

*Churchill Equation*

$$f = 8 \left[ \left( \frac{8}{\text{Re}} \right)^{12} + \frac{1}{(B + C)^{1.5}} \right]^{1/12}$$

$$\text{where } B = \left[ 2.457 \ln \frac{1}{(7/\text{Re})^{0.9} + (0.27\epsilon/D)} \right]^{16}$$

$$C = \left( \frac{37530}{\text{Re}} \right)^{16}$$

**P. Pressure Drop Calculations**

$$\Delta p_p = \frac{f_p L}{ID_p} \frac{\rho_p V_p^2}{2g_c} =$$

$$\Delta p_a = \left( \frac{f_a L}{D_h} + 1 \right) \frac{\rho_a V_a^2}{2g_c} =$$

**Q. Summary of Information Requested in Problem Statement**

**EXAMPLE 7.4.** Water at a temperature of 175°F and a mass flow rate of 5000 lbm/hr is to be used to heat ethylene glycol. The ethylene glycol is available at 90°F with a mass flow rate of 30,000 lbm/hr. A double pipe heat exchanger consisting of a 1<sup>1</sup>/<sub>4</sub>-standard type M copper tubing inside of 2-standard type M copper tubing is to be used. The exchanger consists of two double hairpin exchangers, each 6 ft long. The flow configuration is such that both fluids travel in series throughout. Determine the outlet temperature of both fluids using counterflow and again using parallel flow.

**Discussion**

Water loses energy only to the ethylene glycol and, as heat is transferred, fluid properties change with temperature changes. Outlet temperatures are unknown, so in order to evaluate properties, we use either the inlet temperatures or the average of both inlet temperatures. The fluid with the higher flow rate should be placed in the passage (annular or tubular) having the greater cross-sectional area so that pressure losses are minimized. With the piping arrangement all in series, the system is equivalent to having one 24-ft-long double pipe heat exchanger.

- Assumptions**
1. Steady-state conditions exist.
  2. Fluid properties remain constant and are evaluated at  $132^\circ\text{F}$  [ $\approx (175 + 90)/2$ ].

- Nomenclature**
1.  $T$  refers to the temperature of the warmer fluid.
  2.  $t$  refers to the temperature of the cooler fluid.
  3.  $w$  subscript refers to the warmer fluid.
  4.  $h$  subscript refers to hydraulic diameter.
  5.  $c$  subscript refers to the cooler fluid.
  6.  $a$  subscript refers to the annular flow area or dimension.
  7.  $p$  subscript refers to the tubular flow area or dimension.
  8. 1 subscript refers to an inlet condition.
  9. 2 subscript refers to an outlet condition.
  10.  $e$  subscript refers to equivalent diameter.

**A. Fluid Properties** (interpolated from the property tables in the appendix)

<b>Water</b> 132°F	$\dot{m}_w = 5000 \text{ lbm/hr}$	$T_1 = 175^\circ\text{F}$
	$\rho = 61.8 \text{ lbm/ft}^3$	$C_p = 0.9966 \text{ BTU/lbm}\cdot^\circ\text{R}$
	$k_f = 0.378 \text{ BTU/hr}\cdot\text{ft}\cdot^\circ\text{R}$	$\alpha = 6.12 \times 10^{-3} \text{ ft}^2/\text{hr}$
	$\nu = 0.484 \times 10^{-5} \text{ ft}^2/\text{s}$	$\text{Pr} = 2.84$
<b>Ethylene Glycol</b> 132°F	$\dot{m}_c = 30,000 \text{ lbm/hr}$	$t_1 = 90^\circ\text{F}$
	$\rho = 68.0 \text{ lbm/ft}^3$	$C_p = 0.608 \text{ BTU/lbm}\cdot^\circ\text{R}$
	$k_f = 0.150 \text{ BTU/hr}\cdot\text{ft}\cdot^\circ\text{R}$	$\alpha = 3.61 \times 10^{-3} \text{ ft}^2/\text{hr}$
	$\nu = 5.93 \times 10^{-5} \text{ ft}^2/\text{s}$	$\text{Pr} = 59.0$

**B. Tubing Sizes**

2-std M	$ID_a = 0.1674 \text{ ft}$	$OD_p = 1.375/12 = 0.1146 \text{ ft}$
1 $\frac{1}{4}$ -std M	$ID_p = 0.1076 \text{ ft}$	

**C. Flow Areas**

$$A_p = \pi ID_p^2 / 4 = 0.00909 \text{ ft}^2$$

$$A_a = \pi (ID_a^2 - OD_p^2) / 4 = 0.0117 \text{ ft}^2$$

**D. Fluid Velocities** [Because  $A_a > A_p$ , we route the ethylene glycol (the fluid with the higher flow rate) through the annulus.]

