PHYS4080 Content Notes

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1 Week 1

Ch 8.7 – Cosmological Parameters

8.7.1 – The Standard Cosmological Model from CMB Measurements

The base model. According to the model of inflation in the early universe, our Universe is expected to be spatially flat. Such a model is described by a minimum of six parameters. Two of them characterise the initial density fluctuations (namely an amplitude A and the power-law slope n_s). As a third parameters, the post-reionisation scattering optical depth τ needs to be chosen. The remaining three parameters describe the energy contents and scale of the universe. The three typically used are $\omega_b \equiv \Omega_b h^2$ (subscript b for baryonic matter), $\omega_c \equiv \Omega_c h^2$ (where $\Omega_c = \Omega_m - \Omega_b$ is the density parameter of cold dark matter), and Ω_Λ.

8.7.2 – Consistency and Discrepancies with Other Measurements

Baryonic Acoustic Oscillations. The results from BAO studies of galaxy distributions is perfectly compatible with the cosmological model parameters as determined by WMAP. Furthermore, one can also compare the observed power spectrum of galaxies with the predictions from the standard model (using the best-fit parameters). The agreement again is excellent on large scales; on smaller scales, slight discrepancies appear which mostly likely are due to non-linear effects in structure formation, and thus also in the clustering of galaxies.

8.7.3 – Extensions of the Standard Model

Although the standard model, defined by the six basic parameters mentioned earlier, provides an excellent fit to the CMB data – temperature fluctuation power spectrum out to $l = 2500$, low-l polarisation fluctuations, and the CMB lensing effects – it is worth generalising this model by considering extensions in various ways.

Curvature. The standard model assumes spatial flatness. The CMB data, together with the lensing information and BAO studies, gives a spatial flatness estimate of $(\Omega_m + \Omega_\Lambda - 1) = 0.10^{+0.65}_{-0.62}\%$. Hence, there is very little room for generalising the model away from flatness.

Number of Neutrino Families and their Masses. The standard model assumes that there are three families of neutrinos, and that they are essentially massless. Relaxing one or both of these assumptions, we can see whether the CMB data preferences a non-standard picture of the neutrinos. If the sum of the neutrino masses is significantly larger than the minimum mass required from neutrino oscillations, then they would contribute to the current matter density in the Universe in the form of hot dark matter, and thus affect the shape of the power spectrum. If the number of neutrino families is larger than three, there would be a larger radiation component in the early universe, changing the expansion law and the epoch of matter-radiation equality.

All of this affects the CMB fluctuations.

The CMB and BAO data together yield $\sum m_\nu < 0.28$ eV, and $N_{\text{eff}} = 3.32^{+0.54}_{-0.52}$. This result confirms our picture of particle physics, according to which there are three families of leptons, and thus three kinds of neutrinos.

Big Bang Nucleosynthesis. The standard model assumes that the helium abundance following the first few minutes after the Big Bang is given by the theory of nucleosynthesis, and is therefore a function of $\omega_b \equiv \Omega_b h^2$. This helium abundance then determines the evolution of the number density of free electrons before recombination, since helium recombines earlier than hydrogen because of its higher ionization potential. The value of ω_b, as determined from CMB anisotropies, is in excellent agreement with the results obtained from the observed abundance of helium and deuterium in the Universe. Leaving the helium abundance as a free parameter and using the CMB data to directly predict it, one obtains a result in excellent agreement to BBN, and hence very strongly rules out any exotic model that wants to explain the current helium contents of the Universe solely by nuclear fusion in stars.

8.7.4 – Cosmic Harmony

Hubble Constant. H_0 was determined by means of the distance ladder, particularly using Cepheids. All of the methods of measuring H_0 suggest that the Hubble constant is within 5% of 71 km s^{-1}Mpc^{-1}, or $h \approx 1/\sqrt{2}$.

Contribution of Baryons to the Total Matter Density. The ratio Ω_b/Ω_m is determined from the baryon fraction in clusters of galaxies, from redshift surveys of galaxies, and from the CMB fluctuations, all yielding $\Omega_b/\Omega_m \approx 0.15$.

Ch 8.8 – Dark Energy: Cosmological Constant, or Something Else?

