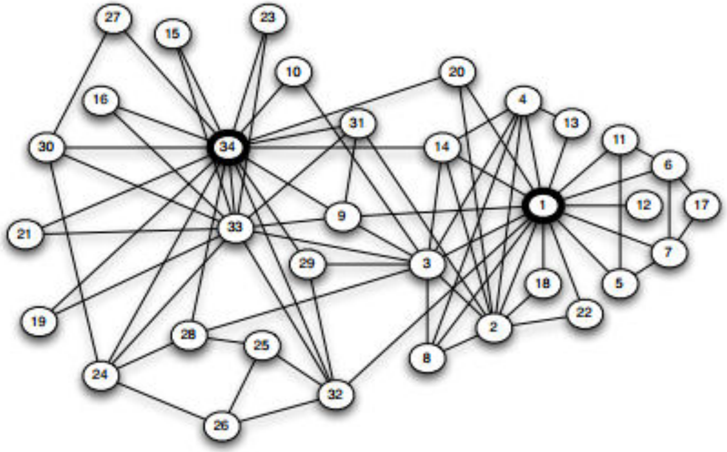
Node Importance

Based on the structure of the network, which are the 5 most important node in the Karate Club friendship network?



Friendship network in a 34-person karate club
[Zachary 1977]

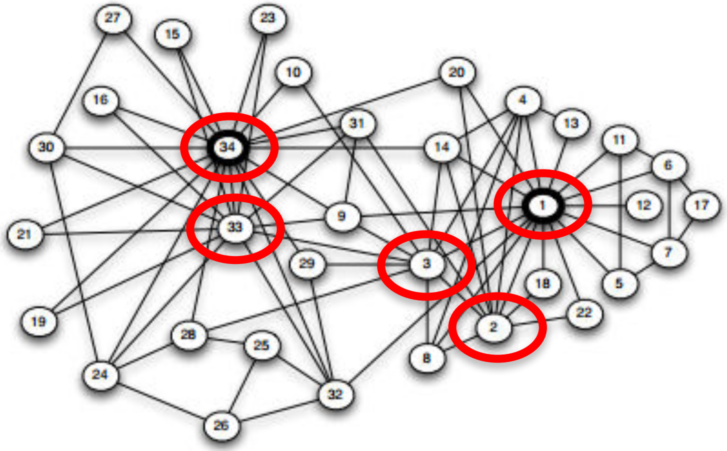
Node Importance

Different ways of thinking about "importance".

Ex. Degree: number of friends.

5 most important nodes are:

34, 1, 33, 3, 2



Friendship network in a 34-person karate club
[Zachary 1977]

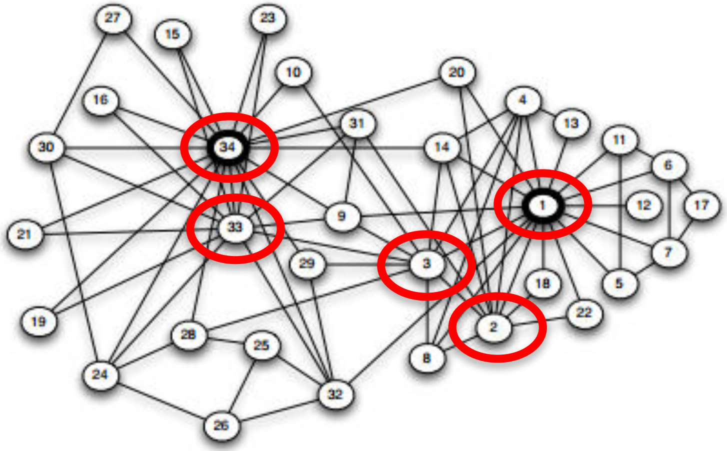
Node Importance

Different ways of thinking about "importance".

Ex. Average proximity to other nodes.

5 most important nodes are:

1, 3, 34, 32, 9



Friendship network in a 34-person karate club [Zachary 1977]

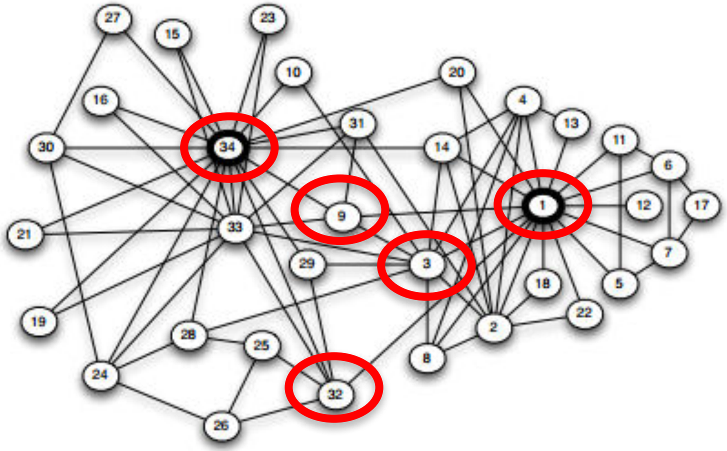
Node Importance

Different ways of thinking about "importance".

