EXPLORING DC TRANSIENTS AND SEMICONDUCTORS

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ABSTRACT

Three different circuits were constructed in order to find the properties and characteristic curves of a capacitor, diode, and transistor. In the RC circuit, the charging equation of the capacitor was found to have a time constant τ = (12.5±.1) s, while the discharging equation was found to have τ = (12.1±.1) s, The differences in the time constant were determined to be due to the resistance of the voltmeter device. The diode circuit's V-i curve was regressed with an R^2 of 0.92 using the Shockley equation, and had a strong fit of R^2=0.96 in forward bias. The measured threshold voltage of (0.67±.8) V conformed with the standard 0.7 V, but the reverse saturation current approached the maximum value of 5 μA. In the third circuit, from the amplification curves, it was found that the h_{FE} of the transistor is (136±4), with an O-C residual of (36±6).

I INTRODUCTION

Semiconductors, which have properties between those of insulators and conductors, are crucial to circuit components. Both diodes, which are akin to one-way valves for electrical current, and transistors, which may be used as switches or amplifiers, rely heavily on them. The function of these components can be verified by constructing circuits and analyzing their performance.

II THEORY

Kirchhoff's Laws state, at a junction in a circuit and in a loop of a circuit, respectively:

$$\sum i_n = 0 \quad [1]$$
(HRW, 2011)

$$\sum V_n = 0 \quad [2]$$
(HRW, 2011)

Where:
V_n = Potential across a load in the circuit (V)
i_n = Current in a wire at the junction (A)

The equations for potential over time for a capacitor in a series RC circuit are:

$$V_C(t) = V_0(1 - e^{-\frac{t}{\tau}}) \quad [3]$$
(HRW, 2011)

$$V_D(t) = V_0\, e^{-\frac{t}{\tau}} \quad [4]$$
(HRW, 2011)

Where:
$V_C(t)$ = Voltage across charging capacitor (V)
$V_D(t)$ = Voltage across discharging capacitor (V)
V_0 = Voltage across initial source (V)
t = Time (s)
τ = Time constant (s)

The time constant of an RC circuit is:

$$\tau = RC \quad [5]$$
(HRW, 2011)

Where:
R = Resistance (Ω)
C = Capacitance (F)

Ohm's Law states that:

$$V = iR \quad [6]$$
(HRW, 2011)

The total resistance of resistors in parallel is given by the following:

$$\frac{1}{R_S} = \sum \frac{1}{R_n} \quad [7]$$

(HRW, 2011)

Where:

R_S = Equivalent resistance (Ω)
R_n = Resistance of each resistor in parallel (Ω)

The current through a diode is given by the Shockley diode equation:

$$I(t) = I_s(e^{\frac{V_D}{nV_T}} - 1) \quad [8]$$

(Knipp, 2006)

Where:

I(t) = Current through diode at time t (A)
I_S = Reverse bias saturation current (A)
V_D = Potential difference at time t (V)
n = Ideality factor
V_T = Thermal voltage (V)

The relationship between the base current and collector current in a transistor is:

$$I_c = \beta I_B \quad [9]$$

(Najmabadi, 2012)

Where:

I_C = Collector current (A)
β = DC amplification factor, or h_{FE}
I_B = Base current (A)

III Method

A 1 mF capacitor with tolerance of 20%, a PN junction diode, and an NPN transistor were purchased pursuant to van Bemmel (2019). A 3 V battery and four 1.5 V batteries were measured with Vernier differential voltage probes. Probes took 50 measurements per second, and were only used if they measured within 10 mV of each other. Each resistor was measured and verified with two multimeters.

For the first experiment, the circuit shown in Fig. 1 was constructed. The R and C values were selected according to [5] for a theoretical time constant of 10 s. When the capacitor is close to full charge, it acts as a large resistor, leading to inaccurate voltmeter readings (van Bemmel, 2019). To circumvent this, potential was measured across the resistor. After each charge, a discharge was quickly done to ensure minimum leakage.

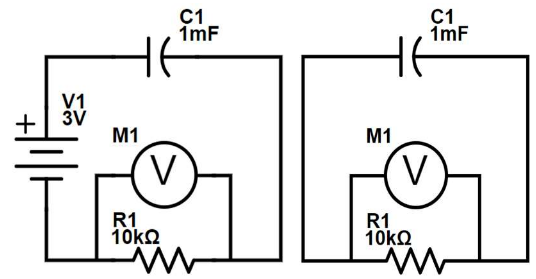

Fig 1. RC Circuits. The circuits for the charging (left) and discharging (right) of a capacitor. The potential across the resistor in the circuit was measured.

