一、Tree Traversal（10 分）。

規定 Input：

輸入一串數字的 Preorder、Inorder（ -999～999）。

規定 Output：

產生 Binary Tree 的圖形，並將三個序顯示出來。

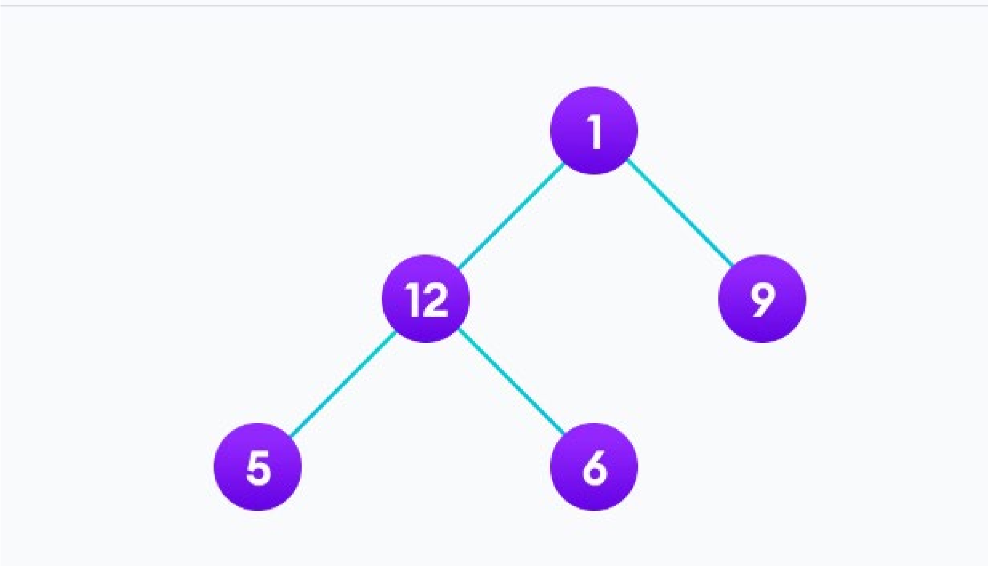
範例 Input：

Preorder：1,12,5,6,9

Inorder：5,12,6,1,9

範例 Output：輸出如下圖。

※樹的 Node 須 pointer 實作並使用圖形化輸出，若使用小黑窗不予計分。



Preorder：1 -> 12 -> 5 -> 6 -> 9

Inorder：5 -> 12 -> 6 -> 1 -> 9

Postorder：5 -> 6 -> 12 -> 9 -> 1

可以用圖形化輸出 tree 和 order。（4 分）

可以解釋並使用演算法。（6 分）

二、請依據輸入的資料建構二元搜尋樹，並將其繪製出來（10 分）。

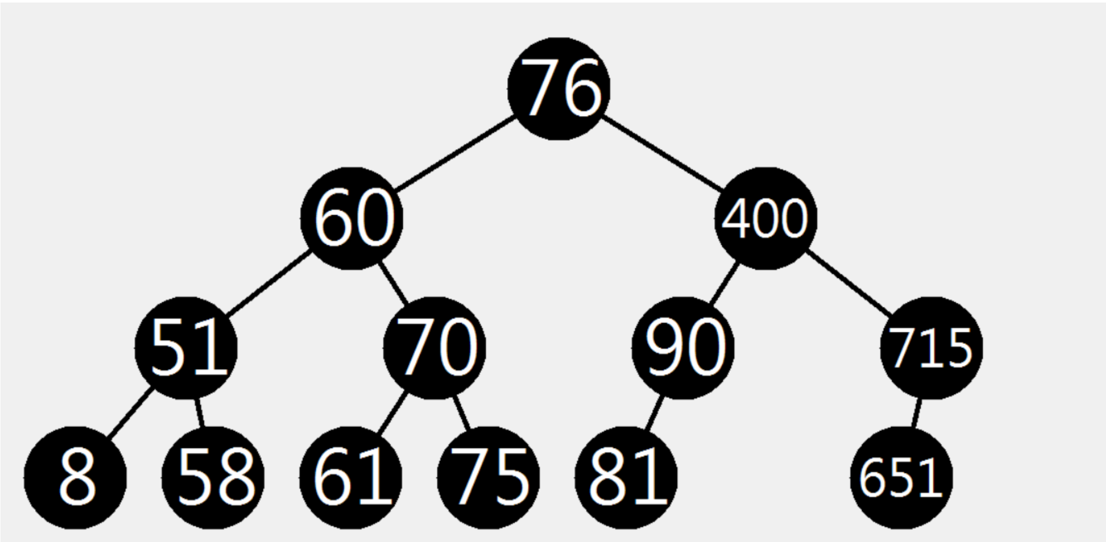
規定 Input：一串以逗號隔開的數字，數字範圍 1～999。

規定 Output：將二元搜尋樹完整、清楚地繪製出來。需使用圖形化視窗輸出，若使用小黑窗以 0 分計算。

範例 Input：76,400,60,51,70,715,90,8,61,75,651,81,58

範例 Output：

節點、數字、連接線的顏色不拘，只要繪製出的二元搜尋樹容易辨識即可。



評分標準：

1. 繪製的結果排序正確且能完整顯示整棵樹。(6 分)
2. 二元搜尋樹觀念與原理能清楚說明。(4 分)

三、AVL Tree（10 分）。

規定 Input：輸入一串數字（ -999 ~ 999 ）。

規定 Output：用三個 order 將排完的 AVL tree 顯示出來。需使用圖形化視窗輸出三個 order（樹不用畫出來），若使用小黑窗以 0 分計算。

範例 Input：10,20,30,50,70,60,80,40,90

範例 Output：Preorder ：50,20,10,30,40,70,60,80,90

Inorder ：10,20,30,40,50,60,70,80,90

Postorder：10,30,40,20,60,80,90,70,50

評分標準：

1.排序結果正確。(6 分)

2. 能清楚說明 AVL Tree 的觀念及原理。(4 分)