Ex. Fraction of shortest paths that pass through node.

5 most important nodes are:

1, 34, 33, 3, 32



Friendship network in a 34-person karate club [Zachary 1977]

Network Centrality

Centrality measures identify the most important nodes in a network:

- Influential nodes in a social network.
- Nodes that disseminate information to many nodes or prevent epidemics.
- Hubs in a transportation network.
- Important pages on the Web.
- Nodes that prevent the network from breaking up.

Centrality Measures

- **Degree centrality**
- **Closeness centrality**
- Betweenness centrality
- Load centrality
- Page Rank
- Katz centrality
- Percolation centrality

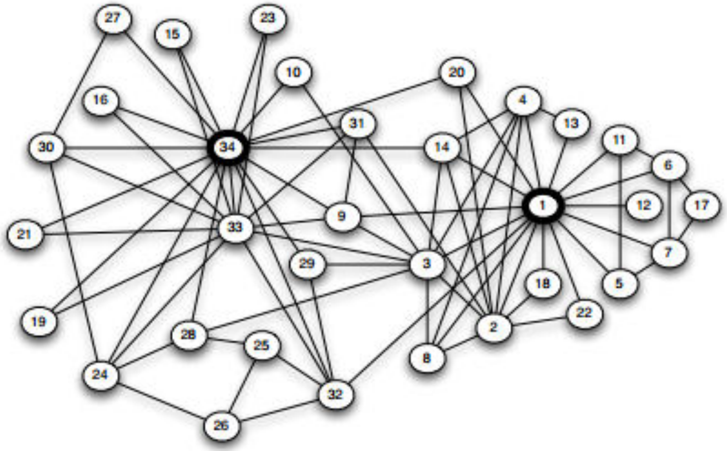
Degree Centrality

Assumption: important nodes have many connections.

The most basic measure of centrality: number of neighbors.

Undirected networks: use degree

Directed networks: use in-degree or out-degree

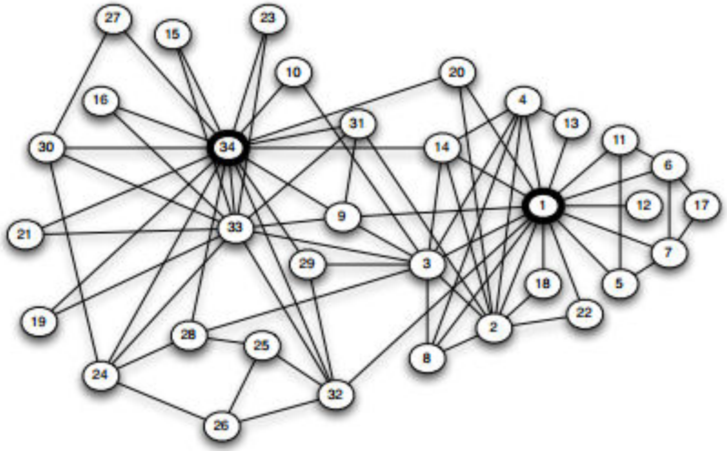


Friendship network in a 34-person karate club
[Zachary 1977]

Degree Centrality – Undirected Networks

$C_{deg}(v) = \frac{d_v}{|N|-1}$, where N is the set of nodes in the network and d_v is the degree of node v.

In: G = nx.karate_club_graph()
In: G = nx.convert_node_labels_to_integers(G,first_label=1)
In: degCent = nx.degree_centrality(G)
In: degCent[34]
Out: 0.515 # 17/33
In: degCent[33]
Out: 0.182 # 6/33



Friendship network in a 34-person karate club
[Zachary 1977]

