PHYS3020 Lab Report 2
Blackbody Radiation

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Abstract

Using a tungsten-bromide lamp with varying voltage supply, we find that an ideal blackbody distribution can be approximated excellently at 7V input. This conclusion was supported via a statistical analysis of data obtained using two independent spectrometers sensitive to the visible and near infrared spectrum. We found that, over a range of 2 to 7 volts, the tungsten lamp displayed colour temperatures (derived from a best fit least squares model) of about 1700K to 2700K, and we derived similar values for effective temperature independently. This approximation of an ideal blackbody was determined to be most valid in the spectral domain of $1000 \leq \lambda \leq 1750$nm independent of supplied voltage. In the process, we derived a value for Boltzmann's constant that was 1.2σ out of agreement with the defined value. After minimisation of error, we refined the calculated value to $k = (1.3808 \pm 0.0019) \times 10^{-23}$ J.K^{-1} which is within 0.01% of the accepted value. Some recommendations were made for future studies which involved using an identical methodology, just with diversified spectral lamps which we argue would further reduce uncertainty in this value of Boltzmann's constant.

1 Introduction

Any body in the universe that has temperature is subject to emit light in a portion of the electromagnetic spectrum – a constraint required by statistical and quantum mechanics that keeps matter in equilibrium. Perhaps the most spectacular examples of these blackbody spectra are the stars, where our Sun emits the light that we see in an approximation to an ideal blackbody spectrum [1] (which lead to a chain of events that has ended in you reading my laboratory report). Since blackbodies are so prominent and fundamental to the workings of the universe, having a solid grounds of understanding is imperative in any hope of doing science in the universe and is of particular importance in the field of astrophysics.
The most perfect approximation to the blackbody spectrum in nature is a remnant of the Big Bang itself: the Cosmic Microwave Background (CMB) [6], with which we don't have any chance of studying with an undergraduate budget [5]. Analysing more imperfect 'blackbodies', however, is a suitable test-case to theory which is ultimately applied to more astronomical examples. Such an approximation to an ideal blackbody is that of a Tungsten-Bromide lamp, which radiates primarily in the infrared spectrum when supplied with a sufficient voltage [5]. In this study, we aim to analyse just how well this lamp approximates a blackbody over a range of supplied voltages, and with that data, make inferences as to the inner workings of the universe and to lamps.

2 Theory

The equation that describes an ideal blackbody as described by Planck [5] is given as

$$r_{bb}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{ch/\lambda kT} - 1} \tag{1}$$

which is dependent (as a variable) only on the temperature of the blackbody and the wavelength being observed. The Planck spectrum is characterised by a single wavelength at which the radiance peaks, given by

$$\lambda_{\max} \simeq 0.2014 \frac{ch}{kT} \tag{2}$$

which can be used to determine Boltzmann's constant given that the temperature is known to high enough precision [5]. Equation (1) is of course an idealisation, and what we observe in practice is the 'real' radiant emittance $r(\lambda, T)$. This disparity in the 'real' vs ideal radiant emittance begs the question as to how well some source approximates a blackbody, which can be quantified via the emissivity ε,

$$\varepsilon(\lambda, T) = \frac{r(\lambda, T)}{r_{bb}(\lambda, T)} \tag{3}$$

The closer to order unity ε is, the more closely a light source approximates an ideal blackbody in distribution. Naively calculating equation (3) will no doubt give a very small number, as the detected radiant emittance is heavily dependent on the conditions inherent in the experimental system (i.e. on the distance of the detector from the lamp, the relative angles, etc). Thankfully, the detected emittance can be arbitrarily scaled by some factor $\mathcal{F}$ so that the observed power most closely resembles that predicted by equation (1). Hence, our method of calculating emissivity is

$$\varepsilon = \frac{\mathcal{F}\ r(\lambda, T)}{r_{bb}(\lambda, T)} \tag{4}$$

where $\mathcal{F}$ is taken to be such that the maximum emissivity of the Tungsten-Bromide lamp at 7V input is equal to 1. With our experimental setup, we measure the radiant emittance of the source relative to that at 7V, and so the actual emissivity we calculate is

$$\varepsilon' = \frac{\varepsilon}{\varepsilon_7} \tag{5}$$

where the dash indicates the relative measurement. In practice, we expect that $\varepsilon_7 \simeq 0.329814$ [5].
If the light source approximates a blackbody sufficiently well and consistently, we'd expect that the emissivity is independent of wavelength, and so $\varepsilon(\lambda, T) \approx \varepsilon(T)$, which can be calculated by taking the mean of the emissivity-wavelength data. For metals, we expect the emissivity to take the form of

