Financial Engineering & Risk Management

Structured Credit: CDO's and Beyond

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Securitization

Recall that securitization is the name given to the process of constructing new securities from the cash-flows generated by a pool of underlying securities

– have already seen examples of securitization in the **mortgage market**.

The economic rationale behind securitization is that it enables the construction of new securities with a **broad range** of **risk profiles**.

A broad range of investors may therefore be interested in these new securities

– even if they had no interest in the underlying securities.

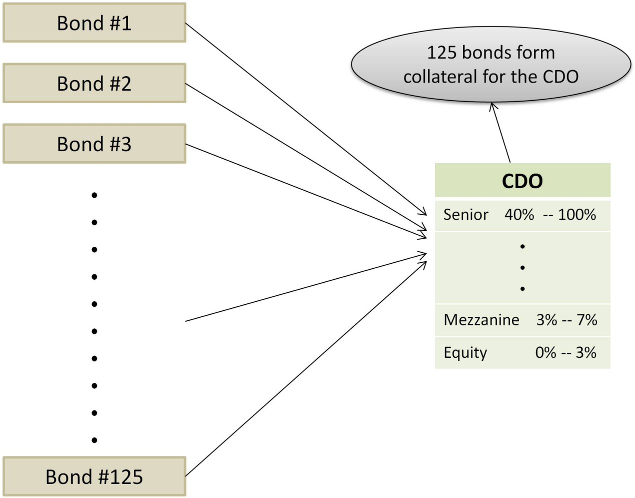
This results in an increased **demand** for the underlying cash-flows

– and so the cost-of-capital is reduced for the issuers of the underlying securities.

Credit default obligations (CDOs) are securities that are constructed from an underlying pool of fixed-income securities

– first issued by banks in the mid-1990's

– originally motivated by regulatory arbitrage considerations to **supply** CDO's to the market.


Bond #1
Bond #2
Bond #3
Bond #125
125 bonds form collateral for the CDO
CDO
Senior 40% -- 100%
Mezzanine 3% -- 7%
Equity 0% -- 3%

Securitization

Collateralized loan obligations (CLOs) are securities that are constructed from an underlying pool of loans.

The **structured credit** market was at the heart of the financial crisis

– terms like ABS, MBS, CDO and ABS-CDO's became standard in the financial (and mainstream) media.

Our introduction to structured credit and CDOs will:

1. Provide further examples of the securitization process
2. Allow us to introduce the (in)famous Gaussian copula model
3. Discuss the difficulties in risk managing structured credit portfolios
4. Emphasize just how crazy some parts of the financial markets had become in the lead-up to the financial crisis.

The mechanics of how CDOs work have much in common with the mechanics of **credit default swaps** (CDSs)

– so worthwhile to review and understand CDSs before studying CDO's.

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The Gaussian Copula Model

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Assumptions

We assume there are N bonds or credits in the reference portfolio.

Each credit has a notional amount of A_i.

If the i^{th} credit defaults, then the portfolio incurs a loss of $A_i \times (1 - R_i)$

- R_i is the recovery rate, i.e., the percentage of the notional amount that is recovered upon default.

From a modeling perspective R_i is often assumed to be fixed and known

- in practice it is random and not known until after a default event has taken place.

We also assume the **risk-neutral distribution** of default time of i^{th} credit is known

- it can be estimated from either credit-default-swap (CDS) spreads or the prices of corporate bonds.

Therefore can compute $q_i(t)$, the risk-neutral probability that the i^{th} credit defaults before time t, for any time t.

The Gaussian Copula Model

We let X_i denote the normalized asset value of the i^{th} credit

– can think of X_i as representing the total value of assets of firm i.

We assume that

$$X_i \;=\; a_i M \;+\; \sqrt{1 - a_i^2}\; Z_i \tag{1}$$

where $M, Z_1, \ldots, Z_N$ are standard IID normal random variables

– note that each X_i is also a standard normal random variable.

Each of the factor loadings, a_i, is assumed to lie in the interval $[0, 1]$.

Clear that Corr$(X_i, X_j) \;=\; a_i a_j$ and that the X_i's are multivariate normally distributed

– with covariance matrix equal to correlation matrix, $\mathbf{P}$, where $\mathbf{P}_{i,j} = a_i a_j$ for $i \neq j$.

The Gaussian Copula Model

Assume that the i^{th} credit has defaulted by time t_i if X_i falls below some threshold value, $\bar{x}_i(t_i)$.

