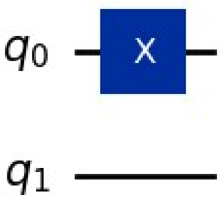
```
import numpy as np
import matplotlib.pyplot as plt
from math import pi
from numpy import arccos, sqrt
from qiskit import transpile, QuantumCircuit, QuantumRegister,
ClassicalRegister
from qiskit_aer import AerSimulator
from qiskit.visualization import plot_histogram, plot_state_city,
plot_bloch_multivector
import qiskit.quantum_info as qi
from qiskit.quantum_info import Statevector
```

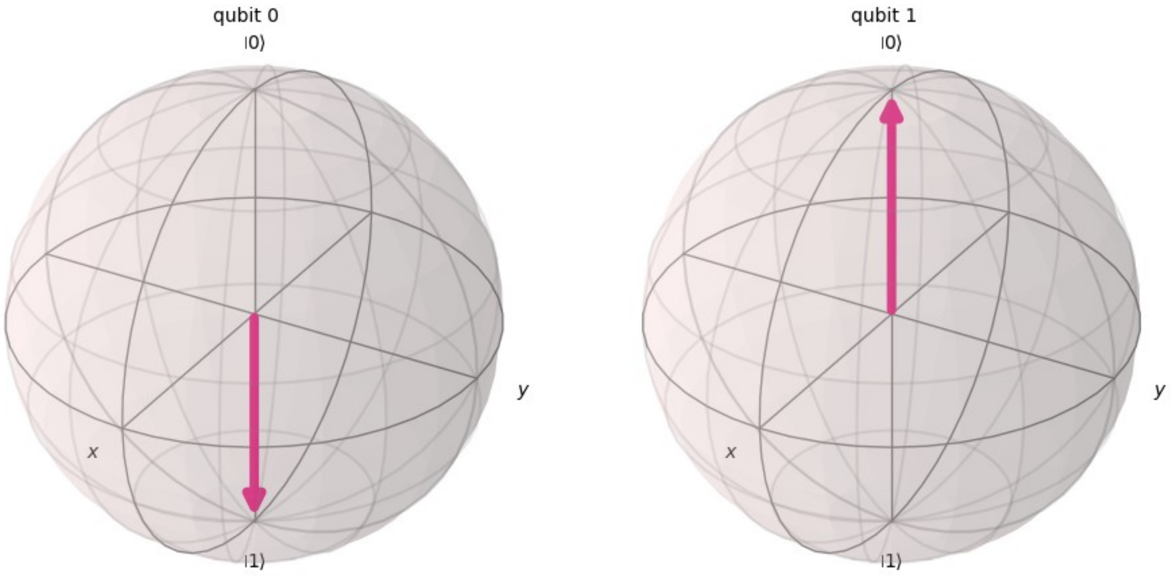
Task 1: Create a non-entangled 2-qubit state

- Create a circuit that puts 2 qubits in a separable (product) state using quantum gates
- Visualize the state vector
- Measure the qubits and display results
- Analyze the results and write down your observations about how you know the qubits are not entangled
- (optional) prove that your state can be written as a tensor product

```
# Part 1: Create a circuit that puts 2 qubits in a separable (product)
state using quantum gates
qc = QuantumCircuit(2)
qc.x(0)
qc.draw("mpl")
```



