M10-L1 Problem 1

In this problem you will implement the K-Means algorithm from scratch, and use it to cluster two datasets: a "blob" shaped dataset with three classes, and a "moon" shaped dataset with two classes.

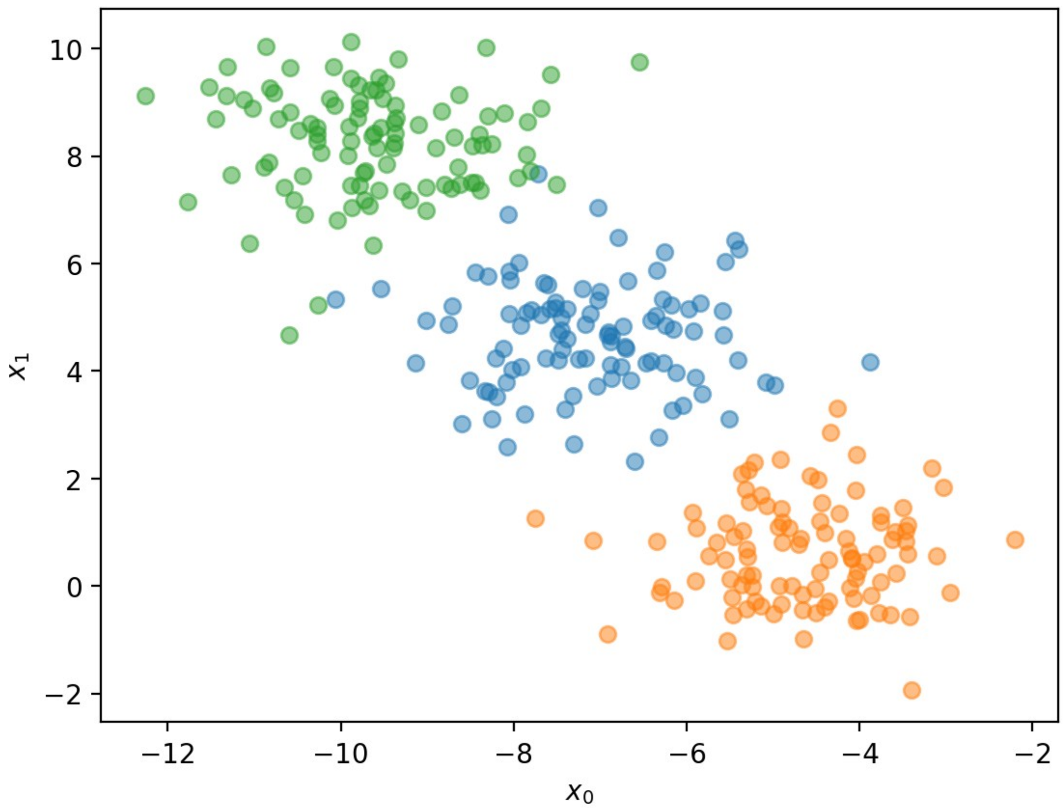
```
import numpy as np
import matplotlib.pyplot as plt

from sklearn.datasets import make_blobs, make_moons

## DO NOT MODIFY
def plotter(x, y, labels = None, centers = None):
    fig = plt.figure(dpi = 200)
    for i in range(len(np.unique(y))):
        if labels is not None:
            plt.scatter(x[labels == i, 0], x[labels == i, 1], alpha = 
0.5)
        else:
            plt.scatter(x[y == i, 0], x[y == i, 1], alpha = 0.5)
    if labels is not None:
        if (labels != y).any():
            plt.scatter(x[labels != y, 0], x[labels != y, 1], s = 100, 
c = 'None', edgecolors = 'black', label = 'Misclassified Points')
    if centers is not None:
        plt.scatter(centers[:,0], centers[:,1], c = 'red', label = 
'Cluster Centers')
    plt.xlabel('$x_0$')
    plt.ylabel('$x_1$')
    if labels is not None or centers is not None:
        plt.legend()
    plt.show()
```

We will use `sklearn.datasets.make_blobs()` to generate the dataset. The `random_state = 12` argument is used to ensure all students have the same data.

```
## DO NOT MODIFY
x, y = make_blobs(n_samples = 300, n_features = 2, random_state = 12)
```

Visualize the data using the `plotter(x,y)` function. You do not need to pass the `labels` or `centers` arguments

```
## YOUR CODE GOES HERE
# visualize the data
plotter(x, y)
```



Now we will begin to create our own K-Means function.

