M12-L1 Problem 1

This problem is intended to demonstrate PCA on a small 2D dataset. This will emphasize how PCs are computed and what they mean.

```
import numpy as np
import matplotlib.pyplot as plt

X = np.array([[2.5, 2.4],[0.5, 0.7],[2.2, 2.9],[1.9, 2.2],[3.1, 3. ],
              [2.3, 2.7],[2., 1.6],[1., 1.1],[1.5, 1.6],[1.1, 0.9]])
```

Computing the Principal Components

First, compute the principal components of the dataset by following these steps:

1. Compute `M` (1×2), the mean of each dimension in `X`
2. Compute `S` (2×2), the covariance matrix of `X` (see `np.cov`)
3. Report `w`, the 2 eigenvalues of `S` (see `np.linalg.eig`)
4. Get `e1` and `e2`, the eigenvectors corresponding to the elements of `w`

The principal components in this problem are then `e1` and `e2`.

```
print('X:\n', X)

# YOUR CODE GOES HERE: Compute M
M = np.mean(X, axis=0)
print('\nMean of each dimension:\n', M)

# YOUR CODE GOES HERE: Compute S
S = np.cov(X, rowvar=False)
print('\nCovariance Matrix:\n', S)


# YOUR CODE GOES HERE: Compute w
w = np.linalg.eig(S)[0]
w = np.real(w)
print('\nEigenvalues of covariance matrix:\n',w)

# YOUR CODE GOES HERE: Compute e1, e2
print('\nPrincipal Components:')
v = np.linalg.eig(S)[1]
e1 = v[:,0]
e2 = v[:,1]
print('e1:',e1)
print('e2:',e2)
```

```
X:
 [[2.5 2.4]
 [0.5 0.7]
 [2.2 2.9]
 [1.9 2.2]
 [3.1 3. ]
 [2.3 2.7]
 [2.  1.6]
 [1.  1.1]
 [1.5 1.6]
 [1.1 0.9]]

Mean of each dimension:
 [1.81 1.91]

Covariance Matrix:
 [[0.61655556 0.61544444]
 [0.61544444 0.71655556]]

Eigenvalues of covariance matrix:
 [0.0490834  1.28402771]

Principal Components:
e1: [-0.73517866  0.6778734 ]
e2: [-0.6778734  -0.73517866]
```

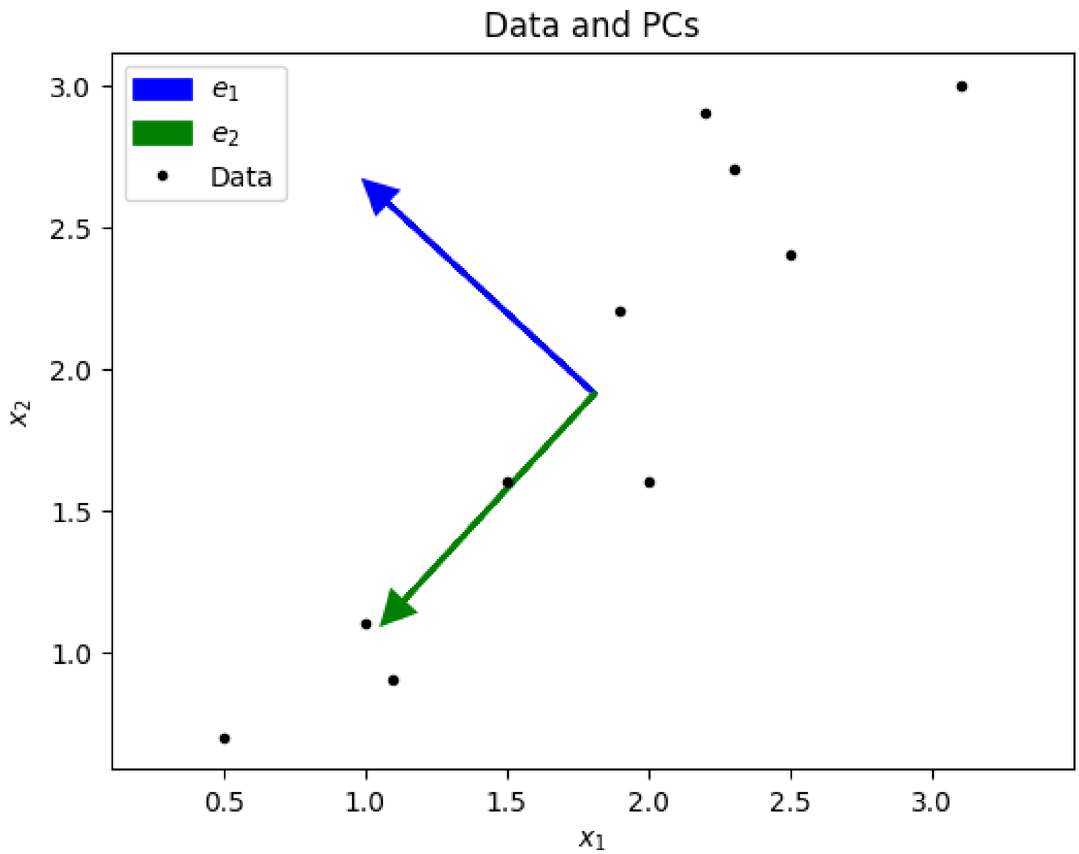
Plotting data with principal components

Complete the code below to plot the original data with principal components represented as unit vector arrows.

```
plt.figure()
plt.title("Data and PCs")

e1, e2 = e1.flatten(), e2.flatten()
plt.arrow(M[0],M[1],e1[0],e1[1], color="blue", linewidth=2,
head_width=0.1, head_length=0.1, label="$e_1$")
plt.arrow(M[0],M[1],e2[0],e2[1], color="green", linewidth=2,
head_width=0.1, head_length=0.1, label="$e_2$")
plt.plot(X[:,0],X[:,1],'.',color="black", label="Data")

plt.xlabel("$x_1$")
plt.ylabel("$x_2$")
plt.legend()
plt.axis("equal")
plt.show()
```



Plotting transformed data

Now, transform the data with the formula $a_i = (x - \mu) \cdot e_i$.

Print the transformed data matrix columns `a1` and `a2`.

Then plot the transformed data on $e_1 - e_2$ axes.

```
# YOUR CODE GOES HERE: Compute a1, a2
a1 = np.dot(X-M,e1)
a2 = np.dot(X-M,e2)

print("a_1 = ",a1)
print("a_2 = ",a2)

plt.figure()
plt.title("Transformed data")

e1, e2 = e1.flatten(), e2.flatten()
# YOUR CODE GOES HERE: Plot transformed data
plt.plot(a1,a2,'.',color="black", label="Data")
plt.legend()
```

```
plt.xlabel("$e_1$")
plt.ylabel("$e_2$")
plt.axis("equal")
plt.show()
```

```
a_1 =  [-0.17511531  0.14285723  0.38437499  0.13041721 -0.20949846
0.17528244
 -0.3498247   0.04641726  0.01776463 -0.16267529]
a_2 =  [-0.82797019  1.77758033 -0.99219749 -0.27421042 -1.67580142 -
0.9129491
  0.09910944  1.14457216  0.43804614  1.22382056]
```