Project: Part 4

24-677 Special Topics: Modern Control - Theory and Design

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Due: Nov 28, 2023, 11:59 pm

Exercise 1

Given: $X_{t+1} = X_t + \delta t\,(\overset{\dot{X}}{\overbrace{\dot{x}_t \cos\psi_t - \dot{y}_t \sin\psi_t}}) + w_t^x$

$Y_{t+1} = Y_t + \delta t\,(\overset{\dot{Y}}{\overbrace{\dot{x}_t \sin\psi_t + \dot{y}_t \cos\psi_t}}) + w_t^y$

$\psi_{t+1} = \psi_t + \delta t\,\dot{\psi}_t + w_t^\psi$

$$x_t = \begin{bmatrix} X_t \\ Y_t \\ \psi_t \\ m_x^1 \\ m_y^1 \\ m_x^2 \\ m_y^2 \\ \vdots \\ m_x^n \\ m_y^n \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{robot's pose} \\ \left.\begin{matrix} \\ \\ \\ \\ \\ \\ \\ \end{matrix}\right\} \text{landmark} \end{matrix} \qquad y_t = \begin{bmatrix} \|m^1 - p_t\| \\ \vdots \\ \|m^n - p_t\| \\ \operatorname{atan2}(m_y^1 - Y_t,\ m_x^1 - X_t) - \psi_t \\ \vdots \\ \operatorname{atan2}(m_y^n - Y_t,\ m_x^n - X_t) - \psi_t \end{bmatrix} + \begin{bmatrix} v_t^1,\ \text{distance} \\ \vdots \\ v_t^n,\ \text{distance} \\ v_t^1,\ \text{bearing} \\ \vdots \\ v_t^n,\ \text{bearing} \end{bmatrix}$$

Find: Derive F_t and H_t for EKF SLAM to estimate the vehicle states X, Y, ψ and feature positions $m^j = [m_x^j \ \ m_y^j]^T$ simultaneously.

Solution: $F_t = \left.\dfrac{\partial f}{\partial x}\right|_{\hat{x}_{t-1|t-1},\,u_t}$, $H_t = \left.\dfrac{\partial h}{\partial x}\right|_{\hat{x}_{t|t-1}}$

$$\left.\begin{aligned} X_t &= f(x_{t-1}, u_t) + w_t^x \to f(x_{t-1}, u_t) = X_{t-1} + \delta_{t-1}(\dot{x}_t \cos\psi_{t-1} - \dot{y}_t \sin\psi_{t-1}) \\ Y_t &= f(y_{t-1}, u_t) + w_t^y \to f(y_{t-1}, u_t) = Y_{t-1} + \delta_{t-1}(\dot{x}_t \sin\psi_{t-1} + \dot{y}_t \cos\psi_{t-1}) \\ \psi_t &= f(\psi_{t-1}, u_t) + w_t^\psi \to f(\psi_{t-1}, u_t) = \psi_{t-1} + \delta_{t-1}\dot{\psi}_{t-1} \end{aligned}\right\} \to F_t = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{t-1|t-1},\,u_t}$$

$$F_t = \left[\begin{array}{ccc:c} 1 & 0 & \delta_{t-1}(-\dot{x}_t \sin\psi_{t-1} - \dot{y}_t \cos\psi_{t-1}) & \\ 0 & 1 & \delta_{t-1}(\dot{x}_t \cos\psi_{t-1} - \dot{y}_t \sin\psi_{t-1}) & 0_{3\times 2n} \\ 0 & 0 & 1 & \\ \hdashline & 0_{2n\times 3} & & I_{2n\times 2n} \end{array}\right]$$

