Information	
Course code and title	MATH3070 Natural Resource Mathematics
Semester	Semester 2, 2023
Type	Online, non-invigilated assignment, under ‘take home exam’ conditions.
Technology	File upload to Blackboard Assignment
Date and time	Your assignment will begin at the time specified by your course coordinator. You have a fixed 170-hour window from this time in which it must be completed. You can access and submit your paper at any time within the 170-hours. Even though you have the entire 170-hours to complete and submit this assessment, the expectation is that it will take students with a strong command of the material around 8 hours to complete. Note that you must leave sufficient time to submit and upload your answers.
Permitted materials	This assignment is open book – all official course materials are permitted. Some materials outside of the course, including published books, journal articles, computer algebra programs, etc. are permitted, but must be cited in your solution where used. Materials which include answers, discussion, or other forms of communication directly related to the questions in this assignment are NOT permitted, and the use of such materials is considered cheating.
Recommended materials	Ensure the following materials are available during the available time: R or similar programming language; bilingual dictionary; phone/camera/scanner; A computer algebra tool, such as wolfram alpha, Mathematica, Maple, Maxima, or MATLAB symbolic solver package may be useful, but is not required.
Instructions	You will need to download the question paper under the assessment section of blackboard. Once you have completed the assignment, upload a single pdf file with your answers to the Blackboard assignment submission link. You may submit multiple times, but only the last uploaded pdf file will be graded. Any computer code must also be submitted as an executable file, e.g. ‘.R’ file. You can print the question paper and write on that paper or write your answers on blank paper (clearly label your solutions so that it is clear which problem it is a solution to) or annotate an electronic file on a suitable device.
Who to contact	Given the nature of this assessment, responding to student queries and/or relaying corrections during the allowed time may not be feasible. If you have any concerns or queries about a particular question or need to make any assumptions to answer the question, state these at the start of your solution to that question. You may also include queries you may have made with respect to a particular question, should you have been able to ‘raise your hand’ in an examination-type setting. If you experience any interruptions during the allowed time, please collect evidence of the interruption (e.g. photographs, screenshots or emails).

	If you experience any technical difficulties during the assignment, contact the Course Coordinator <m.holden1@uq.edu.au>. Note that this is for technical difficulties only.
Late or incomplete submissions	In the event of a **late submission**, you will be required to submit evidence that you completed the assessment in the time allowed. This will also apply if there is an **error in your submission** (e.g. corrupt file, missing pages, poor quality scan). We **strongly recommend** you use a phone camera to take time-stamped photos (or a video) of every page of your paper during the time allowed (even if you submit on time). If you submit your paper after the due time, then you should send details to SMP Exams (exams.smp@uq.edu.au) as soon as possible after the end of the time allowed. Include an explanation of why you submitted late (with any evidence of technical issues) AND time-stamped images of every page of your paper (eg screen shot from your phone showing both the image and the time at which it was taken).
Further important information	You are responsible for managing your multi-factor authentication in this examination. Please check the guidance on How do I MFA before an online exam? Academic integrity is a core value of the UQ community and as such the highest standards of academic integrity apply to assessment, whether undertaken in-person or online. This means: • You are permitted to refer to the allowed resources for this assignment, and you must not use any instances of work that has been submitted previously elsewhere. • You are not permitted to consult any other person – whether directly, online, or through any other means – about any aspect of this assignment during the period that it is available. • If it is found that you have given or sought outside assistance with this assignment, then that will be deemed to be cheating. If you submit your answers after the end of allowed time, the following penalties will be applied to the total mark available for the assessment: • Less than 5 minutes – 5% penalty • From 5 minutes to less than 15 minutes – 20% penalty • More than 15 minutes – 100% penalty These penalties will be applied unless there is sufficient evidence of problems with the system and/or process that were beyond your control. Undertaking this online assignment deems your commitment to UQ's academic integrity pledge as summarised in the following declaration: *"I certify that I have completed this assignment in an honest, fair and trustworthy manner, that my submitted answers are entirely my own work, and that I have neither given nor received any unauthorised assistance on this assignment".*

Q1. [20 points] Consider the following function,

$$f(x) = x + (r-1)x\left(1 - \left(\frac{x}{k}\right)^{\phi}\right),$$

with parameters $r, k, \phi > 0$, and domain $[0, \infty)$.