$$\varepsilon(T) \sim 1 - \exp(-\delta T) \tag{6}$$

where δ is a material-dependent constant (and is given as $(1.50 \pm 0.05) \times 10^{-4}$ K^{-1} for tungsten [5]). Making the substitution $\varepsilon(T) \approx \varepsilon' \cdot \varepsilon_7$ (from equation [5]) into the above equation, and then rearranging, we obtain

$$\exp(-\delta T) \approx 1 - \varepsilon' \cdot \varepsilon_7 \tag{7}$$

$$\ln\left(\frac{\exp(-\delta T)}{\varepsilon_7}\right) \approx \ln\left(\frac{1}{\varepsilon_7} - \varepsilon'\right) \tag{8}$$

$$\Rightarrow \ln\left(\frac{1}{\varepsilon_7} - \varepsilon'\right) \simeq -\delta T - \ln(\varepsilon_7) \tag{9}$$

Equation (9) is of the form of a linear equation, which effectively measures how well the lamp approximates a blackbody at a given temperature. The closer that the data of the form of the left side approximates the relationship given in the right side, the better is represents an ideal black body.

All of the above derivation relies on knowledge of the underlying temperature of the blackbody data. In practice, there are multiple methods of determining this temperature. In the scope of this report, we chose to consider temperature derivations from the best-fit temperature to the data, and a calculated solution given the voltage and current observed from the lamp. The former temperature derivation pertains to the so-called colour temperature, who [3] defines as *"the temperature at which a Planckian radiator would emit radiant energy competent to evoke a color of the same quality as that evoked by the radiant energy from the source in question."* By the same source, we expect that the colour temperature be slightly higher than the actual temperature of the filament, although in the low temperature regime the difference won't be significant.

In order to calculate the temperature of the light-emitting filament, [4] details a polynomial relationship between the resistance in the tungsten filament (relative to a reference value) and the associated temperature,

$$\frac{R(T)}{R_0} = -0.524 + 0.00466T + 2.84 \times 10^{-7}T^2 \tag{10}$$

where $R_0 = 0.84\Omega$ is the reference resistance at 300K [5]. By the fundamental relationship $V = IR$, we can replace $R(T)$ by $R(T) = V/I$ and so

$$0 = \left(-0.524 - \frac{V}{I \cdot R_0}\right) + 0.00466T + 2.84 \times 10^{-7}T^2 \tag{11}$$

Since equation (11) represents a concave-up parabola which has its stationary point in the negative half-space, there will only be one positive root of the equation (and hence only one valid solution for the temperature). The resultant value for temperature given some defined voltage and observed current is

$$T_{\text{calc}} = \max\left(\text{root}\left[\text{eq. (11)}\right]\right) \tag{12}$$

which can be solved using the quadratic formula as a root-finding function.
As for which temperature should be used to calculate the emissivity in equation (4), we'd want to compare to an ideal blackbody of the same colour and radiant energy and so the best-fit (i.e. colour) temperature should be used.

3 Experiment

The apparatus used consisted of a tungsten-bromide lamp within a closed box that has an opening on its side where two independent spectrometers were located. The first spectrometer was sensitive to wavelengths of 300 to 1100nm, while the second spectrometer was sensitive to light in the range of 900 to 1750nm. The spectrometers were connected to both a power supply and a desktop PC with the Avasoft 8 software, which was used as an interface for the data collection. The output data was the merged and normalised results of each spectrometer.

3.1 Method

Calibration Process: To begin with, we calibrated the experimental setup against a 7V reference spectrum.