By earlier assumption must be the case that $\bar{x}_i(t_i) = \Phi^{-1}(q_i(t_i))$ where $\Phi(\cdot)$ is the standard normal CDF.

Let $F(t_1, \ldots, t_N)$ denote the **joint distribution** of the **default times** of the N credits in the portfolio. Then

$$\begin{aligned} F(t_1, \ldots, t_N) &= \mathbf{\Phi_P}\left(\bar{x}_1(t_1), \ldots, \bar{x}_N(t_N)\right) \\ &= \mathbf{\Phi_P}\left(\Phi^{-1}(q_1(t_1)), \ldots, \Phi^{-1}(q_N(t_N))\right) \end{aligned}$$

where $\mathbf{\Phi_P}(\cdot)$ denotes the multivariate normal CDF with mean vector $\mathbf{0}$ and covariance matrix, $\mathbf{P}$.

This is the infamous (1-factor) Gaussian copula model

– a 1-factor model because just 1 random variable, M, driving the dependence between the X_i's

Computing the Portfolio Loss Distribution

In order to price credit derivatives and CDOs in particular, we need to compute the **portfolio loss distribution**.

First note that, **conditional** on M, the N default events are **independent**.

Then conditional on M, the default probabilities are given by

$$q_i(t|M) \;=\; \Phi\left(\frac{\bar{x}_i(t) - a_i M}{\sqrt{1-a_i^2}}\right)$$

where $\Phi(\cdot)$ is the standard normal CDF.

Now let $p^N(l,t)$ denote risk-neutral probability that there are a total of l portfolio defaults before time t.

Then may write

$$p^N(l,t) \;=\; \int_{-\infty}^{\infty} p^N(l,t|M)\;\phi(M)\;dM \tag{2}$$

where $\phi(\cdot)$ is the standard normal PDF.

Computing the Portfolio Loss Distribution

We can easily calculate $p^N(l, t|M)$ using a simple **iterative procedure**:

Can then perform a **numerical integration** on the right-hand-side of (2) to calculate $p^N(l, t)$.

If we assume that the notional, A_i, and the recovery rate, R_i, are **constant** across all credits, then the loss on any given credit will be either 0 or $A(1 - R)$.

Therefore knowing the distribution of the number of defaults **is equivalent** to knowing the distribution of the total loss in the reference portfolio

- so the assumption of constant A_i's and R_i's simplifies calculations
  - we will make this assumption although we could get by without it.

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A Simple Example: Part I

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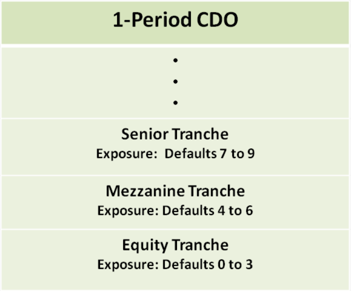
A Simple Example: A 1-Period CDO

We want to find the expected losses in a CDO with the following characteristics:

- Maturity is 1 year
- 125 bonds in the reference portfolio
- Each bond pays a coupon of **one unit** after 1 year if it has not defaulted
- The **recovery rate** on each bond is zero
- There are 3 tranches of interest: the equity, mezzanine and senior tranches



This example is taken from "The Devil is in the Tails: Actuarial Mathematics and the Subprime Mortgage Crisis", by C. Donnelly and P. Embrechts in *ASTIN Bulletin* 40(1), 1-33.

A Simple Example: A 1-Period CDO

We make the simple assumption that the probability, q, of defaulting within 1 year is **identical** across all bonds.

– and that the correlation between each pair of default events is **identical**.

As before X_i is the normalized asset value of the i^{th} credit and now

$$X_i \;=\; \sqrt{\rho} M \;+\; \sqrt{1-\rho}\, Z_i \tag{3}$$

where $M, Z_1, \ldots, Z_N$ are IID normal random variables.

Recall that the i^{th} credit defaults if $X_i \leq \bar{x}_i$. Since probability of default, q, is identical across credits we have

$$\bar{x}_1 = \;\cdots\; = \bar{x}_N = \Phi^{-1}(q).$$

Moreover

$$\begin{aligned} \mathsf{P}(X_i \text{ defaults} \mid M) \;&=\; \mathsf{P}(X_i \leq \bar{x}_i \mid M) \\ &=\; \mathsf{P}(\sqrt{\rho} M + \sqrt{1-\rho}\, Z_i \leq \Phi^{-1}(q) \mid M) \\ &=\; \mathsf{P}\left(Z_i \leq \frac{\Phi^{-1}(q) - \sqrt{\rho} M}{\sqrt{1-\rho}} \mid M \right) \end{aligned}$$

The Conditional Default Distribution

Therefore conditional on M, the total number of defaults is $\text{Bin}(N, q_M)$ so that

$$p^N(l|M) \;=\; \binom{N}{l} q_M^l (1 - q_M)^{N-l} \tag{4}$$

where

$$q_M \;:=\; \Phi\left(\frac{\Phi^{-1}(q) - \sqrt{\rho}M}{\sqrt{1-\rho}}\right)$$

where $\Phi(\cdot)$ is the standard normal CDF and q is the risk-neutral probability of a single name defaulting within 1 year.

