PHYS2041 Assignment 2 Ryan White s4499039

Problem 2.2:

The wavefunction may be written as

$$\psi(x,t) = \psi(x)\varphi(t)$$

where $\varphi(t) = e^{-iVt/\hbar}$

Adding V_0 to V gives $\varphi(t) = e^{-i(V+V_0)t/\hbar} \Rightarrow e^{-iVt/\hbar} e^{-iV_0t/\hbar}$

Therefore, after adding V_0 to the potential energy, the wave function becomes

$$\psi_0(x,t) = \psi(x) e^{-iVt/\hbar} e^{-iV_0t/\hbar}$$

$$= \psi(x)\varphi(t) e^{-iV_0t/\hbar}$$

$$= \psi(x,t) e^{-iV_0t/\hbar}$$

I would suppose that adding this constant wouldn't change the expectation value of any dynamical quantity initially, as the final term goes to 1 as $t \to 0$.

Problem 2.3:

a. The wave function may be normalised at $t=0$. Therefore, the wave function has the form

$$\psi(x,t) = Ce^{-b|x|}$$

The wave function is normalizeable if there exists a constant C such that $\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1$

$$\Rightarrow \int_{-\infty}^{\infty} |Ce^{-b|x|}|^2 dx = C^2 \left(\int_{-\infty}^{0} e^{-2b\times -x} dx + \int_0^\infty e^{-2bx} dx\right) = 1$$

$$= C^2\left(\frac{1}{2b} e^{2bx}\Big|_{-\infty}^{0} + \frac{1}{2b} e^{-2bx}\Big|_0^\infty\right) = 1$$

$$= C^2\left(\frac{1}{2b} + \frac{1}{2b}\right) = 1$$

$$= \frac{C^2}{b} = 1 \Rightarrow C = \sqrt{b}$$

Therefore, the normalised wave function is

$$\psi(x,t) = \sqrt{b}\, e^{-i\omega t} e^{-b|x|}$$

b. The expectation value of x is found as

$$\langle x \rangle = \int_{-\infty}^{\infty} \Psi^* x \Psi \, dx$$

$$= \int_{-\infty}^{\infty} x b e^{-2i\omega t} e^{-b|x|} dx$$

at $t=0$, this is then

$$\langle x \rangle = \int_{-\infty}^{\infty} x b e^{-2b|x|} dx$$

$$= \int_{-\infty}^{0} x b e^{2bx} dx + \int_{0}^{\infty} x b e^{-2bx}$$

$$= \frac{e^{2bx}(2bx-1)}{4b}\Big|_{-\infty}^{0} + -\frac{e^{-2bx}(2bx+1)}{4b}\Big|_{0}^{\infty}$$

$$= \frac{1}{4b} - \frac{1}{4b} = 0$$

for x^2,

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 b e^{-2b|x|} dx$$

$$= \int_{-\infty}^{0} x^2 b e^{2bx} dx + \int_{0}^{\infty} x^2 b e^{-2bx} dx$$

$$= \frac{1}{4b^2} + \frac{4}{4b^2} = \frac{1}{2b^2}$$

c. The standard deviation is found by

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

substituting in the calculated values yields

$$\sigma_x = \sqrt{\frac{1}{2b^2} - 0^2}$$

$$= \frac{1}{\sqrt{2}} \frac{1}{b}$$