Benchmarking Quantum Phase Estimation with Physical Hamiltonians

Lion in a Box

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Topics

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Our Criteria

Overview of our methodology

02

Phase Estimation Algorithms

Differences in the quantum circuits and their advantages

03

Physical Systems

And how we can utilize their structure for benchmarking

04

Conclusions

Implications of the benchmark data

01

Our Criteria

Criteria for Benchmarking

Depth - measured on the longest path of the circuit including initialization and subsequent iterations of U

Number of Gates - Total number of gates used in a circuit, including initialization and

Accuracy - measured by probability of obtaining ground state of different physical system (or associated θ)

02

Phase Estimation Algorithms

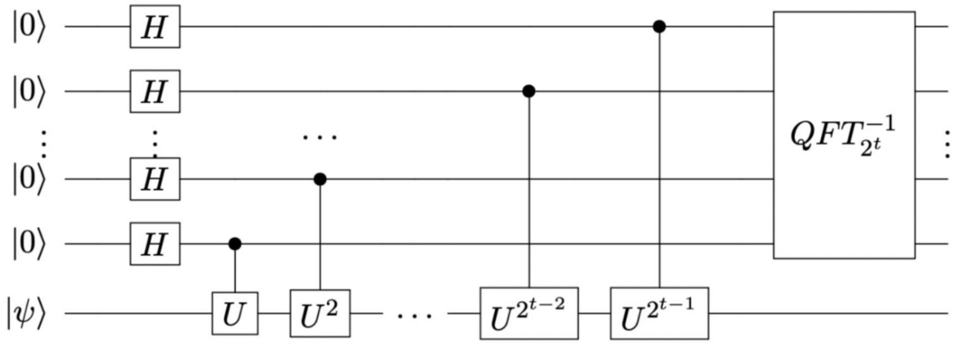
Standard Phase Estimation Algorithm

Overview:

Goal: Given unitary U and its eigenvector, approximate the phase of the eigenvalue, with theta between 0 and 1.

1. Apply H across m measurement qubits to create equal superposition
2. Controlled U operations -> Phase Kickback
3. Inverse QFT
4. Second H to extract phase

- O(2^n) circuit depth for n-bit precision



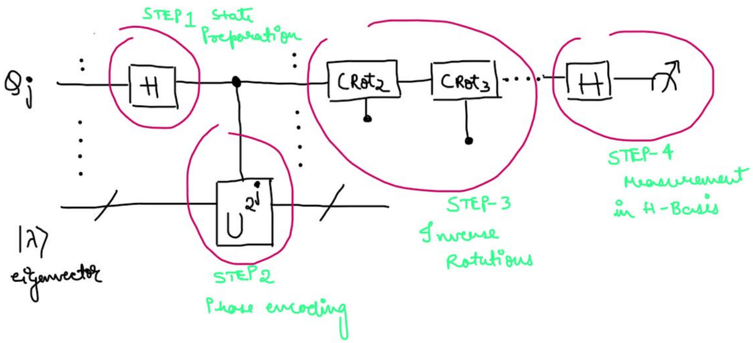
Iterative Phase Estimation Algorithm

Advantages:

Breaks up computation into *k* stages for precision *k*, decreasing the depth of the deepest quantum circuit required

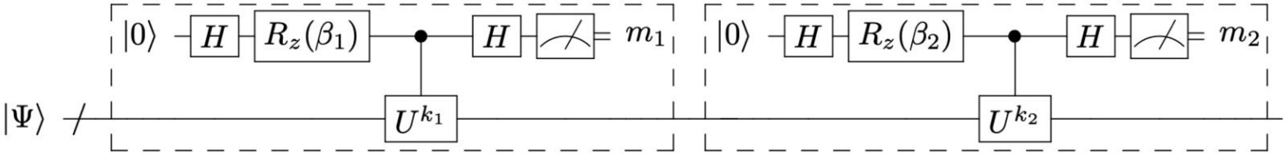
Able to achieve lower error rates on near-term quantum computers compared to standard phase estimation

