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CSE 3500

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Homework 11: Polynomial-Time Reductions

1. ***Give the corresponding instance of the Independent Set Problem. Specify the graph G and the size k.***

Size k = 2

X1 ¬ X1

X2 ¬X­3 X3 X4

The graph above shows that there are two clauses (the triangles) and there are two conflicts as we can see by the blue connecting lines. Since there are two clauses we know that k must be 2.

1. ***Give the corresponding instance of the Set Packing Problem. Specify the elements, the subsets Si and the size k.***

Size k = 2

Sx1= { ¬X1, ¬X3, X2}

S ¬X1 = {X1, X3, X4}

Sx2 = {X1, ¬X­3­}

Sx3={ ¬X3, X4, ¬X­1}

S¬x3={X2, X1,}

SX4={X3, ¬X1}

Corresponding instance of the set packing problem:

Sx2 = {X1, ¬X­3­}

SX4={X3, ¬X1}

Because they have no connections in the graph.

1. ***Give an independent set of size k in the graph G.***

Independent set of size k(2) in the graph G would be {X2, X4}.

1. ***Show how the independent set translates to a set packing using the subsets SI.***

The independent set translates to a set packing using the subsets SI because the size k is 2 you can take the first set that contains X2 and the first set that contains X4 and since they are disjoint they will not have a conflict. X2 and X4 in this case are the independent set elements. This will result in two different subsets with no conflicts if we look at the example in problem number 2.

1. ***Show how the independent set translates to a satisfying assignment to Φ.***

Substituting the independent set {X2, X4} as True (1’s) we get the following equation.

*Φ* = (X1 v 1 v ¬X3) ^ ( ¬X1 v X3 v 1)

1 v and anything simplifies to 1 so the equation now becomes 1 ^ 1 and that simplifies to 1. Thus, we have a satisfying assignment *Φ.*