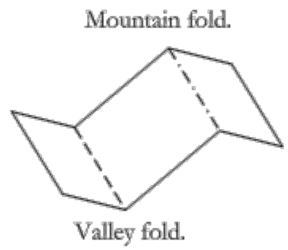
Origami and its Applications By Ryan Y. Batubara

Akira Yoshizawa: “The possibility of creation from paper is infinite”

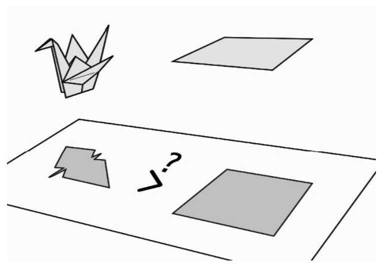
On Origami (折り紙)

Origami is the Japanese art of paper folding. Here, *creases* refer to folds essential to a *model,* opposed to any *fold* which may only be used as reference. Thus, each model corresponds to a unique *crease pattern*, the crease marks left when you unfold the model, up to the kind of fold, as shown right:



Napkin Folding Problem (Geometry)

Define *maximum perimeter* as the largest 2D-perimeter you can measure by projecting a model onto the plane in any direction. Thus, the unit square has *maximum perimeter* of 4, as shown.



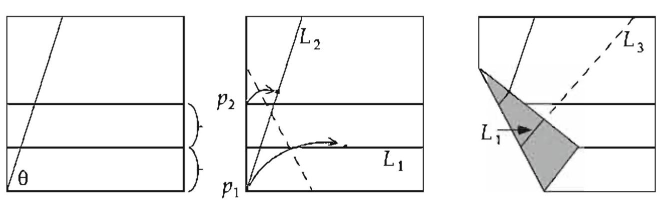
Can you fold a unit square in such a way that the model’s *maximum perimeter* is greater than 4? <u>Experiment with the **pink** paper.</u>

Impossible Trisections (Algebra and Geometry)

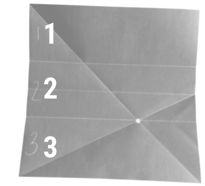
It is impossible to trisect an angle using just a ruler and compass.

Proof sketch: Trisecting 60° is equivalent to computing cos(π/9), which satisfies an irreducible polynomial of degree 3. Since the only numbers that can be produced ruler-and-compass have power of two algebraic degrees, trisection is impossible.

<u>Follow the below construction. What happened?</u>

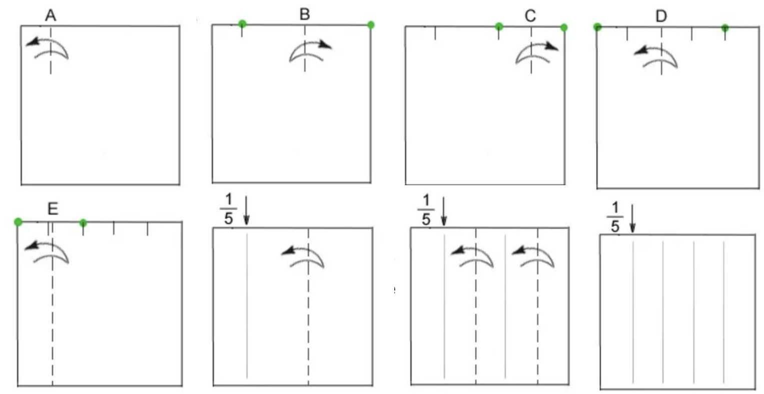


Fujimoto Approximation (Sequences)



You could do a similar to trisect a sheet of paper. *(Surprisingly useful in folding letters into envelopes)*. Now, does the below make perfect fifths of the page? <u>Experiment with the **orange** paper.</u>

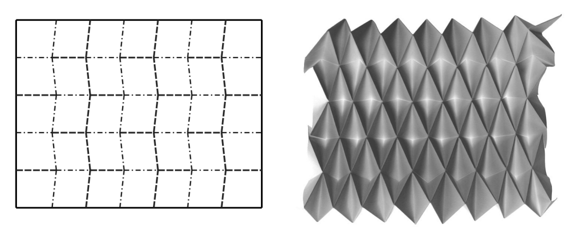
1. Make pinch *A* at approximately ⅕ of the page from the left.
2. Fold the right corner to *A*, making pinch *B*.
3. Fold the right corner to *B*, making pinch *C*.
4. Fold the left corner to *C*, making pinch *D*.
5. Fold the left corner to *D*, making pinch *E*.
6. Since *E* is ⅕ of the page, fold the remaining into halves.