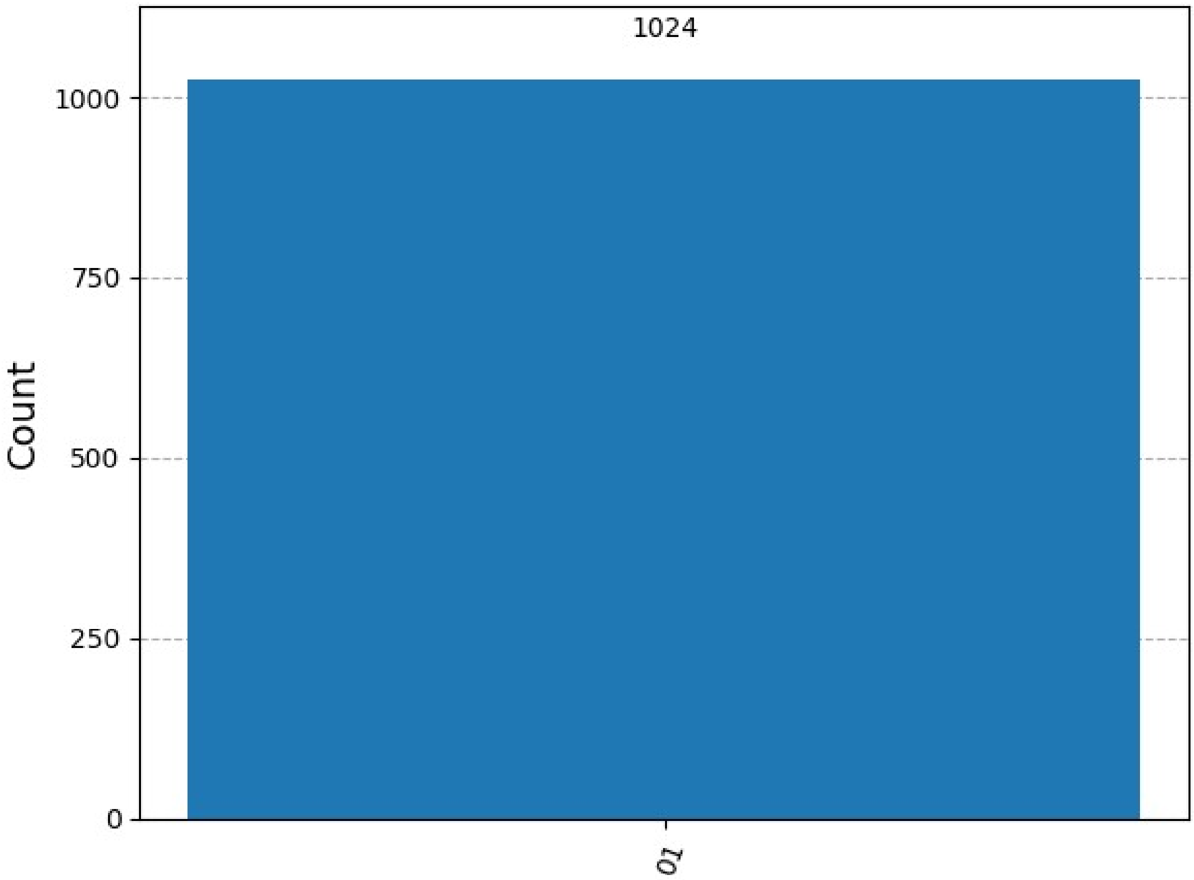
```
# Part 2: Visualize the state vector
statevector = Statevector.from_instruction(qc)
plot_bloch_multivector(statevector)
```



```
# Part 3: Measure the qubits and display results
simulator = AerSimulator()
qc = transpile(qc, simulator)
meas_qc = qc.copy()
meas_qc.measure_all()

result = simulator.run(meas_qc).result()
counts = result.get_counts(meas_qc)
print(counts)
plot_histogram(counts)
```

```
{'01': 1024}
```



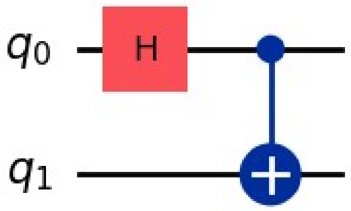
We measure $¿01\rangle$ all 1024 times. This state is not entagled as