First you will write a function `find_cluster(point, centers)` which returns the index of the cluster center closest to the given point.

- `point` is a one dimensional numpy array containing the x_0 and x_1 coordinates of a single data point
- `centers` is a 3×2 numpy array containing the coordinates of the three cluster centers at any given iteration
- **return** the index of the closest cluster center

```
## FILL IN THE FOLLOWING FUNCTION
def find_cluster(point, centers):
    closet_center = np.argmin(np.linalg.norm(centers - point, axis =
1))
    return closet_center
```

Next, write a function `assign_labels(x, centers)` which will loop through all the points in `x` and use the `find_cluster()` function we just wrote to assign the label of the closest cluster center. Your function should return the labels

- `x` is a 300×2 numpy array containing the coordinates of all the points in the dataset
- `centers` is a 3×2 numpy array containing the coordinates of the three cluster centers at any given iteration

- **return** a one dimensional numpy array of length 300 containing the corresponding label for each point in `x`

```
## FILL IN THE FOLLOWING FUNCTION
def assign_labels(x, centers):
    labels = np.zeros(x.shape[0])
    for i in range(x.shape[0]):
        labels[i] = find_cluster(x[i], centers)
    return labels
```

Next, write a function `update_centers(x, labels)` which will compute the new cluster centers using the centroid of each cluster, provided all the points in `x` and their corresponding labels

- `x` is a 300×2 numpy array containing the coordinates of all the points in the dataset
- `labels` is a one dimensional numpy array of length 300 containing the corresponding label for each point in `x`
- **return** a 3×2 numpy array containing the coordinates of the three cluster centers

```
## FILL IN THE FOLLOWING FUNCTION
def update_centers(x, labels):
    cluster_centers = np.zeros((len(np.unique(labels)), x.shape[1]))
    for i in range(len(np.unique(labels))):
        cluster_centers[i] = np.mean(x[labels == i], axis = 0)
    return cluster_centers
```

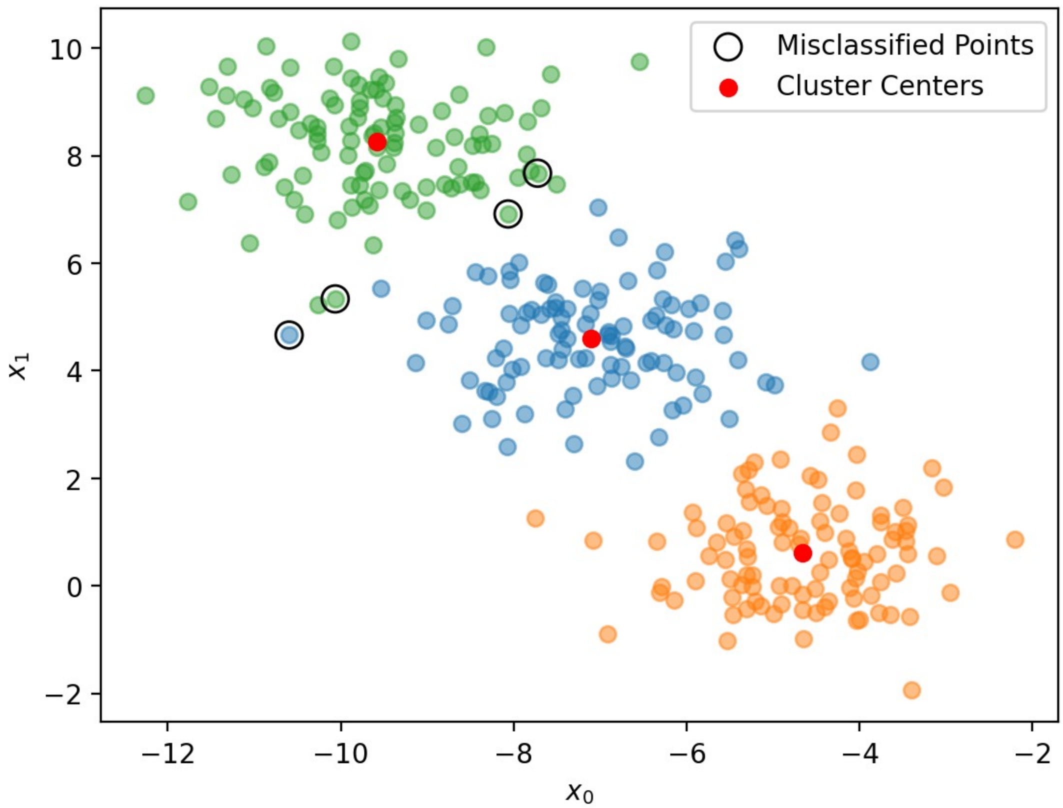
Finally write a function `myKMeans(x, init_centers)` which will run the KMeans algorithm, provided all the points in `x` and the coordinates of the initial cluster centers in `init_centers`. Run the algorithm until there is no change in cluster membership in subsequent iterations. Your function should return both the `labels`, the labels of each point in x, and `centers`, the final coordinates of each of the cluster centers.

