```
1 # MRAC Adaptive controller
 2
 3 # Import libraries
 4 import numpy as np
 5 from base_controller import BaseController
6 from lqr_solver import dlqr, lqr
7 from scipy.linalg import solve_continuous_lyapunov
   , solve_lyapunov, solve_discrete_lyapunov
8 from math import cos, sin
9 import numpy as np
10 from scipy import signal
11
12 class AdaptiveController(BaseController):
13
       """ The LQR controller class.
14
       11 11 11
15
16
17
       def __init__(self, robot, lossOfThurst):
           """ MRAC adaptive controller __init__
18
   method.
19
20
           Initialize parameters here.
21
22
           Args:
23
               robot (webots controller object):
   Controller for the drone.
24
               lossOfThrust (float): percent lost of
   thrust.
25
           11 11 11
26
27
           super().__init__(robot, lossOfThurst)
28
29
30
           # define integral error
31
           self.int_e1 = 0
32
           self.int_e2 = 0
           self.int_e3 = 0
33
           self.int_e4 = 0
34
35
           # flag for initializing adaptive controller
36
37
           self.have_initialized_adaptive = False
```

```
38
39
           # reference model
40
           self.x_m = None
41
           # baseline LQR controller gain
42
43
           self.Kbl = None
44
45
           # Saved matrix for adaptive law computation
46
           self.A_d = None
47
           self.B_d = None
48
           self.Bc_d = None
49
50
           self.B = None
51
           self.Gamma = None
52
           self.P = None
53
54
           # adaptive gain
55
           self.K_ad = None
56
57
       def initializeGainMatrix(self):
           """ Calculate the LQR gain matrix and
58
   matrices for adaptive controller.
59
           11 11 11
60
61
62
           # -----|LQR Controller
63
           # Use the results of linearization to
   create a state-space model
64
65
           # Given parameters
66
           n_p = 12 # number of states
67
           m = 4 # number of integral error terms
68
69
           # robot parameter
70
           self.m = 0.4
71
           self.d1x = 0.1122
           self.d1y = 0.1515
72
73
           self.d2x = 0.11709
74
           self.d2y = 0.128
75
           self.Ix = 0.000913855
```

```
76
            self.Iy = 0.00236242
 77
            self.Iz = 0.00279965
 78
 79
            # constants
 80
            self.q = 9.81
 81
            self.ct = 0.00026
 82
            self.ctau = 5.2e-06
 83
            self.U1_max = 10
 84
            self.pi = 3.1415926535
 85
 86
                        ----- Your Code Here
                ---- #
 87
           # Compute the continuous A, B, Bc, C, D
    and
            # discretized A_d, B_d, Bc_d, C_d, D_d,
 88
   for the computation of LQR gain
 89
 90
           # Matrix A logic
 91
           # Initialize A matrix with zeros ( 16 x 16
            A = np.zeros((n_p + m, n_p + m))
 92
            A[0, 6] = 1; A[1, 7] = 1; A[2, 8] = 1; A[3]
 93
    , 9] = 1; A[4, 10] = 1; A[5, 11] = 1
            A[6, 4] = self.g; A[7, 3] = -self.g
 94
            A[12, 0] = 1; A[13, 1] = 1; A[14, 2] = 1;
 95
    A[15, 5] = 1
            \# A[12, 0] = 1; A[12, 12] = -1; A[13, 1]
 96
    ] = 1; A[13, 13] = -1; A[14, 2] = 1; A[14, 14] = -
    1; A[15, 5] = 1; A[15, 15] = -1
 97
 98
           # Matrix B logic
 99
           # Initialize B matrix with zeros ( 16 x 4
100
            B = np.zeros((n_p + m, m))
            B[8, 0] = 1 / self.m; B[9, 1] = 1 / self.
