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I have read and agree to the collaboration policy. Stephen Woodbury.
Grading: Homework Heavy
Collaborators: none

Assignment 1_3 : Induction

a) Uniform Shuffling

```
UniformShuffle(A)
1    for (i = n down to 1)
2         do
3              j = random integer such that 1<=j<=i
4              exchange A[i] and A[j]
5    return A
```

<u>Runtime Analysis:</u>
Line 1 : O(n)
Line 2 : no cost
Line 3 : O(1)
Line 4 : O(1)
Line 5 : O(1)
Total cost = O(n) * (3 * O(1)) + O(1)
Worst Case Runtime : O(n)

<u>Proof(By Induction)</u>:

I) Base Case :

1) Every element to be moved has a 1/n chance in being selected to be moved to the 0th index. They all have a n-1/n chance of being moved elsewhere throughout the rest of the algorithm (n-1 places other than the 0 position). Base Case 1 is fulfilled and the array is so far uniformly shuffled.

2) Next step is to place an element in the 1's place. Every element has a 1/(n-1) chance of being placed in the one's position and a (n-2)/(n-1) chance of being placed elsewhere. Once again, every element has an equal chance to be sorted, it's uniformly sorted so far.

II) Induction Step :

assume that for all k positions, the array has been uniformly sorted : induction hypothesis

[NTP: to sort array for k+1 position is uniform]

Analyze sorting array for k+1 position. There are n-k elements left to be sorted. The chance of being sorted into the k+1 position for all remaining elements is: 1/(n-k-1) while the probability of being placed into any other position for all other elements is (n-k-2) / (n-k-1). This is still uniformly sorted.

b) Error in Induction

The argument presented is sound for 3 or more buses. However, the argument does not apply when there are only two buses. There must be at least one bus that wasn't removed in the induction step to ensure that the removed buses are leading to the same destination. If there are only two buses, there is no overlap of sub groups formed in the induction step. This means that you can't prove through transitivity that the two buses are heading in the same direction. *The error in this induction proof is not providing for the base case b=2.* If the base case b=2 would have been proven true, then the induction proof would be true. However, since one can show that two buses may not have the same destination, the proof becomes invalid.