PHYS2082 Mystery Planet 1 Report

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1 Introduction

Upon arriving at the HD666123 system, information about the star and possible planet were immediately taken. This data was used in conjunction with the other scientists data from after the emergency displacement to calculate and infer several characteristics about the system as a whole, including the star's properties, the orbit of the planet, and the likely conditions on the surface of the planet.

2 The Star

As the scientist aboard the spacecraft rendezvousing with the HD666123 system, as much information as possible about the star (colloquially referred to as 'Twinky') and possible orbiting satellites was necessary in order to make an informed a decision whether to delve deeper into the system. The apparent position of the star in the plane of the sky was imaged immediately upon arrival, as well as after the 1200 hour emergency displacement to avoid the path of a comet. The angular displacement against the plane of the sky is shown in Figure 1:

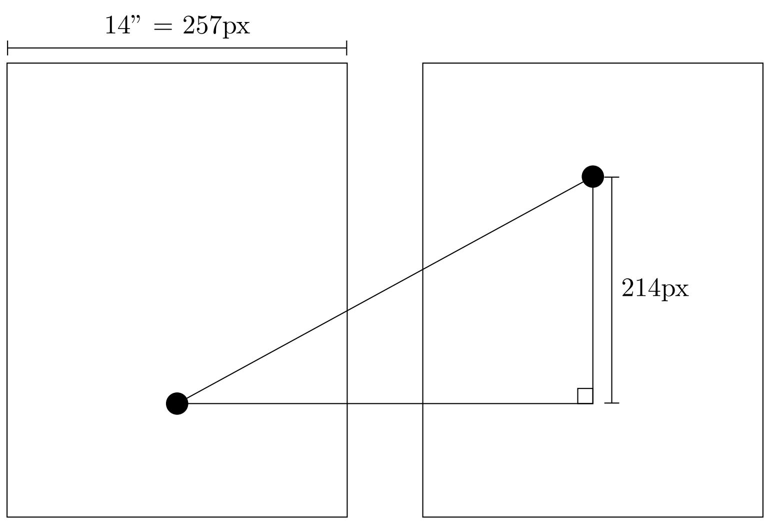

Figure 1: Comparison of Position Before and After Displacement

Since the width of each image was precisely 14 arcseconds (as restricted by the telescope used to image Twinky), the pixel width of the image was used to determine the angular displacement that the star moved by over the 2.1AU spacecraft movement. This angular separation of 214 pixels corresponded to an angular separation of approximately 12" in the plane of the sky. Assuming that the spacecraft's *radial* displacement across the 1200 hours was negligible, a top-down view of Twinky's position relative to the spacecraft was analogous to that shown in Figure 2 (not to scale):

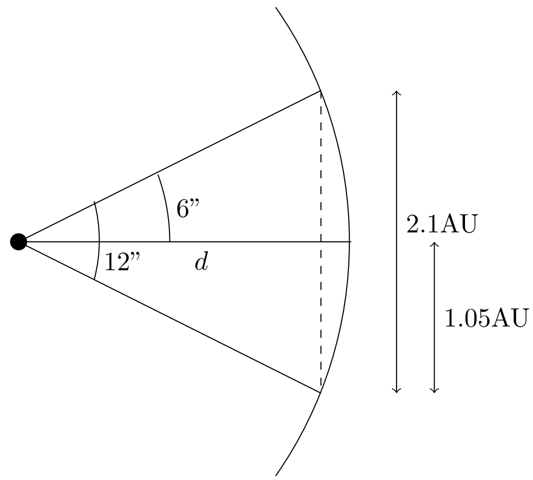

Figure 2: Observed Parallax of Twinky During Displacement

Parallax calculations are well defined, and using these the distance of the spacecraft to the star was calculated:

$$d = \frac{s}{\tan\theta} = \frac{1.05\text{AU}}{\tan(6")} = 36082.5\text{AU} \approx 3.61 \times 10^4\text{AU}$$

