

I have read and agree to the collaboration policy. Stephen Woodbury.
Grading: Homework Heavy
Collaborators: none

Assignment 1_2 : Time Complexity

a) Ranking

Solution:

Ranking[by increasing rate of growth]: gacrepqmobhdfsjltikn

Equivalence classes arranged in increasing order of growth:

EC1:g EC2:a EC3:cr EC4:e EC5:pq EC6:mo EC7:bh

EC8:d EC9:fs EC10:j EC11:l EC12:t EC13:ik EC14:n

b) Statement Validity

Statement 1: Always True

PF: Plug into the definition of big Omega:

$0 \leq c * [\max(f(n), g(n))] \leq f(n) + g(n)$ for $n \geq b$.

This is true if $c=1$ and $b = 0$ (asymptotically non-negative)

Therefore the statement is always true.

Statement 2: Never True

PF: Analyze definition of little omega:

$0 \leq c * g(n) < f(n)$ for all values $c > 0$ and for all $n \geq 0$

Analyze definition of Big O:

$0 \leq f(n) \leq c' * g(n)$ for a value $c' > 0$ and for all $n \geq 0$

For $f(n) = w(g(n))$ and for $f(n) = O(g(n))$,

it must be that $0 \leq f(n) \leq c' * g(n)$ and $0 \leq c' * g(n) < f(n)$

There is no such constant c' that allows this.

Statement 3: Sometimes True

Example where statement is true: $f(n) = x$ and $g(n) = x^2$

$f(n) = O(g(n))$.

Example where statement is false: $f(n)=\sin(n)$ and $g(n)=\cos(n)$.

$\lim_{n \rightarrow \infty} f(n)/g(n) = \text{negative infinity to positive infinity}$. This shows that $f(n)$ doesn't exist in $O(g(n))$ and in turn $g(n)$ can't exist in big Omega $f(n)$.

$\lim_{n \rightarrow \infty} g(n)/f(n) = \text{negative infinity to positive infinity}$. This shows that $g(n)$ doesn't exist in $O(f(n))$ and in turn $f(n)$ can't exist in big Omega $g(n)$.

Therefore statement 3 isn't true here.