加分題

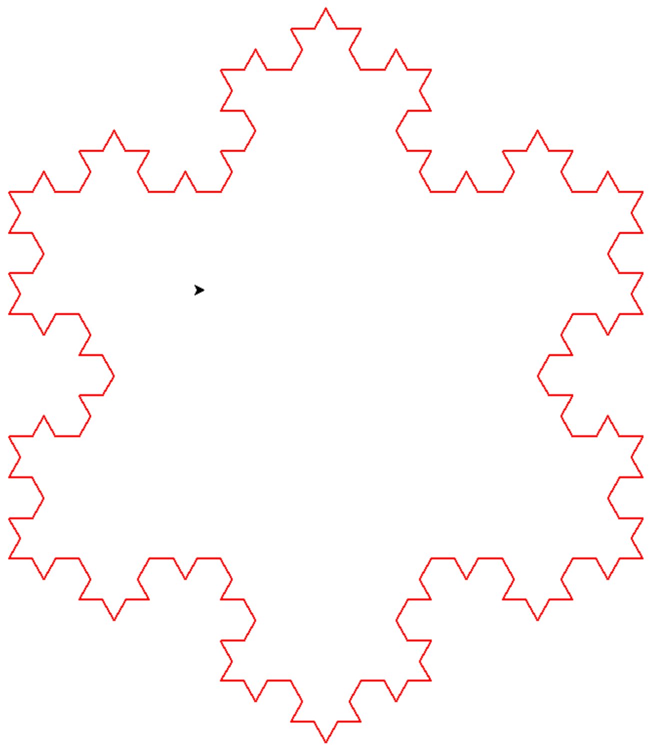
請依據輸入的資料繪製碎形-科赫雪花，並將其繪製出來 (5 分)。

規定 Output：將科赫雪花完整的繪製出來。需使用圖形化視窗輸出，若使用小黑窗以 0 分計算。

科赫雪花參考資料請見附件。

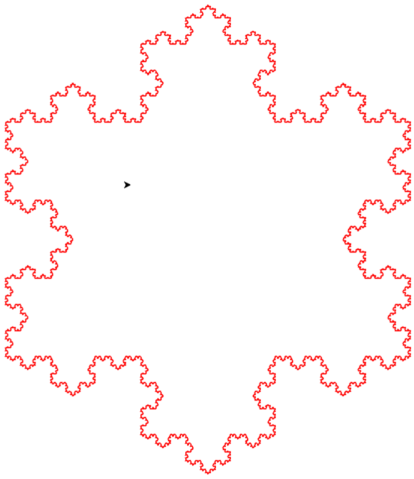
範例 Input：3

範例 Output：



範例 Input：5

範例 Output：



評分標準：

1. 繪製的結果正確且能解釋如何運作。(5 分)

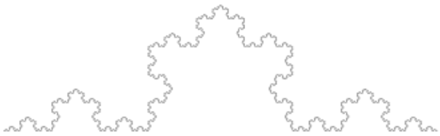
附件

維基百科

科赫曲線

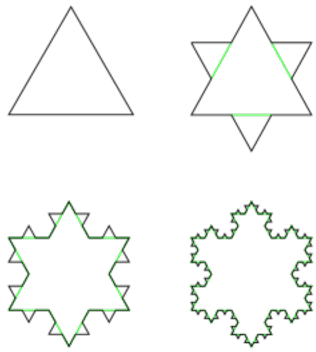
維基百科，自由的百科全書

科赫曲線（英語：Koch curve）是一種碎形。其形態似雪花，又稱科赫雪花（Koch snowflake）、科赫星（Koch star）、科赫島（Koch island）或雪花曲線（Snowflake curve）。其豪斯多夫維是$\log 4/\log 3$。



科赫曲線

它最早出現在瑞典數學家海里格·馮·科赫（Niels Fabian Helge von Koch）的論文《關於一條連續而無切線，可由初等幾何構作的曲線》（1904年，法語原題：*Sur une courbe continue sans tangente, obtenue par une construction géométrique élémentaire*）。



