Stephen Woodbury 6/5/17 CMPS 102
I have read and agree to the collaboration policy. Stephen Woodbury.
Collaborators: none

**Assignment 4_3 : Flow Decomposition**

**a) Prove that every flow has at least one acyclic flow that has the same value, ie, every graph has at least one acyclic flow.**

*Proof:*

*1)* Given a max flow has been found, acyclic or not, means that whatever algorithm we run on our Graph, it must terminate as a max flow was found.

*2)* Let's think about what a cycle in a flow means. A cycle means that the end point is the same location as the start point. Now, let's assume there is an acyclic cycle within our max-flow discovered. Let's say that this cycle is composed of Edges e1, e2,...ek. This is to not lose generality. Let e1's source, v, be the starting point of our cycle.

*3)* If a max-flow includes a cycle, there must an edge somewhere that carries the flow away from the cycle at some point, this is by conservation of flow, it must reach t. If this edge/edges is distributed amongst any number of vertexes from v to vprev (end vertex of e[k-1]), then instead of completing the cycle, the flow may take those edges instead, never completing the cycle. If this edge is instead located at v, then there is no need to even embark on the cycle path if we can take the other path which by definition, must lead us back to t. if these edges are on v and part of vertices between v2-vprev, then they can be split immediately into the other path on v and ao the other paths along the verts of the cycle, still ensuring the cycle won't be complete.

**b) Prove acyclic flow is a combination of a finite number of path Flows**

*Proof:* Using the same reasoning above, we know that our cycle can be avoided by taking a number of branch edges that must stem our flow away from our cycle by order of conservation. This means that every time a branch edge is taken away from our cycle, is another path flow. Since we have a cycle at the termination of our flow, the total number of branch edges is only at most, the number of vertices in our cycle – 1. That means there are at most |V'| - 1 path flows (V' is the number of vertices in cycle). If every branch edge is taken to form a path flow, that means that every acyclic flow is made up of a finite number of path flows.

**c) Showing not all cycles that are part of a max flow are the sum of path flows.**

What if The Edges outside the the cycle were negative and the cycle is necessary to make the flow positive. That would make the cycle necessary.