And so the spacecraft is situated at about 3.61×10^4 Astronomical Units from the star currently. With this distance known, the absolute magnitude of Twinky can be calculated, first by converted from AU to parsec. The distance is approximately $d = 0.175$ pc, and so the absolute magnitude, M, can be found in relation to the apparent magnitude, $m = -6.88$, by

$$d = 10^{(m-M+5)/5} \Rightarrow M = -5\log_{10}(d) + m \approx -3.09$$

And so the absolute magnitude of Twinky is approximately -3.09. The relationship between absolute magnitude and solar luminosity is well defined, and allows for the calculation of the luminosity of a an arbitrary star provided that the absolute magnitude is known:

$$M = M_{\odot} - 2.5\log_{10}\left(\frac{L}{L_{\odot}}\right) \Rightarrow L = L_{\odot}10^{\frac{M_{\odot}-M}{2.5}}$$

Given that the absolute magnitude of Sol is 4.74, the absolute magnitude of Twinky is approx. -3.09 and a Solar luminosity is 3.846×10^{26}W,

$$L = 1355L_{\odot} \approx 1.35 \times 10^3 L_{\odot} \approx 5.19 \times 10^{29}\text{ W}$$

where the value was rounded *down* in this case to help correct for the rounding *up* of the distance from spacecraft-to-Twinky calculation earlier. Twinky being about 13 hundred times more luminous than Sol is expected due to the much higher temperature and mass, and using this luminosity, the radius can be estimated by the Stefan-Boltzmann Equation, approximating Twinky as a blackbody in the process. Since Twinky is a B4-Main Sequence star, a blackbody approximation is reasonably accurate, especially compared to cooler stars such as A or F type main sequence stars (by colour-colour diagram [not shown]). This is of course the total luminosity across the total spectrum of the star. Using the Stefan-Boltzmann equation, the radius of a star is then

$$L = 4\pi R^2\sigma {T_e}^4 \Rightarrow R = \sqrt{\frac{L}{4\pi\sigma {T_e}^4}}$$

With an effective temperature of 17000K, the luminosity calculated above, and the Stefan-Boltzmann constant, this corresponds to a radius of

$$R \approx 2.95 \times 10^9 \text{ m} \approx 4.24 R_\odot$$

which is well within a typical range for a mid-B class star such as Twinky. Following on with the blackbody approximation, the peak emission wavelength for the star was calculated to determine the nature of the radiation incident on a possible planet. Wien's displacement law was used to infer the peak emission wavelength for the effective surface temperature of 17000K:

$$\lambda_{\max} T = 0.002897755 \text{ mK} \Rightarrow \lambda_{\max} \approx 170.46\text{nm}$$

This places the most dominant emission wavelength in the low-mid range of the Ultraviolet spectrum - highly energetic photos which are known to ionise molecules. The radius value was used further along in the report to help calculate the temperature of the planet at which the radio source originated, and the peak emission wavelength used in formulating hypotheses regarding the planet's environment. Before such ideas were discussed, however, many characteristics about the observed "dot" had to be determined first.

3 The Planet's Orbit

As with how the distance from the spacecraft to the star was calculated, the distance from the planet to the star was determined through angular separation values by pixel counting. Figure 2 from the information handout showed a picture of the planet relative to Twinky from the perspective of the spacecraft. The image was quoted as being 0.55 degrees in width, with a corresponding pixel width of 378px. The approximate angular distance between the centres of Twinky and the dot was determined to be 168px, which corresponded to approximately 0.27 degrees, or 4.71×10^{-3}rad.

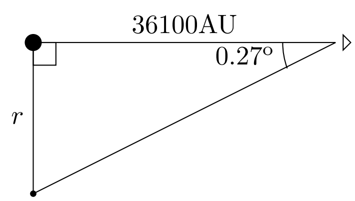

Figure 3: Angular Separation of Twinky and the Planet

Figure 3 to the left shows the system of objects graphically, with the rightmost triangle representing the spacecraft perspective, the large dot Twinky, and the small dot the planet. The simple trigonometric formula to calculate the orbital radius of the planet (before the emergency displacement) is $r = d \tan\theta$, where θ is the aforementioned angular displacement and d the spacecraft 'altitude'. This equation results in a orbital radius of approximately 170AU $\approx 2.54 \times 10^{13}$m.

Now that the planet's orbital radius is known, the distance of the spacecraft from the planet wad determined by simply using pythagoras from the setup in Figure 3. This yielded a distance of 36100.4AU, or just 36100AU when accounting for significant figures. During the emergency displacement, the planet moved around Twinky by 0.13" relative to the spacecraft. A graphical representation of the setup is shown in Figure 4, where D is the distance the planet moved over the 1200 hour period.