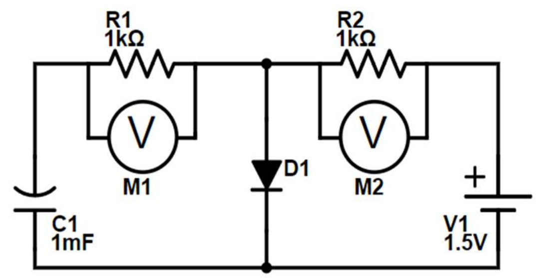

Fig 2. Diode Circuit. The circuit that was constructed to measure the diode's V-i curve from reverse to forward bias.

Then, the circuit shown in Figure 2 was constructed. For each run, the capacitor was charged to reach a potential difference double that of V1. The capacitor was quickly set opposite to V1 to bring the diode from a point of reverse bias to forward bias.

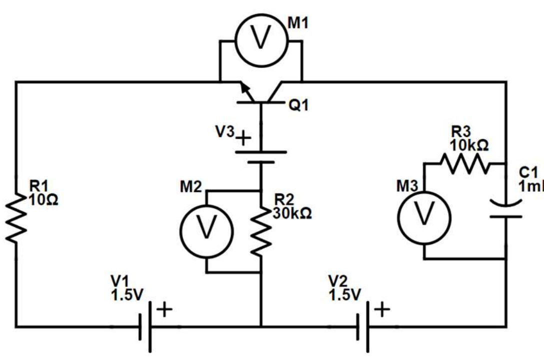

Fig 3. Transistor Circuit. The circuit that was constructed to measure the amplification curve of a basic NPN transistor.

The third circuit was constructed pursuant to Fig 3. To achieve a variable voltage, the capacitor was charged as it was in the second circuit, and then placed in the circuit to discharge. The large difference between R2 and R1 ensured most of the current remained in the circuit's exterior path. The middle path then maintained a near constant base current supplied by V3. To achieve four base currents, four values of V3 were used. A large resistor was put in series with the voltmeter measuring the capacitor to add to the branch's resistance and reduce the error caused by the capacitor's high resistance. This method was verified, as it returned results conforming to theory and to experiment 1.

IV DATA

The following batteries and resistors were used in the constructed circuits:

Index	Circuit	Cell	Differential (±.01 V)
1	1	V1	2.88
2	2	V1	1.42
3	3	V1	1.36
4	3	V2	1.25
5	3	V3	0.26
6	3	V3	1.49
7	3	V3	2.91
8	3	V3	4.37

Fig 4. Battery Measurements. The measured voltages across each battery that was used in a circuit. The cell names refer to those shown in Fig. 1, Fig. 2, and Fig. 3. Indices 5-8 refer to the four voltages used for V3.

Index	Circuit	Resistor	Resistance (Ω)
1	1	R1	10300±100
2	2	R1	1020±10
3	2	R2	990±10
4	3	R1	10.1±.1
5	3	R2	29700±100

Fig 5. Resistor Measurements. The measured resistances across each resistor that was used in a circuit. The cell names refer to those shown in Fig. 1, Fig. 2, and Fig. 3.

V ANALYSIS

In the first circuit, [2] was used to find potential over time across the capacitor. For the charging curve, the data was fit pursuant to [3], producing the following relation:

$$V(t) = (2.870 \pm .005)\ (1 - e^{\frac{-t}{(12.5\pm.1)}}) \quad [10]$$

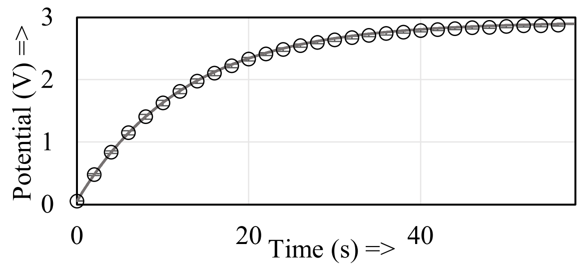

Fig 6. Charging Curve. The potential of the charging capacitor with respect to time. Data was fit pursuant to [3]; the fit had an R^2 value of 0.99. (1.67% of data points shown).

