1) Let $SL_2(\mathbb{R})$ act on

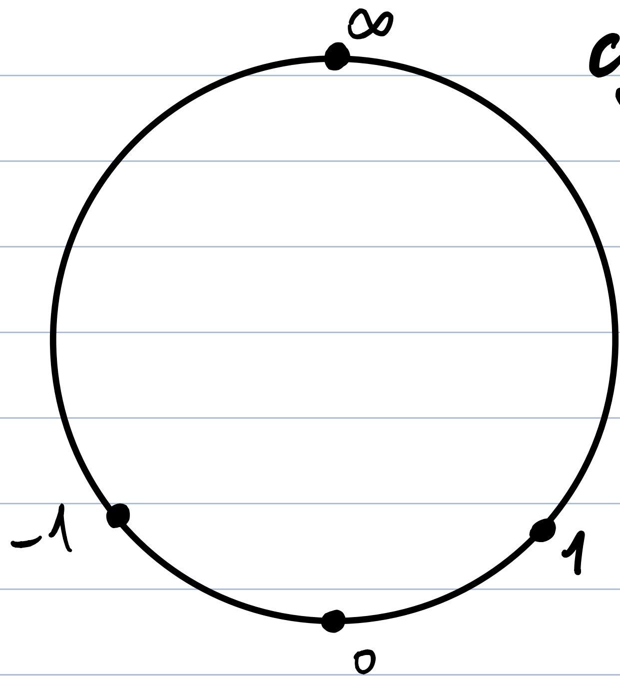
$$\mathbb{H}^2 = \{ z \in \mathbb{C} : \operatorname{Im}(z) > 0 \}$$

by Möbius transformations:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot z = \frac{az+b}{cz+d}.$$

Observe that the action extends continuously to $S^1 = \mathbb{R} \cup \{\infty\}$ (use "calc student" rules to work with ∞.)

a) if (a,b,c) is a triple in $\mathbb{R} \cup \{\infty\}$ s.t. $a<b<c$ in the **cyclic order** on S^1,



show that there exists $g \in SL_2(\mathbb{R})$ w/ $g.(0,1,\infty) = (a,b,c)$.

b) Show that the element g is unique up to multiplication by $-\mathrm{Id}$.

c) Show that if (a,b,c) belong to $\mathbb{Q} \cup \{\infty\}$, then $g \in SL_2(\mathbb{Z})$.

d) Conclude that $SL_2(\mathbb{Z})$ acts transitively on the triangles of the Farey diagram.

e) Compute the stabilizer of a triangle, and of an edge of the Farey diagram

e) ii. Why does each edge belong to exactly two triangles?

f) Does $GL_2(\mathbb{Z})$ act on the Farey diagram?

g) Write down a splitting of $GL_2(\mathbb{Z})$, and of $PSL_2(\mathbb{Z})$

h) What is the minimum index of a free subgroup of $PSL_2(\mathbb{Z})$?

1) What is the minimum rank of a finite-index free subgroup of $PSL_2(\mathbb{Z})$?

2) Let S_2 be a genus 2 surface. Write down two one-edge splittings with $\mathbb{Z}$ edge group.
Write down a splitting with two edges.
Compute presentations for $\pi_1(S_2)$ using your splittings.

3) Gersten's group is

$$\langle a, b, c, t : tat^{-1} = a,\ tbt^{-1} = ba,\ tct^{-1} = ca^2 \rangle.$$

Realize this group as the fundamental group of a graph of groups with one vertex w/ vertex group $\mathbb{Z}^2$ and two edge groups.

4) Describe the action of $BS(1,2) = \langle a, t : tat^{-1} = a^2 \rangle$ on its Bass-Serre tree.

5) Describe a graph of spaces with fundamental group Gersten's group.

(6) Suppose G acts on a tree T. Show that $\forall g \in G$, either

1) g fixes a connected subtree of T or
2) g acts freely.

If you know the terminology, in case 2), show that g has an **axis** in T.

7) Suppose the fixed-point sets of g and h are disjoint. Show that gh acts freely.

8) Suppose G is generated by $\{s_1, \ldots, s_m\}$ and suppose that G acts on a tree T. If each s_i and each $s_i s_j$ fix points of T, prove that G fixes a point.

9) Conclude that triangle groups with all labels finite have property FA.

10) Show that finitely generated torsion groups have property FA.

11) Describe splittings of $\mathbb{Q}$ and $\bigoplus_{i=1}^{\infty} C_2$.

12) Try exercises 1) and 2) in Serre's book **Trees** on p. 2. Find the mistake in 1)

13) Let $A*_C$ be an HNN extension. Write it as a semidirect product $G \rtimes \mathbb{Z}$.

14) Let $(\Gamma, \mathcal{G})$ be a splitting of $\pi_1(S_g)$ with all edge groups $\cong \mathbb{Z}$. What is the maximum number of edges of Γ? What is the euler characteristic of Γ?

15) Suppose Γ is a graph with $\pi_1(\Gamma) \cong F_n$, and suppose Γ has no vertices of valence ≤ 2.
What is the maximum number of edges of Γ?

Repeat for $(\Gamma, \mathcal{G})$ with trivial edge groups, C_2 or trivial vertex groups, $\pi_1 \cong W_n = \ast_{i=1}^{n} C_2$, such that vertices of Γ with valence ≤ 2 have C_2 vertex group.