Does not use an inverse Quantum Fourier Transform



Disadvantages

Computes digits in reverse order from least significant to most significant

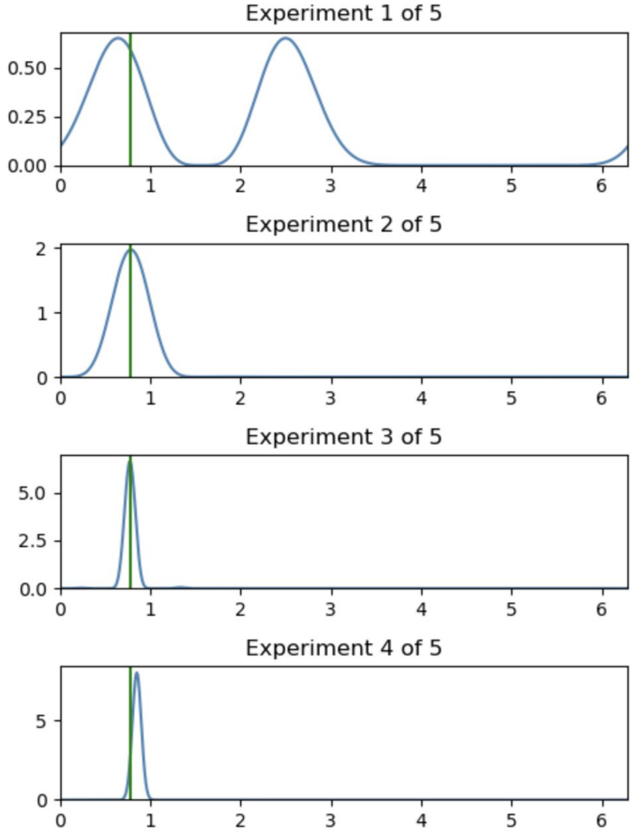


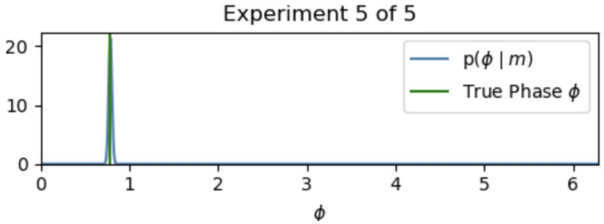
Bayesian Phase Estimation Algorithm

Advantages:
Noise Tamping through Modeling Methods - Depolarizing Noise, Measurement Noise

Less iterations by updating weights - Continuously calculating probability

Disadvantages:
Overhead - quantum/classical hybrid computations
Requires heavier classical post-processing



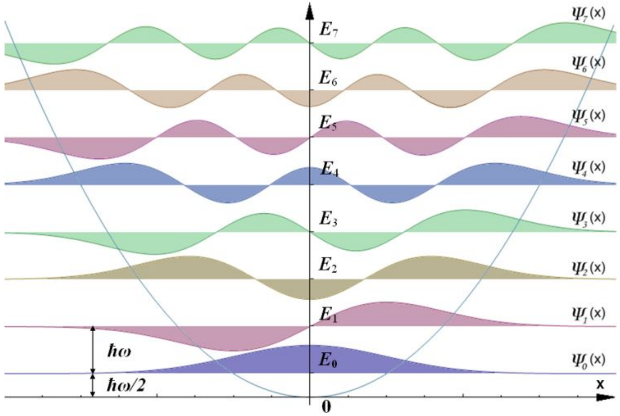


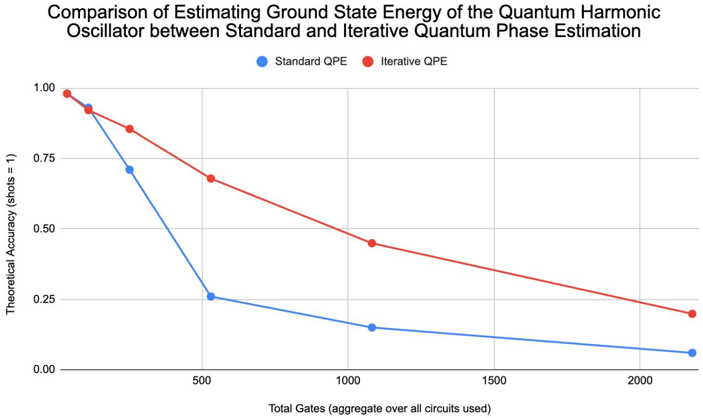
03

Physical Systems

Quantum Harmonic Oscillator Benchmark

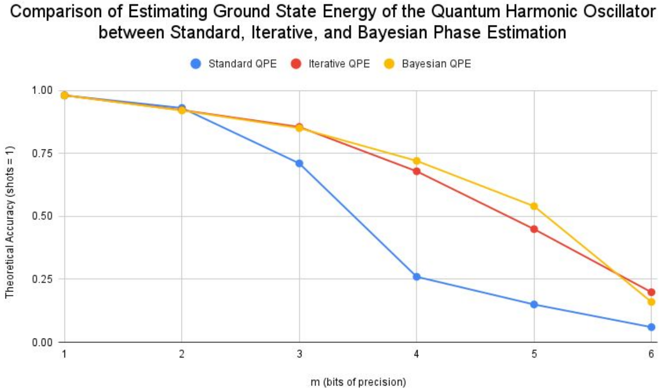
$$H\left|\psi\right\rangle = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + mw^2\hat{x}^2\right)\left|\psi\right\rangle = E\left|\psi\right\rangle$$



