Lec23-interpolation-RyanSponzilli

November 14, 2024

[38]:
```python
import numpy as np
import matplotlib.pyplot as plt
from astropy import table
import scipy.interpolate
```

1 ASTR 310 Lecture 23 - Interpolation

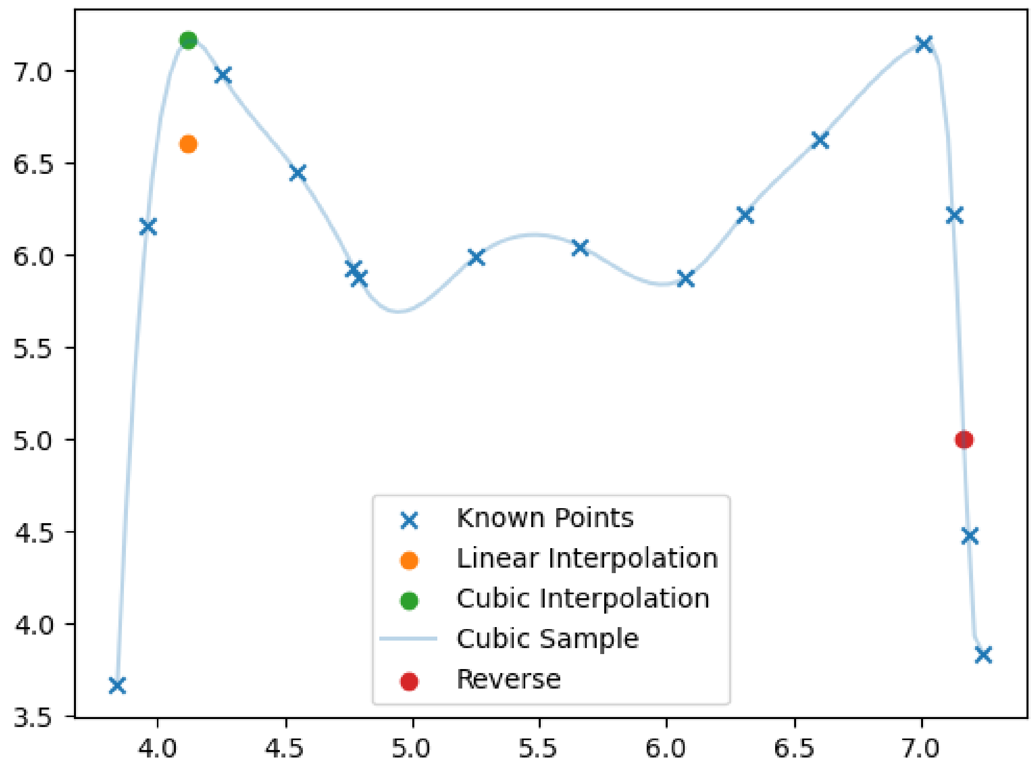
1.0.1 Exercise 1: 1D interpolation, different types

- Read the data from the file cat-profile.csv. (I like np.genfromtxt or astropy Table.read, but you can do whatever works.) [1 pt]
- Select the points between x = 3.8 and 7.3; these are the "known points" around which we will be interpolating. [1 pt]
- Plot the data with nice large symbols so you can easily see the known points. [1 pt]
- Use linear interpolation to find the expected value at x = 4.12. [ans: y = 6.61] Plot that interpolated point on your figure. [1 pt]
- Use cubic spline interpolation to find the expected value at x = 4.12. [ans: y = 7.16] Add that to your figure as well. [1 pt]
- How'd the cubic spline point get way up there? To find out, generate a finely sampled array of 100 x-values spanning the relevant range. Obtain the expected (cubic spline interpolated) values at all of your sample points and plot those as on the figure in a different color. [1 pt]
- Now use linear interpolation to estimate the x value corresponding to y = 5.0 on the right side of the plot. [ans: x = 7.17] If you're not careful, you'll get a goofy answer since this function is not monotonic. Select a subset of the points over which the data are monotonic. Create a new interpolant covering just that monotonic subset, and interpolate. [2 pts]

[105]:
```python
data = table.Table.read("cat-profile.csv")
x = np.array(data["# x"])
y = np.array(data["y"])

known = ((x > 3.8) & (x < 7.3))

interp_lin = scipy.interpolate.interp1d(x[known], y[known], kind='linear')
interp_cub = scipy.interpolate.interp1d(x[known], y[known], kind='cubic')

cubic_sample = np.linspace(x[known][0], x[known][-1], 100)

monotonic = ((x > 7) & (x < 7.3))
```

```
interp_lin_reverse = scipy.interpolate.interp1d(y[monotonic], x[monotonic],␣
    ↪kind='linear')

plt.scatter(x[known], y[known], label="Known Points", marker='x')
plt.scatter(4.12, interp_lin(4.12), label="Linear Interpolation")
plt.scatter(4.12, interp_cub(4.12), label="Cubic Interpolation")
plt.plot(cubic_sample, interp_cub(cubic_sample), label="Cubic Sample", alpha=0.
    ↪3)
plt.scatter(interp_lin_reverse(5), 5, label="Reverse")
plt.legend()
```

[105]: <matplotlib.legend.Legend at 0x171637dd0>



1.0.2 Exercise 2: 1D interpolation error analysis

1.) Write a function that does the following tasks. Given a value Δ, it should construct a uniformly sampled table of x and y values. Let x range from -10 to 10 with a sampling interval of Δ. Let $y = \sin(x)/x$. (There's a function np.sinc but check the definition for a factor of π.) Finally, use `scipy.interpolate.interp1d` to return a piecewise linear interpolant function based on this table. [3 pts]

[89]:
```
def gen_interp(delta):
    x = np.arange(-10, 10, delta)
    y = np.sin(x) / x

    return scipy.interpolate.interp1d(x, y, kind='linear',␣
 ↪fill_value='extrapolate')
```

2.) Call your function to generate an interpolant for $\Delta = 0.5$. Choose 50 random x values between -10 and 10 and use your interpolant to estimate the function values at these points. Plot the interpolated values and the original function using different colors or symbols and make a helpful legend.