Ft

$$
y_t = \begin{bmatrix} \| m^1 - p_t \| \\ \vdots \\ \| m^n - p_t \| \\ \operatorname{atan2}(m_y^1 - Y_t,\ m_x^1 - X_t) - \psi_t \\ \vdots \\ \operatorname{atan2}(m_y^n - Y_t,\ m_x^n - X_t) - \psi_t \end{bmatrix} + \begin{bmatrix} v_t^1\text{, distance} \\ \vdots \\ v_t^n\text{, distance} \\ v_t^1\text{, bearing} \\ \vdots \\ v_t^n\text{, bearing} \end{bmatrix}, \quad H_t = \frac{\partial h}{\partial x}\Big|_{x_{t|t-1}}
$$

$$
\frac{\partial}{\partial x}(|x|) = \frac{x}{|x|}
$$

$$
\frac{\partial}{\partial x}(\arctan x) = \frac{1}{1+x^2}
$$

$$
H_t \in \mathbb{R}^{2n \times (2n+3)}
$$

$$
H_t = \begin{bmatrix}
\frac{-(m_x^1 - X_t)}{\sqrt{(m_x^1 - x_t)^2 + (m_y^1 - y_t)^2}} & \frac{-(m_y^1 - Y_t)}{\sqrt{(m_x^1 - x_t)^2 + (m_y^1 - y_t)^2}} & 0 & \frac{m_x^1 - X_t}{\sqrt{(m_x^1 - x_t)^2 + (m_y^1 - y_t)^2}} & \frac{m_y^1 - Y_t}{\sqrt{(m_x^1 - x_t)^2 + (m_y^1 - y_t)^2}} & 0 & 0 & 0 & 0 & \cdots \\
\frac{-(m_x^2 - X_t)}{\sqrt{(m_x^2 - x_t)^2 + (m_y^2 - y_t)^2}} & \frac{-(m_y^2 - Y_t)}{\sqrt{(m_x^2 - x_t)^2 + (m_y^2 - y_t)^2}} & 0 & 0 & 0 & \frac{m_x^2 - X_t}{\sqrt{(m_x^2 - x_t)^2 + (m_y^2 - y_t)^2}} & \frac{m_y^2 - Y_t}{\sqrt{(m_x^2 - x_t)^2 + (m_y^2 - y_t)^2}} & 0 & 0 & \cdots \\
\vdots & \vdots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\
\frac{-(m_x^n - X_t)}{\sqrt{(m_x^n - x_t)^2 + (m_y^n - y_t)^2}} & \frac{-(m_y^n - Y_t)}{\sqrt{(m_x^n - x_t)^2 + (m_y^n - y_t)^2}} & 0 & \vdots & \vdots & \vdots & \vdots & \cdots & \frac{m_x^n - X_t}{\sqrt{(m_x^n - x_t)^2 + (m_y^n - y_t)^2}} & \frac{m_y^n - Y_t}{\sqrt{(m_x^n - x_t)^2 + (m_y^n - y_t)^2}} \\
\frac{m_y^1 - Y_t}{(m_x^1 - x_t)^2 + (m_y^1 - y_t)^2} & \frac{-(m_x^1 - X_t)}{(m_x^1 - x_t)^2 + (m_y^1 - y_t)^2} & -1 & \frac{-(m_y^1 - Y_t)}{(m_x^1 - x_t)^2 + (m_y^1 - y_t)^2} & \frac{m_x^1 - X_t}{(m_x^1 - x_t)^2 + (m_y^1 - y_t)^2} & 0 & 0 & 0 & 0 & \cdots \\
\frac{m_y^2 - Y_t}{(m_x^2 - x_t)^2 + (m_y^2 - y_t)^2} & \frac{-(m_x^2 - X_t)}{(m_x^2 - x_t)^2 + (m_y^2 - y_t)^2} & -1 & 0 & 0 & \frac{-(m_y^2 - Y_t)}{(m_x^2 - x_t)^2 + (m_y^2 - y_t)^2} & \frac{m_x^2 - X_t}{(m_x^2 - x_t)^2 + (m_y^2 - y_t)^2} & 0 & 0 & \cdots \\
\vdots & \vdots & -1 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\
\frac{m_y^n - Y_t}{(m_x^n - x_t)^2 + (m_y^n - y_t)^2} & \frac{-(m_x^n - X_t)}{(m_x^n - x_t)^2 + (m_y^n - y_t)^2} & -1 & \vdots & \vdots & \vdots & \vdots & \cdots & \frac{-(m_y^n - Y_t)}{(m_x^n - x_t)^2 + (m_y^n - y_t)^2} & \frac{m_x^n - X_t}{(m_x^n - x_t)^2 + (m_y^n - y_t)^2}
\end{bmatrix}
$$

$\# H_t$

Exercise 2.

