A Measurement of Boltzmann's Constant using Video Microscopy of Brownian Motion

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Theory

- μ : linear drag coefficient
- η : coefficient of viscosity
- *a* : radius of particles
- *T* : temp. (K) of particles
- *k* : Boltzmann's constant

- Experiment:
  - Measure mean squared displacements of particles
  - *s* : <R^2>/t

Assuming particles are perfect spheres

$$\mu = 6\pi\eta a \tag{2}$$

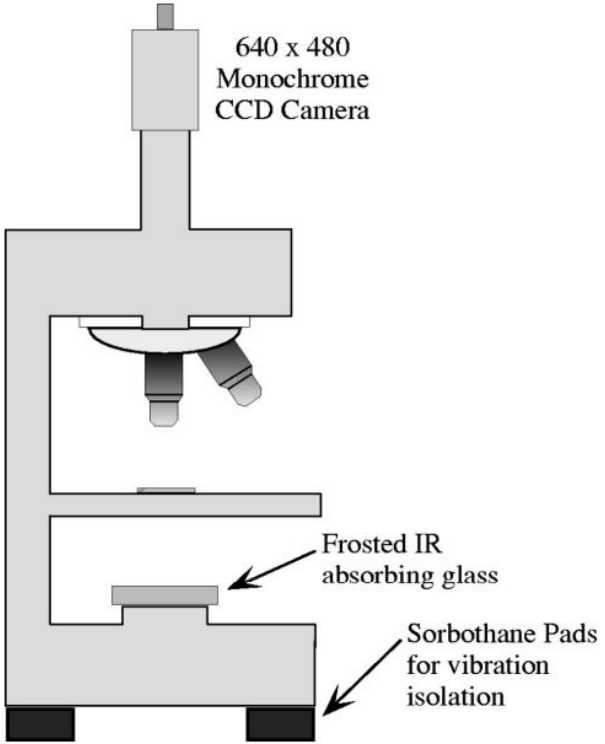
$$\frac{d\langle x^2\rangle}{dt} = \frac{2kT}{\mu} \tag{1}$$

$$\langle R^2\rangle = \frac{4kT}{6\pi\eta a}t \tag{3}$$

$$k = \frac{6\pi\eta as}{4T} \tag{6}$$

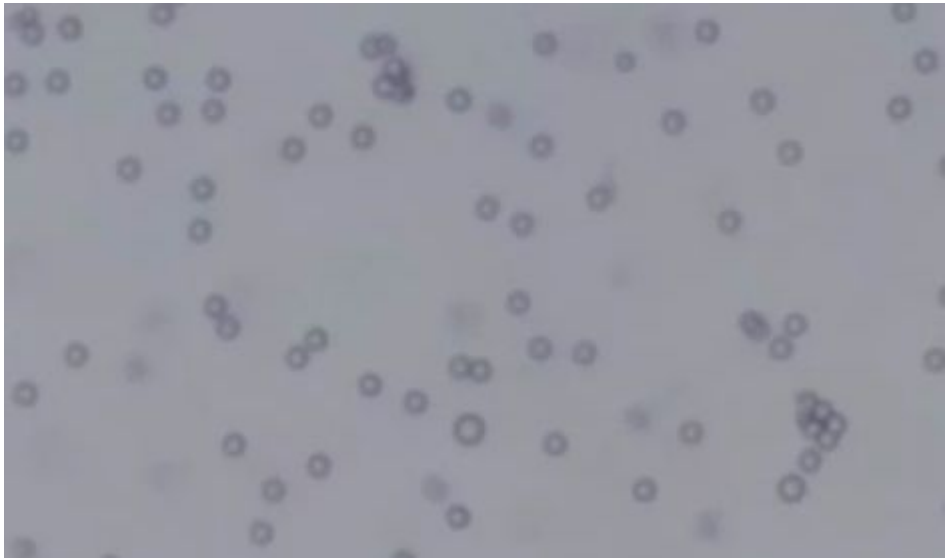
Experiment Setup

- Carboxylate 3μm micron microspheres on slide
- Swift Sw380t 40x-2500x magnification microscope
- Raspberry Pi to project videos of moving particles onto linux computer
- Saved videos for further analysis