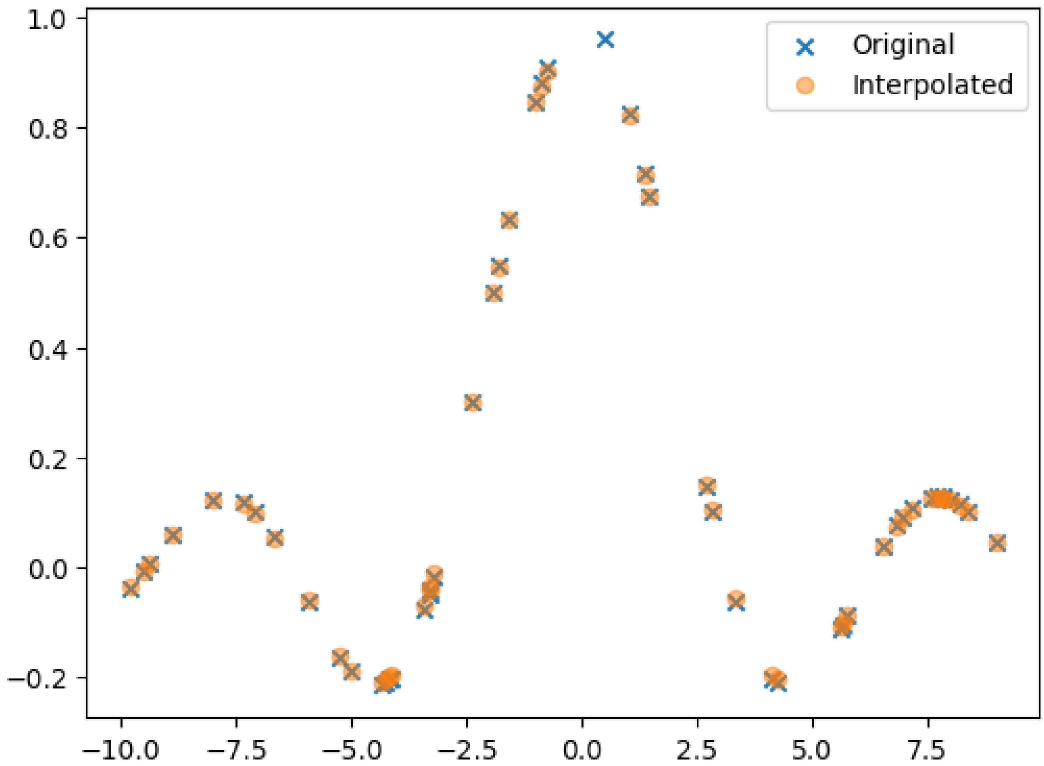
If you're finding these exercises very easy, try making a fancier version of this plot using two panels with `add_axes`, and show the residuals (interpolated value - true value) as well. See the lecture slides for an example.

[4 pts]

[113]:
```
interp = gen_interp(0.5)
x = np.random.random_sample(50) * 20 - 10

y = interp(x)

plt.scatter(x, np.sin(x) / x, label="Original", marker='x')
plt.scatter(x, y, label="Interpolated", alpha=0.5)
plt.legend()
```

```
/var/folders/41/_gkgvhb94wd4156zplzr4cg00000gn/T/ipykernel_76938/2105008982.py:3
: RuntimeWarning: invalid value encountered in divide
  y = np.sin(x) / x
```

[113]:
```
<matplotlib.legend.Legend at 0x1673f1250>
```



3.) Create a loop that varies Δ from 1 to 10^{-4} logarithmically, decreasing by a factor of 3 with each iteration. For each iteration, * generate the interpolant for the corresponding value of Δ, * compute the value of the interpolant at the first random value of x (from part 2), and * compute the error as defined by the normalized difference |interpolated value - true value|/(true value).

Plot the error in the interpolated value against Δ on a log-log plot. For a piecewise linear interpolation, you should expect the error to scale like Δ^2. Plot the line Δ^2 on your figure to compare to your points.

[5 pts]

[133]:
```
deltas = []
errors = []

delta = 1
while delta > 1e-4:
    interp = gen_interp(delta)
    y = interp(x[0])
    error = np.abs(y - (np.sin(x[0])/x[0])) / np.sin(x[0])/x[0]

    deltas.append(delta)
    errors.append(error)
```

```
    delta /= 3

plt.scatter(deltas, errors, label="Errors", c='green')
plt.loglog(np.linspace(1, 1e-4, 100), np.linspace(1, 1e-4, 100)**2,␣
 ↪label="$\Delta^2$ Limit")
plt.legend()
plt.xlim(1, 1e-4)
```

```
<>:16: SyntaxWarning: invalid escape sequence '\D'
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/var/folders/41/_gkgvhb94wd4156zplzr4cg00000gn/T/ipykernel_76938/4204965864.py:1
6: SyntaxWarning: invalid escape sequence '\D'
  plt.loglog(np.linspace(1, 1e-4, 100), np.linspace(1, 1e-4, 100)**2,
label="$\Delta^2$ Limit")
/var/folders/41/_gkgvhb94wd4156zplzr4cg00000gn/T/ipykernel_76938/2105008982.py:3
: RuntimeWarning: invalid value encountered in divide
  y = np.sin(x) / x
```

[133]: (1, 0.0001)