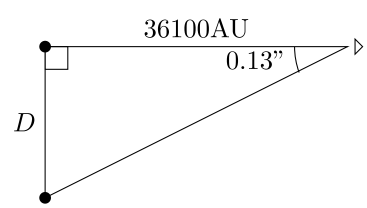

Figure 4: Shift in Planet Position over Spacecraft Displacement

The value of D was calculated to be

$$D = 36100 \times \tan\left(0.13" \times 4.85 \times 10^{-7} \frac{\text{rad}}{\text{arcsec}}\right) \text{ AU} \approx 3.4 \times 10^9\text{m}$$

Over the 1200 period, this displacement corresponded to a velocity of

$$v = \frac{D}{t} = \frac{3.4 \times 10^9\text{m}}{1200 \text{ hours}} = 787\text{m/s} \approx 7.9 \times 10^2\text{m/s}$$

and by a derivation from Kepler's second law, the semi-major axis of an orbit can be found from the instantaneous velocity and radius. Assuming the mass of the planet is negligible compared to Twinky (a suitable approximation, as the mass of the Earth is about a *500 thousandth* that of Sol), $m + M \approx M$, where m is the mass of the planet and M Twinky. Applying this approximation to Kepler's second law and rearranging for semi-major axis of the orbit gives

$$v^2 = G(m+M)\left(\frac{2}{r} - \frac{1}{a}\right) \approx GM\left(\frac{2}{r} - \frac{1}{a}\right) \implies a = \frac{rGM}{2GM - rv^2}$$

Substituting in known values (where $M = 6.8M_{\odot}$) gives a semi-major axis of $a \approx 1.28 \times 10^{13}\text{m} = 85.6\text{AU}$. Knowing this value opened up a range of calculable parameters regarding the planet's orbit. Since the planet was moving at a right angle in it's orbit during the spacecraft's displacement (FIgure 3 in the information handout), the planet was either at aphelion or perihelion. Since $r > a$, this gives $r_a = 170\text{AU}$. The eccentricity of the orbit can be found from the relation $r_a = a(1+e)$. Rearranging for e and substituting in the aphelion and semi-major axis gives an eccentricity of $e = 0.986$. From this, the perihelion was calculated from $r_p = a(1-e) \approx 1.20\text{AU}$. Graphically, these parameters correspond to an orbit shown in Figure 5:

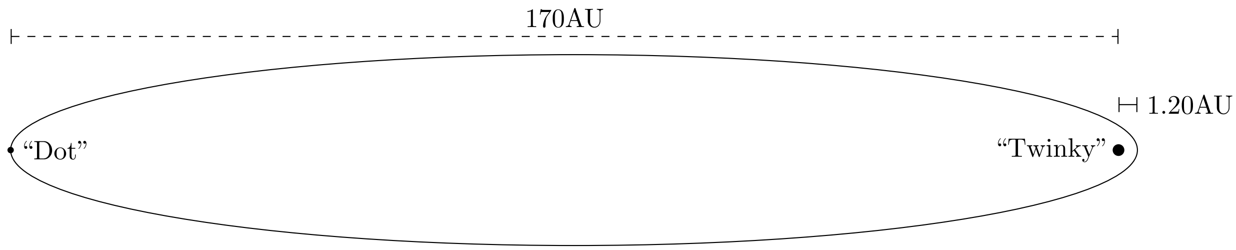

Figure 5: Calculated Orbit of The Planet

From Kepler's third law, the period of the orbit can be calculated by once again making the negligible planet mass approximation. This is

$$P^2 = \frac{4\pi^2}{G(m+M)}a^3 \Rightarrow P \approx \sqrt{\frac{4\pi^2}{GM}a^3}$$

and substituting in known values gives a period of $P \approx 9.58 \times 10^9\text{s} \approx 304$ years.

With so much known about the star and planet, the conditions on and around the planet can be inferred to help make an informed decision on whether to move closer.

