

PHYSICS

CONCEPTS AND CONNECTIONS

BOOK TWO

Brian Heimbecker
Igor Nowikow
Christopher T. Howes
Jacques Mantha
Brian P. Smith
Henri M. van Bemmel

Don Bosomworth, Physics Advisor



IRWIN PUBLISHING

Toronto/Vancouver, Canada

Copyright © 2002 by Irwin Publishing Ltd.

National Library of Canada Cataloguing in Publication Data

Heimbecker, Brian
Physics: concepts and connections two

For use in grade 12
ISBN 0-7725-2938-8

1. Physics. I. Nowikow, Igor. II. Title.

QC23.N683 2002 530 C2002-900508-6

All rights reserved. It is illegal to reproduce any portion of this book in any form or by any means, electronic or mechanical, including photocopy, recording or any information storage and retrieval system now known or to be invented, without the prior written permission of the publisher, except by a reviewer who wishes to quote brief passages in connection with a review written for inclusion in a magazine, newspaper, or broadcast.

Any request for photocopying, recording, taping, or for storing of informational and retrieval systems, of any part of this book should be directed in writing to CANCOPY (Canadian Reprography Collective), One Yonge Street, Suite 1900, Toronto, ON M5E 1E5.

Cover and text design: Dave Murphy/ArtPlus Ltd.

Page layout: Leanne O'Brien, Beth Johnston/ArtPlus Ltd.

Illustration: Donna Guilfoyle, Sandy Sled, Joelle Cottle, Nancy Charbonneau/
ArtPlus Ltd., Dave McKay, Sacha Warunkiw, Jane Whitney

ArtPlus Ltd. production co-ordinator: Dana Lloyd

Publisher: Tim Johnston

Project developer: Doug Panasis

Editor: Lina Mockus-O'Brien

Photo research: Imagineering, Martin Tooke

Indexer: May Look

Published by

Irwin Publishing Ltd.

325 Humber College Blvd.

Toronto, ON M9W 7C3

Printed and bound in Canada

2 3 4 05 04 03 02

We acknowledge for their financial support of our publishing program, the Canada Council, the Ontario Arts Council, and the Government of Canada through the Book Publishing Industry Development Program (BPIDP).

Acknowledgements

The authors and the publisher would like to thank the following reviewers for their insights and suggestions.

Bob Wevers, Teacher, Toronto, Toronto District School Board
Vince Weeks, Teacher, Burlington, Halton District School Board
Peter Mascher, Department of Engineering Physics, McMaster University
Andy Auch, Teacher, Windsor-Essex District School Board
Peter Stone, Teacher, Simcoe County District School Board
George Munro, Teacher, District School Board of Niagara
Brendan Roberts, Teacher, Windsor-Essex Catholic District School Board

To my wife Laurie and my children Alyssa and Emma for making it possible for me to do this one more time.

I would like to thank David Badregon and Vanessa Mann for their contributions to the problems and their solutions.

Brian Heimbecker

I would like to dedicate this book to my family: my wife Jane, my children Melissa and Cameron, my mom Alla, and my brother Alex, as well as all my students.

Special thanks to the students who worked on various aspects of solutions and research: Ashley Pitcher, Roman and Eugene Zassoko, Teddy Lazongas, and Katherine Wetmore.

Igor Nowikow

Dedicated to my wife Marcy and daughter Alison, for their never-ending love and support. In memory of the late Violet Howes and her passion for teaching.

I would like to thank Devin Smith (Queen's University), Kristen Koopmans (McMaster University), Jon Ho (University of Waterloo), and Paul Finlay (University of Guelph) for their solutions to the problems.

Christopher T. Howes

To my wife Lynda for her support and encouragement, and to all my students who make physics fun. I would like to thank Tyler Samson, a student at Confederation Secondary School in Val Caron, for his contribution as a problem solver.

Jacques Mantha

I would like to thank my wife Judy and daughter Erin for their valuable suggestions, and my son Brad for his careful solutions to the problems.

Brian P. Smith

I would like to dedicate my portion of this effort to my wife Nadine for her love and support and to my parents, Hank and Enes, for showing me how to work. Furthermore, I would like to acknowledge these wonderful students who assisted in this effort: Valeri Dessiatnichenko, Mehmod Ul Hassan, Huma Fatima Shabbir, and Kunaal Majmudar.

Henri M. van Bemmel

Table of Contents

To the Student	x	2.6	String-and-pulley Problems	93
	1	2.7	Uniform Circular Motion	98
		2.8	Centripetal Force	103
			Centripetal Force and Banked Curves	106
	4		Centrifugation	107
	5		Satellites in Orbit	109
	5		STSE — The Tape-measure Home Run	112
	7		Summary	114
	7		Exercises	115
	8		Lab 2.1 — Projectile Motion	122
	9		Lab 2.2 — Centripetal Force and Centripetal Acceleration	123
	10		Lab 2.3 — Amusement Park Physics	126
	19	3	Extension: Statics — Objects and Structures in Equilibrium	127
	19	3.1	Keeping Things Still: An Introduction to Statics	128
	20	3.2	The Centre of Mass — The Gravity Spot	128
	24	3.3	Balancing Forces ... Again!	130
	24	3.4	Balancing Torques	134
	32	3.5	Static Equilibrium: Balancing Forces and Torque	139
	33	3.6	Static Equilibrium and the Human Body	148
	34	3.7	Stability and Equilibrium	155
	35	3.8	Elasticity: Hooke's Law	159
	36	3.9	Stress and Strain — Cause and Effect	161
	39	3.9	Stress: The Cause of Strain	161
	44	3.9	Strain: The Effect of Stress	163
	48	3.10	Stress and Strain in Construction	170
	50	3.10	STSE — The Ultimate Effect of Stress on a Structure	172
	52	3.10	Summary	174
	54	3.10	Exercises	175
	55	3.10	Lab 3.1 — Equilibrium in Forces	181
	61	3.10	Lab 3.2 — Balancing Torque	183
	62			
	63	B	Energy and Momentum	185
	64	4	Linear Momentum	188
	64	4.1	Introduction to Linear Momentum	189
	70	4.2	Linear Momentum	189
	71	4.3	Linear Momentum and Impulse	190
	74	4.3	Force-versus-Time Graphs	195
	78	4.4	Conservation of Linear Momentum in One Dimension	199
	85	4.5	Conservation of Linear Momentum in Two Dimensions	203
	89			

4.6	Linear Momentum and Centre of Mass	211	Three Types of Damping	308
	STSE — Recreational Vehicle Safety and Collisions	214	Applications of Damping	309
	Summary	216	Shock Absorbers	309
Exercises		217	STSE — The International Space Station	310
Lab 4.1 — Linear Momentum in One Dimension: Dynamic Laboratory Carts		222	Summary	312
Lab 4.2 — Linear Momentum in Two Dimensions: Air Pucks (Spark Timers)		224	Exercises	313
Lab 4.3 — Linear Momentum in Two Dimensions: Ramp and Ball		227	Lab 6.1 — The Pendulum	316
5 Energy and Interactions		229	7 Angular Motion	317
5.1	Introduction to Energy	230	7.1 Introduction	318
	Isolation and Systems	230	7.2 A Primer on Radian Measure	318
5.2	Work	233	7.3 Angular Velocity and Acceleration	322
	Work from an \vec{F} -versus- Δd Graph	237	Angular Velocity	322
5.3	Kinetic Energy	239	Relating Angular Variables to Linear Ones	323
	Kinetic Energy and Momentum	241	More About Centripetal Acceleration	325
5.4	Gravitational Potential Energy	243	7.4 The Five Angular Equations of Motion	327
5.5	Elastic Potential Energy and Hooke's Law	249	7.5 Moment of Inertia	332
	Conservation of Energy	253	Extension: The Parallel-axis Theorem	337
5.6	Power	255	7.6 Rotational Energy	339
5.7	Elastic and Inelastic Collisions	260	7.7 Rotational Kinetic Energy	342
	Equations for One-dimensional Elastic Collisions	260	7.8 The Conservation of Energy	344
	Graphical Representations of Elastic and Inelastic Collisions	266	7.9 Angular Momentum	347
STSE — The Physics Equation — The Basis of Simulation		270	7.10 The Conservation of Angular Momentum	348
Summary		272	7.11 The Yo-yo	352
Exercises		273	Energy Analysis	352
Lab 5.1 — Conservation of Energy Exhibited by Projectile Motion		280	Force Analysis	352
Lab 5.2 — Hooke's Law		281	STSE — Gyroscopic Action — A Case of Angular Momentum	354
Lab 5.3 — Inelastic Collisions (Dry Lab)		282	Summary	357
Lab 5.4 — Conservation of Kinetic Energy		283	Exercises	358
6 Energy Transfer			Lab 7.1 — Rotational Motion: Finding the Moment of Inertia	365
6.1	Gravity and Energy	284		
	A Comparison of $\Delta E_p = mg\Delta h$ and $E_p = \frac{-GMm}{r}$	285	D C Electric, Gravitational, and Magnetic Fields	367
	Kinetic Energy Considerations	289	8 Electrostatics and Electric Fields	370
	Escape Energy and Escape Speed	290	8.1 Electrostatic Forces and Force Fields	371
	Implications of Escape Speed	292	8.2 The Basis of Electric Charge — The Atom	371
6.2	Orbits	293	8.3 Electric Charge Transfer	373
	Kepler's Laws of Planetary Motion	295	Charging by Friction	374
	Kepler's Third Law for Large Masses	298	Charging by Contact and Induction	375
	Extension: Orbital Parametres	300	8.4 Coulomb's Law	377
6.3	Simple Harmonic Motion — An Energy Introduction	301	The Vector Nature of Electric Forces between Charges	384
	Hooke's Law	303	8.5 Fields and Field-mapping Point Charges	388
6.4	Damped Simple Harmonic Motion	304	Force at a Distance	388
		308	8.6 Field Strength	394
			Coulomb's Law Revisited	395
			Electricity, Gravity, and Magnetism: Forces at a Distance and Field Theory	398

8.7	Electric Potential and Electric Potential Energy	400	Terminology	488
8.8	Movement of Charged Particles in a Field — The Conservation of Energy	404	Phase Shift	490
	The Electric Potential around a Point Charge	409	Simple Harmonic Motion: A Closer Look	491
8.9	The Electric Field Strength of a Parallel-plate Apparatus	414	Simple Harmonic Motion in Two Dimensions	492
	Elementary Charge	415	10.3 Electromagnetic Theory	494
	STSE — Electric Double-layer Capacitors	418	Properties of Electromagnetic Waves	494
	Summary	421	The Speed of Electromagnetic Waves	494
	Exercises	422	The Speed of Light	495
	Lab 8.1 — The Millikan Experiment	430	The Production of Electromagnetic Radiation	497
	Lab 8.2 — Mapping Electric Fields	433	10.4 Electromagnetic Wave Phenomena: Refraction	500
9	Magnetic Fields and Field Theory	435	The Refractive Index, n — A Quick Review	500
9.1	Magnetic Force — Another Force at a Distance	436	Snell's Law: A More In-depth Look	502
9.2	Magnetic Character — Domain Theory	437	Refraction in an Optical Medium	504
9.3	Mapping Magnetic Fields	438	Dispersion	505
9.4	Artificial Magnetic Fields — Electromagnetism	441	The Spectroscope	506
	Magnetic Character Revisited	442	10.5 Electromagnetic Wave Phenomena: Polarization	507
	A Magnetic Field around a Coiled Conductor (a Solenoid)	443	Polarization of Light using Polaroids (Polarizing Filters)	508
9.5	Magnetic Forces on Conductors and Charges — The Motor Principle	447	Malus' Law: The Intensity of Transmitted Light	509
	The Field Strength around a Current-carrying Conductor	451	Polarization by Reflection	511
	The Unit for Electric Current (for Real this Time)	453	Polarization by Anisotropic Crystals	512
	Magnetic Force on Moving Charges	456	Applications of Polarization	514
9.6	Applying the Motor Principle	460	Polarizing Filters in Photography	514
	Magnetohydrodynamics	460	3-D Movies	515
	Centripetal Magnetic Force	461	Radar	516
	The Mass of an Electron and a Proton	462	Liquid Crystal Displays (LCDs)	516
	The Mass Spectrometer	464	Photoelastic Analysis	517
9.7	Electromagnetic Induction — From Electricity to Magnetism and Back Again	467	Polarization in the Insect World	518
	STSE — Magnetic Resonance Imaging (MRI)	472	Polarized Light Microscopy	518
	Summary	474	Measuring Concentrations of Materials in Solution	518
	Exercises	475	10.7 Electromagnetic Wave Phenomena: Scattering	519
	Lab 9.1 — The Mass of an Electron	479	STSE — Microwave Technology: Too Much Too Soon?	522
			Summary	524
			Exercises	525
			Lab 10.1 — Investigating Simple Harmonic Motion	529
			Lab 10.2 — Polarization	530
			Lab 10.3 — Malus' Law	531
D	The Wave Nature of Light	481		
10	The Wave Nature of Light	484	11 The Interaction of Electromagnetic Waves	532
10.1	Introduction to Wave Theory	485	11.1 Introduction	533
	Definitions	485	11.2 Interference Theory	534
	Types of Waves	486	Path Difference	535
10.2	Fundamental Wave Concepts	488		

Two-dimensional Cases	536	12.6 The Bohr Atom	608
11.3 The Interference of Light	537	The Conservation of Energy	609
11.4 Young's Double-slit Equation	538	The Conservation of Angular Momentum	610
11.5 Interferometers	544	Electron Energy	612
Extension: Measuring Thickness using an Interferometer	545	Photon Wavelength	613
Holography	546	Ionization Energy	614
11.6 Thin-film Interference	548	Bohr's Model applied to Heavier Atoms	614
Path Difference Effect	548	The Wave-Particle Duality of Light	614
The Refractive Index Effect	549	12.7 Probability Waves	615
Combining the Effects	549	12.8 Heisenberg's Uncertainty Principle	617
11.7 Diffraction	553	A Hypothetical Mechanical Example of Diffraction	617
Wavelength Dependence	553	Heisenberg's Uncertainty Principle	621
11.8 Single-slit Diffraction	554	and Science Fiction	621
The Single-slit Equation	555	12.9 Extension: Quantum Tunnelling	622
More Single-slit Equations (but they should look familiar)	559	STSE — The Scanning Tunnelling Microscope	624
Resolution	561	Summary	626
11.9 The Diffraction Grating	563	Exercises	627
The Diffraction-grating Equation	564	Lab 12.1 — Hydrogen Spectra	630
11.10 Applications of Diffraction	569	Lab 12.2 — The Photoelectric Effect I	631
A Grating Spectroscope	569	Lab 12.3 — The Photoelectric Effect II	632
Extension: Resolution — What makes a good spectrometer?	569	13 The World of Special Relativity	633
X-ray Diffraction	571	13.1 Inertial Frames of Reference and Einstein's First Postulate of Special Relativity	634
STSE — CD Technology	574	13.2 Einstein's Second Postulate of Special Relativity	637
Summary	576	13.3 Time Dilation and Length Contraction	640
Exercises	578	Moving Clocks Run Slow	640
Lab 11.1 — Analyzing Wave Characteristics using Ripple Tanks	583	Moving Objects Appear Shorter	643
Lab 11.2 — Qualitative Observations of the Properties of Light	586	13.4 Simultaneity and Spacetime Paradoxes	646
Lab 11.3 — Comparison of Light, Sound, and Mechanical Waves	587	Simultaneity	646
Lab 11.4 — Finding the Wavelength of Light using Single Slits, Double Slits, and Diffraction Gratings	588	Paradoxes	647
D E Matter–Energy Interface	589	Spacetime Invariance	649
12 Quantum Mechanics	592	13.5 Mass Dilation	652
12.1 Introduction	593	Electrons Moving in Magnetic Fields	656
Problems with the Classical or Wave Theory of Light	593	13.6 Velocity Addition at Speeds Close to c	659
12.2 The Quantum Idea	594	13.7 Mass–Energy Equivalence	662
Black-body Radiation	595	Relativistic Momentum	663
The Black-body Equation	596	Relativistic Energy	664
12.3 The Photoelectric Effect	598	13.8 Particle Acceleration	668
The Apparatus	598	STSE — The High Cost of High Speed	674
12.4 Momentum and Photons	603	Summary	676
12.5 De Broglie and Matter Waves	606	Exercises	677
		Lab 13.1 — A Relativity Thought Experiment	683
		14 Nuclear and Elementary Particles	685
		14.1 Nuclear Structure and Properties	686
		Isotopes	687
		Unified Atomic Mass Units	687
		Mass Defect and Mass Difference	688

Nuclear Binding Energy and Average Binding Energy per Nucleon	688	Appendix B: Lab Report	752
14.2 Natural Transmutations	690	Lab Report	752
Nuclear Stability	690	Statistical Deviation of the Mean	753
Alpha Decay	691	Appendix C: Uncertainty Analysis	755
Beta Decay	693	Accuracy versus Precision	755
β^- Decay (Electron Emission)	693	Working with Uncertainties	755
β^+ Decay (Positron Emission)	695	Making Measurements with Stated	
Electron Capture and Gamma Decay	695	Uncertainties	755
14.3 Half-life and Radioactive Dating	697	Manipulation of Data with Uncertainties	756
Half-life	697	Addition and Subtraction of Data	756
Radioactive Dating	698	Multiplication and Division of Data	757
14.4 Radioactivity	700	Appendix D: Proportionality Techniques	758
Artificial Transmutations	700	Creating an Equation from a Proportionality	758
Detecting Radiation	703	Finding the Correct Proportionality	
14.5 Fission and Fusion	706	Statement	759
Fission	707	Finding the Constant of Proportionality	
Fission Reactors	710	in a Proportionality Statement	761
The CANDU Reactor	711	Other Methods of Finding Equations	
Fusion	712	from Data	761
Creating the Heavy Elements	715	Appendix E: Helpful Mathematical Equations	
Comparing Energy Sources — A Debate	717	and Techniques	765
14.6 Probing the Nucleus	718	Mathematical Signs and Symbols	765
14.7 Elementary Particles	720	Significant Figures	765
What is matter?	720	The Quadratic Formula	766
What is matter composed of?	721	Substitution Method of Solving Equations	766
The Standard Model	721	Rearranging Equations	766
Leptons	721	Exponents	767
Quarks	723	Analyzing a Graph	767
Hadrons (Baryons and Mesons)	723	Appendix F: Geometry and Trigonometry	768
14.8 Fundamental Forces and Interactions — What holds these particles together?	727	Trigonometric Identities	768
Forces or interactions?	727	Appendix G: SI Units	770
Boson Exchange	728	Appendix H: Some Physical Properties	773
Feynman Diagrams	729	Appendix I: The Periodic Table	774
Quantum Chromodynamics (QCD): Colour Charge and the Strong Nuclear Force	731	Appendix J: Some Elementary Particles	
The Weak Nuclear Force — Decay and Annihilations	736	and Their Properties	775
STSE — Positron Emission Tomography (PET) Summary	739	Numerical Answers to Applying the Concepts	776
Exercises	741	Numerical Answers to End-of-chapter Problems	780
Lab 14.1 — The Half-life of a Short-lived Radioactive Nuclide	747	Glossary	786
Appendices	749	Index	790
Appendix A: Experimental Fundamentals	750	Photograph Credits	798
Introduction	750		
Safety	750		

To the Student

Physics is for everyone. It is more than simply the study of the physical universe. It is much more interesting, diverse, and far more extreme. In physics, we observe nature, seek regularities in the data, and attempt to create mathematical relationships that we can use as tools to study new situations. Physics is not just the study of unrelated concepts, but rather how everything we do profoundly affects society and the environment.

Features



Flowcharts

The flowcharts in this book are visual summaries that graphically show you the interconnections among the concepts presented at the end of each section and chapter. They help you organize the methods and ideas put forward in the course. The flowcharts come in three flavors: Connecting the Concepts, Method of Process, and Putting It All Together. They are introduced as you need them to help you review and remember what you have learned.

EXAMPLE 1



Examples

The examples in this book are loaded with both textual and visual cues, so you can use them to teach yourself to do various problems. They are the next-best thing to having the teacher there with you.



Applying the Concepts

At the end of most subsections, we have included a few simple practice questions that give you a chance to use and manipulate new equations and try out newly introduced concepts. Many of these sections also include extensions of new concepts into the areas of society, technology, and the environment to show you the connection of what you are studying to the real world.

End-of-chapter STSE



Every chapter ends with a feature that deals exclusively with how our studies impact on society and the environment. These articles attempt to introduce many practical applications of the chapter's physics content by challenging you to be conscious of your responsibility to society and the environment. Each feature presents three challenges. The first and most important is to answer and ask more questions about the often-dismissed societal implications of what we do. These sections also illustrate how the knowledge and application of physics are involved in various career opportunities in Canada. Second, you are challenged to evaluate various technologies by performing correlation studies on related topics. Finally, you are challenged to design or build something that has a direct correlation to the topic at hand.

Exercises

EXERCISES

Like a good musician who needs to practise his or her instrument regularly, you need to practise using the skills and tools of physics in order to become good at them. Every chapter ends with an extensive number of questions to give you a chance to practise. Conceptual questions challenge you to think about the concepts you have learned and apply them to new situations. The problems involve numeric calculations that give you a chance to apply the equations and methods you have learned in the chapter. In many cases, the problems in this textbook require you to connect concepts or ideas from other sections of the chapter or from other parts of the book.

Labs

LAB

"Physics is for everyone" is re-enforced by moving learning into the practical and tactile world of the laboratory. You will learn by doing labs that stress verification and review of concepts. By learning the concepts first and applying them in the lab setting, you will internalize the physics you are studying. During the labs, you will use common materials as well as more high-tech devices.

Appendices

The appendices provide brief, concise summaries of mathematical methods that have been developed throughout the book. They also provide you with detailed explanations on how to organize a lab report, evaluate data, and make comparisons and conclusions using results obtained experimentally. They explain uncertainty analysis techniques, including some discussion on statistical analysis for experiments involving repetitive measurements.

We hope that using this book will help you gain greater enjoyment in learning about the world around us.

Toronto, 2002

UNIT

A

Forces and Motion: Dynamics

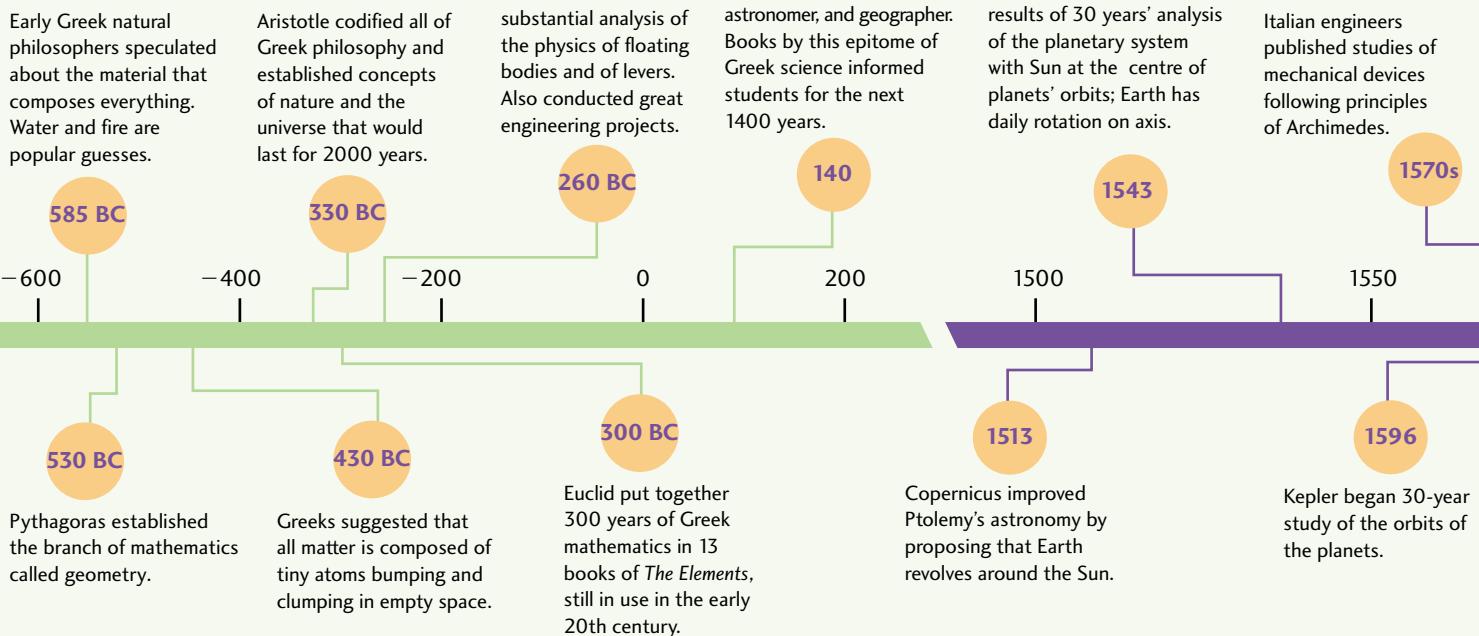
- 1 Kinematics and Dynamics in One Dimension
- 2 Kinematics and Dynamics in Two Dimensions
- 3 Extension: Statics — Objects and Structures in Equilibrium



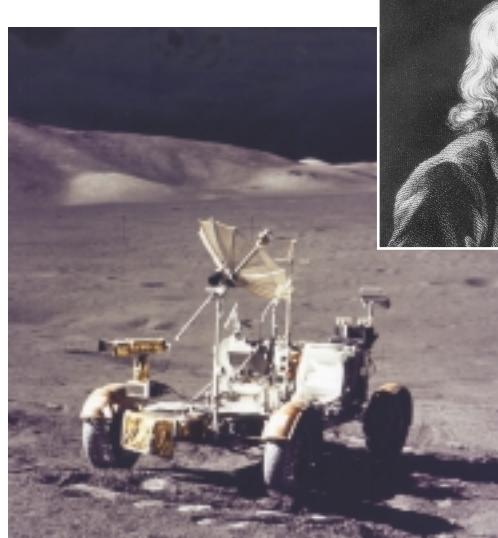
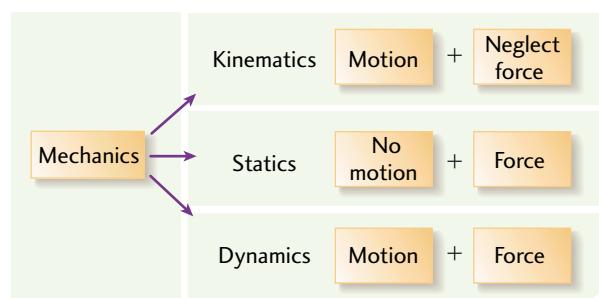
With three simple laws, Newton explained all motion around us. Yet, these laws took thousands of years to formulate. Even though the ancient Greeks made many valuable contributions to mathematics, philosophy, literature, and the sciences, they didn't perform experiments to test all their scientific ideas, which led to some erroneous conclusions.

The classical physics we study today was mostly developed from the mid-16th to the late 19th centuries. The scientific method was formally developed and applied during the Enlightenment (17th and 18th centuries). As a result, many important advances were made in many scientific fields. Nicolas Copernicus (1473–1543), a Polish mathematician, explained the daily motion of the Sun and stars by suggesting that Earth rotates on an axis. Galileo Galilei (1564–1642), an Italian mathematician, experimented extensively to test ancient theories of motion. His famous experiment of dropping two stones, a large one and a small one, from the Tower of Pisa disproved the ancient idea that mass determined the properties of motion. The understanding of celestial mechanics grew quickly with Johannes Kepler (1571–1630), who explained celestial results using Tycho Brahe's data (1546–1601). Sir Isaac Newton (1642–1727) developed the concepts of gravity and laid the foundations of our current concepts of motion in his published book, *Principia Mathematica*. With his three laws and the development of the mathematical methods now called calculus, Newton is responsible for our understanding of dynamics and kinematics. Newton and Galileo created a new approach for scientific analysis — testing and experimentation — which we still use today.

Timeline: The History of Forces and Motion



In this unit, we will learn various methods for studying a variety of forces ranging from simple motion, to motion with friction, to orbital motion. We will also explain the motion of human beings, the development of a variety of vehicles, and the reasons behind the designs of different types of equipment, such as skis and car tires, in terms of the classical laws of physics. This unit lays the foundation for later units on momentum, energy, fields, and modern physics.



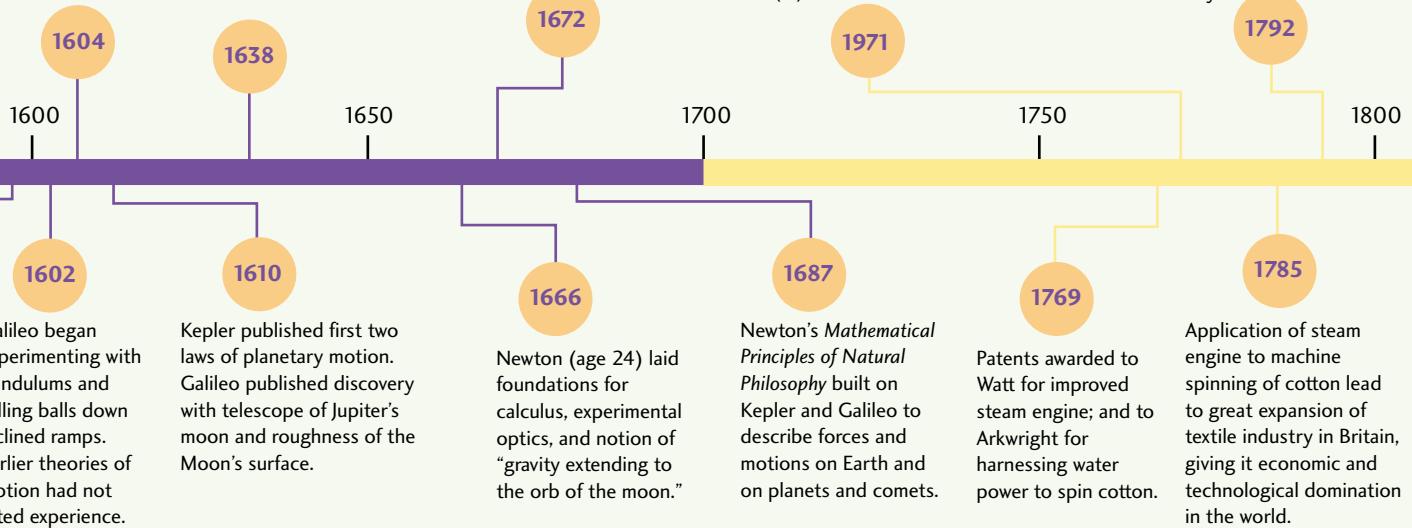
By 1604, Galileo had derived a new theory from analyzing experiments. He found objects fall distances proportional to the square of the time.

After being condemned for Earth's motion in 1633, Galileo published result of a lifetime of motion studies in his *Two New Sciences*.

Huygens in Holland published mechanical study of his new pendulum clock, accurate to 10 s per day — a gigantic improvement.

The 14th General Conference on Weights and Measure picked seven quantities as base quantities, forming the basis of the International System of Units (SI), also called the metric

Republic of France established a new system of weights and measures, defining the metre for the first time. It also tried a 10-h day.

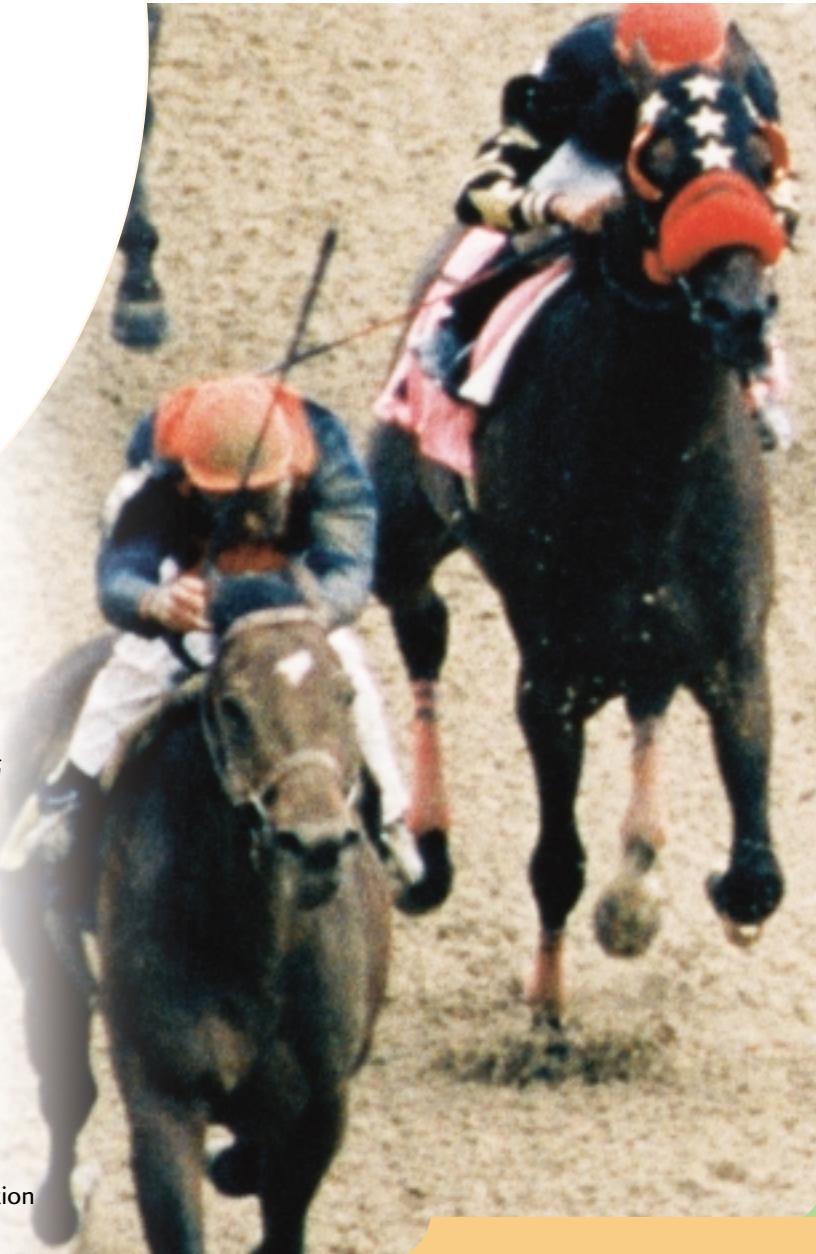


1

Kinematics and Dynamics in One Dimension

Chapter Outline

- 1.1 Introduction
- 1.2 Distance and Displacement
- 1.3 Unit Conversion and Analysis
- 1.4 Speed and Velocity
- 1.5 Acceleration
- 1.6 An Algebraic Description of Uniformly Accelerated Linear Motion
- 1.7 Bodies in Free Fall
- 1.8 A Graphical Analysis of Linear Motion
- 1.9 Dynamics
- 1.10 Free-body Diagrams
- 1.11 Newton's First Law of Motion:
The Law of Inertia
- 1.12 Newton's Second Law of Motion: $\vec{F}_{\text{net}} = m\vec{a}$
- 1.13 Newton's Third Law: Action–Reaction
- 1.14 Friction and the Normal Force
- 1.15 Newton's Law of Universal Gravitation
-  New Respect for the Humble Tire
- LAB** 1.1 Uniform Acceleration: The Relationship between Displacement and Time
- LAB** 1.2 Uniform Acceleration: The Relationship between Angle of Inclination and Acceleration



By the end of this chapter, you will be able to

- analyze the linear motion of objects using graphical and algebraic methods
- solve problems involving forces by applying Newton's laws of motion
- add and subtract vector quantities in one dimension
- solve problems involving Newton's law of universal gravitation

1.1 Introduction

Every day, we observe hundreds of moving objects. Cars drive down the street, you walk your dog through the park, leaves fall to the ground. These events are all part of our everyday experience. It's not surprising, then, that one of the first topics physicists sought to understand was motion.

The study of motion is called **mechanics**. It is broken down into two parts, kinematics and dynamics. **Kinematics** is the “how” of motion, that is, the study of *how* objects move, without concerning itself with *why* they move the way they do. **Dynamics** is the “why” of motion. In dynamics, we are concerned with the causes of motion, which is the study of forces. In the next two chapters, we will consider the aspects of kinematics and dynamics in relation to motion around us.

Fig.1.1 Uniform or non-uniform motion?



Fig.1.2 Moving objects are part of our daily lives



1.2 Distance and Displacement

In any field of study, using precise language is important so that people can understand one another’s work. Every field has certain concepts that are considered the fundamental building blocks of that discipline. When we begin the study of physics, our first task is to define some fundamental concepts that we’ll use throughout this text.

Suppose a friend from your home town asks you, “How do you get to North Bay from here?” You reply, “North Bay is 400 km away.” Is this answer sufficient? No, because you have only told your friend the *distance* to North Bay; you haven’t told her the *direction* in which she should travel.

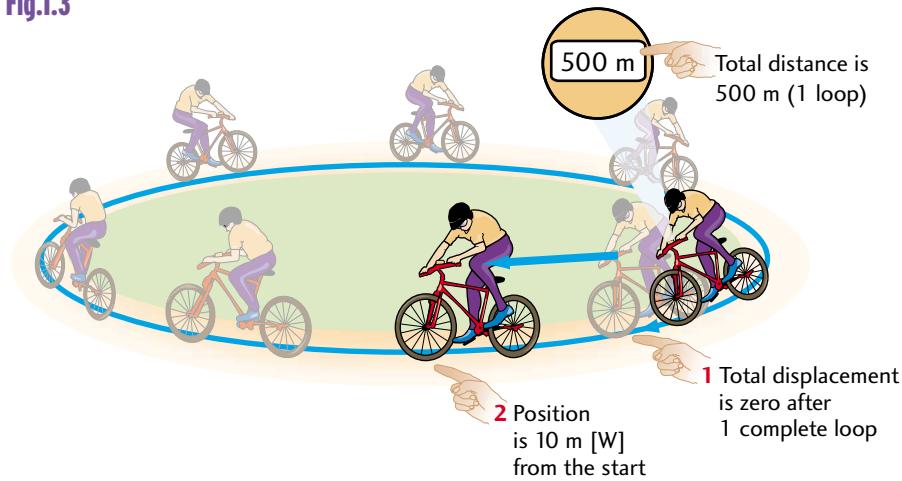
Your answer is a scalar. A **scalar** is a quantity that has a magnitude only, in this case, 400 km. An answer such as “North Bay is 400 km east of here” would answer the question much more clearly. This answer is a vector answer. A **vector** is a quantity that has both a magnitude and a direction. “400 km east” is an example of a *displacement* vector, where the magnitude of the displacement is 400 km and the direction is east. **Displacement** is the change in position of an object. The standard **SI** (*Système International d'Unités*) or **metric** unit is the metre (m), and the variable representing displacement is Δd . Examples of scalars are: 10 minutes, 30°C, 4.0 L, 10 m. Examples of vectors are: 100 km [E], 2.0 m [up], 3.5 m [down]. Displacement is commonly confused with distance. **Distance** is the length of the path travelled and has no direction, so it is a scalar.

EXAMPLE 1

Distance and displacement

A cyclist travels around a 500-m circular track 10 times (Figure 1.3). What is the distance travelled, and what is the cyclist’s final displacement?

Fig.1.3



Position is a vector quantity that gives an object’s location relative to an observer.

Solution and Connection to Theory

The cyclist travels a distance of 500 m each time she completes one lap. Since she completes 10 laps, her total *distance* is 5000 m. To find her *displacement*, we draw a line segment from the starting point to the end point of her motion. Because she starts and ends at the same point, her displacement has a magnitude of zero.

In this example, we obtain very different answers for distance and displacement. It is a good reminder of how important it is to clearly differentiate between vector and scalar quantities.

Defining Directions

In two-dimensional vector problems, directions are often given in terms of the four cardinal directions: north, south, east, and west. For one-dimensional or linear problems, we use the directions of the standard Cartesian coordinate system: vectors to the right and up are positive, and vectors to the left and down are negative.

1.3 Unit Conversion and Analysis

In the past, when the Imperial system of measurement was in common use, it was often necessary to convert from one set of units to another. Today, by using the SI or metric system, conversions between units need only be done occasionally. To convert the speed of a car travelling at 100 km/h to m/s, we multiply the original value by a series of ratios, each of which is equal to one. We set up these ratios such that the units we don't want cancel out, leaving the units of the correct answer. For example,

$$100 \text{ km/h} = \left(\frac{100 \text{ km}}{1 \text{ h}} \right) \left(\frac{1 \text{ h}}{60 \text{ min}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) = 27.8 \text{ m/s}$$

EXAMPLE 2

Unit conversions

How many seconds are there in 18 years?

Solution and Connection to Theory

Let's assume that one year (or annum) has 365 days.

$$18 \text{ a} = 18 \text{ a} \left(\frac{365 \text{ d}}{1 \text{ a}} \right) \left(\frac{24 \text{ h}}{1 \text{ d}} \right) \left(\frac{60 \text{ min}}{1 \text{ h}} \right) \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = 5.7 \times 10^8 \text{ s}$$

There are 5.7×10^8 s in 18 years.

$$1 \text{ m/s} = \frac{0.001 \text{ km}}{\frac{1}{3600} \text{ h}} = 3.6 \text{ km/h}$$

3.6 is a useful conversion factor to remember. To convert m/s to km/h, multiply by 3.6. To convert km/h to m/s, divide by 3.6.

Table 1.1 Prefixes of the Metric System		
Factor	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

1. How many seconds are there in a month that has 30 days?
2. A horse race is 7 furlongs long. How many kilometres do the horses run? (Hint: 8 furlongs = 1 mile, 1 km = 0.63 miles.)
3. Milk used to be sold by the quart. An Imperial quart contains 20 fluid ounces (1 oz = 27.5 mL). How many millilitres of milk are in a quart?



1.4 Speed and Velocity

Fig.1.4 Why doesn't the sign say "Velocity Limit"?



If you were to walk east along Main Street for a distance of 1.0 km in a time of 1 h, you could say that your average velocity is 1.0 km/h [E]. However, en route, you may have stopped to look into a shop window, or even sat down for 10 minutes and had a cold drink. So, while it's true that your average velocity was 1.0 km/h [E], at any given instant, your instantaneous velocity was probably a different value. It is important to differentiate between instantaneous velocity, average velocity, and speed.

Average speed is the total change in *distance* divided by the total elapsed time. Average speed is a scalar quantity and is represented algebraically by the equation

$$v_{\text{avg}} = \frac{\Delta d}{\Delta t} \quad (\text{eq. 1})$$

Average velocity is change in *displacement* over time. Average velocity is a vector quantity and is represented algebraically by the equation

$$\vec{v}_{\text{avg}} = \frac{\vec{\Delta d}}{\Delta t} \quad (\text{eq. 2})$$

Instantaneous velocity is the velocity of an object at a specific time. Note that speed is a scalar and velocity is a vector, but both use the same variable, *v*, and have the same units, m/s. To distinguish velocity from speed, we place an arrow over the velocity variable to show that it's a vector. Similarly, an arrow is placed over the displacement variable, $\vec{\Delta d}$, to distinguish it from distance, Δd . Later, they will be distinguished in the final statement only.

Average and instantaneous velocities can be calculated algebraically. We will revisit these two terms in Section 1.8 using graphical methods.



Fig.1.5 Motion is everywhere



- What is the velocity of the train if it travels a displacement of 25 km [N] in 30 minutes?
- A ship sails 3.0 km [W] in 2.0 h, followed by 5.0 km [E] in 3.0 h.
 - What is the ship's average speed?
 - What is the ship's average velocity?

- The table below shows position–time data for a toy car.

\vec{d} (m) [E]	0	2.0	4.0	6.0	8.0	8.0	9.0	9.0	
t (s)	0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0

- What is the average velocity of the toy car's motion?
- What is the instantaneous velocity of the car at time $t = 5.0$ s?

1.5 Acceleration

The simplest possible type of motion that an object can undergo (short of being at rest) is uniform motion. **Uniform motion** is motion at a constant speed in a straight line. Another name for uniform motion is **uniform velocity**.

When an object's motion isn't uniform, the object's velocity changes. Because velocity is a vector, its magnitude as well as its direction can change. An example of a change of *magnitude only* occurs when a car speeds up as it pulls away from a stoplight. A change in the *direction only* of an object's velocity occurs when a car turns a corner at a constant speed.

Acceleration is the change in velocity per unit time. Velocity can change in magnitude or direction or both. A negative acceleration in horizontal motion is an acceleration to the left. If an object's initial velocity is to the left, the negative acceleration will cause it to *speed up*. If an object's initial velocity is to the right, the negative acceleration will cause it to *slow down*.

Algebraically, we can express acceleration as

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \quad (\text{eq. 3}) \text{ or}$$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} \quad (\text{eq. 4})$$

The SI unit for acceleration is a derived unit; that is, it is a unit created by dividing a velocity unit (such as m/s) by a time unit (such as s), giving

$$\text{units} = \frac{\frac{\text{m}}{\text{s}}}{\text{s}} = \left(\frac{\text{m}}{\text{s}}\right) \times \frac{1}{\text{s}} \text{ or } \frac{\text{m}}{\text{s}^2}$$

Writing acceleration units as m/s² doesn't mean that we have measured a second squared. It is simply a short form for the unit (m/s)/s, which means that the velocity is changing so many m/s each second.

EXAMPLE 3 Vector acceleration

When struck by a hockey stick, a hockey puck's velocity changes from 15 m/s [W] to 10 m/s [E] in 0.30 s. Determine the puck's acceleration. Recall that in our standard coordinate system, we can represent west as negative and east as positive.

Solution and Connection to Theory

Given

$$\vec{v}_1 = -15 \text{ m/s} \quad \vec{v}_2 = +10 \text{ m/s} \quad \Delta t = 0.30 \text{ s}$$

This example is a vector problem, so be sure to take the directions into consideration. We can use the kinematics equation

Fig.1.6 Slapshot!



$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$\vec{a} = \frac{10 \text{ m/s} - (-15 \text{ m/s})}{0.30 \text{ s}}$$

$$\vec{a} = 83 \text{ m/s}^2$$

The puck's acceleration is 83 m/s^2 [E].

The next example deals with negative acceleration.

EXAMPLE 4

Negative acceleration

If the initial velocity of the car in Example 4 had been $-15.0 \frac{\text{m}}{\text{s}}$, an acceleration of -1.0 m/s^2 would mean that the car was speeding up in the negative direction.

Fig.1.7



A car on a drag strip is travelling at a speed of 50 m/s . A parachute opens behind it to assist the car's brakes in bringing the car to rest. Is the acceleration of this car positive or negative? How would its motion change if the acceleration was in the opposite direction?

Solution and Connection to Theory

If we use our standard coordinate system and assume that the initial motion of the car was in the positive direction, its acceleration is in the direction opposite to its initial motion. Therefore, the car's acceleration is negative. If in our example the acceleration of the car is -4.0 m/s^2 , the car is losing 4.0 m/s of speed every second. The negative value for acceleration doesn't mean that the car is going backwards. It means that the car is changing its speed by 4.0 m/s^2 in the negative direction. Since the car was travelling in a positive direction, it is slowing down.

For motion in one dimension, we will designate the direction by using + and – signs. Thus, 12 km [N] becomes $+12 \text{ km}$ (written as 12 km) and 12 km [S] is written as -12 km .

We will also omit vector arrows in the *equations* for displacement, velocity, and acceleration. Instead, we will convey direction by using + and – signs. We will place vector arrows over variables only if the full vector quantity is referred to (e.g., $\vec{d} = 12 \text{ km [N]}$).

1.6 An Algebraic Description of Uniformly Accelerated Linear Motion

Thus far, we have defined two algebraic equations that apply to objects undergoing uniform acceleration. These two equations are

$$v_{\text{avg}} = \frac{\Delta d}{\Delta t} \text{ (eq. 2)} \quad \text{and} \quad a = \frac{v_2 - v_1}{\Delta t} \text{ (eq. 4)}$$

From equation 2, we can isolate Δd :

$$\Delta d = v_{\text{avg}} \Delta t$$

If the acceleration is uniform, $v_{\text{avg}} = \frac{v_1 + v_2}{2}$

and

$$\Delta d = \left(\frac{v_1 + v_2}{2} \right) \Delta t \quad (\text{eq. 5})$$

Even though the vector arrows have been left off of these equations, they are still vector equations! For linear motion, we will leave the vector arrows off, but still indicate direction as positive or negative. In general (i.e., when solving two-dimensional problems), we leave the vector arrows on, otherwise we might forget to add and subtract these values vectorially.

Equations 4 and 5 are both very useful for solving problems in which objects are accelerating uniformly in a straight line. If we look carefully at these two equations, we will notice that many of the variables are common. The only variables not common to both equations are changes in displacement, Δd , and acceleration, a . We can combine equations 4 and 5 by substituting the common variables to form other new and useful equations. First, isolate v_2 in equation 4:

$$v_2 = a\Delta t + v_1 \quad (\text{eq. 6})$$

Now, substitute equation 6 into equation 5:

$$\Delta d = \left(\frac{v_1 + a\Delta t + v_1}{2} \right) \Delta t$$

$$\Delta d = v_1 \Delta t + \frac{1}{2} a \Delta t^2 \quad (\text{eq. 7})$$

The other two possible equations are

$$\Delta d = v_2 \Delta t - \frac{1}{2} a \Delta t^2$$

and

$$v_2^2 = v_1^2 + 2a\Delta d$$

The derivation of these equations is left as an exercise in the Applying the Concepts section. The five equations for uniform linear acceleration are listed in Table 1.2.