The cosmological constant and Einstein's field equation. Einstein's field equation of general relativity can be expressed as

$$G = T \tag{8.43}$$

where G (called the Einstein tensor) describes the curvature of spacetime and thus the effects of gravity, whereas T (the so-called energy-momentum tensor) contains information about the matter and energy density. As it stands, equation (8.43) does not allow a static cosmos, and so Einstein modified his equation to include the cosmological constant which then reads

$$G - \Lambda = T \tag{8.44}$$

With this modification, one can construct a static model. A modern interpretation of the cosmological constant is obtained by slightly rewriting (8.44) as

$$G = T + \Lambda \tag{8.45}$$

where now the Λ-term is seen as a contribution to the source of the gravitational field; it has the same structure as one would get from a constant, uniform vacuum energy density. The difference of the interpretation is that instead of Λ

being a modification of the laws of gravity, it adds a new energy component as a source of gravity.

Time-varying dark energy. There are ideas that the value of the vacuum energy density is actually not a constant in time, but that it may vary. In that case, the dark energy density would be a function of cosmic epoch, $\rho_{DE}(a)$. The question of whether dark energy is compatible with a cosmological constant, or has more complicated properties, is a very interesting one: if it were not constant in time, then it must have a more dynamical origin which would clearly argue against it being another fundamental constant of nature. One therefore considers it as a possible variant of the cosmological constant an equation-of-state of dark energy of the form

$$P_{DE} = w\rho_{DE}c^2 \tag{8.46}$$

where $w = -1$ corresponds to the cosmological constant. In order for this component to potentially lead to an accelerated expansion, the second Friedmann equation requires $w < -1/3$.

In fact, the equation-of-state parameter w does not need to be a constant, and can vary with cosmic epoch. Can one observationally distinguish between the case $w = -1$ for a cosmological constant (or a vacuum energy density that is indistinguishable from a cosmological constant) and the more interesting, dynamical case of $w \neq -1$?. The first major impact of $w \neq -1$ on cosmology would be a change of the expansion rate $H(a)$ of the universe. Second, the growth rate of structure would be affected for a dynamical w.

Lecture 2

Dark matter is the only way to consistently explain rotation curves of galaxies (and the velocity dispersions in elliptical galaxies), the magnitude of gravitational lensing, and cosmological data (which says that $\Omega_{matter} \simeq 0.27$, where big bang nucleosynthesis says $\Omega_{baryonic} \approx 0.04$ and hence $\Omega_{non\text{-}baryonic} \sim 5\Omega_{baryons}$).

What we know about dark matter

Dark matter as we know it must be massive (because of its gravitational interaction), unable to interact via electromagnetism (and hence it's dark), non-baryonic (lest BBN would have accounted for it), cold-ish (in order to allow large scale structure formation), stable on cosmological timescales (otherwise it would have decayed by now), and produced in the right relic abundance in the early universe.

Primordial black holes, massive compact halo objects, and standard model neutrinos all come with their own problems as descriptors of dark matter (MACHOs would be baryonic, and SM neutrinos are too warm). Hence, we turn mainly to Weakly Interacting Massive Particles and axions to describe it.

We don't know what mass these particles are (the possible mass range spans 90 orders of magnitude), their interactions, coupling, etc although many of these properties have been constrained by observations (or lack thereof).

WIMPs

WIMPs as we hypothesise them are dark (no electromagnetic interactions), cold (as they're very massive [~ 10 GeV to ~ 10 TeV]), non-baryonic and stable (and so no problems with BBN or the CMB). The weak force interaction implies some scattering with atomic nuclei, and many WIMPs are Majorana particles (i.e. their own antiparticle) and so their self-annihilation provides a detection channel.

Dark Matter Density Profiles

Rotation curves place good constraints on the density profiles of dark matter within galaxies and clusters, and essentially tell us that dark matter is gravitationally bound, more or less frictionless, and comprise dark matter halos within galaxies. N-body simulations of dark matter halo formation suggesta universal Navarro-Frenk-White profile

$$\rho(r) = \frac{\delta_c \rho_c}{\frac{r}{r_s}(1 + \frac{r}{r_s})^2}$$

or Einasto profile

$$\rho(r) = \rho_s \exp\left(-2n\left[(r/r_s)^{1/n} - 1\right]\right)$$

These profiles may be steepened in the innermost regions by adiabatic contraction or softened by baryonic effects. For low mass galaxies in particular, there is some discrepancy between simulations and the data with sims implying a "cusp" (higher density DM) at small radius, but data implies a "core" (flattening profile) at low radius.

