m2-l2-p3

February 3, 2024

1 M2-L2 Problem 3 (5 points)

When flow is directed across a pin fin heat sink, increasing fluid velocity can improve the heat transfer, making the heat sink more effective.

You have been given a dataset containing measurements for such a scenario, which contains the following: - Input: Reynolds Number of air flowing past the heat sink - Output: Heat transfer coefficient of the heat sink, in W/(m ^{2}) K

Your job is to train a model on this data to predict the heat transfer coefficient, given Reynolds number as input. You will use a high-order polynomial

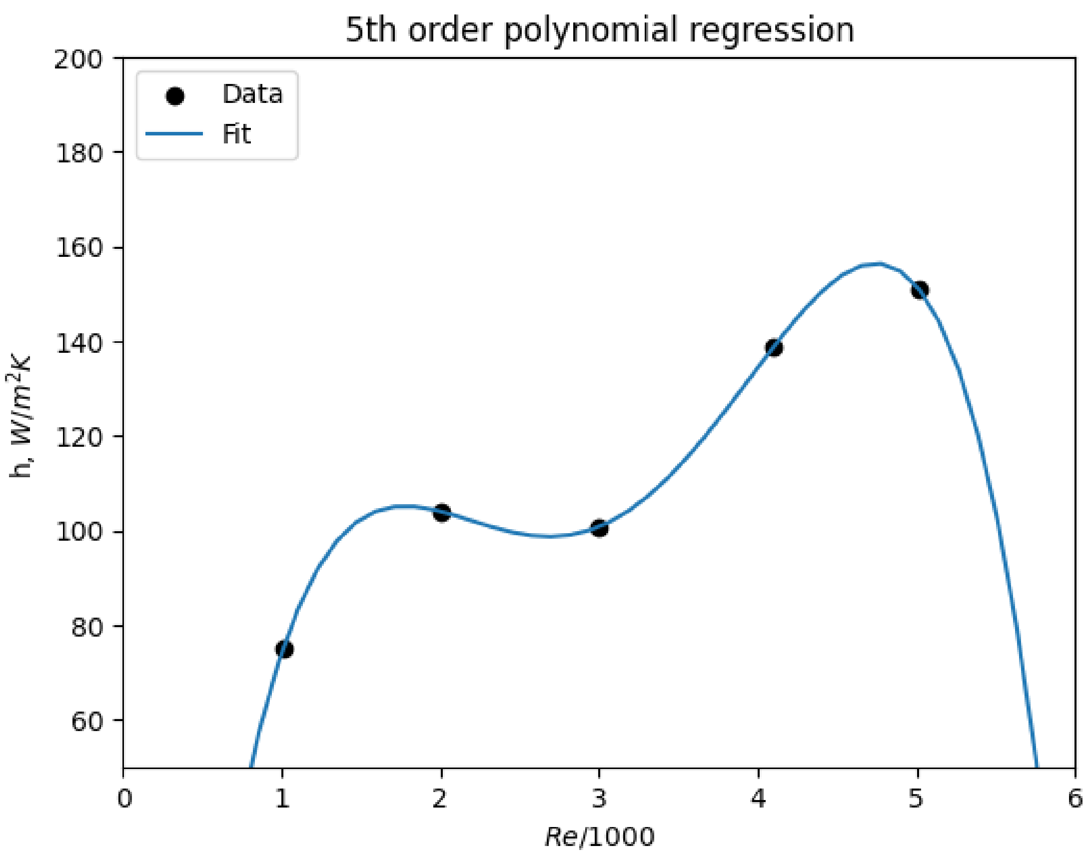
Start by loading the data in the following cell:

[5]:
```
import numpy as np
import matplotlib.pyplot as plt

def plot_data_with_regression(x_data, y_data, x_reg, y_reg, title=""):
    plt.figure()

    plt.scatter(x_data.flatten(), y_data.flatten(), label="Data", c="black")
    plt.plot(x_reg.flatten(), y_reg.flatten(), label="Fit")

    plt.legend(loc="upper left")
    plt.xlabel(r"$Re / 1000$")
    plt.ylabel(r"h, $W/m^2 K$")
    plt.xlim(0,6)
    plt.ylim(50,200)
    plt.title(title)
    plt.show()

deg = 5
x = np.array([1.010,2.000,2.990,4.100,5.020])
y = np.array([75.1, 104.0,100.6,138.8,150.8])
X = np.vander(x,deg)

xreg = np.linspace(0,6)
Xreg = np.vander(xreg,deg)
```

1.1 Least Squares Regression

As we have done for previous problems, we can do least squares regression by computing the pseudo-inverse of the design matrix. Notice how the model performs beyond the training data.

