A Bayesian Probabilistic Model for Quantum State Tomography

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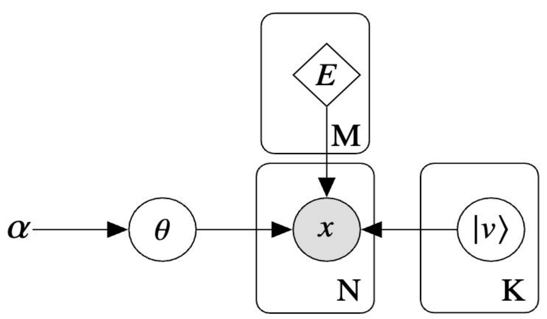
Background: Quantum State Tomography

Suppose we have a quantum device (i.e. quantum computer, laser) and we want to fully characterize how it deviates from its manufacturer's specifications, e.g. what experimental noise it produces. Quantum state tomography can help!

Input: *Many copies of an unknown quantum state + ability to perform arbitrary quantum measurements on them*
Output: *A reconstructed full description of the quantum state (in the form of a density matrix)*

We derive and implement an algorithm for quantum state tomography using the framework of **Bayesian probabilistic modeling with variational inference**.

Overview of Measurement Model



such that…

$$\theta \in \Delta^K \sim \mathrm{Dir}(\alpha)$$

$$\left|v^{(k)}\right\rangle \in \mathbb{C}^d \sim \mathrm{Unif}(\mathbb{C}^d/||\mathbb{C}^d||)$$

$$E_i \in \mathbb{C}^{d\times d}_{\mathrm{psd}} \text{ s.t. } \sum_{i=1}^{M} E_i = \mathbb{I}_d$$

$$\rho := \sum_{k=1}^{K} \theta_k \left|v^{(k)}\right\rangle \left\langle v^{(k)}\right| \in \mathbb{C}^{d\times d}_{\mathrm{psd}}$$

$$x_i \sim \mathrm{Cat}(\mathrm{Tr}[E_1\rho], \ldots, \mathrm{Tr}[E_M\rho])$$

Our model incorporates **Born's rule:** $Pr[E_k|\rho] = \mathrm{Tr}[E_k\rho]$

Methodology:

POVMs (Generalized measurements)

- Quantum states are underspecified by standard basis measurements. To address this, we use generalized quantum measurements called positive operator-valued measures (**POVMs**) that can take on an arbitrarily large range of outcome values.
- An *informationally-complete* (IC) set of POVMs of an *n*-qubit system requires having M>2^{2n} POVMs in the set. We experiment with **randomly generated** sets of POVMs of various sizes (different values of M) below the IC threshold.

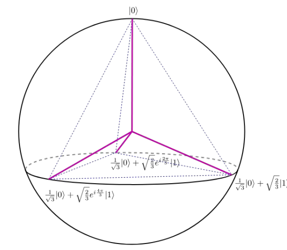
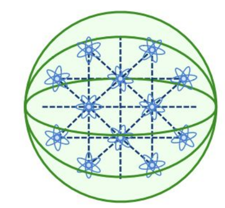


Figure 1: A visual representation of a POVM measurement set on 1 qubit.

Hyperparameters

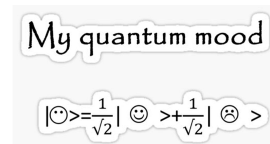
- We sample from a uniform prior over d-dimensional unit length complex vectors by treating the real and imaginary components as separate multivariate normal distributions and then combining and normalizing.
- Structural assumption: density matrices of our unknown states are **approximately low-rank** with a weighting determined by latent variable θ (specified with hyperparameters α (Dirichlet parameter) and K (maximum rank)).