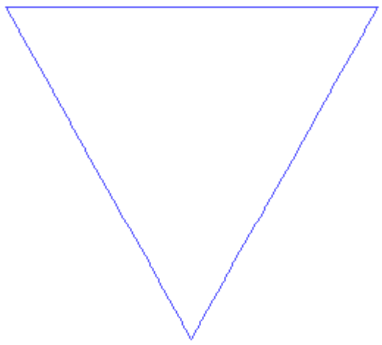
科赫雪花

科赫曲線是de Rham曲線的特例。

給定線段AB，科赫曲線可以由以下步驟生成：

1. 將線段分成三等份（AC,CD,DB）
2. 以CD為底，向外（內外隨意）畫一個等邊三角形DMC
3. 將線段CD移去
4. 分別對AC,CM,MD,DB重複1~3。

科赫雪花是以等邊三角形三邊生成的科赫曲線組成的。科赫雪花的面積是$\frac{2\sqrt{3}(s^2)}{5}$，其中s是原來三角形的邊長。每條科赫曲線的長度是無限大，它是連續而無處可微的曲線。



生成過程

記錄

以L系統：

字符 : F
常數 : +, −
公理 : F++F++F
規則:
F → F−F++F−F

- F ：向前
- - ：左轉60°
- + ：右轉60°

logo源碼

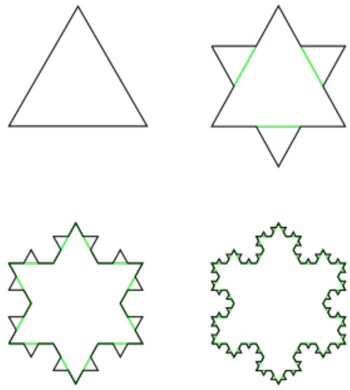
```
rt 30 koch 100.

 to koch :x
   repeat 3 [triline :x rt 120]
 end
 to triline :x
   if :x < 1 [fd :x] [triline :x/3 lt 60 triline :x/3 rt 120 triline :x/3 lt 60 triline :x/3]
 end
```