Table 1.2
The Five Equations Valid for Uniform Linear Acceleration

#	Equation	Δd	a	v_2	v_1	Δt
1	$v_2 = v_1 + a\Delta t$	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
2	$\Delta d = \frac{1}{2}(v_2 + v_1)\Delta t$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
3	$\Delta d = v_1\Delta t + \frac{1}{2}a\Delta t^2$	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
4	$\Delta d = v_2\Delta t - \frac{1}{2}a\Delta t^2$	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
5	$v_2^2 = v_1^2 + 2a\Delta d$	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>

EXAMPLE 5

Choosing the correct equation

A physics teacher accelerates her bass boat from 8.0 m/s to 11 m/s at a rate of 0.50 m/s². How far does the boat travel? Consider forward to be positive.

Solution and Connection to Theory

Given

$$v_1 = 8.0 \text{ m/s} \quad v_2 = 11 \text{ m/s} \quad a = 0.50 \text{ m/s}^2$$

To solve this problem, we must first find an equation from Table 1.2 that contains only the three known variables and the one unknown variable. Usually, only one equation meets these requirements. (Occasionally, we may get lucky and find that more than one equation will work.) For this example, we require equation 5.

$$v_2^2 = v_1^2 + 2a\Delta d \quad (\text{eq. 5})$$

The problem is asking us for the distance travelled. Therefore, we isolate Δd in equation 5:

$$\Delta d = \frac{v_2^2 - v_1^2}{2a}$$

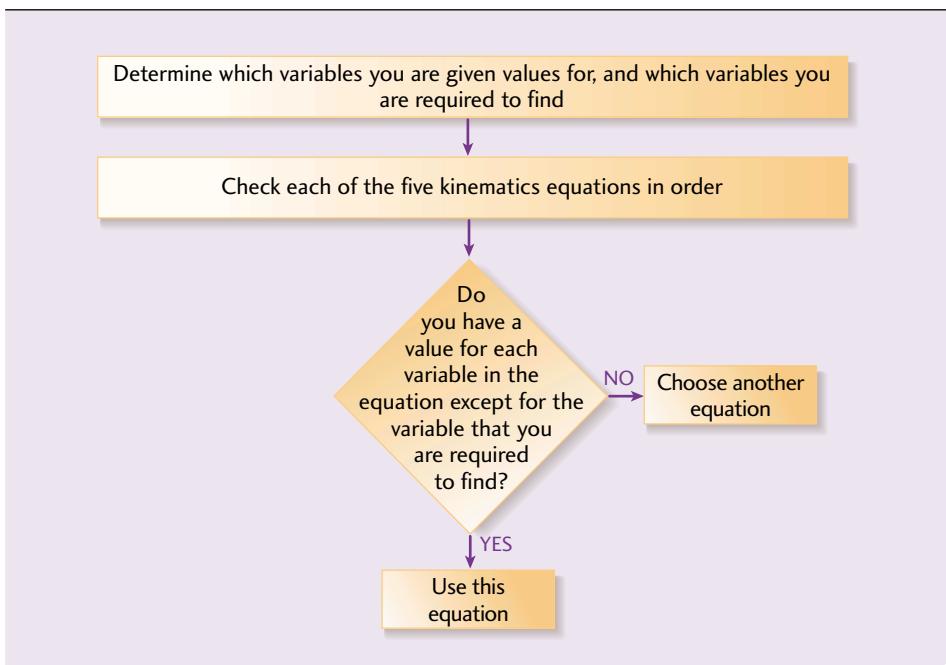
$$\Delta d = \frac{(11 \text{ m/s})^2 - (8.0 \text{ m/s})^2}{2(0.5 \text{ m/s}^2)}$$

$$\Delta d = 57 \text{ m}$$

Therefore, the boat will travel a distance of 57 m.

Figure 1.8 below summarizes how to choose the correct kinematics equation.

Fig.1.8 Choosing Kinematics Equations



EXAMPLE 6 A quadratic solution

Jane Bond runs down the sidewalk, accelerating uniformly at a rate of 0.20 m/s^2 from her initial velocity of 3.0 m/s . How long will it take Jane to travel a distance of 12 m ?

Solution and Connection to Theory

Given

$$a = 0.20 \text{ m/s}^2 \quad v_1 = 3.0 \text{ m/s} \quad \Delta d = 12 \text{ m}$$

The required equation is equation 3.

$$\Delta d = v_1 \Delta t + \frac{1}{2} a \Delta t^2$$

Equation 3 is a quadratic equation for the variable Δt . We will have to solve this problem either by factoring or by using the quadratic formula.

$$0 = \frac{1}{2} a \Delta t^2 + v_1 \Delta t - \Delta d$$

$$0 = (0.1 \text{ m/s}^2) \Delta t^2 + (3.0 \text{ m/s}) \Delta t - 12 \text{ m}$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Fig.1.9 Jane Bond



The Quadratic Equation

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Checking the Units for Δt

$$\begin{aligned} & \frac{\text{m}}{\text{s}} \pm \sqrt{\left(\frac{\text{m}}{\text{s}}\right)^2 - \frac{\text{m}}{\text{s}^2} \times \text{m}} \\ & \frac{\text{m}}{\text{s}^2} \\ & = \frac{\text{m}}{\text{s}} \pm \sqrt{\left(\frac{\text{m}}{\text{s}}\right)^2 - \left(\frac{\text{m}}{\text{s}}\right)^2} \\ & \frac{\text{m}}{\text{s}^2} \\ & = \frac{\text{m}}{\text{s}} = \text{s} \end{aligned}$$

$$\Delta t = \frac{-3.0 \pm \sqrt{(3.0)^2 - 4(0.1)(-12)}}{2(0.1)}$$

$$\Delta t = \frac{-3.0 \pm 3.7}{0.2}$$

Therefore, $\Delta t = 3.5$ s or $\Delta t = -33.5$ s

We use the positive value because time cannot be negative. Therefore, $\Delta t = 3.5$ s. It takes Jane Bond 3.5 s to run 12 m.

EXAMPLE 7

A multiple-step problem

Bounder of Adventure accelerates his massive SUV from rest at a rate of 4.0 m/s^2 for 10 s. He then travels at a constant velocity for 12 s and finally comes to rest over a displacement of 100 m. Assuming all accelerations are uniform, determine Bounder's total displacement and average velocity. Assume that all motion is in the positive direction.

Solution and Connection to Theory

The first step is to break the problem down into simpler parts or stages. This problem asks us to find the total displacement and average velocity. We can solve the problem by first finding the displacement, time, and velocity at each stage of Bounder's trip, then adding the results of each stage together to obtain the final answer. The table below illustrates the different stages of Bounder's trip and the information we are given at each stage.

Stage A

$$\begin{aligned} v_{A_1} &= 0 \\ v_{A_2} &=? \\ a &= 4.0 \text{ m/s}^2 \\ \Delta t &= 10 \text{ s} \end{aligned}$$

Stage B

$$\begin{aligned} v_{B_1} &= v_{B_2} = v_{A_2} \\ \Delta t &= 12 \text{ s} \\ a &= 0 \end{aligned}$$

Stage C

$$\begin{aligned} v_{C_1} &= v_B = ? \\ v_{C_2} &= 0 \\ \Delta d_c &= 100 \text{ m} \end{aligned}$$

Stage A:

Given

$$v_{A_1} = 0 \quad \Delta t_A = 10 \text{ s} \quad a_A = 4.0 \text{ m/s}^2$$

To calculate the final velocity, we can use equation 1 from Table 1.2:

$$v_2 = v_1 + a\Delta t$$

$$v_{A_2} = a\Delta t$$

$$v_{A_2} = (4.0 \text{ m/s}^2)(10 \text{ s})$$

$$v_{A_2} = 40 \text{ m/s}$$

Fig.1.10 A sport utility vehicle (SUV)



To calculate the displacement, we use equation 3:

$$\begin{aligned}\Delta d &= v_1 \Delta t + \frac{1}{2} a \Delta t^2 \\ \Delta d_A &= 0 + \frac{1}{2}(4.0 \text{ m/s}^2)(10 \text{ s})^2 \\ \Delta d_A &= 200 \text{ m}\end{aligned}$$

Stage B:

Given

$$\Delta t = 12 \text{ s}$$

The velocity is constant during this stage, and equal to the final velocity during stage A:

$$\begin{aligned}v_B &= 40 \text{ m/s; therefore,} \\ \Delta d_B &= v_B \Delta t = (40 \text{ m/s})(12 \text{ s}) \\ \Delta d_B &= 480 \text{ m}\end{aligned}$$

Stage C:

Given

$$\Delta d_C = 100 \text{ m} \quad v_{C_2} = 0$$

The initial velocity during stage C is the same as the velocity during stage B because the SUV hasn't slowed down yet; therefore,

$$v_{C_1} = v_B = 40 \text{ m/s}$$

We can calculate the time using equation 2:

$$\Delta d = \frac{1}{2}(v_2 + v_1)\Delta t$$

Isolating Δt , we obtain

$$\Delta t_C = \frac{2\Delta d_C}{v_{C_1} + v_{C_2}}$$

$$\Delta t_C = \frac{2(100 \text{ m})}{40 \text{ m/s}}$$

$$\Delta t_C = 5.0 \text{ s}$$

To find the total displacement, we add the displacements at each stage:

$$\begin{aligned}\Delta d_{\text{tot}} &= \Delta d_A + \Delta d_B + \Delta d_C \\ \Delta d_{\text{tot}} &= 200 \text{ m} + 480 \text{ m} + 100 \text{ m} \\ \Delta d_{\text{tot}} &= 780 \text{ m}\end{aligned}$$

Before we can calculate the average velocity, we need to find the total time of the trip:

$$\begin{aligned}\Delta t_{\text{tot}} &= \Delta t_A + \Delta t_B + \Delta t_C \\ \Delta t_{\text{tot}} &= 10 \text{ s} + 12 \text{ s} + 5.0 \text{ s} \\ \Delta t_{\text{tot}} &= 27 \text{ s}\end{aligned}$$

To find the average velocity, we substitute displacement and time into the velocity equation:

$$v_{\text{avg}} = \frac{\Delta d_{\text{tot}}}{\Delta t_{\text{tot}}}$$

$$v_{\text{avg}} = \frac{780 \text{ m}}{27 \text{ s}}$$

$$v_{\text{avg}} = 29 \text{ m/s}$$

Therefore, Bounder's total displacement is 780 m and his average velocity is 29 m/s.

EXAMPLE 8 A two-body problem

Fred and his friend Barney are at opposite ends of a 1.0-km-long drag strip in their matching racecars. Fred accelerates from rest toward Barney at a constant 2.0 m/s^2 . Barney travels toward Fred at a constant speed of 10 m/s. How much time elapses before Fred and Barney collide?

Solution and Connection to Theory

Given

$$\Delta d = 1000 \text{ m} \quad a_F = 2.0 \text{ m/s}^2 \quad v_{iF} = 0 \quad v_B = -10 \text{ m/s}$$

To solve this problem, we must note two things. First, the distance travelled by Barney plus the distance travelled by Fred must add up to 1000 m. Second, Fred is accelerating uniformly, while Barney is undergoing uniform motion.

We will assume that Fred is moving in the positive direction. At any time Δt , his distance from his starting point is

$$\Delta d = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$\Delta d = 0 + \frac{1}{2} a \Delta t^2$$

$$\Delta d_F = \frac{1}{2} a \Delta t^2$$

$$\Delta d_F = \frac{1}{2} (2 \text{ m/s}^2) \Delta t^2$$

Barney's displacement from the same point is 1000 m plus his displacement at time Δt :

$$\Delta d_B = 1000 \text{ m} + v_B \Delta t$$

$$v_B = -10 \text{ m/s}$$

$$\Delta d_B = 1000 \text{ m} - (10 \text{ m/s}) \Delta t$$

When Fred and Barney meet, their two displacements are equal:

$$\Delta d_F = \Delta d_B$$

$$\frac{1}{2}(2 \text{ m/s}^2) \Delta t^2 = 1000 \text{ m} - (10 \text{ m/s}) \Delta t$$

$$(1 \text{ m/s}^2) \Delta t^2 + (10 \text{ m/s}) \Delta t - 1000 \text{ m} = 0$$

Using the quadratic equation to solve for Δt ,

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta t = \frac{-10 \text{ m/s} \pm \sqrt{(10 \text{ m/s})^2 - 4(1 \text{ m/s}^2)(-1000 \text{ m})}}{2 \text{ m/s}^2}$$

$$\Delta t = \frac{-10 \text{ m/s} \pm 64 \text{ m/s}}{2 \text{ m/s}^2}$$

$$\Delta t = 27 \text{ s} \text{ or } t = -37 \text{ s}$$

Since time is positive, we choose the positive answer. Fred and Barney collide after 27 s.

EXAMPLE 9 Catching a bus

Jack, who is running at 6.0 m/s to catch a bus, sees it start to move when he is 20 m away from it. If the bus accelerates at 1.0 m/s², will Jack overtake it? If so, how long will it take him?

Solution and Connection to Theory

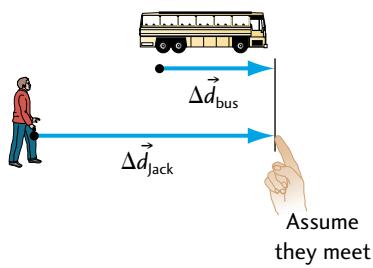
Given

$$v_{\text{Jack}} = 6.0 \text{ m/s} \quad v_{\text{bus}} = 0 \quad a_{\text{bus}} = 1.0 \text{ m/s}^2 \quad a_{\text{Jack}} = 0 \quad \Delta d = 20 \text{ m}$$

We will consider Jack's initial position as our origin and assume that he is running in the positive direction. His displacement at any time Δt is given by

$$\Delta d = v_1 \Delta t + \frac{1}{2} a \Delta t^2$$

$$\Delta d_{\text{Jack}} = (6.0 \text{ m/s}) \Delta t$$

Fig.1.11

The displacement of the bus *from the same origin* at any time Δt is

$$\Delta d_{\text{bus}} = 20 \text{ m} + v_1 \Delta t + \frac{1}{2} a \Delta t^2$$

$$\Delta d_{\text{bus}} = 20 \text{ m} + \frac{1}{2}(1.0 \text{ m/s}^2) \Delta t^2$$

When Jack overtakes the bus, the two displacements are equal:

$$(6.0 \text{ m/s}) \Delta t = 20 \text{ m} + (0.5 \text{ m/s}^2) \Delta t^2$$

$$(0.5 \text{ m/s}^2) \Delta t^2 - (6.0 \text{ m/s}) \Delta t + 20 \text{ m} = 0$$

Using the quadratic equation to solve for Δt ,

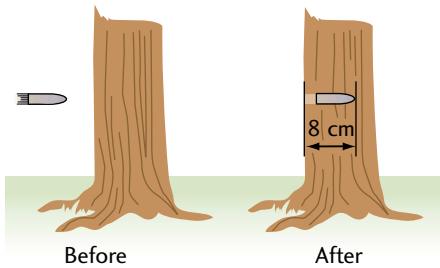
$$\Delta t = \frac{6.0 \text{ m/s} \pm \sqrt{(-6.0 \text{ m/s})^2 - 4(0.5 \text{ m/s}^2)(20 \text{ m})}}{2(0.5 \text{ m/s}^2)}$$

$$\Delta t = \frac{6.0 \text{ m/s} \pm \sqrt{36 \text{ m}^2/\text{s}^2 - 40 \text{ m}^2/\text{s}^2}}{1 \text{ m/s}^2}$$

There are no real roots for this equation; therefore, there is no real time at which Jack and the bus have the same position. Jack will have to walk or wait for the next bus!



1. A CF-18 fighter jet flying at 350 m/s engages its afterburners and accelerates at a rate of 12.6 m/s^2 to a velocity of 600 m/s. How far does the fighter jet travel during acceleration?
2. A butterfly accelerates over a distance of 10 cm in 3.0 s, increasing its velocity to 5.0 cm/s. What was its initial velocity?
3. During a football game, Igor is 8.0 m behind Brian and is running at 7.0 m/s when Brian catches the ball and starts to accelerate away at 2.8 m/s^2 from rest.
 - a) Will Igor catch Brian? If so, after how long?
 - b) How far down the field will Brian have run?
4. A bullet is fired into a tree trunk (Figure 1.12), striking it with an initial velocity of 350 m/s. If the bullet penetrates the tree trunk to a depth of 8.0 cm and comes to rest, what is the acceleration of the bullet?

Fig.1.12

5. A delivery truck accelerates uniformly from rest to a velocity of 8.0 m/s in 3.0 s. It then travels at a constant speed for 6.0 s. Finally, it accelerates again at a rate of 2.5 m/s^2 , increasing its speed for 10 s. Determine the truck's average velocity.
6. While undergoing pilot training, a candidate is put in a rocket sled that is initially travelling at 100 km/h. When the rocket is ignited, the sled accelerates at 30 m/s^2 . At this rate, how long will it take the rocket sled to travel 500 m down the track?
7. A parachutist, descending at a constant speed of 17 m/s, accidentally drops his keys, which accelerate downward at 9.8 m/s^2 .
 - a) Determine the time it takes for the keys to reach the ground if they fall 80 m.
 - b) What is the final velocity of the keys just before they hit the ground?
8. Derive the following equations from first principles:
 - a) $v_2^2 = v_1^2 + 2a\Delta d$
 - b) $\Delta d = v_2\Delta t - \frac{1}{2}a\Delta t^2$

1.7 Bodies in Free Fall

Galileo Galilei (1564–1642), an Italian astronomer and physicist, is credited with being the father of modern experimental science because he combined experiment and calculation rather than accepting the statements of an authority, namely Aristotle, regarding the study of nature. His greatest contributions were in the field of mechanics, especially dynamics. His experiments on falling bodies and inclined planes disproved the accepted Aristotelean idea that a body's rate of descent is proportional to its weight. Galileo's conclusions greatly upset Aristotelean scholars of his day.

The Guinea and Feather Demonstration

Galileo experimented in many different fields. One of his experiments in mechanics involved rolling spheres down a wooden ramp (Figure 1.13b). He found that the square of the time a sphere took to reach the bottom of a ramp was proportional to the length of the ramp. He also observed that the time a sphere took to reach the bottom of the ramp was independent of its mass; that is, less massive objects and more massive objects both reach the bottom of the ramp at the same time when released from the same height. By using ramps inclined at different angles, Galileo extrapolated his findings to a ball falling straight down. He concluded that if two objects of different masses are released from the same height, they will strike the ground at the same time (see Figure 1.14).

Fig.1.13a Galileo Galilei

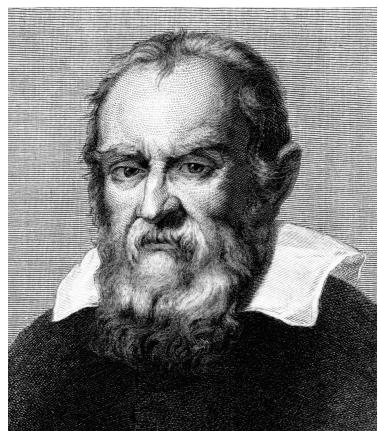


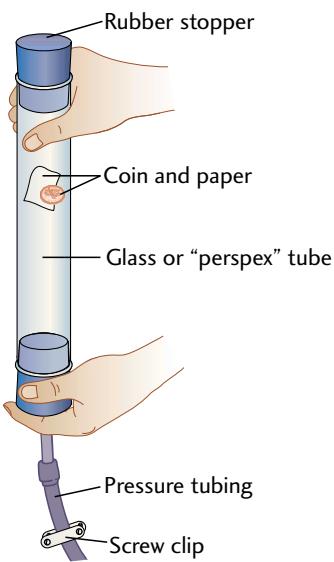
Fig.1.13b The inclined plane used by Galileo Galilei



Fig.1.14 A hammer and a feather are dropped on the Moon. Which will land first?



Fig.1.15 The guinea and feather demonstration



Today, we can easily confirm Galileo's findings by performing the guinea and feather demonstration. A guinea (or any coin) and a feather are placed in a long glass tube with a hole at one end, which is connected to a vacuum pump. If the guinea and feather are allowed to fall through the tube full of air, they will not strike the bottom at the same time. The guinea will land first and the feather will flutter slowly to the bottom due to air resistance. If the vacuum pump is used to remove the air from the tube, both objects will strike the bottom at the same time.

Acceleration due to Gravity

Today we know that when objects are dropped from a height close to Earth's surface, they accelerate downward at a rate of 9.8 m/s^2 . This number is known as the **acceleration due to gravity**. It doesn't depend on the object's mass. For this value to be valid, we must assume that air resistance is negligible and that Earth is a sphere of constant density and radius. In Section 1.15, we will study gravity in greater depth.

EXAMPLE 10 A marble in free fall

Fig.1.16 The CN Tower in Toronto, Ontario



A marble is dropped from the top of the CN Tower, 553 m above the ground.

- How long does it take the marble to reach the ground?
- What is the marble's final speed just before it hits the ground?
- What is the marble's speed at the halfway point of its journey?

Solution and Connection to Theory

Given

$$\Delta d = 553 \text{ m} \quad v_1 = 0 \quad a = g = 9.8 \text{ m/s}^2$$

- We choose down to be the positive direction. To calculate the time, we use the equation

$$\Delta d = v_1 \Delta t + \frac{1}{2} a \Delta t^2$$

Since $v_1 = 0$,

$$\Delta d = \frac{1}{2}a\Delta t^2$$

Isolating Δt , we obtain the equation

$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

$$\Delta t = \sqrt{\frac{2(553 \text{ m})}{9.8 \text{ m/s}^2}}$$

$$\Delta t = 11 \text{ s}$$

Therefore, the marble takes 11 s to reach the ground.

b) To find the final speed, we use the equation

$$v_2^2 = v_1^2 + 2a\Delta d$$

$$v_2 = \sqrt{2(9.8 \text{ m/s}^2)(553 \text{ m})}$$

$$v_2 = 1.0 \times 10^2 \text{ m/s}$$

Therefore, the marble's final speed is $1.0 \times 10^2 \text{ m/s}$.

c) At the halfway point, $d = \frac{553 \text{ m}}{2} = 276.5 \text{ m}$. Using the algebra from b),

$$v_2 = \sqrt{2(9.8 \text{ m/s}^2)(276.5 \text{ m})}$$

$$v_2 = 74 \text{ m/s}$$

Therefore, the marble's speed at the halfway point is 74 m/s.

Fig.1.17 Throwing a baseball straight up

EXAMPLE 11

Maximum height

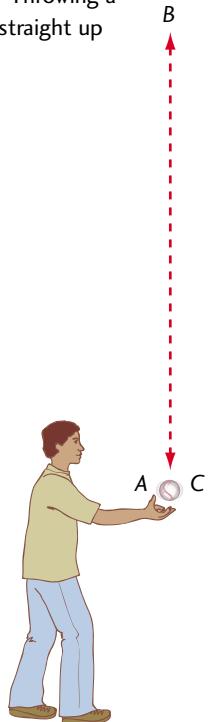
A baseball is thrown straight up in the air, leaving the thrower's hand at an initial velocity of 8.0 m/s.

- How high does the ball go?
- How long will it take the ball to reach maximum height?
- How long will it take before the ball returns to the thrower's hand?

Solution and Connection to Theory

There are three important things to note in this example:

- After the ball is released upward, its acceleration is in the opposite direction of its motion; that is, the ball is moving upward, but acceleration due to gravity is downward. Using our standard coordinate system, we will make acceleration negative.



In this problem, we are ignoring the effects of air resistance.

- 2) At its maximum height, the ball will come to rest. After that, it will fall back down into the thrower's hand. This problem is an example of symmetry because the amount of time it takes the ball to travel upward to maximum height equals the amount of time it takes the ball to fall back down. Also because of symmetry, the velocity with which the ball strikes the thrower's hand equals its initial upward velocity.
- 3) The acceleration is constant in both magnitude and direction for the entire motion. For this reason, the ball slows down as it goes up and speeds up as it falls down.

Given

$$v_1 = 8.0 \text{ m/s} \quad a = -9.8 \text{ m/s}^2 \quad v_2 = 0$$

- a) To find the maximum height of the ball, we use the equation

$$v_2^2 = v_1^2 + 2a\Delta d$$

$$\Delta d = \frac{-v_1^2}{2a}$$

$$\Delta d = \frac{-(8.0 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)}$$

$$\Delta d = 3.27 \text{ m}$$

$$\Delta d = 3.3 \text{ m}$$

Therefore, 3.3 m is the maximum height of the ball.

- b) $v_2 = v_1 + a\Delta t$

$$\Delta t = \frac{v_2 - v_1}{a}$$

$$\Delta t = \frac{0 - 8.0 \text{ m/s}}{-9.8 \text{ m/s}^2}$$

$$\Delta t = 0.82 \text{ s}$$

Therefore, the ball reaches maximum height in 0.82 s.

- c) Because of symmetry, we know that the time to go up equals the time to come down. The time for the ball to go up and come back down is simply twice the answer in b); that is, 1.6 s.

or

For the complete motion (up and down),

$$\Delta d = 0$$

$$v_1 = 8.0 \text{ m/s}$$

$$a = -9.8 \text{ m/s}^2$$

$$\Delta t = ?$$

Using equation 3,

$$\Delta d = v_1 \Delta t + \frac{1}{2} a \Delta t^2$$

$$0 = (8.0 \text{ m/s})\Delta t + (-4.9 \text{ m/s}^2)\Delta t^2$$

$$\Delta t = 0 \text{ or } \Delta t = 1.6 \text{ s}$$

EXAMPLE 12 Throwing a rock upward

A rock is thrown vertically upward from the edge of a cliff at an initial velocity of 10.0 m/s. It hits the beach below the cliff 4.0 s later. How far down from the top of the cliff is the beach? Consider up to be positive.

Solution and Connection to Theory

Given

$$v_1 = 10.0 \text{ m/s} \quad \Delta t = 4.0 \text{ s} \quad a = -9.8 \text{ m/s}^2$$

$$\Delta d = v_1 \Delta t + \frac{1}{2} a \Delta t^2$$

$$\Delta d = (10.0 \text{ m/s})(4.0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(4.0 \text{ s})^2$$

$$\Delta d = 40.0 \text{ m} - 78.4 \text{ m}$$

$$\Delta d = -38.4 \text{ m}$$

Therefore, the beach is 38.4 m below the top of the cliff.

1. An arrow is shot straight up in the air at 80.0 m/s. Find
 - a) its maximum height.
 - b) how long it will take the arrow to reach maximum height.
 - c) the length of time the arrow is in the air.
2. Tom is standing on a bridge 30.0 m above the water.
 - a) If he throws a stone down at 4.0 m/s, how long will it take to reach the water?
 - b) How long will the stone take to reach the water if Tom throws it up at 4.0 m/s?
3. A ball thrown from the edge of a 35-m-high cliff takes 3.5 s to reach the ground below. What was the ball's initial velocity?

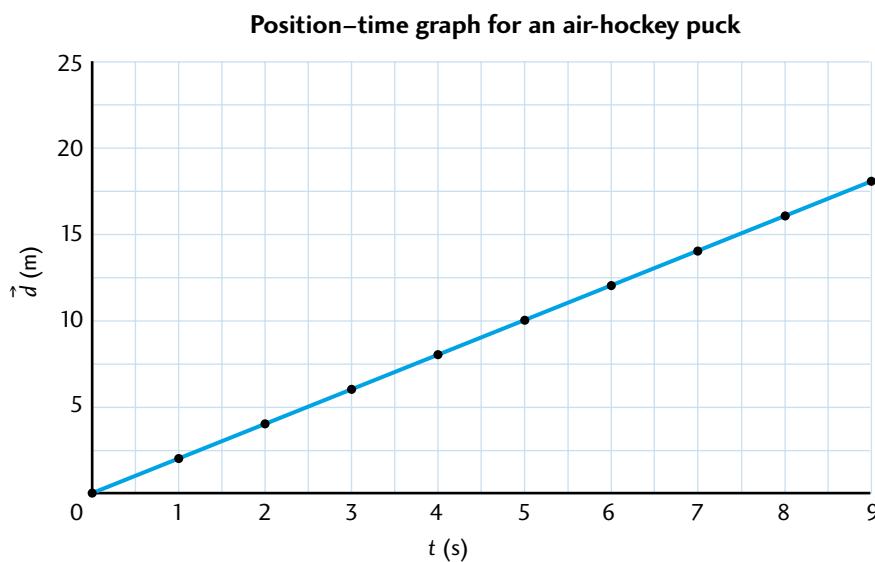


1.8 A Graphical Analysis of Linear Motion

So far, the examples we have studied have been algebraic problems. We have therefore used algebraic solutions. Often in physics, especially while performing experiments, data is presented in graphical form. So, physicists need to be able to analyze graphical data.

There are three main types of graphs used in kinematics: position–time graphs, velocity–time graphs, and acceleration–time graphs. The relationships among these graphs provide us with some of our most powerful analytical tools.

Fig.1.18



Velocity

Figure 1.18 shows the position–time graph for an air-hockey puck moving down the table. This simple example provides us with a considerable amount of information about the motion of the object. Recall that

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{\Delta d \text{ (m)}}{\Delta t \text{ (s)}}$$

$$v = \frac{\Delta d}{\Delta t} \left(\frac{\text{m}}{\text{s}} \right)$$

By calculating the slope of the linear graph, we can determine the velocity of the air-hockey puck in metres per second. From this result, we can conclude that:

The slope of a position–time graph gives the velocity of the object.

If the slope of a position–time graph gives velocity, and uniform motion is constant velocity, then the graph must have a constant slope (i.e., be a straight line). In other words,

If an object is undergoing uniform motion, its position–time graph must be a straight line.

Not all position–time graphs are straight lines. Some are curves, and some are a complex combination of curves and straight lines. Regardless of the graph's shape, the slope of the position–time graph gives the velocity of the object.

Figure 1.19 summarizes the information we can obtain from position–time graphs.

Fig.1.19 Summary of \vec{d} – t Graph Analysis

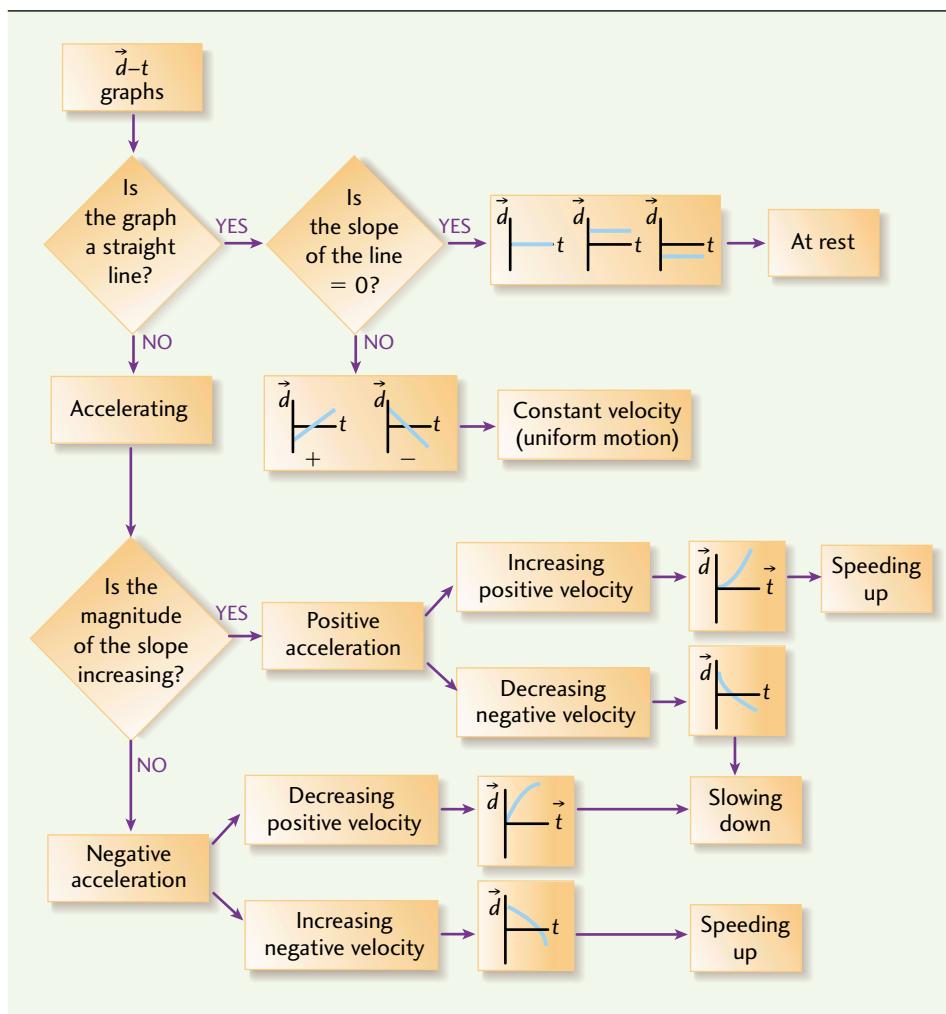


Fig.1.20 The slope of the secant joining A to B is the average velocity of that portion of the motion. That slope lies between the values of the slopes of the tangents at A and B.

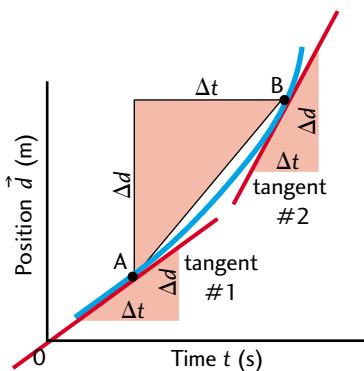


Figure 1.20 shows the slope of the *tangent* at points A and B on an increasing position–time graph. At point B, the velocity of the object (i.e., the slope of the tangent) is greater than at point A. The graph also shows a line joining points A and B. The slope of this *secant* gives us the average velocity between points A and B.

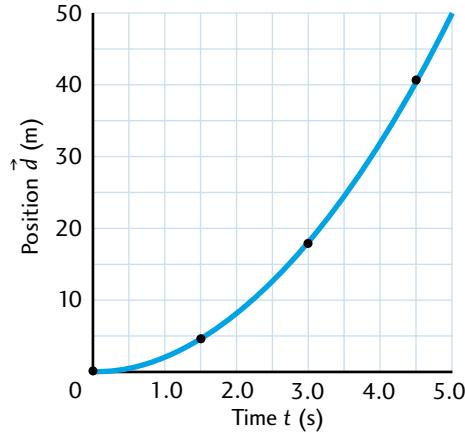
$$v_{\text{avg}} = \frac{\Delta \vec{d}}{\Delta t}$$

Average velocity is the slope of a line connecting two points on a position–time graph. For position–time graphs representing uniform acceleration, the instantaneous velocity of an object can be determined by finding the slope of the tangent to the curve.

EXAMPLE 13 The slope of the tangent on a velocity–time graph

The graph in Figure 1.21 represents the motion of a lime-green AMC Pacer, which has started to roll downhill after its parking brake has disengaged. Using this data, determine the slope of the tangent to the position–time graph at four different points. Then plot the corresponding velocity–time graph, and find its slope. Consider positive values to be down the hill.

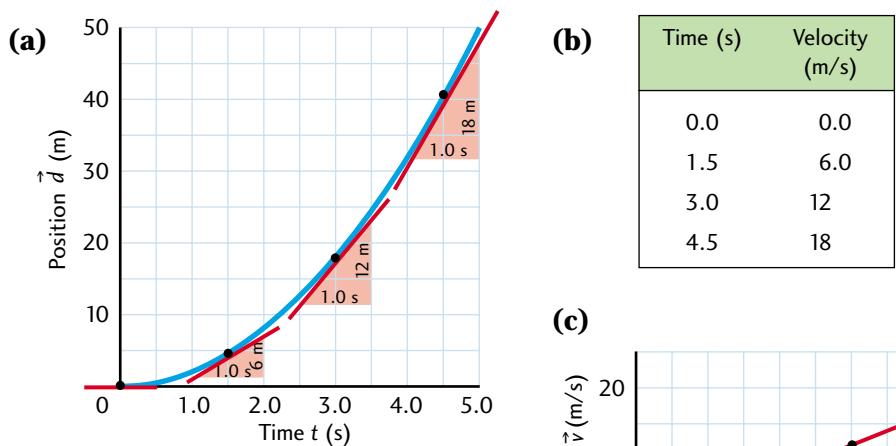
Fig.1.21



Solution and Connection to Theory

When we calculate the slope (i.e., the velocity) at four different points along the curve in Figure 1.22a, we find that these values are increasing. An increasing slope indicates acceleration. Since the velocity–time graph is a straight line (Figure 1.22c), we know that the acceleration is uniform.

Fig.1.22



Now we can find the slope of the velocity–time graph (Figure 1.22c):

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{\Delta v}{\Delta t}$$

$$\text{slope} = \frac{18 \text{ m/s}}{4.5 \text{ s}}$$

$$\text{slope} = 4.0 \text{ m/s}^2 = \text{acceleration}$$

From this example, we have determined that:

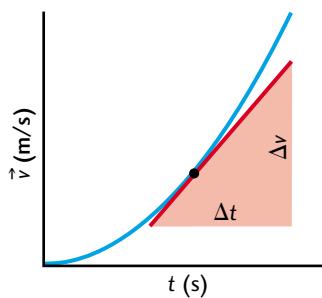
The slope of a straight-line velocity–time graph is the constant acceleration of the object.

By analogy,

If the velocity–time graph is a curve (Fig.1.22d), the slope of its tangent at any given point is the instantaneous acceleration of the object.

What can we learn by finding the area under a velocity–time graph? Let's look at the following example:

Fig.1.22d The slope of a tangent drawn to a point on a \vec{v} – t graph gives the instantaneous acceleration at that time



EXAMPLE 14

The area under a velocity–time graph

What is the area under the graph in Figure 1.23 for the first 3.5 s? (Be sure to include the correct units.)

Fig.1.23

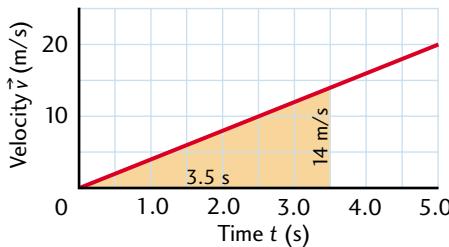
**Solution and Connection to Theory**

Figure 1.23 is a linear, increasing velocity–time graph. The area under this graph is a triangle, which equals half the base times the height:

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(3.5\text{ s})(14\text{ m/s})$$

$$A = 24\text{ m}$$

The unit generated in this example is metres; therefore, we can conclude that:

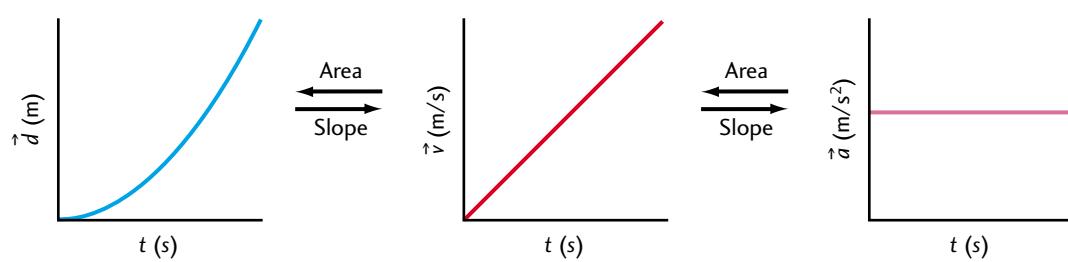
The area under a velocity–time graph is the displacement of the object, \vec{d} .

Similarly,

The area under an acceleration–time graph is the change in velocity of the object, \vec{v} .

Assuming an object starts from rest at the origin, we can summarize our graphical analysis of linear motion in one simple diagram:

Fig.1.24



EXAMPLE 15 Velocity-time graphs

1. From Figure 1.25, what is the instantaneous velocity of the object at each of the following times?

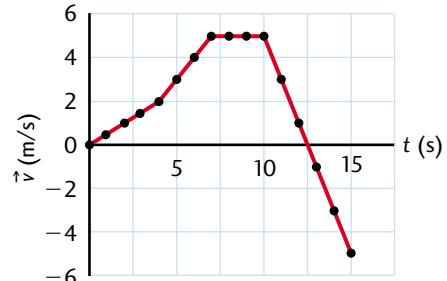
$$t = 4.0 \text{ s}$$

$$t = 8.0 \text{ s}$$

$$t = 12 \text{ s}$$

2. a) What is the average acceleration from time $t = 0$ to $t = 4.0 \text{ s}$?
 b) What is the average acceleration from time $t = 10$ to $t = 15 \text{ s}$?
 3. What is the instantaneous acceleration at $t = 9.0 \text{ s}$?
 4. How far has the object travelled in the first 7.0 s ?

Fig.1.25



Solution and Connection to Theory

1. We can determine the instantaneous velocity by simply reading it off the velocity-time graph. At time $t = 4.0 \text{ s}$, the velocity is 2.0 m/s . At $t = 8.0 \text{ s}$, the velocity is 5.0 m/s . At $t = 12 \text{ s}$, the velocity is 1.0 m/s .
2. Since acceleration is determined by taking the slope of a velocity-time graph, we need to find the slope of the graph at each time interval. For

- a) $t = 0$ to $t = 4.0 \text{ s}$,

$$\text{slope} = \text{acceleration} = \frac{\Delta v}{\Delta t}$$

$$\text{slope} = \frac{2.0 \text{ m/s}}{4.0 \text{ s}}$$

$$\text{slope} = 0.50 \text{ m/s}^2$$

- b) From $t = 10 \text{ s}$ to $t = 15 \text{ s}$, the graph is a descending straight line. We therefore expect to have a negative slope:

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

$$a = \frac{(-5.0 \text{ m/s}) - (5.0 \text{ m/s})}{15 \text{ s} - 10 \text{ s}}$$

$$a = \frac{-10 \text{ m/s}}{5.0 \text{ s}}$$

$$a = -2.0 \text{ m/s}^2$$

The negative acceleration is interesting for two reasons. Above the horizontal time axis, negative acceleration indicates that the object is decreasing in speed (i.e., it is slowing down). At the time axis, the object has a velocity of zero and is at rest for an instant. Finally, below the time axis,

the object is still accelerating in the negative direction, but its speed is increasing in the opposite direction of its original motion (i.e., the object is speeding up backwards). As an example of this type of motion, consider an astronaut who is outside her shuttlecraft and is approaching it with a velocity of 10 m/s [E]. To prevent herself from colliding with the shuttlecraft, she fires a retro-rocket from her rocket pack, which shoots a small amount of hot gas in the easterly direction, causing her to accelerate in the westerly direction. If the rocket pack causes an acceleration of 1.0 m/s² [W], the astronaut would continue to slow down until she came to rest 10 s later. If at that point she shut off the rocket pack, she would remain at rest. If she inadvertently left the rocket pack on, she would continue accelerating in the westerly direction immediately after having come to rest. The astronaut's velocity would increase by 1.0 m/s [W] for each second that the retro-rocket was left burning.

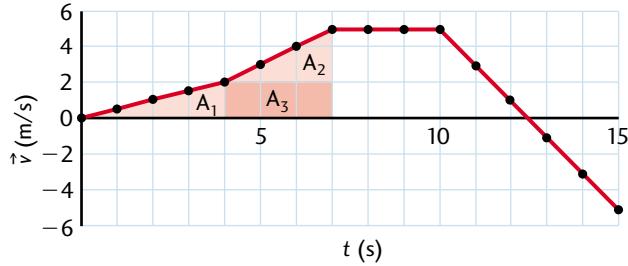
3. The instantaneous acceleration at time $t = 9.0$ s can be found by inspection. The slope of the velocity–time graph gives the acceleration. From $t = 7.0$ s to $t = 9.0$ s, the slope is horizontal, that is, zero. Zero slope means that the object is undergoing uniform motion.
4. To determine the object's displacement, we need to find the area under the graph. In this case, we can simplify the calculation by breaking the area down into a series of triangles and rectangles.

The areas $A_2 + A_3$ in Figure 1.26 form a trapezoid. The area of a trapezoid is the average of the height times the base; that is,

$$A = \frac{(5.0 \text{ m/s} + 2.0 \text{ m/s})}{2} (3.0 \text{ s})$$

$$A = 10.5 \text{ m}$$

Fig.1.26



$$A_{\text{tot}} = A_1 + A_2 + A_3$$

A_1 and A_2 are both triangles and A_3 is a rectangle. We substitute the appropriate equations for each area:

$$A_{\text{tot}} = \frac{1}{2}bh + \frac{1}{2}bh + lw$$

$$A_1 = \frac{1}{2}(4.0 \text{ s})(2.0 \text{ m/s}) = 4.0 \text{ m}$$

$$A_2 = \frac{1}{2}(3.0 \text{ s})(3.0 \text{ m/s}) = 4.5 \text{ m}$$

$$A_3 = (3.0 \text{ s})(2.0 \text{ m/s}) = 6.0 \text{ m}$$

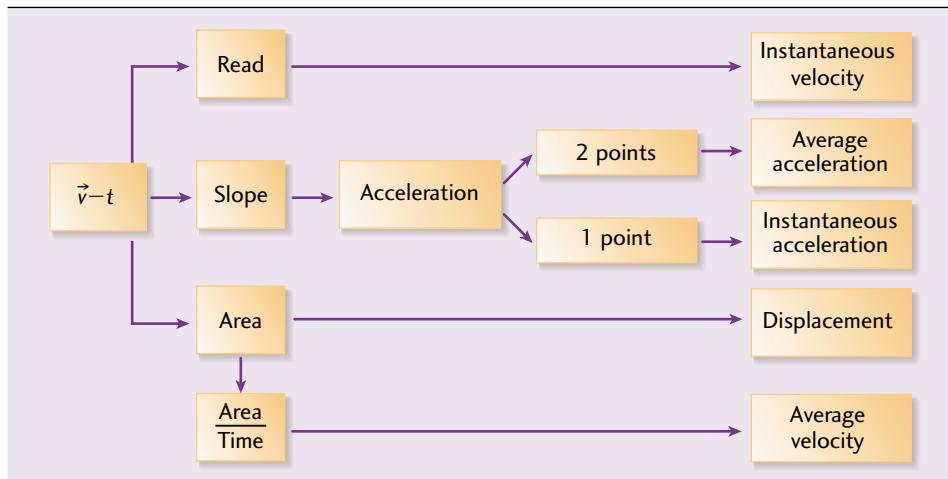
$$A_{\text{tot}} = 4.0 \text{ m} + 4.5 \text{ m} + 6.0 \text{ m}$$

$$A_{\text{tot}} = 14.5 \text{ m}$$

Therefore, the object's displacement in the first 7.0 s is 14.5 m.

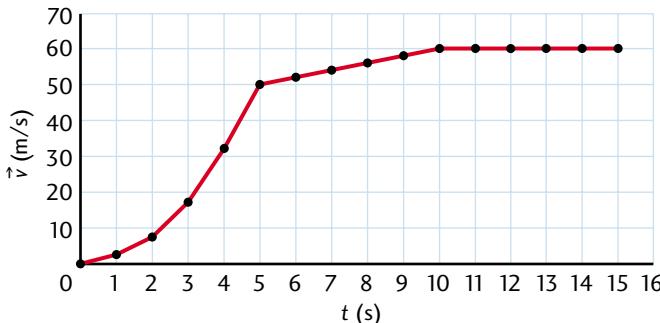
Figure 1.27 summarizes how to obtain information from a velocity–time graph.

Fig.1.27 Information Obtained from a \vec{v} – t Graph



- The \vec{v} – t data in Figure 1.28 are for Puddles, the dog playing at the park.

Fig.1.28



- Determine Puddles' instantaneous acceleration at each of the following points:

$$t = 7.0 \text{ s}$$

$$t = 12 \text{ s}$$

$$t = 3.0 \text{ s}$$

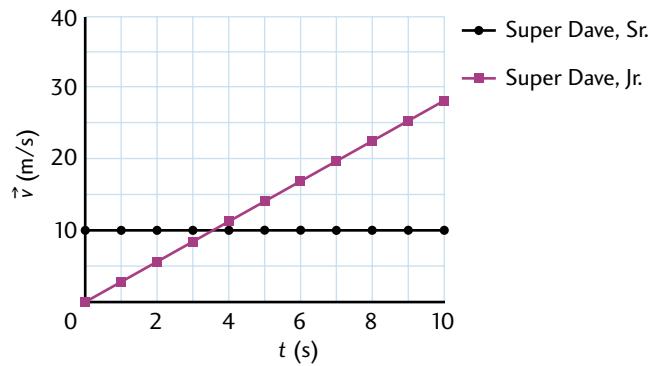
- How far did Puddles run from time $t = 5.0 \text{ s}$ to $t = 13 \text{ s}$?

- Figure 1.29 shows \vec{v} – t data for Super Dave, Sr. and his son, Super Dave, Jr., who are racing their motorcycles on a straight 150-m track.