<b>Water</b>	$V_p = \dot{m}_w / \rho A_p = 2.47 \text{ ft/s}$	<del>_____</del>
<b>Ethylene Glycol</b>	$V_a = \dot{m}_c / \rho A_a = 10.48 \text{ ft/s}$	<del>_____</del>

**E. Annulus Equivalent Diameters**

<b>Friction</b>	$D_h = ID_a - OD_p = 0.0528 \text{ ft}$
<b>Ht Transfer</b>	$D_e = (ID_a^2 - OD_p^2) / OD_p = 0.1299 \text{ ft}$



**F. Reynolds Numbers**

Water  $Re_p = V_p ID_p / \nu = 5.5 \times 10^4$

Ethylene Glycol  $Re_a = V_a D_e / \nu = 2.3 \times 10^4$

**G. Nusselt Numbers**

Water  $Nu_p = 0.023(Re_p)^{4/5} Pr^{0.3} = 194$

Ethylene Glycol  $Nu_a = 0.023(Re_a)^{4/5} Pr^{0.4} = 362$

**H. Convection Coefficients**

Water  $h_i = Nu_p k_f / ID_p = 684$   $h_p = h_i ID_p / OD_p = 642$

Ethylene Glycol  $h_a = Nu_a k_f / D_e = 417 \text{ BTU/hr}\cdot\text{ft}^2\cdot^\circ\text{R}$

**I. Exchanger Coefficient**

$$\frac{1}{U_o} = \frac{1}{h_p} + \frac{1}{h_a} \quad U_o = 253 \text{ BTU/hr}\cdot\text{ft}^2\cdot^\circ\text{R}$$

**J. Outlet Temperature Calculations (Exchanger length  $L = 24 \text{ ft}$ )**

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = 3.66 \quad A_o = \pi OD_p L = 8.64 \text{ ft}^2$$

Counterflow  $E_{\text{counter}} = \exp[U_o A_o (R - 1) / \dot{m}_c C_{pc}] = 1.375$

$$T_2 = \frac{T_1(R - 1) - Rt_1(1 - E_{\text{counter}})}{RE_{\text{counter}} - 1} = 146.1^\circ\text{F}$$

$$t_2 = t_1 + \frac{T_1 - T_2}{R} = 97.9^\circ\text{F}$$

Parallel Flow  $E_{\text{para}} = \exp[U_o A_o (R + 1) / \dot{m}_c C_{pc}] = 1.747$

$$T_2 = \frac{(R + E_{\text{para}})T_1 + Rt_1(E_{\text{para}} - 1)}{(R + 1)E_{\text{para}}} = 146.5^\circ\text{F}$$

$$t_2 = t_1 + \frac{T_1 - T_2}{R} = 97.8^\circ\text{F}$$

Now at this point, we should re-evaluate the fluid properties at new temperatures. So for the water and for the ethylene glycol, we find the average of inlet and outlet temperatures as

$$\text{Water} \quad T = \frac{175 + 146.1}{2} = 160.6^\circ\text{F}$$

$$\text{Ethylene Glycol} \quad t = \frac{90 + 97.8}{2} = 93.9^\circ\text{F}$$

The system is now re-analyzed using properties evaluated at these temperatures. If necessary, a third iteration should be made so that a good estimate of the outlet temperatures is obtained. (With a spreadsheet, many iterations can be made in a very short time.) If this is done, the temperatures we use to find properties are:

Temperature at which to evaluate properties (after several iterations):

$$\text{Water} \quad T = 162.5$$

$$\text{Ethylene Glycol} \quad t = 95.0$$

and the final results are:

#### K. Summary of Requested Information (Outlet temperatures)

Water	$T_2 = 150^\circ\text{F}$	Counterflow
Ethylene Glycol	$t_2 = 97.1^\circ\text{F}$	Counterflow
Water	$T_2 = 150^\circ\text{F}$	Parallel Flow
Ethylene Glycol	$t_2 = 97.1^\circ\text{F}$	Parallel Flow

