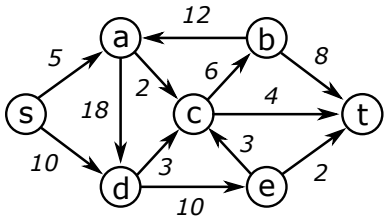
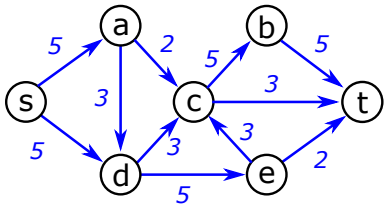
Homework 10: Maximum Flow and Minimum Cut

1. Consider the following flow network.



The following graph shows a maximum flow $\bar{f}$ in the network.



5+2+3 = 10 because those are the inputs to t

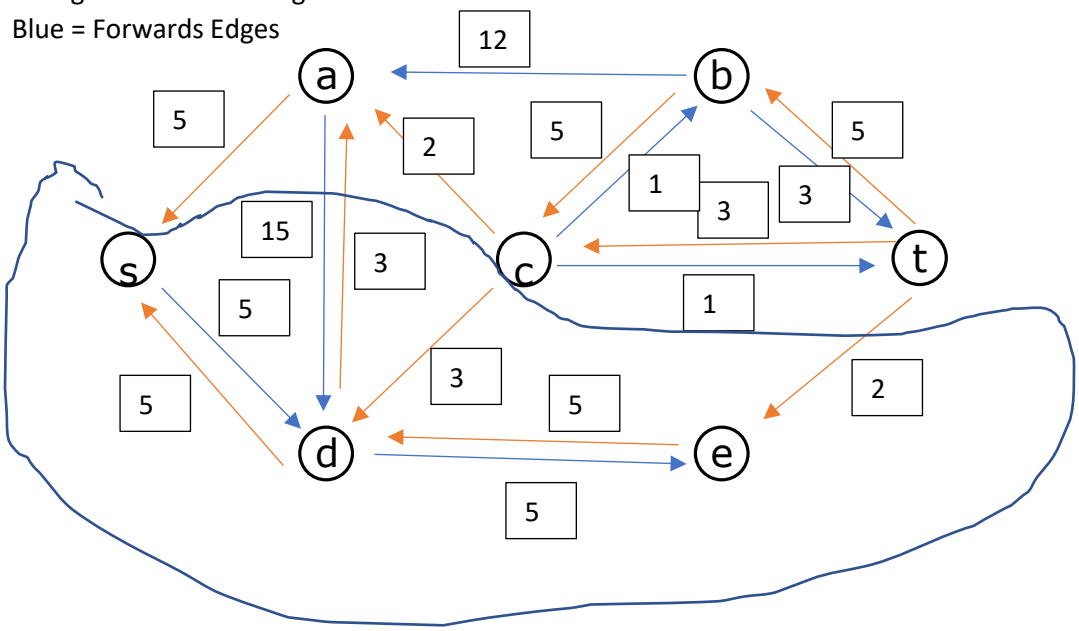
(a) Give the value of the maximum flow $\bar{f}$.

$v(\bar{f})$: 10

(b) Draw the residual graph with respect to the flow $\bar{f}$.

Orange = Backwards Edges

Blue = Forwards Edges



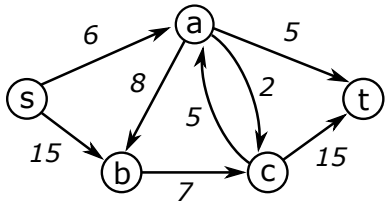
(c) Use the residual graph to identify the minimum cut (A^*,B^*) in the graph.

A*: {s, d, e}

B*: {a, c, b, t}

(d) Give the capacity of the minimum cut (A^*,B^*).
5 + 2 + 3 = 10
$c(A^*,B^*)$: 10

2. Consider the following flow network.



(a) There are 6 possible s-t-cuts in the graph. Compute the capacities of each one of the cuts and fill in the last column of the table.

