Measurement of Boltzmann's Constant Using Video Microscopy of Brownian Motion

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We calculated Boltzmann's constant k using video microscopy of Brownian motion to be within an order of magnitude from the accepted value. Using mean squared displacement of 3μm particles, a fitted linear model can be used to calculate k. We achieved an experimental value of $(5.01 \pm 0.87) \times 10^{-24} J/K$ compared to an accepted value of $1.38 \times 10^{-23} J/K$.

I. INTRODUCTION

The Boltzmann constant, denoted as k, is a fundamental constant in physics. Its precision holds importance for several applications, including nanotechnology and material science. Although there exists established methods for measuring Boltzmann's constant, we propose a method using video microscopy. In this experiment, we tracked the Brownian motion of Carboxylate 3μm Micron Microspheres and measured the displacement of 15 particles. There are 5 particles per trial for a total of 3 trials. Using the mean displacement of these particles, we are able to calculate k to be within a single order of magnitude from the accepted value[3].

II. THEORY

According to Paul Nakroshis's paper[2] on tracking Brownian motion using video microscopy, there are around 10^{11} collisions per second during each trial. Since this is faster than any camera can reasonably capture, it is safe to assume that there will be at least some collisions per frame. The goal of this experiment is to calculate Boltzmann's constant k using measurements of displacements from these collisions. It is important to note that each collision can be assumed to be a new independent event, as they are not related. The average displacement is affected by random impulses and a dissociative drag force. In one dimension the mean squared displacement can be modeled as:

$$\frac{d\langle x^2\rangle}{dt} = \frac{2kT}{\mu} \tag{1}$$

where μ is the linear drag coefficient and T is the temperature of the particles.

The particles are assumed to be perfect spheres, so μ can be calculated as the following:

$$\mu = 6\pi\eta a \tag{2}$$

where a is the radius of the sphere and η is the viscosity of the fluid. Since movements along the X and Y axes are independent, the mean displacement can be found using the following equation:

$$\langle R^2\rangle = \frac{4kT}{6\pi\eta a}t \tag{3}$$

Since $\langle R^2\rangle$ is proportional to $k \cdot t$, a plot of $\langle R^2\rangle$ vs t can be used to find an approximation for Boltzmann's constant.

III. EXPERIMENT SETUP

For this experiment, a Swift Sw380t 40x-2500x Magnification microscope is used to track Brownian motion. The microscope needs to be calibrated in the right range to see the desired 3μm particles. To accomplish this, a 0.001mm microscope calibration slide was inserted into the microscope and a 10/.025 level of magnification was used. The focus was then adjusted with the knob on the left until the ring tick marks could be seen. A diagram of our microscope is given below:

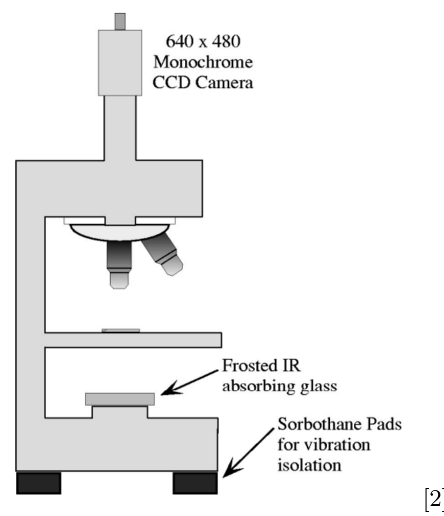
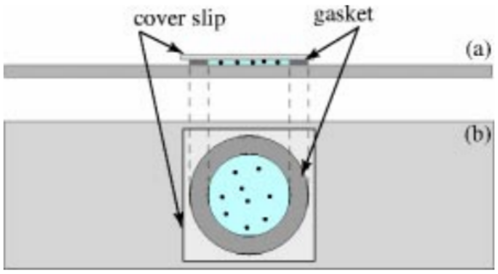


FIG. 1. A visual of the microscope. The camera was attached to a Raspberry Pi for collecting data.

A stable environment is needed to accurately track Brownian motion and prevent outside trembles. EVA foam was initially used for stabilization purposes, but we found that the microscope was more stable without the foam. Slides were then prepared to finish setting up the experiment, which can be seen in Figure 2.