#### L. Heat Balance (as a check on the results)

##### Counterflow

$$\begin{aligned} \text{Water} \quad q_w &= \dot{m}_w C_{pw} (T_1 - T_2) = 34.8 \text{ BTU/s} \\ \text{Ethylene Glycol} \quad q_c &= \dot{m}_c C_{pc} (t_2 - t_1) = 34.8 \text{ BTU/s} \\ \text{Overall} \quad q &= UALMTD = 34.8 \text{ BTU/s} \end{aligned}$$

##### Parallel Flow

$$\begin{aligned} \text{Water} \quad q_w &= \dot{m}_w C_{pw} (T_1 - T_2) = 34.8 \text{ BTU/s} \\ \text{Ethylene Glycol} \quad q_c &= \dot{m}_c C_{pc} (t_2 - t_1) = 34.8 \text{ BTU/s} \\ \text{Overall} \quad q &= UALMTD = 34.8 \text{ BTU/s} \end{aligned}$$



The results show that little difference exists between parallel flow and counterflow for this example. This is not always the case, however. Counterflow is usually the preferred flow configuration.

Outlet temperatures for the first iteration were: water  $T = 146^\circ\text{F}$  and ethylene glycol  $t = 98^\circ\text{F}$  for counterflow. When the properties were re-evaluated at the new temperatures, the final results were (after several iterations): water  $T = 150^\circ\text{F}$  and ethylene glycol  $t = 97^\circ\text{F}$ . In this example, the temperature at which properties were evaluated had a small influence on the outlet temperatures. This occurred for these fluids under the conditions of the problem statement. For some fluids, however (such as oils), properties change more drastically with temperature and evaluating properties at a reasonable temperature becomes very important.

### Temperature Profile

Calculations made using the data of the previous example reveal a number of details worth mentioning. In order to examine the performance of the exchanger, we allow the length to increase and calculate outlet temperatures for both parallel and counterflow configurations. The results are graphed in Figures 7.7 and 7.8.

Figure 7.7 shows temperature variation with length for a counterflow arrangement. Length on the horizontal axis has been allowed to vary to 112 ft, selected because it is the distance required for the fluids to have equal outlet temperatures. The temperature of the warm fluid at inlet is  $175^\circ\text{F}$ ; and as length increases, the temperature decreases, following a curve that is concave downward. The cooler fluid enters the exchanger at  $90^\circ\text{F}$ ; and as length increases, temperature increases, following a curve that is also concave downward. Throughout the exchanger, the cooler fluid must travel a comparatively considerable distance for even a small temperature increase, whereas the warmer fluid experiences a greater temperature change. This is due to what is known as the *capacitance* (i.e.,  $\dot{m}C_p$  product) of the fluids. For the fluids shown,  $\dot{m}_c C_{pc} > \dot{m}_w C_{pw}$ , which means that  $(t_2 - t_1) < (T_1 - T_2)$ , because

$$\dot{m}_c C_{pc}(t_2 - t_1) = \dot{m}_w C_{pw}(T_1 - T_2) = q$$

The curve shapes are typical for this condition.

Figure 7.8 shows temperature profiles for the same exchanger operating in parallel (or unidirectional) flow. Note that even after 112 ft, the outlet temperatures are not equal, nor can they be made equal theoretically, unless the surface area (proportional to length) is infinite. (Actually, for this exchanger, outlet temperatures are equal at a length