101
    Ix; B[10, 2] = 1 / self.Iy; B[11, 3] = 1 / self.Iz
102
           # Matrix Bc logic
103
104
           # Initialize Bc matrix with zeros ( 16 x
   4)
105
            Bc = np.zeros((n_p + m, m))
```

```
106
            Bc[12, 0] = -1; Bc[13, 1] = -1; Bc[14, 2]
    ] = -1; Bc[15, 3] = -1
107
108
            # Combine B and Bc into one matrix
109
            combined_B = np.hstack((B, Bc))
110
111
           # Matrix C logic
112
           # Initialize C matrix with zeros ( 4 x 16
113
            C = np.zeros((m, n_p + m))
            C[0, 0] = 1; C[1, 1] = 1; C[2, 2] = 1; C[3]
114
    , 3] = 1
115
116
           # Matrix D logic
117
           # Zero matrix ( 4 x 4 )
118
            D = np.zeros((m, m))
119
120
           # Discretize the system
121
            sys_discrete = signal.cont2discrete((A,
    combined_B, C, D), self.delT, method='zoh')
122
            \# Extract A_d, B_d, Bc_d, C_d, D_d
123
124
            A_d = sys_discrete[0]
125
            B_d = sys_discrete[1][:, :m] # only take
    the first 4 columns
126
            Bc_d = sys_discrete[1][:, m:] # only take
     the last 4 columns
127
            C_d = sys_discrete[2]
128
            D_d = sys_discrete[3]
129
130
131
                        ----- Your Code Ends Here
132
133
            # Record the matrix for later use
            self.B = B # continuous version of B
134
135
            self.A_d = A_d # discrete version of A
            self.B_d = B_d # discrete version of B
136
137
            self.Bc_d = Bc_d # discrete version of Bc
138
139
                                   Example code
```

```
139
140
           \# \max_{pos} = 15.0
141
           # max_ang = 0.2 * self.pi
142
           \# max vel = 6.0
143
           # max_rate = 0.015 * self.pi
144
           \# max_eyI = 3.
145
146
           \# max_states = np.array([0.1 * max_pos, 0.
    1 * max_pos, max_pos,
147
                                  max_ang, max_ang,
    max_ang,
148
                                  0.5 * max_{vel}, 0.5
     * max_vel, max_vel,
149
                                  max_rate, max_rate,
    max_rate,
150
                                  0.1 * max_{eyI}, 0.1
     * max_eyI, 1 * max_eyI, 0.1 * max_eyI])
151
152
            \# max_inputs = np.array([0.2 * self.U1_max
    , self.U1_max, self.U1_max, self.U1_max])
153
154
           \# Q = np.diag(1/max_states**2)
           # R = np.diag(1/max_inputs**2)
155
           # ----- Example code Ends
156
            # ----- Your Code Here
157
158
           # Come up with reasonable values for Q and
     R (state and control weights)
159
            # The example code above is a good
    starting point, feel free to use them or write you
     own.
160
            # Tune them to get the better performance
161
162
            # referencing the example code above
163
            max_pos = 15.0
164
            max_anq = 0.2 * self.pi
165
            max_vel = 6.0
166
            max_rate = 0.015 * self.pi
167
            max_{eyI} = 3.0
168
```

```
169
           \max_{s} = \min_{s} (0.1 * \max_{s} s)
    * max_pos, max_pos,
170
                                  max_ang, max_ang,
   max_ang,
                                  0.5 * max_vel, 0.5
171
    * max_vel, max_vel,
172
                                  max_rate, max_rate
    , max_rate,
173
                                  0.1 * max_{eyI}, 0.1
    * max_{eyI}, 1 * max_{eyI}, 0.1 * max_{eyI})
174
175
           max_inputs = np.array([0.2 * self.U1_max,
   self.U1_max, self.U1_max, self.U1_max])
176
           Q = np.diag(1 / max_states ** 2)
177
           R = np.diag(1 / max_inputs ** 2)
178
179
180
           # ----- Your Code Ends Here
181
           # solve for LQR gains
182
183
           [K, _, _] = dlqr(A_d, B_d, Q, R)
184
           self.Kbl = -K
185
           [K_CT, _, _] = lqr(A, B, Q, R)
186
           Kbl_CT = -K_CT
187
188
189
           # initialize adaptive controller gain to
   baseline LQR controller gain
           self.K_ad = self.Kbl.T
190
191
192
           # ----- Example code
         ----- #
193
           \# self. Gamma = 3e-3 * np.eye(16)
194
195
          \# Q_{1yap} = np.copy(Q)
           # 0_lyap[0:3,0:3] *= 30
196
          # Q_lyap[6:9,6:9] *= 150
197
198
          # Q_lyap[14,14] *= 2e-3
199
           # ----- Example code Ends
        ----- #
```

```
200
                   ----- Your Code Here
201
            # Come up with reasonable value for Gamma
    matrix and Q_lyap
202
            # The example code above is a good
    starting point, feel free to use them or write you
     own.
