R}} \quad \delta_d = n \left(1 - \frac{0.19}{\sqrt{R}} \right) - \frac{0.66}{\sqrt{R}} \quad (14)$$

Using the data calculated in Appendix 7.4, the Grotrian Diagram for Sodium was shown in Figure 3.

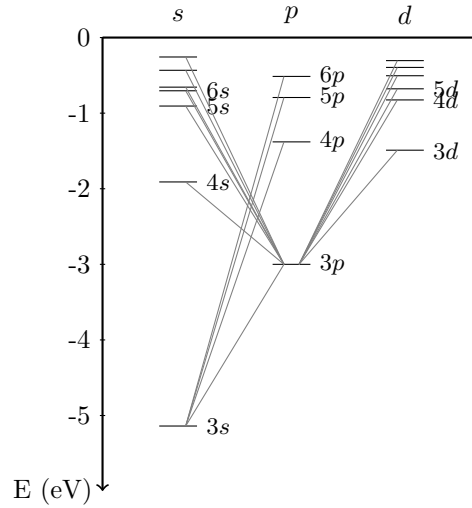


Figure 3: Grotrian Diagram of Sodium

Figure 3 clearly shows the inverse square relationship between valence electron energy and principal quantum number. The Grotrian Diagram also requires that an electron cannot translate to a like-energy shell, and must switch to a neighbouring shell.

6 Conclusions

Through the use of a prism atomic spectrometer, the corresponding wavelengths of the visible Sodium emission spectra were quantified and analysed. With this, the respective energy levels of the valence electron were calculated using a formula derived from Bohr's postulates. The derived energy levels (together with corresponding angular momentum states) aligned within reasonable accuracy to other publications [6].

7 Appendices

7.1 Mercury Emission Lines

Wavelength (nm)	Colour
404.655	violet
407.781	violet
435.835	blue
491.604	blue/green
546.074	green (bright)
576.959	yellow
579.065	yellow

Table 2: Given Mercury Emission Wavelengths
Source: Laboratory - Atomic Spectroscopy Notes

7.2 Mercury Emission Lines for Calibration

Colour	Measured Angle (Degrees)					
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Ave
Yellow	300.20	300.17	300.17	300.17	300.17	300.18
Yellow	300.21	300.18	300.18	300.18	300.18	300.19
Green	300.90	300.90	300.91	300.92	300.91	300.91
Blue/Green	302.60	302.61	302.57	302.55	302.60	302.59
Blue	305.30	305.31	305.24	305.35	305.31	305.30
Violet	307.60	307.53	307.53	307.61	307.52	307.56
Violet	307.71	307.92	307.90	307.88	307.85	307.85

Table 3: Low Pressure Mercury Lamp Spectral Data

7.3 Uncertainty Propagations

7.3.1 Uncertainty Propagation Table

All uncertainties were propagated according to the rules shown in Table 4

Relationship	Uncertainty obtained from
General expression	
$p = f(x, y, z, \dots)$	$(\Delta p)^2 = \left(\frac{\partial f}{\partial x} \Delta x\right)^2 + \left(\frac{\partial f}{\partial y} \Delta y\right)^2 + \left(\frac{\partial f}{\partial z} \Delta z\right)^2 + \dots$
Specific cases	
$p = x + y$	$\Delta p = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
$p = x - y$	$\Delta p = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
$p = x \cdot y$	$\frac{\Delta p}{ p } = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$
$p = \frac{x}{y}$	$\frac{\Delta p}{ p } = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$
$p = Bx$	$\Delta p = B \Delta x$
$p = Ax^n$	$\frac{\Delta p}{p} = n \frac{\Delta x}{x}$
$p = \log x$	$\Delta p = \frac{1}{2.3x} \Delta x$
$p = \sin \theta$	$\Delta p = \cos \theta \Delta \theta$

Table 4: Uncertainty Propagation Formulae

Source: PHYS2020 Linear Regression Document

7.3.2 Example of Uncertainty Propagation

All formulae used for uncertainty propagation were taken from Table 4. The following shows the uncertainty calculation for the trial averages of Table 1.

$$\begin{aligned}
 \Delta \bar{\delta} &= \frac{1}{n} \bar{\delta} \sqrt{\left(\frac{\Delta \delta}{\bar{\delta}}\right)^2} \\
 &= \frac{1}{5} 299.56 \sqrt{\left(\frac{31}{299.55}\right)^2 + \left(\frac{31}{299.52}\right)^2 + \left(\frac{31}{299.60}\right)^2 + \left(\frac{31}{299.58}\right)^2 + \left(\frac{31}{299.57}\right)^2} \\
 &\approx 0.23^\circ
 \end{aligned}$$

