

HW7

Tuesday, November 17, 2020 - Due: Tuesday, November 25, 2020

1. Let \mathcal{C} be an additive \mathbb{C} -category. The radical of \mathcal{C} , denoted by $rad_{\mathcal{C}}$, is defined as:

$$rad_{\mathcal{C}}(X, Y) = \{h \in Hom_{\mathcal{C}}(X, Y) \mid 1_X - g \circ h \text{ is invertible } \forall g \in Hom_{\mathcal{C}}(Y, X)\}.$$

Prove that:

$$rad_{\mathcal{C}}(X, Y) = \{h \in Hom_{\mathcal{C}}(X, Y) \mid 1_Y - h \circ g \text{ is invertible } \forall g \in Hom_{\mathcal{C}}(Y, X)\}.$$

2. (a) Prove $(f \in rad_{\mathcal{C}}(X, Y)) \implies (f \text{ is neither split mono nor split epi})$.
 (b) Show that the converse implication is not true, i.e. find a map f which is neither split mono nor split epi, but $f \notin rad_{\mathcal{C}}(X, Y)$.

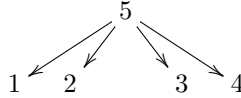
3. (a) Let $M \xrightarrow{f} N$ be a Λ -module homomorphism. Prove that the homomorphism $M \xrightarrow{\begin{bmatrix} f \\ f \end{bmatrix}} N \oplus N$ is not left minimal.

- (b) Prove that the map $M \oplus M \xrightarrow{\begin{bmatrix} 1_M & 1_M \end{bmatrix}} M$ is left minimal.

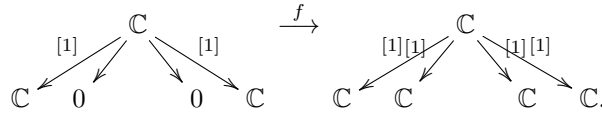
4. Let Q be the quiver $1 \leftarrow 2 \leftarrow 3$.

- (a) Prove that the map $P_1 \xrightarrow{incl} P_3$ is not right almost split map.
 (b) Prove that the map $P_1 \xrightarrow{incl} P_3$ is right minimal map.

5. Let Q be the following quiver:



- (a) Prove that the following representation morphism must be equal to 0 morphism:



- (b) Prove that $P_1 \oplus P_2 \oplus P_3 \xrightarrow{[i_1 \ i_2 \ i_3]} P_5$ is right minimal but is not right minimal almost split map. (Here the maps $i_1 \ i_2 \ i_3$ are the inclusions.)