¿

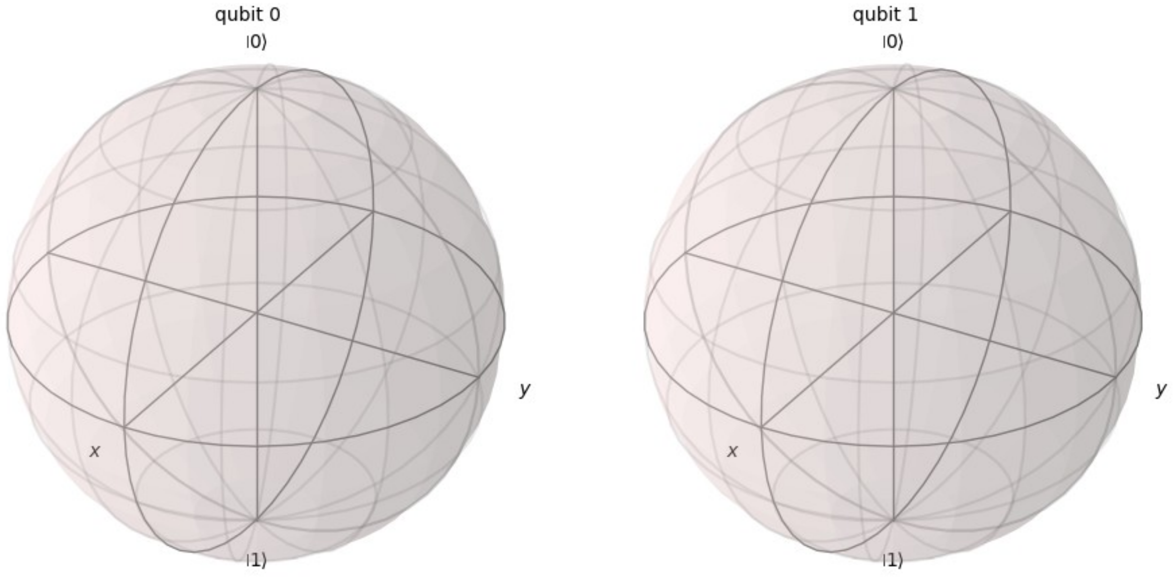
Task 2: Explore the EPR pair

- Create 2 qubit circuit and use gates to construct the Bell State: $¿\psi^{+¿\rangle=\frac{1}{\sqrt{2}}¿¿}$.
- Visualize the state vector
- Measure both of the qubits and show results
- Analyze how these results differ from the previous state
- (optional) prove that this state can’t be written as a tensor product

```
# Part 1: Create 2 qubit circuit and use gates to construct the Bell State: $|\psi^+\rangle = \frac{1}{\sqrt2}(|00\rangle + |11\rangle)$.
qc = QuantumCircuit(2)
qc.h(0)
qc.cx(0, 1)
qc.draw("mpl")
```



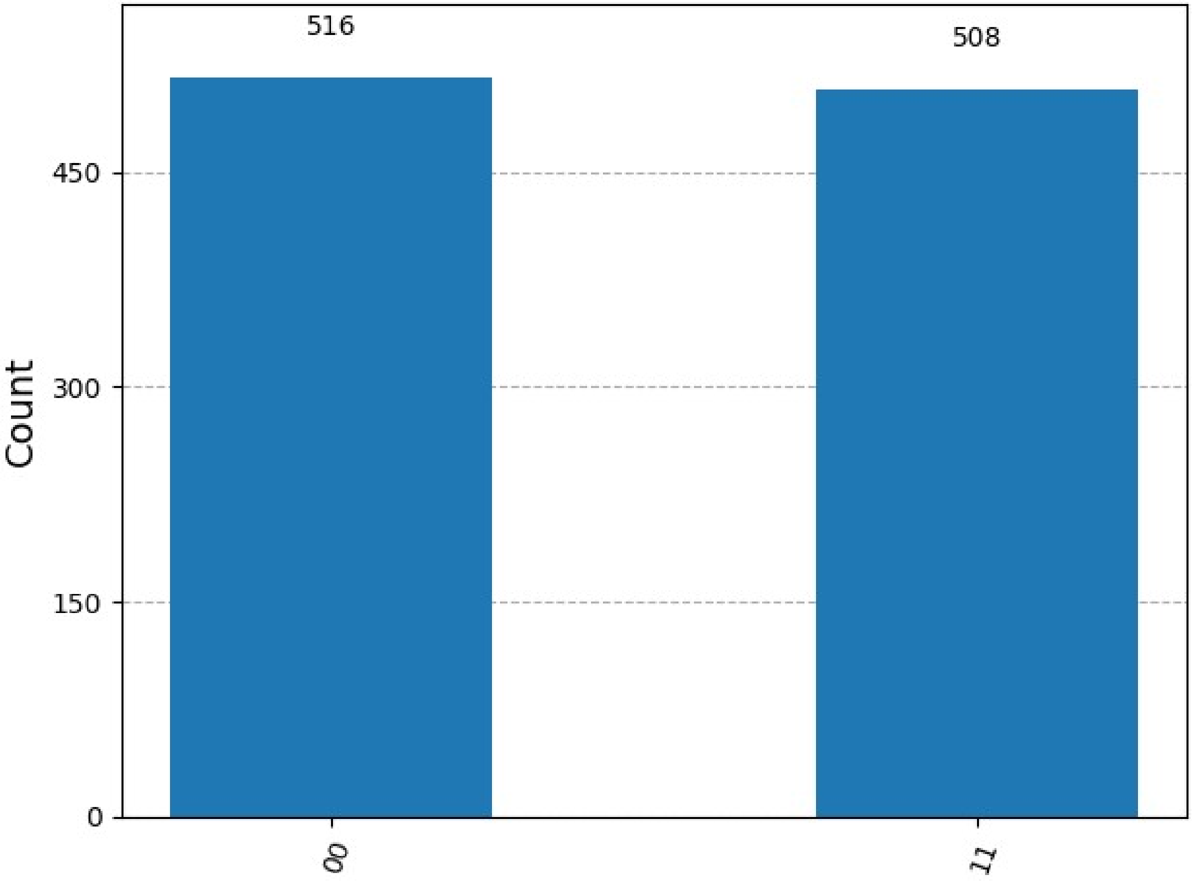
```
# Part 2: Visualize the state vector
statevector = Statevector.from_instruction(qc)
plot_bloch_multivector(statevector)
```



```
# Part 3: Measure both of the qubits and show results
simulator = AerSimulator()
qc = transpile(qc, simulator)
meas_qc = qc.copy()
meas_qc.measure_all()

result = simulator.run(meas_qc).result()
counts = result.get_counts(qc)
print(counts)
plot_histogram(counts)
```

```
{'11': 508, '00': 516}
```



