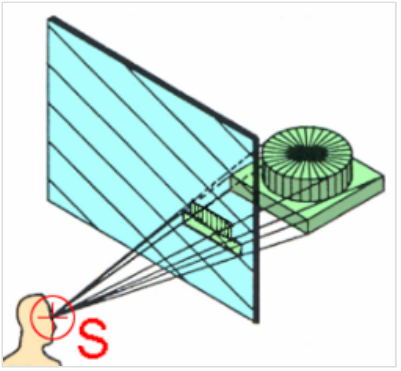
Cg Programming/Unity/Projection for Virtual Reality

User in a CAVE. The user's head position is tracked and the graphics on the walls are computed for this tracked position.

This tutorial discusses off-axis perspective projection in Unity. It is based on Section “Vertex Transformations”. No shader programming is required since only the view matrix and the projection matrix are changed, which is implemented in C#.

The main application of off-axis perspective projection are virtual reality environments such as the CAVE shown in the photo or so-called fish tank VR systems. Usually, the user's head position is tracked and the perspective projection for each display is computed for a camera at the tracked position such that the user experiences the illusion of looking through a window into a three-dimensional world instead of looking at a flat display.

Off-Axis vs. On-Axis Perspective Projection



Off-axis projection is characterized by a camera position S that is not on the symmetry axis of the view plane.

On-axis projection refers to camera positions that are on the symmetry axis of the view plane, i.e. the axis through the center of the view plane and orthogonal to it. This case is discussed in Section “Vertex Transformations”.

In virtual reality environments, however, the virtual camera often follows the tracked position of the user's head in order to create parallax effects and thus a more compelling illusion of a three-dimensional world. Since the tracked head position is not limited to the symmetry axis of the view plane, on-axis projection is not sufficient for most virtual reality environments.

Off-axis perspective projection addresses this issue by allowing for arbitrary camera positions. While some low-level graphics APIs (e.g. older versions of OpenGL) supported off-axis projection, they had much better support for on-axis projection since this was the more common case. Similarly, many high-level tools (e.g. Unity) support off-axis projection but provide much better support for on-axis projection, i.e. you can specify any on-axis projection with some mouse clicks but you need to write a script to implement off-axis projection.

Computing Off-Axis Perspective Projection

Off-axis perspective projection requires a different view matrix and a different projection matrix than on-axis perspective projection. For the computation of the on-axis view matrix, a specified view direction is rotated onto the *z* axis as described in Section “Vertex Transformations”. The only difference for an off-axis view matrix is that this “view direction” is computed as the orthogonal direction to the specified view plane, i.e. the surface normal vector of the view plane.

The off-axis projection matrix has to be changed since the edges of the view plane are no longer symmetric around the intersection point with the (technical) “view direction.” Thus, the four distances to the edges have to be computed and put into a suitable projection matrix. For details, see the description by Robert Kooima in his publication “Generalized Perspective Projection” (http://csc.lsu.edu/~kooima/pdfs/gen-perspective.pdf). The next section presents an implementation of this technique in Unity.

Camera Script

The following script is based on the code in Robert Kooima's publication. There are very few implementation differences. One is that, in Unity, the view plane is more easily specified as a built-in Quad object, which has corners at (±0.5, ±0.5, 0) in object coordinates. Furthermore, the original code was written for a right-handed coordinate system while Unity uses a left-handed coordinate system; thus, the result of all cross products has to be multiplied with -1. Also, the code here takes into account that the camera might be seeing the back-face of the Quad object.

Another difference is that the rotation of the camera's GameObject and the parameter `fieldOfView` are used by Unity for view frustum culling; therefore, the script should set those values to appropriate values. (These values have no meaning for the computation of the matrices.) Unfortunately, this might cause problems if other scripts (namely the script that sets the tracked head position) are also setting the camera rotation. Therefore, this estimation can be deactivated with the variable `estimateViewFrustum` (at the risk of incorrect view frustum culling by Unity).