WIKIPEDIA

Koch snowflake

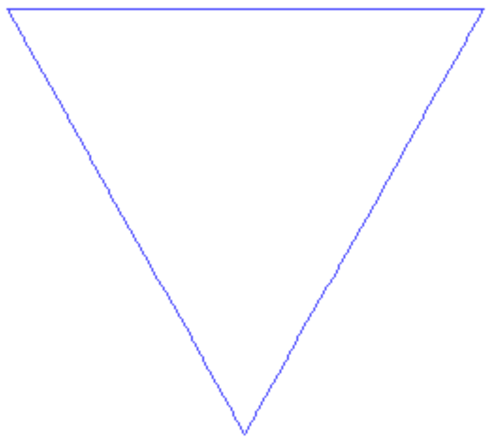
The **Koch snowflake** (also known as the **Koch curve**, **Koch star**, or **Koch island**[1][2]) is a fractal curve and one of the earliest fractals to have been described. It is based on the Koch curve, which appeared in a 1904 paper titled "On a Continuous Curve Without Tangents, Constructible from Elementary Geometry"[3] by the Swedish mathematician Helge von Koch.



The first four iterations of the Koch snowflake

The Koch snowflake can be built up iteratively, in a sequence of stages. The first stage is an equilateral triangle, and each successive stage is formed by adding outward bends to each side of the previous stage, making smaller equilateral triangles. The areas enclosed by the successive stages in the construction of the snowflake converge to $\frac{8}{5}$ times the area of the original triangle, while the perimeters of the successive stages increase without bound. Consequently, the snowflake encloses a finite area, but has an infinite perimeter.

Contents



The first seven iterations in animation

Zooming into the Koch curve

Construction

The Koch snowflake can be constructed by starting with an equilateral triangle, then recursively altering each line segment as follows:

1. divide the line segment into three segments of equal length.

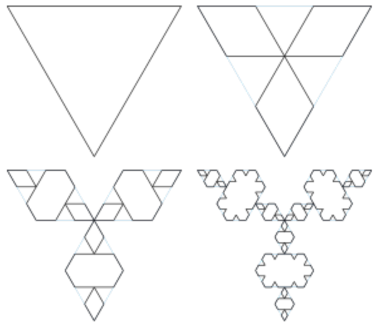
2. draw an equilateral triangle that has the middle segment from step 1 as its base and points outward.
3. remove the line segment that is the base of the triangle from step 2.

The first iteration of this process produces the outline of a hexagram.

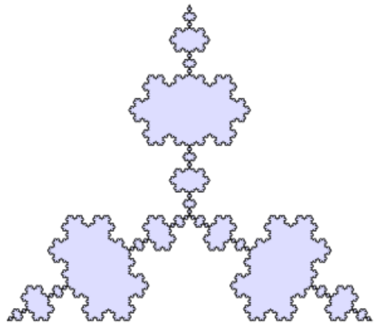
The Koch snowflake is the limit approached as the above steps are followed indefinitely. The Koch curve originally described by Helge von Koch is constructed using only one of the three sides of the original triangle. In other words, three Koch curves make a Koch snowflake.

A Koch curve–based representation of a nominally flat surface can similarly be created by repeatedly segmenting each line in a sawtooth pattern of segments with a given angle.[4]

Koch antisnowflake



First four iterations



Sixth iteration

A fractal rough surface built from multiple Koch curve iterations

Properties

Perimeter of the Koch snowflake

Each iteration multiplies the number of sides in the Koch snowflake by four, so the number of sides after n iterations is given by:

$$N_n = N_{n-1} \cdot 4 = 3 \cdot 4^n \,.$$

If the original equilateral triangle has sides of length s, the length of each side of the snowflake after n iterations is:

$$S_n = \frac{S_{n-1}}{3} = \frac{s}{3^n} \,,$$

an inverse power of three multiple of the original length. The perimeter of the snowflake after n iterations is:

$$P_n = N_n \cdot S_n = 3 \cdot s \cdot \left(\frac{4}{3}\right)^n \,.$$

The Koch curve has an infinite length, because the total length of the curve increases by a factor of $\frac{4}{3}$ with each iteration. Each iteration creates four times as many line segments as in the previous iteration, with the length of each one being $\frac{1}{3}$ the length of the segments in the previous stage. Hence, the length of the curve after n iterations will be $(\frac{4}{3})^n$ times the original triangle perimeter and is unbounded, as n tends to infinity.

