m3-l1-p3

February 9, 2024

1 M3-L1 Problem 3 (5 points)

[19]:
```
import numpy as np
import matplotlib.pyplot as plt


def plot_data(data, c, title="",␣
↪xlabel="$x_1$",ylabel="$x_2$",classes=["",""],alpha=1):
    N = len(c)
    colors = ['royalblue','crimson']
    symbols = ['o','s']

    plt.figure(figsize=(5,5),dpi=120)

    for i in range(2):
        x = data[:,0][c==i]
        y = data[:,1][c==i]

        plt.
↪scatter(x,y,color=colors[i],marker=symbols[i],edgecolor="black",linewidths=0.
↪4,label=classes[i],alpha=alpha)

    plt.legend(loc="upper right")
    plt.xlabel(xlabel)
    plt.ylabel(ylabel)
    ax = plt.gca()
    ax.set_xticklabels([])
    ax.set_yticklabels([])
    plt.xlim([-0.05,1.05])
    plt.ylim([-0.05,1.05])
    plt.title(title)

def plot_contour(predict, mapXY = None):
    res = 500
    vals = np.linspace(-0.05,1.05,res)
    x,y = np.meshgrid(vals,vals)
    XY = np.concatenate((x.reshape(-1,1),y.reshape(-1,1)),axis=1)
```

```
    if mapXY is not None:
        XY = mapXY(XY)
    contour = predict(XY).reshape(res, res)
    plt.contour(x, y, contour)
```

1.1 Generate Dataset

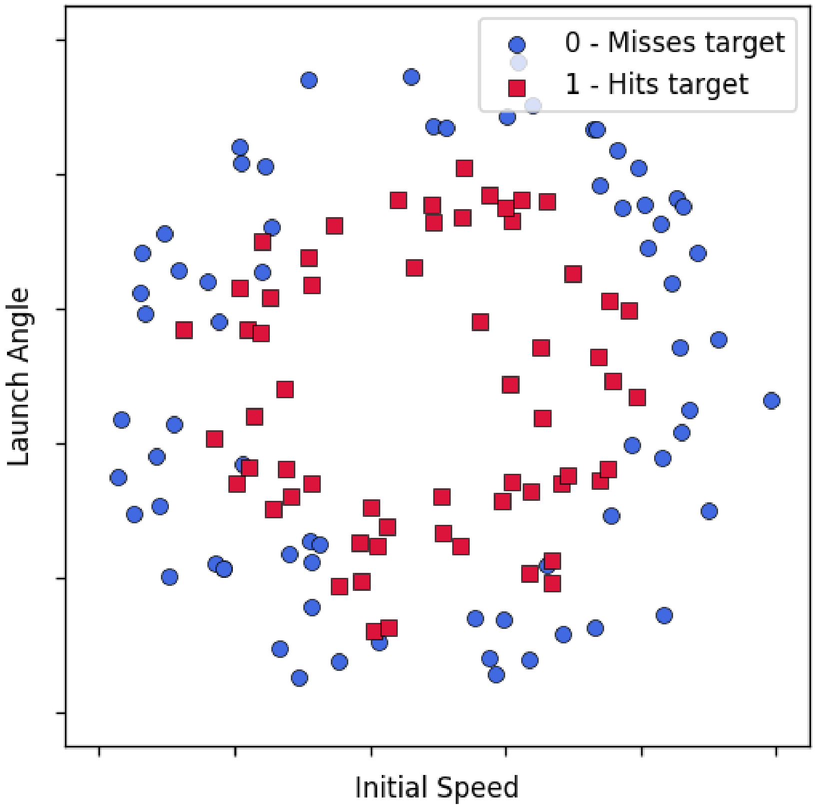
(Don't edit this code.)

[20]:
```
def sample_ring(N,x,y,ro,ri):
    theta = np.random.rand(N)*2*np.pi
    r = np.random.rand(N)
    r = np.sqrt(r*(ro**2-ri**2)+ri**2)
    xs = x + r * np.cos(theta)
    ys = y + r * np.sin(theta)
    return xs, ys

def get_ring_dataset():
    np.random.seed(0)
    c0 = sample_ring(70,0.5,0.5,0.5,0.3)
    c1 = sample_ring(60,0.45,0.47,0.36,0.15)
    xs = np.concatenate([c0[0],c1[0]],0)
    ys = np.concatenate([c0[1],c1[1]],0)
    c = np.concatenate([np.zeros(70),np.ones(60)],0)
    return np.vstack([xs,ys]).T, c
```

[21]:
```
data, classes = get_ring_dataset()
format = dict(xlabel="Initial Speed",ylabel="Launch Angle", classes=["0 -␣
 ↪Misses target", "1 - Hits target"])

plot_data(data, classes, **format)
```



