ENERGY AND FLUID INTERACTIONS IN ROLLING CANS

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ABSTRACT

Two canned soups were rolled down inclined planes in order to find differences in their motion due to their internal fluid behaviour. The can of consommé rolled further than the can of cream soup at a lower angle, while the cream soup would roll further with a higher incline. Using linear and rotational motion relations, the moment of inertia, translational, and rotational energy could be found. On the low and high ramps respectively, it was found that (40±20)% and (40±8)% of the total kinetic energy in the can of consommé was rotational energy. In comparison, for the cream, (64±7)% and (54±5)% of the total kinetic energy was rotational kinetic energy for the low and high ramps, respectively. These findings supported the theory that the consommé would experience a greater shear force against the can walls, while the cream would mostly stick to the surface and result in a higher moment of inertia with sufficient velocity, thus creating greater oscillation in movement.

I INTRODUCTION

Fluid dynamics dictate the motion of gaseous and liquid matter as they are acted upon by external translational and rotational forces. In turn, these laws can describe common events such as weather patterns and ocean flow. Consequently, an understanding of fluid and rotational dynamics is essential to the calculation and creation of models to predict and understand practical phenomena.

II THEORY

The gravitational potential energy of an object on the surface of the Earth is given by:

$$V_g = mgh \qquad [1]$$

(HRW, 2011)

Where:

V_g = Gravitational potential energy (J)
g = Gravitational acceleration on Earth (ms^{-2})
h = Height of object above reference point (m)

The kinetic energy of a rolling object is the sum of its translational and rotational energy, which can be expressed as:

$$T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad [2]$$

(HRW, 2011)

Where:

T = Total kinetic energy (J)
m = Mass (kg)
v = Translational velocity (ms^{-1})
I = Moment of Inertia (kgm^2)
ω = Angular velocity (rad s^{-1})

Translational and rotational velocity in purely rolling objects are related using:

$$v = r\omega \qquad [3]$$

(HRW, 2011)

Where:

r = Radius (m)

In a purely rolling object, the following relation can be found:

$$a = \frac{g\sin\theta}{1+\frac{I}{mr^2}} \quad [4]$$

(HRW, 2011)

Where:

a = Acceleration (ms^{-2})
θ = Angle of incline (rad)

Rearranging [5] and solving for I:

$$I = mr^2\left(\frac{g\,sin\,\theta}{a} - 1\right) \quad [5]$$

III Method

Cans of cream of mushroom soup and beef consommé were purchased pursuant to van Bemmel (2019), each with a listed volume of 284mL. The mass and dimensions of both cans were measured with a digital scale and Vernier caliper. Ramps were made pursuant to van Bemmel (2019), and their dimensions were measured to find the angles of elevation. A board of the ramp's width was placed at the base of the ramp to facilitate the transfer off the ramp. Measurements were effected 30 times pursuant to the law of large numbers (Sternstein, 2017).

Then, meter sticks were taped onto the left and right sides of the board, outlining a path to assert the validity of each run. Runs were voided if the can contacted with either meter stick. The cans were filmed in slow motion rolling down the ramp; the number of rotations and rolling length of the ramp were compared to calculate slipping of the can, which was found to be negligible. The position, velocity, and acceleration were then measured with two Vernier Motion Detector 2s. Each sensor took 20 samples per second. One sensor was placed at the top of the ramp and the other was placed on the end of the flat board to capture the horizontal rolling motion. Sensors were tested prior to usage by calibrating and checking if distances were accurately read by the sensor across its used range. Landmarks of 20, 60, 80, and 100cm were measured by the sensor and checked using a meter stick, and sensors were only used if consistently accurate within 1mm.

IV Data

The dimensions of each can are:

	Consommé	Cream
$Diameter_{top}$ (cm)	6.75±.01	6.737±.008
$Diameter_{bottom}$ (cm)	6.74±.01	6.739±.006
Mass (±.1 g)	340.7	339.4

Fig 1. Dimensions of Consommé and Cream Cans The chart above displays the averages of the measured diameters and masses of each soup can.

The dimensions of the ramp at a low angle and high incline are:

	Low Ramp	High Ramp
Total Length (cm)	115.82±.08	115.82±.08
Rolled Length (cm)	95.85±.05	95.85±.05
Height (cm)	8.39±.06	25.22±.06

Fig 2. Dimensions of Ramp The chart above displays the averages of the measured lengths and heights of the two ramps, measured 30 times with a meter stick.