From the graph, determine the following:

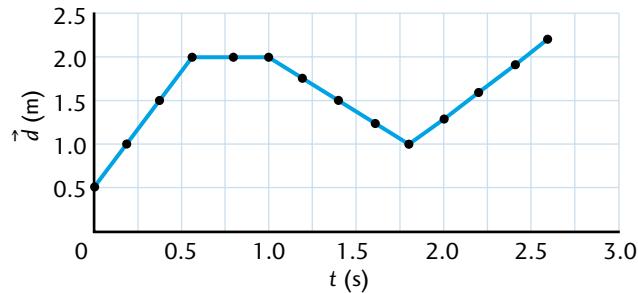
- How long does it take both Super Daves to reach the 50-m mark?
- Who wins the 50-m race, and by how much time?
- Who would have won if the race had been 100 m long?

Fig.1.29



3. Figure 1.30 shows \vec{d} - t data for The Flash, a local jogging enthusiast.

Fig.1.30



- a) Determine the average velocity for each segment of The Flash's motion.
b) What is his average velocity for the entire trip?

Fig.1.31 Understanding crash test results in the lab can save countless lives on our roads

1.9 Dynamics



Dynamics is often called the “why” of motion because it is the study of *why* objects move as opposed to *how* they move. The following terms are very important in the study of dynamics:

A **force** is commonly referred to as a push or pull in a given direction. These forces are called **contact forces**. There are also non-contact forces such as gravity. Force is a vector quantity, and its standard metric unit is the newton (N). In the next few sections, we will study how different types of forces can cause or affect the motion of objects.

Mass is the amount of matter in an object. It is a measure of an object's inertia. The standard SI unit for mass is the kilogram (kg). **Weight**, on the other hand, is the force of gravity acting on an object. The terms *mass* and *weight* are commonly thought to be synonymous, but they are not. Mass is a quantity that doesn't vary with location, whereas weight depends on your location in the universe.

Gravity is the mutual force of attraction between any two objects that contain matter. The magnitude of the force of Earth's gravity (F_g) on an object can be calculated using the following equation:

$$F_g = mg$$

$$1 \text{ N} = (\text{kg})\left(\frac{\text{m}}{\text{s}^2}\right) = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

The symbol g represents Earth's gravitational field strength of 9.80 N/kg. This value is also commonly referred to as the acceleration due to gravity, with units m/s^2 . For this equation to be valid, we must assume that the object is reasonably close to Earth's surface and that Earth is a sphere of uniform mass and radius.

Note: Free-body diagrams only show forces *acting on* an object. They don't show forces exerted by the object.

1.10 Free-body Diagrams

Free-body diagrams (FBDs) are very useful conceptual tools for physics students because they help us isolate the object we wish to study from its environment so that we can examine the forces acting on it. A free-body diagram is created by drawing a circle around the object. The forces acting on the object are represented by arrows pointing away from the circle. For example, if we were to draw a free-body diagram of this textbook sitting at rest on a lab bench, we would draw a circle around the textbook, and draw two arrows representing forces acting on it, as shown in Figure 1.32a. One of the forces is the force of gravity on the book, pulling it downward. The other force is the force due to the bench pushing the book upward. Note that the force applied by the book on the bench downward isn't shown because this force is exerted by the book. This force would only be shown in a free-body diagram of the lab bench.

The forces in Figure 1.32a are equal and opposite; that is, the magnitude of the gravitational force is equal to the magnitude of the upward force due to the lab bench. These two forces are an example of **balanced forces**. When an object is acted on by balanced forces, the forces cancel each other out and the object behaves as though no force is acting on it.

Figure 1.32b is a free-body diagram of a textbook in free fall. Assuming negligible air resistance, the only force acting on the book is the force of gravity downward; there is no balancing upward force. As a result, the force due to gravity on the book is *unbalanced*.

Fig.1.32a

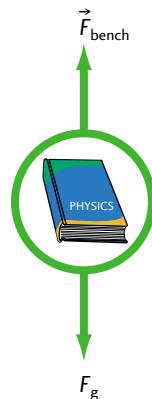
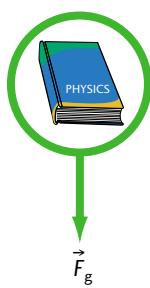


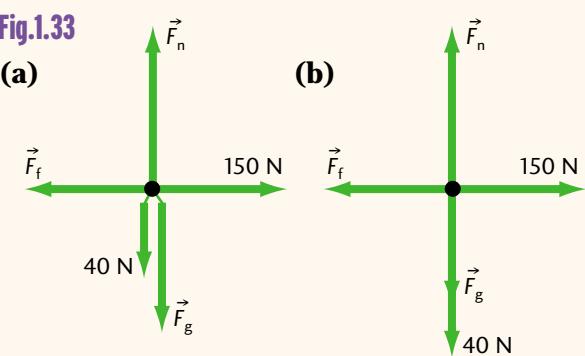
Fig.1.32b



FREE-BODY DIAGRAMS

In more formal physics, the object in a free-body diagram is reduced to a dot representing the object's centre of mass. The forces are shown pointing away from the dot, as in Figure 1.33a . Notice that side-by-side forces are drawn slightly offset. An alternative is to place parallel forces head to tail in a line (Figure 1.33b).

Fig.1.33



1.11 Newton's First Law of Motion: The Law of Inertia

Newton's first law of motion is one of the most important and most commonly misunderstood laws in physics. **Newton's first law** states:

An object will remain at rest or in uniform motion unless acted upon by an external unbalanced force.

In other words, objects at rest or in uniform motion don't require any other forces in order to maintain their current states. An object at rest that is acted on by two balanced forces remains at rest and needs no other force to stay that way. We wouldn't expect the textbook in Figure 1.32a to suddenly start flying around the room unless an additional force was applied to it. For objects in uniform motion, imagine that we take a baseball into outer space and throw it. Once the ball has left our hand and experiences no other forces, it will continue travelling at a constant speed in a straight line away from us forever! No additional force is required to maintain its motion.

EXAMPLE 16

Understanding Newton's first law

Fig.1.34 A Porsche 911

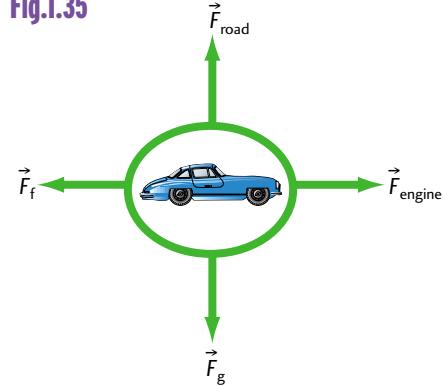


A physics teacher is driving down the highway in her new blue Porsche 911 turbo at a constant velocity of 90 km/h. Why can she not shut the engine off, and travel at a constant speed in a straight line forever (assuming the road is perfectly straight and there is no other traffic)?

Solution and Connection to Theory

Figure 1.35 shows the forces acting on the car. We can assume that the upward force on the car from the road and the downward force of gravity on the car balance, so the net force in that direction is zero. The two horizontal forces, that is, the force of the engine acting on the car and the force of friction from the road and air resistance, are *balanced* if the car is travelling at a constant velocity. The two horizontal forces are unbalanced if the car's engine is shut off. With the engine off, friction will eventually bring the car to rest.

Fig.1.35



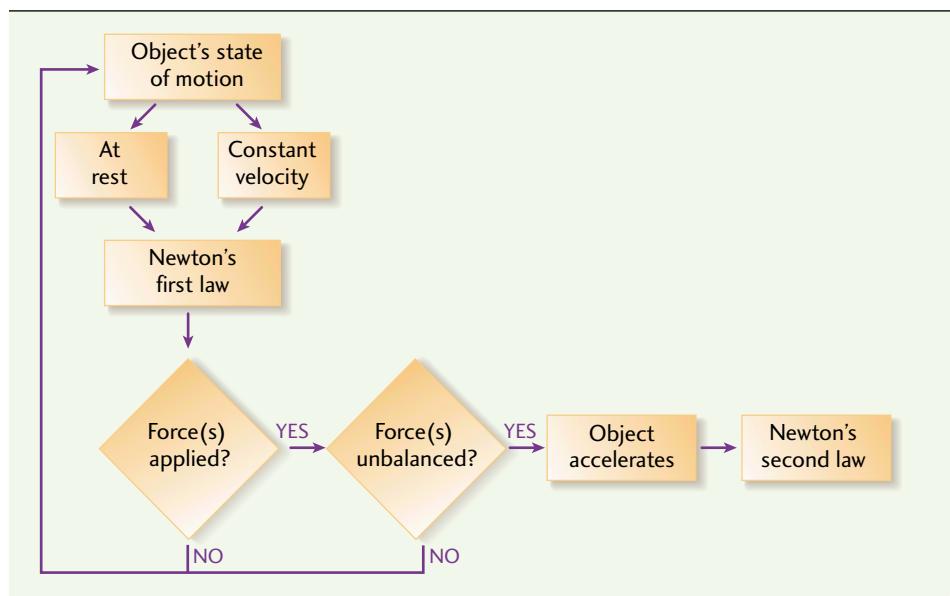
Inertial and Non-inertial Frames of Reference

The point of view from which we observe motion is called a **frame of reference**. It is the stationary “platform” from which we judge or measure all other motion. Situations where Newton’s first law applies are referred to as inertial frames of reference. In an **inertial frame of reference**, the observer is either at rest or travelling at a constant velocity *relative to the frame of reference he or she is observing*. A **non-inertial frame of reference** is accelerating, and Newton’s first law doesn’t apply.

Imagine you are driving in a car down a perfectly straight and perfectly flat road at a constant speed. If you closed your eyes, you would be unaware of any motion. If, all of a sudden, the driver stepped on the gas and caused the car to accelerate, you would feel a force pushing you back into the seat. From your reference frame inside the car, you are accelerating back; therefore, according to Newton’s first law, you require an unbalanced force acting on you. But nothing is pushing you back; so, Newton’s first law doesn’t apply in a non-inertial frame of reference. An observer in an inertial frame of reference on the side of the road would see a force applied to you (i.e., you being pushed by the back of your seat) to accelerate forward along with the car. Therefore, Newton’s first law applies to inertial frames of reference only.

Figure 1.36 summarizes Newton’s first law and the states of motion to which it applies.

Fig.1.36 States of Motion





1. Draw a free-body diagram for each of the following situations, and determine if the forces are balanced or unbalanced. Explain your reasoning in each case.

- a) A goalie kicks a soccer ball from the ground.
- b) An Olympic marksman experiences recoil as his rifle fires.
- c) A penny falls through the water in a wishing well.
- d) An airborne soldier floats to the ground with her parachute open.

1.12 Newton's Second Law of Motion:

$$\vec{F}_{\text{net}} = m\vec{a}$$

In Newton's second law,
 $a \propto F_{\text{net}}$

So, for a given mass, doubling F_{net}
will double a :
 $a \propto \frac{1}{m}$

For a given F_{net} , doubling the mass of
the object halves a .

Newton's second law describes the acceleration produced when an unbalanced force acts on an object. **Newton's second law of motion** can be stated algebraically as

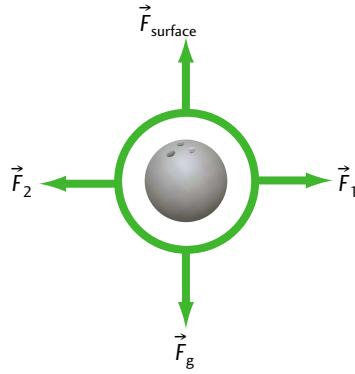
$$\vec{F}_{\text{net}} = m\vec{a}$$

where the net force (\vec{F}_{net}) is the vector sum of all forces acting on the object.

EXAMPLE 17 Newton's second law

What is the acceleration of a 5.0-kg bowling ball that simultaneously experiences a 60-N force east and a 50-N force west? Assume that east is positive.

Fig.1.37



Solution and Connection to Theory

Given

$$m = 5.0 \text{ kg} \quad \vec{F}_1 = 60 \text{ N [E]} \quad \vec{F}_2 = 50 \text{ N [W]}$$

From Newton's second law,

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$$

$$F_{\text{net}} = 60 \text{ N} - 50 \text{ N}$$

$$a = \frac{F_{\text{net}}}{m}$$

$$a = \frac{(60 \text{ N} - 50 \text{ N})}{5.0 \text{ kg}}$$

$$a = 2.0 \text{ m/s}^2$$

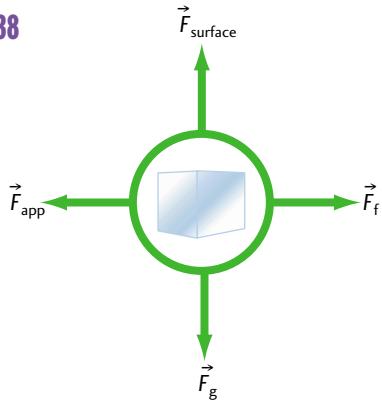
Therefore, the acceleration, \vec{a} , of the bowling ball is 2.0 m/s^2 [E]. In the vertical direction, $F_{\text{surface}} = F_g$ and $F_{\text{nety}} = 0$.

EXAMPLE 18 Frictional force

A 50-kg block of ice experiences an applied horizontal force of 80 N [W] as it accelerates at 1.2 m/s^2 [W] against a force of friction. Determine the magnitude and direction of the frictional force acting on the block of ice.

Solution and Connection to Theory

Fig.1.38



Given

$$m = 50 \text{ kg} \quad \vec{F}_{\text{app}} = 80 \text{ N [W]} \quad \vec{a} = 1.2 \text{ m/s}^2 [\text{W}]$$

Assuming east is positive, $F_{\text{app}} = -80 \text{ N}$ and $a = -1.2 \text{ m/s}^2$

From Newton's second law,

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$\vec{F}_{\text{app}} + \vec{F}_f = m\vec{a}$$

$$\vec{F}_f = m\vec{a} - \vec{F}_{\text{app}}$$

$$F_f = (50 \text{ kg})(-1.2 \text{ m/s}^2) - (-80 \text{ N})$$

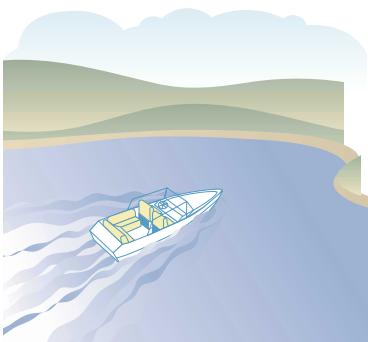
$$F_f = +20 \text{ N}$$

Therefore, the frictional force is 20 N [E]. Notice that it is in the opposite direction to the ice block's motion, as we would expect.

EXAMPLE 19

A multiple-step problem

Fig.1.39



An 800-kg police boat uniformly changes its velocity from 50 km/h [N] to 20 km/h [N] as it enters a harbour. If the acceleration occurs over a displacement of 30 m, what is the frictional force on the boat?

Solution and Connection to Theory

Given

$$\vec{v}_1 = 50 \text{ km/h [N]} \quad \vec{v}_2 = 20 \text{ km/h [N]}$$

$$\text{Assuming north is positive, } v_1 = 50 \text{ km/h} \times \frac{1000 \text{ m}}{3600 \text{ s}} = 13.9 \text{ m/s}$$

$$v_2 = 20 \text{ km/h} \times \frac{1000 \text{ m}}{3600 \text{ s}} = 5.56 \text{ m/s}$$

$$\Delta d = 30 \text{ m} \quad m = 800 \text{ kg} \quad F_f = ?$$

In order to find the force, we must first calculate the acceleration.

Using the equation

$$v_2^2 = v_1^2 + 2a\Delta d$$

and isolating a , we obtain

$$a = \frac{v_2^2 - v_1^2}{2\Delta d}$$

$$a = \frac{(5.56 \text{ m/s})^2 - (13.9 \text{ m/s})^2}{2(30 \text{ m})}$$

$$a = -2.57 \text{ m/s}^2$$

The negative sign means that the acceleration is to the south. Therefore, the acceleration is 2.57 m/s^2 [S].

The force of friction of the water on the boat is the only unbalanced force acting on the boat and is therefore the net force. So,

$$\vec{F}_{\text{net}} = \vec{F}_f$$

$$m\vec{a} = \vec{F}_f$$

$$F_f = (800 \text{ kg})(-2.57 \text{ m/s}^2)$$

$$F_f = -2.1 \times 10^3 \text{ N}$$

Therefore, the frictional force slowing the boat down is $2.1 \times 10^3 \text{ N}$ [S]. Notice once again that the force of friction is in the opposite direction to the boat's motion.

1. A 2.0-kg duck is accelerated by a force of 10 N.
 - a) What is the acceleration of the duck?
 - b) How would the duck's acceleration change if its mass was doubled?
 - c) How would the duck's acceleration change if the force was halved?
2. A 90-kg parachutist in free fall has an acceleration of 6.8 m/s^2 . What is the frictional force provided by air resistance when she is accelerating at this rate?
3. A dart strikes a 0.45-cm-thick dartboard at a velocity of 15 m/s and accelerates uniformly to rest. What force does the dartboard apply to the 80-g dart in bringing it to rest?
4. A 600-kg jet car accelerates from rest under the force of its jet engine. After travelling 1.00 km in 21.0 s, the jet engine shuts off and the jet car eventually comes to rest 1.4 km farther down the track, stopped by the frictional force of the ground. Calculate the force due to the jet engine, and the constant frictional force applied while the car was in motion.

Fig.1.41



5. A 250-g baseball strikes a catcher's mitt with a velocity of 28 m/s. If the catcher's mitt moves backwards 35 cm in bringing the ball to rest, determine the force applied by the mitt on the ball.

Fig.1.40



1.13 Newton's Third Law: Action–Reaction

Consider the following situation: You are standing on a hockey rink, wearing your hockey skates and facing the boards. You apply a 10-N [W] force on the boards. What type of motion will occur due to this force, and in which direction will it occur?

You will move east even though you have applied a force west. This effect is explained by applying **Newton's third law**:

For every action (applied) force, there is an equal and opposite reaction force. The reaction force is equal in magnitude and opposite in direction to the action force.

In our example, you applied a force on the boards of 10 N [W]. According to Newton's third law, the boards apply an equal and opposite reaction force on you of 10 N [E], causing you to accelerate east.

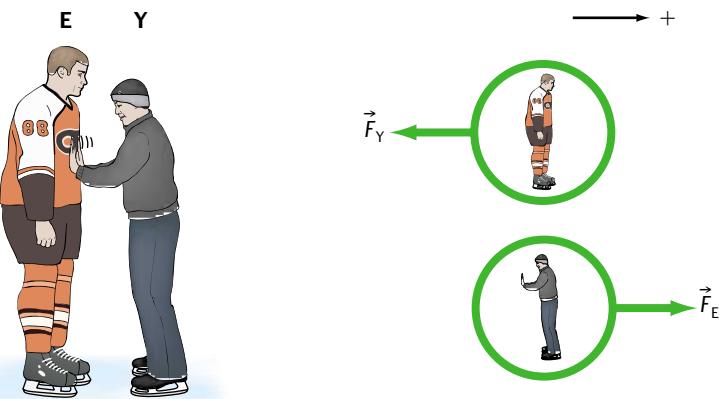
Fig.1.42



EXAMPLE 20**Unbalanced action-reaction forces**

You are standing in the middle of a hockey rink, face to face with Eric Lindros. If you apply a force of 10 N [W] on Eric, who will move and in which direction? What is Eric's force on you, and what are your respective accelerations? Assume that Eric's mass is 100 kg and yours is 70 kg.

Fig.1.43 Action and reaction forces are applied on different objects and require separate FBDs

**Solution and Connection to Theory****Given**

$$m_E = 100 \text{ kg} \quad m_Y = 70 \text{ kg} \quad \vec{F} = 10 \text{ N [W]}$$

Consider the free-body diagram of Eric. Assuming no friction, one force only is applied to him, so

$$a_E = \frac{F}{m_E} = \frac{-10 \text{ N}}{100 \text{ kg}}$$

$$a_E = -0.1 \text{ m/s}^2$$

Eric accelerates at -0.1 m/s^2 [W].

From Newton's third law, the reaction force on you is 10 N [E], so

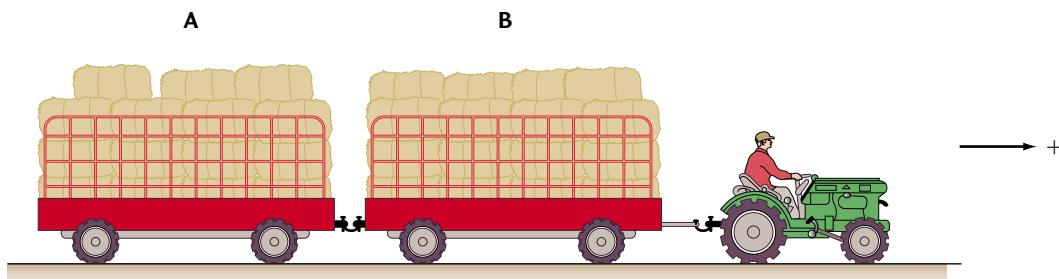
$$a_Y = \frac{F}{m_Y} = \frac{10 \text{ N}}{70 \text{ kg}} = 0.14 \text{ m/s}$$

Your acceleration, \vec{a} , is 0.14 m/s^2 [E].

EXAMPLE 21**Newton's third law**

A tractor pulls two 2000-kg hay wagons, A and B, connected together as shown in Figure 1.44a. If the tractor applies a constant force of 5000 N, determine the acceleration of the two hay wagons, and the force at the point where the two wagons are joined together. Assume no friction.

Fig.1.44a



Solution and Connection to Theory

Given

$$m_A = m_B = 2000 \text{ kg} \quad \vec{F}_T = 5000 \text{ N [E]}$$

Because a constant force is being applied by the tractor, the hay wagons are accelerating. To find the acceleration of both wagons, we use Newton's second law:

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$\vec{F}_{\text{net}} = (m_A + m_B)\vec{a}$$

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{(m_A + m_B)}$$

$$a = \frac{5000 \text{ N}}{(2000 \text{ kg} + 2000 \text{ kg})}$$

$$a = 1.25 \text{ m/s}^2$$

The acceleration of the hay wagons is 1.25 m/s^2 .

To find the force at the junction point of the two wagons, consider wagon A. The only force acting on it is the force from wagon B (see Figure 1.44c).

$$F_{BA} = m_A a$$

$$F_{BA} = (2000 \text{ kg})(1.25 \text{ m/s}^2)$$

$$F_{BA} = 2.5 \times 10^3 \text{ N}$$

Now consider wagon B.

$$a = 1.25 \text{ m/s}^2$$

$$F_{\text{net}} = ma$$

From Figure 1.44d,

$$5000 \text{ N} + F_{AB} = (2000 \text{ N})(1.25 \text{ m/s}^2)$$

$$F_{AB} = -2.5 \times 10^3 \text{ N}$$

\vec{F}_{BA} and \vec{F}_{AB} are two equal and opposite forces, as predicted by Newton's third law.

Fig.1.44b

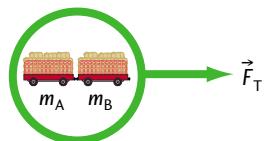


Fig.1.44c

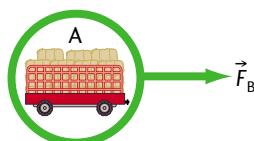


Fig.1.44d

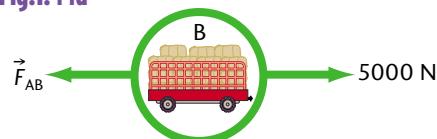


Figure 1.45 summarizes how to solve problems involving Newton's laws of motion.

Fig.1.45 Applying Newton's Laws

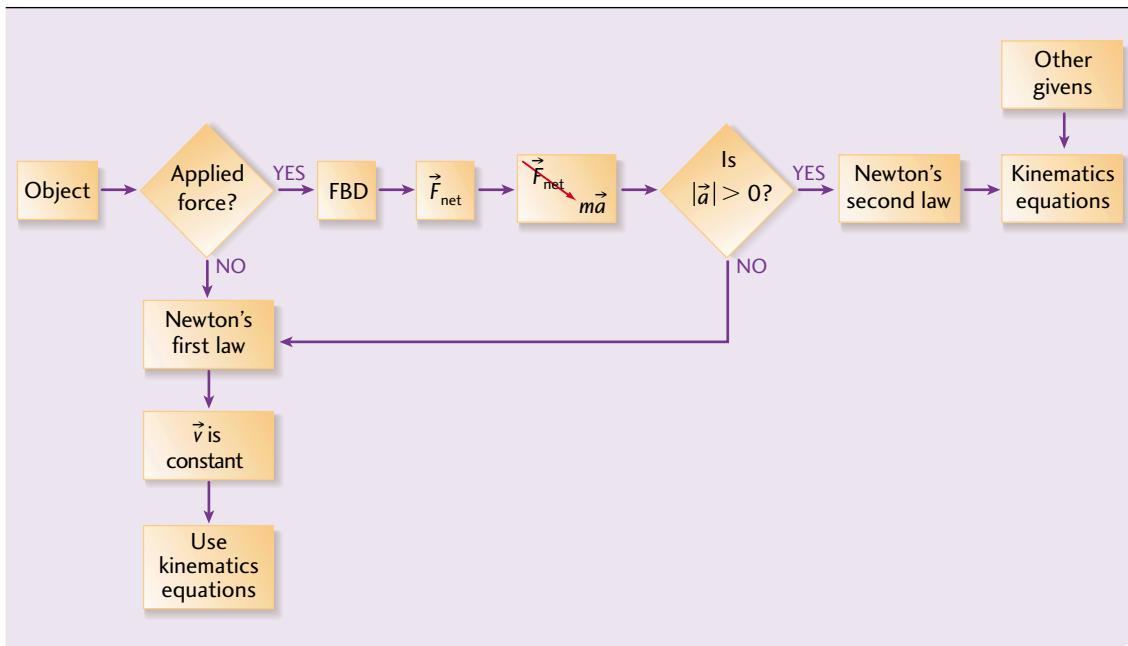


Fig.1.46



1. Identify the action–reaction pairs in each of the following situations. Include the direction of each force (organize in chart form).
 - a) A soccer player kicks a stationary soccer ball east.
 - b) A canoeist pushes the water back with his paddle.
 - c) A child releases a balloon full of air, letting go of the open end.
 - d) An apple hangs from a tree branch.
 - e) A laptop computer sits on a desk.

Fig.1.47 What are the action–reaction pairs here?



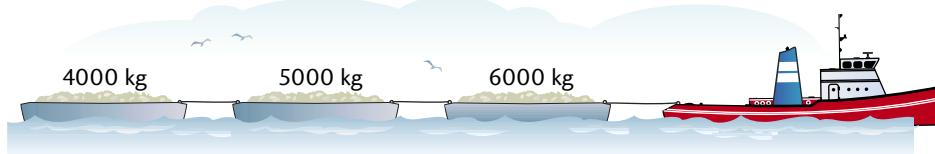
2. Competitive clay target shooters (Figure 1.47) experience Newton's third law every time they compete. Explain what happens in terms of Newton's third law.

3. A tugboat pulls three barges connected end to end with wire cables (Figure 1.48). The barge closest to the tugboat has a mass of 6000 kg. The next-closest barge has a mass of 5000 kg, and the last barge has a mass of 4000 kg.

a) Calculate the force that the tugboat must apply to accelerate the three barges at a rate of 1.5 m/s^2 .

b) Determine the tension in the cable joining each pair of barges.

Fig.1.48

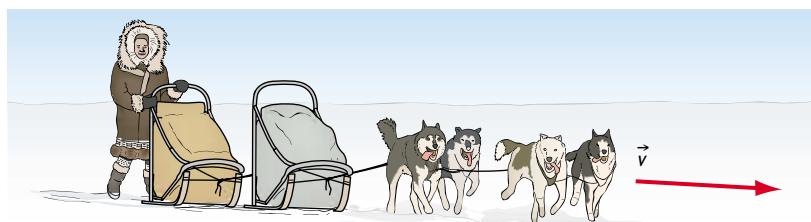


4. A dog team driven by an Inuit hunter pulls two toboggans (Figure 1.49). The dog team can apply a maximum force of 700 N. Each toboggan experiences a constant frictional force of 100 N.

a) Determine the acceleration of the two toboggans, if each has a mass of 300 kg.

b) What is the force in the rope joining the two toboggans together?

Fig.1.49



Recoilless Rifles

A recoilless rifle (Figure 1.50) is a type of cannon that can be used as an antitank weapon. A conventional artillery piece must have a large mass and be securely fixed to the ground to prevent the substantial recoil from moving it out of position. Because a recoilless rifle doesn't recoil, it can have a smaller mass and can be mounted less securely on a jeep or small tripod.

5. How would you design a gun that fires a shell of approximately the same size as a regular cannon but doesn't recoil?

6. Earth applies a gravitational force to the Moon, and the Moon applies an equal and opposite gravitational force to Earth. Why don't these forces cancel each other out? Explain your answer in terms of Newton's laws of motion.

Fig.1.50 What makes this gun recoilless?



1.14 Friction and the Normal Force

Whenever two bodies slide over each other, frictional forces between them develop. Sometimes these forces help us and sometimes they hinder us. Without friction, it would be impossible to make a car start, stop, or turn. However, if we were able to turn friction off once our car was travelling at a constant velocity, we wouldn't need the engine anymore because, according to Newton's first law, we would travel at a constant speed in a straight line.

The microscopic details of the force of friction are still not properly understood. We believe that when two objects are in contact, they make microscopic connections at various points on their surfaces. Even highly polished surfaces are rough and ridged when viewed under a powerful microscope. Because the contact points are so close to each other, intermolecular forces form microscopic welds that must be broken in order for the objects to move apart. These welds continually form and break as the objects move across each other.

Fig.1.51 Friction between two surfaces

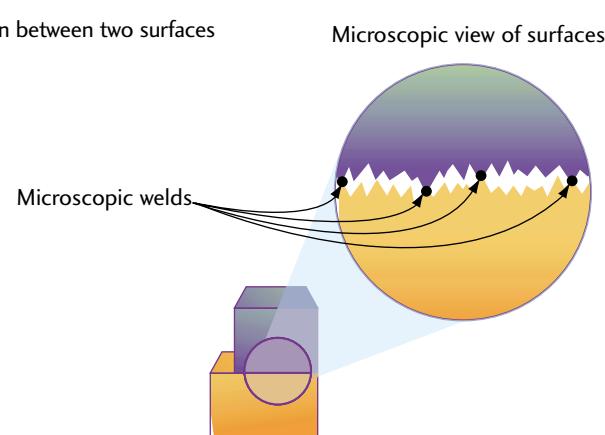


Fig.1.52 A lamp at rest on a table



Consider a lamp sitting on a table. Figure 1.52 is a free-body diagram of the lamp showing the force due to gravity (downward) and the force due to the table (upward). Assuming that the lamp isn't accelerating, these two forces are equal and opposite. The upward force of the table is perpendicular to the table's surface. Any force applied on an object by a surface that is perpendicular to the surface (i.e., normal to the surface) is referred to as a **normal force**, F_n .

The force of friction always acts to oppose the sliding of two surfaces past each other. The magnitude of the force of friction, which depends directly on the normal force, F_n , is given by

$$F_f = \mu F_n$$

where μ (pronounced "mew," like a cat), the **coefficient of friction**, depends on the nature of the surfaces and is found experimentally.

There are two kinds of sliding friction: **static friction** and **kinetic friction**. In general, the force of static friction is greater than the force of kinetic friction. In other words, it is more difficult to begin moving an object at rest than it is to move an object already in motion. For example, when a car is stuck in mud, it is more difficult to get it unstuck than it is to keep it moving once unstuck. To reflect this observation, we use different coefficients of friction, depending on whether the object is at rest or in motion. When an object is at rest (i.e., static), we substitute the **coefficient of static friction**, μ_s , in the friction equation. When an object is in motion, we use the **coefficient of kinetic friction**, μ_k .

EXAMPLE 22 Frictional force

A new homeowner pushes a 150-kg refrigerator across the floor at a constant speed. If the coefficient of kinetic friction is 0.30, what is

- the frictional force on the refrigerator?
- the force applied by the homeowner?

Solution and Connection to Theory

a) Our free-body diagram shows four forces. We know that the fridge is travelling at a constant speed; that is, all forces acting on it are balanced (Newton's first law). We can therefore conclude that the applied force and the frictional force are equal, as are the normal force and the gravitational force. Calculating the frictional force,

$$\vec{F}_{\text{net}_y} = \vec{F}_n + \vec{F}_g$$

$$0 = F_n - F_g, \text{ so } F_n = F_g$$

$$F_f = \mu F_n$$

$$F_f = \mu F_g$$

But $F_g = mg$. Therefore,

$$F_f = \mu_k mg$$

$$F_f = (0.30)(150 \text{ kg})(9.80 \text{ N/kg})$$

$$F_f = 4.4 \times 10^2 \text{ N}$$

The frictional force on the fridge is $4.4 \times 10^2 \text{ N}$.

b) $\vec{F}_{\text{net}_x} = \vec{F}_{\text{app}} + \vec{F}_f$

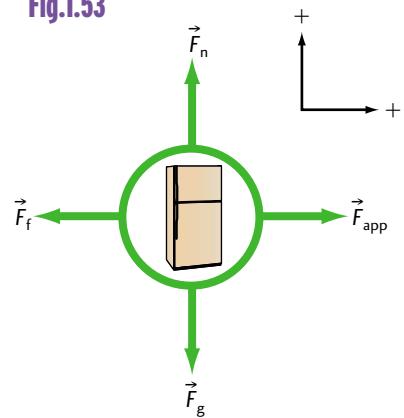
$$0 = F_{\text{app}} - F_f, \text{ so } F_{\text{app}} = F_f$$

The force applied by the homeowner is also $4.4 \times 10^2 \text{ N}$.

The coefficient of friction is the ratio of two forces, the frictional force and the normal force: $\mu = \frac{F_f}{F_n}$. Therefore, μ has no units.

A property of liquids and gases, called **viscosity**, determines the frictional force between two objects sliding over each other when there is a layer of liquid or gas between them. The very low viscosity of air makes the friction between air pucks and a surface almost zero.

Fig.1.53

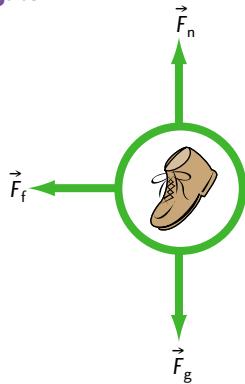


E X A M P L E 2 3 Coming to rest

While cleaning your room, you throw your shoe into the closet. It starts across the floor at a speed of 1.5 m/s. If the shoe has a mass of 200 g, and the coefficient of kinetic friction between the shoe and the floor is 0.15, how far will the shoe travel before coming to rest?

Solution and Connection to Theory

Fig.1.54



Given

$$\vec{v}_1 = 1.5 \text{ m/s} [R] \quad \mu_k = 0.15 \quad m = 0.200 \text{ kg} \quad \vec{v}_2 = 0$$

To the right is positive.

Since the shoe has already left your hand, your hand is no longer capable of applying a force on it. The two vertical forces, the gravitational force and the normal force, balance. Therefore, there is no vertical motion.

In the x direction,

$$F_{\text{net}} = -F_f = -\mu_k F_n$$

$$\text{But } F_n = mg$$

$$F_{\text{net}} = -\mu_k mg$$

$$a = \frac{F_{\text{net}}}{m} = \frac{-\mu_k mg}{m} = -\mu_k g$$

$$a = -(0.15)(9.8 \text{ N/kg})$$

$$a = -1.47 \text{ m/s}^2$$

To calculate the distance the shoe travels before coming to rest, we use the equation

$$v_2^2 = v_1^2 + 2a\Delta d$$

and isolate Δd :

$$\Delta d = \frac{-(v_1^2)}{2a}$$

$$\Delta d = \frac{-(1.5 \text{ m/s})^2}{2(-1.47 \text{ m/s}^2)}$$

$$\Delta d = 0.77 \text{ m}$$

Therefore, the shoe you threw will travel 0.77 m before coming to rest.

EXAMPLE 24 Calculating the force of friction for a lawn mower being pushed

A lawn mower of mass 12 kg is being pushed by a force of 150 N horizontally and 40 N down. If the coefficient of kinetic friction between wheels and grass is 0.9, find the force of friction acting on the lawn mower and the lawn mower's acceleration.

Solution and Connection to Theory

Assume the standard reference system and that motion is to the right.

Given

$$m = 12 \text{ kg} \quad \vec{g} = 9.8 \text{ m/s}^2 \text{ [D]} \quad \mu_k = 0.9 \quad \vec{F}_h = 150 \text{ N [E]} \quad \vec{F}_v = 40 \text{ N [U]}$$

Notice that we have isolated the lawn mower and not the person and lawn mower. If we did that, the two given forces would not be marked on the diagram because they act on the lawn mower only. Because the force of friction involves the normal force, we chose the direction of the normal force first when solving for \vec{F}_{net} . We designate \vec{F}_n as positive.

Vertically,

$$F_{\text{net}} = F_n - F_g - 40 \text{ N}$$

There is no vertical motion. Therefore, $F_{\text{net}} = 0$ and $F_g = mg$.

$$F_n = mg + 40 \text{ N} = (12 \text{ kg})(9.8 \text{ m/s}^2) + 40 \text{ N} = 158 \text{ N}$$

Horizontally, because the lawn mower is moving,

$$F_f = \mu_k F_n = (0.9)(158 \text{ N}) = 142 \text{ N}$$

The force of friction acting on the lawn mower is 142 N.

Having solved for \vec{F}_{net} in the y (vertical) direction, we now solve for \vec{F}_{net} in the direction of motion (horizontal, x) to calculate the lawn mower's acceleration.

$$F_{\text{net}} = 150 \text{ N} - F_f$$

The direction of friction is opposite to the direction of motion. In this case,

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$ma = 150 \text{ N} - 142 \text{ N} = 8.0 \text{ N}$$

$$a = \frac{8.0 \text{ N}}{12 \text{ kg}} = 0.7 \text{ m/s}^2$$

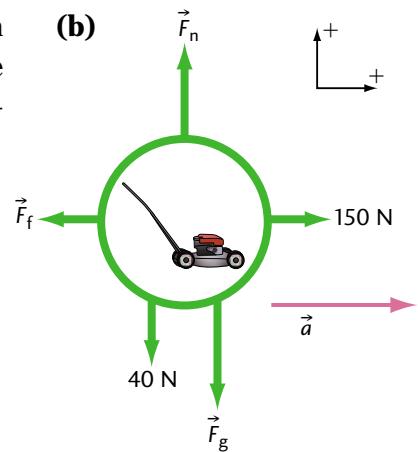
The applied force accelerates the mower at 0.7 m/s^2 [E].

Fig.1.55

(a)



(b)



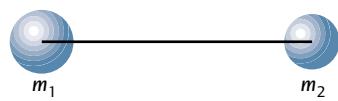


1. **a)** What frictional force is required to bring a skidding 4000-kg truck to rest from 60 km/h in 10 s?
- b)** What minimum coefficient of kinetic friction is required?
2. A toy duck waddles across the floor at a constant speed.
 - a)** How do the magnitudes of the applied force and the frictional force compare to each other?
 - b)** If the coefficient of kinetic friction of the floor is 0.15, what is the maximum forward acceleration the duck can give itself?
3. A 90-kg movie stuntman jumps off the back of a truck moving at 80 km/h, and slides down the road on his protected back. If the coefficient of kinetic friction between his protective suit and the road is 0.60, determine how far the stuntman will slide.

1.15 Newton's Law of Universal Gravitation

Gravity is the mutual force of attraction between any two objects that contain matter, regardless of their size. The strength of the gravitational force between two objects depends on two variables: *mass* and *distance*. If we were to take two physics textbooks into outer space and separate them by 1.0 m, they would accelerate toward each other very slowly. If we separate two planet Earths by 1.0 m between their surfaces, they would accelerate toward each other very rapidly. If we significantly increased the distance separating the two Earths, they would accelerate toward each other much more slowly. Newton expressed this relationship algebraically in his **law of universal gravitation**.

Consider two spheres of mass m_1 and m_2 , separated from their centres by a distance r (see Figure 1.56a). According to the law of universal gravitation, the magnitude of the force of attraction between them is expressed by the equation



$$F_g = \frac{Gm_1m_2}{r^2}$$

where F_g is the gravitational force of m_1 on m_2 , r is the separation of the centres of m_1 and m_2 , and G is the universal gravitational constant.

The universal gravitational constant, G , was first measured by Henry Cavendish in 1798. In his classic experiment, Cavendish used a torsional balance consisting of a horizontal 2-m rod suspended from its centre by a thin wire. A 0.8-kg lead sphere was mounted at each end of the rod. When two larger, 50-kg lead spheres were brought near each of the small spheres, the thin wire twisted slightly due to the forces of attraction between the

Fig.1.56a

large and small spheres (see Figure 1.56b). Cavendish was able to calculate the force required to twist the thin wire, and used it to find the force of attraction between the spheres. He found that the force of attraction between two 1-kg masses 1 m apart is 6.67×10^{-11} N. From Newton's law of universal gravitation,

$$F = \frac{Gm_1m_2}{r^2}$$

Therefore,

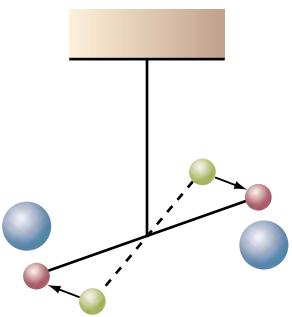
$$G = \frac{Fr^2}{m_1m_2}$$

$$G = \frac{(6.67 \times 10^{-11} \text{ N})(1.00 \text{ m})^2}{(1.00 \text{ kg})(1.00 \text{ kg})}$$

$$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$

This constant should not be confused with g , which is the gravitational field strength, or the acceleration due to gravity.

Fig.1.56b



EXAMPLE 25 The gravitational force between two textbooks

What is the gravitational force between two 1.3-kg textbooks separated from their centres by a distance of 2.0 m?

Solution and Connection to Theory

We will assume that these textbooks are uniform, and that their centres of mass are the centres of the textbooks.

Given

$$m_1 = m_2 = 1.3 \text{ kg} \quad r = 2.0 \text{ m} \quad G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$

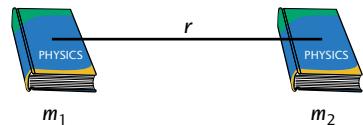
$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.3 \text{ kg})(1.3 \text{ kg})}{(2.0 \text{ m})^2}$$

$$F = 2.8 \times 10^{-11} \text{ N}$$

Because this force is so small, if the two textbooks were released in outer space, the acceleration experienced by each one would also be incredibly small. Note that the forces applied by the books on each other are equal and opposite.

Fig.1.57



Calculating Gravitational Forces

In Section 1.9, we used the equation $F_g = mg$ to calculate the gravitational force. This equation only applies to objects close to Earth's surface. We can derive the value of g using the law of universal gravitation.

Consider an apple of mass m_A near Earth's surface. We express this situation algebraically by the equation

$$F_g = \frac{Gm_E m_A}{r^2}$$

This equation can be rewritten as

$$F_g = m_A \left(\frac{Gm_E}{r^2} \right)$$

If $m_E = 5.98 \times 10^{24}$ kg, $r_E = 6.38 \times 10^6$ m, and $G = 6.67 \times 10^{-11}$ N·m²/kg², then

$$g = \frac{Gm_E}{r^2}$$

$$g = \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.38 \times 10^6 \text{ m})^2}$$

$$g = 9.8 \text{ m/s}^2$$

Therefore,

$$F_g = mg$$

Both equations for gravitation are valid near Earth's surface. As we move farther from Earth's surface, we either have to use the law of universal gravitation or change the value of g to calculate the force of gravity between two objects (see Table 1.3).

Table 1.3 How much does a 60-kg earthling weigh?		
Place	g (N/kg)	Weight (N)
Earth's moon	1.62	97
Mercury	3.61	217
Mars	3.75	225
Venus	8.83	530
Earth	9.81	589

EXAMPLE 26 Gravitational proportionalities

Two spheres of mass m_A and m_B are separated from their centres by a distance r_{AB} . What would happen to the force between the spheres if

- m_A was doubled?
- both masses were doubled?
- the masses were not changed, but the distance between the spheres was doubled?

Solution and Connection to Theory

- Recall that

$$F_{AB_1} = \frac{Gm_A m_B}{r_{AB}^2}$$

If we double m_A , the equation becomes

$$F_{AB_2} = \frac{G(2m_A)m_B}{r_{AB}^2}$$

$$F_{AB_2} = 2\left(\frac{Gm_Am_B}{r_{AB}^2}\right)$$

The value in parentheses is the original force value. Therefore, doubling one of the masses causes the force between them to double.

- b)** If we double both masses,

$$F_{AB_2} = \frac{G(2m_A)(2m_B)}{r_{AB}^2}$$

$$F_{AB_2} = 4\left(\frac{Gm_Am_B}{r_{AB}^2}\right)$$

Doubling both masses causes the force between them to quadruple.

- c)** If we double the distance between the spheres, the equation becomes

$$F_{AB_2} = \frac{Gm_Am_B}{(2r_{AB})^2}$$

$$F_{AB_2} = \frac{1}{4}\left(\frac{Gm_Am_B}{r_{AB}^2}\right)$$

The force is now one-quarter of its original value.

1. What is the force of gravity between two electrons separated by a distance of 1.0 cm? ($m_e = 9.1 \times 10^{-31}$ kg)
2. What is the force of gravity between Earth and the Moon if the Moon's mass is 0.013 times that of Earth?
3. Two spheres of mass m_1 and m_2 are separated by a distance r . What would happen to the force of gravitational attraction between them if
 - a) m_1 was halved and m_2 was quartered?
 - b) m_1 was doubled and r was tripled?
 - c) m_1 , m_2 , and r were all doubled?
4. How far above Earth's surface would you have to go to lose half your weight?
5. Jupiter has a mass of 1.9×10^{27} kg and a radius of 7.2×10^7 m. What is the acceleration due to gravity on Jupiter?





New Respect for the Humble Tire

Canada has some of the most variable weather in the world. Dramatic changes in temperature, combined with rain, snow, and ice, can make driving conditions treacherous. New cars in Canada are usually equipped with all-season radial tires. These tires are designed to provide better traction in the snow than summer tires, while still providing an acceptably good ride during the summer months. All-season radial tires provide a good balance between winter traction and mild weather needs.

Many factors must be considered in designing a quality tire. Two important factors are the values of the coefficients of kinetic and static friction provided by the tire's surface. For driving on dry pavement, we are concerned with the coefficient of static friction only because the tire is constantly rotating while making contact with the road and is not sliding across it. On dry roads, smooth (no-tread) tires, such as those used on racecars, provide a coefficient of static friction of about 0.9. The increased surface area of the tire that makes contact with the road increases the coefficient of static friction.

The value for the coefficient of kinetic friction is usually significantly lower than that of static friction. In some cases, the coefficient of kinetic friction is only 50% of the coefficient of static friction. In an emergency situation, with brakes locked, the coefficient of kinetic friction ultimately determines the car's minimum stopping distance.

When water is on the surface of a road, it acts as a lubricant, which dramatically reduces the coefficient of static friction. As a result, automobile tires are designed with treads that pump the water away from the road, thereby increasing traction. Studies have shown that in snowy or icy conditions, all-season radial tires provide less traction than winter tires and studded tires.

Good tire maintenance is important regardless of tire type or make. Tire pressure must be maintained at the recommended level if the tire treads are to make contact with the driving surface in a way that ensures maximum surface contact. If tires are allowed to wear excessively, insufficient tread depth will prevent tires from channeling water away from the road surface, which decreases the coefficient of static friction and makes maneuvering the car more difficult. Similarly, in an emergency situation without antilock brakes, the coefficient of kinetic friction will be reduced to an even lower value than on a dry surface. On snow and ice, where it may be impossible for tires to make contact with the road surface, tread depth, design, and the type of rubber used are especially important.

Winter tires are designed with treads that are up to 30% deeper than all-season tires. Deeper treads provide greater grip by allowing the tires to eject snow more easily. One reason for loss of traction in all-season tires is that the rubber compounds used tend to become hard at temperatures below -10°C . Winter tires, on the other hand, use special rubber compounds that allow them to stay elastic in temperatures as low as -40°C .

Studded tires have small metal studs embedded in them that provide enhanced traction. They are illegal in Ontario on the grounds that they damage road surfaces, but may be used in all other provinces with some restrictions.

Design a Study of Societal Impact

Every year, thousands of Canadians die in car accidents. Design a study to investigate the societal impact of car accidents. Compare the frequency and severity of the accidents that occur in a population from one season to another. Compare data between two areas, such as Sweden (where studded winter tires are commonplace) and Ontario (where studded tires are illegal). Determine the societal cost of damaged roads caused by studded tires. Do the benefits of using studded tires outweigh the cost?

Design an Activity to Evaluate

Photograph ten different types of tire treads found on cars in your school parking lot or a local tire store. Photograph as broad a selection of tire treads as possible, such as all-season radials, winter tires, and, if possible, studded tires. Look for similarities and differences in tire treads for each of these three types of tires. Using your knowledge of Newton's laws, explain how various features in these treads are used to increase traction, expel snow and water, and help the car maneuver effectively.