7.3.3 Uncertainty Propagation Formula for Wavelength

$$\Delta \lambda = \lambda \sqrt{\left(\frac{\sqrt{\Delta \delta^2 + \Delta c^2}}{\delta - c}\right)^2 + \left(\frac{\Delta m}{m}\right)^2}$$

7.4 Sodium Emission Lines

Wavelength (nm)	Transition	Colour	Energy (eV)
1139	$4s \rightarrow 3p$	infrared	-1.91 ± 0.30
819	$3d \rightarrow 3p$	infrared	-1.49 ± 0.30
592 ± 72	$5s \rightarrow 3p$	red	-0.906 ± 0.41
580 ± 71	$3p \rightarrow 3s$	yellow (bright)	-3.00 ± 0.28
570 ± 71	$4d \rightarrow 3p$	green	-0.825 ± 0.42
540 ± 69	$6s \rightarrow 3p$	green	-0.704 ± 0.43
534 ± 69	$5d \rightarrow 3p$	green	-0.678 ± 0.44
529 ± 69	$7s \rightarrow 3p$	blue-green	-0.656 ± 0.44
497 ± 67	$6d \rightarrow 3p$	blue	-0.505 ± 0.46
483 ± 67	$8s \rightarrow 3p$	violet (weak)	-0.433 ± 0.48
476 ± 66	$7d \rightarrow 3p$	violet (weak)	-0.395 ± 0.49
460 ± 66	$8d \rightarrow 3p$	violet (weak)	-0.305 ± 0.50
452 ± 65	$9s \rightarrow 3p$	violet (weak)	-0.257 ± 0.51
330.24	$4p \rightarrow 3s$	ultraviolet	-1.38
285.28	$5p \rightarrow 3s$	ultraviolet	-0.793
268.04	$6p \rightarrow 3s$	ultraviolet	-0.514

Table 5: Sodium Emission Lines Compared to Transition

The above energy values were calculated according to the following formula (rearranged and expanded from equation (4) in the laboratory notes):

$$E_n = 6.242 \times 10^{18} \left(\frac{ch}{\lambda} + (E_{n-1} \times 1.6022 \times 10^{-19}) \right) \quad (15)$$

Where each value for energy is in electron volts, and $E_1 = -5.139\text{eV}$.