Question: When correlations and default probabilities are not identical why is (4) no longer valid?

Question: How do we calculate $p^N(l|M)$ in that case?

Computing the Expected Tranche Losses

We can compute the **expected loss** on each tranche:

$$
\begin{aligned}
\mathsf{E}_0^{\mathbb{Q}}\left[\text{Equity tranche loss}\right] &= 3 \times \mathsf{P}\left(\text{3 or more defaults by year end}\right) \\
&\quad + \sum_{k=1}^{2} k \times \mathsf{P}\left(k \text{ defaults by year end}\right) \\
\mathsf{E}_0^{\mathbb{Q}}\left[\text{Mezzanine tranche loss}\right] &= 3 \times \mathsf{P}\left(\text{6 or more defaults by year end}\right) \\
&\quad + \sum_{k=1}^{2} k \times \mathsf{P}\left(k+3 \text{ defaults by year end}\right) \\
\mathsf{E}_0^{\mathbb{Q}}\left[\text{Senior tranche loss}\right] &= 3 \times \mathsf{P}\left(\text{9 or more defaults by year end}\right) \\
&\quad + \sum_{k=1}^{2} k \times \mathsf{P}\left(k+6 \text{ defaults by year end}\right)
\end{aligned}
$$

Each probability $\mathsf{P}(\cdot)$ can be calculated by integrating the binomial probabilities with respect to M:

$$p(l) \;=\; \int_{-\infty}^{\infty} p^N(l|M)\;\phi(M)\;dM \tag{5}$$

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A Simple Example: Part II

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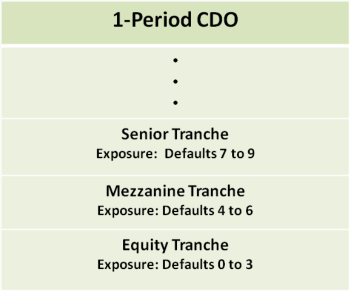
A Simple Example: A 1-Period CDO

We want to find the expected losses in a CDO with the following characteristics:

- Maturity is 1 year
- 125 bonds in the reference portfolio
- Each bond pays a coupon of **one unit** after 1 year if it has not defaulted
- The **recovery rate** on each bond is zero
- There are 3 tranches of interest: the equity, mezzanine and senior tranches



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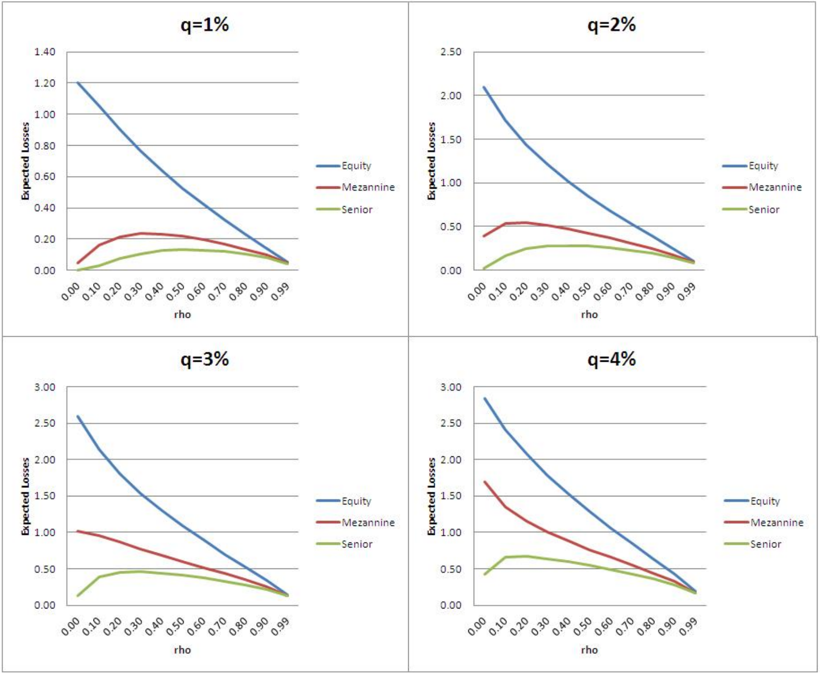


q=1%
q=2%
q=3%
q=4%
Expected Losses
rho
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 0.99
0.00 0.20 0.40 0.60 0.80 1.00 1.20 1.40
0.00 0.50 1.00 1.50 2.00 2.50
0.00 0.50 1.00 1.50 2.00 2.50 3.00
Equity
Mezannine
Senior