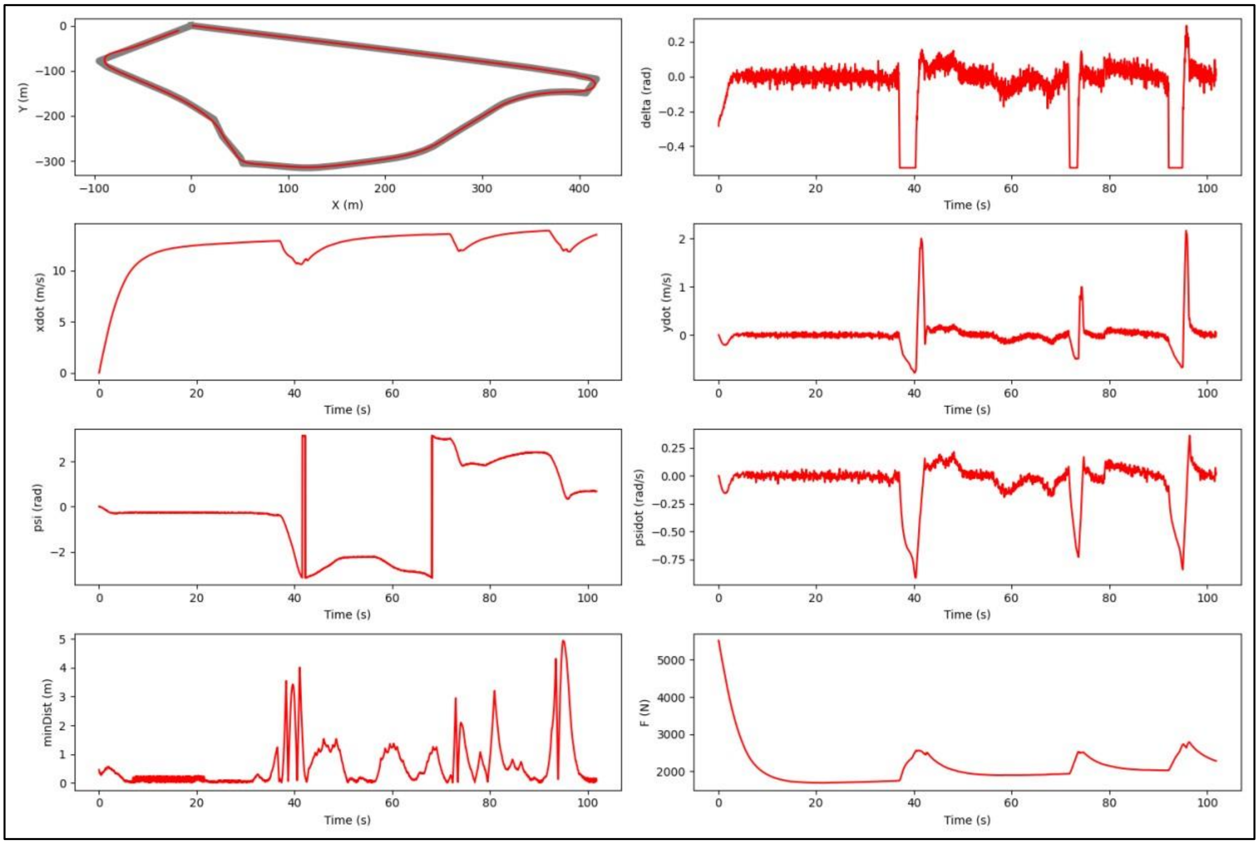
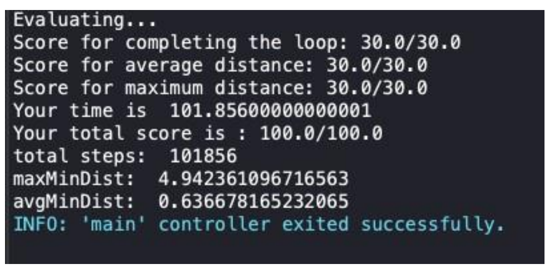