V Analysis

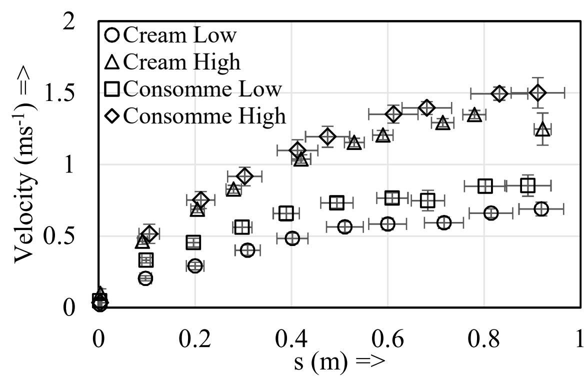

Fig 3. Linear Velocity versus Position. The average linear velocity of both cans rolled from each of the two heights is compared to the distance travelled at that point. Points are shown approximately every 0.1m for clarity.

Conservation of energy states that ΔT is equal to $-\Delta V_g$, and by [1], they begin with approximately equal V_g. Additionally, since the cans have very similar surfaces and masses, they experience similar magnitudes of frictional force. Therefore, the T of both cans should be equal at the same ramp position. However, as shown in Fig. 3, the translational velocity, and thus translational energy, of consommé is always higher than that of cream for a given angle. From [2], it follows that a greater portion of the kinetic energy is imparted as rotational kinetic energy on the can of cream. However, since one can rolls further at each incline, there is an internal force acting against rotation on the one that rolls less. Loss of kinetic energy is often due to friction, which occurs when the fluid rubs against the can's rotating shell.

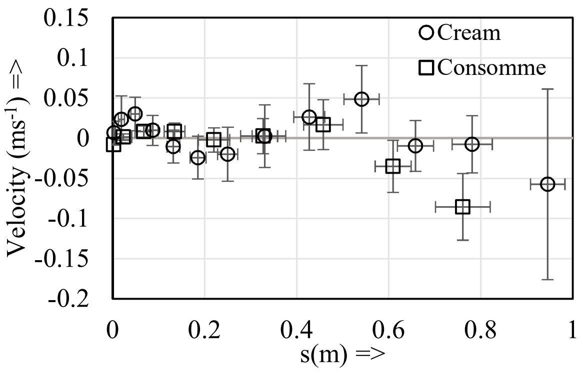

Fig 4. Velocity Difference versus Position. The oscillation found in each soup's linear velocity from the line of best fit as they rolled down the slight-angled ramp plotted against the rolled distance. Every four points are shown for clarity.

This can be understood by analyzing the oscillatory behaviour of can velocity. Since a rigid body has a linear trend in velocity, a linear regression of the cans' velocity can be subtracted from the measured velocity, with the difference being attributed to fluid movement in the cans. From Fig. 4, it is seen that the cream experiences more significant oscillation than the consommé on the low angle ramp. In particular, the cream low oscillation range was (7±5) cms^{-1}, greater than the consommé range of (2±6) cms^{-1}. Similar data was found at the high angle.

The consommé's relative lack of rotation shears against the rotating shell, and the shear stress is proportional to velocity (Princeton, 1996); thus, at high ramp angles that lead to high can velocities, there is a high rate of thermal energy loss. In contrast, the regular oscillation of the cream indicates that with a sufficient incline, the velocity, and thus, centripetal acceleration of the can creates a sufficient force to push the fluid against the can wall, enabling it to revolve about the can's central axis. The cream contributes to the can's forward motion when it revolves towards the front of the can, thus increasing torque, whereas it decreases torque when opposite the can's motion. If the angle of the incline is not high enough, the fluid does not gain enough speed to maintain the required force to complete revolutions, causing the soup to the bottom of the can and resulting in inefficient rolling. These factors cause the consommé to travel further than the cream at a low ramp angle, while the opposite is true at a high angle.

By [5], the moment of inertia of the cans with respect to time is calculated:

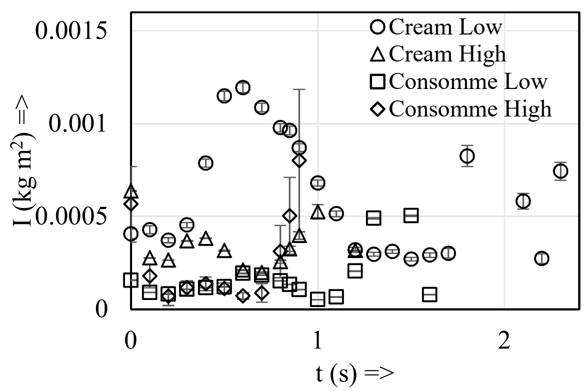

Fig 5. Moment of Inertia versus Time. The rotational energy of each can at each angle is compared to the time. Every other point is shown for clarity.