Some Important Observations

Regardless of the individual default probability, q, and correlation, ρ, we see

$$\mathsf{E}_0^{\mathbb{Q}}\left[\text{Equity tranche loss}\right] \geq \mathsf{E}_0^{\mathbb{Q}}\left[\text{Mezzanine tranche loss}\right] \geq \mathsf{E}_0^{\mathbb{Q}}\left[\text{Senior tranche loss}\right]$$

- only holds when each tranche has same notional exposure
  - in this case 3 units.

The expected losses in the equity tranche are always **decreasing** in ρ.

Mezzanine tranches are often relatively **insensitive** to ρ

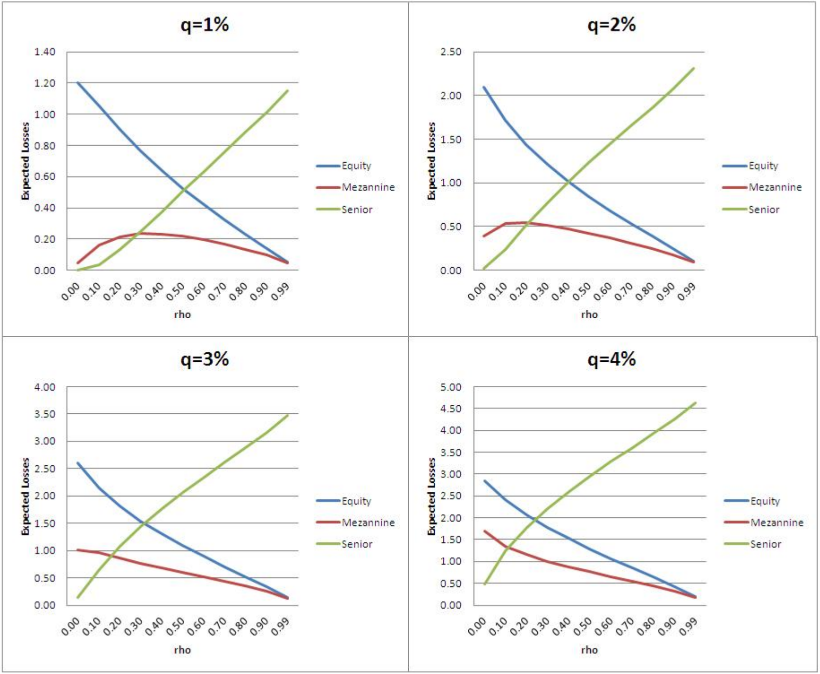
- this has implications when it comes to **model calibration**.

The expected losses in super senior tranches (with upper attachment point of 100% or 125 units in our example) are always **increasing** in ρ

- as evidenced in figure on next slide where the senior tranche is now exposed to all losses between 7 and 125.

Note also that in figure on next slide the total expected losses on the three tranches, i.e. the expected losses on the index, is **independent** of ρ

- this is not an accident!


q=1%
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
Expected Losses
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
0.99
rho
Equity
Mezannine
Senior
q=2%
2.50
2.00
1.50
1.00
0.50
0.00
Expected Losses
rho
Equity
Mezannine
Senior
q=3%
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
Expected Losses
rho
Equity
Mezannine
Senior
q=4%
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
Expected Losses
rho
Equity
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Senior

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The Mechanics of a "Synthetic" CDO Tranche

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The Mechanics of a "Synthetic" CDO Tranche

Recall there are N credits in the reference portfolio.

Each credit has the same notional amount, A,

If the i^{th} credit defaults, then the portfolio incurs a loss of $A \times (1 - R)$

– the recovery rate, R, is assumed fixed, known and constant across credits.

A tranche is defined by the lower and upper attachment points, L and U, respectively.

The tranche loss function, $TL^{L,U}(l)$, for a fixed time, t, is a function of the number of defaults, l, up to time t and is given by

$$TL_t^{L,U}(l) \;:=\; \max\left\{\min\{lA(1-R),\; U\} - L,\; 0\right\}.$$

For a given number of defaults it tells us the loss suffered by the tranche.