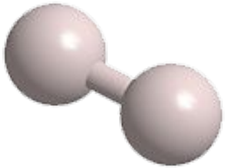


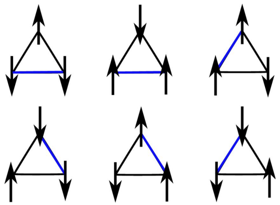
Unitary Time Evolution Operator For QHO

$$U = e^{\frac{-iHt}{\hbar}} = \begin{bmatrix} e^{\frac{-i\hbar\omega t}{2}} & 0 & \cdots & 0 \\ 0 & e^{\frac{-3i\hbar\omega t}{2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & e^{-i\hbar\omega(n+\frac{1}{2})t} \end{bmatrix}$$



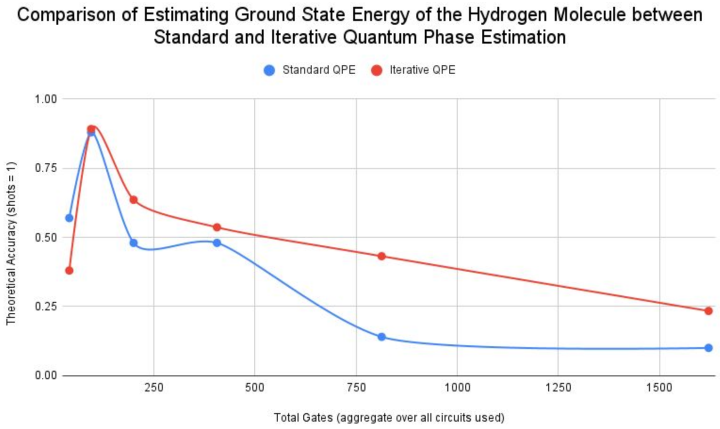
H_2 Molecule and Ising Model





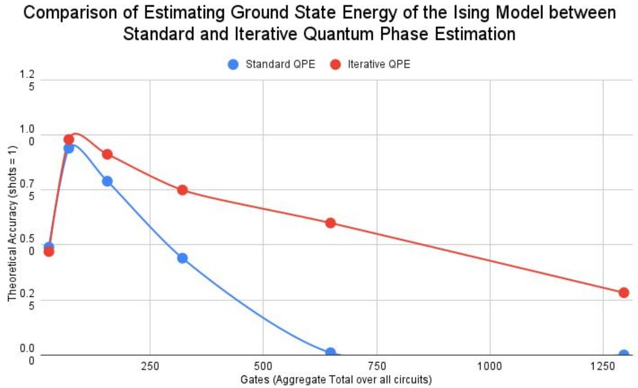
H_2 Molecule Hamiltonian

$$H = h_1 Z_1 + h_2 Z_2 + h_3 Y_1 Y_2 + h_4 Z_1 Z_2 + h_5 I.$$



Triangular Lattice Ising Model

$$H = -\sum_{<i,j>} J_{ij} Z_i Z_j - \sum_i g_i X_i$$



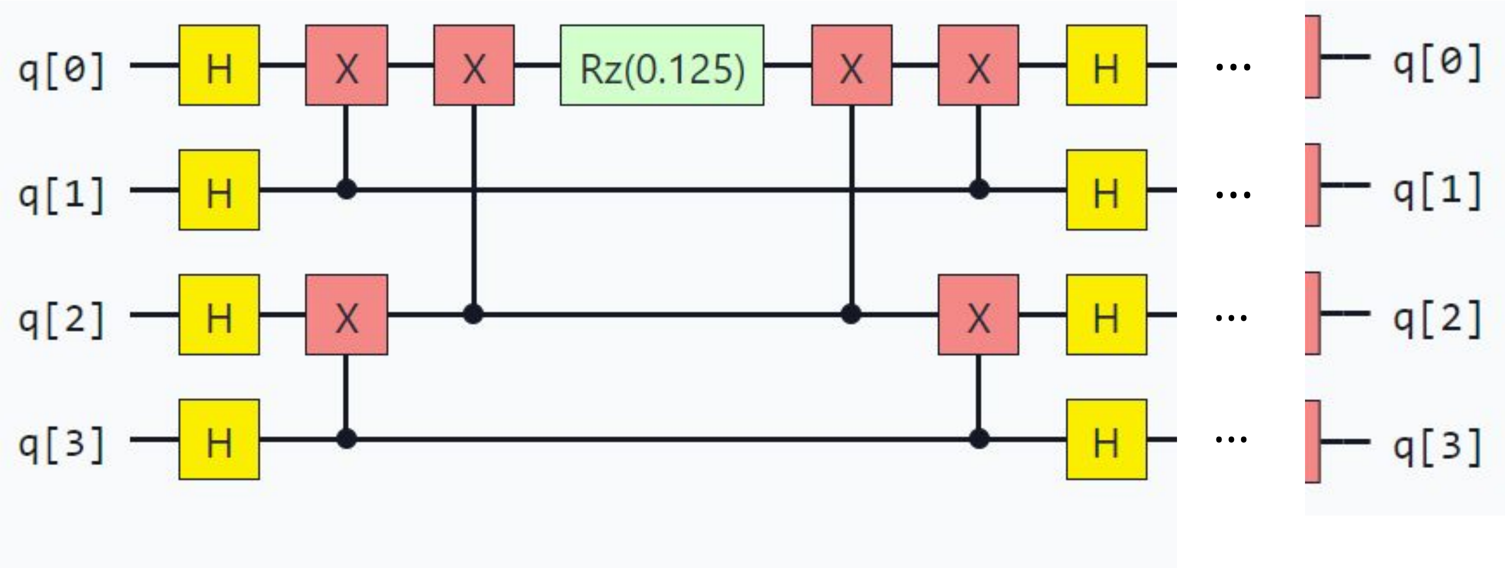
Improving Hamiltonian Simulation

Why this hamiltonian?