Limit of perimeter

As the number of iterations tends to infinity, the limit of the perimeter is:

$$\lim_{n\to\infty} P_n = \lim_{n\to\infty} 3\cdot s\cdot\left(\frac{4}{3}\right)^n = \infty\,,$$

since $\frac{4}{3} > 1$.

An $\frac{\ln 4}{\ln 3}$-dimensional measure exists, but has not been calculated so far. Only upper and lower bounds have been invented. [5]

Area of the Koch snowflake

In each iteration a new triangle is added on each side of the previous iteration, so the number of new triangles added in iteration n is:

$$T_n = N_{n-1} = 3\cdot 4^{n-1} = \frac{3}{4}\cdot 4^n\,.$$

The area of each new triangle added in an iteration is $\frac{1}{9}$ of the area of each triangle added in the previous iteration, so the area of each triangle added in iteration n is:

$$a_n = \frac{a_{n-1}}{9} = \frac{a_0}{9^n}\,.$$

where a_0 is the area of the original triangle. The total new area added in iteration n is therefore:

$$b_n = T_n\cdot a_n = \frac{3}{4}\cdot\left(\frac{4}{9}\right)^n\cdot a_0$$

The total area of the snowflake after n iterations is:

$$A_n = a_0 + \sum_{k=1}^{n} b_k = a_0\left(1+\frac{3}{4}\sum_{k=1}^{n}\left(\frac{4}{9}\right)^k\right) = a_0\left(1+\frac{1}{3}\sum_{k=0}^{n-1}\left(\frac{4}{9}\right)^k\right)\,.$$

Collapsing the geometric sum gives:

$$A_n = a_0\left(1+\frac{3}{5}\left(1-\left(\frac{4}{9}\right)^n\right)\right) = \frac{a_0}{5}\left(8-3\left(\frac{4}{9}\right)^n\right)\,.$$

Limits of area

The limit of the area is:

$$\lim_{n\to\infty} A_n = \lim_{n\to\infty} \frac{a_0}{5} \cdot \left(8 - 3\left(\frac{4}{9}\right)^n\right) = \frac{8}{5} \cdot a_0 \,,$$

since $\frac{4}{9} < 1$.

Thus, the area of the Koch snowflake is $\frac{8}{5}$ of the area of the original triangle. Expressed in terms of the side length s of the original triangle, this is:[6]

$$\frac{2s^2\sqrt{3}}{5}.$$

Solid of revolution

The volume of the solid of revolution of the Koch snowflake about an axis of symmetry of the initiating equilateral triangle of unit side is $\frac{11\sqrt{3}}{135}\pi$.[7]

Other properties

The Koch snowflake is self-replicating with six smaller copies surrounding one larger copy at the center. Hence, it is an irrep-7 irrep-tile (see Rep-tile for discussion).

The fractal dimension of the Koch curve is $\frac{\ln 4}{\ln 3} \approx 1.26186$. This is greater than that of a line $(= 1)$ but less than that of Peano's space-filling curve $(= 2)$.

The Koch curve is continuous everywhere, but differentiable nowhere.

Tessellation of the plane

It is possible to tessellate the plane by copies of Koch snowflakes in two different sizes. However, such a tessellation is not possible using only snowflakes of one size. Since each Koch snowflake in the tessellation can be subdivided into seven smaller snowflakes of two different sizes, it is also possible to find tessellations that use more than two sizes at once.[8] Koch snowflakes and Koch antisnowflakes of the same size may be used to tile the plane.