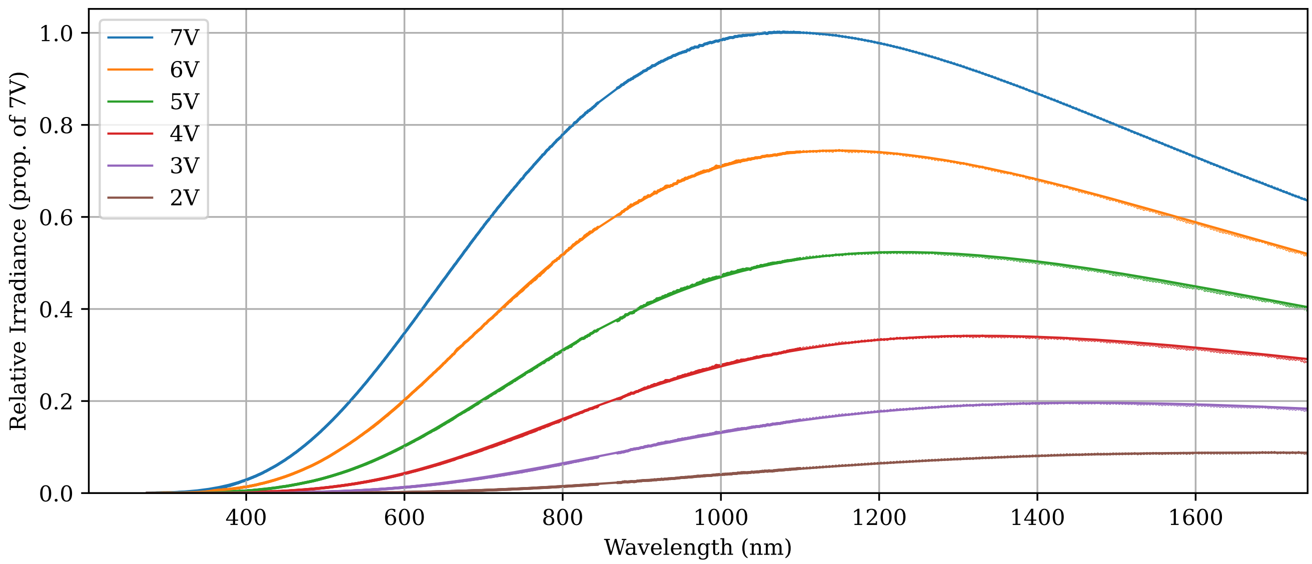

Figure. 1 Blackbody data and fitted curves for each tested voltage. Each voltage set of data was fitted according to the theoretical blackbody spectrum equation using a least squares fitting method. All data points in the range $840 \leq \lambda \leq 868$nm were omitted due to significant instrument error. Ignoring these points would not have affected the fit due to the broad domain of data available.

This involved first turning all of the equipment on, but having the voltage source at 0V so that the lamp was still turned off. We recorded the 'darks' value for the background radiation signature in Avasoft, and then turned the lamp on at a 7V supply. After leaving the lamp to heat up for about a minute, we set the spectrum as the reference spectrum in the software so that all subsequent measurements would be taken relative to this reference. This was, of course, assuming that the spectrum at 7V most closely approximated a blackbody compared to other supplied voltages. Worth noting is that we took the average radiance values across 400 trials, each with 20ms integration time (which was chosen so that we would observe the recommended intensity according to the manual).
We recorded the input voltage and the observed subsequent current to calculate the approximate temperature of the lamp according to equation (12), and input this as the temperature reference in the software.

Data Collection: The above calibration step effectively served as the first data point in the experiment. The radiance values were saved as a `.csv` file, and then the lamp was turned off by setting the voltage to 0. We then recorded the darks once more and saved them in the system. The lamp was then turned on, but only supplied with 6V (i.e. one less volt than before). After waiting about a minute, we saved the radiance data again. This process (turn off, take darks, turn on and wait, record data) was repeated for each subsequent decrease in volts up to a minimum of 1V.

4 Results

The relative irradiance curves (the main data) of the tungsten lamp at different voltages are shown in Figure 1. We see that the data is fit extremely well by a blackbody curve at each voltage, with characteristic temperatures shown in Table 1, compared against temperatures calculated by equation (12) (and data from Table 2).

Voltage (V)	Temperature (K)	
	Best-Fit	Calculated
7	2671.6 ± 0.06	$(2.65 \pm 0.003) \times 10^3$
6	2524.2 ± 0.2	$(2.51 \pm 0.005) \times 10^3$
5	2361.1 ± 0.3	$(2.35 \pm 0.003) \times 10^3$
4	2182.3 ± 0.4	$(2.16 \pm 0.003) \times 10^3$
3	1979.0 ± 0.4	$(1.97 \pm 0.008) \times 10^3$
2	1724.5 ± 0.4	$(1.70 \pm 0.004) \times 10^3$

Table 1 Best fit and calculated temperature values for the irradiance spectrum of the tungsten lamp with varying voltage. Uncertainties in the best-fit temperatures were given in the covariance matrix output by the Python code used to fit the model to the data (using a weighted least squares method). The uncertainty in calculated temperature values were derived from uncertainty propagation laws, and were increased in magnitude when substantial rounding in the main value occurred (to account for significant figure consistency).

