Communication Lower Bounds for Multi-Party Graph Traversal

Quarter 2 Project Proposal

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Abstract

We propose a 7-week project on investigating the lower bounds for communication between agents traversing a given graph. We begin with a hyper-constrained version of the problem to prove the feasibility of the project. We propose extending this model of multi-agent traversal for the project to create increasingly applicable bounds for communication in applications like self-driving cars, multi-agent search problems, transport, and other similar problems. Project deliverables include a paper with formal proofs of our claims and a poster presentation aimed at a more general, applied audience.

Repository: https://github.com/rybplayer/DSCCapstone

1 Introduction 2
2 Motivation 2
3 Problem Statement 3
4 Deliverables 5
5 Proposed Schedule 6
References 6

1 Introduction

Communication complexity involves the study of how much *communication* is needed for multiple parties to jointly compute a function where each party's input is limited. This field is fundamentally different from time complexity as it only seeks to determine how many *bits* need to be exchanged to compute a protocol. In fact, communication complexity may assume its parties each have access to infinite compute resources; we only care about the communication between them.

At its core, communication complexity seeks to determine how difficult it is to compute a function where information is spread out across multiple parties. In our project we will seek to apply communication complexity to a real-world problem, namely that of multi-agent traversal.

2 Motivation

2.1 Background: Multi-Agent Reinforcement Learning

Multi-agent reinforcement learning (MARL) studies how multiple autonomous agents learn to act within a shared environment. Each agent receives observations, selects actions, and updates its policy based on its local experience. Unlike the single-agent setting, however, overall system performance frequently depends on the agents' ability to coordinate their actions. For navigation problems in particular, agents must avoid collisions, resolve conflicts over shared space, and make decisions that depend not only on their own goals but also on the trajectories of others.

To facilitate coordination, many MARL models introduce an explicit communication channel through which agents may exchange messages containing local observations, planned intentions, or compressed representations of future behavior. When communication is unrestricted, agents can often coordinate perfectly by sharing all relevant information. Yet in most practical applications—robotics, autonomous vehicles, drone swarms, and distributed sensor networks—communication is fundamentally limited. Bandwidth, latency, and energy constraints mean that agents cannot transmit arbitrarily large messages at arbitrarily high frequencies.

Despite these real-world constraints, a large portion of the MARL literature either assumes that communication is entirely free or discourages excessive communication using heuristic penalty terms. These approaches are convenient for simulation but fail to provide any principled understanding of the *minimum* communication necessary for successful coordination.

2.2 The Role of Communication Complexity

This mismatch between theoretical assumptions and practical constraints motivates the need for a more rigorous framework for studying communication in multi-agent settings. Communication complexity offers precisely such a framework. In classical communication complexity, one seeks to determine the minimum number of bits that distributed parties must exchange in order to jointly compute a function. By translating multi-agent coordination tasks into appropriate communication problems, we can analyze the *information requirements* inherent in collaborative decision-making.

This perspective provides several key advantages:

- It allows us to distinguish between communication that is merely helpful and communication that is *fundamentally unavoidable*.
- It provides provable lower bounds that any MARL algorithm—regardless of learning method, reward shaping, or policy class—must obey.
- It reveals situations in which heuristic penalty-based methods may either overshoot (forcing too little communication) or undershoot (allowing unrealistic communication levels).
- It provides a baseline for evaluating the efficiency of practical MARL communication protocols.

In simple environments such as grid navigation, communication complexity allows us to quantify how much information the agents must exchange to avoid collisions and reach their destinations. These foundational results serve both as a theoretical guide and as a diagnostic tool: they clarify the inherent difficulty of coordination and highlight the divergence between real-world requirements and common algorithmic assumptions.

Ultimately, the motivation for this work is to bridge the gap between idealized multi-agent learning models and the physical constraints of deployed systems. By grounding MARL coordination in rigorous lower bounds, we obtain a clearer understanding of the informational structure of multi-agent navigation and a principled foundation for designing communication-efficient algorithms.