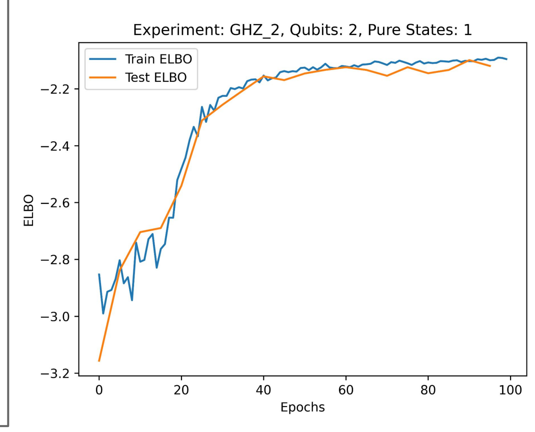


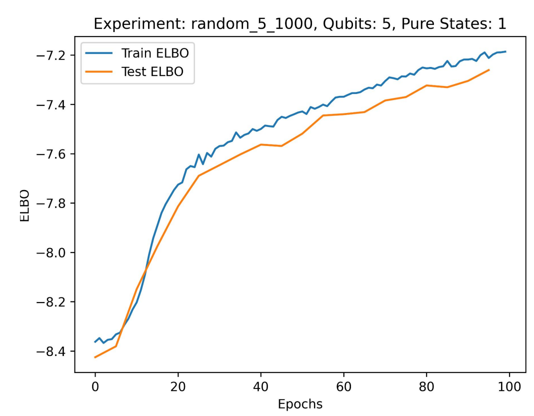
Quantum System

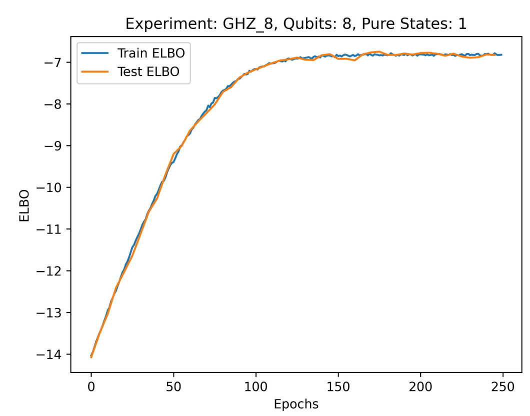
Variational Inference

- Used a mean-field normal variational family and the Adam optimizer to maximize the evidence lower bound (ELBO) and approximate the posterior distribution.



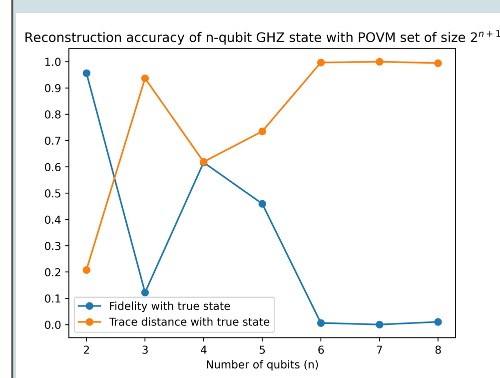


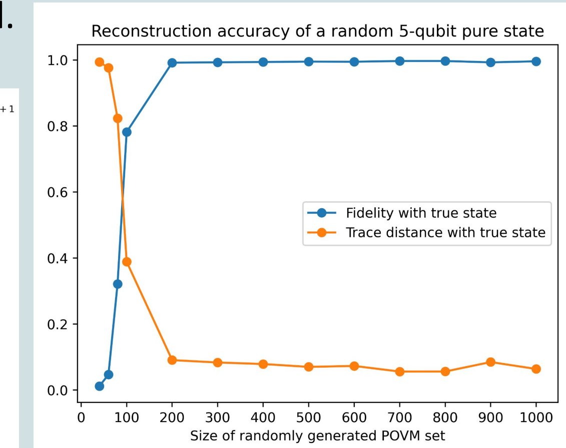




Results and Discussion:

- Our model reconstructs density matrices very well for *small* systems (# of qubits < 6).
- Computation time and optimization challenge become unmanageable for larger quantum states due to # of parameters growing exponentially with # of qubits.
- Though POVM sets of size >2^{2n} are needed for theoretical information-completeness, significantly smaller POVM sets were sufficient to attain >99% reconstruction fidelities in practice with our model.





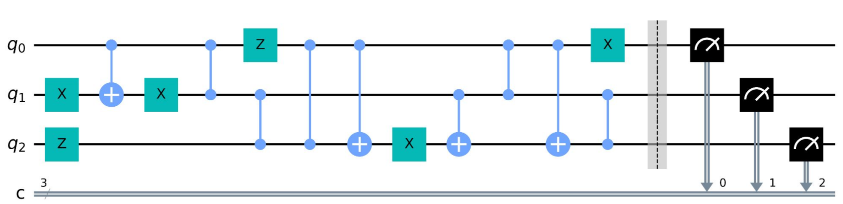
In Progress/Future Directions:

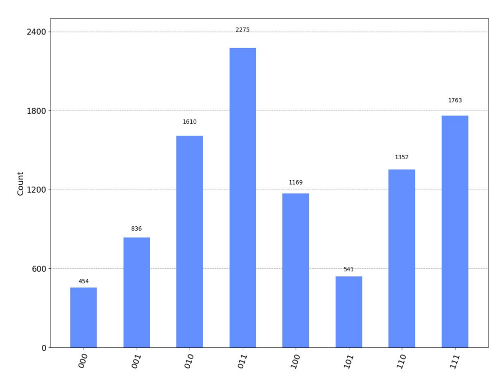
- Developing a more scalable probabilistic model for quantum state tomography, custom variational family
- Exploring MCMC as an alternative to VI, trying different priors
- Using data collected from real quantum computers
- Bayesian neural network quantum state tomography[1]

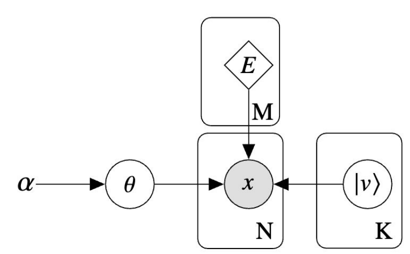
1. We run a (potentially noisy) quantum circuit using a simulator or on a real quantum computer with the IBM Quantum API, and get standard basis or POVM measurement statistics.