<u>Can a similar construction be done to split a page into any *n*ths?</u>

Modular Origami & Miura Map Fold (3D Geometry)

Rigid origami, where sheets cannot be bent and edges behave as hinges, has gained recent popularity in compacting solar panels for spacecraft and satellites. The *Miura Map Fold* is a famous example used in the *Space Flyer Unit,* a 1996 Japanese satellite.

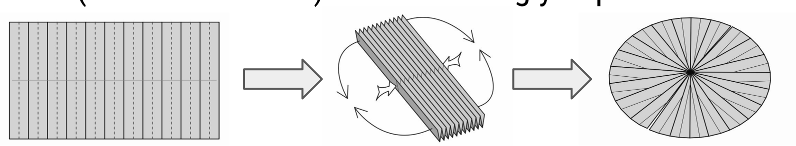


Activities, proofs, and illustrations inspired/taken from:
“Project Origami: Activities for Exploring Mathematics” by Thomas Hull in 2006
“Origami Design Secrets” second edition by Robert Lang in 2012
“Twists, Tilings, and Tessellations” by Robert Lang in 2018

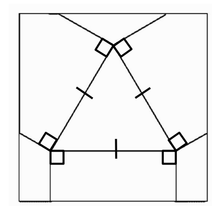
ANSWERS:
Napkin Problem: Yes, and as large as we want given infinitely thin paper.
Impossible Trisections: We trisected, meaning we can solve cubics with origami!
Fujimoto: Perfect iff ∞ iterations; Yes, fold out 2s, then apply general Fujimoto.

Impossible Folds (Geometry)

Some folds (iterated ∞ times) make seemingly impossible flat shapes:



But are some folds impossible? The right pattern is impossible to make flat! Why?



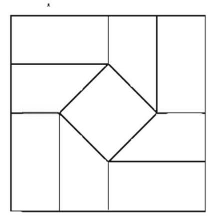
Big-Little-Big Lemma*: Given consecutive angles a,b,c around a flat vertex pattern with a>b, b<c then the creases between these angles cannot have same mountain-valley parity (else paper intersects).*

Flat-Foldability (Geometry and Combinatorics)

There are a finite number of theorems that can tell us whether a single vertex crease pattern is flat-foldable. However, it is NP-hard to determine if multi-vertex patterns are flat foldable, and it remains an open problem what the sufficiency conditions are to be flat-foldable!

Enumerating Folds (Graph Theory)

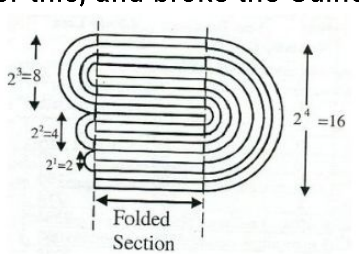
How many different models you can make with this crease pattern? Experiment on the **blue** paper. (Try different combinations of mountains/valleys).



Maekawa-Justin*: The number of valley (V) and mountain (M) folds in a single vertex must differ by 2, i.e. |M - V| = 2. (How does this relate to Euler's V - E + F = 2?)*

Physical Limitations (Physics)

Paper is not infinitely thin; In 2002, Britney Gallivan derived a formula for this, and broke the Guiness record by folding toilet paper 12 times.