Thue–Morse sequence and turtle graphics

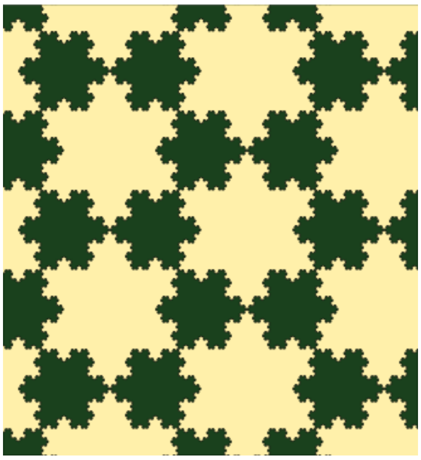
A turtle graphic is the curve that is generated if an automaton is programmed with a sequence. If the Thue–Morse sequence members are used in order to select program states:

- If $t(n) = 0$, move ahead by one unit,

- If $t(n) = 1$, rotate counterclockwise by an angle of $\frac{\pi}{3}$,

the resulting curve converges to the Koch snowflake.

Representation as Lindenmayer system



Tessellation by two sizes of Koch snowflake

The Koch curve can be expressed by the following rewrite system (Lindenmayer system):

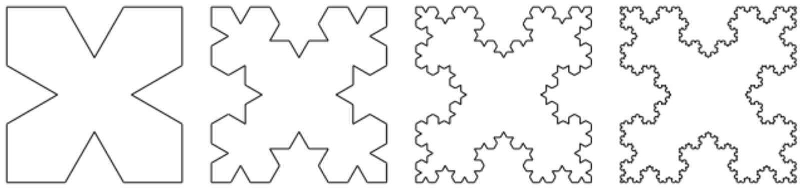
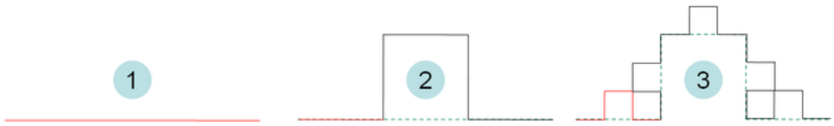
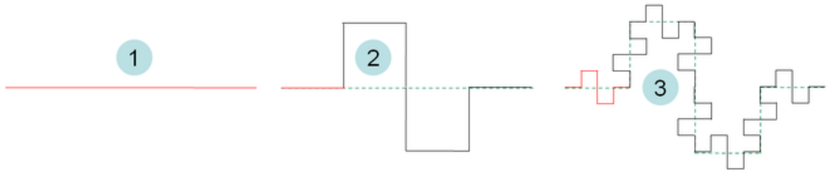
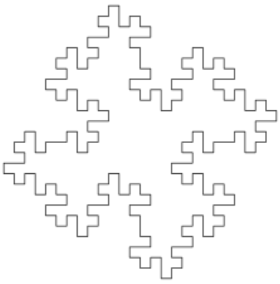
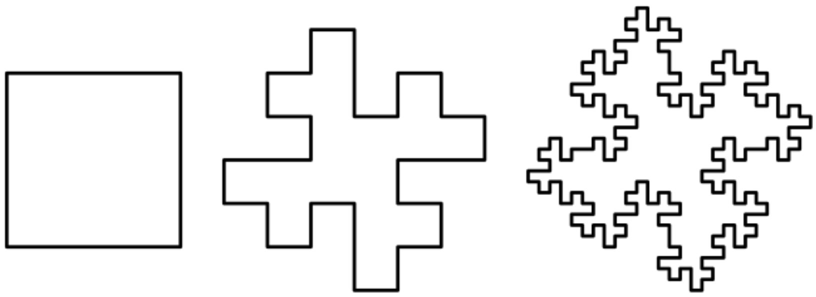
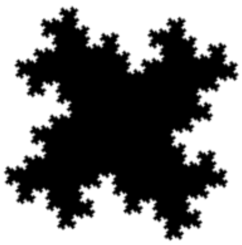
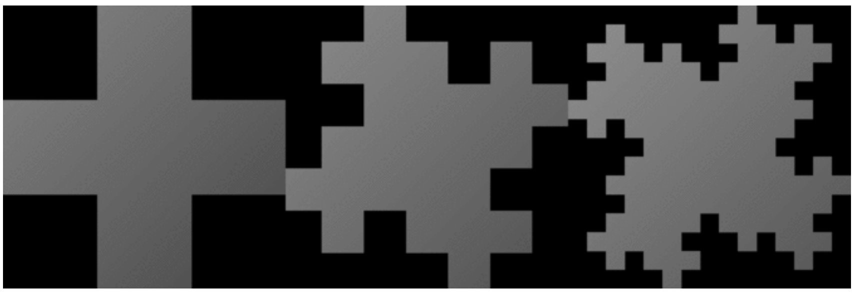
> **Alphabet** : F
> **Constants** : +, −
> **Axiom** : F
> **Production rules**:
> F → F+F--F+F