If the parameter `setNearClipPlane` is set to `true`, the script sets the distance of the near clip plane to the distance between the camera and the view plane plus the value of `nearClipDistanceOffset`. However, if that value is less than `minNearClipDistance` then it is set to `minNearClipDistance`. This functionality is particularly useful when using the script to render mirrors as described in Section “Mirrors”. `nearClipDistanceOffset` should then be a negative number that is as close to 0 as possible while avoiding artifacts.

```
// This script should be attached to a Camera object
// in Unity. Once a Quad object is specified as the
// "projectionScreen", the script computes a suitable
// view and projection matrix for the camera.
// The code is based on Robert Kooima's publication
// "Generalized Perspective Projection," 2009,
// http://csc.lsu.edu/~kooima/pdfs/gen-perspective.pdf

using UnityEngine;

// Use the following line to apply the script in the editor:
[ExecuteInEditMode]

public class ObliqueProjectionToQuad : MonoBehaviour {
    public GameObject projectionScreen;
```

```
public bool estimateViewFrustum = true;
public bool setNearClipPlane = false;
public float minNearClipDistance = 0.0001f;
public float nearClipDistanceOffset = -0.01f;

private Camera cameraComponent;

void OnPreCull () {
    cameraComponent = GetComponent<Camera> ();
    if (null != projectionScreen && null != cameraComponent) {
        Vector3 pa =
            projectionScreen.transform.TransformPoint (
                new Vector3 (-0.5f, -0.5f, 0.0f));
        // lower left corner in world coordinates
        Vector3 pb =
            projectionScreen.transform.TransformPoint (
                new Vector3 (0.5f, -0.5f, 0.0f));
        // lower right corner
        Vector3 pc =
            projectionScreen.transform.TransformPoint (
                new Vector3 (-0.5f, 0.5f, 0.0f));
        // upper left corner
        Vector3 pe = transform.position;
        // eye position
        float n = cameraComponent.nearClipPlane;
        // distance of near clipping plane
        float f = cameraComponent.farClipPlane;
        // distance of far clipping plane

        Vector3 va; // from pe to pa
        Vector3 vb; // from pe to pb
        Vector3 vc; // from pe to pc
        Vector3 vr; // right axis of screen
        Vector3 vu; // up axis of screen
        Vector3 vn; // normal vector of screen

        float l; // distance to left screen edge
        float r; // distance to right screen edge
        float b; // distance to bottom screen edge
        float t; // distance to top screen edge
        float d; // distance from eye to screen

        vr = pb - pa;
        vu = pc - pa;
        va = pa - pe;
        vb = pb - pe;
        vc = pc - pe;

        // are we looking at the backface of the plane object?
        if (Vector3.Dot (-Vector3.Cross (va, vc), vb) < 0.0f) {
            // mirror points along the x axis (most users
            // probably expect the y axis to stay fixed)
            vr = -vr;
            pa = pb;
            pb = pa + vr;
            pc = pa + vu;
            va = pa - pe;
            vb = pb - pe;
            vc = pc - pe;
        }

        vr.Normalize ();
        vu.Normalize ();
        vn = -Vector3.Cross (vr, vu);
        // we need the minus sign because Unity
        // uses a left-handed coordinate system
        vn.Normalize ();

        d = -Vector3.Dot (va, vn);
        if (setNearClipPlane) {
            n = Mathf.Max (minNearClipDistance, d + nearClipDistanceOffset);
            cameraComponent.nearClipPlane = n;
        }
        l = Vector3.Dot (vr, va) * n / d;
        r = Vector3.Dot (vr, vb) * n / d;
        b = Vector3.Dot (vu, va) * n / d;
        t = Vector3.Dot (vu, vc) * n / d;
```

```
Matrix4x4 p = new Matrix4x4 (); // projection matrix
p[0, 0] = 2.0f * n / (r - l);
p[0, 1] = 0.0f;
p[0, 2] = (r + l) / (r - l);
p[0, 3] = 0.0f;

p[1, 0] = 0.0f;
p[1, 1] = 2.0f * n / (t - b);
p[1, 2] = (t + b) / (t - b);
p[1, 3] = 0.0f;

p[2, 0] = 0.0f;
p[2, 1] = 0.0f;
p[2, 2] = (f + n) / (n - f);
p[2, 3] = 2.0f * f * n / (n - f);

p[3, 0] = 0.0f;
p[3, 1] = 0.0f;
p[3, 2] = -1.0f;
p[3, 3] = 0.0f;

Matrix4x4 rm = new Matrix4x4 (); // rotation matrix;
rm[0, 0] = vr.x;
rm[0, 1] = vr.y;
rm[0, 2] = vr.z;
rm[0, 3] = 0.0f;

rm[1, 0] = vu.x;
rm[1, 1] = vu.y;
rm[1, 2] = vu.z;
rm[1, 3] = 0.0f;

rm[2, 0] = vn.x;
rm[2, 1] = vn.y;
rm[2, 2] = vn.z;
rm[2, 3] = 0.0f;

rm[3, 0] = 0.0f;
rm[3, 1] = 0.0f;
rm[3, 2] = 0.0f;
rm[3, 3] = 1.0f;

Matrix4x4 tm = new Matrix4x4 (); // translation matrix;
tm[0, 0] = 1.0f;
tm[0, 1] = 0.0f;
tm[0, 2] = 0.0f;
tm[0, 3] = -pe.x;

tm[1, 0] = 0.0f;
tm[1, 1] = 1.0f;
tm[1, 2] = 0.0f;
tm[1, 3] = -pe.y;

tm[2, 0] = 0.0f;
tm[2, 1] = 0.0f;
tm[2, 2] = 1.0f;
tm[2, 3] = -pe.z;

tm[3, 0] = 0.0f;
tm[3, 1] = 0.0f;
tm[3, 2] = 0.0f;
tm[3, 3] = 1.0f;

// set matrices
cameraComponent.projectionMatrix = p;
cameraComponent.worldToCameraMatrix = rm * tm;
// The original paper puts everything into the projection
// matrix (i.e. sets it to p * rm * tm and the other
// matrix to the identity), but this doesn't appear to
// work with Unity's shadow maps.

if (estimateViewFrustum) {
    // rotate camera to screen for culling to work
    Quaternion q = new Quaternion ();
    q.SetLookRotation ((0.5f * (pb + pc) - pe), vu);
    // look at center of screen
```

```
                cameraComponent.transform.rotation = q;

                // set fieldOfView to a conservative estimate
                // to make frustum tall enough
                if (cameraComponent.aspect >= 1.0f) {
                    cameraComponent.fieldOfView = Mathf.Rad2Deg *
                        Mathf.Atan (((pb - pa).magnitude + (pc - pa).magnitude) /
                            va.magnitude);
                } else {
                    // take the camera aspect into account to
                    // make the frustum wide enough
                    cameraComponent.fieldOfView =
                        Mathf.Rad2Deg / cameraComponent.aspect *
                        Mathf.Atan (((pb - pa).magnitude + (pc - pa).magnitude) /
                            va.magnitude);
                }
            }
        }
    }
}
```