The galactic dark matter halo is *isothermal* to a first approximation and so a WIMP wind is predicted as the solar system moves through the randomly oriented DM particle motion.

Thermal Production

A particle with a larger interaction cross section, $\langle \sigma v \rangle$, can withstand more universe expansion before its abundance 'freezes out' (i.e. its abundance stabilises).

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Ch 7.5 – Non-Linear Structure Evolution

7.5.1 – Model of Spherical Collapse

Assumptions. We consider a spherical region in an expanding universe, with its density $\rho(t)$ enhanced compared to the mean cosmic density $\bar{\rho}(t)$,

$$\rho(t) = [1 + \delta(t)]\bar{\rho}(t) \tag{7.46}$$

For reasons of simplicity we assume that the density within the sphere is homogeneous although, as we'll later see, this is

not really a restriction. The density perturbation is assumed to be small for small t so that it will grow linearly at first, $\delta(t) \propto D_+(t)$, as long as $\delta \ll 1$. If we consider a time t_i which is sufficiently early such that $\delta(t_i) \ll 1$, then according to the definition of the growth factor D_+, $\delta(t_i) = \delta_0 D_+(t_i)$, where δ_0 is the density contrast linearly extrapolated to the present day. It should be mentioned once again that $\delta_0 \neq \delta(t_0)$, because the latter is in general affected by the non-linear evolution.

Let R be the initial *comoving* radius of the overdense sphere; as long as $\delta \ll 1$, the comoving radius will change only marginally. The mass within the sphere is

$$M = \frac{4\pi}{3} R^3 \rho_0 (1 + \delta_i) \approx \frac{4\pi}{3} R^3 \rho_0 \qquad (7.47)$$

Evolution. Due to the enhanced gravitational force, the sphere will expand slightly more slowly than the universe as a whole, which will lead to an increase in its density contrast. This then decelerates the expansion rate (relative to the cosmic rate) even further. If the initial mass density within the sphere is sufficiently large, the expansion of the sphere will come to a halt, i.e., its proper radius $R_{\mathrm{phys}}(t)$ will reach a maximum; after this, the sphere will recollapse. If $t_{\max}$ is the time of maximum expansion, then the sphere will, theoretically, collapse to a single point at time $t_{\mathrm{coll}} = 2t_{\max}$. This relation follows from the time reversal symmetry of the equation of motion.

Special case: Einstein-de Sitter model. In the special case of $\Omega_m = 1$, $\Omega_\Lambda = 0$, the Friedmann equations can be solved to yield a recollapse time at the present epoch corresponding to an overdensity of

$$\delta_c \simeq 1.69 \qquad (7.48)$$

More generally, for a recollapse before redshift z, one needs $\delta_0 \geq \delta_c(1+z)$ (although the critical overdensity is cosmology dependent).

Violent Relaxation and Virial Equilibrium. Of course, the sphere will not really collapse to a single point. In reality, small scale density and gravitational fluctuation will exist within such a sphere which lead to derivations in the particles' tracks from perfectly radial orbits. The particles will scatter on these fluctuations in the gravitational field and will virialize; this process occurs on short time-scales – roughly the dynamical time-scale (i.e. the time it takes for particles to fully cross the sphere). In this case, the virialization is essentially complete at t_{coll}. After that, the sphere will be in virial equilibrium and its average density will be $(1 + \delta_{\mathrm{vir}}) \sim 178$. A conclusion from this consideration is that a massive galaxy cluster with a virial radius of $1.5 h^{-1}\mathrm{Mpc}$ must have formed from the collapse of a region that originally had a comoving radius larger by about an order of magnitude. Such a virialized mass concentration of dark matter is called a ***dark matter halo***.

7.5.2 – Number Density of Dark Matter Halos

The Mass Spectrum. We consider the number density of relaxed dark matter halos in the universe, $n(M, z)$, which is a function of mass M and redshift z.