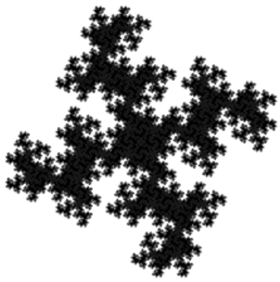
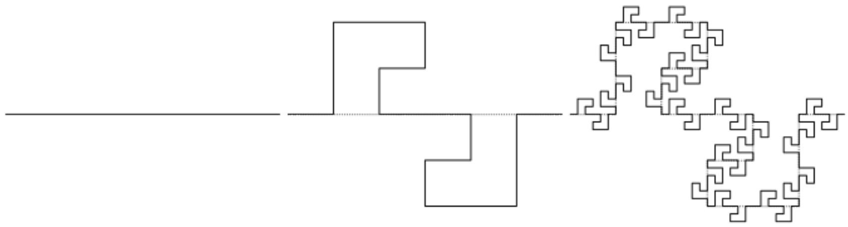
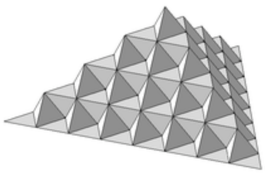
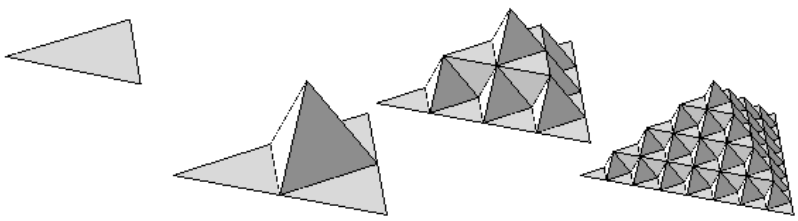
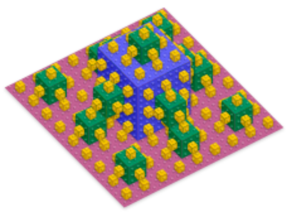
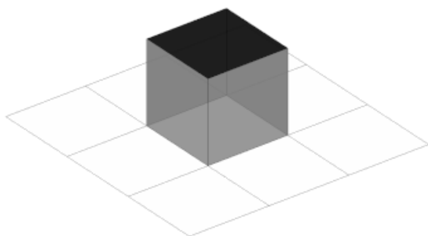
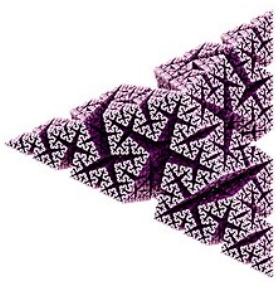
Here, *F* means "draw forward", - means "turn right 60°", and + means "turn left 60°".

To create the Koch snowflake, one would use F--F--F (an equilateral triangle) as the axiom.