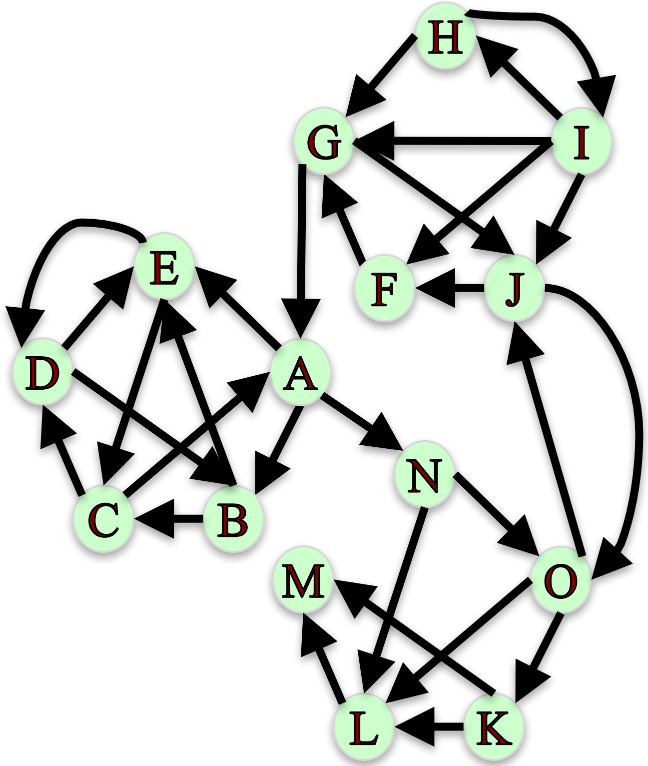
Degree Centrality – Directed Networks

$C_{indeg}(v) = \frac{d_v^{in}}{|N|-1}$, where

N = set of nodes in the network,

d_v^{in} = the in-degree of node v.

In: indegCent = nx.in_degree_centrality(G)
In: indegCent['A']
Out: 0.143 # 2/14
In: indegCent['L']
Out: 0.214 # 3/14



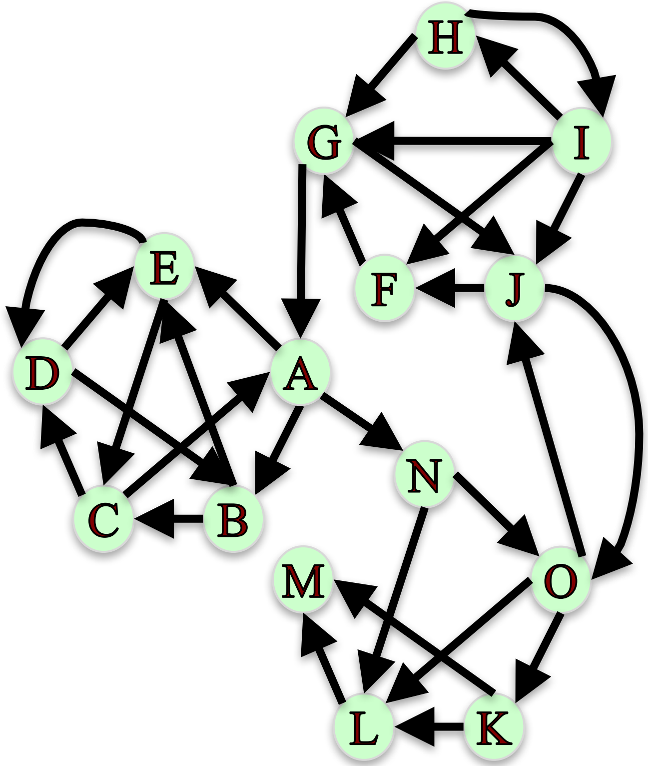
Degree Centrality – Directed Networks

$C_{outdeg}(v) = \frac{d_v^{out}}{|N|-1}$, where

N = set of nodes in the network,

d_v^{out} = the out-degree of node v.

In: outdegCent = nx.out_degree_centrality(G)
In: outdegCent['A']
Out: 0.214 # 3/14
In: outdegCent['L']
Out: 0.071 # 1/14



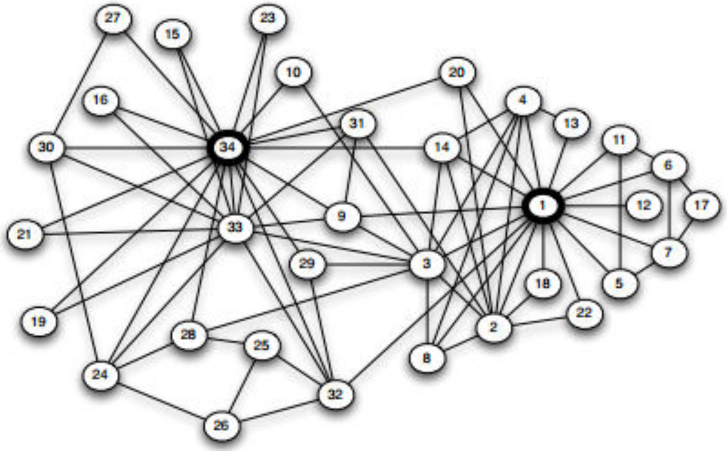
Closeness Centrality

Assumption: important nodes are close to other nodes.

$$C_{close}(v) = \frac{|N|-1}{\sum_{u\in N\setminus\{v\}} d(v,u)},$$ where

N = set of nodes in the network,

$d(v,u)$ = length of shortest path from v to u.