To use this script, choose **Create > C# Script** in the **Project Window**, name the script "ObliqueProjectionToQuad", double-click the new script to edit it, and copy & paste the code from above into it. Then attach the script to your main camera (drag it from the **Project Window** over the camera object in the **Hierarchy Window**). Furthermore, create a Quad object (**GameObject > 3D Object > Quad** in the main menu) and place it into the virtual scene to define the view plane. Deactivate the **Mesh Renderer** of the Quad in the **Inspector Window** to make it invisible (it is only a placeholder). Select the camera object and drag the Quad object to **Projection Screen** in the **Inspector**. The script will be active when the game is started. Add the line

```
[ExecuteInEditMode]
```

as described in the code to make the script also run in the editor.

Note that there are probably some parts of Unity that ignore the new projection matrix and that therefore are unusable in combination with this script.

Older JavaScript code: click to show/hide

Note that this code was written for the built-in Plane object instead of the Quad object.

```
// This script should be attached to a Camera object
// in Unity. Once a Plane object is specified as the
// "projectionScreen", the script computes a suitable
// view and projection matrix for the camera.
// The code is based on Robert Kooima's publication
// "Generalized Perspective Projection," 2009,
// http://csc.lsu.edu/~kooima/pdfs/gen-perspective.pdf

// Use the following line to apply the script in the editor:
// @script ExecuteInEditMode()

#pragma strict

public var projectionScreen : GameObject;
public var estimateViewFrustum : boolean = true;
public var setNearClipPlane : boolean = false;
public var nearClipDistanceOffset : float = -0.01;

private var cameraComponent : Camera;

function LateUpdate() {
   cameraComponent = GetComponent(Camera);
   if (null != projectionScreen && null != cameraComponent)
```

```
{
   var pa : Vector3 =
      projectionScreen.transform.TransformPoint(
      Vector3(-5.0, 0.0, -5.0));
      // lower left corner in world coordinates
   var pb : Vector3 =
      projectionScreen.transform.TransformPoint(
      Vector3(5.0, 0.0, -5.0));
      // lower right corner
   var pc : Vector3 =
      projectionScreen.transform.TransformPoint(
      Vector3(-5.0, 0.0, 5.0));
      // upper left corner
   var pe : Vector3 = transform.position;
      // eye position
   var n : float = cameraComponent.nearClipPlane;
      // distance of near clipping plane
   var f : float = cameraComponent.farClipPlane;
      // distance of far clipping plane

   var va : Vector3; // from pe to pa
   var vb : Vector3; // from pe to pb
   var vc : Vector3; // from pe to pc
   var vr : Vector3; // right axis of screen
   var vu : Vector3; // up axis of screen
   var vn : Vector3; // normal vector of screen

   var l : float; // distance to left screen edge
   var r : float; // distance to right screen edge
   var b : float; // distance to bottom screen edge
   var t : float; // distance to top screen edge
   var d : float; // distance from eye to screen

   vr = pb - pa;
   vu = pc - pa;
   va = pa - pe;
   vb = pb - pe;
   vc = pc - pe;

   // are we looking at the backface of the plane object?
   if (Vector3.Dot(-Vector3.Cross(va, vc), vb) < 0.0)