Example: Suppose $L = 3\%$ and $U = 7\%$ and suppose total portfolio loss is $lA(1 - R) = 5\%$.

Then tranche loss is 2% of total portfolio notional

– or 50% of **tranche notional** $= 7\% - 3\% = 4\%$.

The Mechanics of a "Synthetic" CDO Tranche

When an investor sells protection on the tranche she is guaranteeing to reimburse any realized losses on the tranche to the protection buyer.

In return, the protection seller receives a premium at regular intervals from the protection buyer

– these payments typically take place every three months.

In some cases protection buyer may also make an upfront payment in addition to, or instead of, a regular premium

– often the case for equity tranches which have a lower attachment point of zero.

The fair value of the CDO tranche is that value of the premium (plus upfront payment if applicable) for which the expected value of the premium leg equals the expected value of the default leg

– so just like a swap, the initial value of the position is **zero**.

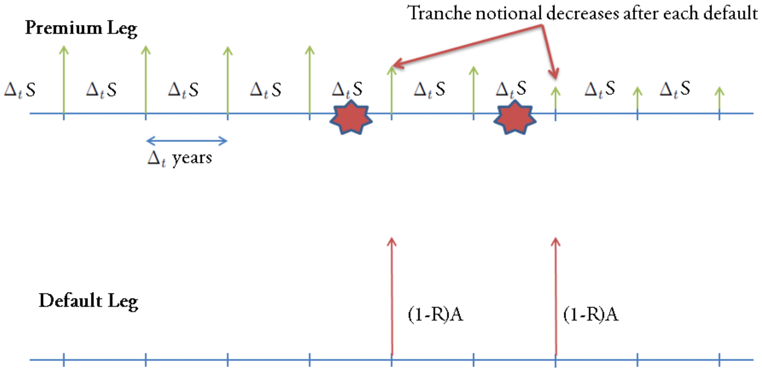
Clearly then the fair value of the CDO tranche depends on the expected value of the tranche loss function at each of the fixed time periods.

The Mechanics of a "Synthetic" CDO Tranche

Indeed, for a fixed time, t, the expected tranche loss is given by

$$\mathsf{E}_0^{\mathbb{Q}}\left[TL_t^{L,U}\right] = \sum_{l=0}^{N} TL_t^{L,U}(l)\; p(l,t)$$

which we can compute using our expression $p(l,t) = \int_{-\infty}^{\infty} p^N(l,t|M)\;\phi(M)\;dM$.



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Computing the Fair Value of a CDO Tranche

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The Fair Value of the Premium Leg

The premium leg represents the premium payments that are paid periodically by the protection buyer to the protection seller.

These payments are made at the end of each time interval and they are based upon the remaining notional in the tranche

– in this sense different to a CDS since the latter contract ends as soon as a default occurs.

Formally, the time $t = 0$ value of the premium leg, $P_0^{L,U}$, satisfies

$$PL_0^{L,U} = S \sum_{t=1}^{n} d_t \Delta_t \left((U - L) - \mathsf{E}_0^{\mathbb{Q}} \left[TL_{t-1}^{L,U} \right] \right) \tag{6}$$

where:

- n is the number of periods in the contract
- d_t is the risk-free discount factor for payment date t
- S is the annualized spread or premium paid to the protection seller
- Δ_t is the accrual factor for date t. **e.g.** $\Delta_t = 1/4$ for quarterly payments.

The Fair Value of the Default Leg

The default leg represents the cash flows paid to the protection buyer upon losses occurring in the tranche.

Formally, the time $t = 0$ value of the default leg, $DL_0^{L,U}$, satisfies

$$DL_0^{L,U} = \sum_{t=1}^{n} d_t \left(\mathsf{E}_0^{\mathbb{Q}} \left[TL_t^{L,U} \right] - \mathsf{E}_0^{\mathbb{Q}} \left[TL_{t-1}^{L,U} \right] \right). \qquad (7)$$

While some programming is required, we can calculate the $\mathsf{E}_0^{\mathbb{Q}} \left[TL_t^{L,U} \right]$'s numerically very quickly using the **Gaussian copula** model

- therefore can easily compute $PL_0^{L,U}$ and $DL_0^{L,U}$
- the principal reason for the model's popularity despite its **many flaws**.