[6]:
```
w = np.linalg.inv(X.T @ X) @ X.T @ y.reshape(-1,1)
yreg = Xreg @ w
plot_data_with_regression(x, y, xreg, yreg, "5th order polynomial regression")
```



1.2 L2 Regularization

Notice that the plot above reveals that our fifth-order model is overfitting to the data. Let's try applying L2 regularization to fix this. In the lecture, the closed-form solution to least squares with L2 regularization was:

$$w = (X'X + \lambda I_m)^{-1} X'y$$

where I_m is the identity matrix, but with zero in the bias row/column instead of 1; λ is regularization strength; X' is the design matrix and y column vector output.

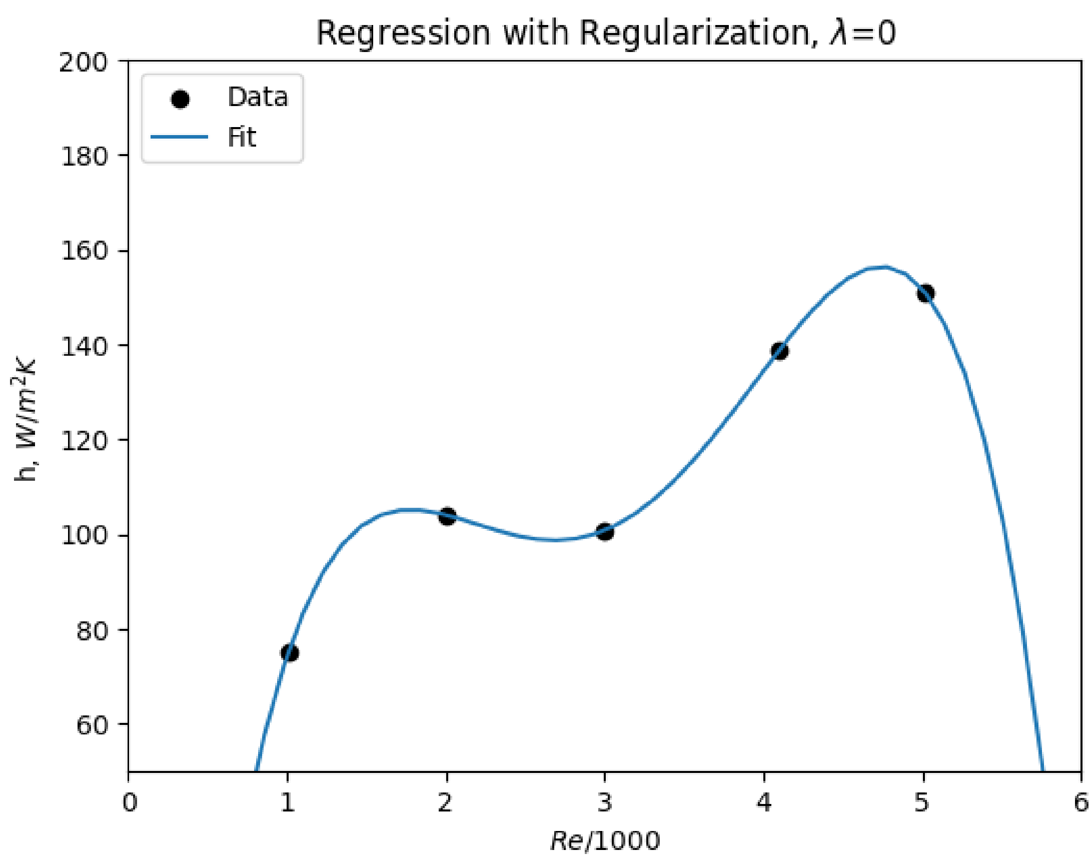
Complete the function below to compute this w for a given lambda:

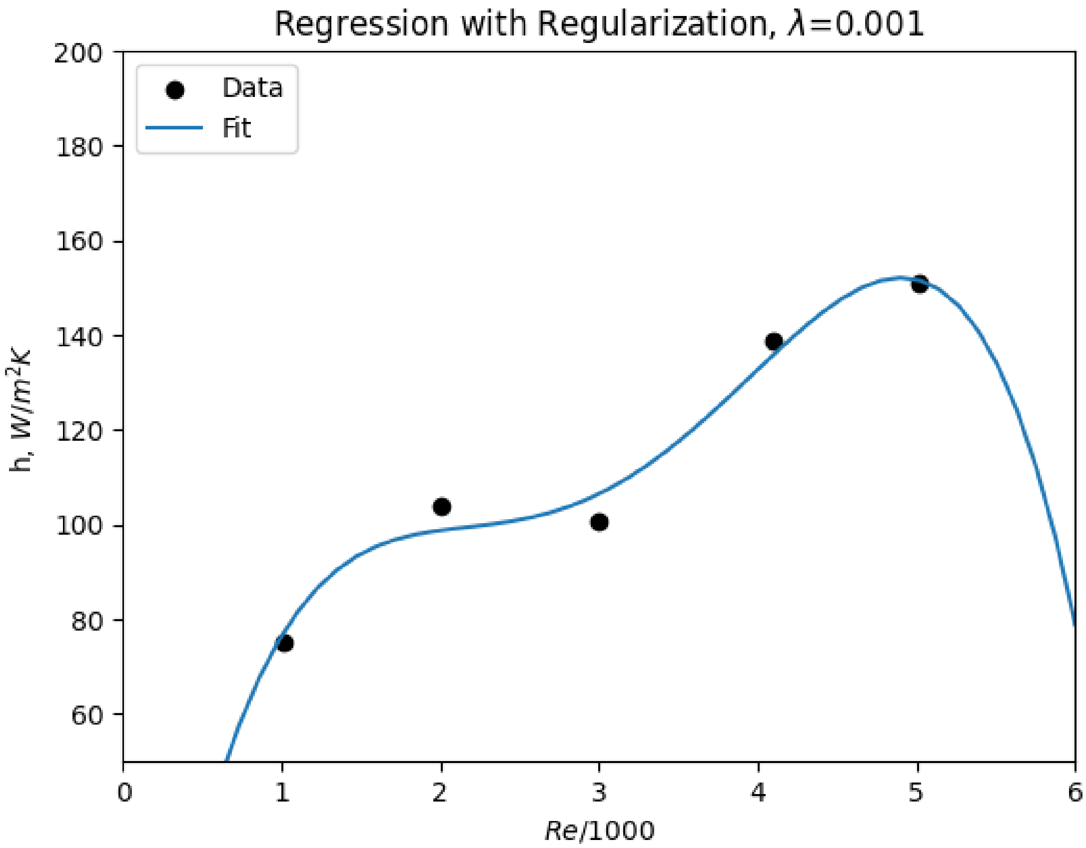
[7]:
```python
def get_regularized_w(L):
    I_m = np.eye(deg)
    I_m[-1,-1] = 0

    # YOUR CODE GOES HERE
    # return regularized w
    regularized_w = np.linalg.inv(X.T @ X + L*I_m) @ X.T @ y.reshape(-1,1)
    return regularized_w
```