Figure 1. The final completion plots.



```
Evaluating...
Score for completing the loop: 30.0/30.0
Score for average distance: 30.0/30.0
Score for maximum distance: 30.0/30.0
Your time is  101.85600000000001
Your total score is : 100.0/100.0
total steps:  101856
maxMinDist:  4.942361096716563
avgMinDist:  0.636678165232065
INFO: 'main' controller exited successfully.
```

Figure 2. The Webots terminal output for completion time.

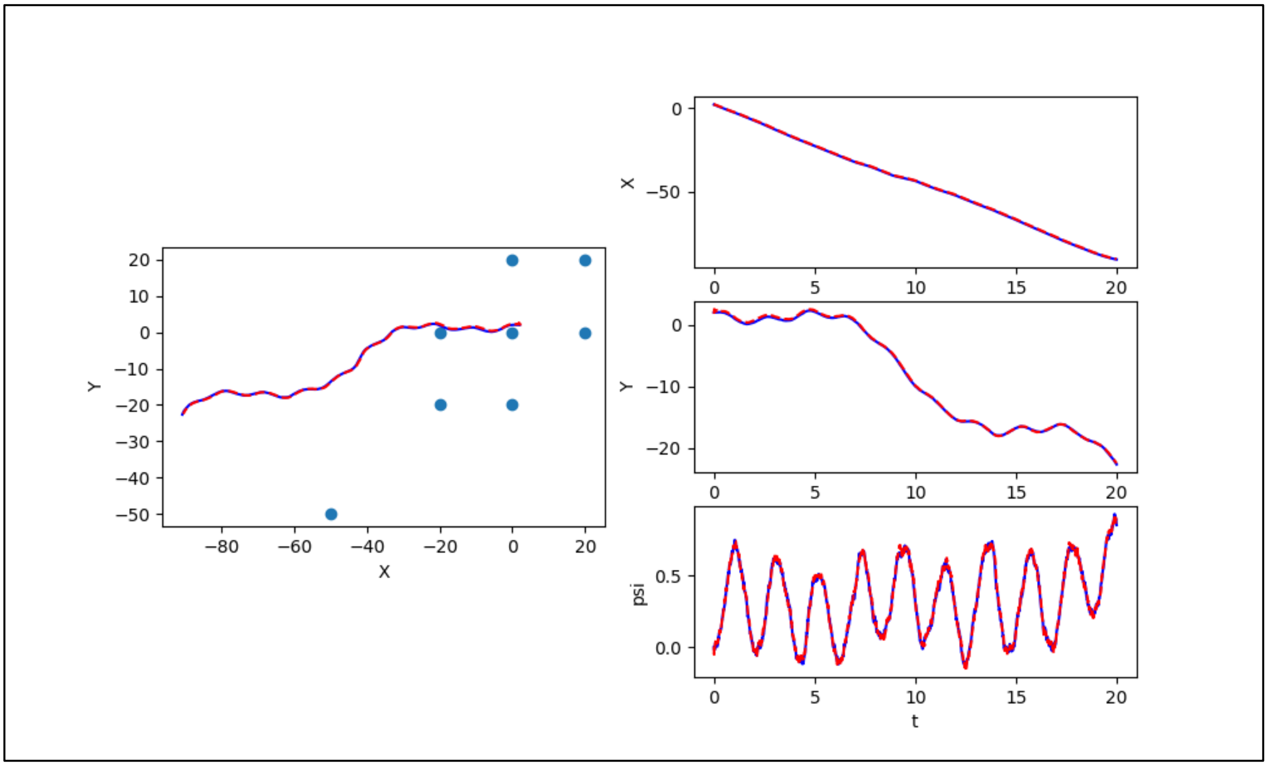


Figure 3. Plots from the efk_slam.py test script.