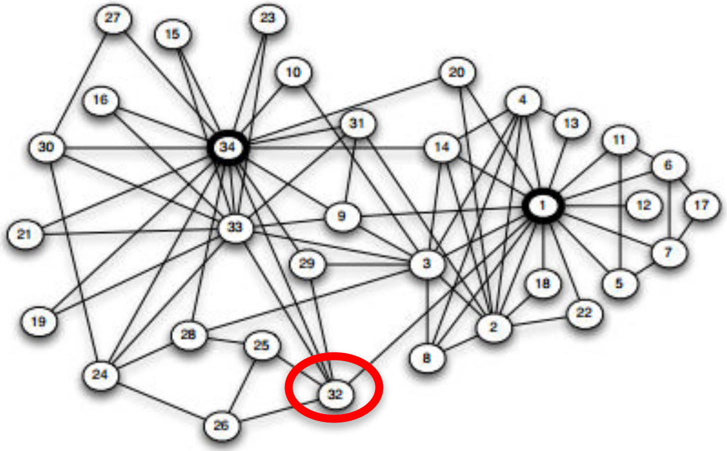
Friendship network in a 34-person karate club
[Zachary 1977]

Closeness Centrality

Assumption: important nodes are close to other nodes.

```
In: closeCent  = nx.closeness_centrality(G)
In: closeCent[32]
Out: 0.541

In: sum(nx.shortest_path_length(G,32).values())
Out: 61
In: (len(G.nodes())-1)/61.
Out: 0.541
```



Friendship network in a 34-person karate club
[Zachary 1977]

Disconnected Nodes

How to measure the closeness centrality of a node when it cannot reach all other nodes?

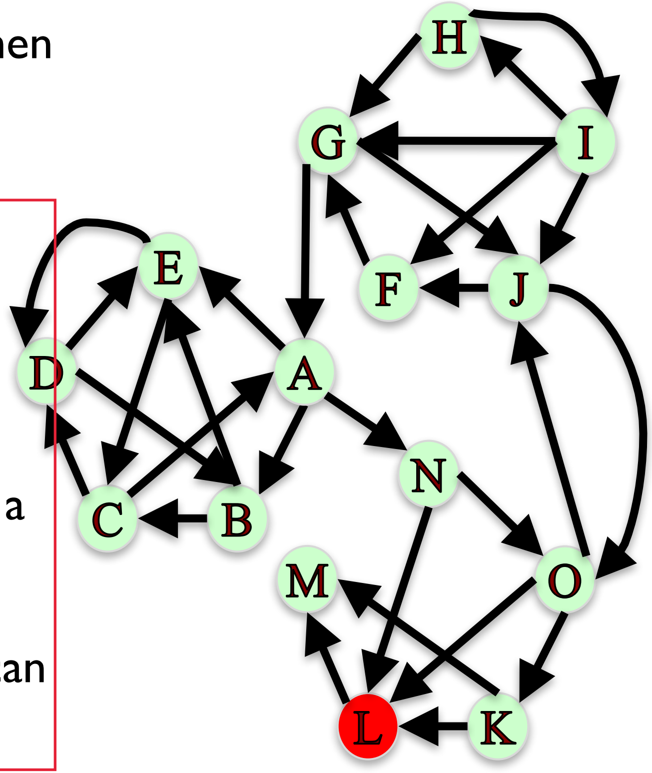
What is the closeness centrality of node L?

Option 1: Consider only nodes that L can reach:

$C_{close}(L) = \frac{|R(L)|}{\sum_{u \in R(L)} d(L,u)}$, where $R(L)$ is the set of nodes L can reach.

$C_{close}(L) = \frac{1}{1} = 1$, since L can only reach M and it has a shortest path of length 1.

Problem: centrality of 1 is too high for a node than can only reach one other node!



Disconnected Nodes

How to measure the closeness centrality of a node when it cannot reach all other nodes?