We find that $n(M, z)$ is a decreasing function of halo mass. For large M, $n(M, z)$ decreases exponentially because sufficiently high peaks (of the initial mass function) become very rare for large distance scales. Therefore, *very* few clusters with $M \gtrsim 2 \times 10^{15} M_\odot$ exist today. At higher redshift, the cut-off in the abundance is at smaller masses, so that massive clusters are expected to be increasingly rare at higher z.

Hierarchical Structure Formation. The Press-Schechter model (i.e. the spherical collapse formation of DM halos) describes a very general property of structure formation in a CDM model, namely that low-mass structure (like galaxy-mass dark halos) format at early times, whereas large mass accumulations evolve only later. This is an example of a ***bottom-up scenario***, in which small structures that form early merge to form large structures.

Ch 7.6 – Properties of Dark Matter Halos

7.6.1 – Profile of Dark Matter Halos

Dark matter halos can be identified in mass distributions generated by numerical simulations. If we define a halo as a spherical region within which the average density is ~ 200 times the critical density at the respective redshift, the mass M of the halo is related to its (virial) radius r_{200} by

$$M = \frac{100 r_{200}^3 H^2(z)}{G} \qquad (7.56)$$

so that each redshift, a unique relation exists between the halo mass and its radius. We can also define the virial velocity V_{200} of a halo as the circular velocity at the virial radius

$$V_{200}^2 = \frac{GM}{r_{200}} \qquad (7.57)$$

We can combine the two above equations to express the halo mass and virial radius as a function of the virial velocity,

$$M = \frac{V_{200}^3}{10 G H(z)}; \qquad r_{200} = \frac{V_{200}}{10 H(z)} \qquad (7.58)$$

Since the Hubble function $H(z)$ increases with redshift, the virial radius at fixed virial velocity decreases with redshift. We also see that r_{200} decreases with redshift at fixed halo mass. Hence, halos at a given mass (or given virial velocity) are more compact at higher redshift than they are today because the critical density was higher in the past.

The NFW Profile. The ***density profile*** of halos averaged over spherical shells seems to have a universal functional form described by

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2} \qquad (7.59)$$

where r_s specifies a characteristic radius, and $\rho_s = 4\rho(r_s)$ determines the amplitude of the density profile. For $r \ll r_s$ we find $\rho \propto r^{-1}$, whereas for $r \gg r_s$ the profile follows $\rho \propto r^{-3}$. Therefore, r_s is the radius at which the slope of the density profile changes. We define the **concentration index** as

$$c \equiv \frac{r_{200}}{r_s} \tag{7.61}$$

where the larger the value of c, the more strongly the mass is concentrated towards the inner regions. We can write the density profile amplitude factor as

$$\rho_s = \frac{200}{3}\rho_{\mathrm{cr}}(z)\frac{c^3}{\ln(1+c) - c/(1+c)}$$

where $\rho_{\mathrm{cr}}(z) = 3H^2(z)/(8\pi G)$ is the critical density at redshift z.

Comparison with Observations. A complication of modelling profiles with NFW (or other) profiles is that baryonic matter is present in the inner regions of galaxies (and clusters), thus contributing to the density, but also that these baryons have modified the density profile of dark matter halos in the course of cosmic evolution. Baryons are dissipative, they can cool, form a disk, and accrete inwards. Conversely, SNe can push gas to larger radii or even drive it out of the halo. The changes in the resulting density distribution of baryons by dissipative processes cause a change of the gravitational potential over time, to which dark matter also reacts. The dark matter profile in real galaxies is thus modified compare to pure dark matter simulations.

7.6.2 – The Shape and Spin of Halos

Halo Shapes. There is no reason for halos to have spherical symmetry. Approximating the surfaces of constant density in a halo by an ellipsoid of semi-axes $a_1 \leq a_2 \leq a_3$, the shape of a halo is characterised by the ratios $s = a_1/a_3$ and $q = a_2/a_3$. If $s = q < 1$, the ellipsoid is prolate, and a halo with $s < 1$ and $q = 1$ is oblate. If all three axes are different, the halo is triaxial.

Numerical simulations show that dark matter halos depend strongly on their formation time and merger history. If two halos of comparable mass collide and merge, the shape of the resulting halo will be strongly prolate. Halos that form early and experience no strong mergers tend to be more spherical.