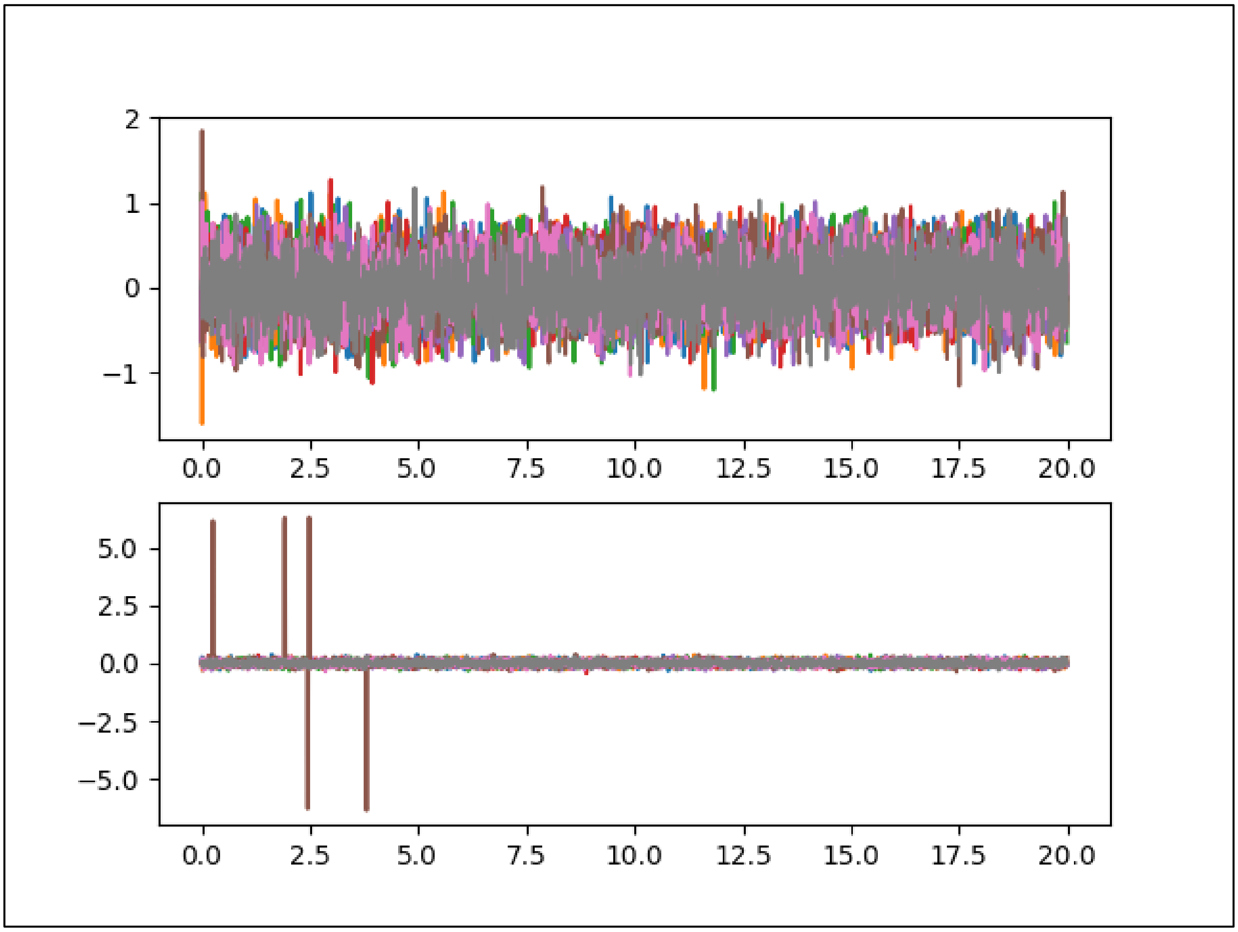


Figure 4. Plots from the efk_slam.py test script.