Here we have a $50:50$ split between measuring ¿$00\rangle$ and ¿$11\rangle$, unlike the previous state. The EPR pair cannot be the tensor product of two qubits. Suppose not, so

¿

where $ |\phi_1\rangle $ and $ |\phi_2\rangle $ are arbitrary single-qubit states. A general single-qubit state can be expressed as:

$$|\phi_1\rangle = a|0\rangle + b|1\rangle \text{and} |\phi_2\rangle = c|0\rangle + d|1\rangle$$

Thus, the tensor product becomes:

$$|\phi_1\rangle \otimes |\phi_2\rangle = (a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle) = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$$

This results in a state of the form:

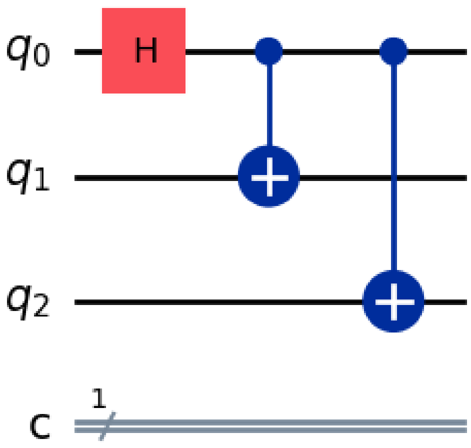
$$|\phi_1\rangle \otimes |\phi_2\rangle = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$$

For $ |\psi\rangle $ to equal $ |\phi_1\rangle \otimes |\phi_2\rangle $, the coefficients must satisfy $ ad = 0 $ and $ bc = 0 $. This implies either $ a = 0 $ or $ d = 0 $ (for $ ad = 0 $), and either $ b = 0 $ or $ c = 0 $ (for $ bc = 0 $). In all combinations, we reach a contradiction.

Task 3: Generalize to GHZ state

- Create 3 qubit circuit and use gates to create the GHZ state: ¿
- Measure all of the qubits and display results
- Analyze results and discuss how entanglement is shared for this state
- Now repeat but only measuring one of the qubits. What did you notice?

