Financial Engineering and Risk Management Part II

A Brief Introduction to Real Options and the Simplico Gold Mine

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The term "real options" is often used to describe investment situations involving non-financial, i.e. *real*, assets together with some degree of *optionality*. For example, an industrialist who owns a factory with excess capacity has an option to increase production that she may exercise at any time. This option might be of particular value when demand for the factory's output increases. The owner of an oilfield has an option to drill for oil that he may exercise at any time. In fact since he can drill for oil in each time period he actually holds an entire series of options. On the other hand if he only holds a lease on the oilfield that expires on a specified date, then he holds only a finite number of drilling options. For a final example, consider a company that is considering investing in a new technology. Taken in isolation, this investment may have a negative net present value (NPV) so that it does not appear to be worth pursuing. However, it may be the case that investing in this new technology affords the company the option to develop more advanced and profitable technology at a later date. As a result, the investment might ultimately be a positive NPV value project that is indeed worth pursuing. These types of investment opportunities are often called *real options* and economics and financial engineering provide useful guidance towards valuing these investments.

The real options example that we consider here is the *Simplico* gold mine example from David Luenberger's *Investment Science*. The goal is to: (a) value a lease on a gold mine and (b) value an option to increase the rate at which gold can be extracted from the mine.

The Simplico Gold Mine

Gold can be extracted from the simplico gold mine at a rate of up to $10,000$ ounces per year at a cost of $\$200$ per ounce. The current market price of gold is $\$400$ and it fluctuates randomly in such a way that it increases each year by a factor of 1.2 with probability $.75$ or it decreases by a factor of $.9$ with probability $.25$, i.e. gold price fluctuations are described by a binomial model with each period corresponding to 1 year. Interest rates are flat at 10% per year. We want to compute the price of a lease on the gold mine that expires after 10 years. It is assumed that any gold that is extracted in a given year is sold at the end of the year at the price that prevailed at the beginning of the year. The gold price binomial lattice is given below.

Gold Price Lattice

											2476.7
										2063.9	1857.5
									1719.9	1547.9	1393.1
								1433.3	1289.9	1161.0	1044.9
							1194.4	1075.0	967.5	870.7	783.6
						995.3	895.8	806.2	725.6	653.0	587.7
					829.4	746.5	671.8	604.7	544.2	489.8	440.8
				691.2	622.1	559.9	503.9	453.5	408.1	367.3	330.6
			576.0	518.4	466.6	419.9	377.9	340.1	306.1	275.5	247.9
		480.0	432.0	388.8	349.9	314.9	283.4	255.1	229.6	206.6	186.0
	400.0	360.0	324.0	291.6	262.4	236.2	212.6	191.3	172.2	155.0	139.5
Date	0	1	2	3	4	5	6	7	8	9	10

The risk-neutral probabilities are found to be $q = 2/3$ and $1 - q = 1/3$. The value of the lease is then computed by working backwards in the lattice below. Because the lease expires worthless the node values at $t = 10$ are all zero. The value 16.9 on the uppermost node at $t = 9$, for example, is obtained by discounting the profits earned at $t = 10$ back to the beginning of the year. We therefore obtain $16.94 = 10,000(2063.9 - 200)/1.1$. The value at a node in any earlier year is obtained by discounting the value of the lease *and* the profit obtained at the end of the year back to the beginning of the year and adding the two quantities together. For example, in year 6 the central node has a value of 12.6 million and this is obtained[1] as

$$12,000,000 \;=\; \frac{10,000(503.9 - 200)}{1.1} \;+\; \frac{q \times 11,500,000 + (1-q) \times 7,400,000}{1.1}.$$

Note that in any year when the price of gold is less than $\$200$, it is optimal to extract no gold and so no profits are recorded for that year. We find the value of the lease at $t = 0$ is 24.1 million.