203
            # Tune them to get the better performance
204
205
            # referencing the example code above
206
            self.Gamma = 3e-3 * np.eye(16)
207
208
            Q_{\text{lyap}} = \text{np.copy}(Q)
209
            Q_{\text{lyap}}[0:3,0:3] *= 30
            Q_{yap}[6:9,6:9] *= 150
210
211
            Q_{yap}[14,14] *= 2e-3
212
            # ----- Your Code Ends Here
213
214
            A_m = A + self.B @ Kbl_CT
215
            self.P = solve_continuous_lyapunov(A_m.T
    , -Q_lyap)
216
217
        def update(self, r):
            """ Get current states and calculate
218
    desired control input.
219
220
            Args:
221
                r (np.array): reference trajectory.
222
223
            Returns:
224
                np.array: states. information of the
    16 states.
225
                np.array: U. desired control input.
226
            11 11 11
227
228
            U = np.array([0.0, 0.0, 0.0, 0.0]).reshape
229
    (-1,1)
230
```

```
231
            # Fetch the states from the BaseController
     method
232
            x_t = super().getStates()
233
234
            # update integral term
235
            self.int_e1 += float((x_t[0]-r[0])*(self.
    delT))
            self.int_e2 += float((x_t[1]-r[1])*(self.
236
    delT))
            self.int_e3 += float((x_t[2]-r[2])*(self.
237
    delT))
            self.int_e4 += float((x_t[5]-r[3])*(self.
238
    delT))
239
240
            # Assemble error-based states into array
241
            error_state = np.array([self.int_e1, self.
    int_e2, self.int_e3, self.int_e4]).reshape((-1,1))
            states = np.concatenate((x_t, error_state
242
    ))
243
244
            # initialize adaptive controller
245
            if self.have_initialized_adaptive == False
246
                print("Initialize adaptive controller"
247
                self.x_m = states
248
                self.have_initialized_adaptive = True
249
            else:
250
                         ----- Your Code Here
251
                # adaptive controller update law
252
                # Update self.K_ad by first order
    approximation:
253
                \# self.K_ad = self.K_ad +
    rate_of_change * self.delT
254
255
                # error term
256
                e = states - self.x_m
257
258
                # adaptive rate of the control gain
259
                rate_of_change = -self.Gamma @ states
```

```
259
     0 e.T 0 self.P 0 self.B
260
                self.K_ad = self.K_ad + rate_of_change
     * self.delT
261
262
                # ----- Your Code Ends
263
264
               # compute x_m at k+1
265
                self.x_m = self.A_d @ self.x_m + self.
    B_d @ self.Kbl @ self.x_m + self.Bc_d @ r
266
                # Compute control input
               U = self.K_ad.T @ states
267
268
           # calculate control input
269
           U[0] += self.g * self.m
270
271
272
            # Return all states and calculated control
     inputs U
273
            return states, U
```