Variants of the Koch curve

Following von Koch's concept, several variants of the Koch curve were designed, considering right angles (quadratic), other angles (Cesàro), circles and polyhedra and their extensions to higher dimensions (Sphereflake and Kochcube, respectively)

Variant (dimension, angle)	Illustration	Construction
≤1D, 60-90° angle	 Cesàro fractal (85°)	The Cesàro fractal is a variant of the Koch curve with an angle between 60° and 90°.  First four iterations of a Cesàro antisnowflake (four 60° curves arranged in a 90° square)
≈1.46D, 90° angle	 Quadratic type 1 curve	 First two iterations
1.5D, 90° angle	 Quadratic type 2 curve	Minkowski Sausage[9]  First two iterations. Its fractal dimension equals $\frac{3}{2}$ and is exactly half-way between dimension 1 and 2. It is therefore often chosen when studying the physical properties of non-integer fractal objects.
≤2D, 90° angle	 Third iteration	Minkowski Island  Four quadratic type 2 curves arranged in a square
≈1.37D, 90° angle	 Quadratic flake	 4 quadratic type 1 curves arranged in a polygon: First two iterations. Known as the "Minkowski Sausage",[10][11][12] its fractal dimension equals $\frac{\ln 3}{\ln \sqrt{5}} = 1.36521$.[13]
≤2D, 90° angle	 Quadratic antiflake	Anti**cross-stitch curve**, the quadratic flake type 1, with the curves facing inwards instead of outwards (Vicsek fractal)

≈1.49D, 90° angle	 Quadratic Cross	Another variation. Its fractal dimension equals $\frac{\ln 3.33}{\ln \sqrt{5}} = 1.49$.
≤2D, 90° angle	 Quadratic island[14]	 Quadratic curve, iterations 0, 1, and 2; dimension of $\frac{\ln 18}{\ln 6} \approx 1.61$
≤2D, 60° angle	 von Koch surface	 First three iterations of a natural extension of the Koch curve in two dimensions.
≤2D, 90° angle	 First (blue block), second (plus green blocks), third (plus yellow blocks) and fourth (plus transparent blocks) iterations of the type 1 3D Koch quadratic fractal	Extension of the quadratic type 1 curve. The illustration at left shows the fractal after the second iteration  Animation quadratic surface
≤3D, any	 Koch curve in 3D	A three-dimensional fractal constructed from Koch curves. The shape can be considered a three-dimensional extension of the curve in the same sense that the Sierpiński pyramid and Menger sponge can be considered extensions of the Sierpinski triangle and Sierpinski carpet. The version of the curve used for this shape uses 85° angles.

Squares can be used to generate similar fractal curves. Starting with a unit square and adding to each side at each iteration a square with dimension one third of the squares in the previous iteration, it can be shown that both the length of the perimeter and the total area are determined by geometric progressions. The progression for the area converges to 2 while the progression for the perimeter diverges to infinity, so as in the case of the Koch snowflake, we have a finite area bounded by an infinite fractal curve.[15] The resulting area fills a square with the same center as the original, but twice the area, and rotated by $\frac{\pi}{4}$ radians, the perimeter touching but never overlapping itself.

The total area covered at the nth iteration is:

$$A_n = \frac{1}{5} + \frac{4}{5}\sum_{k=0}^{n}\left(\frac{5}{9}\right)^k \quad \text{giving} \quad \lim_{n\to\infty} A_n = 2\,,$$

while the total length of the perimeter is:

$$P_n = 4\left(\frac{5}{3}\right)^n a\,,$$

which approaches infinity as n increases.

See also

- List of fractals by Hausdorff dimension
- Gabriel's Horn (infinite surface area but encloses a finite volume)
- Gosper curve (also known as the Peano–Gosper curve or *flowsnake*)
- Osgood curve
- Self-similarity
- Teragon
- Weierstrass function
- Coastline paradox

References

1. Addison, Paul S. (1997). *Fractals and Chaos: An Illustrated Course*. Institute of Physics. p. 19. ISBN 0-7503-0400-6.
2. Lauwerier, Hans (1991). *Fractals: Endlessly Repeated Geometrical Figures*. Translated by Gill-Hoffstädt, Sophia. Princeton University Press. p. 36. ISBN 0-691-02445-6. "Mandelbrot called this a Koch island."
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