Week 5

Ch 7.8 – The Substructure of Halos

Sub-halos of galaxies and clusters of galaxies. Numerical simulations of structure formation in the CDM model show that the mass density in halos is not smooth; instead, they reveal that halos contain numerous halos of much lower mass, so called sub-halos. For instance, a halo with the mass of a galaxy cluster contains hundreds or even thousands of halos with masses that are orders of magnitude lower. Similarly, a galaxy-mass halo will have sub-halos

in the simulations. The presence of substructure over a wide range in mass is a direct consequence of hierarchical structure formation, in which objects of higher mass each contain smaller structure that have been formed earlier in the cosmic evolution.

Such simulations show that of order $\sim 10\%$ of the mass of halos is contained in sub-halos, with galaxy-mass halos having about $\sim 7\%$.

The spatial distribution of the sub-halos is less centrally concentrated than the total mass. The reason for this lies in the fact that the sub-halos whose orbits bring them deep into the potential well of the host halo are subject to strong tidal forces and get disrupted in the course of evolution. Simulations that include gas physics find that disruption becomes weaker if baryons are included – their dissipational nature leads to more compact (and thus more tightly bound) sub-halos which can resist tidal forces for longer.

The 'substructure' problem. The apparent deficit in the number of observed sub-halos within a galaxy-mass halo is considered to be a potential problem of CDM models. One possibility is that many of the sub-halos are extremely non-luminous and have just not yet been found. Indeed, the currently identified sub-halos (such as satellite galaxies) generally have a high velocity dispersion and thus high mass for low luminosities. Their extremely low metallicity argues for a very early epoch of star formation; this is confirmed by the colour-magnitude diagrams for some of the dwarf galaxies, which assign them an age of ~ 13 Gyr.

Warm dark matter as an alternative. The apparent conflict between the abundance of sub-halos and the observed satellite galaxies in the Milky Way can be potentially avoided if the initial power spectrum of density fluctuations has less power on small spatial scales – corresponding to masses of satellite galaxies. In particular, if a WDM particle has a mass of ~ 2 keV, the cut-off in the power spectrum would correspond to the halo mass of dwarf galaxies and all small-scale sub-halos effectively disappear. However, observations of the Lyman-α forest strongly constrain the allowed mass range of WDM particles, and lower limits on the mass of WDM particles obtained from these studies come exceedingly close to the mass needed to substantially reduce the abundance of sub-halos.

Evidence for the presence of CDM substructure in galaxies. Gravitational lensing studies have provided strong constraints on the number and mass of sub-halos within lensing galaxies. This generally favours complex sub-halo structure, rather than a 'smooth', simple mass model consisting of few and sparse halos.

The 'disk of satellite galaxies'. Whereas the abundance of dark matter sub-halos in galaxies no longer presents a serious problem for CDM models of structure formation, the spatial distribution of satellite galaxies around the Milky Way requires more explanation. The 11 classical satellites

of the Milky Way seem to form a planar distribution. Such a distribution would be extremely unlikely if the satellite population was drawn from a near-isotropic probability distribution.

Using models of galaxy formation, the accretion of smaller mass halos onto a high-mass halo occurs predominantly in the direction of the filaments that galaxies preferentially form in within large scale structure. The most massive sub-halos therefore tend to form in a planar distribution, not unlike the one seen in the Milky Way's satellite distribution – solving the problem.

Ch 10.1 – Galaxy Evolution

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In the cold dark matter universe, small density structures formed first, which means that low-mass dark matter halos preceded those of higher mass. This 'bottom-up' scenario of structure formation follows from the shape of the power spectrum of density fluctuations, which itself is determined by the nature of dark matter—namely cold dark matter. The gas in these halos is compressed and heated, the source of heat being the potential energy. If the gas is able to cool by radiative processes, i.e., to get rid of some of its thermal energy and thus pressure, it can collapse into denser structures, and eventually form stars. In order for this to happen, the potential wells have to have a minimum depth, so the resulting kinetic energy of atoms is sufficient to excite the lowest-lying energy levels whose de-excitation then leads to the emission of a photon which yields the radiative cooling. We shall see that this latter aspect is particularly relevant for the first stars to form, since they have to be made of gas of primordial composition, i.e., only of hydrogen and helium.