- `x` is a 300×2 numpy array containing the coordinates of all the points in the dataset
- `init_centers` is a 3×2 numpy array containing the coordinates of the three cluster centers provided to you
- **return** `labels` and `centers` as defined above

```
## FILL IN THE FOLLOWING FUNCTION
def myKMeans(x, init_centers):
    centers = init_centers
    labels = assign_labels(x, centers)
    new_centers = update_centers(x, labels)
    while not np.allclose(centers, new_centers):
        centers = new_centers
        labels = assign_labels(x, centers)
        new_centers = update_centers(x, labels)
    return labels, centers
```

Now use your `myKMeans()` function to cluster the provided data points `x` and set the initial cluster centers as `init_centers = np.array([[-5,5],[0,0],[-10,10]])`. Then use the provided plotting function, `plotter(x,y,labels,centers)` to visualize your model's clustering.

```
## YOUR CODE GOES HERE
# visualize the model's clustering
init_centers = np.array([[-5,5], [0,0], [-10,10]])
labels, centers = myKMeans(x, init_centers)
plotter(x, y, labels, centers)
```



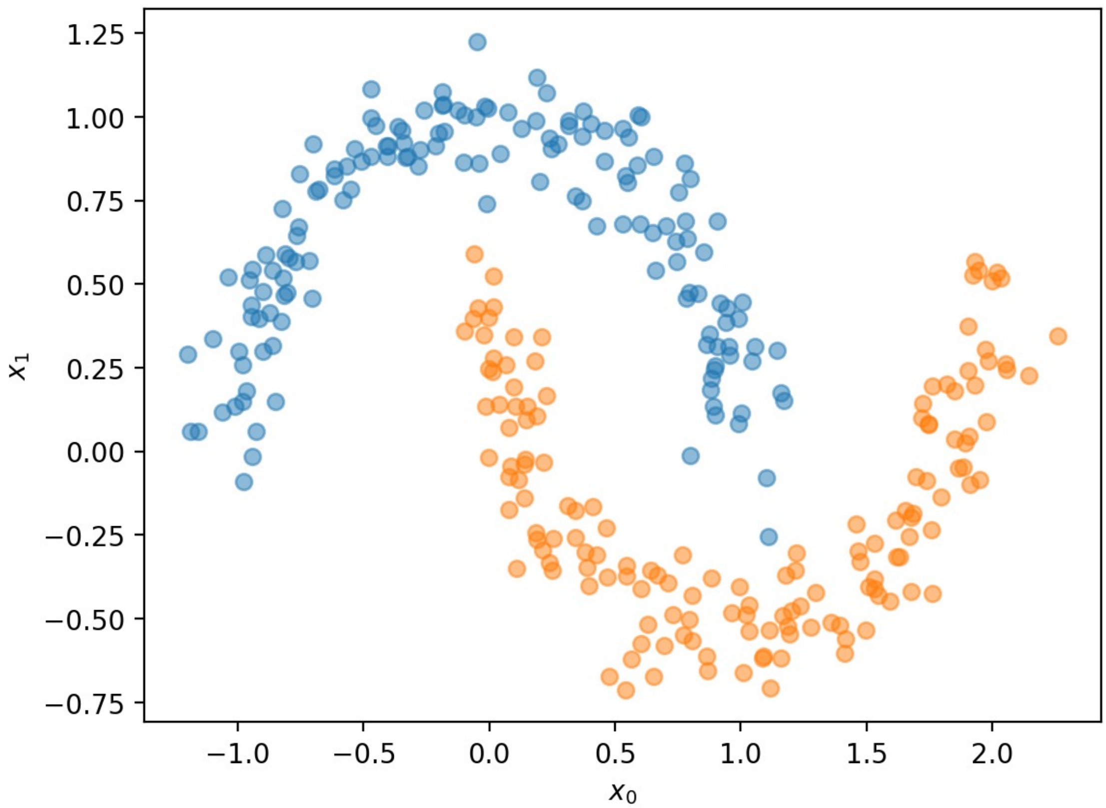
Moon Dataset

Now we will try using our `myKMeans()` function on a more challenging dataset, as generated below.

```
## DO NOT MODIFY
x,y = make_moons(n_samples = 300, noise = 0.1, random_state = 0)
```

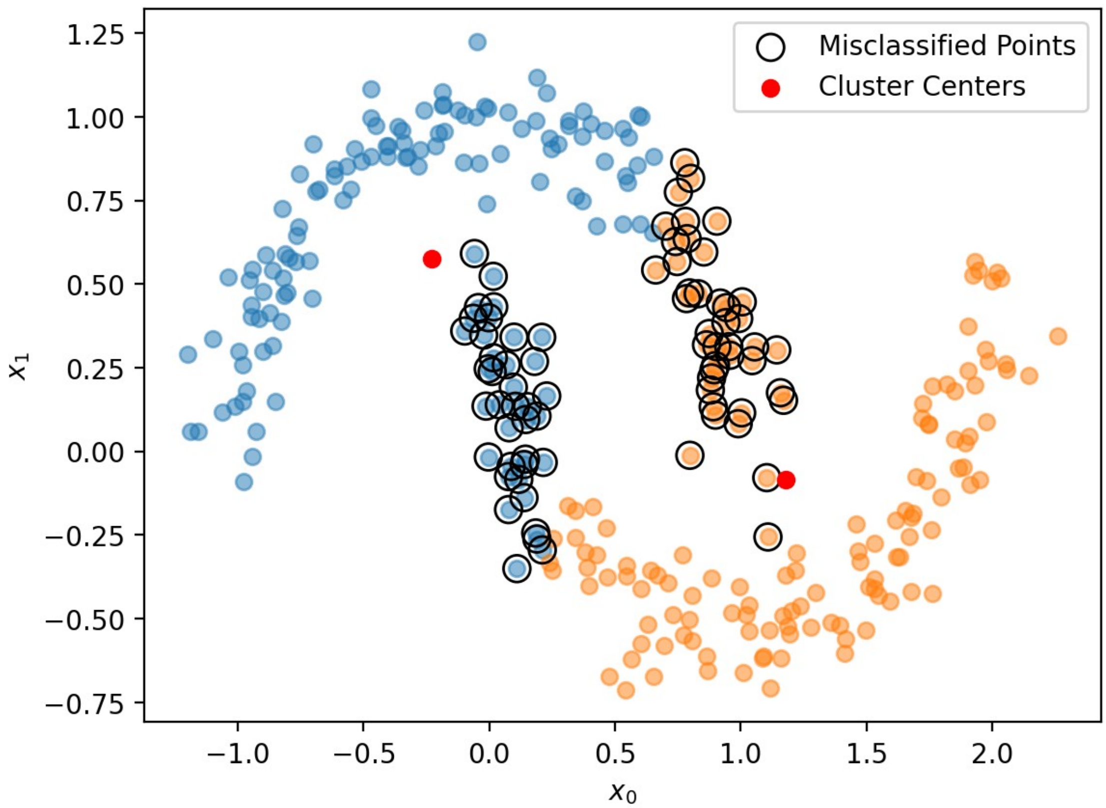
Visualize the data using the `plotter(x,y)` function.

```
## YOUR CODE GOES HERE
# visualize the data
plotter(x, y)
```



Using your `myKMeans()` function and `init_centers = np.array([[0,1],[1,-0.5]])` cluster the data, and visualize the results using `plotter(x,y,labels,centers)`.

```
## YOUR CODE GOES HERE
# visualize the model's clustering
init_centers = np.array([[0,1], [1,-0.5]])
labels, centers = myKMeans(x, init_centers)
plotter(x, y, labels, centers)
```



Misclassified Points
Cluster Centers
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
x_1
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x_0