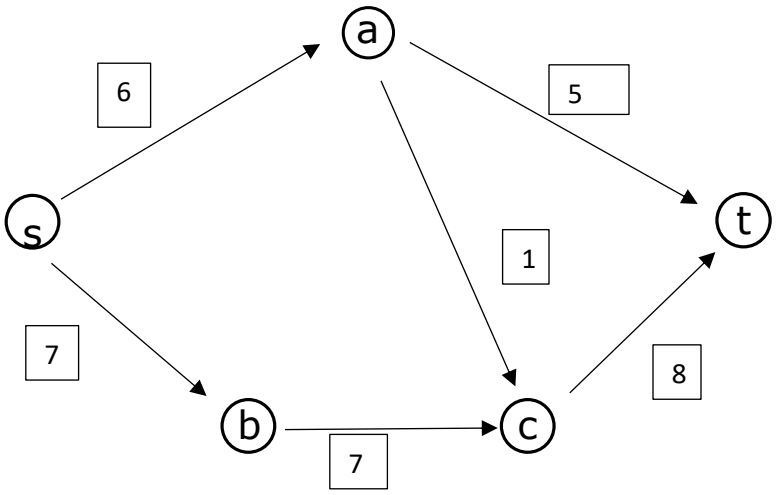
	Cut	A	B	$c(A,B)$
15+ 6 = 21	1	$\{s\}$	$\{a,b,c,t\}$	21
8 + 15 + 5 + 2 = 30	2	$\{s,a\}$	$\{b,c,t\}$	30
7 + 6 = 13	3	$\{s,b\}$	$\{a,c,t\}$	13
5+7+2 = 14	4	$\{s,a,b\}$	$\{c,t\}$	14
6+15+5 = 26	5	$\{s,b,c\}$	$\{a,t\}$	26
15+5 = 20	6	$\{s,a,b,c\}$	$\{t\}$	20

(b) Use the table to identify the minimum cut (A^*,B^*) in the graph.

A*: {s, b}

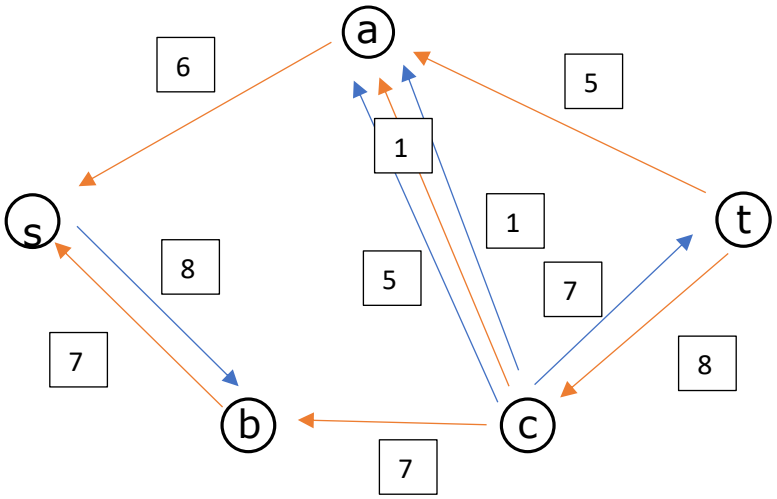
B* {a, c, t}

(c) Use the minimum cut to identify a maximum flow $\bar{f}$. Draw the graph showing the maximum flow $\bar{f}$.

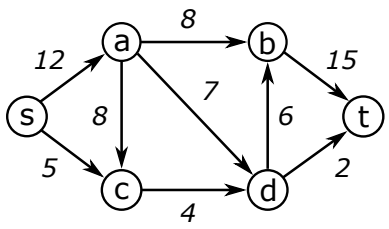


(d) Draw the residual graph $G_{\bar{f}}$ associated with the maximum flow.

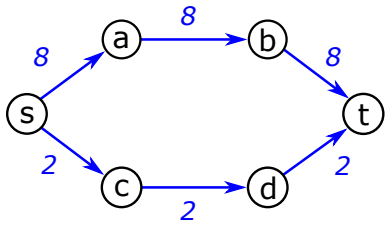
Blue: Forwards Edges
Orange: Backwards Edges



3. Consider the following flow network.



The following graph shows a flow f in the network.