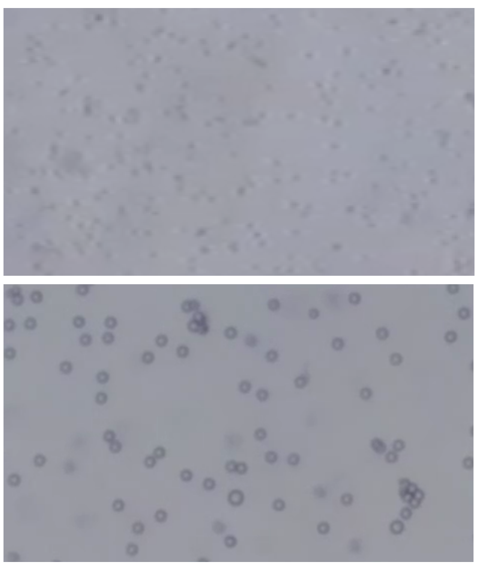
A proper magnification is needed to observe the particles clearly. The knob on the sides of the microscope is turned until the droplets are visible. This differences in



[2]

FIG. 2. Carboxylate $3\mu m$ micron microspheres were fed onto a 25.4mm by 76.2mm by 1mm thick slide using a pipette and covered by a 18mm by 18mm transparent cover. The slide was then carefully placed into the microscope apparatus so data can be taken.

magnification can be seen in Figure 3.



(a)

(b)

FIG. 3. Different levels of magnification affect resolution of the particles. Figure 3b was used in the experiment over Figure 3a.

IV. DATA EXTRACTION

Prior to tracking Brownian motion, temperature was collected, as it can affect Boltzmann's constant in both T and η. The temperature was collected close to the particle to approximate the temperature of the particles. To track Brownian motion, a Raspberry Pi application was used on a 32-bit Linux system to record videos of moving particles. Approximately thirty 30-second videos were taken. Due to resolution and stabilization of certain videos, 3 videos from the same set of droplets were chosen for further analysis. These videos are also chosen to avoid drift from the glass slides when they are initially inserted. After a few minutes, the particles will be less excited, and it is easier to detect Brownian motion. For measuring position of particles, OPS Tracker was used to auto-track positions of particles. The tracker application has units of meters, and a conversion factor can be found within the application given the size of the particles and the coordinates of particles. The data collection process can be completed once data files of X-Y positions of each particle were outputted into a comma-separated .txt file for further analysis.

V. ANALYSIS

After the X and Y positions of the particles are collected, they are analyzed using Python's statistical libraries. To obtain the Boltzmann constant k, mean squared displacement is calculated using equations 4 and 5.

$$[\Delta R_i(t)]^2 = [\Delta x_i(t)]^2 + [\Delta y_i(t)]^2 \tag{4}$$

$$\langle R^2 \rangle = \frac{1}{N}\sum_{i=1}^{N}[\Delta R_i(t)^2] \tag{5}$$

The change in positions, $\Delta x_i(t)$ and $\Delta y_i(t)$, are calculated as the current distance from its initial positions, that is $\Delta x_i(t) = x_i(t) - x_i(0)$ and $\Delta y_i(t) = y_i(t) - y_i(0)$, where i is a particular i^{th} particle.

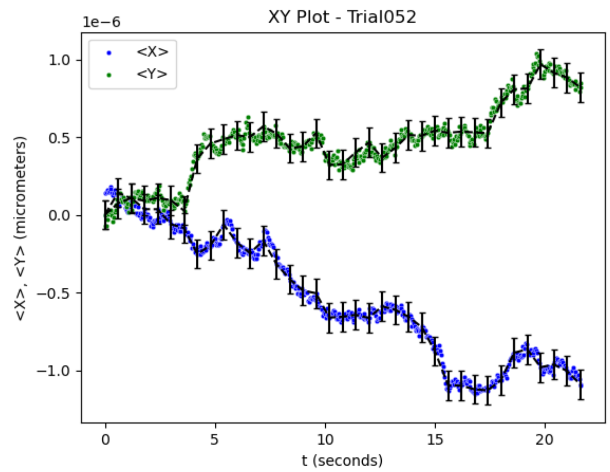


FIG. 4. Average values of X and Y (μm) values plotted against time (seconds) for Trial 052.