1.2 Feature Expansion

Define a function to expand 2 features into more features For the features x_1 and x_2, expand into: - 1 - x_1 - x_2 - x_1^2 - x_2^2 - $\sin(x_1)$ - $\cos(x_1)$ - $\sin(x_2)$ - $\cos(x_2)$ - $\sin^2(x_1)$ - $\cos^2(x_1)$ - $\sin^2(x_2)$ - $\cos^2(x_2)$ - $\exp(x_1)$ - $\exp(x_2)$

[22]:
```
def feature_expand(x):
    x1 = x[:,0].reshape(-1, 1)
    x2 = x[:,1].reshape(-1, 1)

    # YOUR CODE GOES HERE:
    columns = [np.ones_like(x1), x1, x2] # Add all expanded features to this␣
↪list
    columns.append(x1**2)
    columns.append(x2**2)
    columns.append(np.sin(x1))
    columns.append(np.cos(x1))
    columns.append(np.sin(x2))
    columns.append(np.cos(x2))
    columns.append(np.sin(x1)**2)
    columns.append(np.cos(x1)**2)
```

```
    columns.append(np.sin(x2)**2)
    columns.append(np.cos(x2)**2)
    columns.append(np.exp(x1))
    columns.append(np.exp(x2))

    X = np.concatenate(columns, axis=1)
    return X

features = feature_expand(data)
print("Dataset size:", np.shape(data))
print("Expanded dataset size:", np.shape(features))
```

```
Dataset size: (130, 2)
Expanded dataset size: (130, 15)
```

1.3 Logistic Regression

Use SciKit-Learn's Logistic Regression model to learn the decision boundary for this data, using regularization. (The `C` argument controls regularization strength.)

Train this model on your expanded feature set.

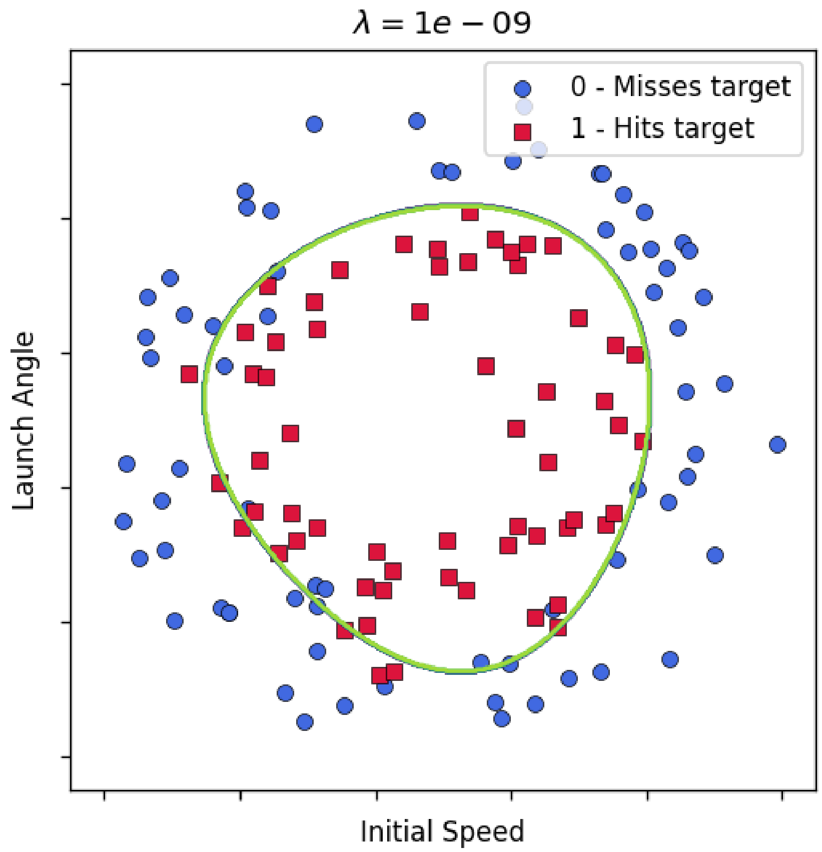
Details about how to use this are here: https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LogisticRegression.html