Using the blackbody fit at 7V, we obtained a scaling parameter of

$$\mathcal{F} \simeq 1.7453 \times 10^{10}$$

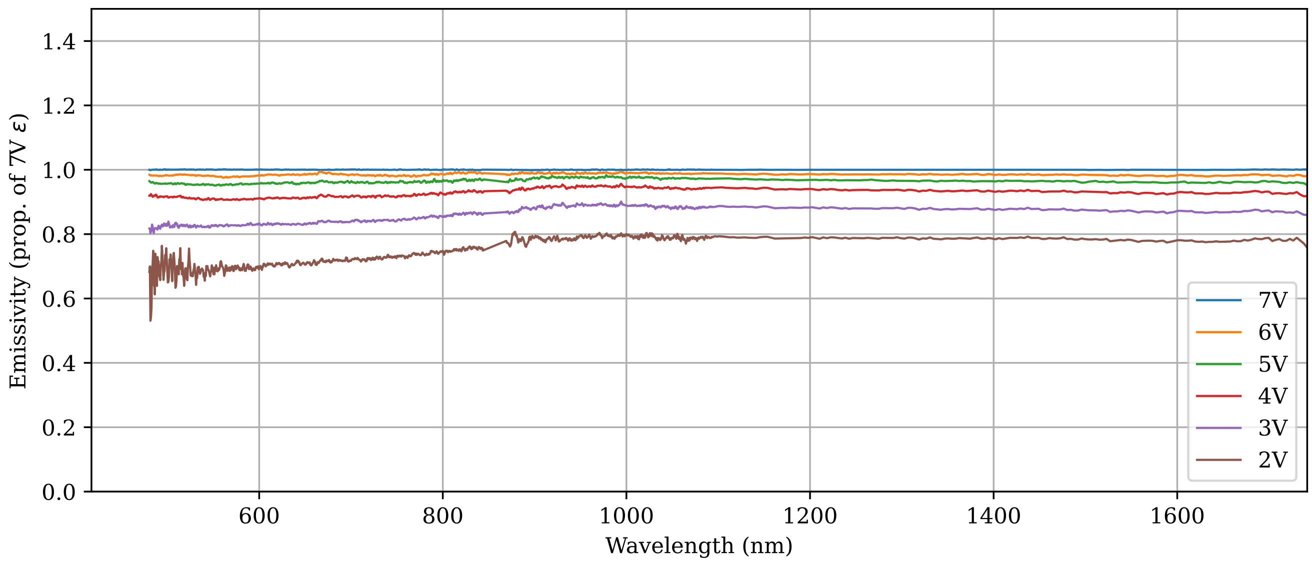

Figure. 2 Emissivity data for each voltage set of data, calculated from the data divided by the theoretical blackbody curve at that wavelength. The data was truncated at a wavelength of 480nm due to significant deviations in the emissivity from the expected range [0, 1], particularly for the lower voltage tests.

in order to match the data in magnitude with that of the idealised blackbody spectrum. This value was used in all subsequent fitting.

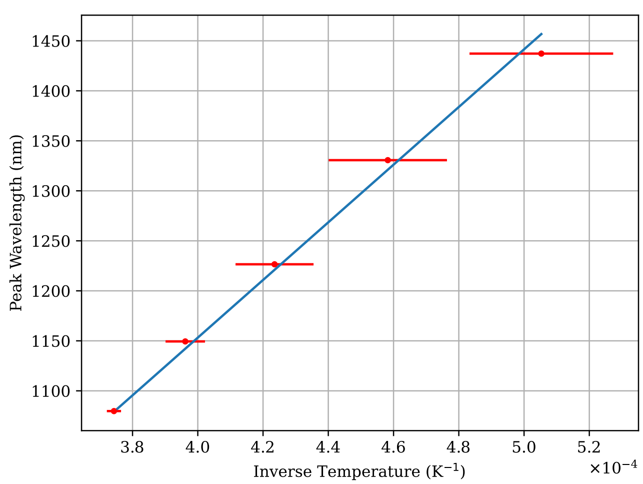

Figure. 3 Peak wavelengths of each voltage distribution against inverse temperature. The temperatures of each point correspond to the best-fit temperature, where the uncertainty was propagated according to well-defined propagation laws. The peak wavelengths of each distribution were taken to be the wavelength at the point of maximum radiance. The trendline was fit with parameters determined from a weighted least-squares fitting of a linear model to the data, where the weighting took into account uncertainty in x data and y data (determined via the values of the points around the chosen maximum).