Once the first stars form in the Universe, the baryons in their cosmic neighborhood get ionized. This reionization at first happens locally around the most massive dark matter halos that were formed; later on, the individual ionized regions begin to overlap, the remaining neutral regions become increasingly small, until the process of reionization is completed, and the Universe becomes largely transparent to radiation, i.e., photons can propagate over large distances in the Universe. The gas in dark matter halos is denser than that in intergalactic space; therefore, the recombination rate is higher there and the gas in these halos is more difficult to ionize. Probably, the ionizing intergalactic radiation has a small influence on the gas in halos hosting a massive galaxy. However, for lower-mass halos, the gas not only maintains a higher ionization fraction, but the heating due to ionization can be appreciable. As a result, the gas in these low-mass halos finds it more difficult to cool and to form stars. Thus, the star-formation efficiency—or the fraction of baryons that is turned into stars—is expected to be smaller in low-mass galaxies.

The mass of halos grows, either by merging processes of smaller-mass halos or by accreting surrounding matter through the filaments of the large-scale density field. The behavior of the baryonic matter in these halos depends on the interplay of various processes. If the gas in a halo can cool, it will sink towards the center. One expects that the gas, having a finite amount of angular momentum like the dark matter halo itself, will initially accumulate in a disk perpendicular to the angular momentum of the gas, as a consequence of gas friction—provided a sufficiently long time of quiescent evolution for this to happen. The gas in the disk then reaches densities at which efficient star formation can set in. In this way, the formation of disk galaxies, thus of spirals, can be understood qualitatively.

As soon as star formation sets in, it has a feedback on the gas: the most massive stars very quickly explode as supernovae, putting energy into the gas and thereby heating it. This feedback then prevents that all the gas turns into stars on a very short time-scale, providing a self-regulation mechanism of the star-formation rate. In the accretion of additional material from the surrounding of a dark matter halo, also additional gas is accreted as raw material for further star formation.

When two dark matter halos with their embedded galaxies merge, the outcome depends mainly on the mass ratio of the halos: if one of them is much lighter than the other, its mass is simply added to the more massive halo; the same is true for their stars. More specific, the small-mass galaxy is disrupted by tidal forces, in the same way as the Sagittarius dwarf galaxy is currently destroyed in our Milky Way, with the stars being added to the Galactic halo. If, on the other hand, the masses of the two objects are similar, the kinematically cold disks of the two galaxies are expected to be disrupted, the stars in both objects obtain a large random velocity component, and the resulting object will be kinematically hot, resembling an elliptical galaxy. In addition, the merging of gas-rich galaxies can yield strong compression of the gas, triggering a burst of star formation, such as we have seen in the Antennae galaxies. Merging should be particularly frequent in regions where the galaxy density is high, in galaxy groups for instance. A large number of such merging and collision processes are detected in galaxy clusters at high redshift.

In parallel, the supermassive black holes in the center of galaxies must evolve, as clearly shown by the tight scaling relations between black hole mass and the properties of the stellar component of galaxies. The same gas that triggers star formation, say in galaxy mergers, can be used to 'feed' the central black hole. If, for example, a certain fraction of infalling gas is accreted onto the black hole, with the rest being transformed into stars, the parallel evolution of black hole mass and stellar mass could be explained. In those phases where the black hole accretes, the galaxy turns into an active galaxy; energy from the active galactic nucleus, e.g., in the form of kinetic energy carried by the jets, can be transmitted to the gas of the galaxy, thereby heating it. This provides another kind of feedback regulating the cooling of gas and star formation.

When two halos merge, both hosting a galaxy with a central black hole, the fate of the black holes needs to be considered. At first they will be orbiting in the resulting merged galaxy. In this process, they will scatter off stars, transmitting a small fraction of their kinetic energy to these stars. As a result, the velocity of the stars on average increases and many of them will be 'kicked out' of the galaxy. Through these scattering events, the black holes lose orbital energy and sink towards the center of the potential. Finally, they form a tight binary black hole system which loses energy through the emission of gravitational waves until they merge.