The Fair Value of the Tranche

The fair premium, S^* say, is the value of S that equates the value of the default leg with the value of the premium leg at the beginning of the contract:

$$S^* \;:=\; \frac{DL_0^{L,U}}{\sum_{t=1}^{n} d_t \Delta_t \left((U-L) - \mathsf{E}_0^{\mathbb{Q}}\left[TL_{t-1}^{L,U} \right] \right)}. \tag{8}$$

As is the case with swaps and forwards, the fair value of the tranche to the protection buyer and seller at initiation is therefore **zero**.

Easy to incorporate any possible upfront payments that the protection buyer must pay at time $t = 0$ in addition to the regular premium payments.

Can also incorporate recovery values and notional values that vary with each credit in the portfolio.

In practice S^* was seen in the market and (8) was used to calibrate an implied correlation parameter

- unfortunately a **different** correlation value is inferred for each tranche
- and sometimes not possible to solve / calibrate an implied correlation parameter at all.

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Cash and Synthetic CDOs

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Cash CDOs

The first CDOs to be traded were all cash CDOs where the reference portfolio actually existed and consisted of corporate bonds that the CDO issuer usually kept on its balance sheet.

Capital requirements meant that these bonds required a substantial amount of capital to be set aside to cover any potential losses.

To reduce these capital requirements, banks converted the portfolio into a series of tranches and sold most of these tranches to investors.

But they usually kept the **equity tranche** for themselves

- thereby keeping most of the economic risk and rewards of the portfolio
- but they also succeeded in dramatically reducing the amount of capital they needed to set aside.

First CDO deals were therefore motivated by regulatory arbitrage considerations.

But cash CDOs must be managed, and the legal documentation can be lengthy

- waterfall structures, credit enhancement etc. are all important features
- the tranches were typically "rated" by the ratings agencies.

Synthetic CDOs

Soon become clear there was an appetite in the market-place for these products

- hedge funds, for example, were keen to buy the riskier tranches
- whereas insurance companies and others sought the AAA-rated senior and super-senior tranches.

This appetite and explosion in the CDS market gave rise to synthetic tranches

- the underlying reference portfolio is no longer a physical portfolio of corporate bonds or loans
- instead it is a **fictitious** portfolio consisting of a number of credits with an associated notional amount for each credit.

Mechanics of Synthetic CDOs

Mechanics of a synthetic tranche are precisely as described earlier.

But they have at least two features that distinguish them from cash CDOs:

(1) With a synthetic CDO it is no longer necessary to tranche the entire portfolio and sell the entire "deal".

e.g. A bank could sell protection on a 3%-7% tranche and never have to worry about selling the other pieces of the reference portfolio

– this is not the case with cash CDOs.

(2) Because the bank no longer owns the underlying bond portfolio, it is no longer hedged against adverse price movements

– it therefore needs to **dynamically hedge** its synthetic tranche position

– typically does so using the CDS markets

– but hedging and risk managing CDO portfolios are very difficult with or without a good pricing model

  – there are simply too many moving parts.

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Pricing and Risk Management of CDO Portfolios

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A Sample Synthetic CDO Portfolio

Index	Tranche Description	L	U	Maturity	Notional ($m's)	Current Price (bps)
CDX IG A	Equity	0%	3%	10 years	100	722
CDX IG A	Mezzanine	3%	7%	7 years	-300	223
CDX IG B	Senior	7%	10%	5 years	-400	85
CDX IG B	Equity	0%	3%	5 years	50	567
CDX IG B	Index	0%	100%	5 years	-400	75
CDX XO D	Mezzanine	5%	10%	3 years	-30	324
CDX XO E	Mezzanine	5%	10%	4 years	45	63
CDX HY F	Equity	0%	10%	10 years	20	1400
CDX HY F	Mezzanine	10%	20%	10 years	-80	250

A Simple Example of a Structured Credit Portfolio

In practice structured credit portfolios could contain many, many positions with:

- different reference portfolios, different maturities and counter-parties
- different trading formats, i.e. upfront and / or a running spread format.

Pricing and Risk Management of CDO Portfolios

Ultimate payoff of such a portfolio is very path dependent with substantial idiosyncratic risk

- very difficult to risk-manage these portfolios properly
- can also be expensive to unwind due to wide bid-offer spreads.

Computing the mark-to-market value of these portfolios can also be very difficult because market prices may non be transparent

- witness, for example, the *"Belly of the Whale Series"* on the *Alphaville* blog of the Financial Times at `http://ftalphaville.ft.com/`
  - apparently the so-called London Whale first came to attention because price levels in the *CDX IG9* index diverged too much from other related price levels.

