Problem 1 (20 points)

Problem Description

As a lecture activity, you performed support vector classification on a linearly separable dataset by solving the quadratic programming optimization problem to create a large margin classifier.

Now, you will use a similar approach to create a soft margin classifier on a dataset that is not cleanly separable.

Fill out the notebook as instructed, making the requested plots and printing necessary values.

You are welcome to use any of the code provided in the lecture activities.

Summary of deliverables:

Functions (described later):

- `soft_margin_svm(X,y,C)`

Results:

- Print the values of w1, w2, and b for the C=0.05 case

Plots:

- Plot the data with the optimized margin and decision boundary for the case C=0.05
- Make 4 such plots for the requested C values

Discussion:

- Respond to the prompt asked at the end of the notebook

Imports and Utility Functions:

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap

from cvxopt import matrix, solvers
solvers.options['show_progress'] = False

def plot_boundary(x, y, w1, w2, b, e=0.1):
    x1min, x1max = min(x[:,0]), max(x[:,0])
    x2min, x2max = min(x[:,1]), max(x[:,1])

    xb = np.linspace(x1min,x1max)
    y_0 = 1/w2*(-b-w1*xb)
    y_1 = 1/w2*(1-b-w1*xb)
    y_m1 = 1/w2*(-1-b-w1*xb)
```

```
    cmap = ListedColormap(["purple","orange"])

    plt.scatter(x[:,0],x[:,1],c=y,cmap=cmap)
    plt.plot(xb,y_0,'-',c='blue')
    plt.plot(xb,y_1,'--',c='green')
    plt.plot(xb,y_m1,'--',c='green')
    plt.xlabel('$x_1$')
    plt.ylabel('$x_2$')
    plt.axis((x1min-e,x1max+e,x2min-e,x2max+e))
```

Load data

Data is loaded as follows:

- X: input features, Nx2 array

- y: output class, length N array

```
data = np.load("data/w4-hw1-data.npy")
X = data[:, 0:2]
y = data[:, 2]
```

Soft Margin SVM Optimization Problem

For soft-margin SVM, we introduce N slack variables ξ_i (one for each point), and reformulate the optimization problem as:

$$\min_{w,b} \frac{1}{2}||w||^2 + C\sum_i \xi_i$$

$$\text{subject to: } y_i\left(w^T x_i + b\right) \geq 1 - \xi_i ; \xi_i \geq 0$$

To put this into a form compatible with `cvxopt`, we will need to assemble large matrices as described in the next section.

Soft Margin SVM function

Define a function `soft_margin_svm(X, y, C)` with inputs:

- `X`: (Nx2) array of input features
- `y`: Length N array of output classes, -1 or 1
- `C`: Regularization parameter

In this function, do the following steps:

1. Create the P, q, G, and h arrays for this problem (each comprised of multiple sub-matrices you need to combine into one)

- `P`: (3+N) x (3+N)

- Upper left: Identity matrix, but with 0 instead of 1 for the bias (third) row/column
  - Upper right (3xN): Zeros
  - Lower left (Nx3): Zeros
  - Lower right: (NxN): Zeros
- `q`: (3+N) x (1)
  - Top (3x1): Vector of zeros
  - Bottom (Nx1): Vector filled with 'C'
- `G`: (N+N) x (N+3):
  - Upper left (Nx3): Negative y multiplied element-wise by [`x1`, `x2`, `1`]
  - Upper right (NxN): Negative identity matrix
  - Lower left (Nx3): Zeros
  - Lower right (NxN): Negative identity matrix
- `h`: (N+N) x (1)
  - Top: Vector of -1
  - Bottom: Vector of zeros

You can use `np.block()` to combine multiple submatrices into one.

1. Convert each of these into cvxopt matrices (Provided)
2. Solve the problem using `cvxopt.solvers.qp` (Provided)
3. Extract the `w1`, `w2`, and `b` values from the solution, and return them (Provided)

```
def soft_margin_svm(X, y, C):
    N = np.shape(X)[0]

    # YOUR CODE GOES HERE
    # Define P, q, G, h
    P = np.zeros((N+3,N+3))
    P[:2,:2] = np.eye(2)

    q = np.zeros(N+3)
    q[3:] = C

    G = np.zeros((N+N,N+3))
    G[:N,:2] = -y[:,None]*X
    G[:N,2] = -y
    G[:N,3:] = -np.eye(N)
    G[N:,:N] = -np.eye(N)
    G[N:,3:] = -np.eye(N)

    h = np.zeros(N+N).reshape(-1,1)
    h[:N] = -1

    z = solvers.qp(matrix(P),matrix(q),matrix(G),matrix(h))
    w1 = z['x'][0]
    w2 = z['x'][1]
```