Speculate as to which of the tires you photographed provides the shortest stopping distance on dry roads, snow, and ice. Use the Internet and electronic and print resources to collect data on the stopping distances for the tires you have chosen. Compare these results to your predictions.

Fig. STSE.1.1 A Michelin X One all-season radial tire



Build a Structure

Make an "ice slide" by coating a piece of plywood with ice. (This activity is most easily done outdoors in winter by wetting a piece of plywood several times during the day.) Obtain different types of used tires from a local tire store. Cut same-size samples of rubber from each tire and place each piece of rubber sequentially on the ice slide. Then increase the angle of inclination of the slide. Determine the coefficient of static friction for each piece of rubber by using the equation

$$\mu = \tan \theta$$

Compare your results for each rubber sample. In general, how does the coefficient of static friction for winter tires on ice compare to that for all-season radials? What controls are necessary for the results of this experiment to be valid?

You should be able to*Understand Basic Concepts:*

- Differentiate between scalar and vector quantities.
- Perform unit conversions and analysis.
- Define and calculate distance and displacement.
- Define and calculate speed and velocity.
- Define and calculate acceleration.
- Describe algebraically the motion of objects undergoing uniform linear acceleration.
- Solve problems using the five kinematics equations.
- Describe the contributions of Galileo to our understanding of physics.
- Define and describe the acceleration due to gravity of objects near the surface of Earth.
- Solve problems involving objects thrown vertically.
- Perform graphical analyses in describing linear motion.
- Determine information from displacement–time, velocity–time, and acceleration–time graphs.
- Differentiate between mass and weight.
- Define Newton's three laws of motion.
- Differentiate between balanced and unbalanced forces.
- Define and describe the frictional force acting on an object.
- Solve linear problems involving friction using Newton's laws.
- Apply Newton's law of universal gravitation to objects close to and far from the surface of Earth.

Develop Skills of Inquiry and Communication:

- Design and perform an experiment to determine the relationship between displacement and time for an object undergoing uniform acceleration.
- Design and perform an experiment to determine the relationship between the angle of inclination and acceleration for an object on an inclined plane.

Relate Science to Technology, Society, and the Environment:

- Investigate the societal benefits of studded tires.
- Design an activity to evaluate which physical characteristics of tires can be changed to increase traction.
- Determine the coefficient of static friction of different tires.
- Appreciate the role of physics in the design of better tires.

Equations

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{d}}{\Delta t}$$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$\Delta \vec{d} = \left(\frac{\vec{v}_1 + \vec{v}_2}{2} \right) \Delta t$$

$$\Delta \vec{d} = \vec{v}_1 \Delta t + \frac{1}{2} \vec{a} \Delta t^2$$

$$\Delta \vec{d} = \vec{v}_2 \Delta t - \frac{1}{2} \vec{a} \Delta t^2$$

$$\vec{v}_2^2 = \vec{v}_1^2 + 2\vec{a}\Delta d$$

$$\vec{F}_g = m\vec{g}$$

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \dots$$

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$F_f = \mu F_n$$

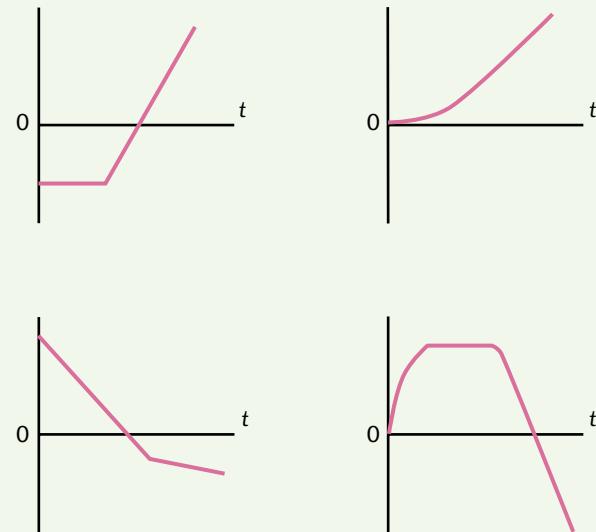
$$F_g = \frac{Gm_1m_2}{r^2}$$

EXERCISES

Conceptual Questions

- Is it possible for an object to be accelerating and at rest at the same time? Explain.
- Does a speedometer measure a car's speed or velocity?
- A penny is dropped into a wishing well from a height of 2 m. The well is 2 m deep, and the penny falls through the water at a constant speed. Sketch a position-time graph and a velocity-time graph describing the motion of the penny.
- Explain the significance of having a negative displacement, a negative velocity, and a negative acceleration.
- Why are the seconds squared in the standard SI unit for acceleration? What is the significance of this notation?
- Figure 1.58 shows possible shapes of graphs. Interpret each graph, first as a \bar{d} - t graph and then as a \bar{v} - t graph, to describe the motion.

Fig.1.58



- A bus drives 1 km up a hill in 5.0 minutes. It then drives down the hill in 4.0 minutes. For the bus, find
 - the average speed up the hill.
 - the average speed down the hill.
 - the average speed for the whole trip.
 - Why is the answer for c) not equal to $\frac{\text{the speed up the hill plus the speed down}}{2}$?

Fig.1.59 Does this craft need its engines burning?



- In science fiction movies, spacecraft are often seen with their engines burning. If you were to fly from planet A to planet B, when during your flight would you be required to burn your engines?
- What is meant by the statement “free-body diagrams show the forces applied on an object, not the forces applied by an object”? Give an example to clarify your answer.
- Draw a free-body diagram of a motorcycle when its brakes are applied as it approaches a red light.
- Write a brief letter to your ten-year-old cousin explaining Newton’s first law of motion. Include an example and a diagram.
- Earth and the Moon apply equal and opposite gravitational forces to each other. Why don’t these two forces cancel each other out?

- 13.** When you fire a rifle, the bullet goes in one direction and you recoil in the opposite direction. Why?
- 14.** A baseball is thrown straight up in the air, then caught on its way down. Prove that the time it takes to go up is equal to the time it takes to fall back down.
- 15.** A ball at the end of a string is swung in a horizontal circle above a person's head at a speed of 2.0 m/s. Is the ball undergoing uniform motion? Explain your answer.

Problems

1.2 Distance and Displacement

- 16.** A flight of stairs is 10 m high. If you were to run up and down the stairs 10 times, determine
- your total distance travelled.
 - your total displacement.

Fig.1.60 A guardsman at Buckingham Palace



- 17.** A guardsman in front of Buckingham Palace (Figure 1.60) marches 15 m [E], followed by 6.0 m [W], and finally 2.0 m [E].
- What is his total distance travelled?
 - What is his total displacement?

1.3 Unit Conversion and Analysis

- 18.** Convert the acceleration due to gravity, g , from its metric units to standard imperial units, feet per second squared. 1 ft = 12 in and 1 in = 2.54 cm.
- 19.** Mariners use a distance measurement called the nautical mile. One nautical mile is 6080 ft. A ship travelling at a speed of one nautical mile per hour is said to be travelling at one knot. What is the speed of a ship travelling at 10 knots
- in kilometres per hour?
 - in metres per second?

- 20.** Astronomers use a distance measurement called the light year. A light year is the distance travelled by light in one Earth year. If light has a speed of 3.0×10^8 m/s, how many centimetres are there in one light year?

1.4 Speed and Velocity

- 21.** Catwoman can run the 100-m dash in 15.4 s. Robin can run the 200-m dash in 28.0 s. Find the average speed of each.
- 22.** The sweep second hand of a clock has a length of 2.0 cm.
- What is the speed of the sweep second hand tip at the 6 o'clock position?
 - What is the velocity of the sweep second hand tip at the 6 o'clock position?
- 23.** A shopper can ride up a moving escalator in 15 s. When the escalator is turned off, the shopper can walk up the stationary escalator in 8.0 s.
- How long would it take the shopper to walk up the moving escalator?
 - Could the shopper walk down the moving escalator to the floor below? If so, how long would it take?

1.5 Acceleration

24. A rabbit, initially hopping at 0.5 m/s, sees a fox and accelerates at a rate of 1.5 m/s^2 for 3.0 s. What is the rabbit's final velocity?
25. How much time does it take for an F-22 fighter jet to accelerate from Mach 1 to Mach 2 at a rate of 50 m/s^2 ? (Hint: speed = Mach number \times speed of sound (332 m/s at 0°C .)
26. A squash ball makes contact with a squash racquet and changes velocity from 15 m/s west to 25 m/s east in 0.10 s. Determine the vector acceleration of the squash ball.

1.6 An Algebraic Description of Uniformly Accelerated Linear Motion

27. Two friends see each other in a grocery store. Initially, they are 50 m apart. The first friend starts walking toward the second friend at a constant speed of 0.50 m/s. At the same instant, the second friend accelerates uniformly from rest at a rate of 1.0 m/s^2 toward the first friend. How long before the two friends can shake hands?
28. Batman is sitting in the Batmobile at a stoplight. As the light turns green, Robin passes Batman in his lime-green Pinto at a constant speed of 60 km/h. If Batman gives chase, accelerating at a constant rate of 10 km/h/s, determine
- how long it takes Batman to attain the same speed as Robin.
 - how far Batman travels in this time.
 - how long it takes for Batman to catch up to Robin.
29. A child is running at her maximum speed of 4.0 m/s to catch an ice-cream truck, which is stopped at the side of the road. When the child is 20 m from the truck, the ice-cream truck starts to accelerate away at a rate of 1.0 m/s^2 . Does the child catch the truck?
Note: This problem can be solved either graphically or algebraically.

30. An Olympic athlete wants to complete the 4000-m run in less than 12.0 minutes. After exactly 10.0 minutes of running at a constant speed, she still has 800 m to go. If she then accelerates at a rate of 0.40 m/s^2 ,

- how much longer will it take her to complete the race?
- will she achieve her desired time?

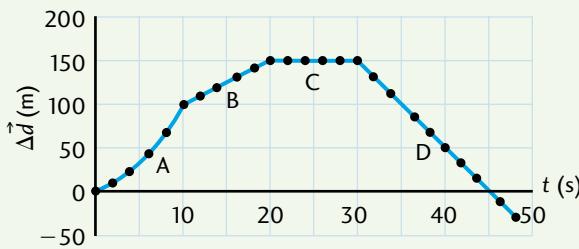
1.7 Bodies in Free Fall

31. A falling flowerpot takes 0.20 s to fall past a window that is 1.9 m tall. From what height above the top of the window was the flowerpot dropped?
32. A person standing on the roof of a building throws a rubber ball down with a velocity of 8.0 m/s.
- What is the acceleration (magnitude and direction) of the ball?
 - Does the ball slow down while falling?
 - After 0.25 s, how far has the ball fallen?
33. A hot-air balloon is rising upward with a constant velocity of 4.0 m/s. As the balloon reaches a height of 4.0 m above the ground, the balloonist accidentally drops a can of pop over the edge of the basket. How long does it take the pop can to reach the ground?
34. A stone is dropped off a cliff of height h . At the same time, a second stone is thrown straight upward from the base of the cliff with an initial velocity \vec{v}_i . Assuming that the second rock is thrown hard enough, at what time t will the two stones meet?

1.8 A Graphical Analysis of Linear Motion

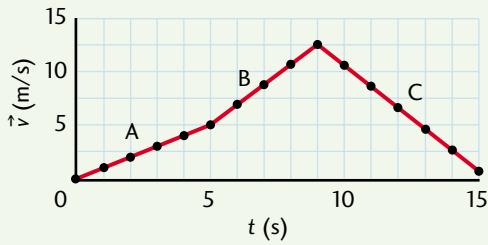
35. Consider the displacement–time graph for a running jackrabbit in Figure 1.61.

Fig.1.61



- a) In which parts of the graph is the jackrabbit undergoing uniform motion?
 b) In which parts of the graph is the jackrabbit undergoing uniform acceleration?
 c) What is the average velocity during segments B, C, and D?
 d) What is the instantaneous velocity of the jackrabbit at $t = 6.0\text{ s}$?
 e) What is its instantaneous velocity at $t = 25\text{ s}$?
 f) Interpret the negative slope in segment D.
 g) What is the jackrabbit's displacement after 42 s?
36. The velocity–time graph in Figure 1.62 is for a car on a drag strip.

Fig.1.62



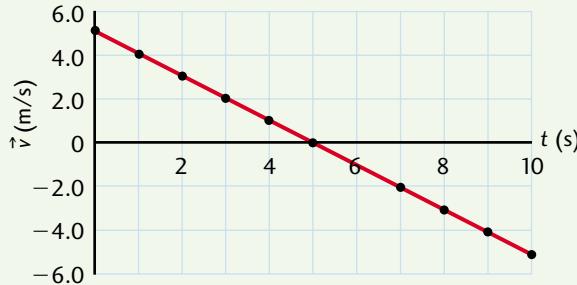
- a) Determine the car's acceleration during each of the three segments shown.
 b) Interpret the negative acceleration.
 c) How far did the car travel during its 15-s trip?

37. A skateboarder is riding his skateboard up and down the sides of an empty hemispherical swimming pool. The velocity–time graph in Figure 1.63b describes his motion as he goes from the bottom of the pool up to ground level and back down again.

Fig.1.63a



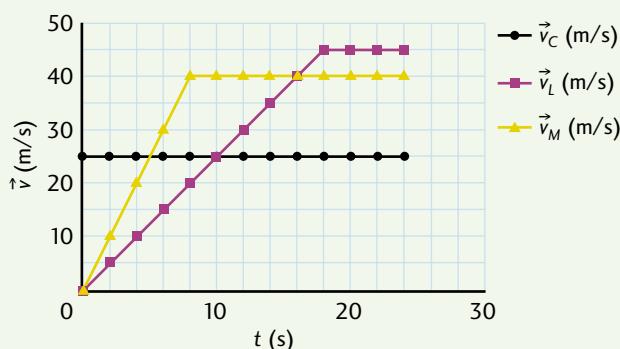
Fig.1.63b



- a) Explain which part of the graph describes the skateboarder's upward motion.
 b) Explain which portion of the graph describes his downward motion.
 c) What kind of motion is the skateboarder undergoing in both of these situations?
 d) At what point on the graph is the skateboarder at rest? Where is he at rest in the swimming pool?
 e) Calculate the skateboarder's acceleration.

38. The Three Stooges, Curly, Larry, and Moe, are having a motorcycle race. Figure 1.64 is the velocity–time graph of their motion.

Fig.1.64

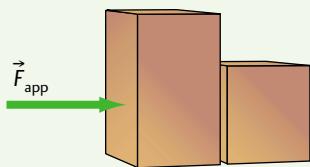


- a) What is the instantaneous acceleration of each Stooge at $t = 4.0$ s?
- b) How far has each Stooge cycled at $t = 4.0$ s?
- c) If the race takes place on a 600-m track, who wins?

1.10 Free-body Diagrams

- 39. Draw a free-body diagram for a baseball at the instant it is struck by a baseball bat.
- 40. Two large boxes are side by side, as shown in Figure 1.65. A force is applied to the box on the left such that both boxes accelerate to the right. Draw a free-body diagram for each box. Include the force of friction.

Fig.1.65



- 41. A baby is bouncing up and down in a Jolly Jumper, as shown in Figure 1.66. Draw a free-body diagram for the baby.

Fig.1.66 A child in a Jolly Jumper



42. A textbook tossed across a lab bench eventually slows down due to friction. Draw a free-body diagram of the textbook just after it hits the bench.

1.11 Newton's First Law of Motion: The Law of Inertia

- 43. For each of the following situations, draw free-body diagrams showing all the forces. Compare the magnitudes of the forces on each FBD.
 - a) You are in an elevator that is at rest on the second floor of a building.
 - b) You are in an elevator that is moving from the second floor to the third floor at a constant speed.
 - c) The cable of the elevator you are in has just broken.
 - d) You are in a car driving at 50 km/h when all of a sudden, you hit a patch of black ice.
 - e) You are in an F-14 *Tomcat* at rest on the flight deck of an aircraft carrier. Suddenly, the catapult is released and you are rapidly launched off the ship.

1.12 Newton's Second Law of Motion: $\vec{F}_{\text{net}} = \vec{m}\vec{a}$

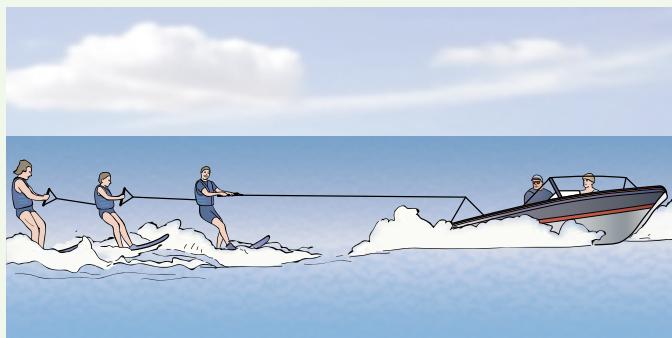
- 44. A pickup truck travelling at 50 km/h strikes a tree. During the collision, the front end of the truck is compressed and the driver comes to rest after travelling a distance of 0.60 m. What is the average acceleration of the driver during the collision? Express your answer in terms of g , the acceleration due to gravity.
- 45. A booster rocket causes a shuttlecraft of mass 10 000 kg to accelerate from 100 m/s to 150 m/s over a distance of 1.00 km. Determine the force applied by the booster rocket to the shuttlecraft.
- 46. Canadian astronaut Chris Hadfield is approaching his shuttlecraft, *Atlantis*, at a velocity of 0.50 m/s. If Chris's mass with equipment is 200 kg and the retro-rockets on his space suit provide a force of 400 N, how long will it take Chris to come to rest?

- 47.** An applied force accelerates mass A at a rate of 6.0 m/s^2 . The same force applied to mass B accelerates the mass at a rate of 8.0 m/s^2 . If the same force were used to accelerate both masses together, what would the resulting acceleration be?

**1.13 Newton's Third Law:
Action–Reaction**

- 48.** A hammer drives a nail a distance of 1.3 cm in 0.10 s . If the hammer has a mass of 1.8 kg , determine the force applied by the hammer to the nail. Determine the force applied by the nail to the hammer.
- 49.** Five 200-kg cows are standing on a steel plate. Determine the upward force applied by the steel plate to the cows.

Fig.1.67



- 50.** A motorboat is pulling three water skiers connected in series, as shown in Figure 1.67. The water skiers' masses are 75 kg , 80 kg , and 100 kg . If the boat applies a force of $10\,000 \text{ N}$, assuming no friction, determine
a) the acceleration of the water skiers.
b) the force applied by each water skier on the other skiers.

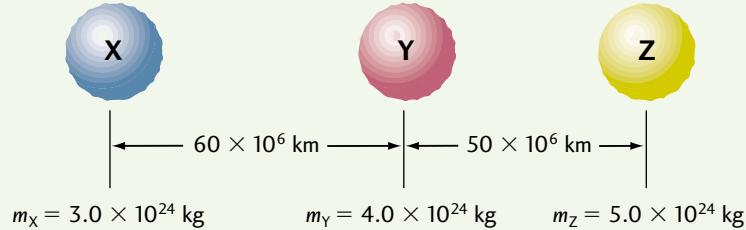
1.14 Friction and the Normal Force

- 51.** Determine the frictional force applied to a 2.0-kg horizontally sliding block if the coefficient of kinetic friction is 0.16 .
- 52.** A pizza box thrown across the room strikes the floor with a horizontal velocity of 2.0 m/s . If the 300-g box encounters a floor with a coefficient of kinetic friction of 0.3 , how far will the box slide before coming to rest?
- 53.** What force must a jet boat's engine apply in order to accelerate the boat from 50 km/h to 100 km/h in 6.0 s ? The mass of the boat is 800 kg and the coefficient of kinetic friction is 0.30 .

1.15 Newton's Law of Universal Gravitation

- 54.** What is the force of gravity between two supertankers, each of mass $300\,000 \text{ kg}$, if they are separated by a distance of 1.0 km ? What acceleration would each tanker experience due to this attraction?
- 55.** What would the acceleration due to gravity on Earth be if its mass was doubled, assuming the same density?
- 56.** Three planets, X, Y, and Z, are situated as shown in Figure 1.68. What is the net gravitational force on planet Z?

Fig.1.68



- 57.** A 100-kg astronaut is in a spacecraft 300 km above Earth's surface. What is the force of gravity on him at this location?



Uniform Acceleration: The Relationship between Displacement and Time

Introduction and Theory

Galileo Galilei performed experiments in fundamental mechanics in the 1600s using a grooved ramp and a sphere. In this experiment, we will duplicate one of Galileo's experiments using more modern equipment.

Purpose

To determine the relationship between displacement and time for an object undergoing uniform acceleration

Hypothesis

Based on what you already know, predict the relationship between displacement and time for an object undergoing uniform acceleration.

Equipment

Stopwatch
Metre stick
Masking tape
Graph paper
Bricks
Dynamics cart

Inclined plane, ramp or table, approximately 2 m long, that can be set at an angle

Procedure

1. Set up the ramp at an angle of inclination of approximately 5° .
2. Place the dynamics cart at the top of the ramp such that all four wheels are on the ramp. Mark a zero point at the front of the cart using masking tape in such a way that it will not interfere with the motion of the cart going down the ramp.
3. Starting at the zero point, measure down the ramp and mark positions at distances of 10 cm, 40 cm, 90 cm, and 160 cm.

Data

1. Simultaneously release the dynamics cart from the zero point and start the stopwatch. Stop the stopwatch as soon as the front of the dynamics cart reaches the 10-cm mark.

Record this distance–time data in a data table and repeat this measurement three more times. Record your values.

2. Repeat step 1 for positions 40 cm, 90 cm, and 160 cm.
3. Calculate an average time value for each of your displacement trials and record it in the data table.

Uncertainty

Assign an instrumental uncertainty to the metre stick you are using. Estimate the uncertainty in your time measurements based on your personal reaction time and the stopwatch you are using. Include these uncertainties in your data table.

Analysis

1. Plot a displacement-versus-average time graph. Draw a line or curve of best fit.
2. Manipulate your displacement–time data so that you end up with a straight-line graph.
3. Calculate the slope of this straight-line graph.

Discussion

1. Describe your initial displacement–time graph.
2. What is the relationship between your displacement and time data values?
3. What must be the relationship between displacement and time to give you a straight-line graph?
4. What was the slope of your straight-line graph? (Don't forget the units!)
5. What is the significance of the slope of your displacement–time graph?
6. Write the equation that describes your straight-line graph.

Conclusion

State the relationship you found between displacement and time for an object undergoing uniform acceleration.



Uniform Acceleration: The Relationship between Angle of Inclination and Acceleration

Purpose

To determine the relationship between the acceleration due to gravity for an object on an inclined plane, and the angle of inclination

Equipment

Tickertape apparatus: power supply, clacker or spark gap timer

Tickertape

Metre stick

Graph paper

Masking tape

C-clamp

Bricks

Dynamics cart

Inclined plane or ramp (approximately 2 m long) that can be set at an angle

Note: This lab can also be performed using other measuring devices such as a motion sensor and computer interface, an air track and photo gates, or a stopwatch.

Procedure

1. Set the ramp at an angle of inclination of approximately 5° .
2. Attach the measuring device you will be using to the top of the ramp.
3. Allow the dynamics cart to roll down the inclined plane, accelerating uniformly.
4. Record the tickertape results in a data table.
5. Repeat the procedure four more times, each time inclining the ramp at a different angle.

Data

For each angle of inclination,

1. measure the height, h , of the top of the inclined plane from the horizontal. Also measure the length, L , of the inclined plane. Record both values in a data table.
2. Analyze your data to determine the time and speed of the cart.
3. Plot a speed–time graph. From the graph, determine the acceleration of the dynamics cart. Record this value in a data table.
4. Calculate the height-to-length ratio ($\frac{h}{L}$) for each height of the inclined plane. Record these numbers in your data table.

Uncertainty

Assign an instrumental uncertainty to the metre stick you are using. Estimate the uncertainty in your acceleration value based on the measuring device you used. Record these values in your data table.

Analysis

Plot a graph of acceleration versus $\frac{h}{L}$, with acceleration as the dependent variable.

Discussion

1. Describe your acceleration-versus- $(\frac{h}{L})$ graph.
2. What is the slope of this graph?
3. What is the equation of this graph?
4. Rewrite your equation using a trigonometric function.
5. Explain how Galileo used this method to determine the acceleration due to gravity.

Conclusion

State the relationship you found between the acceleration of an object on an inclined plane and the angle of inclination.

Kinematics and Dynamics in Two Dimensions



Chapter Outline

- 2.1 Vectors in Two Dimensions
- 2.2 Relative Motion
- 2.3 Projectile Motion
- 2.4 Newton's Laws in Two Dimensions
- 2.5 The Inclined Plane
- 2.6 String-and-pulley Problems
- 2.7 Uniform Circular Motion
- 2.8 Centripetal Force
- The Tape-measure Home Run
- LAB** 2.1 Projectile Motion
- LAB** 2.2 Centripetal Force and Centripetal Acceleration
- LAB** 2.3 Amusement Park Physics

By the end of this chapter, you will be able to

- add and subtract vectors in two dimensions
- analyze the motion of projectiles in two dimensions
- solve problems involving Newton's laws in two dimensions
- solve problems involving inclined planes
- investigate the centripetal accelerations of objects moving in uniform circular motion
- solve problems involving centripetal force

2.1 Vectors in Two Dimensions

In Chapter 1, we learned that a vector is a quantity that has both a magnitude and a direction. Vectors can be represented as directed line segments. Throughout this chapter, we will use vector addition and vector subtraction to solve problems.

In two dimensions, we will use scalar components in the x and y directions. As in one-dimensional kinematics, we will convey direction using the + and – signs. No vector arrows will be used unless referring to a vector diagram or a quantity that has both magnitude and direction (e.g., $\vec{d} = 12 \text{ km } [N30^\circ E]$).

Vector Addition

From Pythagoras' theorem,

$$x^2 = y^2 + z^2$$

where $x = \pm\sqrt{y^2 + z^2}$

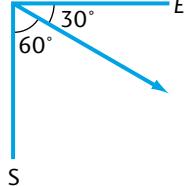
$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Vector Direction

Fig. 2.1b



The direction of this vector can be expressed as [E30°S] or [S60°E], which is read “point south and turn 60 degrees east.”

If two vectors are perpendicular, we can add them using Pythagoras' theorem.

EXAMPLE 1

Two perpendicular vectors

An ant walks 10 cm [E] across a picnic table, then turns and walks 15 cm [N]. What is the ant's total displacement?

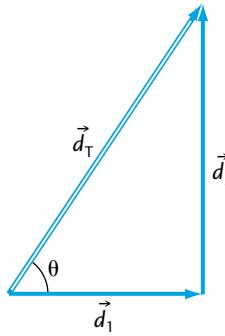
Solution and Connection to Theory

Given

$$\vec{d}_1 = 10 \text{ cm } [E] \quad \vec{d}_2 = 15 \text{ cm } [N]$$

$$\vec{d}_T = \vec{d}_1 + \vec{d}_2$$

Fig. 2.1a



First we determine the resultant's magnitude.

$$d_T^2 = d_1^2 + d_2^2$$

$$d_T^2 = \sqrt{d_1^2 + d_2^2}$$

$$d_T = \sqrt{(10 \text{ cm})^2 + (15 \text{ cm})^2}$$

$$d_T = 18 \text{ cm}$$

For the resultant's direction,

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \theta = \frac{d_2}{d_1}$$

$$\theta = \tan^{-1}\left(\frac{15 \text{ cm}}{10 \text{ cm}}\right)$$

$$\theta = 56^\circ$$

The final displacement is $\vec{d}_T = 18 \text{ cm} [\text{E}56^\circ\text{N}]$ or $18 \text{ cm} [\text{N}34^\circ\text{E}]$.

EXAMPLE 2

Addition of two generalized vectors in two dimensions

A sailboat travels $20 \text{ km} [\text{E}25^\circ\text{N}]$, and then moves $45 \text{ km} [\text{N}40^\circ\text{W}]$. What is the sailboat's total displacement?

Solution and Connection to Theory

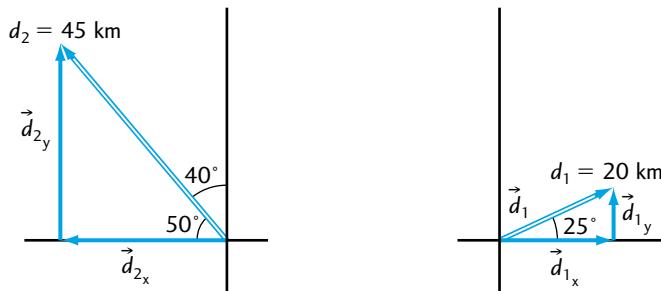
Given

$$\vec{d}_1 = 20 \text{ km} [\text{E}25^\circ\text{N}] \quad \vec{d}_2 = 45 \text{ km} [\text{N}40^\circ\text{W}]$$

$$\vec{d}_T = \vec{d}_1 + \vec{d}_2$$

Method 1: Vector Addition by Components

Fig.2.2



Using the cosine function, we can describe d_{1x} as follows:

$$\cos 25^\circ = \frac{d_{1x}}{d_1}$$

$$d_{1x} = d_1 \cos 25^\circ$$

By analogy, $d_{2x} = -d_2 \cos 50^\circ$

To find the y components, we use the sine function. For d_1 ,

$$\sin 25^\circ = \frac{d_{1y}}{d_1}$$

$$\text{or } d_{1y} = d_1 \sin 25^\circ$$

By analogy, $d_{2y} = d_2 \sin 50^\circ$

The vector sum of the x components is

$$\vec{d}_{T_x} = \vec{d}_{1_x} + \vec{d}_{2_x}$$

$$d_{T_x} = d_1 \cos 25^\circ - d_2 \cos 50^\circ$$

$$d_{T_x} = (20 \text{ km})\cos 25^\circ - (45 \text{ km})\cos 50^\circ$$

$$d_{T_x} = 18.13 \text{ km} - 28.93 \text{ km}$$

$$d_{T_x} = -10.80 \text{ km}$$

$$\text{Therefore, } \vec{d}_{T_x} = 10.80 \text{ km [W]}$$

Notice that we have carried four significant digits in this answer to reduce the chance of rounding error. The final answer will be rounded to two significant digits, which is correct for the numbers given.

The vector sum of the y components is

$$\vec{d}_{T_y} = \vec{d}_{1_y} + \vec{d}_{2_y}$$

$$d_{T_y} = d_1 \sin 25^\circ + d_2 \sin 50^\circ$$

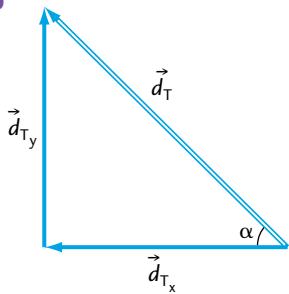
$$d_{T_y} = (20 \text{ km})\sin 25^\circ + (45 \text{ km})\sin 50^\circ$$

$$d_{T_y} = 8.452 \text{ km} + 34.47 \text{ km}$$

$$d_{T_y} = 42.92 \text{ km}$$

$$\text{Therefore, } \vec{d}_{T_y} = 42.92 \text{ km [N].}$$

Fig.2.3



To find the magnitude of the displacement, we use Pythagoras' theorem (Figure 2.3),

$$d_T^2 = d_{T_x}^2 + d_{T_y}^2$$

$$d_T = \sqrt{d_{T_x}^2 + d_{T_y}^2}$$

$$d_T = \sqrt{(10.80 \text{ km})^2 + (42.92 \text{ km})^2}$$

$$d_T = 44 \text{ km}$$

To find the direction, we use the tangent function. From Figure 2.3,

$$\tan \alpha = \frac{d_{T_y}}{d_{T_x}}$$

$$\alpha = \tan^{-1}\left(\frac{42.92 \text{ km}}{10.80 \text{ km}}\right)$$

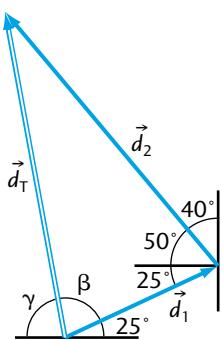
$$\alpha = 76^\circ$$

The x component is negative and the y component is positive; therefore, the angle is [W76°N].

The total displacement is $\vec{d}_T = 44 \text{ km [W76°N]}$.

Method 2: Vector Addition Using the Sine and Cosine Laws

Fig.2.4a



Using the cosine law,

$$d_T^2 = d_1^2 + d_2^2 - 2d_1d_2 \cos 75^\circ$$

$$d_T = \sqrt{(20 \text{ km})^2 + (45 \text{ km})^2 - 2(20 \text{ km})(45 \text{ km})\cos 75^\circ}$$

$$d_T = 44 \text{ km}$$

Using the sine law,

$$\frac{d_2}{\sin \beta} = \frac{d_T}{\sin 75^\circ}$$

$$\beta = \sin^{-1}\left(\frac{(45 \text{ km})\sin 75^\circ}{44 \text{ km}}\right)$$

$$\beta = 79^\circ$$

To find the angle from the horizontal, γ , we subtract from 180° (see Figure 2.4a):

$$\gamma = 180^\circ - \beta - 25^\circ$$

$$\gamma = 76^\circ$$

Therefore, $\vec{d}_T = 44 \text{ km}$ [W76°N]. This answer is the same as the one we obtained using the component method.

TRIGONOMETRIC EQUATIONS

In Figure 2.4b, the angles of the triangle are labeled using uppercase letters, and the sides opposite them are labeled using lowercase letters. Whenever the lengths of two sides and the contained angle (the angle between them) are known, the **cosine law** is used.

$$c^2 = a^2 + b^2 - 2ab \cos C.$$

Similarly, if we know the value of an angle, the side opposite it, plus another angle or side, we can use the **sine law**.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Fig.2.4b

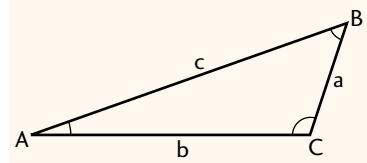
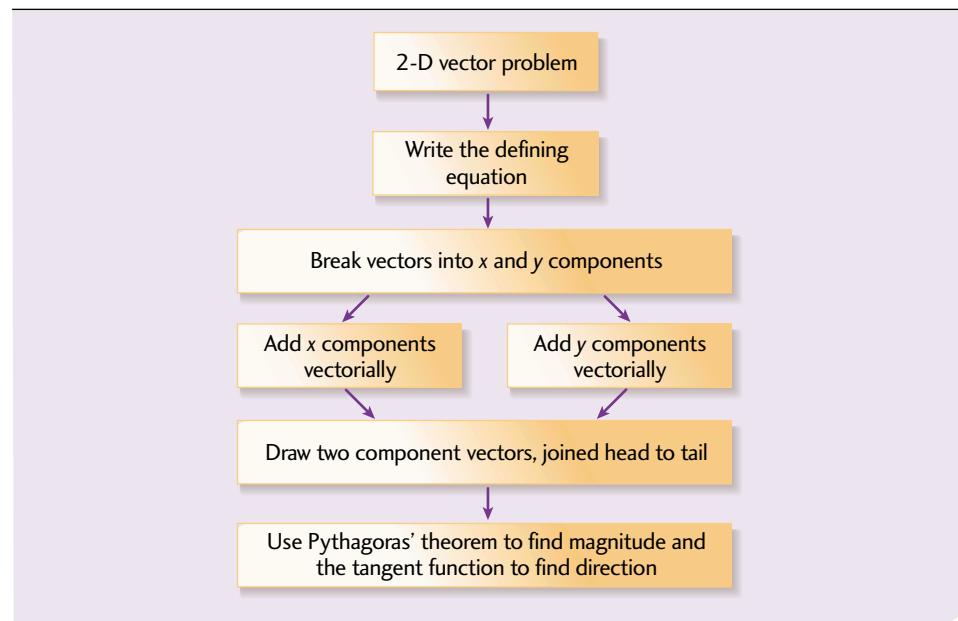


Figure 2.5 summarizes the steps for solving vector addition problems using the component method.

Fig.2.5 Vector Addition by Components



E X A M P L E 3 **Vector acceleration**

Fig.2.6 Can this bus accelerate without changing its speed?



A school bus changes its velocity from 10 m/s [W] to 10 m/s [W30°S] in 5.0 s. Determine the vector acceleration of the school bus.

Solution and Connection to Theory

Given

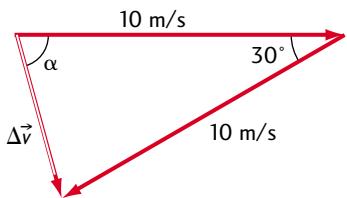
$$\vec{v}_1 = 10 \text{ m/s [W]} \quad \vec{v}_2 = 10 \text{ m/s [W30}^{\circ}\text{S]} \quad t = 5.0 \text{ s}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\Delta \vec{v} = 10 \text{ m/s [W30}^{\circ}\text{S]} - 10 \text{ m/s [W]}$$

This problem involves vector subtraction. We can convert it to a vector addition problem by adding the negative of \vec{v}_1 :

$$\Delta \vec{v} = 10 \text{ m/s [W30}^{\circ}\text{S]} + 10 \text{ m/s [E]}$$

Fig.2.7

To find the *magnitude* of Δv , we use the cosine law:

$$c^2 = a^2 + b^2 - 2ab \cos 30^\circ$$

$$\Delta v = \sqrt{(10 \text{ m/s})^2 + (10 \text{ m/s})^2 - 2(10 \text{ m/s})(10 \text{ m/s})\cos 30^\circ}$$

$$\Delta v = 5.18 \text{ m/s}$$

To find the *direction* of Δv , we use the sine law:

$$\frac{10 \text{ m/s}}{\sin \alpha} = \frac{\Delta v}{\sin 30^\circ}$$

$$\sin \alpha = \frac{(10 \text{ m/s}) \sin 30^\circ}{5.18 \text{ m/s}}$$

$$\alpha = 75^\circ$$

Therefore, $\vec{\Delta v} = 5.18 \text{ m/s [E}75^\circ\text{S]}$.

$$\vec{a} = \frac{\vec{\Delta v}}{\Delta t}, \text{ so}$$

$$\vec{a} = \frac{5.18 \text{ m/s [E}75^\circ\text{S]}}{5.0 \text{ s}}$$

Therefore, the acceleration is $1.0 \text{ m/s}^2 \text{ [E}75^\circ\text{S]}$

Note that the value for acceleration is non-zero. Even though the speed (magnitude of velocity) hasn't changed, the school bus has still undergone acceleration because its direction has changed.

Figure 2.9 summarizes the rules for subtracting vectors.

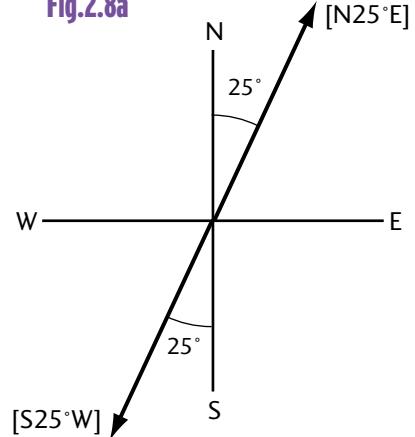
Fig.2.9 Vector Subtraction

Write equation involving subtraction and enter values

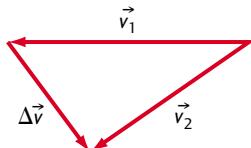
Change sign of negative value to positive and change direction by 180° (e.g., north becomes south)

Add new vectors by components or sine and cosine laws

A vector pointing in one direction can be made to point in the opposite direction by rotating it 180° . For example, a vector pointing [N25°E] rotated 180° now points [S25°W]. Therefore, subtracting a vector pointing [N25°E] is the same as adding a vector pointing [S25°W].

Fig.2.8a

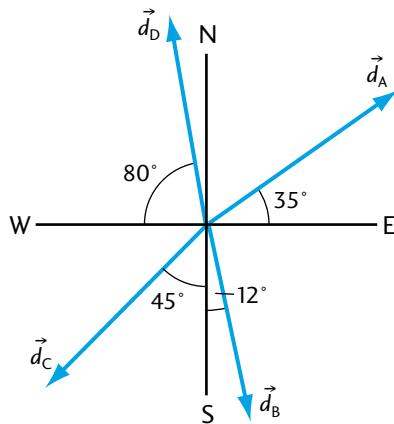
An alternative method of vector subtraction is to join the two vectors tail to tail:

Fig.2.8b



1. Describe the direction for each of the following vectors in two ways.

Fig. 2.10



2. Break each of the following vectors into their x and y components.

- a) $\Delta\vec{d} = 50 \text{ m } [\text{S}14^\circ\text{E}]$
- b) $\vec{v} = 200 \text{ m/s } [\text{W}30^\circ\text{S}]$
- c) $\vec{a} = 15 \text{ m/s}^2 [\text{E}56^\circ\text{N}]$

- 3. A cold duck slides down a snow-covered hill inclined at an angle of 25° to the horizontal. If the duck's speed down the incline is a constant 5.0 m/s , determine the horizontal and vertical components of its velocity.
- 4. A hot-air balloon is rising at a velocity of 3.0 m/s . At the same time, the wind is blowing it horizontally with a velocity of 4.0 m/s . According to an observer on the ground, what is the velocity of the balloon?
- 5. Add each of the following sets of vectors together using the method indicated.
 - a) $\vec{v}_1 = 50 \text{ m/s } [\text{W}36^\circ\text{N}], \vec{v}_2 = 70 \text{ m/s } [\text{E}20^\circ\text{S}]$ (both methods)
 - b) $\Delta\vec{d}_1 = 28 \text{ m } [\text{W}37^\circ\text{S}], \Delta\vec{d}_2 = 40 \text{ m } [\text{W}53^\circ\text{N}]$ (sine and cosine laws)
 - c) $\vec{F}_1 = 140 \text{ N } [\text{W}], \vec{F}_2 = 200 \text{ N } [\text{E}30^\circ\text{N}], \vec{F}_3 = 100 \text{ N } [\text{S}35^\circ\text{W}]$ (component method. Why?)
- 6. When a handball strikes a vertical wall, its velocity changes from $25 \text{ m/s } [\text{S}15^\circ\text{E}]$ to $30 \text{ m/s } [\text{S}40^\circ\text{W}]$. Determine the handball's change in velocity.

2.2 Relative Motion

All motion is relative; that is, motion must be measured relative to a **frame of reference**. For example, if you're sitting in a rowboat that is floating down a river with the current, your friend sitting on the shore may see you moving downstream with a velocity of $10 \text{ m/s } [\text{W}]$. Relative to your friend's frame of reference, you have a velocity of $10 \text{ m/s } [\text{W}]$. On the other hand, relative to a passenger sitting in your boat, you have a velocity of zero; you're not moving because you are both at rest in your common frame of

reference, the boat. Relative to *your* frame of reference in the boat, your friend on the shore is moving at 10 m/s [E]. This example is a simple one-dimensional example of relative motion. In this section, we will study two-dimensional relative motion problems.

Relative Velocity Problems

We will examine a number of examples where an object is moving through a medium, like air or water, which is in turn moving relative to Earth or the ground. In order to keep these velocities distinct, we will use a series of subscripts.

\vec{v}_{og} is the velocity of the person or *object* relative to the *ground*, or ground velocity.

\vec{v}_{mg} is the velocity of the *medium* the person or object is in, relative to the *ground*.

\vec{v}_{om} is the velocity of the *object* or person relative to the *medium* it is in; for example, the velocity of a swimmer relative to the water around him.

The equation that relates these three velocities is

$$\vec{v}_{og} = \vec{v}_{mg} + \vec{v}_{om}$$

EXAMPLE 4 A river-crossing problem: Part A

Fig.2.11

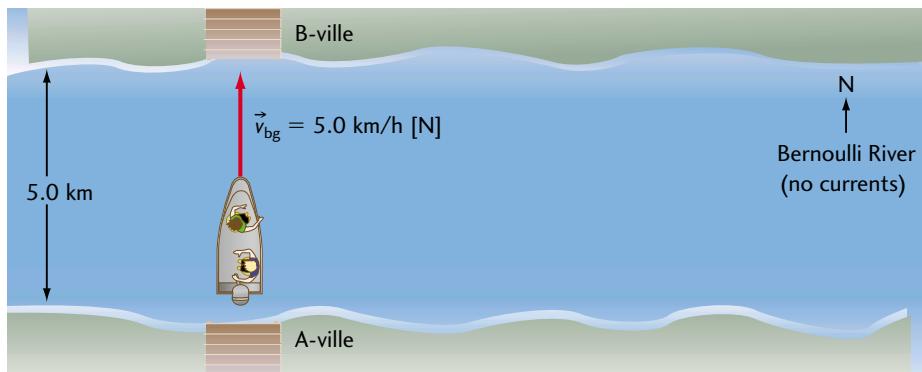


A physics teacher wants to cross the Bernoulli River. He hops into his bass boat at A-ville and drives straight, due north, toward B-ville with a velocity of 5.0 km/h. If the river is 5.0 km wide, how long does it take the teacher to reach the other side?

Solution and Connection to Theory

Since the boat isn't accelerating, we can use the defining equation for speed.

Fig.2.12a



Given

$$\Delta d = 5.0 \text{ km} \quad v_{bg} = 5.0 \text{ km/h}$$

$$v_{bg} = \frac{\Delta d}{\Delta t}$$

$$\Delta t = \frac{\Delta d}{v_{bg}}$$

$$\Delta t = \frac{5.0 \text{ km}}{5.0 \text{ km/h}}$$

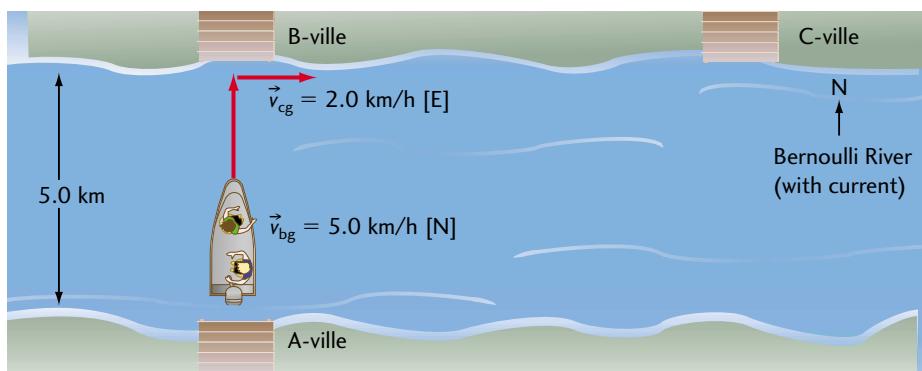
$$\Delta t = 1.0 \text{ h}$$

The teacher takes 1.0 h to reach B-ville.

A river-crossing problem: Part B

Let's introduce a current, flowing at 2.0 km/h [E], which will prevent the boat from landing at B-ville by pushing it farther east. Instead, the boat will land at C-ville, as shown in Figure 2.12b. How does the current affect the time required to cross the river? Does the boat take the same amount of time, a shorter period of time, or a longer period of time?

Fig.2.12b



Solution and Connection to Theory

It will take the boat exactly the same amount of time to reach the other shore, regardless of whether a current is present or not. Time isn't affected because the boat's velocity [N] and the current's velocity [E] are perpendicular to each other. Therefore, the boat's velocity has no components in the same (or in the opposite) direction as the current's velocity and vice versa. The two velocities therefore have no effect on each other. Since the boat is travelling at the same velocity due north as in the Part A of this example, and the distance across the river hasn't changed, it will take the same amount of time to cross the river.

A river-crossing problem: Part C

Given that it still takes 1.0 h to reach the other shore, how far is it from B-ville to C-ville?

Solution and Connection to Theory

The only velocity causing the boat to move downstream from B-ville to C-ville is the river current's velocity. Therefore, we can calculate the distance by using the defining equation for speed and substituting the current's speed, represented by the subscript *cg*:

$$v_{\text{cg}} = \frac{\Delta d}{\Delta t}$$

$$\Delta d = v_{\text{cg}} \Delta t$$

$$\Delta d = (2.0 \text{ km/h})(1.0 \text{ h})$$

$$\Delta d = 2.0 \text{ km}$$

Therefore, the distance between B-ville and C-ville is 2.0 km.

Notice that in this problem, no vector diagrams or vector addition is required. As we have shown, the entire problem can be solved using scalars. We would only need to use vectors if we wanted to determine the ground velocity, \vec{v}_{bg} , of the boat; that is, the velocity of the boat relative to a person standing on the ground (or shore). To solve the problem, we add the two perpendicular vectors, \vec{v}_{bc} (the velocity of the boat relative to the current) and \vec{v}_{cg} (the velocity of the current relative to the ground) using Pythagoras' theorem and the tangent function. This calculation is shown in Figure 2.12c.

$$\vec{v}_{bg} = \vec{v}_{bc} + \vec{v}_{cg}$$

$$v_{bg}^2 = v_{bc}^2 + v_{cg}^2$$

$$v_{bg} = \sqrt{(5.0 \text{ km/h})^2 + (2.0 \text{ km/h})^2}$$

$$v_{bg} = 5.4 \text{ km/h}$$

$$\tan \beta = \frac{v_{cg}}{v_{bc}}$$

$$\beta = \tan^{-1}\left(\frac{2.0 \text{ km/h}}{5.0 \text{ km/h}}\right)$$

$$\beta = 22^\circ$$

$$\vec{v}_{bg} = 5.4 \text{ km/h [N}22^\circ\text{E]}$$

Therefore, the boat's ground velocity is 5.4 km/h [N22°E].