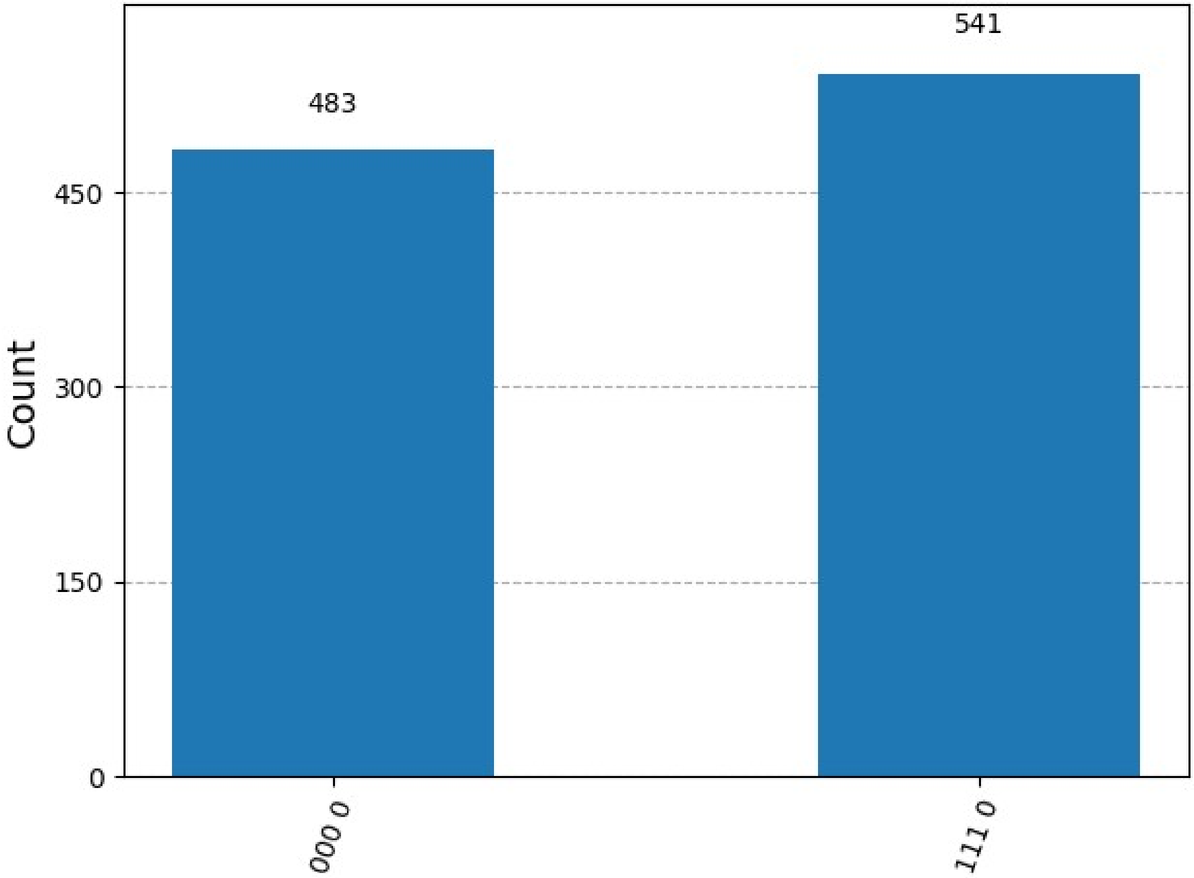
```
# Part 1: Create 3 qubit circuit and use gates to create the GHZ state: $|GHZ\rangle = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle$
qc = QuantumCircuit(3,1)
qc.h(0)
qc.cx(0, 1)
qc.cx(0, 2)
qc.draw("mpl")
```



```
# Part 2: Measure all of the qubits and display results
simulator = AerSimulator()
qc = transpile(qc, simulator)
meas_qc = qc.copy()
meas_qc.measure_all()

result = simulator.run(meas_qc).result()
counts = result.get_counts(meas_qc)
print(counts)
plot_histogram(counts)
```

```
{'111 0': 541, '000 0': 483}
```

