Financial Engineering and Risk Management

Liquidity, trading costs and portfolio execution

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What is liquidity?

A liquid security is one that

- can be traded quickly (quick execution),
- with little price impact (low slippage),
- and in large quantities (deep order book)

Measures of liquidity

- volume based measures: Trading volume, Turnover ($= \frac{\text{Trading volume}}{\text{Shares outstanding}}$)
- cost based measures: Percentage bid-ask spread, VWAP (volume weighted average price), Loeb price impact function, Kissel-Glantz price impact function

Trading cost function

Typically assumed to be separable across assets: ignores cross-asset price impact.

Kissel-Glantz function: Cost $c(Q)$ of trading Q shares

$$c(Q) = \left(a_1 \left(\frac{100\,|Q|}{V} \right)^{\beta} + a_2 \sigma_i + a_3 \right) |PQ|$$

where

- V = average daily traded volume of the asset
- σ = volatility of the asset
- P = price/share of the asset before the trade

Estimated using regression of the form:

$$\frac{c(Q_t)}{|P_t Q_t|} = a_1 \left(\frac{100\,|Q_t|}{V_t} \right)^{\beta} + a_2 \sigma_t + a_3 + \epsilon$$

where Q_t is volume traded and P_t is the price per share before trade at time t

Liquidity and portfolio selection

Two approaches

- Do usual portfolio selection and account for liquidity in executing trades
- Incorporate liquidity concerns directly into portfolio selection
- Best practice: Do both ...

Mean-variance-liquidity optimization problem: current position $\boldsymbol{y} \in \mathbb{R}^n$, $n = \#$stocks

$$\begin{array}{ll} \max & \boldsymbol{\mu}^\top \boldsymbol{x} - \frac{\lambda}{2} \cdot \boldsymbol{x}^\top \boldsymbol{V} \boldsymbol{x} - \eta \cdot C(\boldsymbol{x}, \boldsymbol{y}) \\ \text{s.t.} & \mathbf{1}^\top \boldsymbol{x} = \mathbf{1}^\top \boldsymbol{y} \end{array}$$

Trading cost

$$\begin{aligned} C(\boldsymbol{x}, \boldsymbol{y}) &= \sum_{i=1}^{n} \left(a_1 \left(\frac{100\,|x_i - y_i|}{P_i V_i} \right)^{\beta} + a_2 \sigma_i + a_3 \right) |x_i - y_i| \\ &= \sum_{i=1}^{n} a_1 \left(\frac{100}{P_i V_i} \right)^{\beta} |x_i - y_i|^{1+\beta} + (a_2 \sigma_i + a_3)\, |x_i - y_i| \end{aligned}$$

Show how to solve this optimization problem in `trading.xlsx`.

Another approach

Reference: Lo, Petrov and Wierzbicki. "It's 11pm – Do you know where your liquidity is? The mean-variance-liquidity frontier"

Let $\tilde{\ell}_{it}$ denote a measure of liquidity where high values implies more liquidity.

- Normalized liquidity measure

$$\ell_i = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{\tilde{\ell}_{it} - \min_{i',t'} \tilde{\ell}_{i't'}}{\max_{i',t'} \tilde{\ell}_{i't'} - \min_{i't'} \ell_{i't'}}\right)$$

Assume all wealth in cash. Formulate three different optimization problems

- Liquidity filtered portfolio selection

$$\begin{array}{ll} \text{max} & \boldsymbol{\mu}^\top \boldsymbol{x} - \frac{\lambda}{2}\cdot \boldsymbol{x}^\top \boldsymbol{V}\boldsymbol{x} \\ \text{s.t.} & \mathbf{1}'\boldsymbol{x} = 1, \quad x_i = 0, i \in \{k : \ell_k \geq \bar{L}\} \end{array}$$

- Mean-variance-liquidity objective

$$\begin{array}{ll} \text{max} & \boldsymbol{\mu}^\top \boldsymbol{x} - \frac{\lambda}{2}\cdot \boldsymbol{x}^\top \boldsymbol{V}\boldsymbol{x} + \eta \cdot \sum_{i=1}^{n} \ell_i \, |x_i| \\ \text{s.t.} & \mathbf{1}'\boldsymbol{x} = 1 \end{array}$$

Lo et al approach (contd)

- Liquidity constrained portfolio

$$\begin{array}{rl} \max & \boldsymbol{\mu}^\top \boldsymbol{x} - \frac{\lambda}{2} \cdot \boldsymbol{x}^\top \boldsymbol{V} \boldsymbol{x} \\ \text{s.t.} & \mathbf{1}'\boldsymbol{x} = 1, \quad \sum_{i=1}^{n} \ell_i \frac{|x_i|}{\sum_j |x_j|} \geq \bar{L} \end{array}$$

Note that the expression for overall liquidity of the portfolio is different here. This is to prevent short positions in illiquid stocks canceling long positions in (other) illiquid stocks.

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Optimal Execution

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Optimal Execution of Single Stock

Goal: Sell a total of X shares over at most T trades

Trading strategy

- n_j = shares sold in the j-th trade.
- $\boldsymbol{n} = (n_1, \ldots, n_T)$ = execution sequence.
- $X = \sum_{j=1}^{T} n_j$
- $x_0 = X$, $x_k = X - \sum_{j=1}^{k} n_j$ = holdings at the end of j-th trade

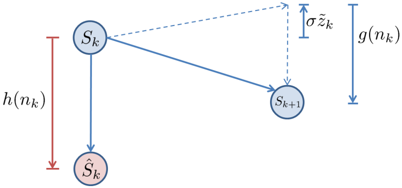
Trades impact the price

- $\hat{S}_k$ = price per share received for the n_k shares sold in the k-th trade
- $\hat{S}_k$ is a function of the trades $(n_1, \ldots, n_k)$.
- Note that it depends on the trade n_k as well.