**TABLE C.6. Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia).**

**Water Vapor or Steam** (gas constant = 461.5 J/(kg·K) = 85.78 ft·lbf/(lbm·°R);  $\gamma = C_p/C_v = 1.333$ )

Temp K	°R	Density $\rho$ kg/m <sup>3</sup> lbm/ft <sup>3</sup>		Specific Heat $C_p$		Kinematic Viscosity $\nu$ m <sup>2</sup> /s × 10 <sup>5</sup> ft <sup>2</sup> /s × 10 <sup>4</sup>		Thermal Conductivity $k$		Thermal Diffusivity $\alpha$ m <sup>2</sup> /s × 10 <sup>4</sup> ft <sup>2</sup> /hr		Prandtl Number Pr
				J kg·K	BTU lbm·°R			W m·K	BTU hr·ft·°R			
380	684	0.586	0.0366	2 060	0.492	2.16	2.33	0.024 6	0.0142	0.293 6	0.789	1.060
400	720	0.554	0.0346	2 014	0.481	2.42	2.61	0.026 1	0.0151	0.233 8	0.906	1.040
450	810	0.490	0.0306	1 980	0.473	3.11	3.35	0.029 9	0.0173	0.307	1.19	1.010
500	900	0.440	0.0275	1 985	0.474	3.86	4.16	0.033 9	0.0196	0.386	1.50	0.996
550	990	0.400	0.0250	1 997	0.477	4.70	5.06	0.037 9	0.0219	0.475	1.84	0.991
600	1080	0.0365	0.0228	2 026	0.484	5.66	6.09	0.042 2	0.0244	0.573	2.22	0.986
650	1170	0.338	0.0211	2 056	0.491	6.64	7.15	0.046 4	0.0268	0.666	2.58	0.995
700	1260	0.314	0.0196	2 085	0.498	7.72	8.31	0.050 5	0.0292	0.772	2.99	1.000
750	1350	0.293	0.0183	2 119	0.506	8.88	9.56	0.054 9	0.0317	0.883	3.42	1.005
800	1440	0.274	0.0171	2 152	0.514	10.20	10.98	0.059 2	0.0342	1.001	3.88	1.010
850	1530	0.258	0.0161	2 186	0.522	11.52	12.40	0.063 7	0.0368	1.130	4.38	1.019

Example of reading values: kinematic viscosity at 380 K is  $\nu = 2.16 \times 10^{-5} \text{ m}^2/\text{s}$

Source: Data taken from a number of references.



TABLE B.5. Properties of saturated liquids: water  $H_2O$ .

Temp °C	°F	Specific Gravity	Specific Heat $C_p$		Kinematic Viscosity $\nu$		Thermal Conductivity $k$		Thermal Diffusivity $\alpha$		Prandtl Number Pr
			$\frac{J}{kg \cdot K}$	$\frac{BTU}{lbm \cdot ^\circ R}$	$m^2/s \times 10^7$	$ft^2/s \times 10^6$	$\frac{W}{m \cdot K}$	$\frac{BTU}{hr \cdot ft \cdot ^\circ R}$	$m^2/s \times 10^7$	$ft^2/hr \times 10^3$	
0	32	1.002	4 217	1.0074	17.88	19.25	0.552	0.319	1.308	5.07	13.6
20	68	1.000	4 181	0.9988	10.06	10.83	0.597	0.345	1.430	5.54	7.02
40	104	0.994	4 178	0.9980	6.58	7.08	0.628	0.363	1.512	5.86	4.34
60	140	0.985	4 184	0.9994	4.78	5.14	0.651	0.376	1.554	6.02	3.02
80	176	0.974	4 196	1.0023	3.64	3.92	0.668	0.386	1.636	6.34	2.22
100	212	0.960	4 216	1.0070	2.94	3.16	0.680	0.393	1.680	6.51	1.74
120	248	0.945	4 250	1.015	2.47	2.66	0.685	0.396	1.708	6.62	1.446
140	284	0.928	4 283	1.023	2.14	2.30	0.684	0.395	1.724	6.68	1.241
160	320	0.909	4 342	1.037	1.90	2.04	0.670	0.393	1.729	6.70	1.099
180	356	0.889	4 417	1.055	1.73	1.86	0.675	0.390	1.724	6.68	1.004
200	392	0.866	4 505	1.076	1.60	1.72	0.665	0.384	1.706	6.61	0.937
220	428	0.842	4 610	1.101	1.50	1.61	0.572	0.377	1.680	6.51	0.891
240	464	0.815	4 756	1.136	1.43	1.54	0.635	0.367	1.639	6.35	0.871
260	500	0.785	4 949	1.182	1.37	1.48	0.611	0.353	1.577	6.11	0.874
280	537	0.752	5 208	1.244	1.35	1.45	0.580	0.335	1.481	5.74	0.910
300	572	0.714	5 728	1 368	1.35	1.45	0.540	0.312	1.324	5.13	1.109

$$\beta = 0.18 \times 10^{-3} / K = 0.10 \times 10^{-3} / ^\circ R$$

Example of reading values: kinematic viscosity at  $17^\circ C$  is  $\nu = 17.88 \times 10^{-7} m^2/s$

Source: Data taken from Handbook of Heat and Mass Transfer, by E. R. G. Eckert and R. M. Drake, Jr., Taylor & Francis Group—Washington Publishing Co., New York, N. Y. 1972. Used with permission of the publisher.