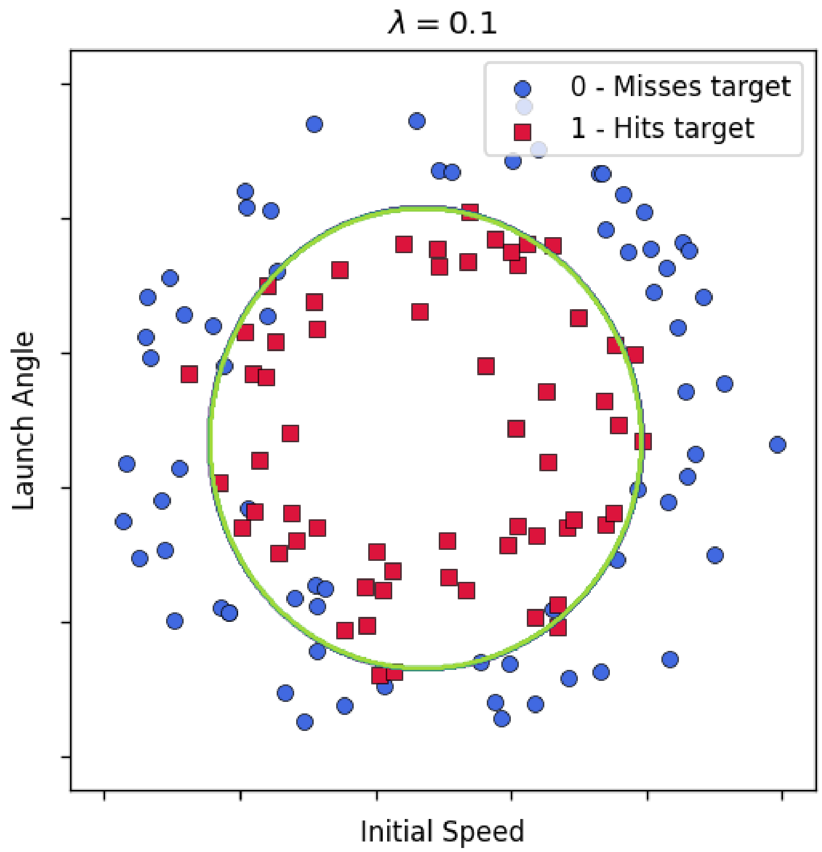
Notes: - λ is related to sklearn's regularization strength C by: $\lambda = 1/C$ - You may want to increase the maximum number of iterations

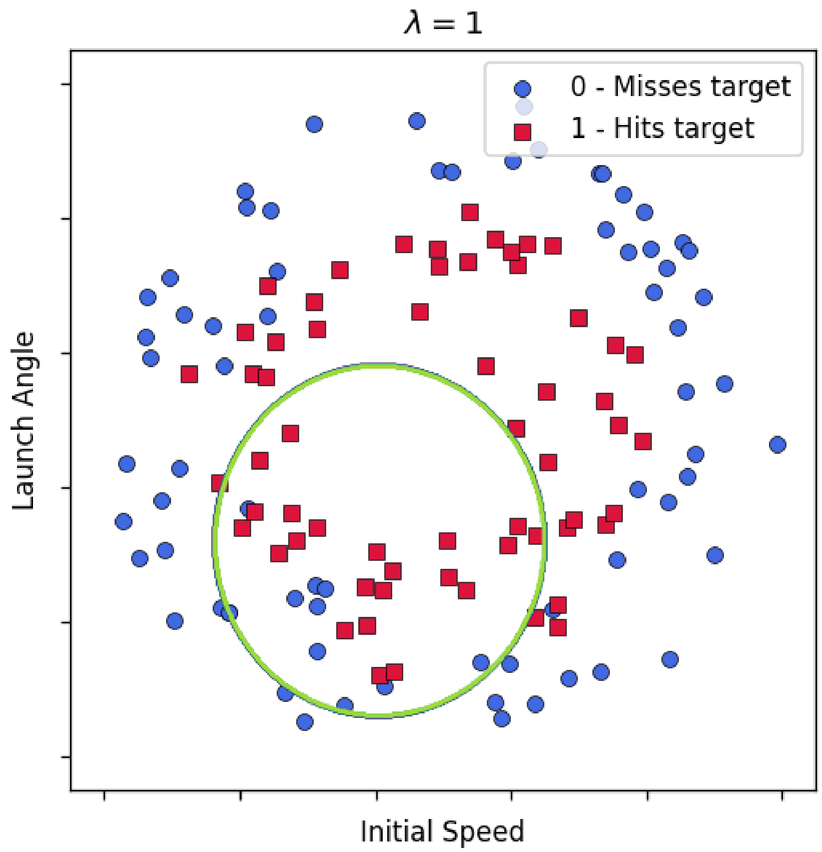
[23]:
```
from sklearn.linear_model import LogisticRegression

def get_logistic_regressor(features, classes, L = 1):
    # YOUR CODE GOES HERE
    # - Instantiate model with regularization
    # - Fit model to expanded data
    model = LogisticRegression(C=1/L)
    model.fit(features, classes)
    return model
```

[24]:
```
for L in [1e-9, 1e-1, 1]:
    model = get_logistic_regressor(features, classes, L)
    plot_data(data, classes, **format, title=f"$\lambda={L}$")
    plot_contour(model.predict, feature_expand)
    plt.show()
```


$\lambda = 1e - 09$
0 - Misses target
1 - Hits target
Launch Angle
Initial Speed


$\lambda = 0.1$
0 - Misses target
1 - Hits target
Launch Angle
Initial Speed



As λ increases, note what happens to the decision boundary. Why does this occur?

AS λ increases, the decision boundary becomes smoother and simpler with less overfitting but higher bias. This is due to an increase in λ leads to the increase in regularization strength.