Price impact model

Price impact has two components:

- Temporary price impact: impact of a trade n_k on its own price per share $\hat{S}_k$
- Permanent price impact: impact of a trade n_k on future prices $\hat{S}_\ell$, $\ell > k$



Price increases on buy orders.

- S_k = price just before k-th trade
- $\hat{S}_k$ = price for the k-th trade

Temporary price impact function $h(n)$

$$\hat{S}_k = S_k - h(n_k)$$

Permanent price impact function $g(n)$

$$S_{k+1} = S_k + \sigma z_k - g(n_k)$$

$\{z_k\}$ IID standard Normal random variables.

Total revenue from execution

Capture of the trading strategy: total revenue of the trading strategy

$$\begin{aligned}
\sum_{k=1}^{T} \hat{S}_k n_k &= \sum_{k=1}^{T} \left(S_k - h(n_k)\right) n_k \\
&= \sum_{k=1}^{T} \Big(S_1 + \sum_{j=1}^{k-1} \left(\sigma \tilde{z}_j - g(n_j)\right)\Big) n_k - \sum_{k=1}^{T} h(n_k) n_k \\
&= S_1 \Big(\sum_{k=1}^{T} n_k\Big) + \sigma \sum_{k=1}^{T} \tilde{z}_k x_k - \sum_{k=1}^{T} g(n_k) x_k - \sum_{k=1}^{T} h(n_k) n_k
\end{aligned}$$

Expected cost (or slippage) of the trading strategy:

$$C(\boldsymbol{n}) = \sum_{k=1}^{T} g(n_k) x_k + \sum_{k=1}^{T} h(n_k) n_k$$

Risk ($\equiv$ variance) of the trading strategy: $V(\boldsymbol{n}) = \sigma^2 \sum_{k=1}^{T} x_k^2$

Optimal execution frontier

Optimal execution optimization problem

$$\min_{\boldsymbol{n}} \; C(\boldsymbol{n}) + \rho \cdot V(\boldsymbol{n})$$

The time horizon T is also an important variable to be optimized.

Typical choices for g and h

- Linear permanent price impact: $g(v) = \gamma v$
- Nonlinear temporary price impact: Kissel-Glantz function

$$h(n) = \left(a_1 \left(\frac{100\,|n|}{V} \right)^{\beta} + \left(a_2 \sigma_i + a_3 \right) \right) \mathsf{sign}(n)$$

Show how to solve this optimization problem in `trading.xlsx`.

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Portfolio Execution

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Portfolio execution

Goal: Move from portfolio $\boldsymbol{x} \in \mathbb{R}^n$ to portfolio $\boldsymbol{y} \in \mathbb{R}^n$ over T trades

- The trade horizon T is a variable.
- Effect of cross-asset price impact.
- Trade-off between market and limit orders.
- Impact of dark pools.
- Want the portfolio to remain balanced.

All of these aspects are not fully understood.

Similar issues come up in high frequency trading

- Where to place orders in the limit book?
- How to manage the risk of the inventory (portfolio)?

Cross-asset price impact

There is increasing evidence of cross-asset price impact

- trades in one asset influence the prices of other (even unrelated) assets.

Models for cross-asset price impact

- Market-makers attempt to learn the fundamental value of one asset from the order flow of related assets.
- Speculators strategically trade in several assets in order to reduce trading costs, manage risk, and the direct price impact.
- Market-makers respond to the speculators strategy into account when setting prices.

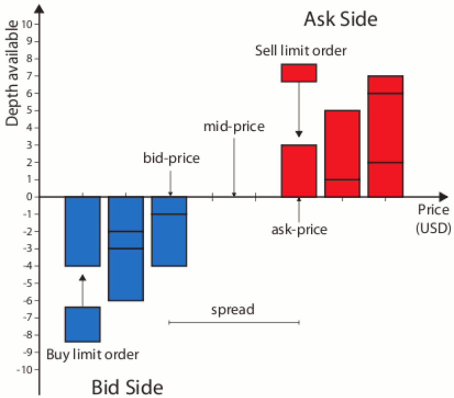
Cross-asset price impact (contd)

Some features of the cross-asset price impact

- cross-price impact is often negative.
- both direct and cross-price impact are smaller when speculators are more numerous.
- greater when marketwide dispersion of beliefs is higher.
- greater among stocks dealt by the same specialist

Market vs Limit Orders

A limit book is a database that keeps orders, (i.e. buy orders or bids, and sell orders or offers) on price-time-priority basis.



Market vs Limit Orders

A market order picks up orders from the limit order starting from the bid or offer price, i.e. the best buy or sell order in the limit book.

- Immediate execution
- Low revenue from selling and high cost for buying

A limit order places the order on the limit book at a specified price

- Execution time is uncertain
- Revenue (resp. cost) is higher (resp. lower)

Whether or where to place the limit order is a function of:

- patience: opportunistic vs. immediate trade
- volatility of the stock price
- discount given to liquidity providers

Lit vs dark pools

Dark pools of liquidity are venues where blocks of shares can be bought or sold without revealing either the size of the trade or the identity until the trade is filled. By contrast, regular exchanges are called lit pools.

Consequently, dark pool trading avoiding market impact. But the volume of trade executed is uncertain.

Dark pools have recently attracted controversy.

- By some estimates dark pools are attracting $11 - 12\%$ of the volume of certain securities. This is impacting price discovery, and publicly traded prices may no longer be "fair".
- Dark pools have a winner's curse problem – a buyer who buys a big block in a dark pool is better off letting the volume come to an exchange and buy it after the price impact.
- In 2009 the SEC introduced new measures to increase the transparency of dark pools, "so investors get a clearer view of stock prices and liquidity"