*Homework 10:* Maximum Flow and Minimum Cut

1. Consider the following flow network.

c

*10*

*10*

*2*

s

*5*

*8*

*2*

t

*6*

e

*3*

b

a

d

*18*

*12*

*3*

*4*

The following graph shows a maximum flow *f*¯in the network.

c

*5*

*5*

*2*

s

*5*

*5*

*2*

t

*5*

e

*3*

b

a

d

*3*

*3*

*3*

5+2+3 = 10 because those are the inputs to t

* 1. Give the value of the maximum flow *f*¯.

*v*(*f*¯): 10

* 1. Draw the residual graph with respect to the flow *f*¯.

Orange = Backwards Edges

Blue = Forwards Edges

12

a b

15

5

2

5

1

5

3

3

s c

1

5

5

2

5

3

3

t

d e

5

* + 1. Use the residual graph to identify the minimum cut (*A*∗*,B*∗) in the graph.

A\*: {s, d, e}

B\*: {a, c, b, t}

* + 1. Give the capacity of the minimum cut (*A*∗*,B*∗).

5 + 2 + 3 = 10

*c*(*A*∗*,B*∗): 10

1. Consider the following flow network.

*7*

*15*

*6*

s

*5*

*5*

*15*

t

*2*

c

b

a

*8*

(a) There are 6 possible *s*-*t*-cuts in the graph. Compute the capacities of each one of the cuts and fill in the last column of the table.

|  |  |  |  |
| --- | --- | --- | --- |
| Cut | *A* | *B* | *c*(*A,B*) |
| 1 | {*s*} | {*a,b,c,t*} | 21 |
| 2 | {*s,a*} | {*b,c,t*} | 30 |
| 3 | {*s,b*} | {*a,c,t*} | 13 |
| 4 | {*s,a,b*} | {*c,t*} | 14 |
| 5 | {*s,b,c*} | {*a,t*} | 26 |
| 6 | {*s,a,b,c*} | {*t*} | 20 |

15+ 6 = 21

8 + 15 + 5 + 2 = 30

7 + 6 = 13

5+7+2 = 14

6+15+5 = 26

15+5 = 20

1. Use the table to identify the minimum cut (*A*∗*,B*∗) in the graph.

A\*: {s, b}

B\* {a, c, t}

1. Use the minimum cut to identify a maximum flow *f*¯. Draw the graph showing the maximum flow *f*¯.

a

5

6

s

1

8

7

t

b c

7

(d) Draw the residual graph *Gf*¯ associated with the maximum flow.

Blue: Forwards Edges

Orange: Backwards Edges

a

1

5

6

s

7

8

1

5

8

7

t

b c

7

3. Consider the following flow network.

*8*

*4*

*7*

*5*

*12*

s

*15*

*2*

b

t

d

*6*

c

a

*8*

The following graph shows a flow *f* in the network.

*8*

*2*

*2*

*8*

s

*8*

*2*

b

t

d

c

a

1. Draw the residual graph with respect to the flow *f*.

8

Blue: Forwards Edges

Orange: Backwards Edges

a b

7

7

8

8

4

s

6

8

3

2

2

2

t

c d

2

1. Find all *s*-*t*-paths in the residual graph and the bottleneck value for each one of them.

|  |  |  |  |
| --- | --- | --- | --- |
| *s*-*t*-path | nodes |  | *bottleneck* |
| 1 | s, a, d, b, t |  | 4 |
| 2 | s, c, d, b, t |  | 2 |
| 3 | s, a, c, d, b,t |  | 2 |

1. Continue the Ford-Fulkerson algorithm to find the maximum flow *f*¯. Draw the graph showing the maximum flow *f*¯.

8

a b

12

14

12

s

2

4

4

t

c d

4

(d) Use the maximum flow to identify the minimum cut (*A*∗*,B*∗) in the graph.

*A*\*:{s, c}

*B*∗:{a, b, d,t}

Min Cut = 12 + 4 = 16

Hand in your solution on HuskyCT, in pdf format, no later than December 1 at 11:59 PM.