Pricing and Risk Management of CDO Portfolios

Risk-management for structured credit portfolios is also very challenging:

1. **Scenario analysis** is certainly difficult
   - what are the main risk factors?
   - what are reasonable stress levels for these factors?
   - how do we re-evaluate the portfolio in a given scenario?
2. The **Greeks** can be computed and used to risk-manage a portfolio
   - but they don't work well either and are model dependent
   - witness the fallout from the downgrade of *Ford* and *General Motors* in May 2005 as described in the Wall Street Journal on 12^{th} Sep 2005 and available at:

   `http://online.wsj.com/article/0,,SB112649094075137685,00.html`

   - but **don't believe** every thing you read!

Liquidity risk and market endogeneity are key risks that must be considered.

Over-reliance on ratings agencies, models, the behavior of organizations etc. all played an important part in the crisis.

A Brief Aside on Copulas

The most famous model for pricing structured credit securities is the Gaussian copula model.

There has also been enormous criticism aimed at this model

– most (if not all of it) justified

– but **nothing** we didn't know well before the crisis!

There has been much academic work on building better and more sophisticated models

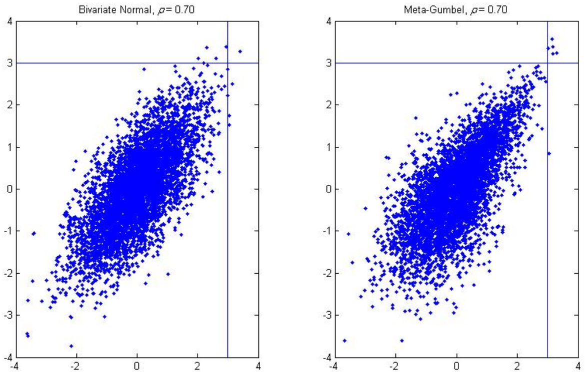
– but none of them are really satisfactory.

Aside: A common fallacy is that the marginal distributions and correlation matrix are sufficient for describing the joint distribution of a multivariate distribution.

This is **not true**! Correlation only measures linear dependence

– and the next slide provides a counter-example.

A Brief Aside on Copulas



- Each figure shows 5000 simulated points from the **Bivariate Normal** and **Meta-Gumbel** distributions.
- Both bivariate distributions have standard normal marginal distributions.
- In each case the correlation is $.7$.
- But Meta-Gumbel is much more likely to see large **joint** moves.

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CDO-Squared's and Beyond

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CDO-Squared's and Beyond

It should already be clear that structured credit portfolios consisting of CDO tranches can be difficult to risk manage.

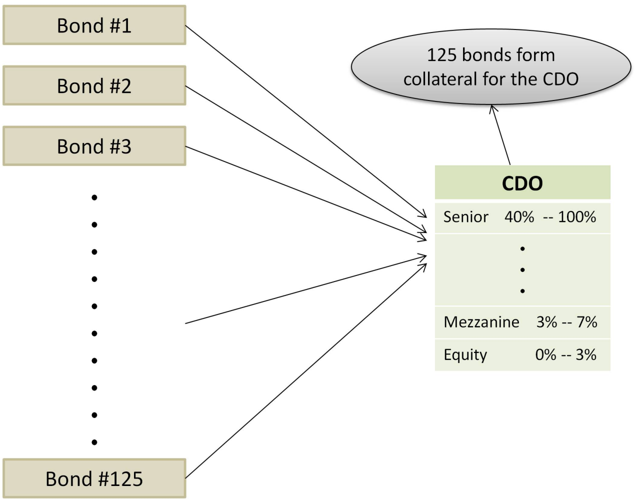
But at least there is a solid risk-sharing motivation for the creation of CDOs

- true for securitization in general.

But the structured credit market quickly ran amok with the creation and trading of ever more complex securities

- for example, products such as CDO-squared's or CDO^2 were soon developed
    - they were difficult to justify economically
    - and provided great examples of **product risk**, **model risk**, **legal risk**, etc.