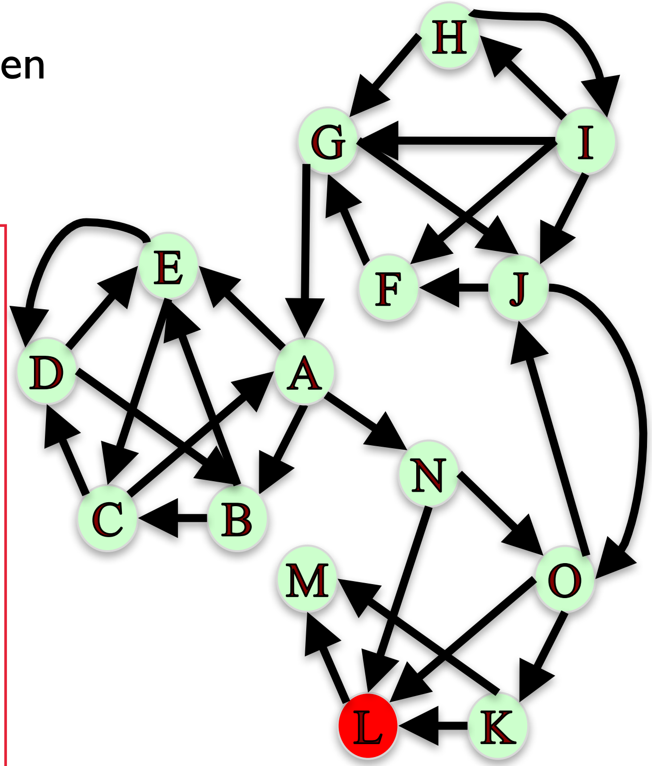
What is the closeness centrality of node L?

Option 2: Consider only nodes that L can reach and normalize by the fraction of nodes L can reach:

$$C_{close}(L) = \left[\frac{|R(L)|}{|N-1|}\right] \frac{|R(L)|}{\sum_{u \in R(L)} d(L,u)}$$

$$C_{close}(L) = \left[\frac{1}{14}\right] \frac{1}{1} = 0.071$$

Note that this definition matches our definition of closeness centrality when a graph is connected since $R(L) = N - 1$



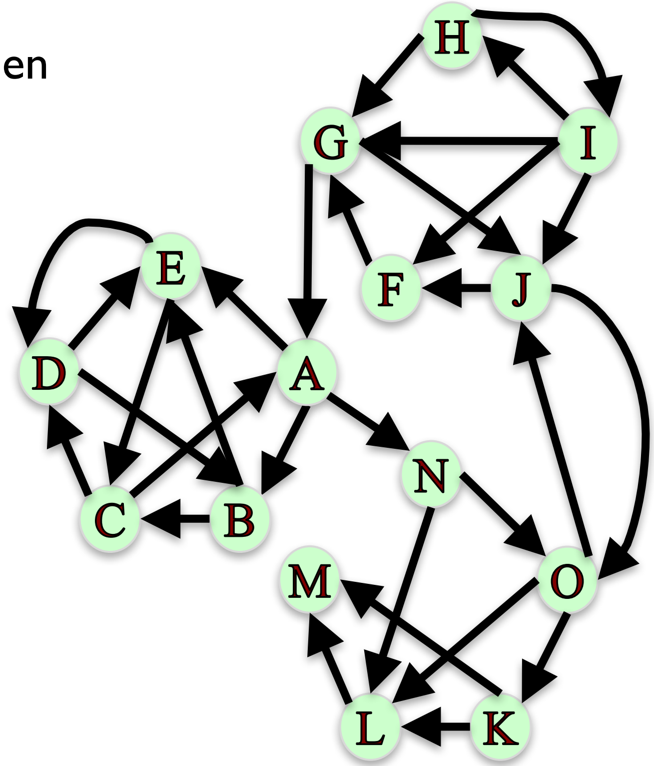
Disconnected Nodes

How to measure the closeness centrality of a node when it cannot reach all other nodes?

What is the closeness centrality of node L?

In: closeCent = nx.closeness_centrality(G, normalized = False)
In: closeCent['L']
Out: 1

In: closeCent = nx.closeness_centrality(G, normalized = True)
In: closeCent['L']
Out: 0.071



Summary

Centrality measures identify the most important nodes in a network:

Degree Centrality

Assumption: important nodes have many connections.

$$C_{deg}(v) = \frac{d_v}{|N| - 1}$$

nx.degree_centrality(G)
nx.in_degree_centrality(G)
nx.out_degree_centrality(G)

Closeness Centrality

Assumption: important nodes are close to other nodes.

$$C_{close}(L) = \left[\frac{|R(L)|}{|N-1|}\right] \frac{|R(L)|}{\sum_{u \in R(L)} d(L,u)}$$

nx.closeness_centrality(G, normalized = True)