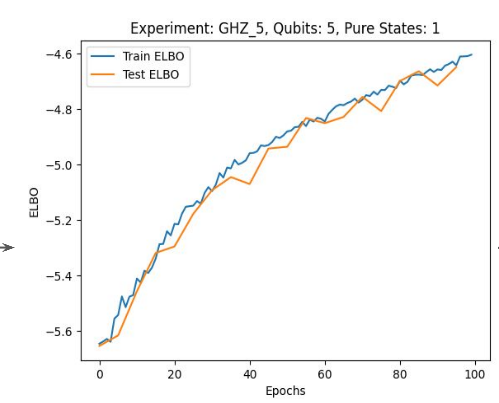
2. We use the measurements obtained as our observed data and do approximate posterior inference with the probabilistic model.

3. Using the posterior estimates of the latent variables, we reconstruct the density matrix of the quantum state, completing the tomography.









$$\hat{\rho} = \begin{bmatrix}
0.13 & -\frac{4}{25}-\frac{3i}{100} & \frac{7}{100}+\frac{i}{25} & \frac{1}{10}+\frac{9i}{100} & -\frac{3}{50}+\frac{19i}{100} & -\frac{1}{25}+\frac{i}{10} & -\frac{1}{20}-\frac{i}{50} & \frac{3}{100}-\frac{i}{20} \\
-\frac{4}{25}+\frac{3i}{100} & 0.21 & -\frac{1}{10}-\frac{i}{25} & -\frac{3}{20}-\frac{9i}{100} & \frac{3}{100}-\frac{i}{4} & \frac{1}{50}-\frac{7i}{50} & \frac{7}{100}+\frac{i}{50} & -\frac{3}{100}+\frac{7i}{100} \\
\frac{7}{100}-\frac{i}{25} & -\frac{1}{10}+\frac{i}{25} & \frac{3}{50} & \frac{9}{100}+\frac{i}{50} & \frac{3}{100}+\frac{13i}{100} & \frac{1}{100}+\frac{7i}{100} & -\frac{1}{25} & -\frac{i}{25} \\
\frac{1}{10}-\frac{9i}{100} & -\frac{3}{20}+\frac{9i}{100} & \frac{9}{100}-\frac{i}{50} & \frac{4}{25} & \frac{9}{100}+\frac{i}{5} & \frac{1}{20}+\frac{11i}{100} & -\frac{3}{50}+\frac{i}{50} & -\frac{1}{100}-\frac{3i}{50} \\
-\frac{3}{50}-\frac{19i}{100} & \frac{3}{100}+\frac{i}{4} & \frac{3}{100}-\frac{13i}{100} & \frac{9}{100}-\frac{i}{5} & 0.31 & \frac{17}{100}+\frac{i}{100} & -\frac{1}{100}+\frac{9i}{100} & -\frac{9}{100}-\frac{3i}{100} \\
-\frac{1}{25}-\frac{i}{10} & \frac{1}{50}+\frac{7i}{50} & \frac{1}{100}-\frac{7i}{100} & \frac{1}{20}-\frac{11i}{100} & \frac{17}{100}-\frac{i}{100} & 0.09 & \frac{i}{20} & -\frac{1}{20}-\frac{i}{100} \\
-\frac{1}{20}+\frac{i}{50} & \frac{7}{100}-\frac{i}{50} & -\frac{1}{25} & -\frac{3}{50}-\frac{i}{50} & -\frac{1}{100}-\frac{9i}{100} & -\frac{i}{20} & \frac{1}{50} & \frac{i}{50} \\
\frac{3}{100}+\frac{i}{20} & -\frac{3}{100}-\frac{7i}{100} & \frac{i}{25} & -\frac{1}{100}+\frac{3i}{50} & -\frac{9}{100}+\frac{3i}{100} & -\frac{1}{20}+\frac{i}{100} & -\frac{i}{50} & 0.03
\end{bmatrix}$$

[1]Torlai, G., Mazzola, G., Carrasquilla, J. *et al.* Neural-network quantum state tomography. *Nature Phys* 14, 447–450 (2018). https://doi.org/10.1038/s41567-018-0048-5