Financial Engineering and Risk Management

Real options

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Option theory applied to operations

Option theory can be used to evaluate non-financial investment and decision problems.

Examples:

- Valuation of a lease on a gold mine.
- Valuation of an equipment upgrade option.
- Valuation of a drug development process.
- Valuation of a manufacturing firm that has the option of contracting its facilities to other manufacturers.
- Valuation of a tolling contract on a power plant.
- Valuation of a gas storage facility.

Real Options paradigm

A project has many sources of uncertainty

- Market uncertainties: prices, demand
- Industry uncertainties: M & A, innovations
- Technical uncertainties: R & D
- Organizational uncertainties: key personnel
- Political uncertainties: regulations, wars

Manage uncertainty by adding flexibility via options.

Project management is all about managing the flexibility.

Will explore valuation of some simple real options in this module.

Simplico Gold Mine Case

Example 12.7 from Investment Science by Luenberger.

Goal: Evaluate the value of a 10 year lease on a gold mine

- Price of gold.
  - Current price of gold $= \$400$ per ounce.
  - Each year the price increases by a factor $u = 1.2$ with probability $p = 0.75$ or decreases by a factor $d = 0.9$ with probability $1 - p = 0.25$.
- Cost of extracting gold $C = \$200$ per ounce.
- Maximum rate of gold extraction $G = 10,000$ ounces/year.
- Interest rate $r = 10\%$
  - Risk neutral probability $q = \frac{(1+r)-d}{u-d} = \frac{2}{3}$
  - Implicitly assume that we can buy and short sell gold.

Convention: Cash flows occur at the end of year. Revenue is determined by the price at the beginning of the year.

Simplico Gold Mine Pricing

- Current time $t = 0$. Lease ends at $t = 10$.
- States in each year are given by the gold prices at the beginning of the year.
- $V_t(s) =$ Value in state s at the beginning of year t.
- Since lease ends in 10 years, $V_{10}(s) = 0$ for all s.

Recursion to compute $V_t(s)$

$$
\begin{aligned}
V_t(s) &= \frac{\text{Revenue in year } t + \mathbb{E}_t^Q[V_{t+1}]}{1+r} \\
&= \frac{\max\{s - C, 0\}\, G + \big(q V_{t+1}(us) + (1-q)\, V_{t+1}(ds)\big)}{1+r}
\end{aligned}
$$

- Pricing $\Rightarrow$ Expectation with respect to risk-neutral probability.
- It is optimal to produce at G when $s > C$.
- Assumes that there is no cost for shutting down operations in any year.

Simplico with equipment enhancement option

Equipment upgrade details

- Cost of upgrade $= \$4$ million
- New rate of production $G_{up} = 12,500$ ounces/year
- New cost per ounce $C_{up} = 240$/ounce

This upgrade is an option that can be exercised any time over the lease period

- Once the upgrade is in place, it applies for all future years.
- When the lease ends, the upgraded equipment reverts to the mine owners.

What is the value of this upgrade option?

Pricing the upgrade option

Define

$$V_t^{\text{up}}(s) = \text{Value of mine in state } s \text{ and date } t \text{ with equipment upgrade in place}$$

Same computation as before but with new parameters G_{up} and C_{up}.

The upgrade option is an American option that pays $V_t^{\text{up}}(s) - \$4\text{M}$ if exercised in state s on date t. Once exercised, the equipment is upgraded and the mine is operated according to the policy corresponding to V_t^{up}.

Define $U_t(s) =$ Value in state s and date t with upgrade option

$$U_t(s) = \max\left\{ \underbrace{V_t^{\text{up}}(s) - 4\text{M}}_{\text{exercise option}}, \underbrace{\frac{\max\{s-C,0\}G + \big(qU_{t+1}(us) + (1-q)U_{t+1}(ds)\big)}{1+r}}_{\text{continue}} \right\}$$

Note: $C = 200$ and $G = 10,000$

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Valuation of natural gas and electricity related options

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Valuation of a natural gas storage facility

Caverns can be used to store gas and profit from temporal variation of price.

Goal: Evaluate the value of a lease on a cavern with capacity C.

Dynamics of gas storage:

- $I_t =$ gas stored in the cavern on day t.
- $z_t \geq 0$ (resp. ≤ 0) $=$ gas pumped out of (resp. into) the cavern.
- $f(z, I) =$ loss of gas if z units are pumped out when the gas volume in the cavern is I. Most of this loss is because gas is used to drive the pumps.
- $P_t =$ Price of gas at time t

Optimization problem

$$\begin{array}{rl} \max_{\{z_t\}} & \mathbb{E}_0^Q \left[\sum_{t=1}^T e^{-rt} P_t \Big(z_t - f(z_t, I_t) \Big) \right] \\ \text{such that} & I_{t+1} = I_t - z_t, \quad I_t \in [0, C] \\ & z_t \text{ function of past prices and inventory level } I_t \end{array}$$

The expectation has to be with respect to the risk-neutral distribution.

Valuation of a gas storage facility (contd)

Dynamic program for solving the gas pricing problem

$$V_t(I,P) = \max_z \left\{ P\big(z - f(z,I)\big) + e^{-r}\mathbb{E}_t^Q\left[V_{t+1}(I-z, P_{t+1})\right]\right\}$$

- Can use a binomial lattice for the price P; however, one has to enumerate all possible inventory levels I.
- Can use approximate dynamic programming where the value function is approximated by factors.
- Can also use simulation-based optimization.

Tolling agreements on a gas-fired power plant

Optimal policy for a company renting a two regime gas power plant over $[0, T]$.

Details:

- Shut down mode: rent K
- Low capacity mode: output $\underline{Q}$, gas consumption $\underline{H}$, rent $\underline{K}$
- High capacity mode: output $\overline{Q}$, gas consumption $\overline{H}$, rent $\overline{K}$

State of the plant: $s_t \in \{0, 1\}$, $0 \equiv$ off and $1 \equiv$ on.

- Ramp-down cost C_d
- Ramp-up cost C_u
- Actions a in state s_t: $a \in \{0, 1\}$, $0 \equiv$ off and $1 \equiv$ on.

$$V_t(s_t, P_t, G_t) \quad = \quad \text{Optimal profit over } [t, T] \text{ with current state } s_t, \text{ price of electricity} = P_t, \text{ and price of gas} = G_t$$

Tolling agreements on a gas-fired power plant

Notation:

- $c(s, a)$: cost of taking action a when the plant is in state s
- $u(s, a)$: new state of the plant when action a is taken in state s

$$\begin{aligned} c(0,0) &= K \\ c(0,1) &= C_u + K \\ c(1,0) &= C_d + K \\ c(1,1) &= \max\left\{P_t\overline{Q} - G_t\overline{H} - \overline{K}, P_t\underline{Q} - G_t\underline{H} - \underline{K}\right\} \end{aligned} \qquad \begin{aligned} u(0,0) &= 0 \\ u(0,1) &= 1 \\ u(1,0) &= 0 \\ u(1,1) &= 1 \end{aligned}$$

Note that when the plant is on, one has the choice to run it at the low or high capacity mode.

Dynamic program

$$V_t(s_t, P_t, G_t) = \max_a \left\{c(s_t, a) + e^{-r}\mathbb{E}_t^Q\left[V_t(u(s_t, a), P_{t+1}, G_{t+1})\right]\right\}$$

Can solve this using a binomial lattice for the gas price G_t and electricity price P_t, and two states for the plant in every node of the lattice.