```
    b  = z['x'][2]

    return w1, w2, b
```

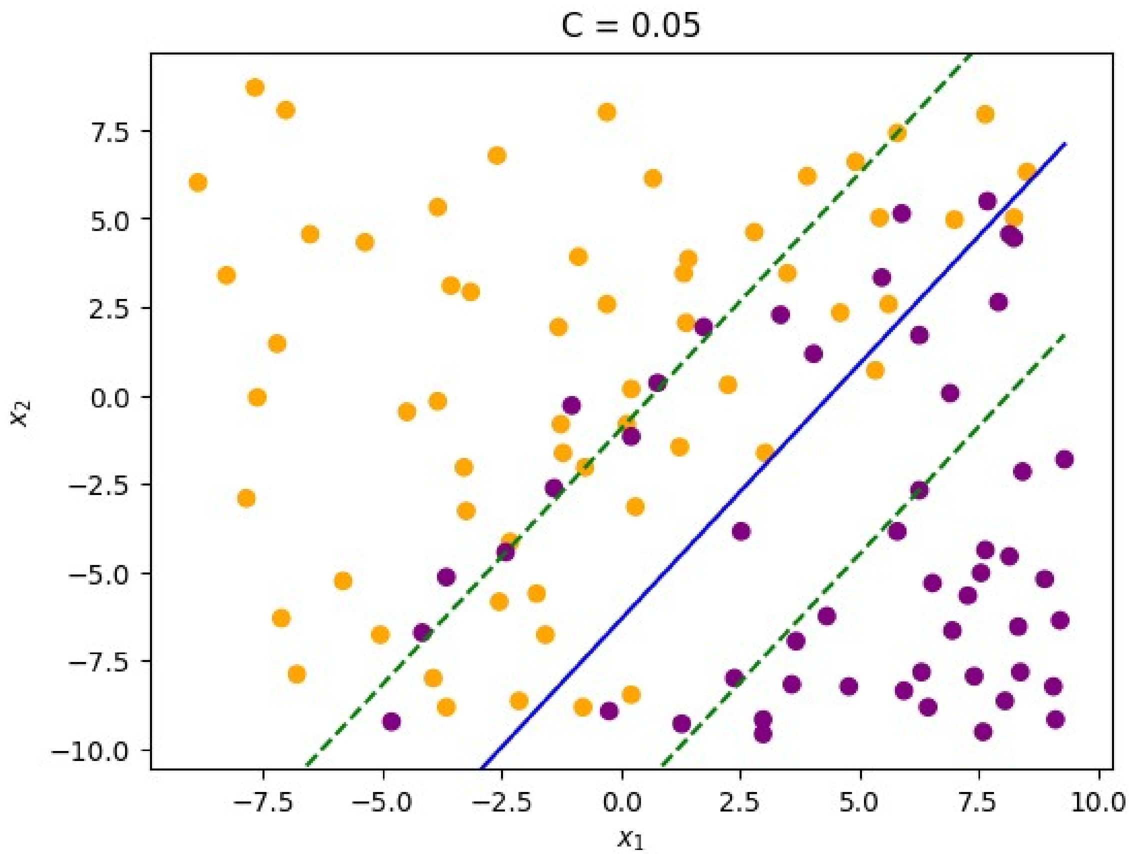
Demo: C = 0.05

Run the cell below to create the plot for the N = 0.05 case

```
C = 0.05
w1, w2, b = soft_margin_svm(X, y, C)
print(f"\nSolution\n--------\nw1: {w1:8.4f}\nw2: {w2:8.4f}\n b: {b:8.4f}")