$$H = \frac{1}{8}\begin{pmatrix} X \otimes X \otimes X \otimes X - X \otimes X \otimes Y \otimes Y \\ +X \otimes Y \otimes X \otimes Y + X \otimes Y \otimes Y \otimes X \\ +Y \otimes X \otimes X \otimes Y + Y \otimes X \otimes Y \otimes X \\ -Y \otimes Y \otimes X \otimes X + Y \otimes Y \otimes Y \otimes Y \end{pmatrix}$$

Improving Hamiltonian Simulation

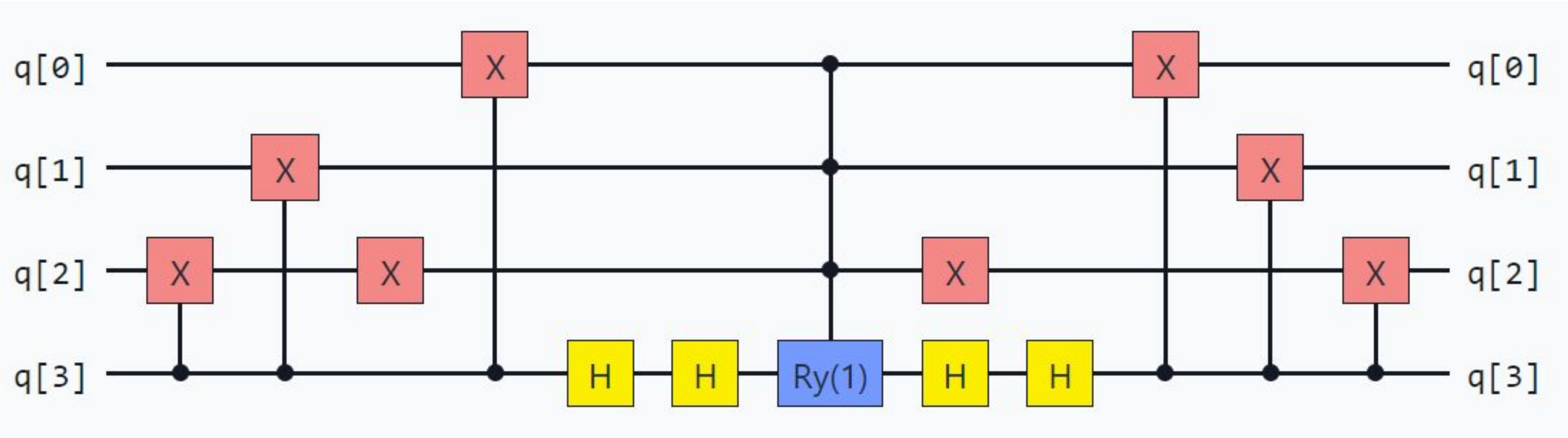
#A- Commutativity of Pauli Matrices + Basis change



Deep!

Improving Hamiltonian Simulation

#B- Outer Product and Rank Exploitation



Shallow

Improving Hamiltonian Simulation

Assumption: state preparation to an eigenstate

$$|0000> \longrightarrow |++++>$$

QFP gives null phase...
> Improve assumption on eigenstate of Hamiltonian

Thank you!