3 Problem Statement

3.1 Model Definition

We take a grid, upon which there are players which wish to traverse to their destination. We wish to use this model to find the minimum communication required for each player to arrive at it's destination. We make the following assumptions:

1. Each player has a pre-defined endpoint
2. The players each wish to minimize the number of edges traversed to reach their destination
3. The graph edges have no weights

4. There are no obstacles. In other words, each player can traverse any edge freely
5. Each player has full knowledge of the grid
6. All player start and end points are unique
7. The players are traversing a grid
8. There exits a shared random string
9. All players move 1 edge at a time
10. There are 2 players
11. The map is known in advance of the algorithm's design
12. All players can communicate
13. No two players may occupy the same node at the same time
14. Both players take their turn at the same time

Using this model and varying the assumptions we are able to prove some lower bounds on the communication complexity for this problem. We start by using all listed assumptions, and find a $\Omega(1)$ lower bound for this problem

3.2 Lower bound with all Assumptions

We start with the following claim: Both players can reach their endpoints without collisions using $\Omega(1)$ bits of communication

We propose a simple zero communication protocol for both players to traverse the grid. Since each player can see the start and endpoints of the other player, they can both identify all possible paths which minimize the distance traveled. Since the players are attempting to traverse only between two points, there will always be a pair of paths following the shortest distance which intersects a maximum of once. Player collision can then be determined entirety by the intersection point.

If there are shortest paths with no intersection point, than it is trivial for both players to simply follow those paths and not intersect. Otherwise, one of two cases emerges. In the first case, all possible points of intersection are equidistant from the player start, in which case one player (which one can be agreed upon in advance) simply holds for a turn, while the other player moves, thus guaranteeing that they will not intersect at the same time.

If non-equidistant points exist, both players can use a shared protocol to pick a point of intersection, and follow one of the paths which intersects at that point. Since the point is not equidistant, one player will reach it before the other, thus guaranteeing they do not visit it at the same time.

4 Deliverables

4.1 Project Goals

Therefore, the goal of our project is to consider relaxations of the above problem statement, and examine the communication complexity for each. Some relaxations we can consider, and their analogy to the application of "self driving cars" is as follows:

- There are obstacles at nodes, and they are only visible to players who are adjacent to the node of that obstable. This mimics the idea of random roadblocks noticed by the cars.
- Edges have weights representing the cost of travelling. The goal is to minimize the total cost of travel. This mimics the idea of some roads being more congested than others.
- There are n players. The self-driving cars may form a taxi network.
- The games are continuous; every time an endpoint is reached, a new start and end is assigned to each player. This mimics the idea of an taxi service.

4.2 Actionables

Therefore, this project can be summarized into three specific, actionables:

1. Define new models or problems in the field.
2. Write a paper that outlines the formal, technical communication complexity bounds for a given multi-agent traversal problem with certain constraints.
3. Give a poster presentation of the key results of the paper, with the intention of making the results applicable to a wider audience.

Therefore, our work will remain theoretically grounded while remaining applicable for the wider computing community.

4.3 Deliverables

We have three main deliverables:

1. A paper presenting actionable item 1 and 2.
2. A research poster of our key results, for actionable 3.
3. A website containing the paper and poster information, for actionable 3.

To be more specific, the paper will take the form of a research paper, where we define the problems we are working on, prove bounds on these problems, and show how they model situations in the field of multi-agent graph traversal. The paper is our primary project output.

5 Proposed Schedule

We allocate two weeks to reading to ensure we understand the subdomain and are not needlessly duplicating results.

Week	Focus	Ciro	Darren	Ivy	Ryan
1	Field Background Knowledge	Read and present 1)	Read and present 1)	Read and present 2)	Read and present 2)
2	Field Background Knowledge	Read and present 3)	Read and present 4)	Read and present 3)	Read and present 4)
3	Define the simplest case and prove it	Each proves a lemma (incomplete information)	Each proves a lemma (randomness)	Each proves a lemma (intersecting paths)	Each proves a lemma (duplicate endpoints)
4	Consider obstacles	Each proves a lemma	Each proves a lemma	Each proves a lemma	Each proves a lemma
5	Consider weights on edges	Each proves a lemma	Each proves a lemma	Each proves a lemma	Each proves a lemma
6	Consider more players	Each proves a lemma	Each proves a lemma	Each proves a lemma	Each proves a lemma
7	Wrap up	Writer	Figures	Equation	Figures

The annotation 1) refers to Bourgain, Freedman and Shapiro (2005), 2) to Min, Chatziafratis and Jordan (2023), 3) to Arii and Iwama (1992), and 4) to Klavins (2004)

References

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