plt.figure()
plot_boundary(X,y,w1,w2,b,e=1)
plt.title(f"C = {C}")
plt.show()
```

```
Solution
--------
w1:  -0.2685
w2:   0.1857
 b:   1.1785
```



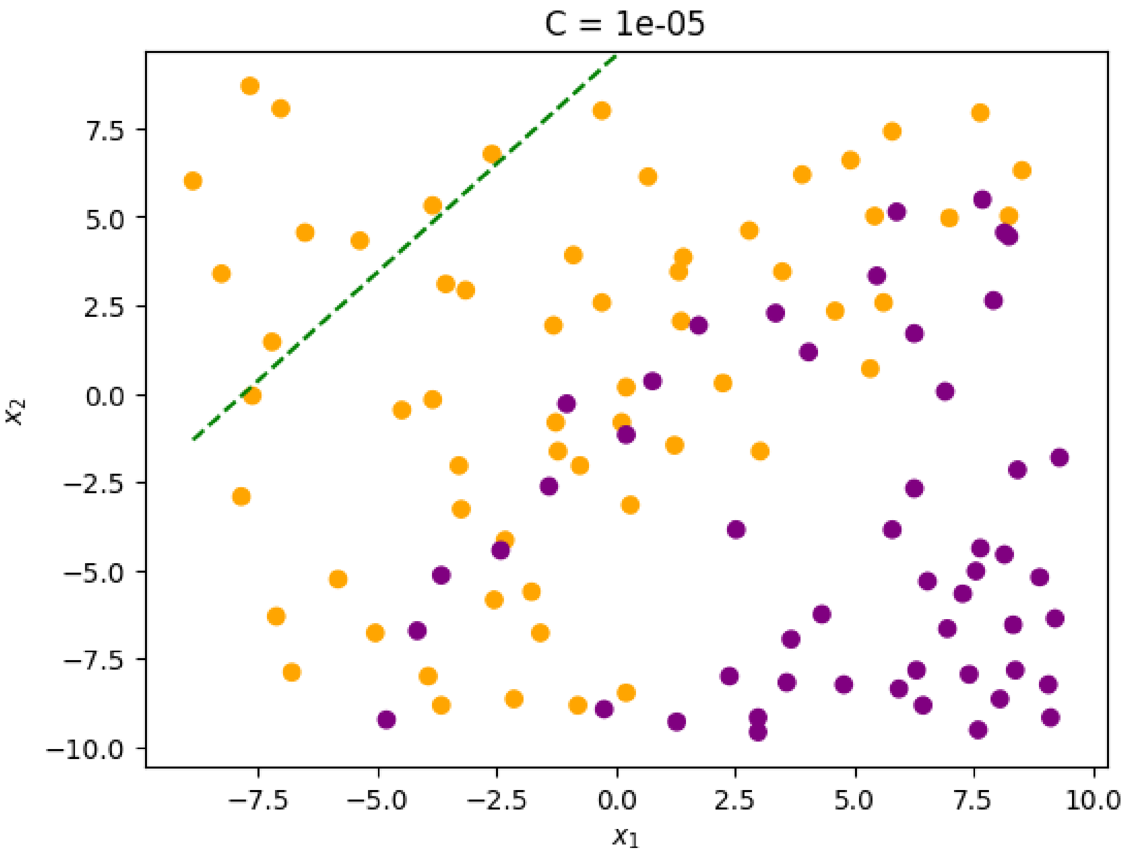
Varying C

Now loop over the C values [1e-5, 1e-3, 1e-2, 1] and generate soft margin decision boundary plots like the one above for each case.

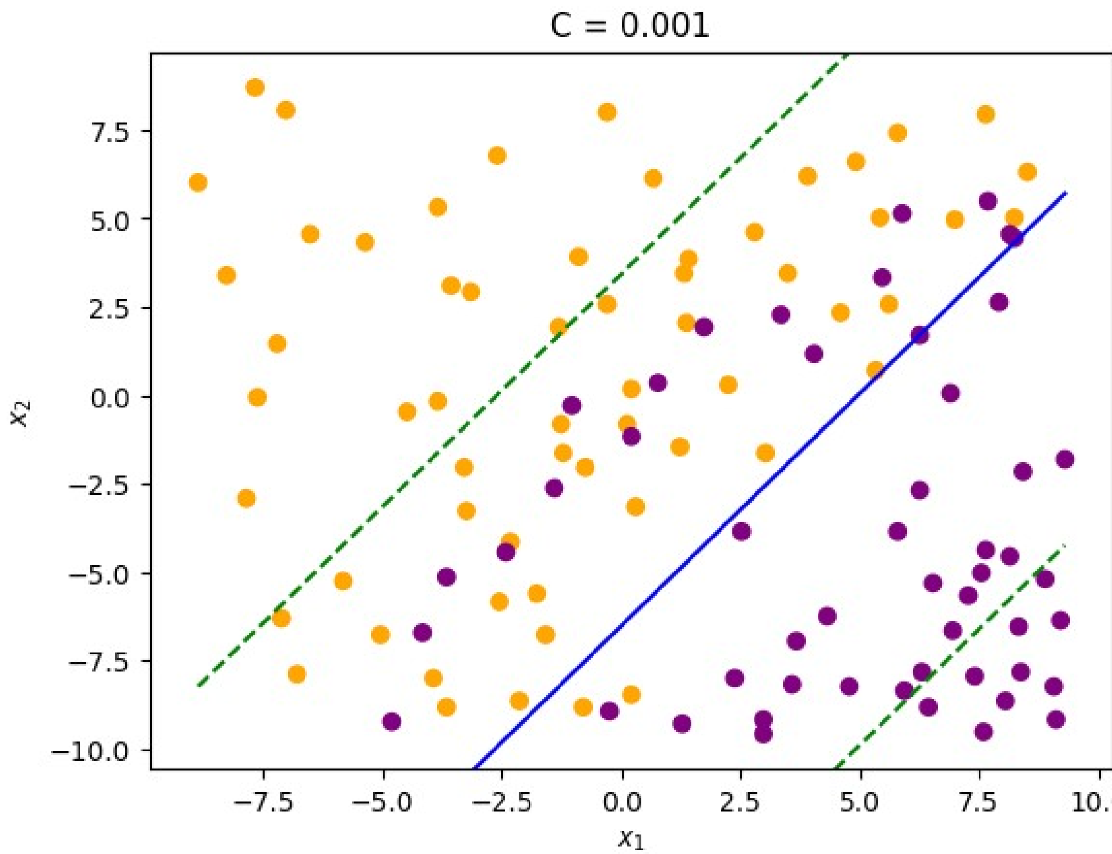
```
# YOUR CODE GOES HERE
# loop over different values of C [1e-5, 1e-3, 1e-2, 1] and plot the decision boundary
C_values = [1e-5, 1e-3, 1e-2, 1]
for C in C_values:
    w1, w2, b = soft_margin_svm(X, y, C)
    print(f"\nSolution\n--------\nw1: {w1:8.4f}\nw2: {w2:8.4f}\n b: {b:8.4f}")
    plt.figure()
    plot_boundary(X,y,w1,w2,b,e=1)
    plt.title(f"C = {C}")
    plt.show()
```