Fig.2.12c

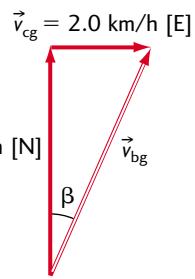


Figure 2.13 summarizes how to solve problems where the object's heading is perpendicular to the medium velocity.

Fig.2.13 Solving Problems Involving Perpendicular Vectors



Two perpendicular vectors, magnitude and direction given

Solve for time using $v = \frac{\Delta d}{\Delta t}$
(scalar problem)

Solve for ground velocity using Pythagoras' theorem and tangent function (vector problem)

Problems Involving Non-perpendicular Vectors

E X A M P L E 5

A boat-navigation problem

The physics teacher from Example 4 Part B wants to go to B-ville, which is directly north of A-ville. To do so, the bass boat must be aimed upstream to compensate for the current, as shown in Figure 2.14a. The current velocity is 2.0 km/h [E] and the boat's speed is 5.0 km/h.

- a) In which direction must the boat be pointed in order to land at B-ville?
- b) What is the ground velocity of the boat?
- c) How long will it take the boat to reach the other shore?

Solution and Connection to Theory

Let \vec{v}_{cg} be the velocity of the current relative to the ground, let \vec{v}_{bc} be the velocity of the boat relative to the water, and let \vec{v}_{bg} be the velocity of the boat relative to the ground. The triangle in Figure 2.14a is a right-angle triangle. Therefore, we can determine the direction in which the boat must be pointed by using the cosine function:

Given

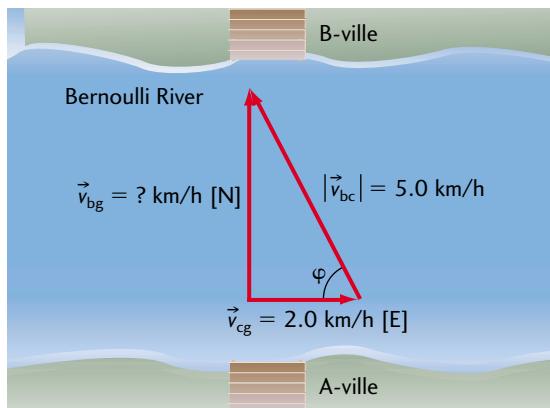
$$\vec{v}_{cg} = 2.0 \text{ km/h [E]} \quad \vec{v}_{bc} = 5.0 \text{ km/h [want to go north]}$$

a) for direction, $\cos \varphi = \frac{v_{cg}}{v_{bc}}$

$$\varphi = \cos^{-1}\left(\frac{2.0 \text{ km/h}}{5.0 \text{ km/h}}\right)$$

$$\varphi = 66^\circ$$

Fig. 2.14a



In Figure 2.14a, \vec{v}_{bc} shows the direction in which the boat must be pointed in order to land at B-ville; that is, [W66°N] or [N24°W].

b) To determine the magnitude of the ground velocity, v_{bg} ,

$$v_{bc}^2 = v_{cg}^2 + v_{bg}^2$$

$$v_{bg}^2 = v_{bc}^2 - v_{cg}^2$$

$$v_{bg} = \sqrt{(5.0 \text{ km/h})^2 - (2.0 \text{ km/h})^2}$$

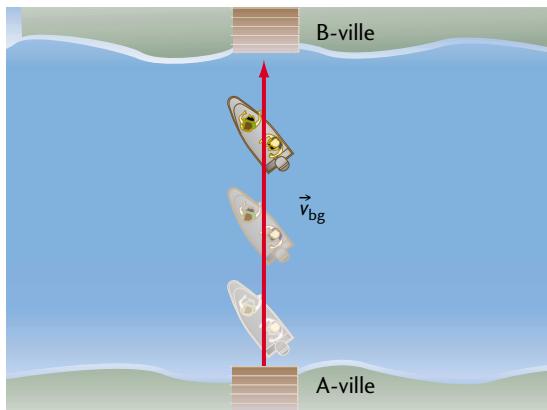
$$v_{bg} = 4.58 \text{ km/h} \text{ (The extra digit is for further calculations.)}$$

From Figure 2.14a, we can see that the direction of the ground velocity is north.

Therefore, the ground velocity of the bass boat is 4.58 km/h [N].

- c) If you were observing the boat, you would see the situation illustrated in Figure 2.14b.

Fig. 2.14b



Note that the magnitude of the ground velocity is less than the magnitude of the boat's velocity relative to the water. As a result, it will take longer to cross than if the boat was pointed straight across the river. This time, however, the teacher lands at B-ville. The time to cross the river can be calculated as follows:

$$v_{bg} = \frac{\Delta d}{\Delta t}$$

$$\Delta t = \frac{(5.0 \text{ km})}{(4.58 \text{ km/h})}$$

$$\Delta t = 1.1 \text{ h}$$

In Example 4, it took the teacher 1.0 h to cross the river. Therefore, it now takes him an extra 0.1 h to cross the river.

EXAMPLE 6 A classic air-navigation problem

A pilot wishes to fly her Cesna 441 due north. There is a wind from the west at 20.0 km/h. If the plane can fly at a velocity of 150 km/h in still air,

- a) what is the plane's heading (i.e., in which direction should the pilot point the plane)?
- b) what is the plane's ground velocity?

Fig. 2.15a What factors affect air navigation?



Solution and Connection to Theory

Given

$\vec{v}_{wg} = 20.0 \text{ km/h [E]}$ (a wind from the west blows east)

$$\vec{v}_{ow} = 150 \text{ km/h [?]} \quad \vec{v}_{og} = ? \text{ km/h [N]}$$

a) To determine the plane's heading, we solve for γ (Figure 2.15b):

$$\cos \gamma = \frac{V_{wg}}{V_{ow}}$$

$$\gamma = \cos^{-1}\left(\frac{20.0 \text{ km/h}}{150 \text{ km/h}}\right)$$

$$\gamma = 82^\circ$$

Therefore, by looking at Figure 2.15b, we can see that the plane's heading is [W82°N].

b) To calculate the magnitude of the ground velocity,

$$V_{ow}^2 = V_{og}^2 + V_{wg}^2$$

$$V_{og} = \sqrt{V_{ow}^2 - V_{wg}^2}$$

$$V_{og} = \sqrt{(150 \text{ km/h})^2 - (20.0 \text{ km/h})^2}$$

$$V_{og} = 149 \text{ km/h}$$

We are given that the plane's direction is north. Therefore, the plane's ground velocity is 149 km/h [N].

Fig.2.15b

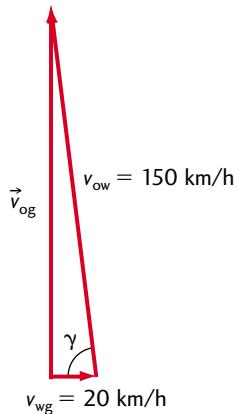
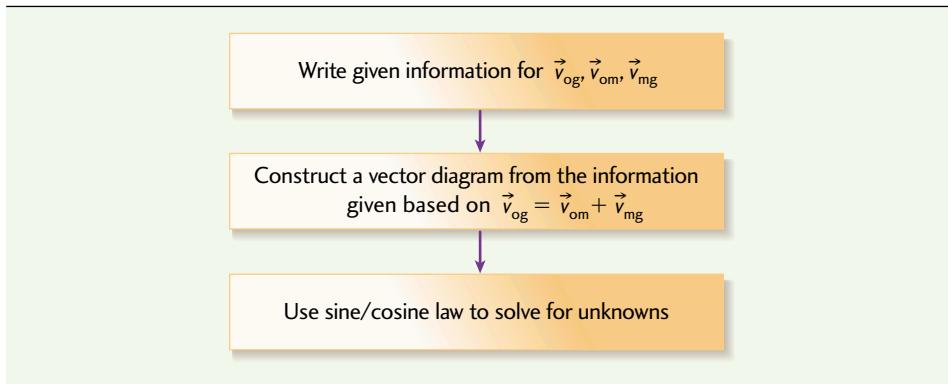


Figure 2.16 summarizes how to solve relative velocity problems.

Fig.2.16 The Overall Picture



1. A ship's captain wants to sail east. Her ship experiences a current of 5.0 km/h [N]. The ship's engines can produce a speed of 20 km/h.
 - What is the ship's required heading?
 - What is the ground velocity of the ship?
 - If the ship travels a total distance of 100 km, how long is the trip?



2. A boy and girl race their canoes across a 500-m-wide river to a location due north of their starting point. Both young people can paddle their canoes at a velocity of 3.0 m/s in still water. The boy paddles in a northerly direction, while the girl aims her canoe slightly upstream so that she travels directly north as she paddles. If the current in the river is 0.50 m/s [W], determine
- a) the girl's heading.
 - b) the time it takes for each person to reach the opposite shore.
 - c) the distance between the boy and girl when they reach the opposite shore.
 - d) As soon as he reaches the shore, the boy starts to run toward the girl's landing site at a speed of 5.0 m/s. Who wins the race?
3. A large cruise ship is moving with a velocity of 10 km/h [E] relative to the water. A passenger jogging on deck moves with a velocity of 6.0 km/h [N] relative to the ship. What is the jogger's velocity relative to the water?

Fig.2.17



4. Terry, the three-year-old terror, rides his tricycle down the sidewalk at a velocity of 0.50 m/s [N]. As he passes his sister, who is 5.0 m east of his position, Terry throws a peanut-butter-and-jam sandwich at her.
- a) If Terry can throw the sandwich at a maximum velocity of 2.0 m/s, in which direction must he throw it in order to hit his sister?
 - b) How much time does his sister have to get out of the way?

2.3 Projectile Motion

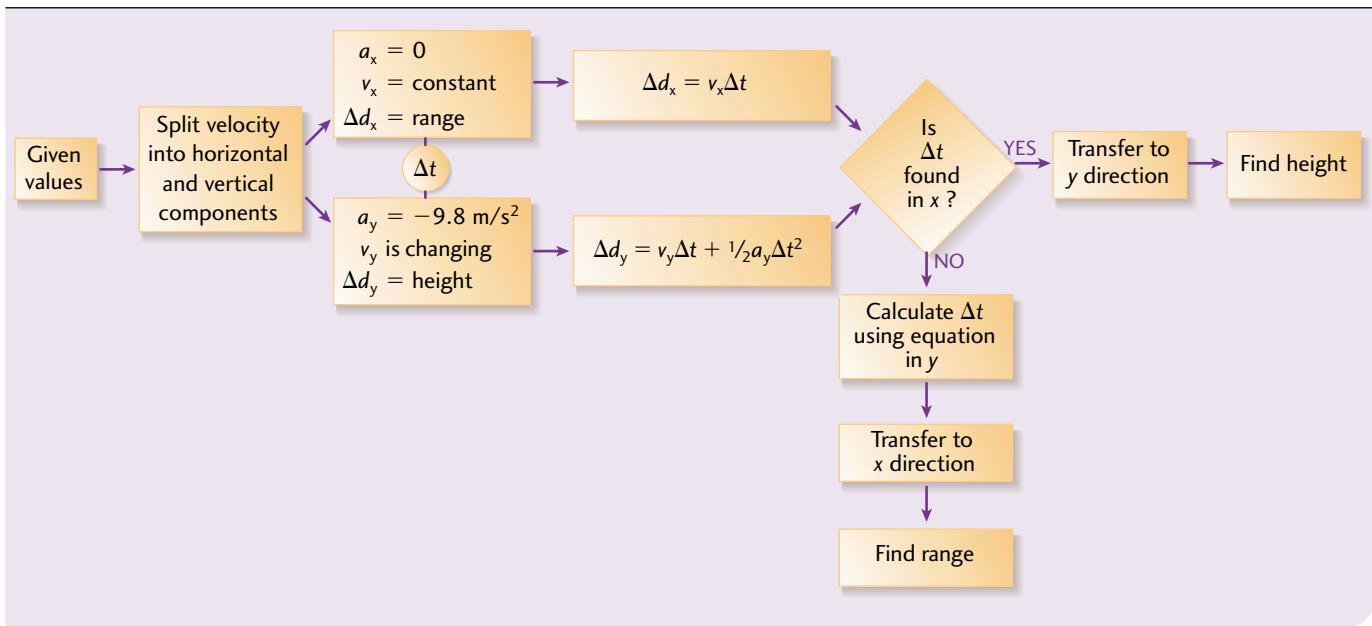
Range is the horizontal distance travelled by a projectile.

In projectile motion, the projectile travels at a constant velocity in the horizontal direction only. In the vertical direction, however, all projectiles accelerate downward at 9.8 m/s^2 due to the force of gravity. A projectile therefore experiences uniform horizontal motion as well as vertical acceleration (assuming no air resistance).

Figure 2.18 summarizes the method of solving projectile motion problems.

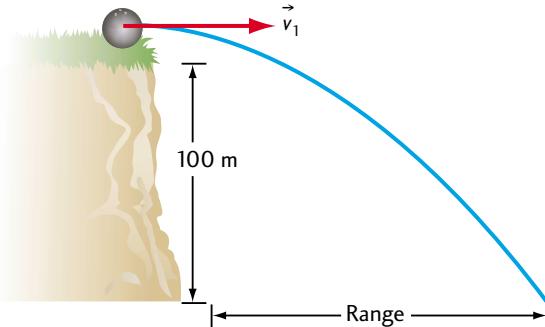


Fig.2.18 Solving Projectile Motion Problems



EXAMPLE 7 A horizontal projectile

Fig.2.19



A bowling ball is rolled off the top of a cliff with an initial horizontal velocity of 6.0 m/s (Figure 2.19). If the cliff is 100 m above the ground, determine

- the ball's time of flight (i.e., the time taken to reach the ground).
- the ball's range (i.e., the horizontal distance travelled by the ball).
- the final velocity of the ball just before it strikes the ground.

Solution and Connection to Theory

In projectile motion problems, the horizontal and vertical components of motion are considered separately. The common variable is the time of flight; that is, the length of time the object is in the air is the same for both vertical and horizontal components.

For projectile motion problems, we use the directions of the Cartesian coordinate system: directions of objects going up or to the right are positive, while those going down or to the left are negative.

Given

$$v_{1x} = 6.0 \text{ m/s} \quad a_x = 0 \quad v_{1y} = 0 \quad a_y = -9.8 \text{ m/s}^2 \quad \Delta d_y = -100 \text{ m}$$

- a) To determine the time of flight from the vertical components, we use the equation

$$\Delta d_y = v_{1y}\Delta t + \frac{1}{2}a_y\Delta t^2$$

Since the ball originally rolled off the cliff horizontally, its initial vertical velocity is zero; that is, $v_{1y} = 0$. Therefore,

$$\Delta d_y = \frac{1}{2}a_y\Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

$$\Delta t = \pm \sqrt{\frac{2(-100 \text{ m})}{-9.8 \text{ m/s}^2}}$$

We choose the positive root because time must be positive.

$$\Delta t = 4.52 \text{ s}$$

- b) To calculate the range, we use the equation

$$\Delta d_x = v_{1x}\Delta t + \frac{1}{2}a_x\Delta t^2$$

Since this projectile has no horizontal acceleration, $a_x = 0$. Therefore,

$$\Delta d_x = v_{1x}\Delta t$$

$$\Delta d_x = (6.0 \text{ m/s})(4.52 \text{ s})$$

$$\Delta d_x = 27 \text{ m}$$

Therefore, the range of the bowling ball is 27 m.

- c) To calculate the final velocity, we must first determine the final velocity components. For the horizontal motion, the final velocity is equal to the initial velocity because there is no acceleration:

$$v_{1x} = v_{2x} = 6.0 \text{ m/s}$$

For the vertical motion, the object is accelerating.

$$v_{2y}^2 = v_{1y}^2 + 2a_y\Delta d_y$$

Since the ball is rolled off the cliff, its initial motion is horizontal. Therefore, its vertical velocity $v_{1y} = 0$. So,

$$v_{2y}^2 = v_{1y}^2 + 2a_y\Delta d_y$$
$$v_{2y} = \sqrt{2(-9.8 \text{ m/s}^2)(-100 \text{ m})}$$

$$v_{2y} = 44.3 \text{ m/s}$$

For the final velocity,

$$v_f = \sqrt{(6.0 \text{ m/s})^2 + (44.3 \text{ m/s})^2}$$

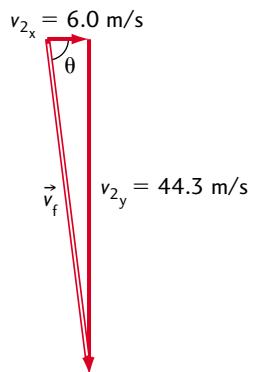
$$v_f = 45 \text{ m/s}$$

$$\tan \theta = \frac{44.3 \text{ m/s}}{6.0 \text{ m/s}}$$

$$\theta = 82^\circ$$

Therefore, the ball's final velocity is 45 m/s [R82°D].

Fig.2.20

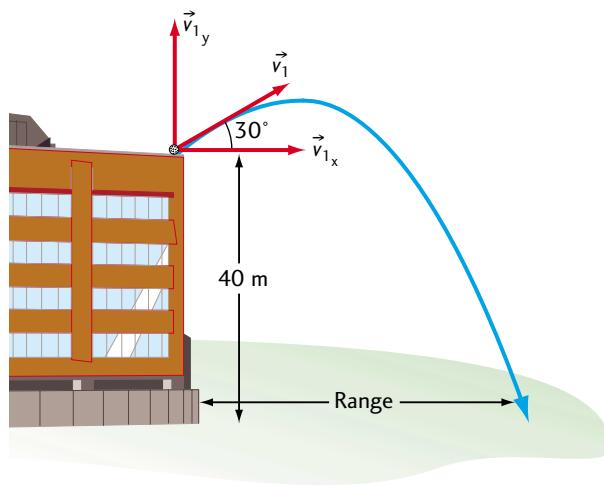


EXAMPLE 8 A projectile launched at an angle

A golf ball is launched from the roof of a school with a velocity of 20 m/s at an angle of 30° above the horizontal. If the roof is 40 m above the ground, calculate

- a) the ball's time of flight.
- b) the ball's horizontal displacement.

Fig.2.21



Solution and Connection to Theory

Given

$$v_1 = 20 \text{ m/s} \quad \theta = 30^\circ \quad a_x = 0 \quad a_y = -9.8 \text{ m/s}^2 \quad \Delta d_y = -40 \text{ m}$$

$$v_{1x} = v_1 \cos 30^\circ$$

$$v_{1y} = v_1 \sin 30^\circ$$

$$v_{1x} = 17.3 \text{ m/s}$$

$$v_{1y} = 10.0 \text{ m/s}$$

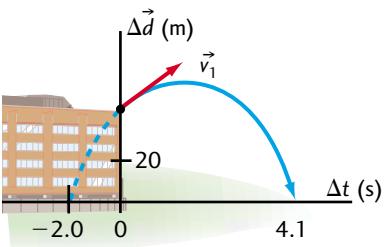
From the given information, we can determine the time of flight by considering vertical motion only.

Negative Time

We can give a meaning to the negative square root for time found in problems like Example 8. The quadratic equation, which describes a parabola, has two roots: $\Delta t = -2.0 \text{ s}$ and $\Delta t = 4.1 \text{ s}$.

These values are the two points at which the parabola intersects the time axis. If the golf ball had been launched from the ground so that its velocity 40 m up was 20 m/s, 30° above the horizontal, it would have taken 2.0 s to reach that point. Mathematically, the earlier part of the motion is the part of the parabola to the left of the vertical axis, as shown in Figure 2.22.

Fig. 2.22



a) For the vertical components,

$$\Delta d_y = v_{1y}\Delta t + \frac{1}{2}a_y\Delta t^2$$

$$0 = \frac{1}{2}a_y\Delta t^2 + v_{1y}\Delta t - \Delta d_y$$

$$0 = (-4.9 \text{ m/s}^2)\Delta t^2 + (10.0 \text{ m/s})\Delta t + 40 \text{ m}$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta t = \frac{-10 \text{ m/s} \pm \sqrt{(10.0 \text{ m/s})^2 - 4(-4.9 \text{ m/s}^2)(40 \text{ m})}}{2(-4.9 \text{ m/s}^2)}$$

$$\Delta t = \frac{-10 \text{ m/s} \pm 29.7 \text{ m/s}}{-9.8 \text{ m/s}^2}$$

$$\Delta t = -2.0 \text{ s} \text{ or } \Delta t = 4.1 \text{ s}$$

We take the positive Δt because negative time is not permitted. Therefore, the golf ball's time of flight was 4.1 s.

b) To determine the horizontal displacement,

$$\Delta d_x = v_{1x}\Delta t + \frac{1}{2}a_x\Delta t^2$$

Since there is no horizontal acceleration, $a_x = 0$.

$$\Delta d_x = v_{1x}\Delta t$$

$$\Delta d_x = (17.3 \text{ m/s})(4.1 \text{ s})$$

$$\Delta d_x = 71 \text{ m}$$

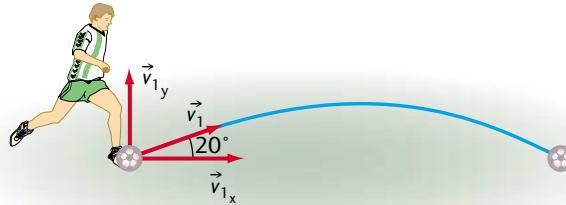
Therefore, the golf ball will travel 71 m horizontally.

EXAMPLE 9 A projectile with a vertical displacement of zero

Fig. 2.23



Fig. 2.24



A soccer player kicks a ball from the ground with a velocity of 15 m/s. If the ball is kicked at an angle of 20° above the horizontal,

- a) what is the ball's time of flight?
- b) how far will the ball travel horizontally before striking the ground?

Solution and Connection to Theory

This problem is an example of a special case. Since the soccer ball starts and finishes its motion at ground level, its vertical displacement is zero, which means that we don't need to use the quadratic equation to determine the time of flight.

a) Given

$$\vec{v}_i = 15 \text{ m/s} [\text{E}20^\circ\text{N}] \quad a_x = 0 \quad a_y = -9.8 \text{ m/s}^2 \quad \Delta d_y = 0 \text{ m}$$

$$v_{1x} = v_i \cos 20^\circ$$

$$v_{1x} = 14.1 \text{ m/s}$$

$$v_{1y} = v_i \sin 20^\circ$$

$$v_{1y} = 5.13 \text{ m/s}$$

For the vertical components, we use the equation

$$\Delta d_y = v_{1y}\Delta t + \frac{1}{2}a_y\Delta t^2$$

Since $\Delta d_y = 0$,

$$0 = v_{1y}\Delta t + \frac{1}{2}a_y\Delta t^2$$

Because $\Delta t \neq 0$, we can divide both sides by Δt .

$$0 = v_{1y} + \frac{1}{2}a_y\Delta t$$

$$\Delta t = \frac{-(2v_{1y})}{a_y}$$

$$\Delta t = \frac{-2(5.13 \text{ m/s})}{-9.8 \text{ m/s}^2}$$

$$\Delta t = 1.047 \text{ s}$$

Therefore, the ball's time of flight is 1.0 s.

Range

We can derive an equation for the range of a projectile in the special case where it starts and ends at the same level: $\Delta d_y = 0$.

$$\Delta d_x = (v \sin \theta)\Delta t$$

$$\sin \theta = \frac{\Delta d_x}{v\Delta t}$$

$$\Delta d_y = 0 = (v \cos \theta)\Delta t - \frac{1}{2}g\Delta t^2$$

$$\cos \theta = \frac{g\Delta t^2}{2v\Delta t}$$

Combining these two equations, we obtain

$$\sin \theta \cos \theta = \frac{\Delta d_x g \Delta t^2}{2v^2 \Delta t^2}$$

$$2\sin \theta \cos \theta = \frac{\Delta d_x g}{v^2}$$

But $2\sin \theta \cos \theta = \sin 2\theta$

$$\text{Range} = R = \Delta d_x = \frac{v^2 \sin 2\theta}{g}$$

b) For the horizontal components,

$$\Delta d_x = v_{1x}\Delta t + \frac{1}{2}a_x\Delta t^2$$

Since $a_x = 0$,

$$\Delta d_x = v_{1x}\Delta t$$

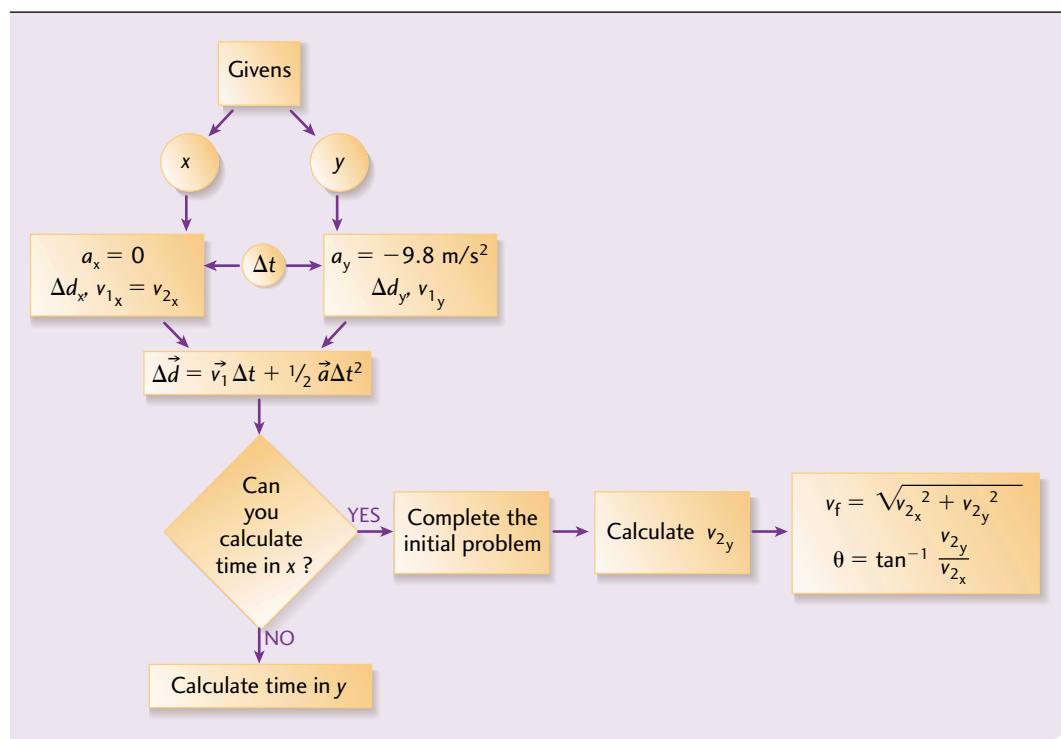
$$\Delta d_x = (14.1 \text{ m/s})(1.047 \text{ s})$$

$$\Delta d_x = 15 \text{ m}$$

Therefore, the ball's range is 15 m.

Figure 2.25 summarizes the steps in calculating the final velocity of a projectile.

Fig.2.25 Projectile Motion Overview



- 1.** A helicopter flying horizontally at a velocity of 25 m/s drops a mailbag from a height of 15 m to a letter carrier waiting on the ground below.
- How long will the bag take to fall to the ground?
 - How far in advance of the letter carrier must the bag be released so that it lands at her feet?
- 2.** Blasto the Magnificent is fired from a cannon inclined at 40° to the horizontal (Figure 2.26). If Blasto leaves the cannon at a speed of 35 m/s,
- how long will it take him to reach his maximum height?
 - how far will he travel horizontally?

3. A fighter jet going 350 km/h dives at an angle of 25° to the vertical. If it drops a bomb from a height of 200 m,
- how far will the bomb travel horizontally?
 - what will be the velocity of the bomb just before it hits the ground?
4. A golfer strikes a golf ball at an angle of 17° above the horizontal. With what velocity must the ball be struck in order to reach the green, which is a horizontal distance of 250 m from the golfer at the same height?

Fig.2.26



2.4 Newton's Laws in Two Dimensions

We are now ready to solve some other types of problems in two dimensions. These problems will be either in the horizontal or the vertical plane.

EXAMPLE 10 Newton's laws in two dimensions (balanced forces)

A barge is being pulled through a canal by two horses, as shown in Figure 2.27a. If each horse applies a force of 5000 N, determine the frictional force applied by the water as the barge moves at a constant speed.

Fig.2.27a

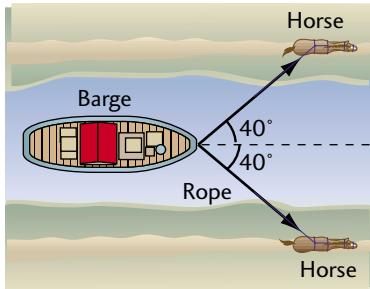
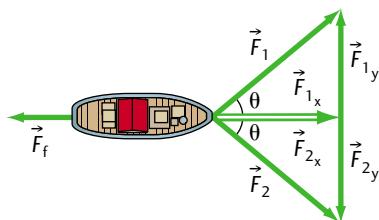


Fig.2.27b



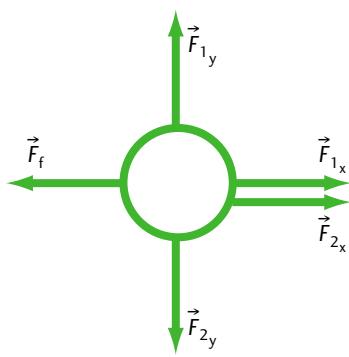
Solution and Connection to Theory

Given

$$F_1 = F_2 = 5000 \text{ N} \quad \theta = 40^\circ$$

The free-body diagram in Figure 2.27b has two forces, labeled \vec{F}_1 and \vec{F}_2 , applied on the barge, pulling to the right. Each of these vectors has two perpendicular components: \vec{F}_{1x} , \vec{F}_{1y} and \vec{F}_{2x} , \vec{F}_{2y} . Since the forces applied by each horse are equal, and the angles at which the forces are applied relative to the canal are equal, by symmetry, F_{1y} and F_{2y} are both $(5000 \text{ N})\sin 40^\circ$ in opposite directions. As a result, the y-component forces cancel and the net force on the barge is strictly forward. So, the net force is the sum of the x components.

Fig.2.27c



Newton's first law states that if the barge is travelling at a constant velocity, all forces must be balanced. Therefore, F_{1x} and F_{2x} (applied to the right) must be balanced in magnitude by the frictional force, F_f (applied to the left).

For the y components,

$$\begin{aligned}\vec{F}_{\text{net}y} &= \vec{F}_{1y} + \vec{F}_{2y} \\ F_{\text{net}y} &= F_1 \sin 40^\circ - F_2 \sin 40^\circ \\ F_{\text{net}y} &= 0\end{aligned}$$

For the x components,

$$\vec{F}_{\text{net}x} = \vec{F}_{1x} + \vec{F}_{2x} + \vec{F}_f$$

Since $F_{1x} = F_{2x}$,

$$F_{\text{net}x} = 2F_1 \cos 40^\circ - F_f$$

$$F_{\text{net}x} = ma_x = 0$$

$$0 = 2(F_1 \cos 40^\circ) - F_f$$

$$F_f = 2(5000 \text{ N}) \cos 40^\circ$$

$$F_f = 7.7 \times 10^3 \text{ N}$$

Therefore, the frictional force applied by the water on the barge is $7.7 \times 10^3 \text{ N}$ [left].

EXAMPLE 11 Newton's laws in two dimensions (vertical plane)

Fig.2.28a

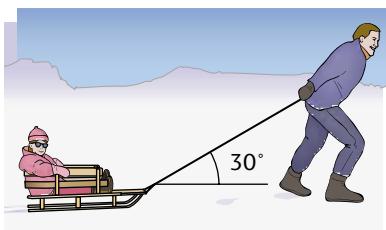
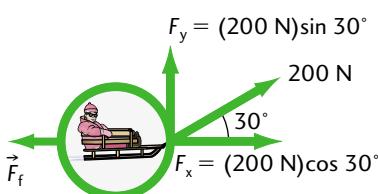


Fig.2.28b



A father pulls a child on a sled across the snow, as shown in Figure 2.28a. The child and sled have a combined mass of 50 kg. If the snow has a coefficient of kinetic friction of 0.28 and the father applies a force of 200 N along the handle of the sled 30° above the horizontal, determine the acceleration of the child and sled.

Solution and Connection to Theory

Given

$$m = 50 \text{ kg} \quad \mu_k = 0.28 \quad \vec{F} = 200 \text{ N} [\text{R}30^\circ \text{ U}]$$

In Figure 2.28b, the applied force, \vec{F} , has been broken down into its perpendicular components. Assuming that the child and sled accelerate horizontally only, then, according to Newton's first law, all of the vertical forces must be balanced.

For the vertical motion, according to Newton's first law, since the object is not accelerating vertically, the sum of the forces upward must be balanced by the sum of the forces downward. Algebraically, the scalar equation is

$$F_{\text{net}} = 0 = F_n + F \sin 30^\circ - F_g$$

$$F_n = mg - F \sin 30^\circ$$

$$F_n = 3.90 \times 10^2 \text{ N}$$

For the horizontal motion, we apply Newton's second law:

$$F_{\text{net}_x} = ma$$

$$F_x - F_f = ma$$

$$a = \frac{F_x - F_f}{m} \quad (\text{eq. 1})$$

$$F_x = F \cos 30^\circ \quad (\text{eq. 2})$$

$$F_f = \mu_k F_n$$

$$F_f = (0.28)(3.90 \times 10^2 \text{ N})$$

$$F_f = 1.09 \times 10^2 \text{ N}$$

Substituting this value into equation 1,

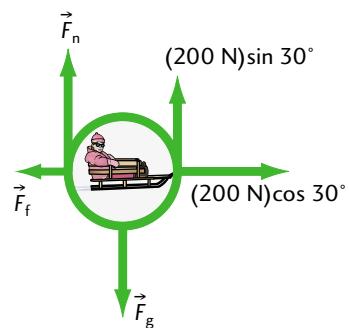
$$a = \frac{F \cos 30^\circ - F_f}{m}$$

$$a = \frac{(200 \text{ N}) \cos 30^\circ - 1.09 \times 10^2 \text{ N}}{50 \text{ kg}}$$

$$a = 1.3 \text{ m/s}^2$$

Therefore, the acceleration of the child and sled is 1.3 m/s^2 [right].

Fig.2.28c



EXAMPLE 12 Newton's laws in two dimensions (horizontal plane)

Mr. Wharf accidentally fires two rockets on his shuttlecraft at the same time. The first rocket applies a force of 1000 N [E 25° S], and the second rocket applies a force of 1200 N [N 40° W]. If the shuttlecraft has a mass of $5.0 \times 10^4 \text{ kg}$, determine the vector acceleration it will experience.

Solution and Connection to Theory

Given

$$\vec{F}_1 = 1000 \text{ N} [\text{E}25^\circ\text{S}] \quad \vec{F}_2 = 1200 \text{ N} [\text{N}40^\circ\text{W}] \quad m = 5.0 \times 10^4 \text{ kg}$$

Fig.2.29a

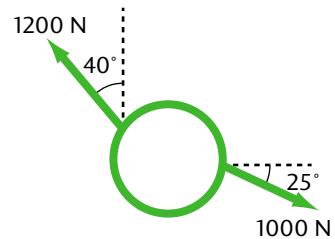


Fig.2.29b

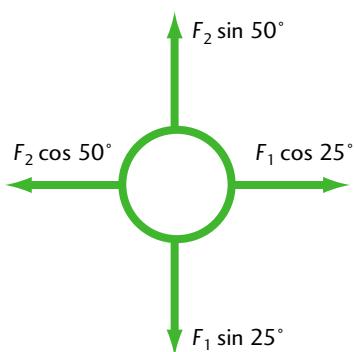
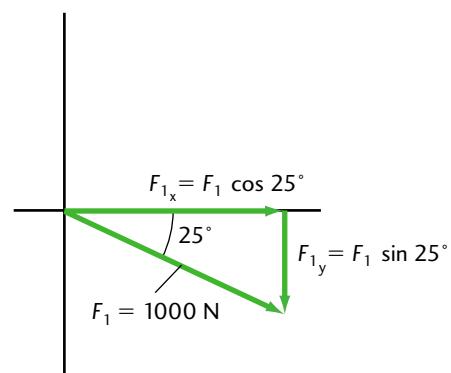
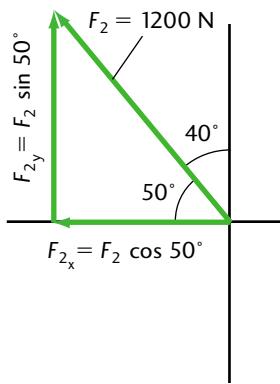


Fig.2.29c



Adding the x components,

$$\vec{F}_{\text{net}_x} = \vec{F}_{1x} + \vec{F}_{2x}$$

$$F_{\text{net}_x} = (1000 \text{ N})\cos 25^\circ - (1200 \text{ N})\cos 50^\circ$$

$$F_{\text{net}_x} = 135 \text{ N}$$

Adding the y components,

$$\vec{F}_{\text{net}_y} = \vec{F}_{1y} + \vec{F}_{2y}$$

$$F_{\text{net}_y} = (1200 \text{ N})\sin 50^\circ - (1000 \text{ N})\sin 25^\circ$$

$$F_{\text{net}_y} = 497 \text{ N}$$

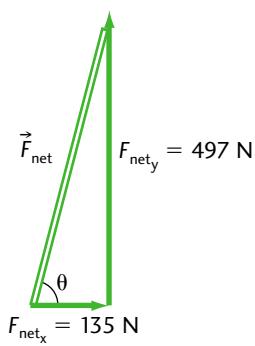
Now we add the resultants of the x and y components using Pythagoras' theorem to find the final resultant:

$$F_{\text{net}} = \sqrt{F_{\text{net}_x}^2 + F_{\text{net}_y}^2}$$

$$F_{\text{net}} = \sqrt{(135 \text{ N})^2 + (497 \text{ N})^2}$$

$$F_{\text{net}} = 515 \text{ N}$$

Fig.2.29d



To find the angle of the resultant force, we use the tangent function:

$$\tan \theta = \frac{F_{\text{net}_y}}{F_{\text{net}_x}}$$

$$\theta = \tan^{-1}\left(\frac{497 \text{ N}}{135 \text{ N}}\right)$$

$$\theta = 75^\circ$$

Therefore, the resultant force is $\vec{F}_{\text{net}} = 515 \text{ N} [\text{E}75^\circ\text{N}]$.

To find the acceleration, from $\vec{F} = m\vec{a}$,

$$\vec{a} = \frac{515 \text{ N} [\text{E}75^\circ\text{N}]}{5.0 \times 10^4 \text{ kg}}$$

$$\vec{a} = 1.0 \times 10^{-2} \text{ m/s}^2 [\text{E}75^\circ\text{N}]$$

Therefore, the acceleration of Mr. Wharf's shuttlecraft is $1.0 \times 10^{-2} \text{ m/s}^2 [\text{E}75^\circ\text{N}]$.

1. Two people are pushing horizontally on a 200-kg stove at the same time. The first person applies a force of 200 N [N] and the second person applies a force of 300 N [W]. If μ_k for the stove is 0.23, what is the resulting acceleration of the stove?
2. Three politicians are having a tug-of-war on a voter. If the force exerted by each politician on the 80-kg voter is $\vec{F}_1 = 25 \text{ N } [\text{S}16^\circ\text{E}]$, $\vec{F}_2 = 35 \text{ N } [\text{N}40^\circ\text{E}]$, and $\vec{F}_3 = 45 \text{ N } [\text{W}]$, determine
 - a) the net force acting on the voter.
 - b) the acceleration of the voter.
3. Two players kick a soccer ball at the same time. If one player applies a force of 100 N [N25°W] and the 250-g ball experiences an acceleration of 200 m/s^2 [W15°S], determine the magnitude and direction of the force applied by the second player.
4. A gardener pushes down on the handle of a lawnmower, applying a force of 250 N. The handle is inclined at an angle of 45° to the horizontal. If the coefficient of kinetic friction between the wheels of the lawnmower and the ground is 0.40, what is the acceleration of the 20-kg lawnmower?

Fig. 2.30



2.5 The Inclined Plane

Since many real-life situations occur either on a ramp, a hill, or some other form of incline, inclined-plane problems are very common. Inclined-plane problems can be solved using coordinate rotation; that is, using axes that are parallel and perpendicular to the *incline itself*.

EXAMPLE 13 An inclined-plane problem

A girl sits at the top of a frictionless snow-covered hill on her inner tube. If the hill is inclined at an angle of 25° to the horizontal, what will be the girl's acceleration due to gravity?

Solution and Connection to Theory

Given

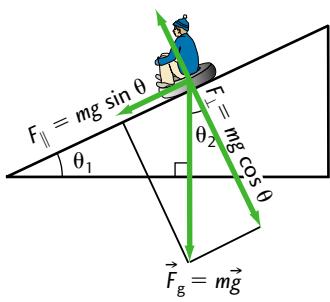
$$\theta = 25^\circ$$

Because the girl is on an inclined plane, the value of her acceleration will be less than 9.8 m/s^2 .

Fig. 2.31



Fig.2.32a



The right side of θ_1 is perpendicular to the left side of θ_2 . The left side of θ_1 is perpendicular to the right side of θ_2 . Therefore, $\theta_1 = \theta_2 = \theta$.

Fig.2.32b

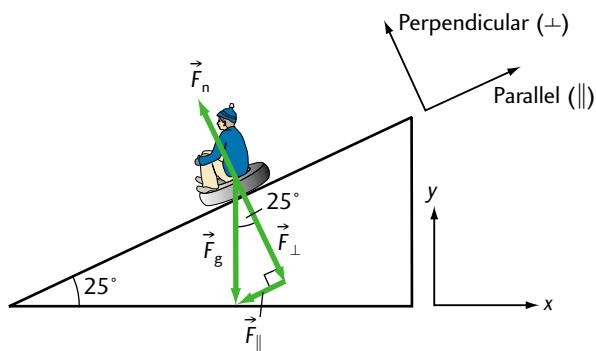


Figure 2.32b shows the force due to gravity acting on the girl, broken down into components. The first component is perpendicular to the incline and is labeled \vec{F}_\perp . The other force that is perpendicular to the incline is the normal force, \vec{F}_n . It is the force of the incline on the girl. \vec{F}_\perp and \vec{F}_n are equal and opposite (balanced), so they don't affect the girl's motion if there is no friction. The only unbalanced force is the component of the force of gravity that is parallel to the incline, \vec{F}_{\parallel} . This force is the net force acting on the girl. It can be described trigonometrically in terms of the gravitational force, \vec{F}_g ; that is, we can write \vec{F}_{\parallel} as a component of \vec{F}_g . Now we can use Newton's second law to determine the acceleration of the girl:

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$F_{\parallel} = ma$$

Substituting $F_g \sin 25^\circ$ for F_{\parallel} , we get

$$F_g \sin 25^\circ = ma$$

Since $F_g = mg$, then

$$mg \sin 25^\circ = ma$$

$$a = g \sin 25^\circ$$

$$a = (9.8 \text{ m/s}^2) \sin 25^\circ$$

$$a = 4.1 \text{ m/s}^2$$

Therefore, the girl's acceleration is 4.1 m/s^2 down the incline.

EXAMPLE 14 An inclined-plane problem with friction

Evil Kinevil is driving his motorcycle up a ramp inclined at 30° to the horizontal before jumping over a row of cars. If there's a constant frictional force of 1000 N on the ramp, determine the force that the motorcycle engine must apply to accelerate the motorcycle up the ramp at $\frac{1}{3}g$. (Assume that Evil and the motorcycle have a combined mass of 250.0 kg.)

Solution and Connection to Theory

Given

$$F_f = 1000 \text{ N} \quad a = \frac{1}{3}g = \frac{9.8 \text{ m/s}^2}{3} = 3.27 \text{ m/s}^2 \quad m = 250.0 \text{ kg}$$

Assuming up the ramp is positive,

$$\vec{F}_{\text{net}} = \vec{F}_{\text{engine}} + \vec{F}_{\parallel} + \vec{F}_f = m\vec{a}$$

$$F_{\text{engine}} - F_{\parallel} - F_f = ma$$

Isolating F_{engine} and substituting $mg \sin 30^\circ$ for F_{\parallel} , the equation becomes

$$F_{\text{engine}} = ma + mg \sin 30^\circ + F_f$$

$$F_{\text{engine}} = (250 \text{ kg})(3.27 \text{ m/s}^2) + (250 \text{ kg})(9.8 \text{ m/s}^2)\sin 30^\circ + 1000 \text{ N}$$

$$F_{\text{engine}} = 3.04 \times 10^3 \text{ N}$$

Therefore, the engine must apply a force of $3.04 \times 10^3 \text{ N}$.

Fig.2.33a

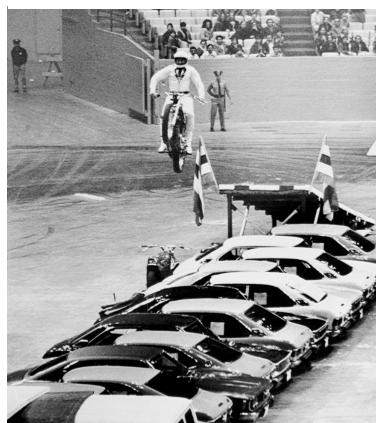
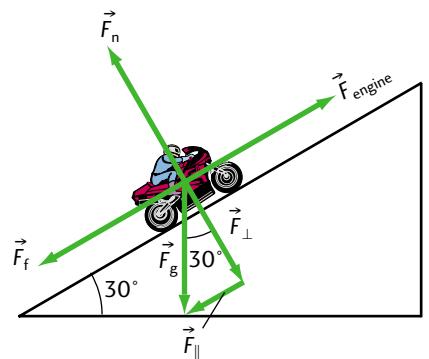


Fig.2.33b



EXAMPLE 15 Determining the coefficient of static friction

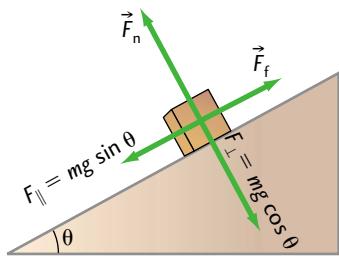
We can determine the coefficient of static friction experimentally by placing a small block of wood on the surface of a piece of plywood and slowly increasing the angle of inclination of the plywood until the block of wood just begins to move. At this instant, all forces acting on the block are balanced; therefore, the block will move at a constant speed. We can then determine the coefficient of static friction for a block sliding down an inclined plane at a constant speed.

Solution and Connection to Theory

Given

Since the block is moving at a constant speed, all forces acting on it are balanced.

Fig. 2.33c



For the components perpendicular to the incline,

$$F_{\text{net}} = 0$$

$$F_n - mg \cos \theta = 0$$

$$F_n = mg \cos \theta$$

Since $F_f = \mu_s F_n$,

$$F_f = \mu_s mg \cos \theta$$

For the components parallel to the incline,

$$F_{\text{net}} = 0$$

$$F_f - mg \sin \theta = 0$$

$$F_f = mg \sin \theta$$

Combining the two equations,

$$\mu_s mg \cos \theta = mg \sin \theta$$

$$\mu_s = \frac{\sin \theta}{\cos \theta}$$

$$\mu_s = \tan \theta$$

We can determine the coefficient of static friction for an inclined plane by simply measuring the angle of inclination, θ .



1. A copy of *Physics: Concepts and Connections* leaves the printing press and slides down a 4.0-m-long ramp into the arms of an eager physics student. If the ramp is inclined at an angle of 25° to the horizontal and has a coefficient of kinetic friction of 0.10, how long will it take the 2.0-kg textbook to reach the student?
2. The plastic case from your least-favourite CD recording is flung up a frictionless ramp, inclined at an angle of 20° to the horizontal. If the case leaves your hand at a speed of 4.0 m/s, how long will it take before the case comes to rest?
3. A skateboarder slides down a frictionless ramp inclined at an angle of 30° to the horizontal. He then slides across a frictionless horizontal floor and begins to slide up a second incline at an angle of 25° to the horizontal. The skateboarder starts at a distance of 10 m from the bottom of the first incline. How far up the second incline will he go if the coefficient of kinetic friction on the second incline is 0.10?
4. Batman is driving the Batmobile down a hill coming from the Bat Cave. The hill is inclined at an angle of 30° to the horizontal and has a coefficient of kinetic friction of 0.28. What force must the Batmobile's engine apply to cause the Batmobile to accelerate at $0.60g$?

2.6 String-and-pulley Problems

To solve a string-and-pulley problem, we need to draw a free-body diagram for each object being considered, determine the direction of motion of each object, then write an algebraic description of Newton's second law for each object. We then solve the equations for the unknown variables. In this section, we will assume throughout that the pulleys are massless and frictionless, and that the strings are infinitely strong, massless, and never stretch.

EXAMPLE 16

Strings and pulleys for a frictionless horizontal surface

Two 5.0-kg masses are connected as shown in Figure 2.34a. Determine the acceleration of the system and the tension in the rope if the tabletop is frictionless.

Solution and Connection to Theory

Given

$$m_1 = m_2 = 5.0 \text{ kg}$$

The first step is to draw a free-body diagram for each of the two masses (Figure 2.34b).