Data Extraction

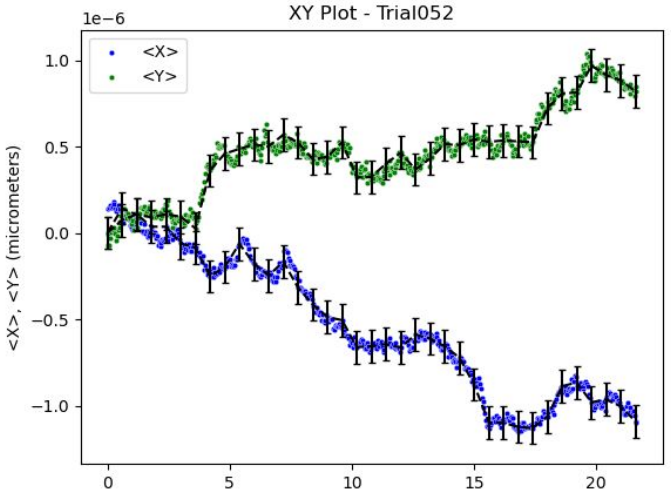
- Using recorded videos
- Tracker used to extract X-Y coordinates
- Export data for further analysis and linear model on Python

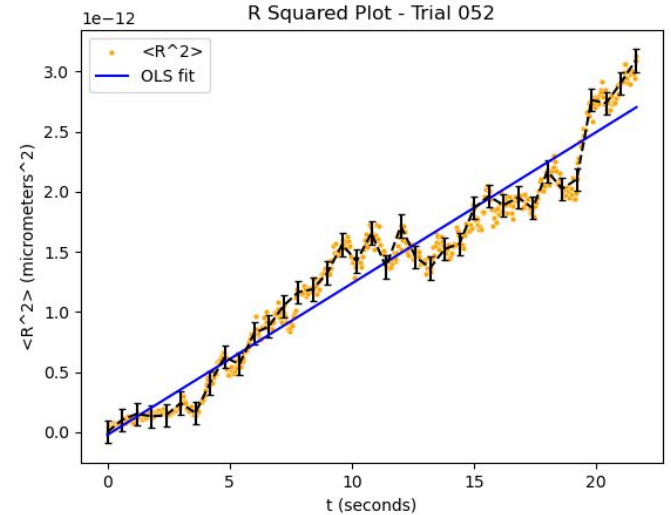


Results

Pitre's value: 1.38 × 10^−23 J/K *

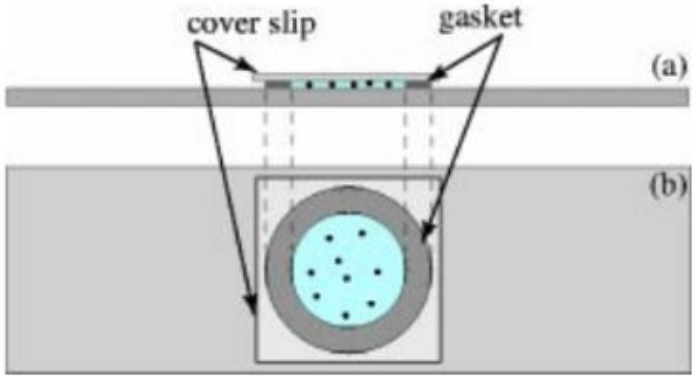
Average experimental value: (5.01 ± 0.87) × 10^−24 J/K over 3 trials





*Pitre, L. New measurement of the boltzmann constant k by acoustic thermometry of helium-4 gas. *Metrologia* 54, 6 (2017).

Thank you!
Q+A?



[2]

FIG. 2. Carboxylate $3\mu m$ micron microspheres were fed onto a 25.4mm by 76.2mm by 1mm thick slide using a pipette and covered by a 18mm by 18mm transparent cover. The slide was then carefully placed into the microscope apparatus so data can be taken.

$$\eta = 910 \pm 15 \mu Pas \tag{7}$$
$$a = 3 \pm 0.01 \mu m \tag{8}$$
$$T = 297.15 \pm 0.3K \tag{9}$$

The value of η is based on temperature T and is given by Kestin's chemistry paper[1]. The results from the three trials are then aggregated to find the average value for s:

$$s = 0.12 \pm 0.02 \mu m^2/s \tag{10}$$