TABLE C.1. Physical properties of gases at room temperature and pressure.

Gas	Density $\rho$		Specific Heat $C_p$		Kinematic Viscosity $\nu$		Gas Constant $R$		$\gamma = C_p/C_v$
	$kg/m^3$	$lbm/ft^3$	$\frac{J}{kg \cdot K}$	$\frac{BTU}{lbm \cdot ^\circ R}$	$m^2/s \times 10^6$	$ft^2/s \times 10^5$	$\frac{J}{kg \cdot K}$	$\frac{ft \cdot lbf}{lbm \cdot ^\circ R}$	
Air	1.177	0.0735	1 005.7	0.240	15.68	16.88	0.026 24	0.01516	1.4
Carbon dioxide	1.797	0.1122	871	0.208	8.321	8.957	0.016 572	0.009575	1.3
Nitrogen	1.142	0.0713	1 040.8	0.2486	15.63	16.82	0.026 20	0.01514	1.4
Oxygen	1.300	0.0812	920.3	0.2198	15.86	17.07	0.026 76	0.01546	1.4

Example of reading values: kinematic viscosity of air is  $\nu = 15.68 \times 10^{-6} m^2/s$

MW = molecular weight (engineering units) = molecular mass (SI units)

Gas constant  $R = R_u/MW$ ;  $R_u = 1545 \text{ ft} \cdot \text{lbf}/\text{lbmol} \cdot ^\circ R = 49,700 \text{ ft} \cdot \text{lbf}/\text{slugmol} \cdot ^\circ R = 8\,312 \text{ N} \cdot \text{m}/(\text{mol} \cdot K)$

1 BTU = 778 ft·lbf

Source: Data taken from a number of sources; see Bibliography at the end of the text.



Table B.3 • Properties of ethylene glycol

TABLE B.3. Properties of saturated liquids: ethylene glycol  $C_2H_4(OH)_2$ .

Temp °C	Temp °F	Specific		Kinematic		Thermal		Prandtl Number Pr
		Specific Gravity	Heat $C_p$ $\frac{J}{kg \cdot K}$	Viscosity $\nu$ $\frac{m^2}{s} \times 10^6$	Conductivity $k$ $\frac{W}{m \cdot K}$	Diffusivity $\alpha$ $\frac{m^2}{s} \times 10^8$	Thermal Diffusivity $\alpha$ $\frac{ft^2}{hr} \times 10^3$	
0	32	1.130	2294	57.53	0.548	61.92	0.242	0.140
20	68	1.116	2382	19.18	0.569	20.64	0.249	0.144
40	104	1.101	2474	8.69	0.591	9.35	0.256	0.148
60	140	1.087	2562	4.75	0.612	5.11	0.260	0.150
80	176	1.077	2650	2.98	0.633	3.21	0.261	0.151
100	212	1.058	2742	2.03	0.655	2.18	0.263	0.152

$$\beta = 0.65 \times 10^{-3}/K = 0.36 \times 10^{-3}/^{\circ}R$$

Example of reading values: kinematic viscosity at 0°C is  $\nu = 57.53 \times 10^{-6} m^2/s$

Source: Data taken from Analysis of Heat and Mass Transfer, by E. R. G. Eckert and R. M. Drake, Jr., Taylor & Francis Group—Hemisphire Publishing Corp., New York, © 1987. Used with permission of the publisher.

Table B.4 • Properties of glycerol

TABLE B.4. Properties of saturated liquids: glycerol  $C_3H_5(OH)_3$ .

Temp °C	Temp °F	Specific		Kinematic		Thermal		Prandtl Number Pr
		Specific Gravity	Heat $C_p$ $\frac{J}{kg \cdot K}$	Viscosity $\nu$ $\frac{m^2}{s} \times 10^3$	Conductivity $k$ $\frac{W}{m \cdot K}$	Diffusivity $\alpha$ $\frac{m^2}{s} \times 10^8$	Thermal Diffusivity $\alpha$ $\frac{ft^2}{hr} \times 10^3$	
0	32	1.276	2261	8.31	0.540	8.95	0.282	0.163
10	50	1.270	2319	3.00	0.554	3.23	0.284	0.164
20	68	1.264	2386	1.18	0.570	1.27	0.286	0.156
30	86	1.258	2445	0.50	0.584	0.54	0.287	0.150