## 1 Proof

$$f(t) = \frac{\int_0^a \sigma'_{zz}(t) 2\pi r \,\mathrm{d}r}{\pi a^2} \tag{1}$$

$$= \frac{2}{a^2} \int_0^a \left[ -p + C_{13} \left( \frac{\partial u}{\partial r} + \frac{u}{r} + C_{13} \varepsilon \right) \right] r \, \mathrm{d}r \tag{8}$$

$$u' = -\frac{\varepsilon r'}{2} \left[ 1 - \frac{(C'_{11} + C'_{12} - 2C'_{13}) \frac{I_1[\sqrt{s}]}{\sqrt{s}}}{C'_{11}I_0[\sqrt{s}] - 2\frac{I_1[\sqrt{s}]}{\sqrt{s}}} \right]$$
(10)

$$p' = \frac{C'_{11} \left( C'_{11} + C'_{12} - 2C'_{13} \right)}{2} \left[ \frac{I_0[\sqrt{s}r'] - I_0[\sqrt{s}]}{C'_{11} I_0[\sqrt{s}] - 2\frac{I_1[\sqrt{s}]}{\sqrt{s}}} \right]$$
(11)

$$f(t) = \frac{\int_0^a \sigma'_{zz}(t) 2\pi r \, dr}{\pi a^2}$$
 (2)

$$= \frac{2}{a^2} \int_0^a \left[ -p + C_{13} \left( \frac{\partial u}{\partial r} + \frac{u}{r} + C_{13} \varepsilon \right) \right] r \, \mathrm{d}r \tag{8}$$

$$u' = -\frac{\varepsilon r'}{2} \left[ 1 - \frac{(C'_{11} + C'_{12} - 2C'_{13}) \frac{I_1[\sqrt{s}]}{\sqrt{s}}}{C'_{11}I_0[\sqrt{s}] - 2\frac{I_1[\sqrt{s}]}{\sqrt{s}}} \right]$$
 (Cohen Eqn10)

$$= -\frac{\varepsilon r'}{2} \qquad \text{if } \begin{array}{c} \nu_{31} = 0.5 \text{ so} \\ C'_{11} + C'_{12} - 2C'_{13} = 0 \end{array}$$
 (3)

$$p' = \frac{C'_{11} \left( C'_{11} + C'_{12} - 2C'_{13} \right)}{2} \left[ \frac{I_0[\sqrt{s}r'] - I_0[\sqrt{s}]}{C'_{11} I_0[\sqrt{s}] - 2\frac{I_1[\sqrt{s}]}{\sqrt{s}}} \right]$$
(11)

$$=0 if {}^{\nu_{31}=0.5 \text{ so}}_{C'_{11}+C'_{12}-2C'_{13}=0} (4)$$