

24-787: Machine Learning and Artificial Intelligence for Engineers

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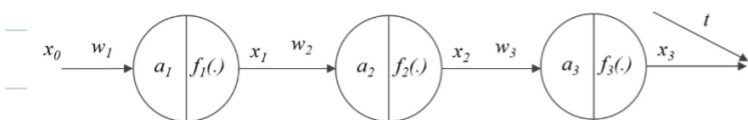
Homework 8

Due: Mar 23, 2024

Concept Questions:

Problem 1

Given: $x_0=2$, $w_1=-1$, $w_2=3$, $w_3=7$, linear (identity) activation function, $L=\frac{1}{2}e^T e$, $t=-40$



Find: $\frac{\partial L}{\partial w_3}$, $\frac{\partial L}{\partial w_2}$, $\frac{\partial L}{\partial w_1}$

Equations: $\frac{\partial L}{\partial w_3} = \delta_3 x_2$, where $\delta_3 = -(t - x_3)f'_3(a_3)$, $a_3 = w_3 x_2$

$\frac{\partial L}{\partial w_2} = \delta_2 x_1$, where $\delta_2 = \delta_3 w_3 f'_2(a_2)$, $a_2 = w_2 x_1$

$\frac{\partial L}{\partial w_1} = \delta_1 x_0$, where $\delta_1 = \delta_2 w_2 f'_1(a_1)$, $a_1 = w_1 x_0$

$x_3 = x_0 \cdot w_1 \cdot w_2 \cdot w_3$, $x_2 = x_0 \cdot w_1 \cdot w_2$, $x_1 = x_0 \cdot w_1$

Solutions:

$$x_1 = x_0 \cdot w_1 \rightarrow x_1 = 2(-1) \rightarrow x_1 = -2$$

$$x_2 = x_0 \cdot w_1 \cdot w_2 \rightarrow x_2 = 2(-1)(3) \rightarrow x_2 = -6$$

$$x_3 = x_0 \cdot w_1 \cdot w_2 \cdot w_3 \rightarrow x_3 = 2(-1)(3)(7) \rightarrow x_3 = -42$$

$$a_1 = w_1 x_0 \rightarrow a_1 = (-1)(2) \rightarrow a_1 = -2$$

$$a_2 = w_2 x_1 \rightarrow a_2 = 3(-2) \rightarrow a_2 = -6$$

$$a_3 = w_3 x_2 \rightarrow a_3 = 7(-6) \rightarrow a_3 = -42$$

$\left(\frac{\partial L}{\partial w_3}\right)$

$$\frac{\partial L}{\partial w_3} = \delta_3 x_2 \rightarrow \frac{\partial L}{\partial w_3} = -(t - x_3)f'_3(a_3)x_2 \rightarrow \frac{\partial L}{\partial w_3} = -(-40 - (-42))(1)(-6) = \frac{\partial L}{\partial w_3} = 12$$

$$\frac{\partial L}{\partial w_2} = \delta_2 x_1 \rightarrow \frac{\partial L}{\partial w_2} = \delta_3 w_3 f'_2(a_2)x_1 \rightarrow \frac{\partial L}{\partial w_2} = (-2)(7)(1)(-2) \rightarrow \frac{\partial L}{\partial w_2} = 28$$

$$\frac{\partial L}{\partial w_1} = \delta_1 x_0 \rightarrow \frac{\partial L}{\partial w_1} = \delta_2 w_2 f'_1(a_1)x_0 \rightarrow \frac{\partial L}{\partial w_1} = (-14)(3)(1)(2) \rightarrow \frac{\partial L}{\partial w_1} = -84$$