Fig.2.34b

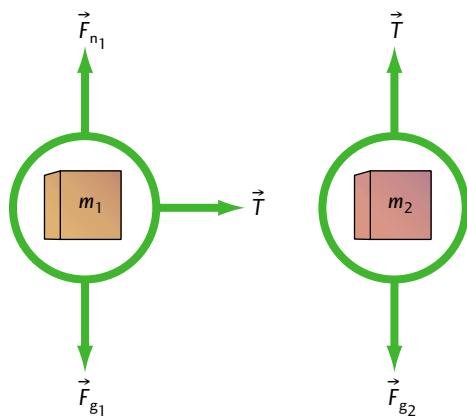
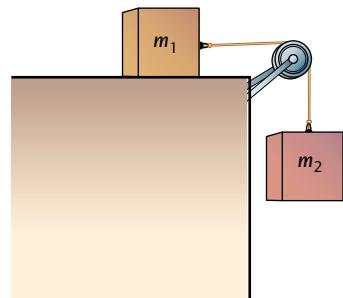


Fig.2.34a



Notice that the free-body diagram for m_1 has two vertical forces. The normal force upward is balanced by the gravitational force downward. As a result, these two forces will not affect the acceleration of m_1 . The only force causing m_1 to accelerate is the tension, \vec{T} . Mass m_2 , on the other hand, has two forces that could cause it to accelerate. We expect m_2 to move downward. Therefore, the gravitational force must be greater than the tension in the rope.

The second step is to apply Newton's second law to each mass. We will then have two equations that we can solve for acceleration and tension.

Alternative Solution

If we assume that all the internal forces balance, then the only force causing the system to accelerate is the weight of m_2 ; that is, F_{g2} , which accelerates m_1 and m_2 . Therefore,

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$F_{g2} = (m_1 + m_2)a$$

$$a = \frac{m_2 g}{m_1 + m_2}$$

$$a = 4.9 \text{ m/s}^2$$

For m_1 ,

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$T = m_1 a \quad (\text{eq. 1})$$

For m_2 ,

$$F_{g2} - T = m_2 a$$

$$m_2 g - T = m_2 a \quad (\text{eq. 2})$$

Adding equation 1 and equation 2,

$$m_2 g = (m_1 + m_2) a$$

$$a = \frac{m_2 g}{m_1 + m_2}$$

Since $m_1 = m_2$,

$$a = \frac{m_2 g}{2m_2}$$

$$a = \frac{g}{2}$$

$$a = 4.9 \text{ m/s}^2$$

The acceleration of the system is 4.9 m/s^2 .

To determine the tension, we can substitute our acceleration value into equation 1 or equation 2. Substituting into equation 1,

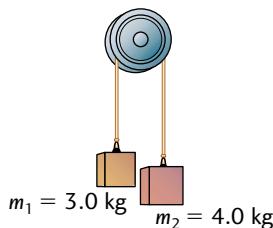
$$T = (5.0 \text{ kg})(4.9 \text{ m/s}^2)$$

$$T = 24 \text{ N}$$

The tension in the rope is 24 N.

EXAMPLE 17 A vertical string-and-pulley problem

Fig. 2.35a



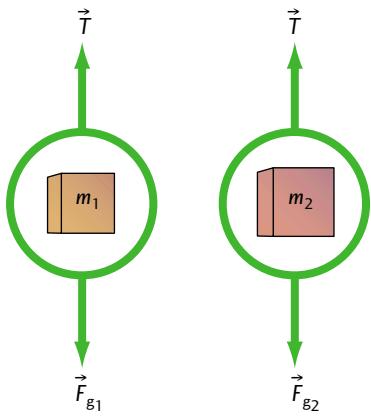
A 3.0-kg mass and a 4.0-kg mass are suspended from a frictionless pulley, as shown in Figure 2.35a. Determine the system's acceleration and the tension in the rope.

Solution and Connection to Theory

Given

$$m_1 = 3.0 \text{ kg} \quad m_2 = 4.0 \text{ kg}$$

Fig. 2.35b



From Figure 2.35b, since m_2 is more massive than m_1 , m_2 will go downward. Therefore, the gravitational force acting on m_2 must be greater than the tension of the rope pulling it upward. Using Newton's second law, for m_1 , $\vec{T} + \vec{F}_{g1} = \vec{F}_{\text{net}}$

$$T - m_1 g = m_1 a \quad (\text{eq. 1})$$

$$\text{For } m_2, \vec{F}_{g2} + \vec{T} = \vec{F}_{\text{net}}$$

$$m_2 g - T = m_2 a \quad (\text{eq. 2})$$

Adding equation 1 and equation 2,

$$m_2 g - m_1 g = m_1 a + m_2 a$$

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$$a = \frac{(4.0 \text{ kg} - 3.0 \text{ kg})(9.8 \text{ m/s}^2)}{3.0 \text{ kg} + 4.0 \text{ kg}}$$

$$a = 1.4 \text{ m/s}^2$$

The acceleration of the system is 1.4 m/s^2 .

Substituting into equation 1 to solve for T ,

$$T = m_1 a + m_1 g$$

$$T = m_1(a + g)$$

$$T = (3.0 \text{ kg})(1.4 \text{ m/s}^2 + 9.8 \text{ m/s}^2)$$

$$T = 34 \text{ N}$$

The tension in the rope is 34 N.

Alternative Solution

The total mass of the system is

$$m_T = m_1 + m_2$$

$$\vec{F}_{\text{net}} = \vec{F}_{g2} + \vec{F}_{g1}$$

$$F_{\text{net}} = F_{g2} - F_{g1} \text{ (right)}$$

$$a = \frac{F_{\text{net}}}{m_T} = \frac{F_{g2} - F_{g1}}{m_1 + m_2}$$

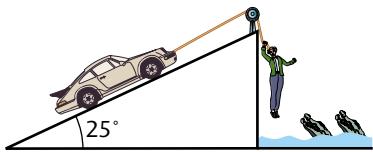
$$a = \frac{(4.0 \text{ kg})(9.8 \text{ m/s}^2) - (3.0 \text{ kg})(9.8 \text{ m/s}^2)}{3.0 \text{ kg} + 4.0 \text{ kg}}$$

$$a = 1.4 \text{ m/s}^2$$

EXAMPLE 18

Strings, pulleys, and an inclined plane

Fig.2.36



Jane Bond is suspended in the unfortunate frictionless situation shown in Figure 2.36. If Jane has a mass of 75 kg and her car has a mass of 1500 kg, determine

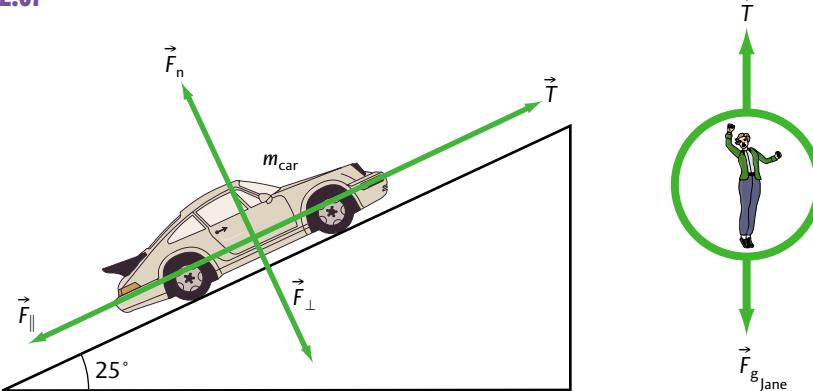
- the direction of Jane's motion.
- Jane's acceleration.
- the tension in the rope.

Solution and Connection to Theory**Given**

$$m_{\text{Jane}} = 75 \text{ kg} \quad m_{\text{car}} = 1500 \text{ kg}$$

- a) There is a tug-of-war going on between Jane and her car. Jane's gravitational force is pulling her down. The parallel component of the car's gravitational force, \vec{F}_{\parallel} , is trying to pull the car down. The stronger force will cause both bodies to accelerate in its direction.

Fig.2.37



To calculate which force is stronger, we use Newton's second law:

$$F_{\parallel} = F_{g,\text{car}} \sin 25^{\circ}$$

$$F_{\parallel} = m_{\text{car}} g \sin 25^{\circ}$$

$$F_{\parallel} = (1500 \text{ kg})(9.8 \text{ m/s}^2) \sin 25^{\circ}$$

$$F_{\parallel} = 6212 \text{ N}$$

$$F_{g,\text{Jane}} = m_{\text{Jane}} g$$

$$F_{g,\text{Jane}} = (75 \text{ kg})(9.8 \text{ m/s}^2)$$

$$F_{g,\text{Jane}} = 735 \text{ N}$$

F_{\parallel} is greater than Jane's gravitational force; therefore, the car goes down the ramp and Jane goes up.

b) To calculate the acceleration,

for m_{car} ,

$$F_x - T = m_{\text{car}}a \quad (\text{eq. 1})$$

for m_{Jane} ,

$$T - m_{\text{Jane}}g = m_{\text{Jane}}a \quad (\text{eq. 2})$$

Adding equation 1 and equation 2,

$$F_x - m_{\text{Jane}}g = (m_{\text{car}} + m_{\text{Jane}})a, \text{ where } F_x = m_{\text{car}}g \sin 25^\circ$$

Isolating a and substituting, we obtain

$$a = \frac{m_{\text{car}}g \sin 25^\circ - m_{\text{Jane}}g}{m_{\text{car}} + m_{\text{Jane}}}$$

$$a = \frac{g(m_{\text{car}} \sin 25^\circ - m_{\text{Jane}})}{m_{\text{car}} + m_{\text{Jane}}}$$

$$a = \frac{(9.8 \text{ m/s}^2)[(1500 \text{ kg}) \sin 25^\circ - 75 \text{ kg}]}{1500 \text{ kg} + 75 \text{ kg}}$$

$$a = 3.48 \text{ m/s}^2$$

Jane's acceleration is 3.48 m/s^2 [up].

c) To calculate the tension in the rope, we substitute the acceleration from part b) into equation 2:

$$T = m_{\text{Jane}}(a + g)$$

$$T = (75 \text{ kg})(3.48 \text{ m/s}^2 + 9.8 \text{ m/s}^2)$$

$$T = 996 \text{ N}$$

Therefore, the tension in the rope is $1.0 \times 10^3 \text{ N}$.

Alternative Solution

$$(m_1 + m_2)a = m_{\text{car}}g \sin \theta - m_{\text{Jane}}g$$

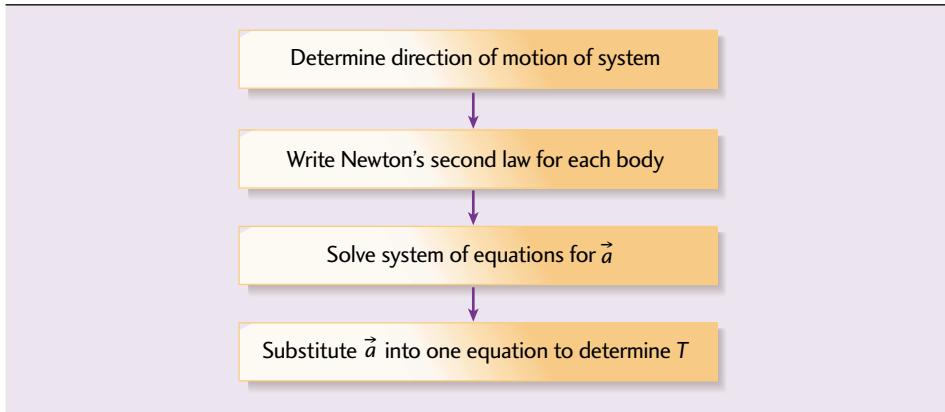
$$a = \frac{m_{\text{car}}g \sin \theta - m_{\text{Jane}}g}{m_1 + m_2}$$

$$a = \frac{(1500 \text{ kg})(9.8 \text{ m/s}^2) \sin 25^\circ - (75 \text{ kg})(9.8 \text{ m/s}^2)}{1500 \text{ kg} + 75 \text{ kg}}$$

$$a = 3.48 \text{ m/s}^2$$

Figure 2.38 summarizes the method of solving string-and-pulley problems.

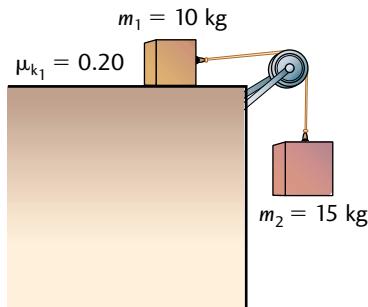
Fig.2.38 Solving String-and-pulley Problems



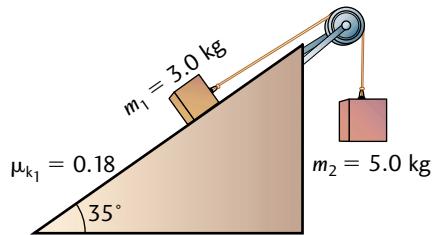


1. For each of the following systems, determine the acceleration and the tension in each rope.

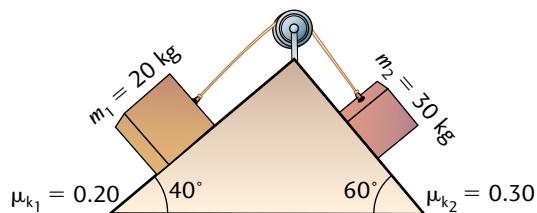
a) Fig.2.39



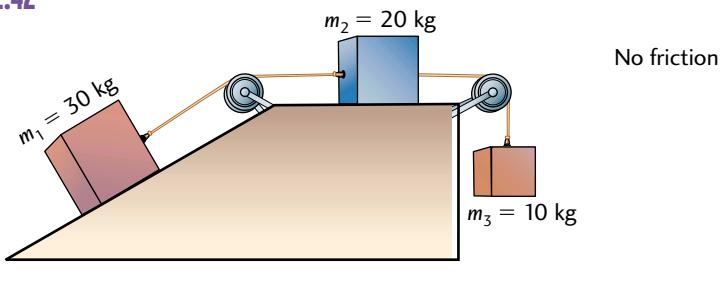
b) Fig.2.40



c) Fig.2.41



d) Fig.2.42



2.7 Uniform Circular Motion

A special kind of two-dimensional problem involves objects undergoing uniform circular motion. **Uniform circular motion** is motion in a circle at a constant speed.

EXAMPLE 19**An analogue stopwatch — calculating average acceleration**

A track coach starts an analogue stopwatch and allows it to run for one minute. If the sweep second hand on the stopwatch is 2.0 cm in length, determine

- the speed of the sweep second hand.
- the velocity of the sweep second hand at the 15-s point.
- the velocity of the sweep second hand at the 30-s point.
- the average acceleration of the sweep second hand between the 15-s and 30-s points.

Solution and Connection to Theory**Given**

$$r_{\text{stopwatch}} = 2.0 \times 10^{-2} \text{ m} \quad t = 15 \text{ s}$$

- a)** To determine the speed of the sweep second hand, we can use the defining equation for speed,

$$v = \frac{d}{t}$$

which can be modified to

$v = \frac{C}{T}$, where C is the circumference of the stopwatch, in metres, and T is the period, in seconds. $C = 2\pi r$; therefore,

$$v = \frac{2\pi r}{T}$$

$$v = \frac{2\pi(2.0 \times 10^{-2} \text{ m})}{60 \text{ s}}$$

$$v = 2.1 \times 10^{-3} \text{ m/s}$$

- b)** Now that we know the speed of the sweep second hand, we only need to determine the direction for velocity. At $t = 15 \text{ s}$, the second hand is moving down. Therefore, the velocity of the sweep second hand at $t = 15 \text{ s}$ is $\vec{v}_{15} = 2.1 \times 10^{-3} \text{ m/s}$ [D].
- c)** Similarly, at $t = 30 \text{ s}$, the sweep second hand is moving left. Therefore, the velocity is $\vec{v}_{30} = 2.1 \times 10^{-3} \text{ m/s}$ [L].
- d)** To determine the average vector acceleration, we can use our defining equation for acceleration,

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_{30} - \vec{v}_{15}}{\Delta t}$$

Recall that subtracting a vector is the same as adding its opposite!

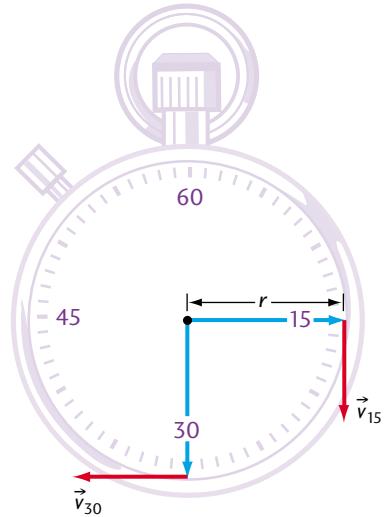
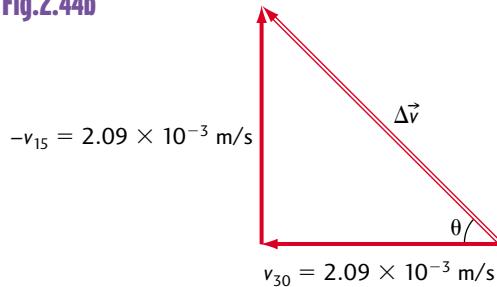
Fig.2.43**Fig.2.44a**

Fig. 2.44b



In Figure 2.44b,

$$\Delta\vec{v} = \vec{v}_{30} + (-\vec{v}_{15})$$

$$\Delta v = \sqrt{(2.09 \times 10^{-3} \text{ m/s})^2 + (2.09 \times 10^{-3} \text{ m/s})^2}$$

$$\Delta v = 2.96 \times 10^{-3} \text{ m/s}$$

$$\tan \theta = 1.0$$

$$\theta = 45^\circ$$

$$\Delta\vec{v} = 2.96 \times 10^{-3} \text{ m/s [L45}^\circ \text{ U]}$$

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}_{\text{avg}} = \frac{2.96 \times 10^{-3} \text{ m/s [L45}^\circ \text{ U]}}{15 \text{ s}}$$

$$\vec{a}_{\text{avg}} = 2.0 \times 10^{-4} \text{ m/s [L45}^\circ \text{ U]}$$

Therefore, the average acceleration of the sweep second hand between the 15-s and 30-s points is $2.0 \times 10^{-4} \text{ m/s}^2$ [L45° U].

Alternative diagram
(tail-to-tail method)

Fig. 2.44c

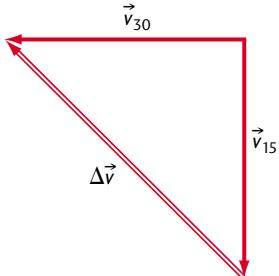
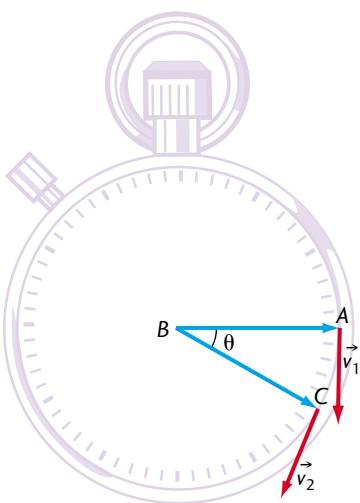


Fig. 2.45a



From this example, we can note two things. First, remember that acceleration is a vector. The magnitude of the velocity (i.e., the speed) need not change in order for acceleration to occur; it is sufficient that there is a change in direction, as in this case. As a result, we have acceleration.

Second, what is the direction of the average acceleration? In Example 19, the direction is [L45° U]. This value is the average acceleration over the 15-s time interval. Therefore, it is also the direction of the instantaneous acceleration at the midpoint of the arc between $t = 15 \text{ s}$ and $t = 30 \text{ s}$. More precisely, it is the direction of the instantaneous acceleration at $t = 22.5 \text{ s}$. At this point, the instantaneous acceleration is directed toward the centre of the circle; that is, the acceleration is centre-seeking or **centripetal**.

Whenever an object is undergoing uniform circular motion, it undergoes centripetal acceleration (i.e., directed toward the centre of the circle).

The *magnitude* of centripetal acceleration is constant, as in Example 19, but its *direction* changes every instant so that it is always directed toward the centre of the circle. Our task now is to derive an algebraic equation for the magnitude of centripetal acceleration.

Figure 2.45a shows a stopwatch with a sweep second hand and two velocity vectors, \vec{v}_1 and \vec{v}_2 , at times t_1 and t_2 , respectively. As the sweep second hand rotates from point A to point C in Figure 2.45a, it sweeps out an angle, θ . Figure 2.45b shows both vectors, \vec{v}_1 and \vec{v}_2 , drawn from the same point. The angle γ represents the change in the velocity vector's direction as the second hand's velocity changes from point A to point C. Since the radii BA and BC are both perpendicular to the tangential velocities \vec{v}_1 and \vec{v}_2 ,

$$\gamma + \phi = 90^\circ \quad \text{and} \quad \phi + \theta = 90^\circ$$

Therefore, angles γ and θ must be equal.

Fig.2.45c

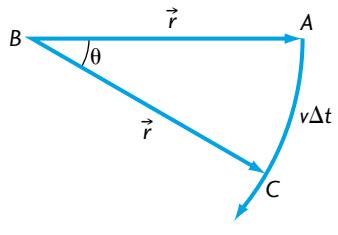
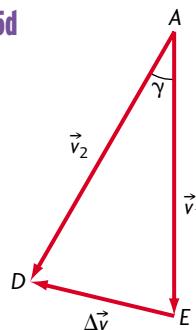


Fig.2.45d



As the second hand sweeps through a change in time Δt , it sweeps through an arc of length $v\Delta t$ (Figure 2.45c). As Δt approaches zero, the angle θ also approaches zero, and the arc swept out by the second hand gets smaller and smaller and eventually approaches a straight line. The triangle DAE, shown in Figure 2.45d, is an isosceles triangle. The vector $\Delta\vec{v}$ is its base. Because angles γ and θ are equal, triangles CBA and DAE are similar triangles. Since $v_1 = v_2 = v$,

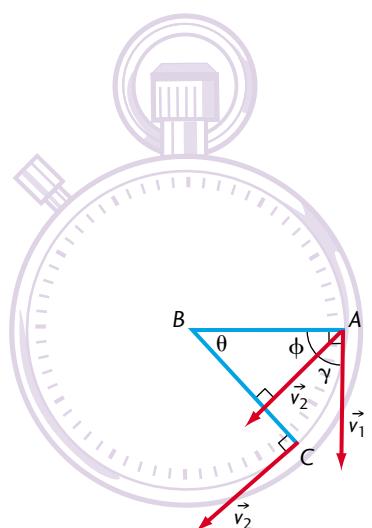
$$\frac{\Delta v}{v} = \frac{v\Delta t}{r} \quad \text{or} \quad \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

But $a = \frac{\Delta v}{\Delta t}$ is the equation for acceleration. Therefore, the magnitude of centripetal acceleration, a_c , is

$$a_c = \frac{v^2}{r} \quad (\text{eq. 1})$$

For objects undergoing uniform circular cyclic motion, we can also find the centripetal acceleration in terms of frequency and period.

Fig.2.45b



In our sweep-second-hand example, we can describe the speed of the second hand as

$$v = \frac{d}{t}$$

Since the distance travelled equals the circumference of the circle, $C = 2\pi r$, and the time taken is equal to the period of rotation, T , we can state that

$$v = \frac{2\pi r}{T} \quad (\text{eq. 2})$$

Substituting equation 2 into equation 1, we obtain

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$a_c = \frac{4\pi^2 r^2}{r T^2}$$

$$a_c = \frac{4\pi^2 r}{T^2} \quad (\text{eq. 3})$$

Also, since $T = \frac{1}{f}$ (where f represents the frequency of rotation),

$$a_c = 4\pi^2 r f^2 \quad (\text{eq. 4})$$



1. A racecar enters a circular curve of radius 30 m at a constant speed of 25 m/s. Determine the car's centripetal acceleration.
2. A bicycle wheel of radius 1.3 m undergoes 25 rotations in 60 s. Determine the centripetal acceleration of a point on the wheel.
3. A carnival ride rotates children on swings about a vertical axis (Figure 2.46). Describe the effect on the centripetal acceleration of a child as
 - a) the speed is doubled.
 - b) the radius is doubled.
 - c) the radius is halved.
4. The Moon orbits Earth with a period of approximately 27.3 days. If the distance from Earth to the Moon is approximately 3.8×10^5 km,
 - a) what is the magnitude of the Moon's centripetal acceleration?
 - b) in which direction is the Moon's centripetal acceleration?
 - c) what causes this centripetal acceleration?
5. The outer edge of a 120-mm-diameter CD-ROM experiences an acceleration of 1.6 m/s^2 . What is the speed of the CD-ROM?
6. Space stations can produce "artificial gravity" by rotating. A space station is built in the shape of a bicycle wheel of diameter 500 m. How many times each day should the space station rotate for an astronaut to experience an acceleration equal to the acceleration due to gravity on Earth?

Fig. 2.46



2.8 Centripetal Force



Fig.2.47

An object undergoing uniform circular motion experiences centripetal acceleration. From Newton's second law, the centripetal acceleration must be caused by an unbalanced force. Whenever an object travels in a circle at a constant speed, it must have a force acting on it that is perpendicular to its velocity. The **centripetal force** is the net force; that is, the vector sum of all forces acting on the object. If the net force becomes zero, inertia will cause the object to move off at a constant speed in a straight line.

Since

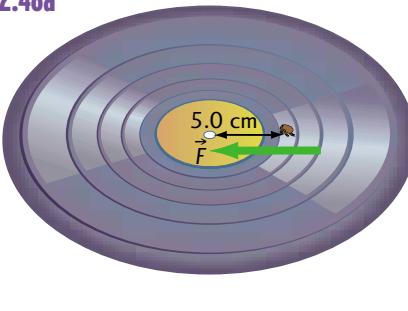
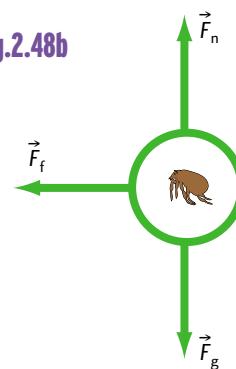
$$\vec{F}_c = \vec{F}_{\text{net}} = m\vec{a}_c$$

we can derive three forms of \vec{F}_{net} for circular motion by substituting the three derived equations for centripetal acceleration (magnitude only):

$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{m4\pi^2 r}{T^2}$$

$$F_c = m4\pi^2 rf^2$$

EXAMPLE 20**Centripetal force in the horizontal plane****Fig.2.48a****Fig.2.48b**

A 0.20-g flea sits at a distance of 5.0 cm from the centre of a rotating LP record. If the record rotates at 77 rpm, what centripetal force must be provided by friction to cause the flea to maintain its uniform circular motion?

Solution and Connection to Theory**Given**

$$m = 0.20 \times 10^{-3} \text{ kg} \quad r = 5.0 \times 10^{-2} \text{ m} \quad f = 77 \text{ rpm} = 1.28 \text{ Hz}$$

$$\vec{F}_{\text{net}} = \vec{F}_f$$

$$F_c = F_f$$

$$F_c = m4\pi^2rf^2$$

$$F_c = (0.20 \times 10^{-3} \text{ kg})4\pi^2(5.0 \times 10^{-2} \text{ m})(1.28 \text{ Hz})^2$$

$$F_c = 6.5 \times 10^{-4} \text{ N}$$

Therefore, friction must create a centripetal force of 6.5×10^{-4} N.

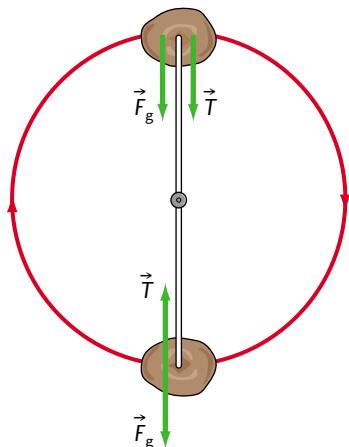
EXAMPLE 21**Centripetal force in the vertical plane**

A 25-g chestnut with a hole drilled through its centre is hanging from a long massless shoelace. A child spins the chestnut in a vertical circle at a speed of 4.0 m/s. If the shoelace is 0.80 m long, determine the tension in the shoelace at the top and bottom of the circle.

Solution and Connection to Theory**Given**

$$m = 0.25 \text{ kg} \quad v = 4.0 \text{ m/s} \quad r = 0.80 \text{ m}$$

Recall that the centripetal force is the vector sum of all forces acting on an object undergoing circular motion. This force can be expressed algebraically as

Fig.2.49a

$$\vec{F}_{\text{net}} = \vec{T} + \vec{F}_g$$

$$\vec{F}_c = \vec{T} + \vec{F}_g$$

$$\vec{T} = \vec{F}_c - \vec{F}_g$$

where \vec{T} is the tension in the shoelace, \vec{F}_c is the centripetal force, and \vec{F}_g is the gravitational force on the chestnut.

At the top of the circle:

According to our standard coordinate system, both \vec{F}_c and \vec{F}_g are downward forces, as indicated by their negative signs below. Expanding the third equation, we obtain

$$T_{\text{top}} = \frac{-mv^2}{r} - (-mg)$$

$$T_{\text{top}} = \frac{-mv^2}{r} + mg$$

$$T_{\text{top}} = \frac{-(0.25 \text{ kg})(4.0 \text{ m/s})^2}{0.80 \text{ m}} + (0.25 \text{ kg})(9.8 \text{ m/s}^2)$$

$$T_{\text{top}} = -2.6 \text{ N}$$

The tension at the top of the circle is -2.6 N or 2.6 N [down].

At the bottom of the circle:

Again using our standard coordinate system, the gravitational force will have a negative sign because it is directed downward, and the centripetal force will have a positive value because it's applied upward toward the centre of the circle:

$$\vec{F}_c = \vec{T} + \vec{F}_g \longrightarrow F_c = T - F_g$$

$$T_{\text{bottom}} = \frac{mv^2}{r} + mg$$

$$T_{\text{bottom}} = \frac{(0.25 \text{ kg})(4.0 \text{ m/s})^2}{0.80 \text{ m}} + (0.25 \text{ kg})(9.8 \text{ m/s}^2)$$

$$T_{\text{bottom}} = 7.4 \text{ N} \text{ [up]}$$

The tension at the bottom of the circle is 7.4 N [up].

There are two things of note in this solution. First, note the directions of the tensions at the top and bottom of the circle. At the top of the circle, the tension is downward, toward the centre of the circle. At the bottom of the circle, the tension is upward, also toward the centre of the circle. In each case, the chestnut is being pulled toward the centre.

Fig.2.49b

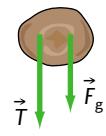


Fig.2.49c



At the top, the tension force can become zero because gravity provides the necessary force to turn the object. In this case, the speed at which the object turns is the minimum speed required to keep the object moving in a circle.

Second, the magnitudes of the tensions at the top and bottom of the circle are not the same. At the top of the circle, gravity applies a downward force toward the centre of the circle, providing part of the centripetal force. As a result, the rope can apply a smaller force than would otherwise be needed to keep the chestnut moving in a circle. At the bottom, on the other hand, the shoelace not only applies a force upward to provide the centripetal force, it must also apply an upward force to balance gravity. So, the tension in the shoelace is greater at the bottom of the circle than at the top.

Centripetal Force and Banked Curves

When a car travels along a curve, the centripetal force is usually provided by the frictional force between the car's tires and the road's surface. To reduce the reliance on friction, we can incline, or bank, the curve relative to the horizontal. This method is used in car races on circular or oval tracks and on highway on- and off-ramps. For a given banked curve, there is one speed at which the centripetal force is provided strictly by a component of the normal force. At this speed, the object doesn't require a frictional force to undergo uniform circular motion.

EXAMPLE 22 Banked curves

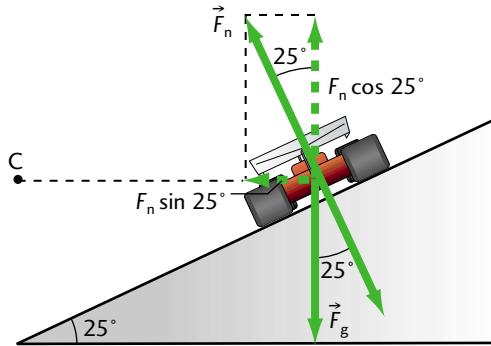
A racecar travels along a banked curve at a speed of 120 km/h. It doesn't depend on the force of friction to keep it on the track. If the turn is banked at an angle of 25° to the horizontal, what is its radius of rotation?

Solution and Connection to Theory

Given

$$v = 120 \text{ km/h} = 33.3 \text{ m/s} \quad \theta = 25^\circ$$

Fig.2.50



In Figure 2.50, the only two forces acting on the car as it travels along the curve are the gravitational force and the normal force. The normal force has two components: the vertical component is balanced by gravity, whereas the horizontal component is unbalanced. This component is the centripetal force because it acts toward the centre of motion, labeled C.

For the vertical forces,

$$\begin{aligned} F_{\text{net}_y} &= 0 \\ F_{n_y} &= F_g \\ F_{n_y} &= mg \\ F_n \cos 25^\circ &= mg \quad (\text{eq. 1}) \end{aligned}$$

For the horizontal forces,

$$\begin{aligned} F_{n_x} &= F_c \\ F_n \sin 25^\circ &= \frac{mv^2}{r} \quad (\text{eq. 2}) \end{aligned}$$

Dividing equation 2 by equation 1,

$$\frac{F_n \sin 25^\circ}{F_n \cos 25^\circ} = \frac{\frac{mv^2}{r}}{mg}$$

$$\tan 25^\circ = \frac{v^2}{rg}$$

$$r = \frac{v^2}{g \tan 25^\circ}$$

$$r = \frac{(33.3 \text{ m/s})^2}{(9.8 \text{ m/s}^2) \tan 25^\circ}$$

$$r = 2.4 \times 10^2 \text{ m}$$

Therefore, the radius of rotation is $2.4 \times 10^2 \text{ m}$.

Centrifugation

In laboratories, it is often necessary to separate one material from another. In many cases, if left to stand for long periods of time, substances such as sand and rocks will settle to the bottom of a test tube due to the force of gravity. This effect is called **sedimentation**. In some cases, when the particles of a given substance are of small mass, it may take too long to wait for substances to separate by sedimentation. In such cases, a centrifuge is often used. A **centrifuge** is a device that separates substances suspended in a liquid by spinning a sample of liquid very quickly around

Fig.2.51a



Fig.2.51b



an axle. The test tubes are placed symmetrically about a vertical axle. They are usually mounted in a cradle that allows their bottom ends to pivot outward. As the vertical axle starts to rotate at low speed, the test tubes are positioned vertically. As the speed increases, they progressively lift higher and higher until they approach the horizontal. Any small denser particles found in the liquid travel in a straight line inside the test tube, obeying Newton's first law. The liquid in the test tube applies a centripetal force on these particles to keep them moving in a circle. Eventually, as the speed increases, the liquid is unable to apply a great enough force to maintain the particles' circular motion, and the tiny particles will continue to move in a straight line until they reach the bottom of the test tube. The test tube itself then provides the centripetal force that keeps the particles moving in a circle. After running the centrifuge at high speed for a period of time, the particles become clumped together at the bottom of the test tube.

Modern ultracentrifuges must be very precisely balanced. In some cases, they can run at frequencies as high as 60 000 rpm. An improperly balanced centrifuge can have catastrophic results.

Satellites in Orbit

The Moon has been orbiting Earth for millions of years (Figure 2.52a). Human-made Earth satellites, however, have only been around since 1958. Canada is one of the world leaders in satellite technology. In November 2000, Canada launched the *Anik F1* satellite (Figure 2.52b). Upon launch, it was the most advanced telecommunications satellite in the world.

Anik F1 was launched from French Guiana on an *Ariane* rocket. It is currently in **geosynchronous Earth orbit (GEO)**. GEO is an orbit approximately 19 400 nautical miles (35 900 km) above Earth's surface at the equator, in which a payload completes one Earth orbit in a 24-hour period, holding a fixed position relative to Earth.

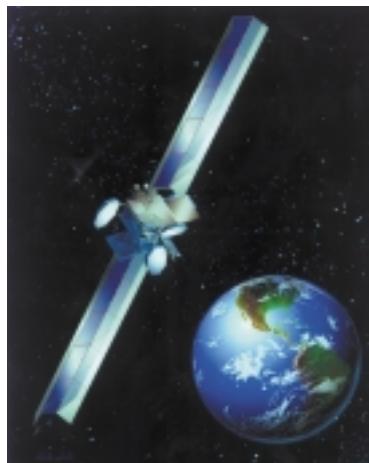
Geosynchronous orbit is also known as geostationary orbit.

Placing a satellite in geosynchronous Earth orbit requires a sufficient force, with sufficient speed, to transport the satellite to Earth orbit. If its speed is too fast, the satellite will miss Earth orbit and, according to Newton's first law, travel at a constant speed in a straight line until acted upon by an unbalanced force. If its speed is too slow, the satellite will crash to Earth due to Earth's gravitational force.

Fig. 2.52a The Moon has been orbiting Earth for millions of years



Fig. 2.52b The *Anik F1* satellite



EXAMPLE 23 Geosynchronous Earth orbit

The *Anik F1* satellite has a mass of 3021 kg. How high above the equator must the satellite be in order to maintain geosynchronous Earth orbit? Earth's period is 23 hours, 56 minutes, and 4 seconds.

Solution and Connection to Theory

Given

$$m_s = 3021 \text{ kg} \quad m_E = 5.98 \times 10^{24} \text{ kg} \quad r_E = 6.38 \times 10^6 \text{ m}$$

$$T = 23 \text{ h} \left(\frac{3600 \text{ s}}{\text{h}} \right) + 56 \text{ min} \left(\frac{60 \text{ s}}{\text{min}} \right) + 4 \text{ s} = 8.61 \times 10^4 \text{ s}$$

To solve this problem, we note that Earth's gravitational attraction provides all the necessary centripetal force to keep the satellite in orbit. Second, we will assume that the satellite's period of rotation is the same as that of Earth.

$$\vec{F}_g = \vec{F}_c$$

$$\frac{Gm_E m_s}{r^2} = \frac{m_s 4\pi^2 r}{T^2}$$

$$r^3 = \frac{Gm_E T^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{Gm_E T^2}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(5.98 \times 10^{24} \text{ kg})(8.61 \times 10^4 \text{ s})^2}{4\pi^2}}$$

$$r = 4.22 \times 10^7 \text{ m}$$

This distance is the distance from Earth's centre to the satellite. To determine the satellite's distance above Earth's surface, we subtract Earth's radius from this value.

$$r = 4.22 \times 10^7 \text{ m} - 6.38 \times 10^6 \text{ m}$$

$$r = 3.58 \times 10^7 \text{ m}$$

Therefore, the satellite must be $3.58 \times 10^7 \text{ m}$ above the equator in order to maintain GEO.



1. A 10-kg child is riding a merry-go-round of radius 5.0 m. If the merry-go-round completes 20 rotations in three minutes,
 - a) at what speed does the rider rotate?
 - b) what is the centripetal force on the child?
 - c) what provides this centripetal force?
2. If Tarzan (of mass 60 kg) has a speed of 4 m/s at the bottom of his swing on a 2.5-m vine, find the tension in the vine.
3. A child spins a bucketful of water in a vertical circle by using a piece of rope attached to the bucket. If the rope is 1.2 m long, at what speed must the bucket move so that, at the top of its path, there is no tension in the rope?
4. A racecar driver drives her 1500-kg car around a circular turn, which is banked at an angle of 20° to the horizontal. If the car is travelling around the frictionless curve of radius 100 m,
 - a) draw a free-body diagram of the situation.
 - b) what is the car's speed?
 - c) what is providing the centre-seeking force on the car?
 - d) What would change in this problem if the car were travelling at a higher speed?
 - e) What other force could provide a centre-seeking force in a real-life situation?
5. Earth and the Moon are separated at their centres by a distance of 3.4×10^8 m. Determine the period of the Moon's rotation about Earth.

Fig.2.54



6. The Hubble Telescope (Figure 2.54) is in orbit 600 km above Earth's surface. At what speed is the telescope travelling?
7. In 1969, the *Apollo 8* command module orbited 190 km above the Moon's surface. Given that the Moon is 0.013 times the mass of Earth, determine how long it took the command module to orbit the Moon ten times. The Moon's radius is 1.74×10^6 m.

Fig.2.53





The Tape-measure Home Run

Fig.STSE.2.1 In 1998, Mark McGwire hit 70 home runs! During his career, he averaged a home run for every 12 hits. He also hit some of the longest homers on record.



Hitting a pitched baseball (especially a curved pitch) at a major-league baseball game is the most technically difficult task for a professional athlete. The ball takes half a second to arrive at the plate, during which time the batter must swing his bat and hit it in a direction that will allow the ball to remain in play.

For over 100 years, the sight of a baseball player hitting a ball over the outer fences of the stadium has been one of the most dramatic events in sports. In 1927, Babe Ruth hit 60 home runs in 152 games. In 1961, Roger Maris hit 61 home runs in 162 games. In 1998, Mark McGwire hit 62 home runs in 142 games.

How far do these baseballs travel? It depends on the stadium. Each stadium has its own standards. Some officials record the distance from home plate to the point where the ball lands, while others are interested in how far the ball would have travelled if the stands had not gotten in the way. No one has ever hit a ball out of Yankee Stadium in New York City, but baseballs routinely sail out of Comiskey Park and Wrigley Field in Chicago and Tiger Stadium in Detroit. In the 1950s, the era of Willy Mays and Mickey Mantle, outfield fences were 15 m deeper than in most stadiums today.

What about the winds? Barry Bonds played for years at San Francisco's Candlestick Park, which had strong swirling winds that likely affect a baseball's range. On the other hand, it took him only a couple of years of playing at the new Pacific Bell Park to hit his new record, possibly because this stadium, although still subject to winds off the Pacific Ocean, is built at sea level so ground structures play a much greater role in reducing these effects.

Who was the best home run hitter of all time? What principles can we apply to put the performances of these athletes on equal footing?

The trajectory of a hit baseball cannot be solved using the range equation,

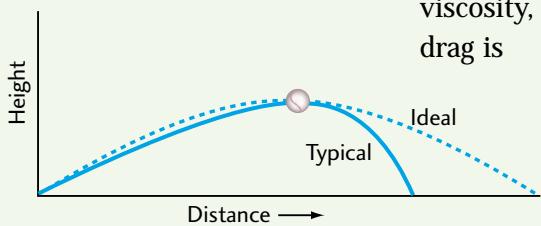
$$R = \frac{v^2 \sin 2\theta}{g}$$

because air drag on the ball slows it down. The path appears more like that shown in Figure STSE.2.2.

Drag is a mechanical force generated by a solid object moving through a fluid (liquid or gas). The amount of drag on an object depends on its shape and size, its velocity and inclination to fluid flow, and the compressibility, viscosity, and mass of the fluid moving past the object. The equation for drag is

$$F_{\text{drag}} = \rho A v^2 C_D$$

Fig.STSE.2.2 The real range of a baseball is shorter than its ideal range because of air drag



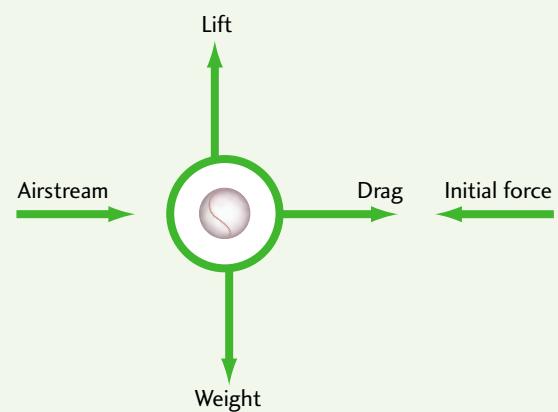
where ρ is the density of air, A is the reference area of the object experiencing drag, v is the object's velocity, and C_D is the drag coefficient. For a sphere of radius r , $C_D = 6\pi\eta r$, where η is the viscosity of the medium in g/cm.

Design a Study of Societal Impact

Discuss with baseball fans of all ages who they think was the best home-run hitter of all time. You might interview the sports caster of a local media network or a teacher who is knowledgeable in this sport. What do you think of the policies of how home runs are measured? Consider contacting various teams to find out how home runs are measured in their stadium.

Sometimes, baseball stadiums are adapted to suit home-run hitters. Is this practice fair? Think of other examples of such practices in other sports.

Fig.STSE.2.3 The forces acting on a ball being thrown



Design an Activity to Evaluate

How can we compensate for the differences among stadiums? Design an activity that will use the flight of a baseball, possibly hit in the schoolyard, and filmed as a model for what is seen on TV in the major leagues. Study the factors that affect the flight of a baseball. How does wind change the distance it travels? You should be able to estimate the distance with some accuracy based on the initial speed of the ball as it leaves the bat, its direction of travel, and the wind speed and direction.

Either manually or using a spreadsheet, use the drag equation to determine which variables affect the range of a projectile the most. Draw a graph of each version of your equation. Which graph comes closest to the ideal range curve in Figure STSE.2.2?

Build a Structure

Design and build a sighting device that will allow a person sitting in a stadium at a specific location of your choosing (but it must be the same for the entire game) to give a reasonably accurate estimate of the distance that a particular home run will travel. Measure the exact position of certain objects in a local ballpark or stadium. Then, using similar triangles, calculate the range of the projectile using your sighting device.

You should be able to*Understand Basic Concepts:*

- Add vectors in two dimensions using components, and sine and cosine laws.
- Determine the x and y components of a vector at an angle.
- Subtract vectors in two dimensions using components, and sine and cosine laws.
- Calculate the vector acceleration of an object in two dimensions.
- Solve relative motion problems in two dimensions.
- Describe and solve projectile motion problems in two dimensions using kinematics equations.
- Use free-body diagrams to isolate and analyze objects and the external forces acting on them in two dimensions.
- Distinguish between inertial and non-inertial frames of reference.
- Describe the motion of an object in two dimensions using Newton's laws.
- Determine the acceleration of an object in two dimensions using Newton's second law.
- Calculate the net force acting on an object in two dimensions.
- Solve problems involving accelerating bodies on inclined planes.
- Solve string and pulley problems, including those that involve inclined planes.
- Define and describe centripetal acceleration.
- Solve problems involving objects undergoing uniform circular motion, and calculate their centripetal acceleration.
- Define centripetal force.
- Solve problems involving objects undergoing uniform circular motion, and determine the forces acting upon them, in both horizontal and vertical planes.
- Solve problems involving objects and astronomical bodies in Earth orbit, and analyze the forces acting on these objects.
- Explain the operation of the laboratory centrifuge.

Develop Skills of Inquiry and Communication:

- Predict the motion of projectiles, and perform an experiment to confirm your predictions.
- Investigate, through experimentation, the relationships among centripetal acceleration, radius of orbit, and the frequency of an object in uniform circular motion.
- Describe, or construct prototypes of, technologies based on the concepts and principles related to circular motion.

Relate Science to Technology, Society, and the Environment:

- Investigate how knowledge of physics can be beneficial to athletes.
- Investigate how equipment and stadium modifications have increased athlete performance.
- Design and construct a sighting device.