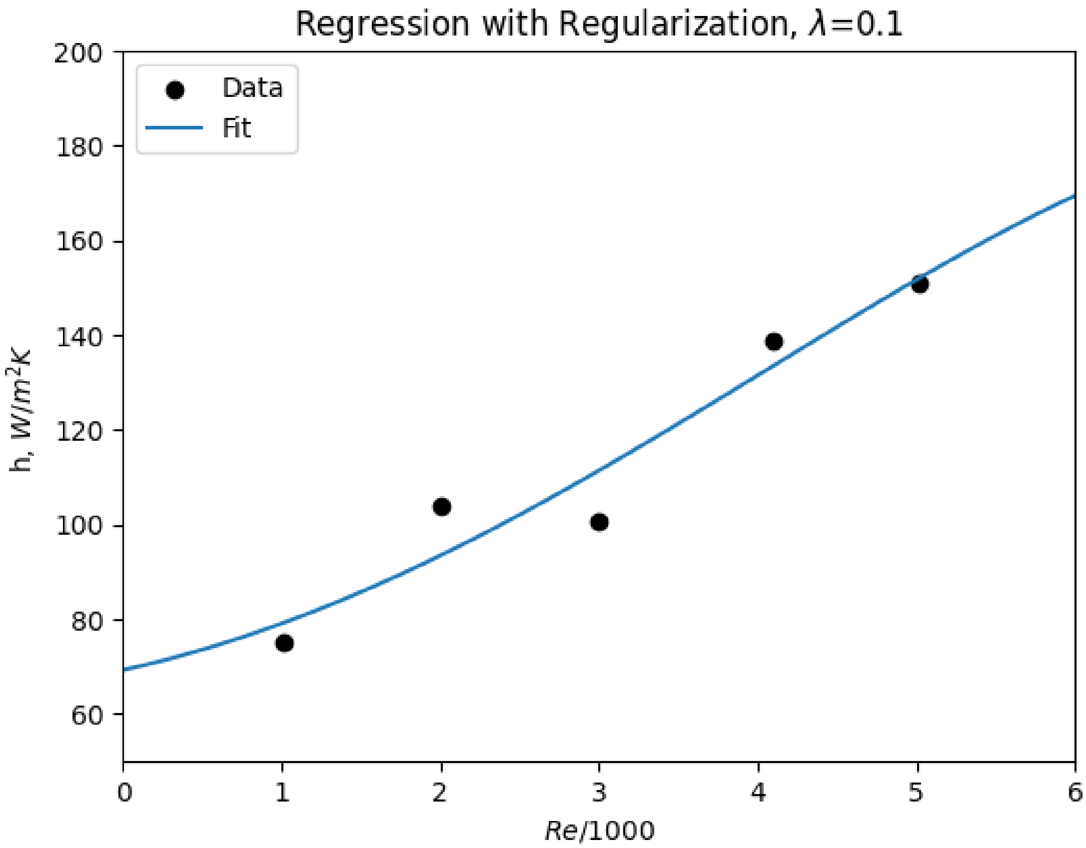
1.3 Testing different lambda values

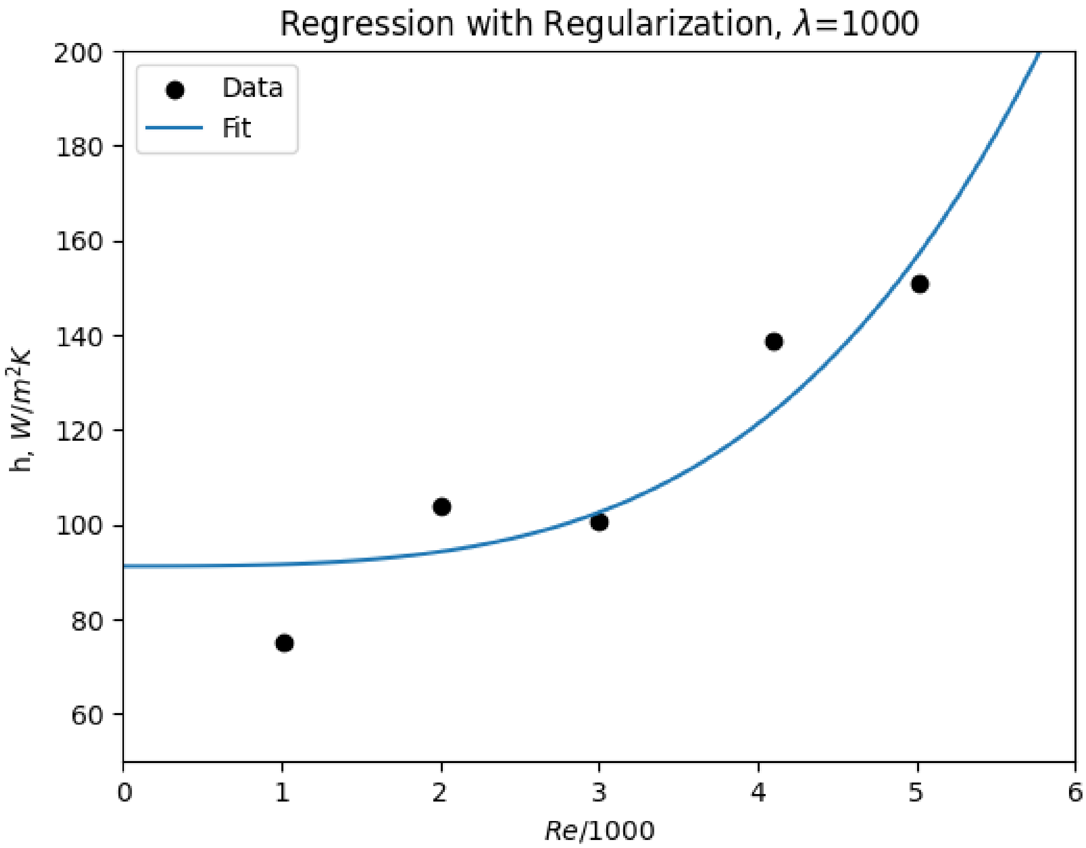
With the above function written, we can compute w for some different values of lambda and decide which is qualitatively best.

[8]:
```python
for L in [0,.001,0.1,1000]:
    w = get_regularized_w(L)
    yreg = Xreg @ w
    plot_data_with_regression(x, y, xreg, yreg,
                              f"Regression with Regularization, $\lambda$={L}")
```




Regression with Regularization, λ=0.001
Data
Fit
h, W/m^2K
$Re/1000$
200
180
160
140
120
100
80
60
0
1
2
3
4
5
6


Regression with Regularization, λ=0.1
Data
Fit
h, W/m^2K
$Re/1000$
200
180
160
140
120
100
80
60
0
1
2
3
4
5
6



1.4 Model Selection

Which value of lambda appears to yield the "best" model?

When lambda is 0.1, the model appears to yield the best.