Before discussing CDO^2, recall first how a CDO is constructed . . .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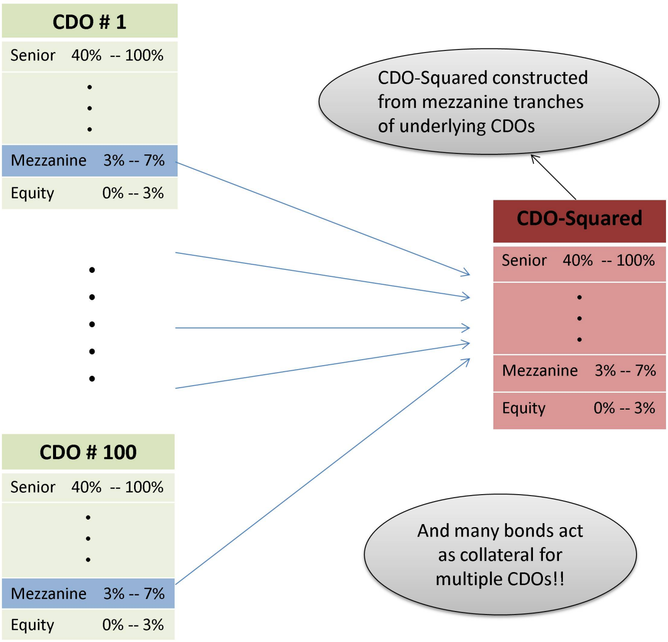

CDO # 1
Senior 40% -- 100%
Mezzanine 3% -- 7%
Equity 0% -- 3%
CDO-Squared constructed from mezzanine tranches of underlying CDOs
CDO-Squared
Senior 40% -- 100%
Mezzanine 3% -- 7%
Equity 0% -- 3%
CDO # 100
Senior 40% -- 100%
Mezzanine 3% -- 7%
Equity 0% -- 3%
And many bonds act as collateral for multiple CDOs!!

Creating a CDO^2

Suppose now we have 100 different CDO's.

We can construct a new CDO using the mezzanine tranches, for example, of these 100 CDO's

– and create a (synthetic) pool of mezzanine tranches.

This pool now forms the collateral for a new CDO

– which we call a **CDO^2**!

The CDO^2 only incurs losses when the mezzanine tranche in one of the 100 underlying CDO's incurs losses

– so a mezzanine tranche of a CDO now plays the role of a bond (or CDS).

Note that many of the same bonds act as collateral for many of the underlying CDO's.

The Joys of CDO-Squared's

Question: How would you price and risk-manage a CDO^2?

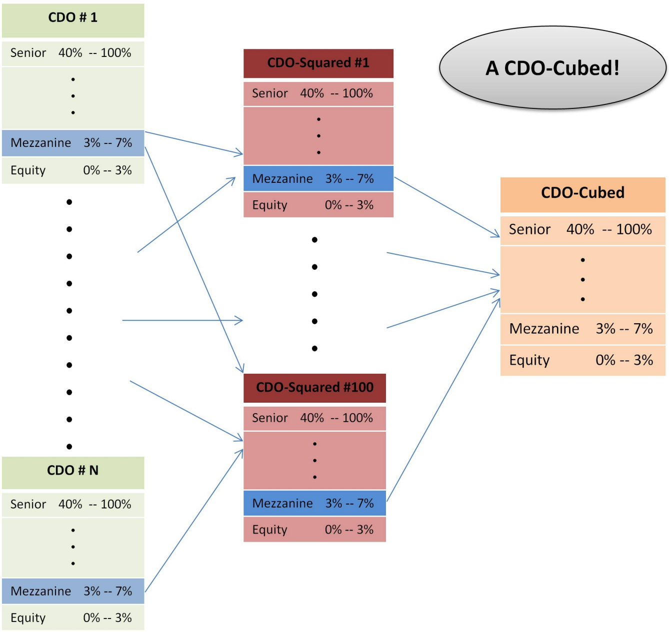
Some considerations ...

1. The legal contract governing each of the mezzanine tranches in the underlying portfolio of CDO's is ≈ 150 pages long
   - so only need to read approx $100 \times 150 = 15,000$ pages of legal documents
   - must also read the contract governing the CDO^2, of course.
2. How would you keep track of CDO^2 performance?
   - many thousands of lines of computer code required!

Question: How would we perform a scenario analysis?

Question: How would we estimate its VaR or CVaR?

But why stop there!? There are also ABS-CDO's, CDO-cubed's, a.k.a. CDO^3 and more ...



CDO # 1
Senior 40% -- 100%
Mezzanine 3% -- 7%
Equity 0% -- 3%
CDO # N
Senior 40% -- 100%
Mezzanine 3% -- 7%
Equity 0% -- 3%
CDO-Squared #1
Senior 40% -- 100%
Mezzanine 3% -- 7%
Equity 0% -- 3%
CDO-Squared #100
Senior 40% -- 100%
Mezzanine 3% -- 7%
Equity 0% -- 3%
A CDO-Cubed!
CDO-Cubed
Senior 40% -- 100%
Mezzanine 3% -- 7%
Equity 0% -- 3%