7.5 $1/\sqrt{-E}$ vs n Plot for Angular Momentum States

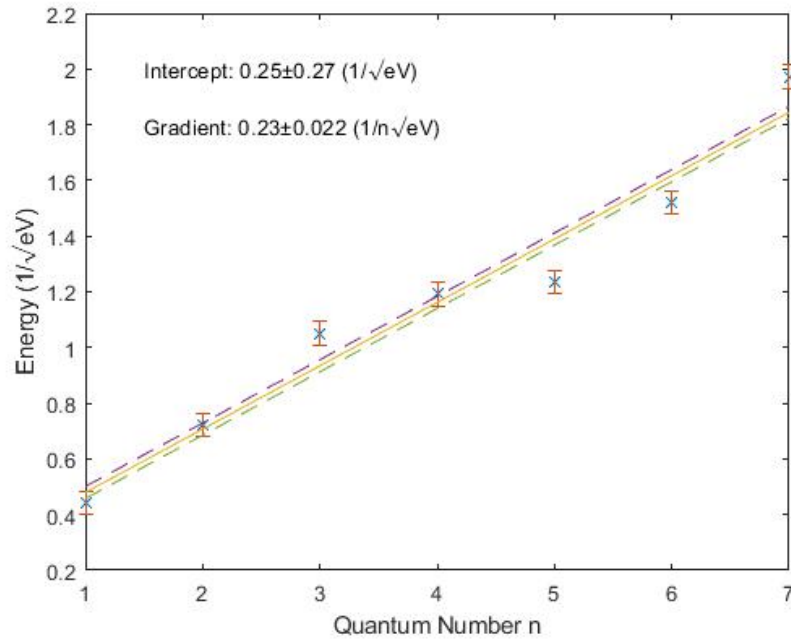


Figure 4: $1/\sqrt{-E}$ vs n for s Angular Momentum State

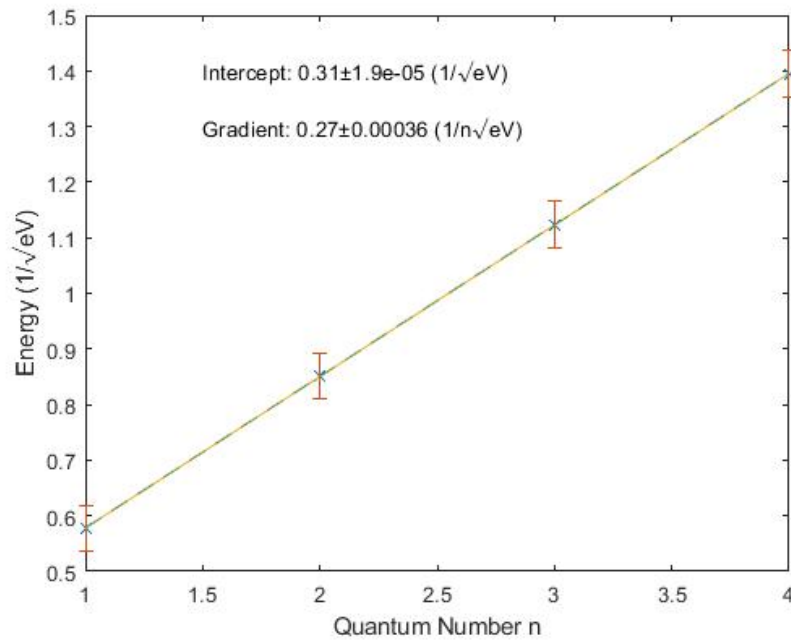


Figure 5: $1/\sqrt{-E}$ vs n for p Angular Momentum State

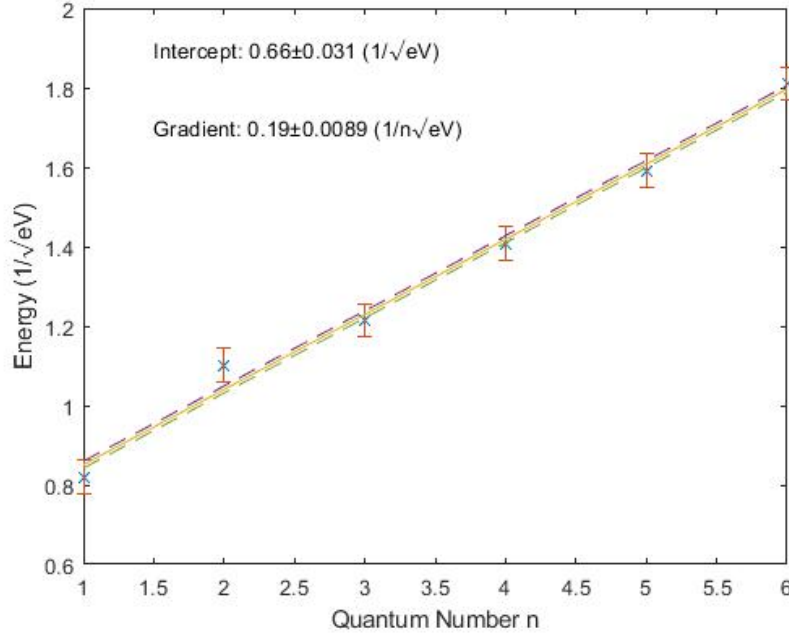


Figure 6: $1/\sqrt{-E}$ vs n for d Angular Momentum State

7.6 Quantum Defect Example Calculation

$$\begin{aligned}
 E_n &= E_{ns} \\
 R \cdot \frac{1}{(n - \delta_s)^2} &= \frac{1}{(0.23n + 0.25)^2} \\
 0.23n + 0.25 &= \sqrt{R}(n - \delta_s) \\
 &= n\sqrt{R} - \delta_s\sqrt{R} \\
 \delta_s &= n \left(1 - \frac{0.23}{\sqrt{R}} \right) - \frac{0.25}{\sqrt{R}}
 \end{aligned}$$

References

- [1] Suto, K 2018, '*Potential Energy of the Electron in a Hydrogen Atom and a Model of a Virtual Particle Pair Constituting the Vacuum*', Applied Physics Research; Vol. 10, No. 4, pp 93-101, viewed 23/09/2020, DOI:<10.5539/apr.v10n4p93>
- [2] Trypsteen, M, Walker, R, 2017, '*Chapter 4 - Types and Function of Dispersive Elements*'. Spectroscopy for Amateur Astronomers, Cambridge University Press, pp. 24-35, DOI:<<https://doi.org/10.1017/9781316694435.005>>
- [3] Knight, R.D. (2017). '*Physics for scientists and engineers: a strategic approach with modern physics (4th ed.)*'. Boston: Pearson.
- [4] Walter, F, 1998. '*A Primer on Quantum Numbers and Spectroscopic Notation*', Accessed on 23/09/2020, Available at: <<http://www.astro.sunysb.edu/fwalter/AST341/qn.html>>

- [5] Campbell, S., D'Orazi, V., Yong, D. et al. 2013. '*Sodium content as a predictor of the advanced evolution of globular cluster stars*'. Nature 498, 198–200 (2013). <https://doi.org/10.1038/nature12191>
- [6] Acousto-optic devices for optical signal processing and quantum computing - Scientific Figure on ResearchGate. Available from: https://www.researchgate.net/figure/Approximate-energy-level-diagram-for-Sodium-Naafter-7-The-arrows-represent_fig4_234799492 [accessed 25 Sep, 2020]