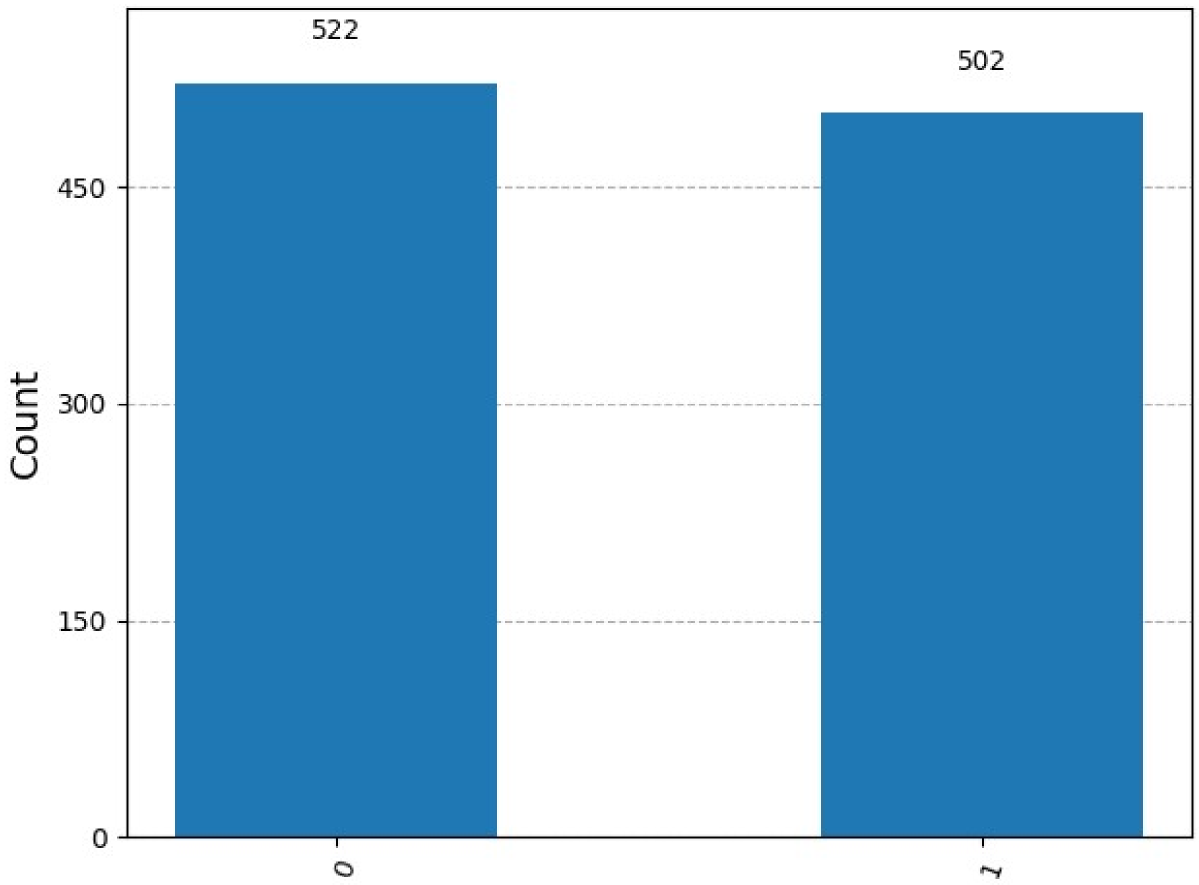


Here we have a $50:50$ split between measuring ¿$000\rangle$ and ¿$111\rangle$, so the entanglement is shared across all qubits. That is, knowing one qubit is a 1 tells us the other qubits are also 1.

```
# Part 4: Now repeat but only measuring one of the qubits. What did you notice?
meas_qc = qc.copy()
meas_qc.measure(0, 0)

result = simulator.run(meas_qc).result()
counts = result.get_counts(meas_qc)
print(counts)
plot_histogram(counts)
```

```
{'1': 502, '0': 522}
```

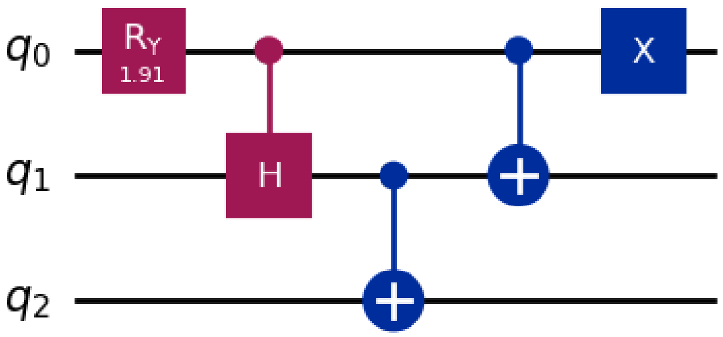


Here we have a 50:50 split between measuring ¿0⟩ and ¿1⟩ on the first qubit. Indeed, knowing one qubit is a 1 tells us the other qubits are also 1.

Task 4: Create W States

- Create the 3 qubit W State using quantum gates:
- Measure all three qubits
- Analyze the results. What do you notice? What’s different?