Lease Value (in millions)

											0.0
										16.9	0.0
									27.8	12.3	0.0
								34.1	20.0	8.7	0.0
							37.1	24.3	14.1	6.1	0.0
						37.7	26.2	17.0	9.7	4.1	0.0
					36.5	26.4	18.1	11.5	6.4	2.6	0.0
				34.2	25.2	17.9	12.0	7.4	3.9	1.5	0.0
			31.2	23.3	16.7	11.5	7.4	4.3	2.1	0.7	0.0
		27.8	20.7	15.0	10.4	6.7	4.0	2.0	0.7	0.1	0.0
	24.1	17.9	12.9	8.8	5.6	3.2	1.4	0.4	0.0	0.0	0.0
Date	0	1	2	3	4	5	6	7	8	9	10

Suppose now that it is possible to enhance the extraction rate to $12,500$ ounces per year by purchasing new equipment that costs $\$4$ million. Once the new equipment is in place then it remains in place for all future years. Moreover the extraction cost would also increase to $\$240$ per year with the enhancement in place and at the end of the lease the new equipment becomes the property of the original owner of the mine. The owner of the lease therefore has an *option* to install the new equipment at any time and we wish to determine the value of this option. To do this, we first compute the value of the lease **assuming that the new equipment is in place at** $t = 0$. This is done in exactly the same manner as before and the values at each node and period are given in the lattice below.

[1] We have rounded the numbers to one decimal place to make the lattice easier to read. Of course the associated Excel workbook, *simplico.xlsx*, does not do this.

Lease Value Assuming Enhancement in Place (in millions)

											0.0
										20.7	0.0
									33.9	14.9	0.0
								41.4	24.1	10.5	0.0
							44.8	29.2	16.8	7.2	0.0
						45.2	31.2	20.0	11.3	4.7	0.0
					43.5	31.0	21.0	13.2	7.2	2.8	0.0
				40.4	29.3	20.4	13.4	8.0	4.1	1.4	0.0
			36.4	26.6	18.7	12.5	7.7	4.1	1.8	0.4	0.0
		31.8	23.3	16.3	10.8	6.5	3.4	1.3	0.2	0.0	0.0
	27.0	19.5	13.5	8.6	4.9	2.3	0.8	0.1	0.0	0.0	0.0
Date	0	1	2	3	4	5	6	7	8	9	10

We see that the value of the lease at $t = 0$ assuming that the new equipment is in place (the $\$4$ million cost of the new equipment has not been subtracted) is $\$27$ million. We now value the **option** to install the new equipment as follows. We construct yet another lattice (shown below) that, starting at $t = 10$, assumes the new equipment is **not** in place, i.e. we begin by assuming the *original* equipment and parameters apply. We work backwards in the lattice, computing the value of the lease at each node as before but now with one added complication: after computing the value of the lease at a node we compare this value, A say, to the value, B say, at the corresponding node in the lattice where the enhancement was assumed to be in place. If $B - \$4$million $\geq A$ then it is optimal to install the equipment at this node, if it has not already been installed. We also place $\max(B - \$4m,\ A)$ at the node in our new lattice. We continue working backwards in this manner, determining at each node whether or not the new equipment should be installed if it hasn't been installed already. The new lattice is displayed below.

Lease Value with Option for Enhancement (in millions)

											0.0
										16.9	0.0
									29.9*	12.3	0.0
								37.4*	20.1*	8.7	0.0
							40.8*	25.2*	14.1	6.1	0.0
						41.2*	27.2*	17.0	9.7	4.1	0.0
					39.5*	27.0*	18.1	11.5	6.4	2.6	0.0
				36.4*	25.6	17.9	12.0	7.4	3.9	1.5	0.0
			32.6	23.5	16.7	11.5	7.4	4.3	2.1	0.7	0.0
		28.6	20.9	15.0	10.4	6.7	4.0	2.0	0.7	0.1	0.0
	24.6	18.0	12.9	8.8	5.6	3.2	1.4	0.4	0.0	0.0	0.0
Date	0	1	2	3	4	5	6	7	8	9	10

We see that the value of the lease with the option is $\$24.6$ million, slightly greater than the value of the lease without the option. Therefore the value of the option is approximately[2] $24.6 - 24.1 = \$500k$. Entries in the lattice marked with a '*' denote nodes where it is optimal to install the new equipment, i.e. to exercise the option, if it hasn't already been installed. Note that the values at these nodes are the same as the values at the

[2] More precisely, the value is actually $\$558,595$ which can be seen by inspecting the *simplico.xlsx* Excel workbook.

corresponding nodes in the preceding lattice less $4 million.

The *simplico* gold mine is an interesting real options problem as all of the uncertainty or randomness comes from gold price uncertainty and is therefore **financial**. In practice, however, real options problems tend to have both financial and non-financial uncertainty. We could easily introduce non-financial uncertainty in the *simplico* example by introducing uncertainty regarding the amount of gold that can be extracted or by introducing stochastic storage or extraction costs for the gold.