   {
      // mirror points along the z axis (most users
      // probably expect the x axis to stay fixed)
      vu = -vu;
      pa = pc;
      pb = pa + vr;
      pc = pa + vu;
      va = pa - pe;
      vb = pb - pe;
      vc = pc - pe;
   }

   vr.Normalize();
   vu.Normalize();
   vn = -Vector3.Cross(vr, vu);
      // we need the minus sign because Unity
      // uses a left-handed coordinate system
   vn.Normalize();

   d = -Vector3.Dot(va, vn);
   if (setNearClipPlane)
   {
      n = d + nearClipDistanceOffset;
      cameraComponent.nearClipPlane = n;
   }
   l = Vector3.Dot(vr, va) * n / d;
   r = Vector3.Dot(vr, vb) * n / d;
   b = Vector3.Dot(vu, va) * n / d;
   t = Vector3.Dot(vu, vc) * n / d;

   var p : Matrix4x4; // projection matrix
   p[0,0] = 2.0*n/(r-l);
   p[0,1] = 0.0;
   p[0,2] = (r+l)/(r-l);
   p[0,3] = 0.0;
```

```
        p[1,0] = 0.0;
        p[1,1] = 2.0*n/(t-b);
        p[1,2] = (t+b)/(t-b);
        p[1,3] = 0.0;

        p[2,0] = 0.0;
        p[2,1] = 0.0;
        p[2,2] = (f+n)/(n-f);
        p[2,3] = 2.0*f*n/(n-f);

        p[3,0] = 0.0;
        p[3,1] = 0.0;
        p[3,2] = -1.0;
        p[3,3] = 0.0;

        var rm : Matrix4x4; // rotation matrix;
        rm[0,0] = vr.x;
        rm[0,1] = vr.y;
        rm[0,2] = vr.z;
        rm[0,3] = 0.0;

        rm[1,0] = vu.x;
        rm[1,1] = vu.y;
        rm[1,2] = vu.z;
        rm[1,3] = 0.0;

        rm[2,0] = vn.x;
        rm[2,1] = vn.y;
        rm[2,2] = vn.z;
        rm[2,3] = 0.0;

        rm[3,0] = 0.0;
        rm[3,1] = 0.0;
        rm[3,2] = 0.0;
        rm[3,3] = 1.0;

        var tm : Matrix4x4; // translation matrix;
        tm[0,0] = 1.0;
        tm[0,1] = 0.0;
        tm[0,2] = 0.0;
        tm[0,3] = -pe.x;

        tm[1,0] = 0.0;
        tm[1,1] = 1.0;
        tm[1,2] = 0.0;
        tm[1,3] = -pe.y;

        tm[2,0] = 0.0;
        tm[2,1] = 0.0;
        tm[2,2] = 1.0;
        tm[2,3] = -pe.z;

        tm[3,0] = 0.0;
        tm[3,1] = 0.0;
        tm[3,2] = 0.0;
        tm[3,3] = 1.0;

        // set matrices
        cameraComponent.projectionMatrix = p;
        cameraComponent.worldToCameraMatrix = rm * tm;
        // The original paper puts everything into the projection
        // matrix (i.e. sets it to p * rm * tm and the other
        // matrix to the identity), but this doesn't appear to
        // work with Unity's shadow maps.

        if (estimateViewFrustum)
        {
           // rotate camera to screen for culling to work
           var q : Quaternion;
           q.SetLookRotation((0.5 * (pb + pc) - pe), vu);
               // look at center of screen
           cameraComponent.transform.rotation = q;

           // set fieldOfView to a conservative estimate
           // to make frustum tall enough
           if (cameraComponent.aspect >= 1.0)
           {
```

```
            cameraComponent.fieldOfView = Mathf.Rad2Deg *
               Mathf.Atan(((pb-pa).magnitude + (pc-pa).magnitude)
               / va.magnitude);
         }
         else
         {
             // take the camera aspect into account to
             // make the frustum wide enough
             cameraComponent.fieldOfView =
                Mathf.Rad2Deg / cameraComponent.aspect *
                Mathf.Atan(((pb-pa).magnitude + (pc-pa).magnitude)
                / va.magnitude);
         }
      }
   }
}
```