```
Solution
--------
w1:  -0.0025
```

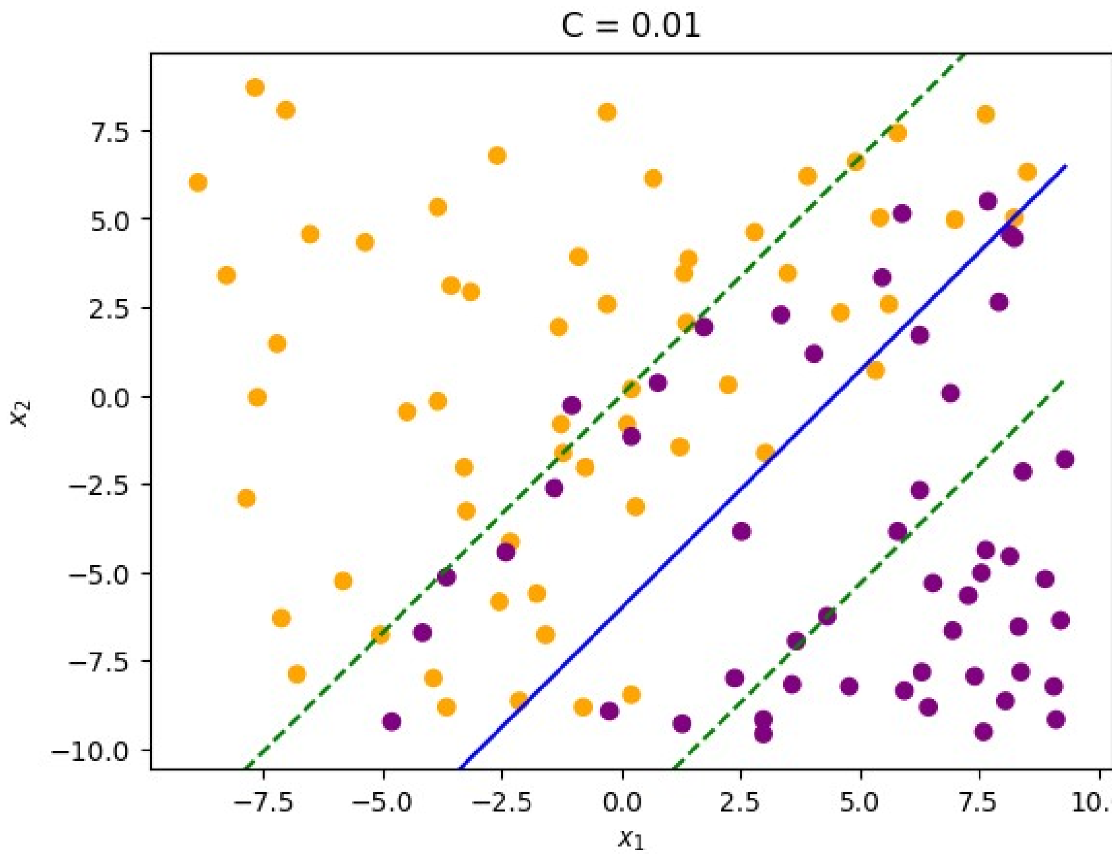
```
w2:   0.0020
 b:   0.9808
```



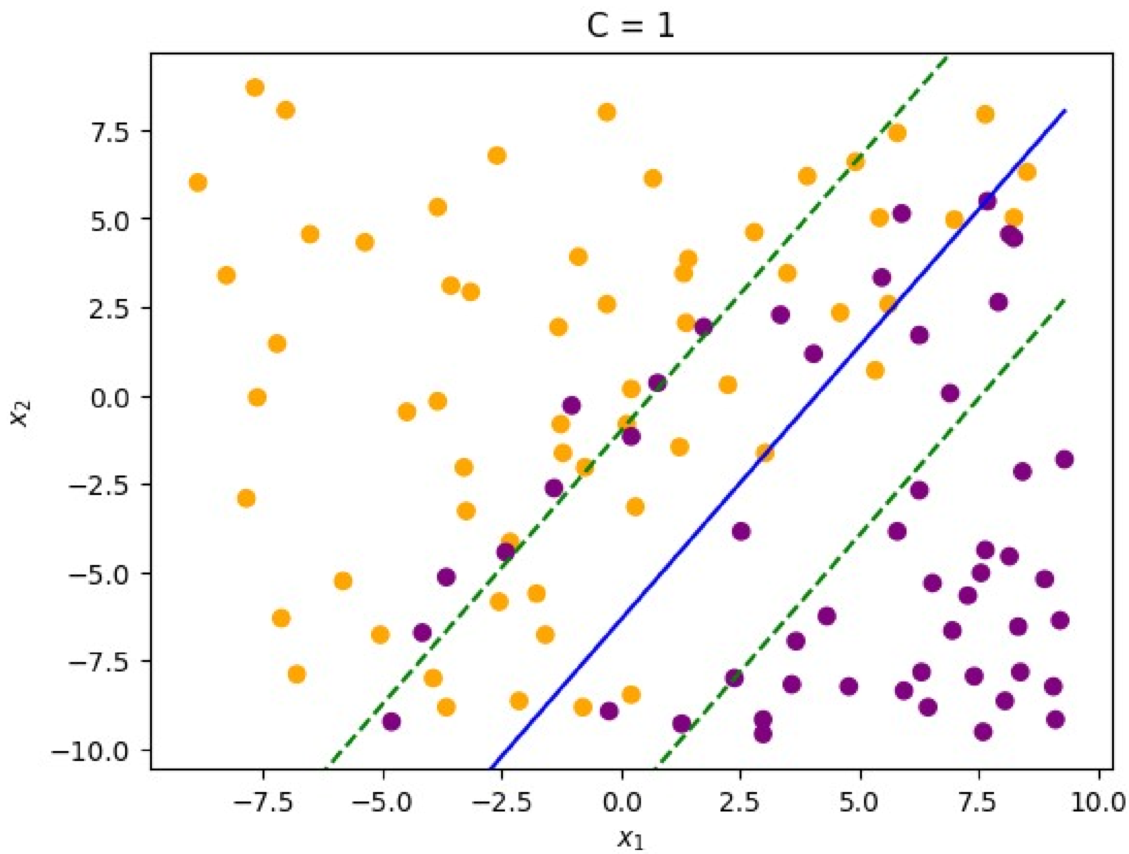

```
Solution
--------
w1:  -0.1323
w2:   0.1006
 b:   0.6563
```




```
Solution
--------
w1:  -0.2231
w2:   0.1661
 b:   1.0017
```




```
Solution
--------
w1:  -0.2899
w2:   0.1873
 b:   1.1899
```



Discussion

Please write a sentence or two discussing what happens to the decision boundary and margin as you vary C, and try to provide some rationale for why.

As the C value varies, the decision boundary and margin also change. In the above case, with a smaller C value, the decision boundary becomes less accurate and the margins between support vectors also get larger. With a larger C value, the decision boundary becomes more accurate and the margins get smaller between support vectors. The above observation is due to higher C values allowing a more complex decision boundary but can lead to overfitting and misclassifications. Lower C values lead to larger margins and better generalization to unseen data but may result in underfitting.