For the discharging curve, the data was fit to the general form in [4], to produce the following discharging relation:

$$V(t) = (2.883 \pm .006)\ e^{\frac{-t}{(12.1\pm.1)}} \quad [11]$$

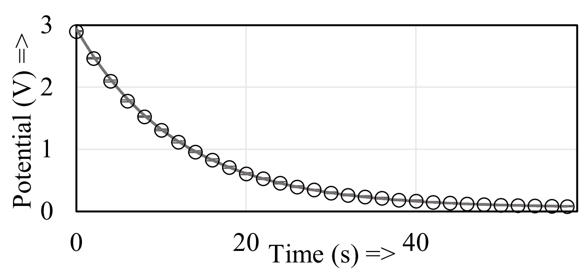

Fig 7. Discharging Curve. The potential of the discharging capacitor with respect to time. Data was fit pursuant to [4]; the fit had an R^2 value of 0.99 (1.67% of data points shown).

The time constants from [10] and [11] were compared to theoretical values. From [5], the time constant is (10±2) s, which is lower than both measured values. It is posited that this discrepancy is due to parasitic capacitance, which is a capacitive effect due to neighbouring electrical components with a voltage differential, such as with alligator clips used in circuit wiring (Heath, 2018).

Furthermore, by comparing [10] to [11], it is seen that τ differs nontrivially when charging compared to discharging. It was found that when the capacitor was discharged solely through the voltmeter by attaching the meter in series with a resistor, the differential would drop by (0.102±.008) V in the first 15 seconds. From [10], the charging circuit's capacitance was (1.21±.02) mF. Thus, the probe resistance was R_{probe} = (340±70) kΩ.

Then, using [7], the equivalent resistance of R1 and R_{probe} in parallel was found to be (10000±200) Ω. From this, the expected time constant considering voltmeter resistance is found to be τ = (12.1±.4) s. This value for the discharging time constant is close to the measured value in [11]. Thus, it is shown quantitatively that the difference in charging and discharging times is largely due to capacitor discharge through the voltmeter.

In the second circuit, the potential and current across each resistor were found with [2] and [6], respectively. In the top junction, the current into the diode was found by [1], and plotted against the voltmeter readings. The plot was regressed to a function in the form of [8], producing the following relation:

$$I(t) = (1.0 \pm .3)\left(e^{(31.1 \pm .4)V_D} - 1\right)10^{-14} \quad [12]$$

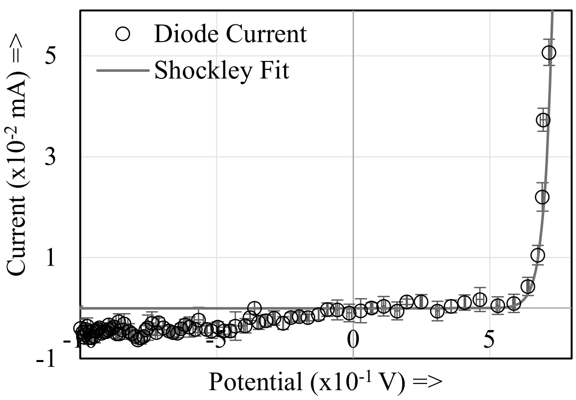

Fig 8. V-I Curve of Diode. The average current through the diode with respect to the potential difference across it. A fit is shown using the Shockley diode equation, with an R^2 value of 0.92. (3.33% of data points shown).

Analyzing O-C residuals for the regression, the R^2 value was given as 0.92. However, when considering only the forward bias region, the R^2 was 0.96, indicating that the Shockley diode equation is a strong model of diode behaviour. The estimated threshold voltage is also pursuant to the typical magic number of 0.7 V for silicon diodes. A piecewise-linear model regression was conducted on the linear region which gave a threshold voltage of (0.67±.08) V.

The majority of residuals appear in the reverse bias region, where the saturation current is much greater than the regressed (1.0±.3)E-14 A value. The unideal leakage of the diode may be influenced by temperature changes, which increase the flux of minority carriers, or by poor manufacturing. However, the NTE588 datasheet allows a maximum reverse current of 5 μA (NTE, 2013), so this saturation current is in reason.