By plotting the peak wavelengths against the inverse of temperature, we effectively have a method of calculating Boltzmann's constant. By equation (2), we would expect the gradient of such a trend to be $m \simeq 0.2014ch/k \Rightarrow k \simeq 0.2014ch/m$.
With Figure 3, we directly fit the above relation to obtain a value of Boltzmann's constant of

$$k = (1.3877 \pm 0.0059) \times 10^{-23} \text{ J.K}^{-1} \tag{13}$$

which is in fair agreement with the literature. This value is approximately 1.2σ out of agreement with the defined value of 1.380649×10^{-23} J.K^{-1}, and the discrepancy is likely the result of the large uncertainty in the low voltage (and hence low temperature) data. Recommendations for future studies are discussed in the Discussion section.

Plotting the emissivity according to equation (5) yields the curves seen in Figure 2. We immediately see that higher voltages approximate a blackbody much better over all plotted wavelengths, with significant deviation only beginning at around the 4V mark. While this is true for the plotted values, this wasn't what was seen across all *observed* values, where the emissivity began to fluctuate even for higher voltages at lower wavelengths. We then conclude that the tungsten lamp approximates a blackbody most effectively in the spectral range $1000 \leq \lambda \leq 1750$nm (where a lower bound of 1000nm results from the rough position where lower voltage emissivities are no longer wavelength dependent, and an upper bound of 1750nm from the truncation of data). At higher voltages, this lower bound can be relaxed to roughly 450nm as the emissivity has only a very weak wavelength dependence in that domain (for

$V \geq 5$V).
To get a better measure of how well the tungsten lamp approximates a blackbody, we can plot temperature and emissivity values according to equation (9), as is shown in Figure 4.

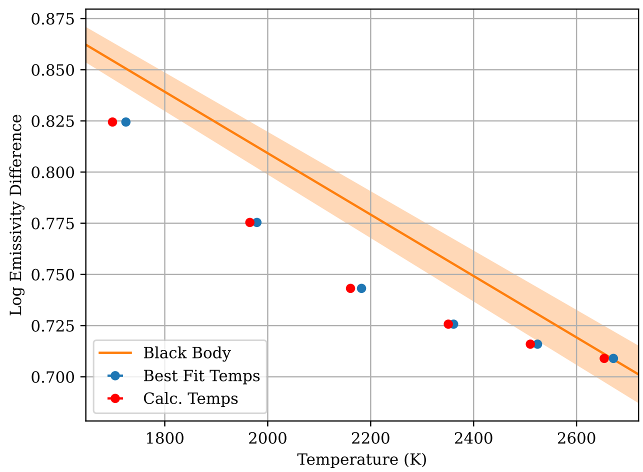

Figure. 4 Log emissivity of of the tungsten-bromide lamp at varying temperature. The temperatures of each point could, in principle, be calculated by two independent methods (the best fit temperature of the data [Figure 1] and the calculated temperature given by eq. (12)), and so were both plotted for comparison. y-values of each point were calculated according to equation (9). Negligible error was assumed for the log emissivity data, and the inferred error for temperature is too small to be resolved on this scale (where [4] quotes an error of order 0.1% in the temperature calculations, and the best fit temperatures were of an order of magnitude smaller at worst). The ideal black body fit is of the form of equation (9), with $\delta = (1.50 \pm 0.05) \times 10^{-4}$ K^{-1} as in [2], and $\varepsilon_7 = 0.329814$ as in [5].

In the above plot, the closer the data points are to the solid orange line effectively corresponds to how well the tungsten lamp approximates a blackbody at that temperature. Comparing this with the best-fit uncertainty values in Table 1 supports this, where the lower uncertainties correspond to temperature points closer to the ideal trend in the figure. Overall this agrees with and supports previous figures in that the lamp is most accurate to a blackbody in spectrum when supplied with 7V.

5 Discussion

Due to the sensitivity of the equipment and careful preparation of the apparatus by the laboratory tutors, most physical sources of error were minimised. As mentioned earlier, there was a significant error in the first spectrometer (sensitive to 300nm to 1100nm wavelengths) where it would record almost no data in the range of $840 \leq \lambda \leq 868$nm. As such, this data was omitted from all subsequent analysis, but it was taken that this had negligible effect on the model parameters due to the low uncertainty in the resulting fit.