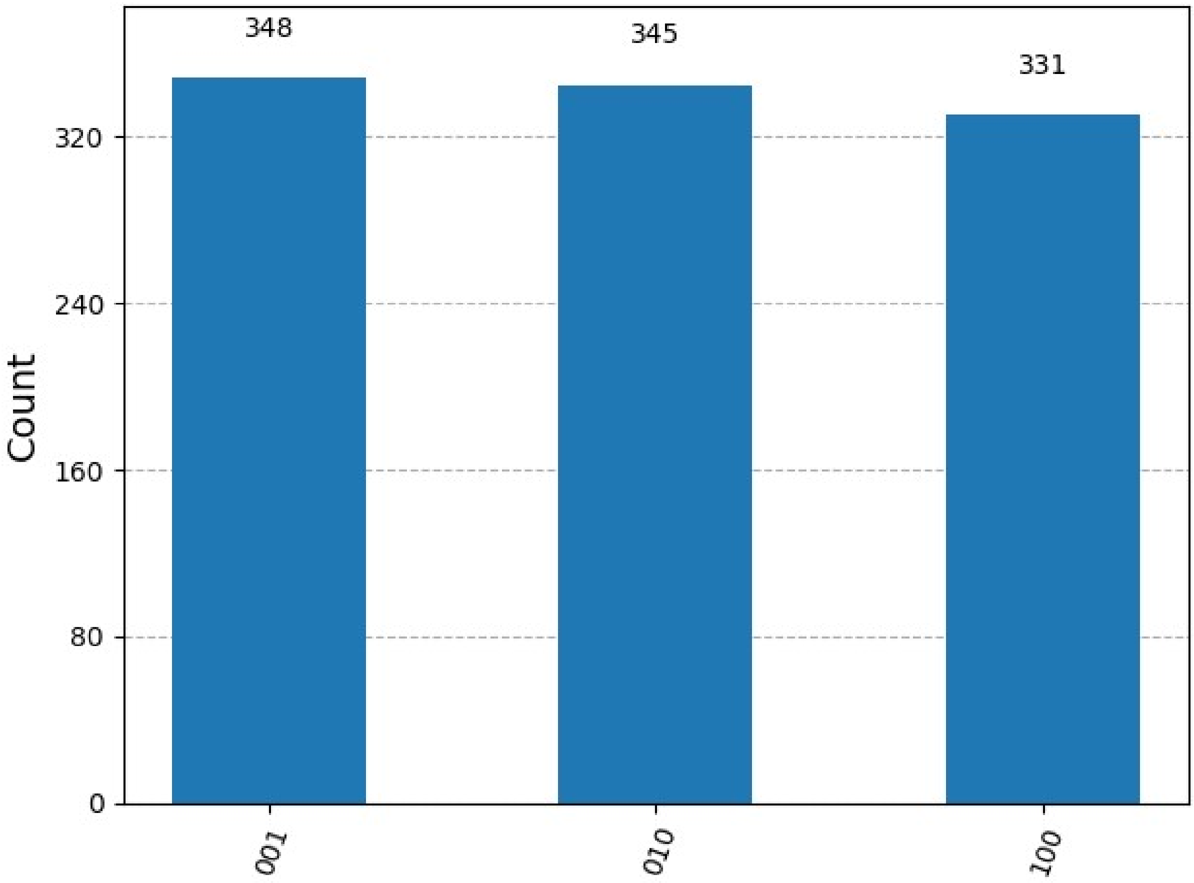
```
# Part 1: Create the 3 qubit W State using quantum gates
qc = QuantumCircuit(3)
qc.ry(2 * arccos(1/sqrt(3)), 0)
qc.ch(0, 1)
qc.cx(1, 2)
qc.cx(0, 1)
qc.x(0)
qc.draw("mpl")
```



```
# Part 2: Measure all three qubits
simulator = AerSimulator()
qc = transpile(qc, simulator)
meas_qc = qc.copy()
meas_qc.measure_all()

result = simulator.run(meas_qc).result()
counts = result.get_counts(meas_qc)
print(counts)
plot_histogram(counts)
```

```
{'100': 331, '001': 348, '010': 345}
```



Here we see an even triple split between measuring ¿001⟩, ¿010⟩, and ¿100⟩. This tells us all these three states are in equal superposition. This differs from the previous circuits, which only had two states in superposition.