4 Conditions on the Planet

The photometry data shown in the handout sheet table shows a clear decline in the Twinky luminosity with increasing wavelength from the visible onwards, as expected by a planck curve with a peak emission line in the ultraviolet. Assuming that the planet reflects all light in the visible and near-infrared spectrum, the emission spectrum of the planet is shown in Figure 6:

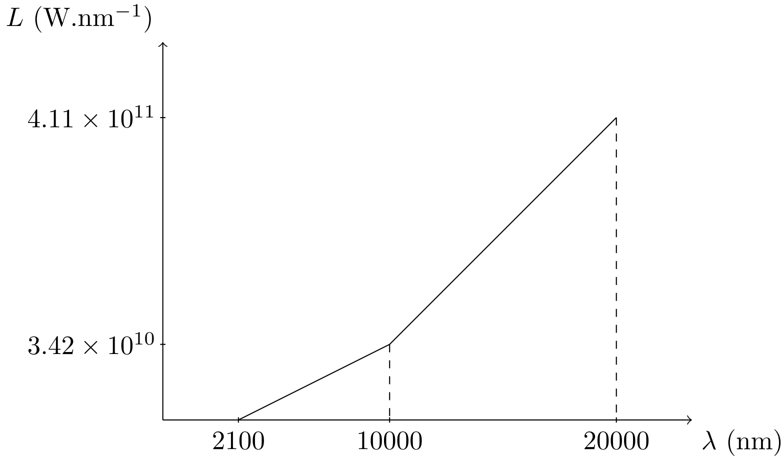

Figure 6: Approximated Luminosity Curve of The Planet (Not Scale)

By using the trapezoidal rule, the luminosity across these wavelengths can be found. First, however, approximating the planet as blackbody allows the peak emission line to be found. If the planet is a blackbody in equilibrium, it will absorb all incident energy and re-emit it, thus having an albedo of $A = 0$. This provides an upper bound on the temperature of a planet through

$$T_P = T_s(1 - A)^{1/4}\sqrt{\frac{R_s}{2D}}$$

where T_s and R_s are the temperatures and radius of the star respectively, and D the orbital radius. With the orbital radius of $D = r_a$ and other known parameters, setting $A = 0$ gives an upper bound temperature of $T_P \approx 130$K. In reality, the albedo of the planet will have some effect on this, albeit a small one due to the quarter power that minimises most of the effect for $A \ll 1$. The peak emission line of the planet at this upper bound temperature can be found using Wien's displacement law, yielding a peak at approximately 22300nm. As this number is relatively close to the far right value in Figure 6, the total luminosity was approximated as being double that of that in the figure above. While this was an ambitious approximation, it would provide a more valuable final temperature than an upper bound would offer. Using this approximation, it was assumed that all (or at least the majority) of visible light was reflected from the surface. Looking at the B band specifically, the dot has a luminosity of $L_B = 9.39 \times 10^{12}$W. With this in mind, the radius of the planet was estimated using the incident flux from the star unto the planet, and solving for the radius necessary to reflect the amount of light seen. The luminosity of an object is related to it's flux by

$$F = \frac{L}{4\pi R^2}$$

where r is the distance from the object, and flux is measured in W/m^2. Since the luminosity of Twinky in the blue band is 3.11×10^{26}W, the flux at 170AU is

$$F = \frac{3.11 \times 10^{26}}{4\pi(2.54 \times 10^{13})^2} \approx 0.0384 \text{ W/m}^2$$

Since only a cross section of a sphere will be *fully* illuminated by the star (as opposed to a hemisphere as one might expect), effectively only one quarter of the planet's surface will be reflecting and absorbing energy from the star. Since flux is defined as the average across the entire surface, the flux with respect to the planet is then $F_p = \frac{1}{4}F_I$, where F_I is the incident flux (from Twinky). Using the same definition of flux as above, the radius of the planet can be estimated:

$$F = \frac{L}{4\pi R^2} \Rightarrow R = \sqrt{\frac{L}{\pi F_I}} = \sqrt{\frac{9.39 \times 10^{12}}{\pi(0.0384)}} \approx 8.82 \times 10^6\text{m}$$

With the planet radius of approx. 8 820km (about $1.3R_{\oplus}$), the albedo can then be calculated and eventually a more accurate temperature estimated. Firstly, the total irradiance across the planet's surface is found by the incident flux (across all wavelengths) multiplied by the planet's cross section. Substituting values gives

$$Ir = \frac{L}{4\pi d^2}\pi R^2 = \frac{5.19 \times 10^{29}}{4(2.54 \times 10^{13})^2} \times (8.82 \times 10^6)^2 \approx 1.56 \times 10^{16}\text{W}$$

The $L/(4\pi d^2)$ component of the above calculation is the flux (from all wavelengths) being shone upon the planet's upper atmosphere (not necessarily the surface, as any atmosphere would absorb and scatter some fraction of radiation). This fraction yields a flux of approximately 201 W/m^2, which is a small fraction of the radiation earth's upper atmosphere experiences (approx 1361 W/m^2 [1]). Worth noting is that Sol's emission spectrum peaks in the visible, while Twinky peaks in the ultraviolet, so a much higher amount of ultraviolet light as a proportion of total radiation is experienced at the top of any atmosphere present.