Even though the uncertainty was low at *worst*, the lower voltage data fits were not quite precise enough to yield the accepted value of Boltzmann's constant within uncertainty. We suspect that the data point pertaining to the 2V trial had a significant effect on the derived value, and it can be seen in Figure 3 as being the only point to the right of the linear fit – lowering the fit gradient and hence effectively increasing the derived value of Boltzmann's constant above the accepted value. As a test of this, we computed the value of Boltzmann's constant ignoring this last point as

$$k = (1.3808 \pm 0.0019) \times 10^{-23} \text{ J.K}^{-1} \tag{14}$$

which is well within uncertainty of the accepted value, as expected. In future studies, we'd recommend using different elemental lamps (in conjunction with the tungsten lamp used in this experiment) that are optimised to approximate blackbodies at lower and/or higher voltages (and hence temperatures) in order to get more data to plot on a figure analogous to Figure 3.

Aside from this, we spent slightly more time in the data collection phase of the experiment to average over more data in an effort to reduce outlier effects. We allude to this in the methodology section of the report, in that we chose 400 trials to average over instead of the minimum required ~ 50 for an acceptable signal-to-noise data sample.

By observation of Figure 2, it's clear that the tungsten-bromide lamp is an excellent approximation to an ideal blackbody at 7V input power. This validates the fundamental assumption made prior to data collection that setting the reference spectrum as 7V is a suitable approximation to an ideal curve.

6 Conclusions

By analysing the emission spectrum of a tungsten-bromide lamp supplied with different voltages, we assessed the effectiveness of approximating a thermal lamp as an ideal blackbody. In this case, we found that when supplied with 7V, the tungsten lamp was an excellent approximation and was still a good blackbody for even as low as 5V. At lower voltages, the emissivity was subject to non-negligible deviations from order unity with respect to wavelength.
From the data obtained, we inferred the temperature of the lamp both from the colour temperature of the data (i.e. the best fit temperature using the model) and by using resistance-temperature relations from the literature. These two methods were completely independent from each other, and the derived temperature results were consistent with the underlying theory – it was found that colour temperature was consistently, but only slightly, larger than the effective temperature.

Using the peak observed wavelength in the spectral data, we also obtained an experimentally derived value of Boltzmann's constant using Wien's law. The originally derived value using all of the data was out of agreement with the model, but after discussion of error, we obtained a more accurate value that aligned excellently with the accepted value.

For future studies we recommended using different spectral lamps that are effective at different voltages (and hence temperatures) to get a more accurate value of Boltzmann's constant. By doing this, the error due to wavelength-dependent accuracy would be minimised, since our data suffered some error at lower temperature distributions.
We expect that the obtained blackbody distributions and values of Boltzmann's constant would be of suitable use in an astrophysical context to describe observed phenomena in the universe.

References

[1] Dale A. Ostlie Bradley W. Carroll. *An Introduction to Modern Astrophysics. 2nd Edition.* Pearson Addison Wesley, 2007. ISBN: 0-321-44284-9.

[2] M.A. Bramson. *Infrared Radiation: A Handbook for Applications.* New York: Plenum Press, 1968. URL: http://link.springer.com/book/10.1007%5C%2F978-1-4757-0911-7.

[3] Raymond Davis. "A Correlated Color Temperature for Illuminants". In: *Bureau of Standards Journal of Research* 20.4 (1931), pp. 659–681. DOI: 10.6028/jres.007.039.

[4] Charles de Izarra and Jean-Michel Gitton. "Calibration and temperature profile of a tungsten filament lamp". In: *European Journal of Physics* 31.4 (June 2010), pp. 933–942. DOI: 10.1088/0143-0807/31/4/022. URL: https://doi.org/10.1088/0143-0807/31/4/022.

[5] Taras Plakhotnik. *PHYS3020 STATISTICAL MECHANICS Laboratory - Blackbody Radiation Experiment.* 2020.

[6] Peter Schneider. *Extragalactic Astronomy and Cosmology: An Introduction. 2nd Edition.* Springer, 2014. DOI: 10.1007/978-3-642-54083-7.

7 Appendices

7.1 Appendix 1 - Supplementary Data

Voltage (V)	Current (A)
7.00 ± 0.01	0.602 ± 0.002
6.00 ± 0.01	0.551 ± 0.002
5.00 ± 0.01	0.496 ± 0.002
4.00 ± 0.01	0.438 ± 0.002
3.00 ± 0.01	0.367 ± 0.002
2.00 ± 0.01	0.290 ± 0.002
1.00 ± 0.01	0.194 ± 0.002

Table 2 Input voltage and observed current data for each trial in the experiment. Uncertainty was determined based off of the minimum precision inherent in the data, and also any slight fluctuation in the observed value across data-taking.