M7-L1 Problem 2

In this problem, you will explore what happens when you change the weights/biases of a neural network.

Neural networks act as functions that attempt to map from input data to output data. In training a neural network, the goal is to find the values of weights and biases that minimize the loss between their output and the desired output. This is typically done with a technique called backpropagation; however, here you will simply note the effect of changing specific weights in the network which has been pre-trained.

First, load the data and initial weights/biases below:

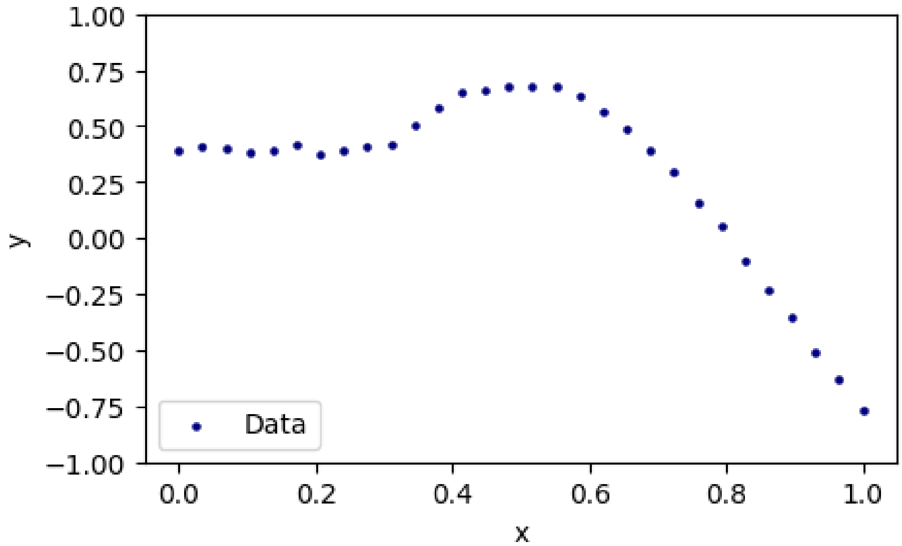
```
import numpy as np
import matplotlib.pyplot as plt

x = np.array([0.        , 0.03448276, 0.06896552, 0.10344828,
0.13793103,0.17241379, 0.20689655, 0.24137931, 0.27586207,
0.31034483,0.34482759, 0.37931034, 0.4137931 , 0.44827586,
0.48275862,0.51724138, 0.55172414, 0.5862069 , 0.62068966,
0.65517241,0.68965517, 0.72413793, 0.75862069, 0.79310345,
0.82758621,0.86206897, 0.89655172, 0.93103448, 0.96551724,
1.        ]).reshape(-1,1)
y = np.array([ 0.38914369,  0.40997345,  0.40282978,  0.38493705,
0.394214  ,0.41651437,  0.37573321,  0.39571087,  0.41265936,
0.41953955,0.50596807,  0.58059196,  0.6481607 ,  0.66050901,
0.67741369,0.67348567,  0.67696078,  0.63537378,  0.56446933,
0.48265412,0.39540671,  0.29878237,  0.15893846,  0.05525194, -
0.10070259,-0.23055219, -0.35288448, -0.51317604, -0.63377544, -
0.76849408]).reshape(-1,1)

weights = [np.array([[-5.90378086,  0,  0 ]]).T,
           np.array([[ 0.8996511 ,  4.75805319, -0.95266992],[-
0.99667812, -0.89303165,  3.19020423],[-1.65213421, -2.93268438,
2.61332701]]).T,
           np.array([[ 1.71988943, -1.56198034, -3.31173131]])]

biases = [np.array([ 2.02112296, -3.47589349, -1.11586831]),
np.array([ 1.35350721, -0.11181542, -4.0283719 ]),
np.array([0.51626399])]

plt.figure(figsize=(5,3))
plt.scatter(x,y,s=5,c="navy",label="Data")
plt.legend(loc="lower left")
plt.ylim(-1,1)
plt.xlabel("x")
plt.ylabel("y")
plt.show()
```



MLP Function

Copy in your MLP function (and all necessary helper functions) below. Make sure it is called `MLP()`. In this case, you can plug in `x`, `weights`, and `biases` to try and predict `y`. Make sure you use the sigmoid activation function after each layer (except the final layer).

```
# YOUR CODE GOES HERE
# perceptron layer function
def perceptron_layer(x, weight, bias):
    a = np.dot(x, weight.T) + bias
    return a

# sigmoid function
def sigmoid(x):
    return 1./(1.+np.exp(-x))

# MLP function
def MLP(x, weights, biases):
    # YOUR CODE GOES HERE
    for i in range(len(weights)-1):
        x = sigmoid(perceptron_layer(x, weights[i], biases[i]))
    # not applying sigmoid to the last layer
    x = perceptron_layer(x, weights[-1], biases[-1])
    return x
```

Varying weights

The provided network has 2 hidden layers, each with 3 neurons. The weights and biases are shown below. Note the weights w_a and w_b -- these are left for you to investigate:

$$ \underline{x; (N\times 1)} \rightarrow \sigma\left(w = \begin{bmatrix} -5.9\ \
boldsymbol{w_a}\ \boldsymbol{w_b}\ \end{bmatrix}

;b = \begin{bmatrix} 2.02\ -3.48\ -1.12\ \end{bmatrix}'

