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CSE 3500

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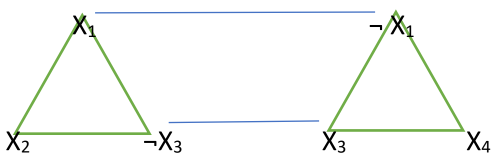
Homework 11: Polynomial-Time Reductions

1.) ***Give the corresponding instance of the Independent Set Problem. Specify the graph G and the size k.***

Size k = 2




The graph above shows that there are two clauses (the triangles) and there are two conflicts as we can see by the blue connecting lines. Since there are two clauses we know that k must be 2.

2.) ***Give the corresponding instance of the Set Packing Problem. Specify the elements, the subsets $S_i$ and the size k.***

Size k = 2

$S_{x1}$= { $\neg X_1$, $\neg X_3$, $X_2$}

S $\neg X_1$ = {$X_1$, $X_3$, $X_4$}

$S_{x2}$ = {$X_1$, ¬X3}

$S_{x3}$={ $\neg X_3$, $X_4$, $\neg X_1$}

S¬x3={$X_2$, $X_1$,}

S$X_4$={$X_3$, $\neg X_1$}

Corresponding instance of the set packing problem:

$S_{x2}$ = {$X_1$, ¬X3}

S$X_4$={$X_3$, $\neg X_1$}

Because they have no connections in the graph.

3.) ***Give an independent set of size k in the graph G.***

Independent set of size k(2) in the graph G would be {$X_2$, $X_4$}.

4.) ***Show how the independent set translates to a set packing using the subsets $S_I$.***

The independent set translates to a set packing using the subsets $S_I$ because the size k is 2 you can take the first set that contains $X_2$ and the first set that contains $X_4$ and since they are disjoint they will not have a conflict. $X_2$ and $X_4$ in this case are the independent set elements. This will result in two different subsets with no conflicts if we look at the example in problem number 2.

5.) ***Show how the independent set translates to a satisfying assignment to Φ.***

Substituting the independent set {$X_2$, $X_4$} as True (1's) we get the following equation.

$\Phi = (X_1 \vee 1 \vee \neg X_3) \wedge (\neg X_1 \vee X_3 \vee 1)$

1 v and anything simplifies to 1 so the equation now becomes 1 ^ 1 and that simplifies to 1. Thus, we have a satisfying assignment *Φ*.