The more massive halos corresponding to groups and clusters only form in the more recent cosmic epoch. In those regions of space where at a later cosmic epoch a cluster will form, the galaxy-mass halos form first—the larger-scale overdensity corresponding to the proto-cluster promotes the formation of galaxy-mass halos, compared to the average density region in the Universe; this is the physical origin of galaxy bias. Therefore, one expects the oldest massive galaxies to be located in clusters nowadays, explaining why most massive cluster galaxies are red. In addition, the large-scale environment provided by the cluster affects the evolution of galaxies, e.g., through tidal stripping of material.

Ch 10.2 – Gas in Dark Matter Halos

10.2.1 – The Infall of Gas During Halo Collapse

Gas Heating. As long as the fractional overdensities are small, the spatial distribution of baryons and dark matter are expected to be very similar in the very early halo formation. However, when the spherical halo collapses, the behaviour of both components must be very different: dark matter is collisionless and baryons are collisional (meaning that friction prevents gas from crossing through a gas distribution). Thus, as the halo collapses, the potential energy of the gas is transformed into heat through the frictional processes. Furthermore, the pressure of the gas can prevent it from falling into the dark matter potential well.

In the case of (approximate) spherical symmetry, the gas had already settled down in the inner part of the halo and gas pressure balances gravitational force. As the outer part of the halo collapses, gas falls onto this gas distribution. The infall speed is much higher than the sound velocity of the (cold) infalling gas, and a shock front develops. Inside this front, the gas is hot and almost all of its kinetic energy gets converted into heat.

Hence, a collapsed halo should contain hot gas with a temperature depending on the halo radius and mass. Such hot gas is seen in galaxy groups and clusters through X-ray emission. Galaxy mass halos have a characteristic gas temperature of $\sim 10^6$K and so are much more difficult to observe (since they're too cool to emit X-rays and most atoms are fully ionized at that temperature). Nevertheless, some significant fraction of the hot gas in galaxy-mass halos does not stay hot and must cool to form stars.

10.2.2 – Cooling of Gas

Optically thin gas can cool by emitting radiation, e.g. by scattering between electrons and nuclei in an ionized gas (bremsstrahlung) or collisional excitations (and relaxation and emission of photons).

Cooling function. Although elements heavier than helium have a small abundance in number, they can dominate the gas cooling due to the rich energy spectrum of many-electron atoms. Hence, more enriched gas finds it easier to cool.
Atomic gas cannot cool efficiently for temperatures $T \lesssim 10^4$K due to the lack of charged particles in the gas. Molecules, on the other hand, have a rich spectrum of energy levels at low temperatures and can lead to efficient cooling. This is why star formation occurs in molecular clouds, where gas can efficiently cool and thereby compress to high densities.

Cooling time. Once we know the rate at which gas loses its energy, we can calculate the cooling time (the time it takes the gas [at constant cooling rate] to lose all of its energy). If this cooling time is longer than the age of the Universe, then the gas essentially stays at the same temperature and is unable to collapse towards a halo center. For most regions within galaxy clusters, this is the case, and cooling is only efficient towards cluster centers.

Conditions for efficient cooling. If the cooling time is shorter than the free-fall time (given by equation (10.5)), then gas falls freely towards the center of a halo essentially unaffected by gas pressure.

$$t_{\text{ff}} = \sqrt{\frac{3\pi}{32G\rho}} \qquad (10.5)$$

If the cooling time is much longer than the free-fall time, the gas at *best* sinks to the center at a rate given by the cooling rate.