Stereoscopic Cameras

If the positions of the left and right camera for a stereoscopic display are known, the script can be applied to each of the cameras separately. However, if a Unity `Camera` – let's call it `mycam` – is used for stereo rendering, the position in `mycam.transform.position` specifies the midpoint between the left and right camera. In this case, the position of the left camera may be obtained (in a 4-dimensional vector) with `mycam.GetStereoViewMatrix(Camera.StereoscopicEye.Left).inverse.GetRow(3)` and the position of the right camera may be obtained correspondingly with `mycam.GetStereoViewMatrix(Camera.StereoscopicEye.Right).inverse.GetRow(3)`. These positions may then be used to set up two separate cameras for the off-axis projection.

In some applications of off-axis projection (e.g. mirrors, portals, or magic lenses), the off-axis cameras might render into render textures, which are then used to texture surfaces. In the case of stereo rendering, there are usually two render textures (one for each eye). Thus, texturing with the resulting render textures usually has to use the correct render texture. To this end, Unity provides the built-in shader variable `unity_StereoEyeIndex` which is 0 for the left eye and 1 for the right eye. For example, a shader might read a color `leftColor` from the render texture for the left eye and a color `rightColor` from the render texture for the right eye. Then the shader expression `lerp(leftColor, rightColor, unity_StereoEyeIndex)` computes the correct color when using the render textures for stereo rendering. Complete shader code for this approach is included in Section “Mirrors”.

Summary

In this tutorial, we have looked at:

- uses of off-axis perspective projection and differences to on-axis perspective projection
- the computation of view and projection matrices for off-axis perspective projection
- an implementation of this computation and its limitations in Unity

Further reading

If you want to know more

- about the on-axis view matrix and the on-axis projection matrix, you should read the description in Section “Vertex Transformations”.

- about the implemented algorithm, you should read Robert Kooima's publication “Generalized Perspective Projection” (http://160592857366.free.fr/joe/ebooks/ShareData/Generalized%20Perspective%20Projection.pdf).

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