Again assuming that the planet reflects all visible and near-infrared light, the reflected luminosity (from photometry data) is

$$\begin{aligned} Re &= \frac{2100 - 500}{2 \times 2}(9.39 \times 10^{12} + 6.42 \times 10^{10}) \text{ W} \\ &\approx 3.78 \times 10^{15} \text{ W} \end{aligned}$$

This approximation isn't ideal, since not all visible light will be reflected, and light from other wavelengths *will* be reflected, but the errors will somewhat cancel each other out. As a result, the estimate for albedo (and consequently temperature) won't be exact, but a good ballpark figure. Since the albedo is calculated as reflected light over incident light, the albedo of the planet at it's 170AU position is

$$A = \frac{Re}{Ir} = \frac{3.78 \times 10^{15} \text{ W}}{1.56 \times 10^{16} \text{ W}} \approx 0.242$$

This corresponds to a temperature of

$$T_P = T_s(1 - A)^{1/4}\sqrt{\frac{R_s}{2D}} \approx 122\text{K}$$

All conditions so far have been calculated regarding the planet at 170AU, however the perihelion is at a mere 1.20AU. Using the same formulae as above (with $A = 0.242$) results in a perihelion temperature of approx 1440K, and an incident flux of 1.28×10^6 W/m^2.

To speculate how the conditions on the surface compare to that of Earth, the extreme temperatures and high UV flux suggests that life as we know it could not exist on the *surface*. The above perihelion temperature assumes that albedo is constant throughout the entire orbit, when in reality liquid and solid compounds on the surface at aphelion would evaporate as the planet moves closer to the star, contributing to a higher albedo and a greenhouse effect, thereby making the planet even hotter. As such, the 1440K temperature could be taken as a *lower* bound on the perihelion temperature. I would propose that the atmosphere would be devoid of typical radiation shielding molecules, such as ozone, etc, due to the sheer amount of ionising radiation that would destroy the molecules before they might replenish from natural/organic processes.

5 Summary

Through the data gathered from the various officers on the ship, Twinky, a B4-class star of 6.8 Solar masses, was found to have a radius of about 4.24 Solar radii, a luminosity of approx 1350 Solar luminosities and an absolute magnitude of -3.09. The spacecraft was determined to be approx 36100AU from the star as measured through parallax methods. Since the star's peak luminosity was found to be about 170nm (in the ultraviolet), life as we know it close to the star was ruled out. A planet was found to be in a distant orbit from the star, with a semi-major axis of 85.6AU and eccentricity of $e = 0.986$. This corresponded to an aphelion of 170AU and a perihelion of 1.20AU, as well as a period of about 304 years. At aphelion, the planet was found to have an albedo of about 0.242, which resulted in a temperature of 122K. At this distance, the flux on the upper atmosphere was found to be about a sixth of that that Earth experiences, although with a higher proportion of ionising radiation. At perihelion, the planet was inferred to have a temperature of *at least* 1440K, with an incident flux of almost one thousand times that of Earth. Due to this, it was concluded that life was we know it has no chance of sustaining itself on this planet. While it is safe to travel to the planet in it's current position (assuming sufficient radiation shielding), the planet closer to the star will not be safe for the spacecraft. That said, the planet's radius was found to be 1.3 Earth radii, and assuming it has similar terrestrial density, would have manageable surface gravity.

In conclusion, it is safe to approach the planet at it's current position but it is unlikely that there will be life as we know it that had originated on the planet.

References

[1] Coddington, O.; Lean, J. L.; Pilewskie, P.; Snow, M.; Lindholm, D. (22 August 2016). "A Solar Irradiance Climate Data Record". *Bulletin of the American Meteorological Society.* doi:10.1175/bams-d-14-00265.1