Equations

$$R = \frac{v^2 \sin 2\theta}{g}$$

$$\vec{v}_{og} = \vec{v}_{om} + \vec{v}_{mg}$$

$$F_f = \mu F_n$$

$$a_c = \frac{v^2}{r} \quad a_c = \frac{4\pi^2 r}{T^2} \quad a_c = 4\pi^2 r f^2$$

$$F_c = \frac{mv^2}{r} \quad F_c = \frac{m4\pi^2 r}{T^2} \quad F_c = m4\pi^2 r f^2$$

$$F_{g\parallel} = mg \sin \theta$$

$$F_{g\perp} = mg \cos \theta$$

$$\mu = \tan \theta$$

$$\frac{v^2}{gR} = \tan \theta$$

Conceptual Questions

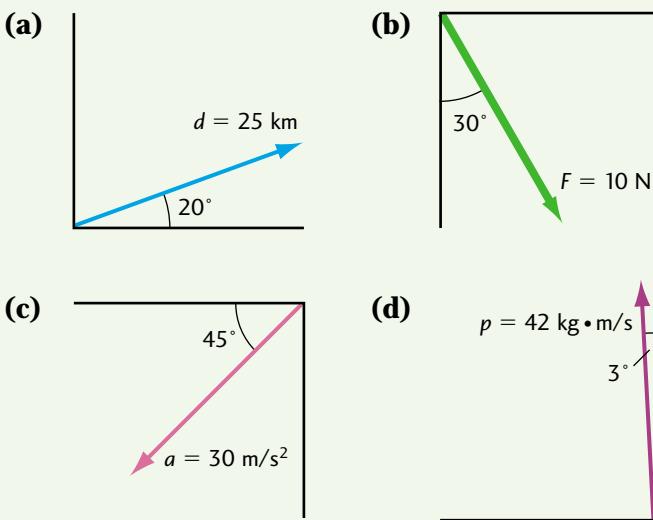
1. A textbook is sitting on top of a table. Why will the frictional force between the table and the textbook not cause the textbook to move?
2. Is it possible to swing a mass at the end of the string in a horizontal circle above your head? Explain your answer.
3. When an object rests on a horizontal surface, how can you be certain that the normal force is balanced by the gravitational force? How would you know if these two forces were not balanced?
4. North, south, east, and west can be used to describe directions in two-dimensional physics problems. How would you describe directions in three dimensions (e.g., in which direction would a vector coming out of the plane of this page be pointing)?
5. Two identical bullets are at the same height. One bullet is fired horizontally from a rifle at a velocity of 1000 m/s over level ground. The other bullet, released at the same instant, falls straight down. How long does it take each bullet to reach the ground? Explain your answer.
6. Write a brief letter to your cousin Wolfgang explaining why a river current's velocity doesn't affect the amount of time it takes to paddle a canoe across a river. In your letter, describe which variables determine the length of time required to cross the river.
7. A 100-kg sofa needs to be moved across a level floor. The coefficient of static friction between the sofa and the floor is 0.40. Two physics students decide to apply a force \vec{F} on the sofa. One student recommends that the force be applied upward at an angle θ above the horizontal. The other student recommends that the force be applied downward at an angle θ below the horizontal. Explain which student has the better idea and why.
8. A baseball is thrown straight up in the air. Describe the baseball's velocity and acceleration at each of the following points:
 - a) half-way up.
 - b) at its maximum height.
 - c) half-way down.
9. In baseball, after a pitcher has released the ball, it will accelerate downward due to gravity. To compensate for this downward motion, the pitcher stands on a mound that is raised relative to the rest of the field. If you were to play baseball on the Moon, would you still need a mound? If so, how would its height compare to a mound on Earth?
10. On Earth, an athlete can jump a horizontal distance of 1.8 m from a standing start. How far could she jump on a planet that has one-half the acceleration due to gravity on Earth?
11. When you ride a bicycle down a wet road after removing the fenders, you'll get a wet stripe down your back. Why?
12. The last cycle in a washing machine is always the spin cycle, during which the drum rotates at high speed about a vertical axis. Explain how the spin cycle removes water from clothing.
13. NASA uses an aircraft, the Vomit Comet, to train astronauts. If flown correctly, for a brief period of time, the astronauts will feel weightless. Describe how the aircraft should be flown in order to achieve weightlessness.

Problems

2.1 Vectors in Two Dimensions

- 14.** In your notebook, break each of the following two-dimensional vectors into perpendicular components.

Fig. 2.55



- 15.** A 10-m-long crane is inclined at an angle of 40° to the horizontal. If the Sun is directly overhead,
- what is the length of the crane's shadow on the ground?
 - how high above the ground is the top of the crane?
- 16.** A skier accelerates at a rate of 4.0 m/s^2 down a ski hill inclined at 35° . What are the vertical and horizontal components of her acceleration?
- 17.** A pizza delivery truck drives 2.0 km [W] , followed by $3.0 \text{ km [W}20^\circ\text{N]}$. What is the total displacement of the delivery truck?
- 18.** A projectile is launched with a horizontal velocity of 10 m/s and a vertical velocity of 20 m/s . What is the magnitude and direction of the projectile's initial velocity?

- 19.** Add the following displacements using the component method:

$$\vec{d}_1 = 20 \text{ cm [N]}, \vec{d}_2 = 50 \text{ cm [S}35^\circ\text{E]}, \\ \vec{d}_3 = 100 \text{ cm [W}15^\circ\text{S]}$$

- 20.** A tennis ball's initial velocity is 30 m/s [S] . When struck by a tennis racquet, its velocity becomes $28 \text{ m/s [N}30^\circ\text{W]}$. Determine the ball's change in velocity.

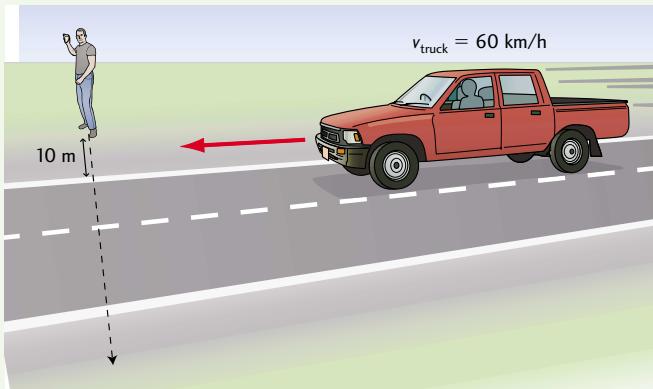
- 21.** A billiard ball with an initial velocity of $2.0 \text{ m/s [S}30^\circ\text{E]}$ strikes the bumper of a billiard table and reflects off it at a velocity of $1.8 \text{ m/s [N}30^\circ\text{E]}$. If the interaction with the bumper takes 0.10 s , determine the vector acceleration of the billiard ball.

2.2 Relative Motion

- 22.** A swimmer, who can swim at a maximum speed of 1.8 km/h , swims straight north across a river of width 0.80 km . If the river's current is 0.50 km/h [E] ,
- how long does it take the swimmer to cross the river?
 - how far downstream will the swimmer land?
 - what is the swimmer's ground velocity?
- 23.** If the swimmer in problem 22 decided to change his direction so as to go straight north, determine
- his heading.
 - his ground velocity.
 - the amount of time it would take him to cross the river.

- 24.** A concerned parent wants to throw a forgotten lunch bag into the back of his daughter's passing pickup truck. The parent is standing 10 m north of a road that runs east–west. If the parent can throw the bag at a speed of 2.0 m/s and the speed limit on the road is 60 km/h, how far east of the parent must the westbound truck be when the bag is released?

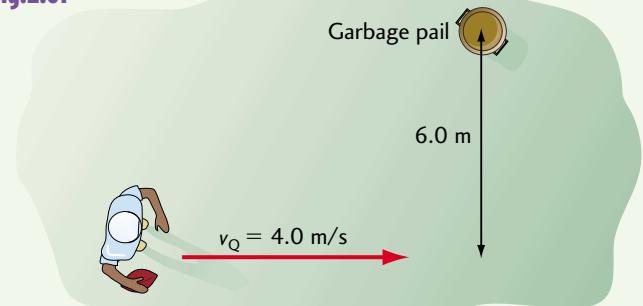
Fig. 2.56



- 25.** A helicopter pilot wishes to fly east. There's a wind from the north at 20 km/h. If the helicopter can fly at a speed of 150 km/h in still air, in which direction must the pilot point the helicopter in order to fly east (i.e., what is the pilot's heading)?
- 26.** A ship's captain wishes to sail his ship northeast. A current is moving his ship with a velocity of 5.0 km/h [S]. If the ship has a maximum speed of 30 km/h, what is the ship's required heading?
- 27.** A cruise ship is sailing north at a speed of 10 km/h. A passenger walks along the deck with a velocity of 0.5 m/s toward the stern of the ship. She then turns toward port and walks to the railing at the same speed. Determine the passenger's velocity for both motions
- relative to the ship.
 - relative to the water.
- 28.** A high-school football quarterback is practising by throwing a football into a garbage pail. The quarterback runs along a line 6.0 m away from the garbage pail at a speed of 4.0 m/s. If

the quarterback can throw the football at a speed of 5.0 m/s,

Fig. 2.57



- how far in advance of the garbage pail must the quarterback release the ball if the ball is thrown perpendicular to the direction in which he's running?
- how long will it take the football to reach the garbage pail?
- what is the football's ground velocity?

- 29.** The quarterback in the problem 28 decides to practise in a different way. This time, he runs along the same path, 10 m away from the garbage pail, and releases the football just as he passes the garbage pail.

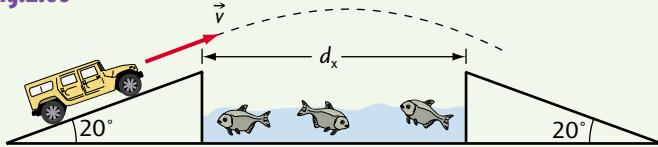
- In which direction must he throw the football so that it lands in the garbage pail?
- How long does it take the football to reach the garbage pail this time?
- What is the football's ground velocity?

2.3 Projectile Motion

- 30.** Blarney, the orange dinosaur, throws a Nerf™ ball horizontally out of an open window with a velocity of 3.0 m/s. If the window is 10 m above the ground, how far away from the building must Blarney's friend stand to catch the ball at ground level?
- 31.** A rock thrown horizontally from the top of a water tower lands 20.0 m from the base of the tower. If the rock was initially thrown at a velocity of 10.0 m/s,
- how high is the water tower?
 - What is the final velocity of the rock?

- 32.** A bag of mail is catapulted from the top of a building 200 m above the ground with a velocity of 20 m/s at an angle of 15° above the horizontal. If the mail is to land on the roof of another building 100 m away, how tall is the second building?
- 33.** A tourist taking the train from Toronto, Ontario to Montreal, Quebec accidentally drops a cup of coffee from a height of 1.3 m. The train is travelling at 180 km/h.
- How long does it take the cup of coffee to hit the floor?
 - Where does the cup land relative to the tourist?
 - How much closer to Montreal is the cup when it strikes the floor compared to when it was dropped?
- 34.** Bounder of Adventure is trying to cross a piranha-infested pool of water in his Humvee. He races up a ramp inclined at 20° to the horizontal at a speed of 30 m/s. There is an identical ramp on the other side of the pool. What is the maximum width of the pool that Bounder of Adventure can successfully cross?

Fig.2.58



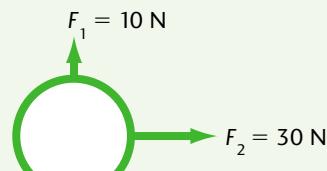
- 35.** A soccer ball is kicked from the ground at an angle θ above the horizontal. Show that the equation $h = 0.25R \tan \theta$ represents the maximum height of the ball, where h is the height and R is the range.
- 36.** A baseball player makes perfect contact with the ball, striking it 45° above the horizontal at a point 1.3 m above the ground. His home-run hit just clears the 3.0-m wall 130 m from home plate. With what velocity did the baseball player strike the ball?

2.4 Newton's Laws in Two Dimensions

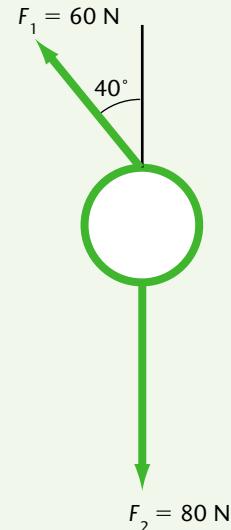
- 37.** Determine the net force for each of the following situations:

Fig.2.59

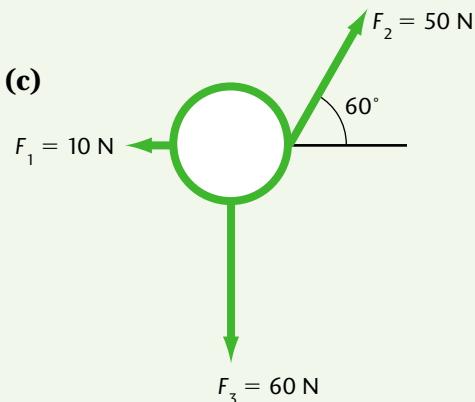
(a)



(b)

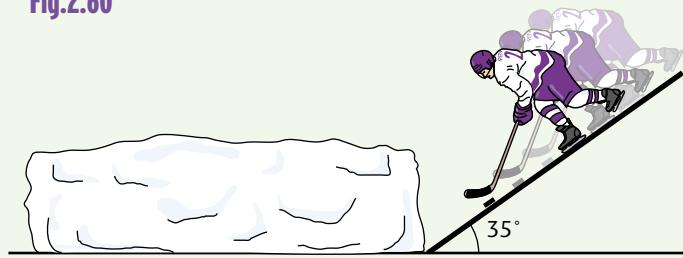


(c)



- 38.** Three movers are applying forces $\vec{F}_1 = 100 \text{ N}$ [W 20° N], $\vec{F}_2 = 200 \text{ N}$ [E 40° S], and $\vec{F}_3 = 300 \text{ N}$ [S] on a 300-kg grand piano. If μ_k for the piano is 0.10, use the component method to determine
- the net force acting on the piano.
 - the acceleration of the piano.
- 39.** A worker drags a 20-kg bag of cement across a floor by applying a force of 100 N at an angle of 50° to the horizontal. If the coefficient of kinetic friction between the cement bag and the floor is 0.30, determine the acceleration of the bag. (Be sure to draw a free-body diagram.)
- 40.** A 0.25-kg hockey puck sliding across the ice with an initial velocity of 12 m/s [S] is struck by a hockey stick with a force of 300 N [N 25° E]. If the hockey stick interacts with the puck for 0.20 s, determine the puck's final velocity.
- 41.** A 100-kg baseball player slides into home plate. If the coefficient of kinetic friction is 0.50,
- what is the frictional force acting on the baseball player?
 - If the baseball player comes to rest in 1.3 s, what was her initial speed?
- 42.** A person tosses his car keys on top of his dresser with an initial velocity of 2.0 m/s. How far will the keys slide across the dresser if the coefficient of kinetic friction between the two surfaces is 0.30?
- 43.** While mopping the deck, a sailor pushes with a force of 30 N down on the handle of his mop at an angle of 45° to the horizontal. If the mop accelerates horizontally at 1.0 m/s^2 and the coefficient of kinetic friction is 0.10, what is the mass of the mop?
- 45.** Two children are having a toboggan race down a frictionless hill inclined at 30° to the horizontal. The children's masses are 20 kg and 40 kg.
- What is the acceleration of each child?
 - Which child reaches the bottom of the hill first?
- 46.** A rescue worker slides a box of supplies from rest down a hill to a group of trapped campers. The hill is inclined at 25° to the horizontal and is 200 m long. If the coefficient of kinetic friction on the hill is 0.45,
- what is the acceleration of the box as it goes down the hill?
 - at what speed does the box reach the bottom of the hill?
 - how long does it take the box to reach the bottom of the hill?
- 47.** Boom-Boom Slapshot, Canadian hockey star, slides down a 50-m-long ice-covered hill on his hockey skates. The frictionless hill is inclined at 35° to the horizontal. Once he reaches the bottom of the hill, the ice is covered with deep snow that has a coefficient of kinetic friction of 0.50. How far into the snow will Boom-Boom go before coming to rest?

Fig. 2.60



2.5 The Inclined Plane

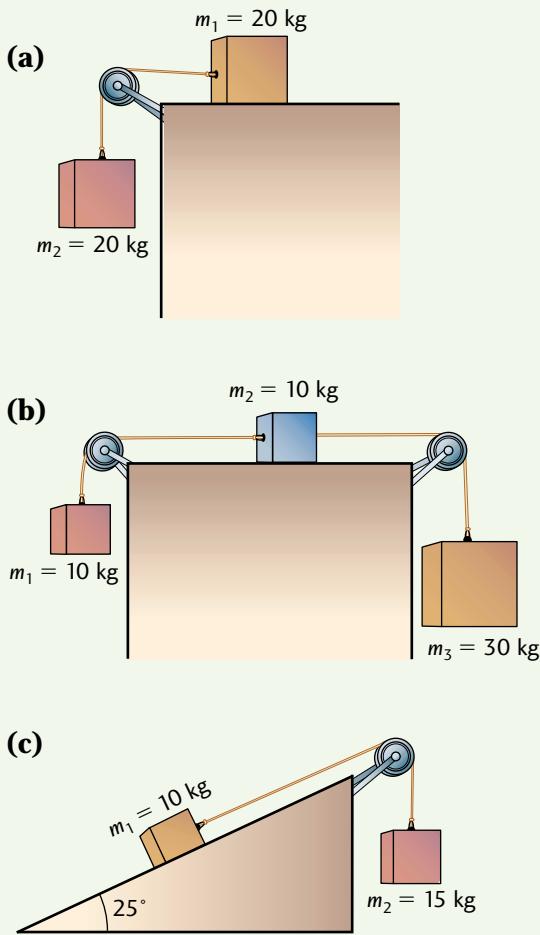
- 44.** The coefficient of static friction between a box and an inclined plane is 0.35. What is the minimum angle required for the box to begin sliding down the incline?

- 48.** Spot the Wonder Cow has strapped on her roller blades and rocket pack and is standing at the bottom of a hill inclined at 20° to the horizontal. If Spot's rocket pack provides a force of 2000 N and Spot has a mass of 250 kg, how long will it take her to reach the top of a 250-m-long hill?

2.6 String-and-pulley Problems

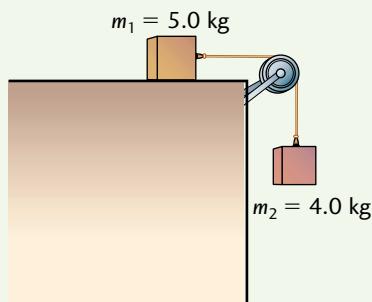
- 49. a)** For each frictionless situation in Figure 2.61, determine the acceleration of the system and the tension in each rope:
b) Repeat for $\mu_k = 0.2$.

Fig.2.61



- 50.** Determine the acceleration of the system in Figure 2.62 if the coefficient of kinetic friction for the tabletop is 0.10.

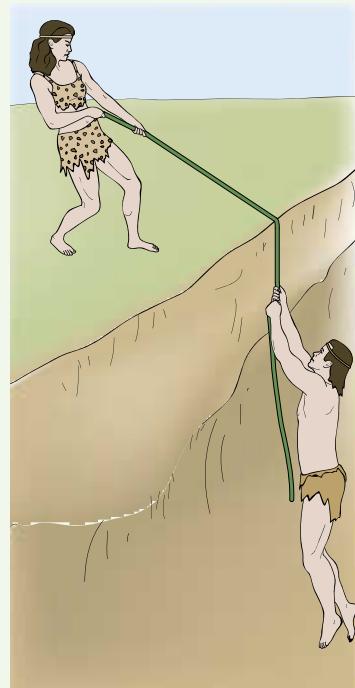
Fig.2.62



- 51.** In Figure 2.62, what coefficient of friction would be required to prevent the system from moving?

- 52.** Tarzana ($m = 65 \text{ kg}$) is trying to rescue Tarzan ($m = 80 \text{ kg}$), who has fallen over the edge of a cliff (Figure 2.63). Tarzana is standing on a horizontal frictionless surface 15 m from the edge of the cliff. Determine how long it will take before Tarzana reaches the edge of the cliff, starting from rest.

Fig.2.63



2.7 Uniform Circular Motion

- 53.** David spins a sling in a horizontal circle above his head. What would happen to the period of rotation if he applied the same force and the length of the sling was

- a)** doubled?
b) halved?

- 54.** The drum in a clothes dryer has a diameter of 0.70 m and completes one rotation every 0.42 s.

- a)** What is the centripetal acceleration of the drum?
b) Why do the clothes not fly toward the centre of the clothes dryer?

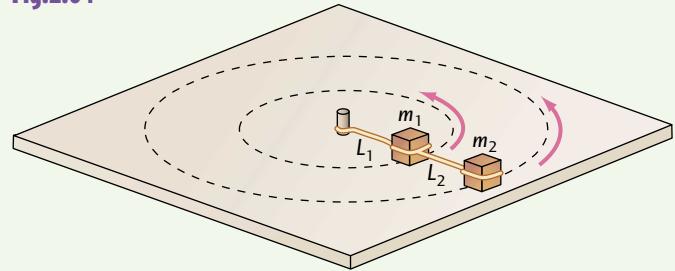
- 55.** Earth is approximately 1.5×10^{11} m from the Sun. If Earth orbits the Sun with a period of 365 days, determine Earth's centripetal acceleration.
- 56.** What is the maximum speed at which a 1500-kg car can round a curve on a flat road if the radius of the curve is 90 m and the coefficient of static friction is 0.50? Is it necessary to know the mass of the car to solve this problem?
- 57.** A 1000-kg Indy car travels around a curve banked at 25° to the horizontal. If the radius of the curve is 80 m, at what speed must the car be travelling if no friction is present?

2.8 Centripetal Force

- 58.** Roller-coaster riders on the "Vomit" go through a vertical loop of radius 10 m. At what minimum speed must a "Vomit" car travel so that the riders don't fall out?
- 59.** A clock's pendulum is 60 cm long with a bob at the end of mass 500 g. Determine the maximum tension in the pendulum rod when the bob is
 - at rest.
 - swinging at a speed of 2.4 m/s.
- 60.** A 2.0-kg mass is attached to the end of a 3.0-m-long rope and spun in a vertical circle at a speed of 6.6 m/s. Determine the maximum and minimum tensions in the rope.

- 61.** As a pilot comes out of a dive in a circular arc, she experiences an upward acceleration of 9.0 gs.
- If the pilot's mass is 60 kg, what is the magnitude of the force applied to her by her seat at the bottom of the arc?
 - If the speed of the plane is 330 km/h, what is the radius of the plane's arc?
- 62.** Earth is a satellite of the Sun with an orbit radius of approximately 1.5×10^{11} m.
- What is the Sun's mass?
 - If the Sun's radius is 6.96×10^8 m, how does the Sun's density compare with Earth's density?
- 63.** A block of mass m_1 is attached to a rope of length L_1 , which is fixed at one end to a table. The mass moves in a horizontal circle supported by a frictionless table. A second block of mass m_2 is attached to the first mass by a rope of length L_2 . This mass also moves in a circle, as shown in Figure 2.64. If the period of the motion is T , find the tension in each rope. (Assume all ropes are massless.)

Fig.2.64



Purpose

To design and construct a device that will experimentally confirm the projectile motion equations in this chapter; specifically, to compare theoretical and experimental values for time of flight, range, and maximum height

Hypothesis

Using your previous knowledge, predict values for time of flight, range, and maximum height for projectiles launched at different angles. Predict how your real-life values will compare to your theoretical values. Give reasons for any potential differences.

Equipment**Provided by the instructor:**

Stopwatch	Safety goggles	Metre stick
Tape measure	Masking tape	

Provided by the student:

Bean balloon projectile launcher
Bean balloon

Procedure

1. Design and construct a device that will **safely** launch “bean balloons” (i.e., small water balloons containing dried beans, rice, etc.).
2. Launchers must be able to launch bean balloons at different angles relative to the horizontal. Launchers must also be capable of firing projectiles at different speeds.
3. The targets will be paper plates placed flat on the floor in front of your launcher. Your teacher will give you the distance to the targets.
4. Using a spreadsheet or calculator, generate theoretical data that will predict theoretical values for time of flight, range, and maximum height given the speed of the projectile, vertical height of launch, and angle of launch. Record these theoretical values in a data table.
5. Launches must be based on theoretical data (no guessing!), and will be compared to experimental values.
6. Launches will take place in an area such as a gymnasium, which is of known length and height. Be sure that none of your launches hits the ceiling or walls.

7. Students launching balloons must wear safety goggles for protection.

Data

1. For each of the four required target distances, experimentally determine a value for time of flight, range, and a maximum height. To determine an approximate maximum height, stick a piece of masking tape to the wall a known height above the floor. Using the launcher, fire a bean balloon parallel to the wall and estimate the distance from the masking tape to the place where the bean balloon reaches its maximum height.
2. Record your experimental values in your data table. Compare them to the theoretical values obtained using the projectile motion equations. Calculate a percent error for each experimental value.

Uncertainty

Assign an instrumental uncertainty for the metre stick you used. Estimate the uncertainty in your time measurements based on your personal reaction time and the stopwatch you used. Estimate the significance of air resistance in the results of your experiment. Include these uncertainties in your data table.

Analysis

Describe the methods you used to measure time of flight, range, and maximum height.

Discussion

1. How did you design your projectile launcher so that it was able to launch bean balloons at different angles and different velocities?
2. How did you calibrate your projectile launcher to successfully hit targets at different distances?
3. How could you have modified your calibration process to minimize the effect of air resistance?
4. What modifications could you make to your bean balloons to minimize the effect of air resistance?
5. How closely do the theoretical equations for projectile motion match your experimental results? Give reasons for any discrepancies.

Conclusion

Are the theoretical equations for projectile motion true, within experimental error?



Centripetal Force and Centripetal Acceleration

Purpose

To determine the relationships between centripetal force (centripetal acceleration) and radius, period, and frequency

Hypothesis

Using your previous knowledge, predict the relationship between centripetal force (centripetal acceleration) and frequency, period, and radius.

Equipment

Rubber lab stopper

Standard mass set

1 small paper clip

2.0 m of fine fishing line or nylon thread

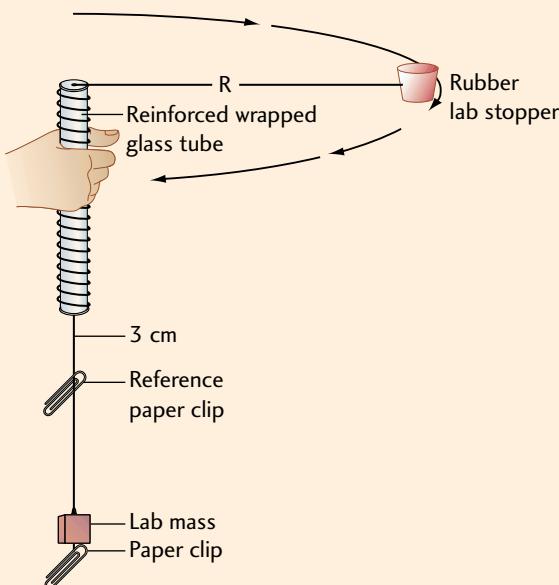
Glass tube (wrapped with masking tape to prevent breakage)

Metre stick

Stopwatch

Safety goggles

Fig. Lab.2.1



Procedure

Part A: Centripetal Force (Acceleration) and Period

- Working in pairs, tie a rubber stopper securely to the end of the 2.0-m string. Pass the string through the glass tube, as shown

in Figure Lab.2.1. Tie a secure loop at the opposite end of the string to act as a support for the standard lab masses.

- Attach a small standard mass to the loop at the end of your string.
- Pull the string so that the rubber stopper is 1.5 m from the top of the glass tube.
- With the string pulled taut, place the paper clip about 3.0 cm below the glass tube, between the glass tube and the standard mass.
- Make sure that both partners are wearing safety goggles.**
- Holding on to the standard mass at the end of the string, swing the rubber stopper in a horizontal circle above your head such that the paper clip between the stopper and the glass tube remains 3.0 cm away from the end of the tube. If the paper clip starts to go downward, then the radius of your circle is less than 1.5 m. The stopper is spinning too slowly and you need to increase the speed of rotation. If the paper clip goes upward and hits the bottom of the tube, then the radius of your circle is greater than 1.5 m. The stopper is spinning too quickly and you need to decrease the speed of rotation. Finding the optimal speed of rotation will take some practice.
- Once you are proficient at swinging, the partner with the stopwatch can start counting rotations and measuring the time for a set number of rotations (e.g., the time for five rotations, ten rotations, etc.).
- Add a different standard mass to the end of the string and repeat step 7 for the same number of rotations. As the mass is changed, the person doing the swinging will need to change the speed of rotation to maintain the reference paper clip in the position required.
- Continue this procedure for at least six different masses.

10. Calculate the gravitational force for each mass used. Recall that in this experiment, the gravitational force is equal to the centripetal force.
11. Create a data table showing the centripetal force, the number of rotations, time, period, and frequency for all six data sets.

Part B: Centripetal Force and Radius

In this part of the experiment, we will calculate the centripetal force and the radius of the circle with frequency as the constant.

1. Attach a 200-g mass to the end of the string.
2. Adjust the string to provide a radius of rotation of 0.750 m.
3. Swing the rubber stopper in a horizontal circle, as in Part A.
4. Record the number of rotations and time in a data table, as in Part A. Calculate the frequency and the frequency squared, and record these numbers in your data table.
5. Repeat steps 1 to 4 for radii of 1.00 m, 1.25 m, and 1.50 m.

Uncertainty

Assign an instrumental uncertainty for the metre stick you used. Estimate the uncertainty in your time measurements based on your personal reaction time and the stopwatch you used. Include these uncertainties in your data table.

Analysis

Part A: Centripetal Force

(Acceleration) and Period

1. Plot a graph of centripetal force versus period.
2. Determine the relationship between centripetal force and period.

Table Lab.2.1

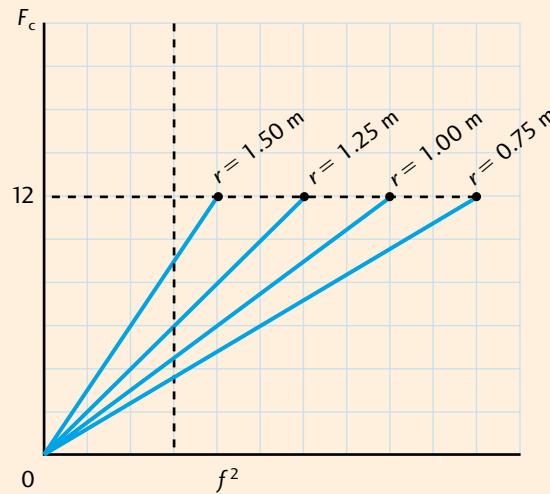
Radius (m)	Number of rotations	Time (s)	Frequency (Hz)	Frequency ² (Hz ²)
0.75				
1.00				
1.25				
1.50				

3. Plot a graph of centripetal force versus frequency.
4. Determine the relationship between centripetal force and frequency.

Part B: Centripetal Force and Radius

1. Plot a graph of centripetal force versus frequency squared. This graph will look odd because you only have four points on your graph. **Don't connect the points!** Instead, draw a line from each point to the origin (because the centripetal force for a frequency of 0 Hz is zero). You will get four straight lines, as shown in Figure Lab.2.2.

Fig. Lab.2.2



2. Draw a vertical line on your graph. This vertical line will cross each of the four lines plotted on your graph. By reading vertically up to each line plotted, you will be able to read across to the central force axis and determine the values of centripetal force for a constant f^2 value for each corresponding radius.
3. Record these four sets of centripetal-force-versus-radius data in a new data table.
4. Plot a graph of centripetal force versus radius for a constant f^2 , with centripetal force as the dependent variable.
5. Determine the relationship between centripetal force and radius.

Discussion

1. Write a proportionality statement for, and the equation describing the relationship between, centripetal force and
 - a) frequency.
 - b) period.
 - c) radius.
2. How do your results from question 1 compare to those in your hypothesis?
3. From your results, what centripetal force would be required to rotate the rubber stopper in a horizontal circle of radius 1.5 m with a frequency of 8.0 Hz?

Conclusion

Are the theoretical equations for centripetal force (acceleration) true, within experimental error?

Amusement park rides are designed to thrill and entertain. By providing us with situations vastly different from our everyday experiences, they allow us to safely experience velocities and accelerations that we would otherwise be unable to attain. Whether you are a fan of roller coasters, free-fall rides, or whether you prefer something more subdued, amusement parks give us a change from everyday life. Through an understanding of physics, we can better appreciate how these rides provide us with so much excitement.

Fig.Lab.2.3



Fig.Lab.2.4



Research

Carry out research on the rides at an amusement park. Current print or online resources may be used for collecting information. Choose one amusement park ride and analyze one part or motion of the ride. Investigate the forces and accelerations that occur. Describe them in writing by applying your knowledge of kinematics and dynamics.

Design and Construct

Design and construct a model of the portion of the amusement park ride that you researched in the previous section. Your model should be constructed so as to show the physics involved. Use a free-body diagram and the appropriate algebra to describe the forces and accelerations involved in the ride. Explain why the ride you chose is so thrilling.

Extension: Statics — Objects and Structures in Equilibrium



Chapter Outline

- 3.1** Keeping Things Still: An Introduction to Statics
- 3.2** The Centre of Mass — The Gravity Spot
- 3.3** Balancing Forces ... Again!
- 3.4** Balancing Torques
- 3.5** Static Equilibrium: Balancing Forces and Torque
- 3.6** Static Equilibrium and the Human Body
- 3.7** Stability and Equilibrium
- 3.8** Elasticity: Hooke's Law
- 3.9** Stress and Strain — Cause and Effect
- 3.10** Stress and Strain in Construction
- The Ultimate Effect of Stress on a Structure
- LAB** **3.1** Equilibrium in Forces
- LAB** **3.2** Balancing Torque

By the end of this chapter, you will be able to

- understand the concepts of balancing forces and torques to maintain objects in static equilibrium
- relate the concepts of centre of mass and torque to the stability of an object
- define and solve problems based on the principle of stress and strain

3.1 Keeping Things Still: An Introduction to Statics

Thus far, we have studied the physics of motion and forces, or **dynamics**. But there are many situations when we don't want things to move, like the snow-covered roof of a barn, or the bridge across the river, or the new deck you just built. **Statics** is the physics of keeping objects still by applying forces to them. This branch of physics involves one of the key components of Newton's first law of motion.

Newton's first law of motion: All objects remain in a state of rest or continue to move at a constant velocity unless acted upon by an external unbalanced force.



Fig.3.1 The bridge over the Tacoma Narrows in Washington State collapsed in 1940 due to an inherent design flaw. The structure began to oscillate in resonance with a gale. For information on similar disasters, visit <www.irwinpublishing.com/students>.

The words "static" and "equilibrium" come from the Latin for "at rest" and "equal forces," respectively.

An object will stay at rest if no unbalanced forces act on it; that is, *if all the forces on the object balance each other*. When we use objects, we apply forces to them. For example, when we cross a bridge, we apply a force on it. But a bridge that remains at rest only when no forces act on it wouldn't be a very good bridge. Bridges are built to withstand great forces without collapsing. *Statics* is the study of the application of appropriate forces in order to balance all forces, keeping the object still, or in **static equilibrium**. The study of statics is very important for careers in structural design, such as architecture and engineering. The collapse of the Tacoma Narrows Bridge (Figure 3.1) reminds us of what can happen when an inherent design flaw in a structure prevents it from remaining in static equilibrium.

Our study of Newton's first law has dealt mainly with uniform velocity, or **dynamic equilibrium**. In this chapter, we will turn our attention to **static equilibrium**, the aspect of Newton's first law where all forces applied on an object lead to no acceleration and zero velocity. But before we can discuss static equilibrium, we need to learn about the centre of mass, a concept that will help us simplify complex situations.

3.2 The Centre of Mass — The Gravity Spot

In our study of traditional mechanics and dynamics, we have mainly examined one type of motion — translation — where an object moves from one place to another. In Chapter 7 (Angular Motion), we will study a force that causes an object to move in a curved path or rotate. Here, we will consider forces on objects around us that tend to cause both translation (movement from one place to another) and rotation, as illustrated in Figure 3.2. These forces must be balanced in order to maintain static equilibrium.

In order to simplify our study of the static equilibrium, we will consider the mass of an object to be concentrated at its centre of mass.

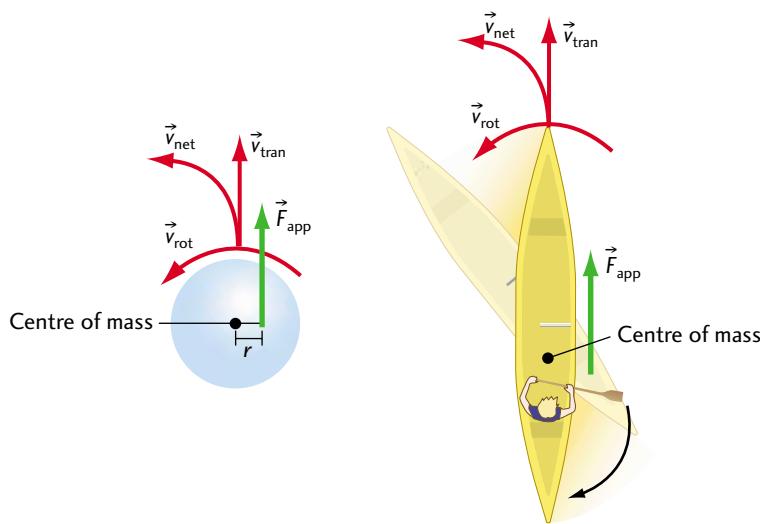


Fig.3.2 Forces cause translation and rotation. Two people paddling a canoe not only share the work, but also balance the rotational effect of each canoeist paddling separately. The dual effect of translation and rotation is the reason why a canoe may move erratically when there is only one person at the stern (Figure 3.3).

The **centre of mass** is a single point at which the entire mass of a body is considered to be concentrated. For uniform, regularly shaped objects such as a sphere, the centre of mass is its geometric centre, as shown in Figure 3.4a. For more oddly shaped objects, like the triangular block in Figure 3.4b, the centre of mass is located at its balance point in any gravitational field. The centre of mass is also referred to as the **centre of gravity**, the point at which the force of gravity acts on a complex or oddly shaped object. Like a balance point, the force of gravity on the mass is equal on both sides of an object's centre of gravity.

One way to determine the balance point of a three-dimensional object is to hang it randomly from at least three different points, as shown in Figure 3.6. The point of intersection of all three plumb lines is the object's centre of gravity. This point is also the object's centre of mass. We will use the concept of centre of mass in later sections of this chapter to determine where the force of gravity acts on an object to cause translation or rotation.

Fig.3.4 The centre of mass is the balance point of the object. In two dimensions, the balanced mass on either side means that equal forces of gravity balance the object. In three dimensions, there must be an equal amount of mass surrounding this point. The centre of mass may be difficult to locate in an oddly shaped figure like the triangular block.

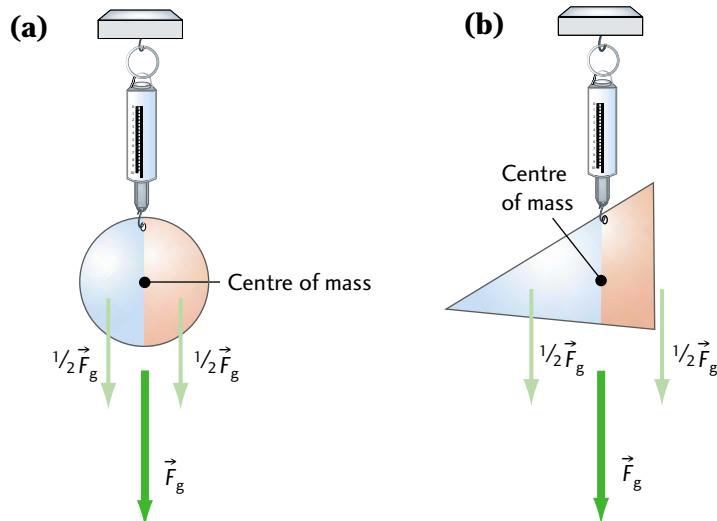


Fig.3.3 A single canoeist must apply specialized strokes and sit in a different position to eliminate any rotational effects and move the canoe along a relatively straight path to where he or she wants to go



Fig.3.5 Where is the centre of mass of this “balanced” rock?

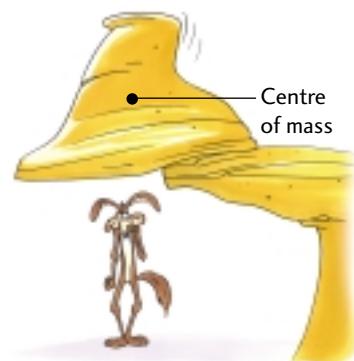
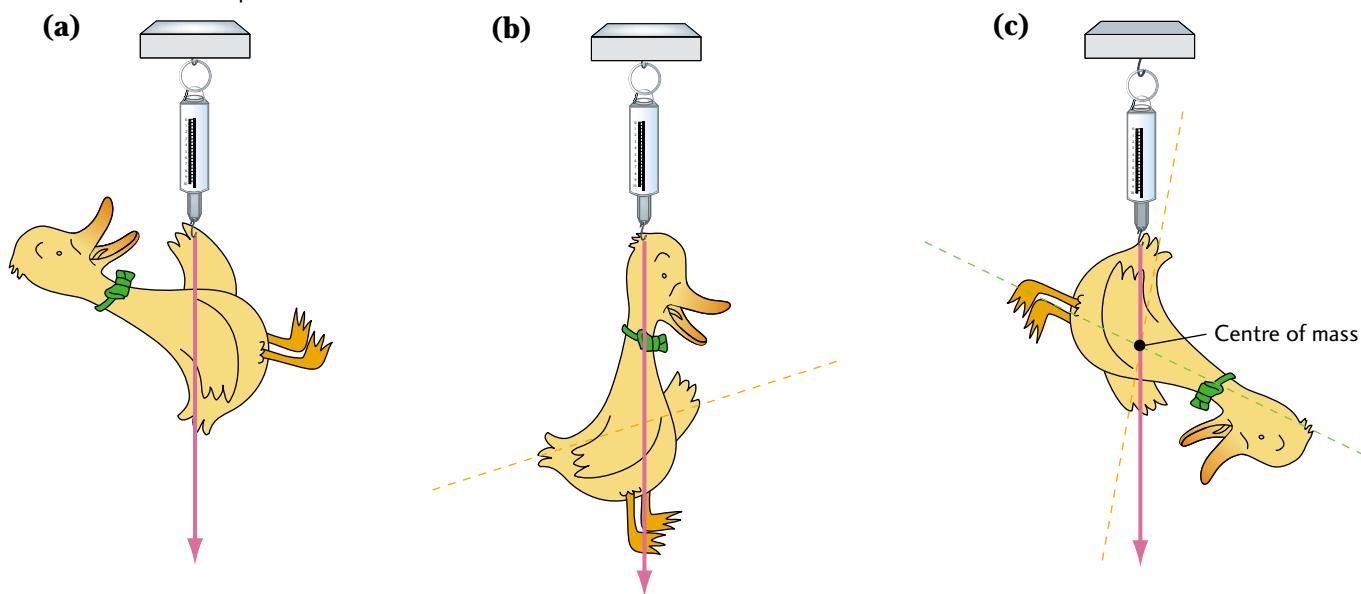


Fig. 3.6 The centre of mass is the point of intersection of all the plumb lines



3.3 Balancing Forces ... Again!

Forces *tend* to cause translation and rotation, depending on where they act with respect to the centre of mass. How does this tendency relate to statics, the study of *no* motion?

An object can be in a state of translational static equilibrium under only two circumstances: either when no forces are applied to it, or when the forces applied to it acting through the centre of mass all balance one another. Mathematically, static equilibrium can be expressed as follows.

The first condition for static equilibrium:

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = 0$$

where \vec{F}_{net} is the sum of all forces acting on an object through the centre of mass (for statics, it is zero), and \vec{F}_1 to \vec{F}_n are the independent external forces applied to an object.

The following example will help you remember how to solve force problems using vector addition and free-body diagrams.

EXAMPLE 1 Static equilibrium: balanced forces

Figure 3.7a illustrates a typical case of static equilibrium: several children are playing a parachute game during a physical education class.

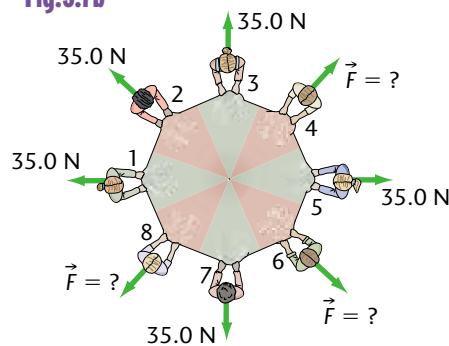
- a) What force must child 8 apply if all children in the circle are applying the same 35.0-N force outward in order to keep the parachute in translational static equilibrium?

- b)** What extra force would child 4 and child 6 each need to apply if their teacher was playing the game in position 1 and applied a force twice that of a typical child?

Fig. 3.7a



Fig. 3.7b



Solution and Connection to Theory

- a)** Each force that has an equivalent force applied in the opposite direction can be balanced because the pair will add vectorially to zero. As a result, vectors 1 and 5, 2 and 6, and 3 and 7 cancel each other out. Child 8 must therefore provide the same 35.0-N force in the opposite direction of child 4 in order to keep the entire parachute still (in translational static equilibrium).
- b)** Child 5 still balances 35 N of force from the teacher, and child 4 and child 6 must each apply their 35-N forces to balance their respective opposing partners, child 2 and child 8. The simplified free-body diagram is shown in Figure 3.8. Notice that the force arrows originate from the object's centre of mass.

For static translational equilibrium, the sum of all the force vectors must be zero when they act through the centre of mass.

The forces applied by child 4 and child 6 must have the same magnitude in order to balance the force applied by the teacher in the opposite direction. In Figure 3.9, we have taken the information from our FBD and created a scaled vector diagram. We can now solve the problem by measuring with a ruler and a protractor, or by calculation using components or trigonometry.

Fig. 3.8

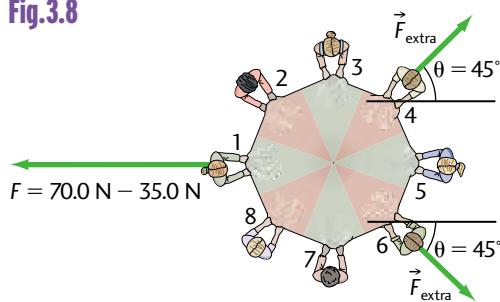


Fig. 3.7c If the force of gravity, \vec{F}_g [down], isn't canceled out by the normal force, \vec{F}_n [up], then the object accelerates upward or downward.

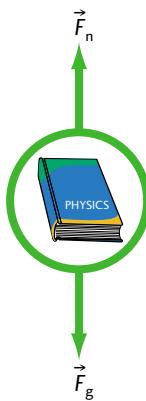
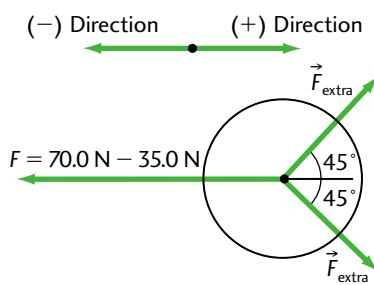


Fig.3.9



We can also solve this problem using the cosine law to find the vector magnitude, followed by the sine law to find the vector direction.

Both applied forces from child 4 and child 6 may be broken down into east–west (E–W) and north–south (N–S) components, as shown in Figure 3.9. The (N–S) components, given by the expression $F_{\text{extra}} \sin \theta$, cancel one another because the parachute remains vertically stationary. Therefore, the extra 35.0 N of westward force from the teacher must be balanced by the two eastward force components of each child, $F_{\text{extra}} \cos \theta$. Substituting into the equation for static equilibrium, we obtain

$$2(F_{\text{extra}} \cos \theta) + (-35.0 \text{ N}) = 0$$

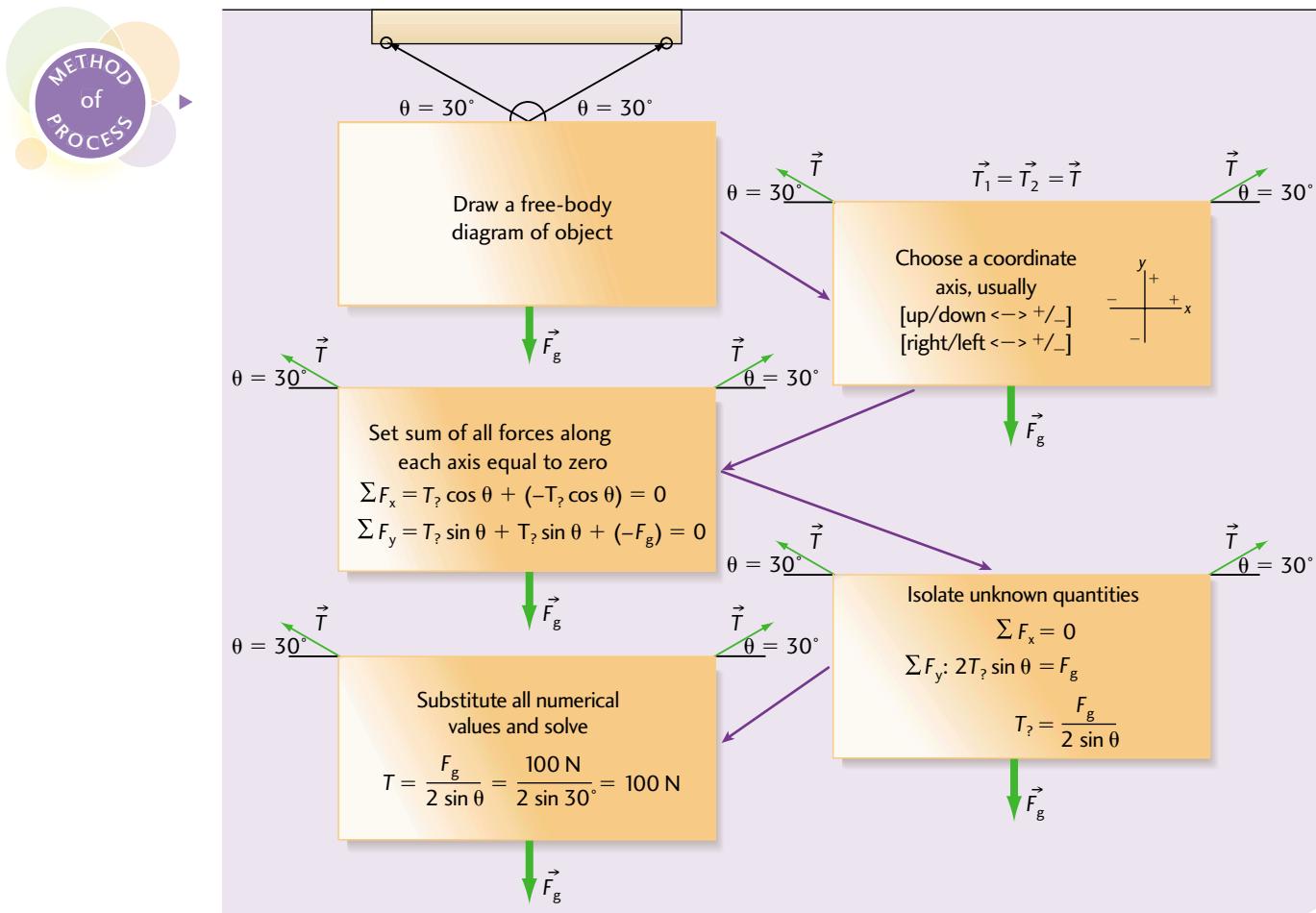
$$F_{\text{extra}} = \frac{35.0 \text{ N}}{2 \cos \theta}$$

$$F_{\text{extra}} = 24.7 \text{ N}$$

Child 4 and child 6 each need to apply an additional force of 24.7 N in order to balance the force applied by the teacher. Therefore, they each apply a total force of $35.0 \text{ N} + 24.7 \text{ N} = 59.7 \text{ N}$.