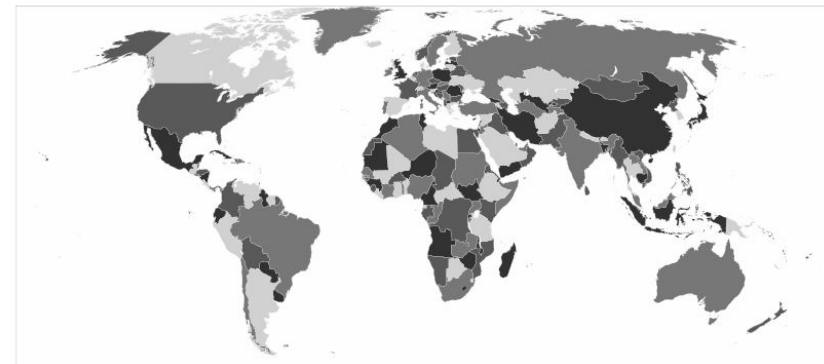
$$L = \frac{\pi t}{6}(2^n + 4)(2^n - 1)$$

$$W = \pi t 2^{3(n-1)/2}$$

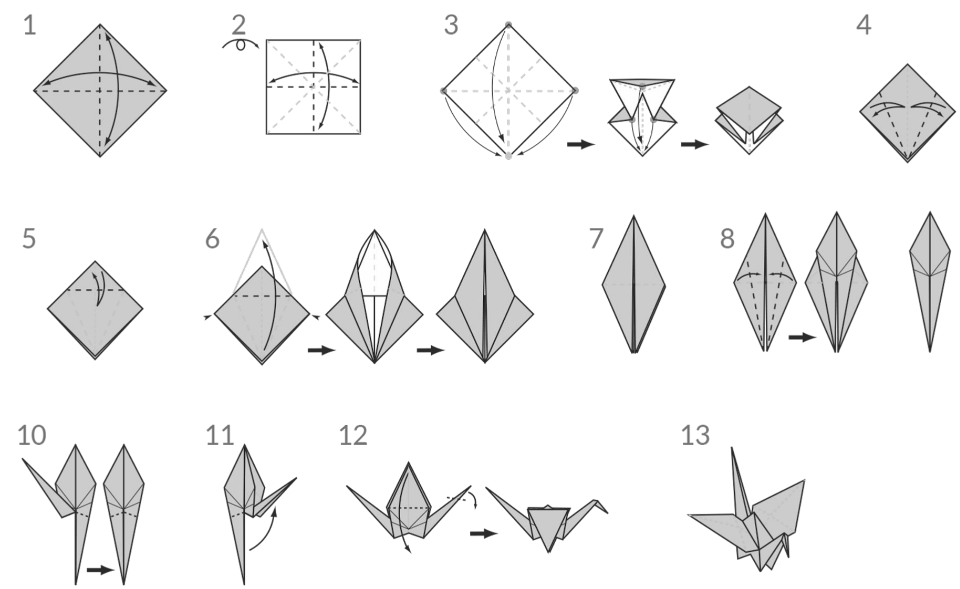
L is length, W is width, n is folds, t is thickness

Coloring Cranes (Graph Theory)

The famous *Four Color Theorem* states at *most* 4 colors are needed to color any map so that no two adjacent regions have the same color:



Fold a crane with the **yellow** paper, then unfolding it to form a crease pattern. Shade the faces of the pattern. How many colors did it take? Now, fold the crane back up, **where the grey side of the paper is the side you shaded; the final product has no yellow.** What do you notice?



This result gives a very fast way for a computer to check the direction in which a flat origami crease pattern will face when folded. Thus it is a powerful shortcut in computational origami.

Activities, proofs, and illustrations inspired/taken from:
"Project Origami: Activities for Exploring Mathematics" by Thomas Hull in 2006
"Origami Design Secrets" second edition by Robert Lang in 2012
"Twists, Tilings, and Tessellations" by Robert Lang in 2018

ANSWERS:
Impossible Folds: By lemma, triangle requires alternating mountain-valley.
Enumerating Folds: There are 6, by repeated application of the theorem.
Coloring Cranes: All origami (even 3D models) are two-face-colorable.