[1], [2], and [6] were used to calculate the CE voltage, CE current, and base current in the third circuit. It is known that the transient current has exponential properties (Lee, 2017). Furthermore, it is seen that a portion of the CE current is linear. The current is thus fit as a piecewise function and is plotted against potential difference below:

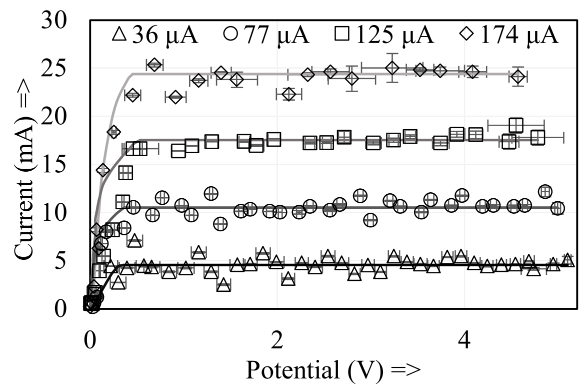

Fig 9. Amplification Curves. The amplification curves for four different base currents are shown. The legend indicates base current for each curve. (25.0% of data points shown).

Fig. 8 illustrates three regions. In the saturation region, the transistor acts as a forward biased diode with little resistance, as shown from 0 V to approximately 0.5 V. The collector current is largely independent of the base. In the cutoff region below the amplification curve, both ends of the transistor are forward-biased, restricting both I_B and I_C to approximately 0 mA and preventing amplification. The active region, between saturation and cutoff, is where the model for the collector current has zero slope, and I_C can be modelled pursuant to [9] (Walkey, 2004). Analyzing the active region using the measured base currents and the listed h_{FE} of (100±5) for the given transistor (LGE, 2005), a set of maximums for I_C are found. A comparison to the measured value can then be made.

Index	Base Current (±2μA)	Calculated Maximum I_C (mA)	Measured Maximum I_C (mA)
1	36	3.6±.4	4.55±.01
2	77	7.7±.6	10.52±.02
3	125	12.5±.8	17.54±.04
4	174	17±1	24.39±.06

Fig 10. Base and Collector Current. The four measured base currents, along with their respective expected and measured values of collector currents.

O-C residuals and the measured h_{FE} value of the transistor can then be calculated:

Index	Base Current (±2μA)	Residuals for I_C (mA)	Measured h_{FE}
1	36	0.9±.4	126±7
2	77	1.4±.1	137±4
3	125	5.0±.8	140±3
4	174	7±1	140±2

Fig 11. O-C Values of Transistor. The O-C residuals for the transistor's collector currents for each I_B, along with the resulting measured h_{FE}.

It can be seen that there are nontrivial discrepancies in the O-C residuals of I_C. However, it is shown that the measured h_{FE} values remain relatively constant. Taking an average yields a functional β = (136±4). Comparing this to the provided value of (100±5) yields an O-C residual of (36±6). Since transistor h_{FE} values often stray from the literature value (Cook, 2015), it is likely that the O-C residuals arose mostly from an h_{FE} value varying from the stated value.

VI Sources of Error

It was assumed that components had trivial temperature changes over time. However, resistors dissipate thermal energy when current is run through them (HRW, 2011), which may be nontrivial. This may lead to higher resistances, and in turn a higher experimental τ value in the first circuit, and higher thermal voltage in the second circuit.

Ideal batteries and wires have no resistance. However, in the real world, both have small internal resistances, effectively acting as resistors in series. By [5], this would result in a greater τ value.

VII Conclusion

In the simple RC circuit, the charging and discharging equations of the capacitor were found to have τ values of (12.5±.1) s and (12.1±.1) s, respectively. This lack of symmetry in the charging and discharging circuits likely arose from the voltmeter resistance when the capacitor discharged.

For the diode circuit, the V-i curve of a PN-junction diode was fitted to produce $I(t) = (1.0 \pm .3)\left(e^{(31.1\pm.4)V_D} - 1\right)10^{-14}$. The forward bias R^2 was 0.96, indicating a strong fit, and the (0.67±.8) V threshold voltage of the piecewise-linear model agreed with the standard 0.7 V. The reverse saturation current was higher than the regressed (10±3) fA, but stayed within the 5 μA max for the diode.

In the third circuit, the four different collector and base currents in the active region were used in conjunction with [9] to solve for a measured value of β = (136±4), which has O-C residuals of (36±6) when compared to the literature value.

VIII Sources

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