\right)\rightarrow

\underline{(N\times 3)} \rightarrow \sigma\left(w = \begin{bmatrix} 0.9 & -1. & -1.65\ 4.76 & -0.89 & -2.93\ -0.95 & 3.19 & 2.61\ \end{bmatrix}

;b = \begin{bmatrix} 1.35\ -0.11\ -4.03\ \end{bmatrix}'

\right)\rightarrow

\underline{(N\times 3)} \rightarrow \sigma\left(w = \begin{bmatrix} 1.72\ -1.56\ -3.31\ \
end{bmatrix}'

;b = \begin{bmatrix} 0.52\ \end{bmatrix}'

\right)\rightarrow \underline{\hat{y}; (N\times 1)} $$

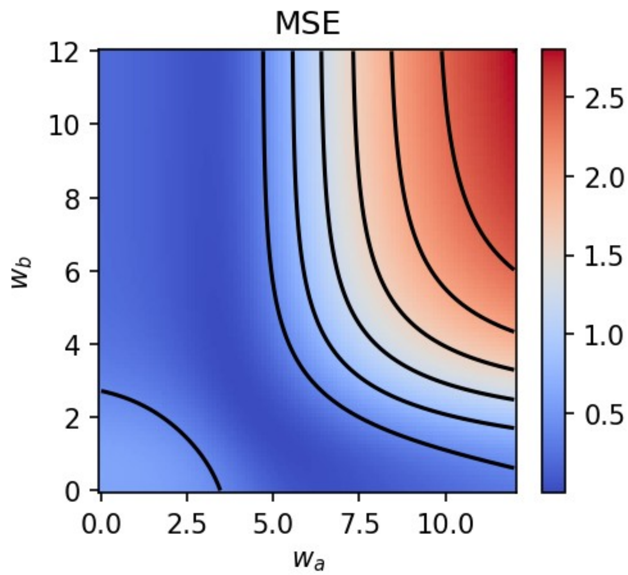
We can compute the MSE for each combination of (w_a, w_b) to see where MSE is minimized.

```
def MSE(y, pred):
    return np.mean((y.flatten()-pred.flatten())**2)

vals = np.linspace(0,12,100)
was, wbs = np.meshgrid(vals,vals)
mses = np.zeros_like(was.flatten())

for i in range(len(was.flatten())):
    ws, bs = weights.copy(), biases.copy()
    ws[0][1,0] = was.flatten()[i]
    ws[0][2,0] = wbs.flatten()[i]
    mses[i] = MSE(y, MLP(x, ws, bs))
mses = mses.reshape(was.shape)

plt.figure(figsize = (3.5,3),dpi=150)
plt.title("MSE")
plt.contour(was,wbs,mses,colors="black")
plt.pcolormesh(was,wbs,mses,shading="nearest",cmap="coolwarm")
plt.xlabel("$w_a$")
plt.ylabel("$w_b$")
plt.colorbar()
plt.show()
```




```
%matplotlib inline
from ipywidgets import interact, interactive, fixed, interact_manual,
Layout, FloatSlider, Dropdown


def plot(wa, wb):
    ws, bs = weights.copy(), biases.copy()
    ws[0][1,0] = wa
    ws[0][2,0] = wb

    xs = np.linspace(0,1)
    ys = MLP(xs.reshape(-1,1), ws, bs)


    plt.figure(figsize=(10,4),dpi=120)

    plt.subplot(1,2,1)
    plt.contour(was,wbs,mses,colors="black")
    plt.pcolormesh(was,wbs,mses,shading="nearest",cmap="coolwarm")
    plt.title(f"$w_a = {wa:.1f}$;  $w_b = {wb:.1f}$")
    plt.xlabel("$w_a$")
    plt.ylabel("$w_b$")
    plt.scatter(wa,wb,marker="*",color="black")
    plt.colorbar()

    plt.subplot(1,2,2)
    plt.scatter(x,y,s=5,c="navy",label="Data")
    plt.plot(xs,ys,"r-",linewidth=1,label="MLP")
    plt.title(f"MSE = {MSE(y, MLP(x, ws, bs)):.3f}")
    plt.legend(loc="lower left")
    plt.ylim(-1,1)
    plt.xlabel("x")
```

```
    plt.ylabel("y")

    plt.show()


slider1 = FloatSlider(
    value=0,
    min=0,
    max=12,
    step=.5,
    description='wa',
    disabled=False,
    continuous_update=True,
    orientation='horizontal',
    readout=False,
    layout = Layout(width='550px')
)

slider2 = FloatSlider(
    value=0,
    min=0,
    max=12,
    step=.5,
    description='wb',
    disabled=False,
    continuous_update=True,
    orientation='horizontal',
    readout=False,
    layout = Layout(width='550px')
)



interactive_plot = interactive(
    plot,
    wa = slider1,
    wb = slider2
    )
output = interactive_plot.children[-1]
output.layout.height = '500px'

interactive_plot
```


```
{"model_id":"d67e866732c2403286652e39ffea65e9","version_major":2,"version_minor":0}
```


Questions

1. For $w_a = 4.0$, what value of w_b gives the lowest MSE (to the nearest 0.5)?

- *ANSWER:* For $w_a = 4.0$, w_b of 3.0 gives the lowest MSE.

1. For the large values of w_a and w_b, describe the MLP's predictions.

- *ANSWER:* For the large values of w_a and w_b, the MLP's predictions before less and less accurate with larger MSE values.