(a) [2 points] Plot $f(x)$ vs. x when ϕ equals $0.5, 1$ and 2 (with $r = 2$ and $k = 100$). Your plot should contain three curves. Write a sentence explaining the biological meaning of increasing the value of ϕ in the model,

$$x_{t+1} = f(x_t), \tag{1}$$

where x_t is the population size at time t.

(b) [1 point] Derive an expression for proliferation in the model given by equation (1).

(c) [3 points] Derive expressions for the equilibria in the model given by equation (1). Include conditions for when each equilibrium exists and is biologically meaningful.

(d) [5 points] For each equilibrium, derive conditions for when it is stable and conditions for when it is unstable.

(e) [4 points] Derive optimal equilibrium escapement in terms of r, ϕ, and k. As a reminder, when determining optimal escapement, we consider models of the form,

$$x_{t+1} = f(x_t - h_t),$$

where h_t is the harvest in year t. Escapement in year t is $S_t = x_t - h_t$. You may use any theorems presented in class.

(f) [5 points] Consider data, $\{X_0, X_1, ..., X_n\}$, for population size in years 0 to n. Assume $X_i \leq k$ for all $i \in \{1, 2, ..., n\}$ and that the values of ϕ and k are known (and therefore fixed). Derive an expression for the least squares estimate of r as a function of the data, ϕ, and k.

Q2. [20 points] The government of a hypothetical country banned fishing of a species after the population collapsed. Since then, there have been anecdotal reports that the population has recovered. The government is considering reopening the fishery.

Prior biological knowledge: From historical observations, prior to the population being fished, the managers know that the population's carrying capacity is 100 tons of fish. The government consulted biologists and bioeconomists who recommended using the population dynamic model in Q1 with $\phi = 0.8$.

The exact value of the growth rate for this species is unknown. However, the government acquired a data set from a nearby university for a related species tracking the population size in 10 tanks over one year. The tanks had unlimited food, no predators, and ample space for the fish to reproduce. The biologists think the two species probably have similar growth rates at low densities. The data is presented in the file `TankStudy.csv`. There are 10 rows in the file. Each entry is the population size in a tank at the end of the study (one year). All tanks started with 2 individuals, and you may assume all individuals have the same mass.

Government data: The reef underwent long-term monitoring from the closure in 2008 until 2023. Provided with the assignment is a time series of population biomass data (tons of fish) from 2008 to 2023. See `GovernmentData.csv`.

Write a summary/report ($\leq$ half a page of text) stating and justifying a recommendation for a limit on the yearly quota (total catch) in this fishery for when it reopens. In your report, you should estimate r given the data, present the associated uncertainty in your estimate, and discuss optimal harvest given the uncertainty in the system. You will be graded on the clarity and logical flow of your report and the completeness and correctness of your supporting calculations, code, and figures. Attach supporting calculations, code, and figures (not counted against your half-page limit) and label and refer to the figures and calculations in your report.

You must use the population dynamic model from question Q1 to receive full marks.

You are welcome to use any programming language you like. If you use `R` you may modify any scripts provided to you in this course to help you answer the question. Submit your code as a separate `.R` file along with your solutions to this assignment. If you use a different programming language, you must upload the appropriate script or program and also upload a README file that documents directions for running the code. The script must be viewable in a general text editor. A README file is not required for `.R` scripts.

Q3. [10 points]

(a) In what way does modelling biology (e.g., an ecosystem model) present challenges different from physics? Provide 2 reasons. (2 marks)

(b) Provide 2 reasons why modellers use size rather than species as the basis for ecosystem modelling? (2 marks)

(c) Explain what allometry is, and include an example. (3 marks)

(d) Explain why allometry is frequently used to estimate parameters in ecosystem models. (3 marks)