Math

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1 Problem 1

a in all forms purely changes the shape and direction of the parabola. c in the expanded form is the y-intercept. d in the vertex form is the x coordinate of the vertex. h in vertex form is the y coordinate of the vertex. x_1 and x_2 in the factored form represent the two x-intercepts. Increasing b while a is positive moves the vertex of the parabola left while maintaining the same shape and y-intercept. Decreasing b while a is positive moves the vertex of the parabola right while maintaining the same shape and y-intercept. Increasing b while a is negative moves the vertex of the parabola right while maintaining the same shape and y-intercept. Decreasing b while a is positive moves the vertex of the parabola left.

2 Problem 2

A quadratic does not have a factored form if $4ac > b^2$

3 Problem 3

x_1 and x_2 are solutions to the equation $0 = ax^2 + bx + c$.

4 Problem 4

4.1 Expanded to Vertex

We start by expanding the $a(x-d)^2$ term:
$a(x-d)^2 = a(x^2 - 2xd + d^2)$
$= ax^2 - 2axd + ad^2$
The ax^2 is good and matches the expanded form. We want the $-2axd$ to equal $+bx$. We can do this by setting d to $\frac{-b}{2a}$.
Next for the h term. To get the h term we first want to remove the ad^2 from the previous expansion and add c. So $h = c - ad^2$. Since we already know $d = \frac{-b}{2a}$ we can plug that in and get $h = c - \frac{-ab^2}{4a^2}$. We can simplify to $h = c - \frac{-b^2}{4a} = \frac{4ac+b^2}{4a}$

4.2 Expanded to Factored

a is the same in both forms. x_1 and x_2 are the two solutions to the quadratic formula $\frac{-b\pm\sqrt{b^2-4ac}}{2a} = 0$.

4.3 Vertex to Expanded

a is the same in both forms. $b = -2da$ and $c = h + \frac{-ab^2}{4a^2}$.

4.4 Vertex to Factored

a is the same in both forms. x_1 and x_2 are the two solutions to $0 = a(x+d)^2+h$

4.5 Factored to Expanded

a is the same in both forms. $b = ax_1x_2$, $c = x_1x_2$.

4.6 Factored to Vertex

a is the same in both forms. We already know $d = \frac{-b}{2a}$ and $h = c - \frac{-ab^2}{4a^2}$. We also know $b = ax_1x_2$ and $c = x_1x_2$. If we plug these into the first to equations we get $d = \frac{-(ax_1x_2)}{2a} = \frac{-x_1x_2}{2a}$ and $h = x_1x_2 - \frac{-ab^2}{4a^2}$.

5 Problem 5

Let (x_0, y_0) be a solution to the equation $y = f(x)$. If you change the x to $x - s$, increasing x_0 by s to account for the decrease by s would also be a solution. Therefore $x_0 + s, y_0$ is a solution to $y = f(x - s)$. If you change y to $y - t$, if you added t to x_0 it would cancel out the t, therefore $(x_0, y_0 + t)$ is a solution. If we combine the two we find that $(x_0 + s, y_0 + t)$ is always a solution to $y - t = f(x - s)$ if (x_0, y_0) is a solution to $y = f(x)$. This means that if you increase s, every point moves right, and if you increase t every point moves up.