Then with [2] and [3], rotational kinetic energy with respect to time was found:

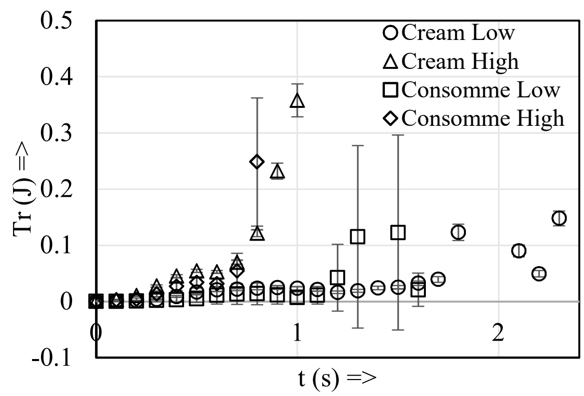

Fig 6. Rotational Energy versus Time. The rotational energy of each can at each angle is compared to the distance rolled by the can. Every other point is shown, and some extreme outliers were removed for clarity.

Finally, the total kinetic energy of the cans with respect to time is found using [2]:

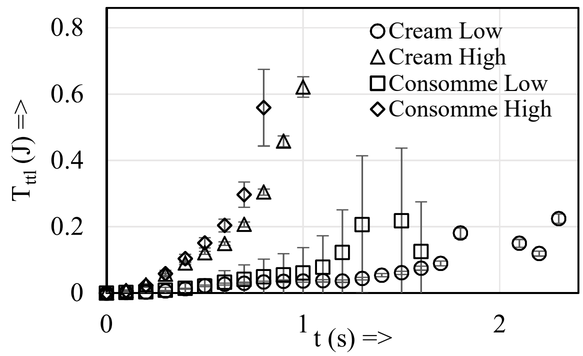

Fig 7. Total Kinetic Energy versus Time. The kinetic energy of each can at each angle is compared to the distance rolled by the can. Every other point shown for clarity.

Fig. 7 shows that the kinetic energy in the cream can as it leaves a high ramp is higher than that of the consommé's, while the opposite is true at a low angle. Specifically, the cream T_{ttl} reaches (0.62±.03) J and the consommé reaches (0.56±.09) J.

The higher exit energy of the cream is consistent with the results of it rolling further from high incline, and suggests the full participation of the fluid in rotation with less energy loss. Dividing rotational energy by total kinetic energy for each case, it was found that (64±7)% and (54±5)% of the cream's kinetic energy is rotational on the low and high ramp, respectively. Contrasting, (40±20)% and (40±8)% of the consommé's energy was in the form of rotational energy for the low and high ramps, respectively.

VI Sources of Error

Much of the analysis that was conducted utilized data from motion sensors on the velocity and acceleration of the cans. However, these values are generally computed as derivatives of the position function, and are therefore less precise and accurate (Vernier, 2019). As a result, even over large data sets, there may be outliers with large uncertainties that are impossible or do not follow the data's overall trend. Furthermore, high uncertainty combined with the manipulation of small measured values resulted in low signal-to-noise ratios. This uncertainty is apparent in Fig. 4, for example, when plotting the velocity oscillation.

Furthermore, despite efforts to allow the can to roll smoothly off the ramp, there was a slight bump at the end of the slope. In passing over this, there was a disturbance of the can's motion which could potentially be a source of the irregular data near the end of many runs.

VII Conclusion

Each soup was found to impact the conversion of potential energy into kinetic energy on each can in substantially differing manners. Specifically, (64±7)% and (54±5)% of the cream's kinetic energy was rotational energy on the low and high ramp, respectively, compared to (40±20)% and (40±8)% in the consommé can.

The theory that the consommé stays near the bottom while the cream revolves around the central axis of the can due to its higher viscosity was supported by the velocity graphs, with much higher oscillations for the cream soup. This theory explains the discrepancy in distances travelled by each can. At low inclines, the velocity is low, so the cream is unable to maintain its revolution around the cans central axis and energy is used inefficiently; at high velocities, the cream was able to maintain its revolution, while thermal energy was lost in the consommé can to friction from shear stress.

VIII Sources

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