Difference between galaxies and groups/clusters. In sufficiently massive halos with $M_g \gtrsim 10^{12} M_\odot$, the small efficiency of gas cooling prevents gas from collapsing to the center and forming stars there. At smaller masses, cooling is effective to enable rapid gas collapse. This dividing line in mass is about the mass which distinguishes galaxies from groups and clusters. In the latter, only a small fraction of the baryons is turned into stars (and this is generally contained within the galaxies of the group). In contrast, a large fraction of baryons in galaxies is concentrated towards the center, as visible in their stellar distribution. Thus, the difference between galaxies and groups/clusters is their efficiency to turn baryons into stars, and this difference is explained with the different cooling efficiencies of matter halos (as in figure 10.4).
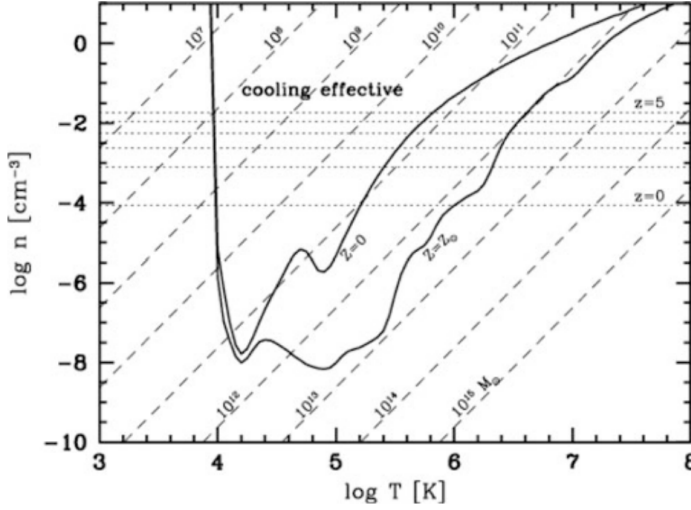
Figure 1: Solid curves are for gas metallicities of $Z = 0$ and $Z = Z_\odot$. Dotted lines show the density of halos formed at the specified redshifts. For gas densities and temperatures above the curve, cooling is efficient, whereas cooling time is longer than the free-fall time below the curves.

Low-mass halos. A halo with gas mass $\sim 10^{7.5} M_\odot$ lies inside the cooling curve only at *very* high redshift, i.e. when the corresponding density in a halo is very high. Therefore, gas can cool, and stars form, in halos of this mass only if they formed early enough. We therefore expect that the stars in such low-mass halos are very old.

matter halos which contain baryons. In order to form stars, the gas in the halos needs to be able to cool further. Here, we are dealing with the particular situation of the first galaxies, whose gas is metal-free, so metal lines cannot contribute to the cooling. The cooling function of primordial gas is much smaller than that of enriched material; in particular, the absence of metals means that even slow cooling through excitation of fine-structure lines cannot occur, as there are no atoms with such transitions present. Thus, cooling by the primordial gas is efficient only above $T \gtrsim 2 \times 10^4$K. Therefore, atomic hydrogen is a very inefficient coolant for these first halos, insufficient to initiate the formation of stars.

The importance of molecular hydrogen. Besides atomic hydrogen and helium, the primordial gas contained a small fraction of molecular hydrogen. Whereas in enriched gas, molecular hydrogen is formed on dust particles, and the primordial gas had no dust, and so H_2 must have formed in the gas phase itself (rendering its abundance very small). Despite its very small density, H_2 dominated the cooling rate of primordial gas at temperatures below $T \sim 10^4$K.
By means of H_2, the gas can cool in halos with a mass of $M \gtrsim 5 \times 10^4 M_\odot$ at $z \sim 20$. In these halos, stars may then be able to form. Such stars, which at the same mass presumably have a much higher temperature and luminosity (due to their zero metallicity) and thus a shorter lifetime, are called ***population III stars***. DUe to their high temperature, they are much more efficient sources of ionizing photons than stars with 'normal' metallicity.

Week 6

Ch 10.3 – Reionization of the Universe

10.3.1 – The First Stars

Understanding reionization (currently understood to have occurred around $z \sim 10$) is directly linked to studying the first generation of stars.

The Jeans Mass. There exists a certain mass of a halo for which pressure forces are unable to prevent the infall of gas into a potential well,

$$M > M_J \equiv \frac{\pi^{5/2}}{6} \left(\frac{c_s^2}{G} \right)^{3/2} \frac{1}{\sqrt{\bar{\rho}}} \qquad (10.6)$$

where $\bar{\rho}$ is the mean cosmic matter density, and c_s is the speed of sound in the gas. The Jeans mass depends on the temperature of the gas, expressed through the sound speed c_s. As a function of redshift, the Jeans mass can be calculated as

$$M_J = 5.7 \times 10^3 \left(\frac{\Omega_m h^2}{0.15} \right)^{-1/2} \left(\frac{\Omega_b h^2}{0.022} \right)^{-3/5} \left(\frac{1+z}{10} \right)^{3/2} M_\odot$$
$$(10.8)$$

Cooling of the Gas. The Jeans criterion is a necessary condition for the formation of proto-galaxies, i.e. dark