A single pixel is used as the measurement error for tracking values of X and Y. The resulting error bars can be seen in Figure 4. A line is then fitted onto a scatter plot of $\langle R^2 \rangle$ vs t using ordinary least square method, as seen in Figure 5.

The slope of the line is needed to calculate Boltzmann's constant using the equation below:

$$k = \frac{6\pi\eta a s}{4T} \tag{6}$$

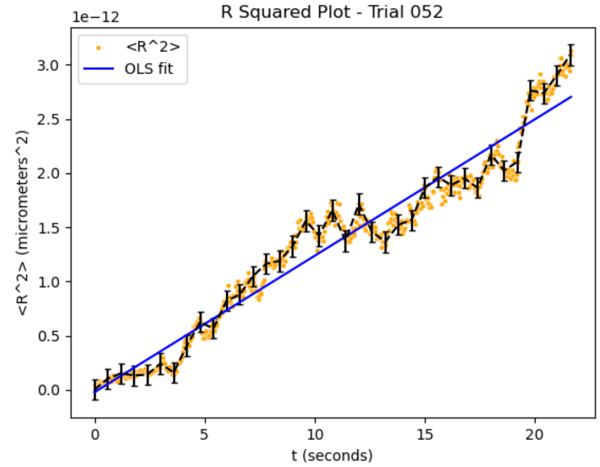


FIG. 5. $\langle R^2 \rangle$ (μm^2) values plotted against time (seconds) for Trial 052.

where s is the slope of the fitted line. The values used in the experiment are listed below:

$$\eta = 910 \pm 15 \mu Pas \tag{7}$$

$$a = 3 \pm 0.01 \mu m \tag{8}$$

$$T = 297.15 \pm 0.3 K \tag{9}$$

The value of η is based on temperature T and is given by Kestin's chemistry paper[1]. The results from the three trials are then aggregated to find the average value for s:

$$s = 0.12 \pm 0.02 \mu m^2/s \tag{10}$$

There are a few systemic errors such as shaking of apparatus and temperature change. To reduce such errors, we stabilize the apparatus as best as we could. We did not observe huge vibrations in the videos so the shaking of the apparatus did not seem to affect our experimental results profusely. Data was also collected in a stable temperature room within 10 minutes of each trial, so the results are likely not affected by a sudden change in temperature. There are, however, measurement errors in the video microscopy and the various variables in the experiment, which can affect the results. Such errors are alluded to in Figure 4, and accounted for using the equation below:

$$\left(\frac{\Delta k}{k}\right)^2 = \left(\frac{\Delta \eta}{\eta}\right)^2 + \left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta s}{s}\right)^2 + \left(\frac{\Delta T}{T}\right)^2 \tag{11}$$

Using equation 6 to solve for k and converting units to J/K, an mean experimental value for Boltzmann constant k is achieved: $(5.01 \pm 0.87) \times 10^{-24} J/K$.

VI. CONCLUSION

The experimental value for Boltzmann's constant k using video microscopy of Brownian motion is $(5.01 \pm 0.87) \times 10^{-24} J/K$. Our results are 64% away, and within an order of magnitude from the accepted value of $1.38 \times 10^{-23} J/K$[3]. Though the results of our experiment is within an order of magnitude of the accepted value, the video microscopy method can be prone to many systemic and measurement error. Various related work can be a better substitute for measuring Boltzmann's constant k with better accuracy and less error.

ACKNOWLEDGMENTS

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[1] Kestin, J., Sokolov, M., and Wakeham, W. A. Viscosity of liquid water in the range 8°C to 150°C. *Journal of Physical and Chemical Reference Data 7*, 3 (07 1978), 941–948.

[2] Nakroshis, P., Amoroso, M., Legere, J., and Smith, C. Measuring Boltzmann's constant using video microscopy of Brownian motion. *American Journal of Physics 71*, 6 (06 2003), 568–573.

[3] Pitre, L. New measurement of the boltzmann constant k by acoustic thermometry of helium-4 gas. *Metrologia 54*, 6 (2017).