(a) Draw the residual graph with respect to the flow f.

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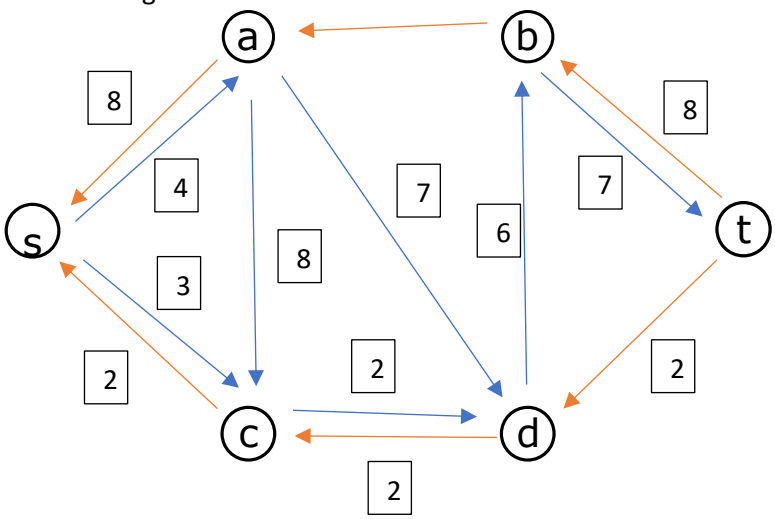
Blue: Forwards Edges

Orange: Backwards Edges

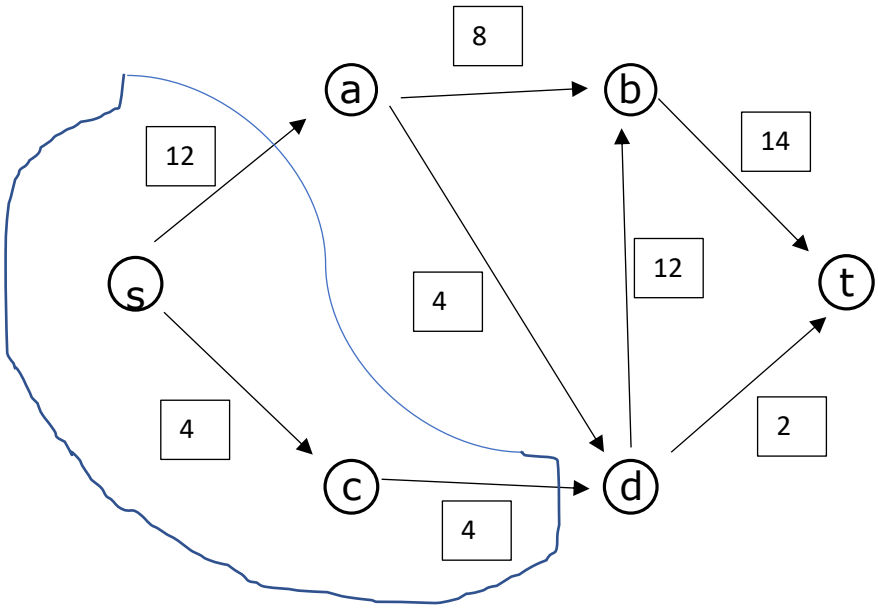


(b) Find all s-t-paths in the residual graph and the bottleneck value for each one of them.

s-t-path	nodes	*bottleneck*
1	s, a, d, b, t	4
2	s, c, d, b, t	2

3	s, a, c, d, b,t	2

(c) Continue the Ford-Fulkerson algorithm to find the maximum flow $\bar{f}$. Draw the graph showing the maximum flow $\bar{f}$.



(d) Use the maximum flow to identify the minimum cut (A^*, B^*) in the graph.

A^*: {s, c}

B^*:{a, b, d,t}

Min Cut = 12 + 4 = 16

Hand in your solution on HuskyCT, in pdf format, no later than December 1 at 11:59 PM.