Figure 3.10 summarizes the procedure for solving translational equilibrium problems.

Fig.3.10 Procedure for Solving Translational Equilibrium Problems



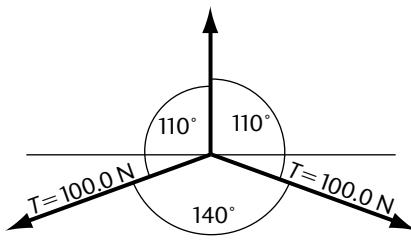
Static equilibrium occurs when forces such as tension/compression, gravity, friction, magnetism, electrostatics, and even elastic forces involving Hooke's law directed through the centre of mass are all balanced. The following problems illustrate some of these possibilities.

1. A guy wire with a tension of 1.0×10^4 N and at an angle of 60° from the ground is attached to the top of a hydroelectric pole, as shown in Figure 3.11. What are the horizontal and vertical components of the force exerted by the wire at the top of the pole in order to maintain the system in static equilibrium?
2. A canoe is tethered to a car with ropes, as shown in Figure 3.12a. What is the tension in the vertical rope if the junction (assumed massless) is held at static equilibrium by the two lower ropes, each with a tension of 100.0 N?

Fig.3.12a



Fig.3.12b



3. A bag of clothespins hung in the middle of a 3.00-m clothesline causes the line to dip 1.5° below the horizontal at each end.
 - a) Draw a free-body diagram for this situation.
 - b) How far does the centre of the line dip when the bag of clothespins is hung on it?
 - c) What is the mass of the bag of clothespins if the tension in the line is 85.0 N?
4. Forces can also be applied by compressing or tensing a rigid object such as a beam. Two beams support a 4.0-kg pail of water above an open well, as shown in Figure 3.13.
 - a) How much compression force is exerted on each beam by the water pail?
 - b) What outward force do the two beams exert on the well's wall?
 - c) What additional vertical compression is exerted on the bricks under the beams?
5. Review some of the Applying the Concepts problems or end-of-chapter exercises in this chapter and find at least three cases of static equilibrium. If you are having difficulty, change some of the variables so that the situation represents a static situation.



Fig.3.11

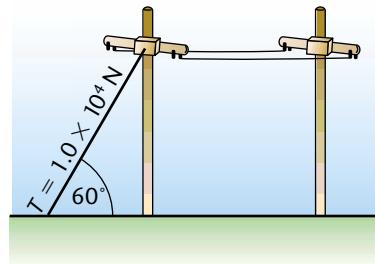
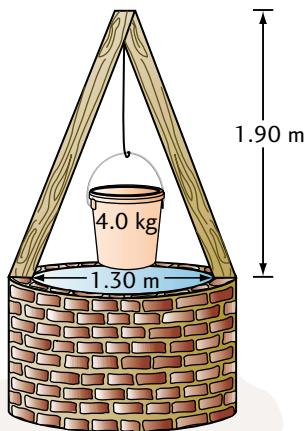


Fig.3.13



6. A boat of mass 400.0 kg is on a trailer at an angle of 30° , as shown in Figure 3.14. There is a coefficient of static friction of 0.25 between the boat and the trailer rollers. What must be the tension in the cable to keep the boat in static equilibrium?

Fig. 3.14

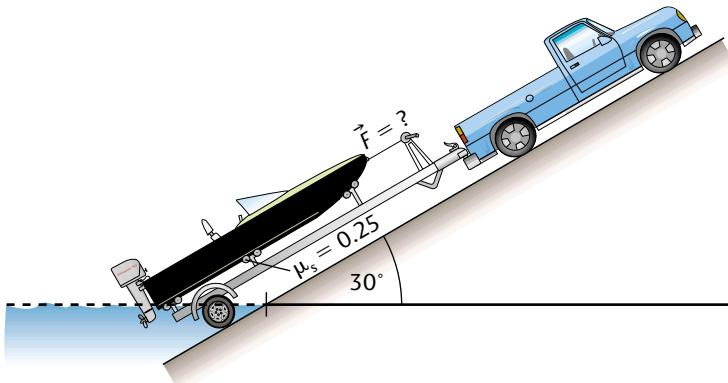


Fig. 3.15 Forces not directed through the centre of mass from a sweep stroke tend to cause rotation rather than translation

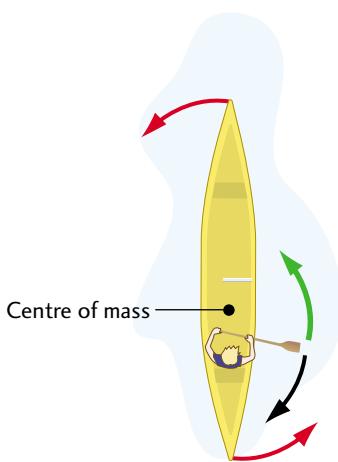
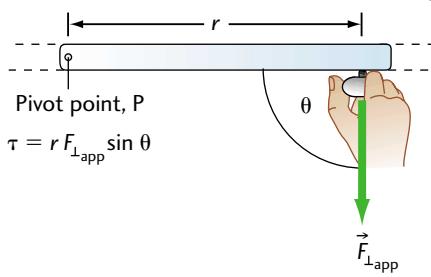


Fig. 3.16



3.4 Balancing Torques

When a force applied on an object isn't directed through the object's centre of mass, then the force *rotates* the object as well as translating it. Figure 3.15 shows how a force could either translate or rotate our canoe, depending on where along the canoe it's applied. For example, a large sweep stroke will rotate the canoe more than a long straight stroke applied close to the body of the canoe.

The rotational effect caused by a force is called **torque**, τ , or **moment of force**. You apply torque when you open a bottle of water, tighten a screw, turn on a water tap, or turn the steering wheel of your car. To examine the factors that affect torque, let's consider the case of opening a stiff door. We know that in order to open this door, we need to apply a force. But where on the door should we apply it and in which direction? Let's do a quick demonstration. Stand this textbook on its bottom edge and open the front cover 90° . Now open and close the front cover by applying forces at different positions and at different angles. Pushing the cover at 90° to the surface at a position farthest away from the pivot point (the spine) rotates the cover with a minimum effort. Similarly, the placement of a doorknob farthest away from the hinges minimizes the force required because it maximizes the torque applied (see Figure 3.16).

If the applied force is *not* at 90° to the radius, we need to substitute its perpendicular component, $F \sin \theta$, where θ is the angle between the applied force and the radius (see Figure 3.17); that is, $F_{\perp app} = F_{app} \sin \theta$.

Torque τ or moment of force is given by the expression

$$\tau = rF_{\perp \text{app}} = rF_{\text{app}} \sin \theta$$

where r is the perpendicular distance (in metres) between the place where the force is applied and the pivot point (point of rotation), $F_{\perp \text{app}}$ is the applied force, in newtons, at 90° to the surface, θ is the angle between the surface and the applied force, and τ is the torque in newton-metres (N·m).

Torque is a vector quantity because it is the cross product of the vector quantities \vec{r} and \vec{F} .

$$\vec{\tau} = \vec{r} \times \vec{F}$$

By definition, the direction of the cross product is perpendicular to the plane defined by the two base vectors, \vec{r} and \vec{F} (see Figure 3.18). The direction of $\vec{\tau}$ can be found using the right-hand rule, as described in Figure 3.19. Even though the right-hand rule can be used to describe the direction of the torque vector, in calculations involving a constant axis of rotation in one plane, direction is not required. We will use the clockwise direction for positive (+) rotation and the counterclockwise direction for negative (-) rotation. Let's use a door example to illustrate how the torque equation and its direction are applied.

EXAMPLE 2

Calculating torque

Hannah, a lively collie, wants to go outside. She pushes the door with a 45.0-N force at an angle of 5° from the perpendicular, 60.0 cm from the hinges. What perpendicular force is she applying to the door and what is the final torque?

Solution and Connection to Theory

Given

$$F_{\text{app}} = 45.0 \text{ N} \quad r = 60.0 \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 0.600 \text{ m}$$

$$\theta = 90^\circ - 5^\circ = 85^\circ$$

$$F_{\perp \text{app}} = F_{\text{app}} \sin \theta$$

$$F_{\perp \text{app}} = (45.0 \text{ N}) \sin 85^\circ$$

$$F_{\perp \text{app}} = 44.8 \text{ N}$$

The perpendicular force that Hannah applies is 44.8 N.

To calculate the torque Hannah applies to the door, we use the equation
 $\tau = rF_{\text{app}} \sin \theta$

Fig.3.17

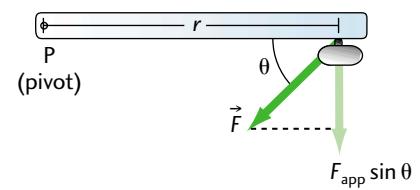


Fig.3.18 The cross product vector is 90° to the plane described by the two vectors involved (\vec{F} and \vec{r})

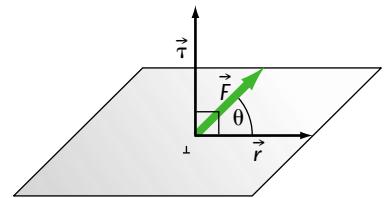
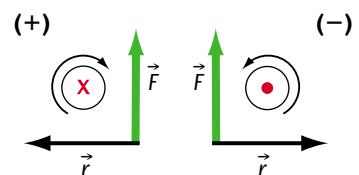


Fig.3.19 The Right-hand Rule for the Direction of Torque

- 1) With the two vectors, \vec{r} and \vec{F} , placed tail to tail, point the fingers of your right hand in the direction of \vec{r} .
- 2) Rotate your right hand so that the palm is pointing toward the force vector, \vec{F} .
- 3) The thumb points in the direction of the torque, $\vec{\tau}$.

⊗ (into page) represents fletching (the feathers at the end of an arrow) moving away from us. ⊖ (out of page) represents the tip of the arrow coming toward us.



The unit for torque, N·m, is dimensionally equivalent to the unit for work, the joule. Work is the dot product while torque is the cross product. To discern between torque and work, the unit N·m is used for torque and the unit J (joule) is used for work, even though both units are equivalent.

Substituting the given values, we obtain

$$\tau = (0.600 \text{ m})(45.0 \text{ N})\sin 85^\circ$$

$$\tau = 26.9 \text{ N}\cdot\text{m}$$

Therefore, Hannah applies a torque of 26.9 N·m to the door in the direction that the door rotates to open.

For the examples in this text, we will assume that the twist or direction of torque is in the same plane as a page of text. In the next example, we will illustrate how the direction of torques is applied.

EXAMPLE 3

Calculating total torque

Two girls are applying torque to a steering wheel (40.0 cm in diameter) of a bumper car during an amusement park ride. The girl on the left applies a force of 10.0 N [up], while the girl on the right pulls directly down with a force of 15.0 N. What net torque are both girls applying to the steering wheel?

Solution and Connection to Theory

Given

$$r = 0.200 \text{ m} \text{ (for both girls)} \quad \vec{F}_{\text{girl1}} = 10.0 \text{ N [up]}$$
$$\vec{F}_{\text{girl2}} = 15.0 \text{ N [down]} \quad \tau = ?$$

The torques being applied by both girls are turning the steering wheel clockwise in the same plane as the steering wheel. We will consider the direction of the torque to be clockwise and positive. We can now add the individual torques of the two girls to find the total torque on the steering wheel.

$$\vec{\tau}_{\text{total}} = \vec{\tau}_{\text{girl1}} + \vec{\tau}_{\text{girl2}}$$

$$\tau_{\text{total}} = rF_{\text{girl1}} \sin \theta + rF_{\text{girl2}} \sin \theta$$

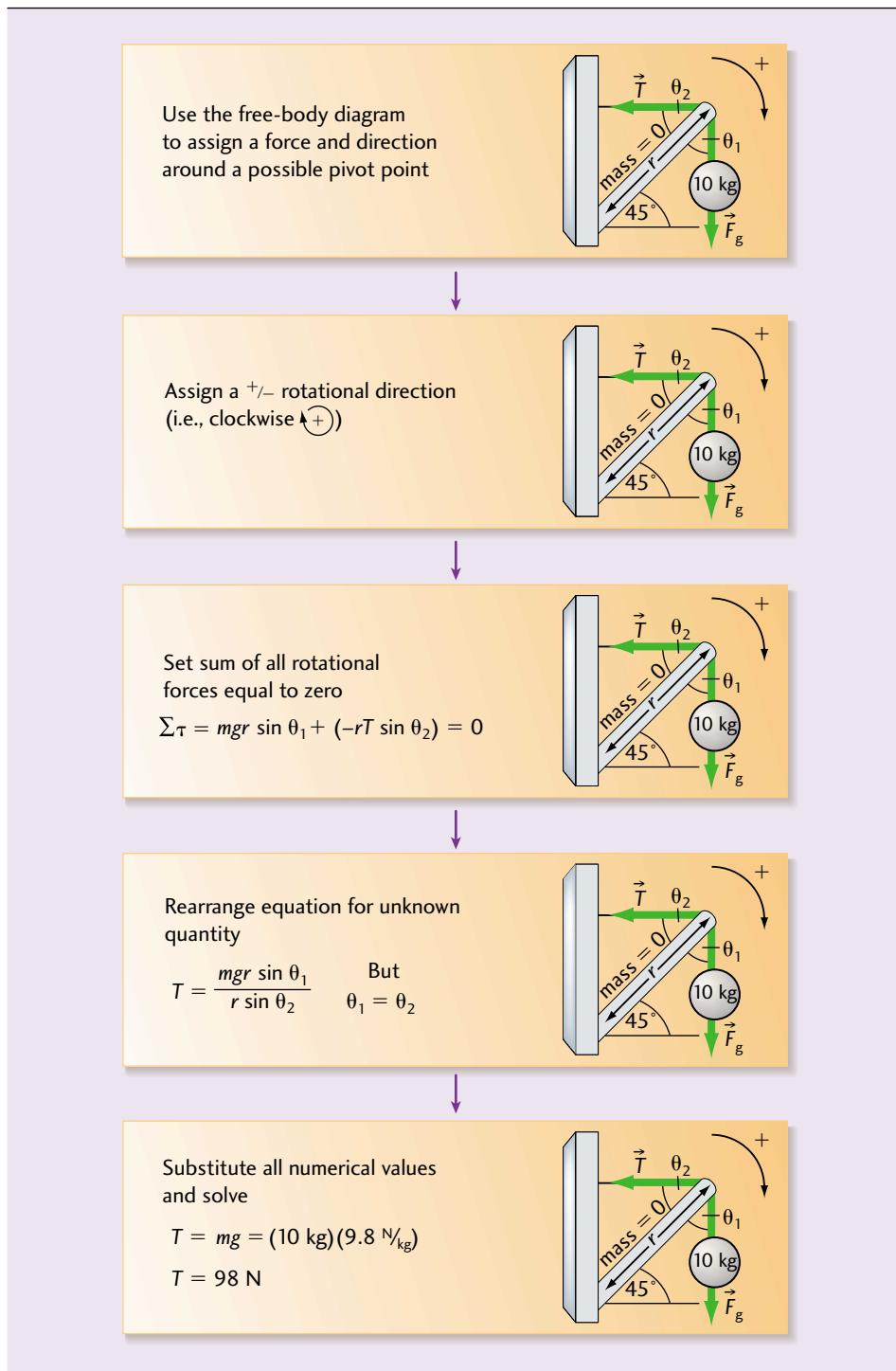
$$\tau_{\text{total}} = (0.200 \text{ m})(10.0 \text{ N})\sin 90^\circ + (0.200 \text{ m})(15.0 \text{ N})\sin 90^\circ$$

$$\tau_{\text{total}} = 5.0 \text{ N}\cdot\text{m}$$

The total torque applied to the steering wheel by both girls is 5.0 N·m. Because it is a positive number, the torque is in the clockwise direction.

Figure 3.20 summarizes the procedure for solving rotational equilibrium problems.

Fig.3.20 Procedure for Solving Rotational Equilibrium Problems





- The trunk of an old Cadillac Eldorado, 1.50 m long and open to an angle of 50° above the horizontal, can only be closed by someone with a mass greater than 45.0 kg hanging vertically from the end of the open lid.
 - Draw a labelled diagram of this situation. Be sure to note any forces applied.
 - What is the minimum amount of torque required to close this trunk?
- The locks at Port Severn, Ontario, are operated by lock personnel who manually push cranks, as shown in Figure 3.21.

Fig. 3.21 The manual locks at Port Severn, Ontario

(a)



(b)

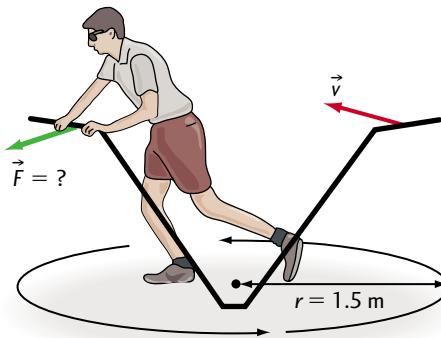
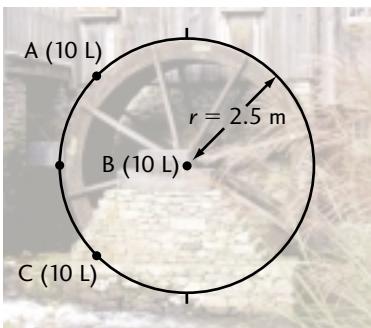


Fig. 3.22

(a)



(b)



- If the torque required to start turning the lock mechanism is $2.0 \times 10^3 \text{ N}\cdot\text{m}$, what force must be exerted by one lock operator?
- The two arms on the mechanism are for two operators. What would the addition of a second operator do for the torque required to operate the lock? Of what benefit would a second operator be for
 - the original operator?
 - the average boater waiting for the lock?
- A water wheel is an engine/turbine that is driven by falling water. The torque is generated by water filling compartments on one side of the wheel (see Figure 3.22) that are pulled down by the force of gravity. The wheel has an effective radius of 2.5 m, and each of the eight equally spaced compartments holds 10.0 L of water.
 - What is the force of gravity acting on each of the water compartments?
 - Which position, A, B, or C, provides the most torque to turn the wheel?
 - What torque is produced at each of the three positions?
 - The total torque is the sum of the individual torques. How could you increase the total torque applied to this water wheel?

3.5 Static Equilibrium: Balancing Forces and Torque

We know that forces tend to cause either translational or rotational motion, depending on the direction and position of the force applied with respect to the centre of mass. Now we turn our attention to one specific effect of an application of force: static equilibrium, or no motion at all. To achieve true static equilibrium, two conditions must be met. First, to avoid translation (moving from place to place), the net force directed through the centre of mass of the object must be zero. Second, to avoid rotation, the net torque on the object must also be zero. These two conditions for static equilibrium are summarized in Table 3.1.

Table 3.1
Conditions for Static Equilibrium — Summary

Condition			
1 $\vec{F}_{\text{net}} = 0$ $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = 0$	If $\vec{F}_{\text{net}} = 0$, there is no translational acceleration/motion		
2 $\vec{\tau}_{\text{net}} = 0$ $\vec{\tau}_{\text{net}} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \dots + \vec{\tau}_n = 0$	If $\vec{\tau}_{\text{net}} = 0$, there is no rotational motion		

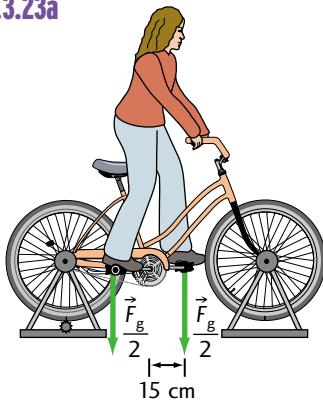
In this text, torque direction is not required.

The application of both these conditions simultaneously is useful in solving more complicated problems. Let's do some examples.

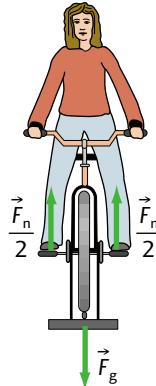
EXAMPLE 4 Static equilibrium

While training on a stationary bicycle, an athlete takes a break by just standing on the pedals, as shown in Figure 3.23a. Her 384-N weight is applied equally on the two pedals, each with a radius of 15.0 cm.

Fig. 3.23a



- a) How do we know that the first condition for static equilibrium (no translation) has been met?
- b) Show that the second condition for static equilibrium (no net rotation of the pedals and crank) has been met.

Fig.3.23b

Solution and Connection to Theory

Given

$$F_g = 384 \text{ N} \quad r = 15.0 \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 0.150 \text{ m} \quad \tau_{\text{net}} = ?$$

- a)** From the problem statement, we know that the athlete is neither falling vertically to the ground, nor rising any higher. Therefore, we can assume that the force of gravity is balanced by the normal force. Let's consider up as positive. The relationship between the vertical forces acting on the pedals and crank is shown in Figure 3.23b. We can substitute these forces into the equation:

$$\vec{F}_{\text{net}} = \vec{F}_n + \vec{F}_g$$

$$F_{\text{net}} = F_n - F_g$$

$$F_{\text{net}} = 384 \text{ N} - 384 \text{ N}$$

$$F_{\text{net}} = 0 \text{ N}$$

The two forces have the same magnitude but the opposite direction; therefore, the net force on the athlete is zero.

- b)** To find the total torque, we need to define a positive rotational direction (clockwise) around a pivot point. The crankshaft between the pedals is the point of rotation for this mechanism. The total torque on the bicycle can be written as

$$\vec{\tau}_{\text{net}} = \vec{\tau}_1 + \vec{\tau}_2$$

$$\tau_{\text{net}} = \tau_{\rightarrow} - \tau_{\leftarrow}$$

$$\tau_{\text{net}} = rF_{\rightarrow}\sin\theta - rF_{\leftarrow}\sin\theta$$

$$\tau_{\text{net}} = (0.150 \text{ m})(192 \text{ N})\sin 90^\circ - (0.150 \text{ m})(192 \text{ N})\sin 90^\circ$$

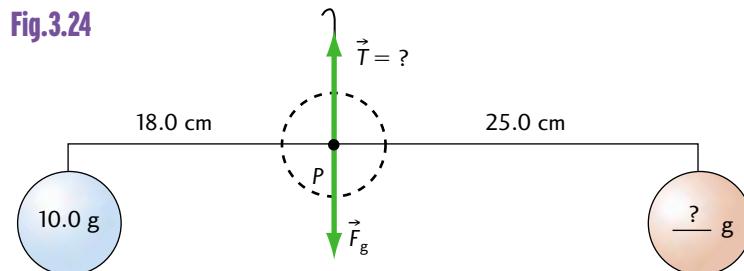
$$\tau_{\text{net}} = 28.8 \text{ N}\cdot\text{m} - 28.8 \text{ N}\cdot\text{m}$$

$$\tau_{\text{net}} = 0 \text{ N}\cdot\text{m}$$

The net torque on the pedals due to the athlete is zero. In other words, the pedals don't turn if the athlete's weight is balanced between them.

EXAMPLE 5 The torque involved in balancing a mobile

A mobile is an artistic piece with figures hanging on it from horizontally balanced massless rods, as shown in Figure 3.24. The 10.0-g figure on the left at 18.0 cm from the pivot point balances another figure at 25.0 cm from the pivot point to maintain static equilibrium of the mobile.

Fig.3.24

- a)** What is the mass of the figure on the right?
b) What is the tension in the single string supporting the entire mobile?

Solution and Connection to Theory

Given

$$m_{\text{left}} = 10.0 \text{ g} \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) = 0.010 \text{ kg} \quad m_{\text{right}} = ?$$

$$r_{\text{left}} = 18.0 \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 0.180 \text{ m} \quad r_{\text{right}} = 25.0 \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 0.250 \text{ m}$$

- a)** If the positive rotational direction is clockwise, then the net torque on the mobile can be written as

$$\vec{\tau}_{\text{net}} = \vec{\tau}_1 + \vec{\tau}_2$$

Because the mobile isn't rotating, we can assume that all torques are balanced. Therefore, we can write

$$\tau_{\text{net}} = \tau_{\rightarrow} + (-\tau_{\leftarrow}) = 0$$

$$\tau_{\text{net}} = rF_{\rightarrow} \sin \theta - rF_{\leftarrow} \sin \theta = 0$$

Substituting the given values into the equation, we obtain

$$\tau_{\text{net}} = (0.250 \text{ m})(F_{\rightarrow}) \sin 90^\circ - (0.180 \text{ m})(0.010 \text{ kg})(9.8 \text{ N/kg}) \sin 90^\circ = 0$$

$$\tau_{\text{net}} = (0.250 \text{ m})(F_{\rightarrow}) - 0.0176 \text{ N}\cdot\text{m} = 0$$

In order for the condition of static equilibrium to be met, τ_{net} must equal zero. Therefore,

$$(0.250 \text{ m})(F_{\rightarrow}) - 0.0176 \text{ N}\cdot\text{m} = 0$$

$$F_{\rightarrow} = \frac{0.0176 \text{ N}\cdot\text{m}}{0.250 \text{ m}}$$

$$F_{\rightarrow} = 7.04 \times 10^{-2} \text{ N}$$

F_{\rightarrow} is positive; therefore, its direction is clockwise. So,

$$F_{\rightarrow} = 7.04 \times 10^{-2} \text{ N} [\text{clockwise}]$$

Now that we know the force applied to the figure on the right, we can calculate its mass using the equation

$$F = mg$$

$$F_{\rightarrow} = m_{\text{right}} g$$

$$m_{\text{right}} = \frac{F_{\rightarrow}}{g}$$

$$m_{\text{right}} = \frac{7.04 \times 10^{-2} \text{ N}}{9.8 \text{ N/kg}}$$

$$m_{\text{right}} = 7.2 \times 10^{-3} \text{ kg or } 7.2 \text{ g}$$

Therefore, a 7.2 g figure balances the mobile.

b) No translation means that all forces are balanced. From Figure 3.24,

$$\vec{T} + \vec{F}_g = 0$$

$$T - m_T g = 0$$

$T = m_T g$, where up is positive.

$$T = (10.0 \times 10^{-3} \text{ kg} + 7.2 \times 10^{-3} \text{ kg})(9.8 \text{ N/kg})$$

$$T = 0.236 \text{ N}$$

The tension in the string is 0.236 N [up].

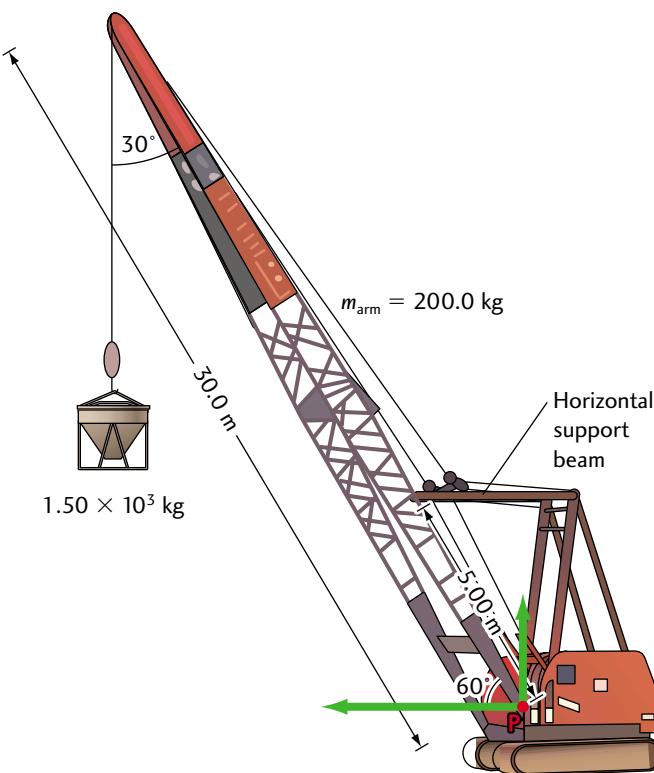
Not all situations involve objects that are horizontal when in static equilibrium, like the pedals on our stationary bike or the mobile. The next example shows how the forces and torques of objects being lifted by a construction crane change.

EXAMPLE 6

Torques applied on a construction crane

A 30.0-m-long construction crane of mass 200.0 kg is supported at an angle of 60° above the horizontal by a horizontal support beam 5.00 m from its base, as shown in Figure 3.25.

Fig.3.25



The mass of $1.50 \times 10^3 \text{ kg}$ hanging from the crane and the crane's arm are held in static equilibrium by a torque provided by the support beam and a force applied at the base. Find the tension in the support beam and the vertical and horizontal reaction forces at the base of the crane's arm.

Solution and Connection to Theory

Given

$$L_{\text{arm}} = 30.0 \text{ m} \quad m_{\text{arm}} = 200.0 \text{ kg} \quad m_{\text{load}} = 1.50 \times 10^3 \text{ kg}$$
$$r_{\text{beam}} = 5.00 \text{ m} \quad \theta = 90^\circ - 60^\circ = 30^\circ$$

The first step in solving this type of problem is to decide which force to calculate first. All forces tend to cause translation and rotation, and the rotational forces can be determined from the torques applied.

Let's choose the base of the crane's arm as our pivot point. From Figure 3.25, we can identify three torques: 1) the torque due to the weight of the crane's arm (counterclockwise), 2) the torque due to the load weight (counterclockwise), and 3) the torque of the horizontal support beam (clockwise). Because the crane's arm is in static equilibrium (as stated in the problem), all of the torques acting on it are balanced. Therefore, we can write the statement

$$\vec{\tau}_{\text{net}} = \vec{\tau}_{\text{arm}} + \vec{\tau}_{\text{load}} + \vec{\tau}_{\text{beam}} = 0$$

The next step is to isolate each torque acting around the pivot point. Let's consider the centre of mass of the crane's arm to be a single point in the centre of the arm. Taking counterclockwise to be a positive rotation,

$$\tau_{\text{arm}} = r_{\text{arm}}(m_{\text{arm}})g \sin \theta$$

$$\tau_{\text{arm}} = (15.0 \text{ m})(200.0 \text{ kg})(9.8 \text{ N/kg}) \sin 30^\circ$$

$$\tau_{\text{arm}} = 1.470 \times 10^4 \text{ N}\cdot\text{m}$$

$$\tau_{\text{load}} = r_{\text{load}}(m_{\text{load}})g \sin \theta$$

$$\tau_{\text{load}} = (30.0 \text{ m})(1.50 \times 10^3 \text{ kg})(9.8 \text{ N/kg}) \sin 30^\circ$$

$$\tau_{\text{load}} = 2.205 \times 10^5 \text{ N}\cdot\text{m}$$

Therefore, the total counterclockwise torque on the arm is

$$1.470 \times 10^4 \text{ N}\cdot\text{m} + 2.205 \times 10^5 \text{ N}\cdot\text{m} = 2.352 \times 10^5 \text{ N}\cdot\text{m}$$

For static equilibrium, the support beam must provide a clockwise (negative) torque equal in magnitude to the total counterclockwise torque of $2.35 \times 10^5 \text{ N}\cdot\text{m}$ to keep the arm from rotating such that

$$\tau_{\text{net}} = \tau_{\leftarrow} + (-\tau_{\rightarrow}) = 0$$

$$\tau_{\leftarrow} = \tau_{\rightarrow}$$

$$r_{\text{beam}} F_{\text{beam}} \sin \theta = \tau_{\rightarrow}$$

$$F_{\text{beam}} = \frac{\tau_{\rightarrow}}{r_{\text{beam}} \sin \theta}$$

$$F_{\text{beam}} = \frac{2.35 \times 10^5 \text{ N}\cdot\text{m}}{(5.00 \text{ m}) \sin 60^\circ} \quad (\text{From Figure 3.25, the angle between the support beam and the crane's arm is } 60^\circ.)$$

$$F_{\text{beam}} = 5.43 \times 10^4 \text{ N [horizontal]}$$

The tension in the support beam must be $5.43 \times 10^4 \text{ N}$.

Because there is no translational motion, we can find the horizontal and vertical components of the force on the pivot point at the base of the crane, as illustrated in Figure 3.26a.

Fig.3.26a

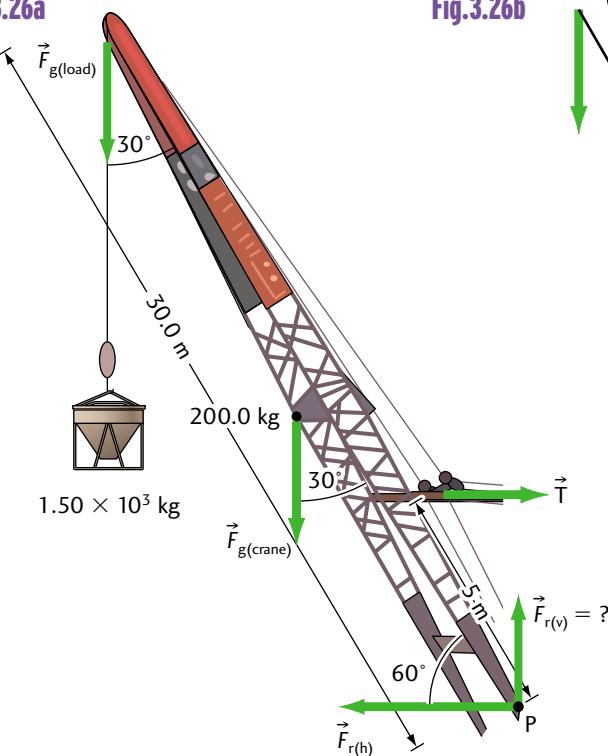
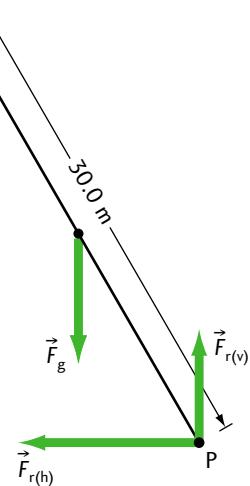


Fig.3.26b



In order for the crane to remain in static equilibrium, both F_h and F_v must balance all other horizontal and vertical forces.

Vertically,

$$\vec{F}_{v(\text{net})} = \vec{F}_{r(v)} + \vec{F}_{g(\text{arm})} + \vec{F}_{g(\text{load})} = 0$$

If up is positive, then

$$F_{r(v)} - F_{g(\text{arm})} - F_{g(\text{load})} = 0$$

$$F_{r(v)} = F_{g(\text{arm})} + F_{g(\text{load})}$$

$$F_{r(v)} = (200 \text{ kg})(9.8 \text{ N/kg}) + (1.50 \times 10^3 \text{ kg})(9.8 \text{ N/kg})$$

$$F_{r(v)} = (1.960 \times 10^3 \text{ N}) + (1.470 \times 10^4 \text{ N})$$

$$F_{r(v)} = 1.666 \times 10^4 \text{ N}$$

Therefore, the vertical force is $1.67 \times 10^4 \text{ N}$ [up].

Horizontally,

$$\vec{F}_{h(\text{net})} = \vec{F}_{r(h)} + \vec{T} = 0$$

If right is positive, then

$$F_{r(h)} + T = 0$$

$$F_{r(h)} = -T$$

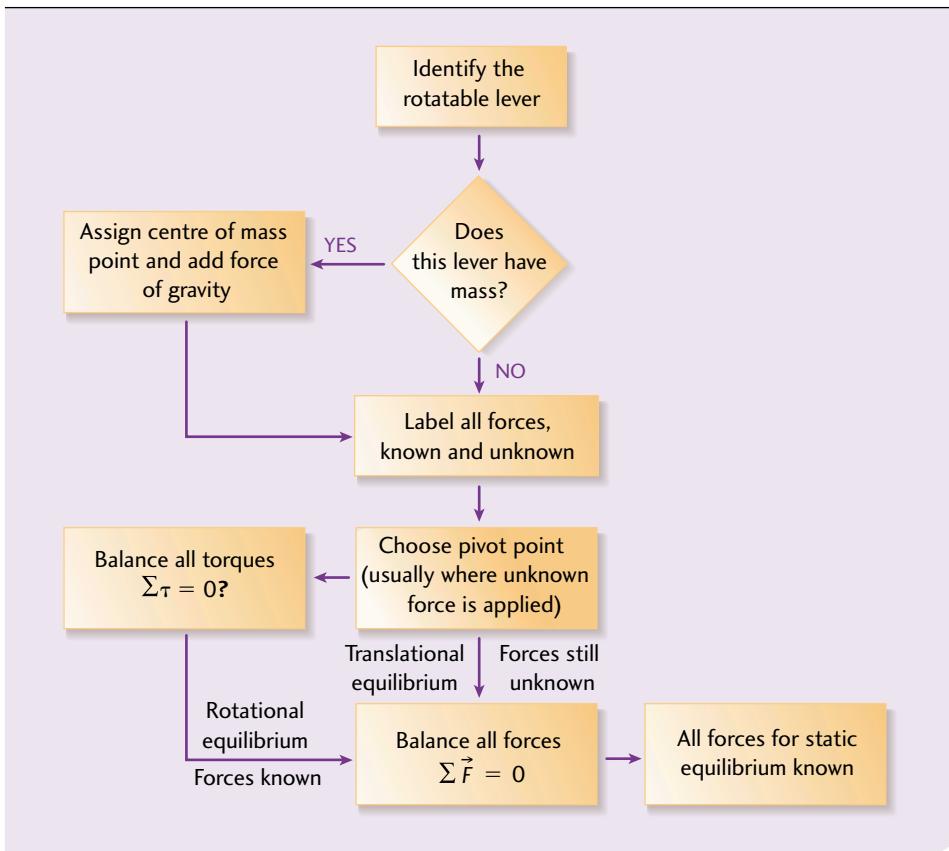
$$F_{r(h)} = -(+5.432 \times 10^4 \text{ N})$$

$$F_{r(h)} = -5.43 \times 10^4 \text{ N}$$

Therefore, the horizontal force applied to the bottom of the crane is $5.43 \times 10^4 \text{ N}$ [left]. Both the vertical and horizontal reaction forces must be applied to the bottom of the crane's arm to stop it from translating in either direction.

Figure 3.27 summarizes the steps for solving torque problems.

Fig.3.27 Method for Solving Torque Problems



In the following example, we will use the concept of static equilibrium to find the centre of mass/gravity of an object.

EXAMPLE 7 The centre of mass of a person

We can determine a person's centre of mass by having the person lie on a specially designed platform supported by two spring scales, as shown in Figure 3.28a.

Fig.3.28a

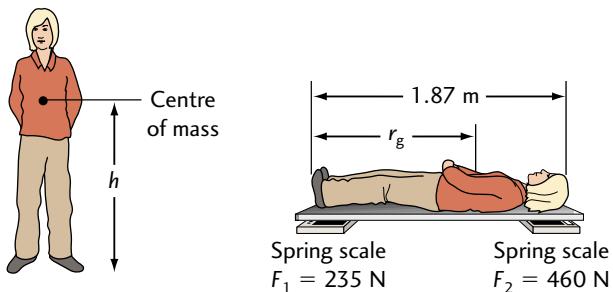
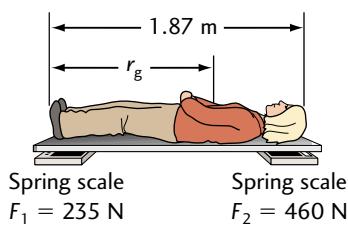


Fig.3.28a

If the left and right scales read 235 N and 460 N, respectively, for a person 1.87 m tall, where is the person's centre of mass?

Solution and Connection to Theory

Given

$$F_1 = 235 \text{ N} \quad F_2 = 460 \text{ N} \quad h = 1.87 \text{ m} \quad r_{g(\text{cm})} = ?$$

Choosing the left end of the lever in Figure 3.28a as the pivot point and clockwise as the positive rotational direction, we can balance the torques from the two spring scale forces [up] and the force of gravity [down]. Because the person is stationary, we can express the net torque as

$$\vec{\tau}_{\text{net}} = \vec{\tau}_{\text{scale1}} + \vec{\tau}_g + \vec{\tau}_{\text{scale2}} = 0$$

where τ_{scale1} and τ_{scale2} are the torques applied at the left and right ends of the lever, respectively, and τ_g is the torque applied by the person's weight through the centre of mass. But $\tau_{\text{scale1}} = 0$ because the distance, r , from the pivot point is zero. Therefore,

$$\vec{\tau}_{\text{net}} = \vec{\tau}_g + \vec{\tau}_{\text{scale2}} = 0 \text{ and}$$

$$\tau_g - \tau_{\text{scale2}} = 0$$

$$r_g F_g \sin \theta = r_{\text{scale2}} F_{\text{scale2}} \sin \theta$$

$$r_g = \frac{r_{\text{scale2}} F_{\text{scale2}} \sin \theta}{F_g \sin \theta}$$

$$\text{But } \sin \theta = \sin 90^\circ = 1$$

Therefore,

$$r_g = \frac{r_{\text{scale2}} F_{\text{scale2}}}{F_g}$$

The total weight of the person is

$$\vec{F}_g = \vec{F}_{\text{scale1}} + \vec{F}_{\text{scale2}}$$

$$F_g = 235 \text{ N} + 460 \text{ N}$$

$$F_g = 695 \text{ N}$$

Substituting the values into our equation, we obtain

$$r_{g(\text{cm})} = \frac{(1.87 \text{ m})(460 \text{ N})}{695 \text{ N}}$$

$$r_{g(\text{cm})} = 1.24 \text{ m}$$

Therefore, the centre of mass/gravity is 1.24 m from the person's feet.

The centre of mass of the human body is the point at which we can consider all our body mass to be concentrated. It is also the point around which the body rotates in free space. Figure 3.28b shows an Olympic diver rotating around her centre of mass.

The idea of body symmetry was captured by Italian Renaissance painter Leonardo da Vinci in his drawing *Vitruvian Man* (Figure 3.28c). The centre of mass of this figure is a point along the midline of the torso.

Fig.3.28b Any object, such as our Olympic diver, that tumbles or rotates in the air pivots around its centre of mass

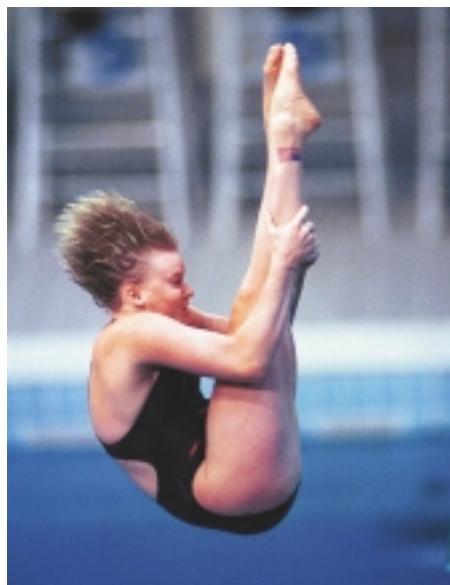
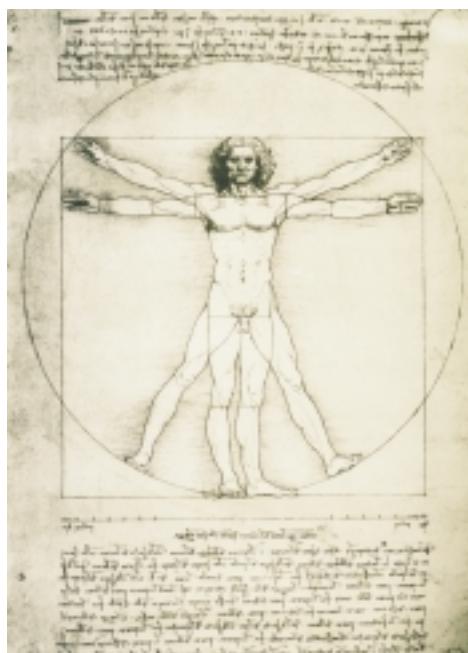


Fig.3.28c Leonardo da Vinci's *Vitruvian Man*



1. A 45.0-kg boy walks along a 3.00-m-long wooden plank of mass 20.0 kg that overhangs a partially completed deck by 0.75 m. How far away from the deck edge can the boy walk before the plank tips?
2. Two children of masses 45 kg and 30 kg are playing on a teeter-totter of length 4.0 m and mass 30.0 kg, pivoted at its centre. The heavier child sits 1.75 m from the centre of the teeter-totter that is 0.50 m from the ground.
 - a) What torque does the teeter-totter apply to each of its sides? With this knowledge, how could you simplify any further calculations?
 - b) Where would the lighter child have to sit from the centre of the teeter-totter to balance properly?
 - c) What percentage of either child's torque is lost between the horizontal position and maximum height?
3. A window of mass 5.00 kg and length 0.75 m is being held open at an angle of 40° from the vertical by a duck pushing it out horizontally from the bottom, as shown in Figure 3.29.
 - a) How much force must be provided to hold the window statically?
 - b) What horizontal and vertical reaction forces must the hinges at the top provide when the window is held open?



Fig.3.29

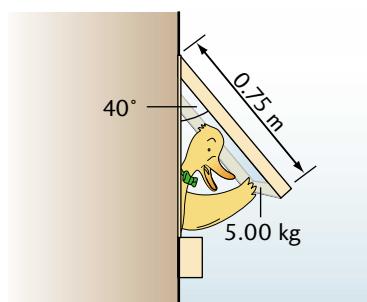
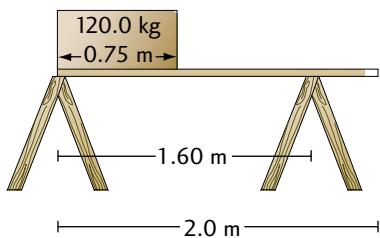


Fig.3.30



4. A block of stone of mass 120.0 kg is being supported by two sawhorses and a rigid plank of mass 5.0 kg, as shown in Figure 3.30. What reaction force must each sawhorse provide to support the block?

3.6 Static Equilibrium and the Human Body

One of the most practical applications of statics is the human body. Our bone structure gives us inherent flexibility of movement at the joints. When muscle fibres contract, they provide **tensile forces** that act through tendons connecting muscle to bone at **points of insertion**. These forces provide torque for our arms, shoulders, knees, and legs to rotate as levers about our joints. The application of simple concepts of static equilibrium in the examples in this section will help us understand the stresses and strains on various parts of our bodies. Table 3.2 presents simplified force-and-pivot drawings for several body parts.

Table 3.2
Force and Pivot Points on the Human Body

Elbow

Fig.3.31a

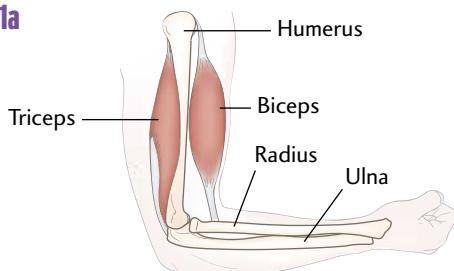
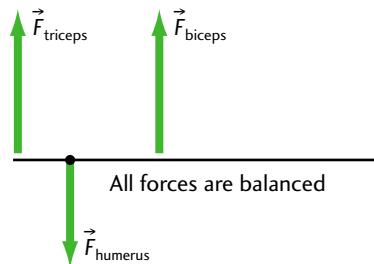


Fig.3.31b



Knee

Fig.3.32a

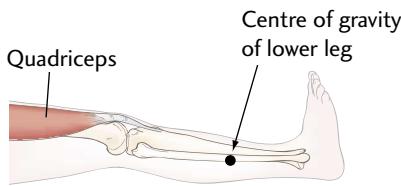


Fig.3.32b

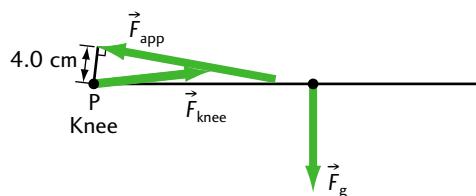


Table 3.2 (cont'd)
Force and Pivot Points on the Human Body

Spine

Fig.3.33a

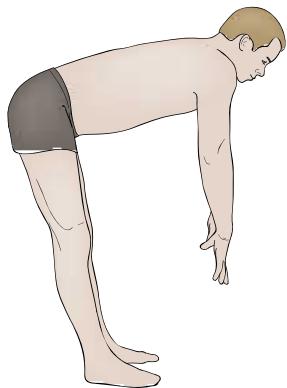
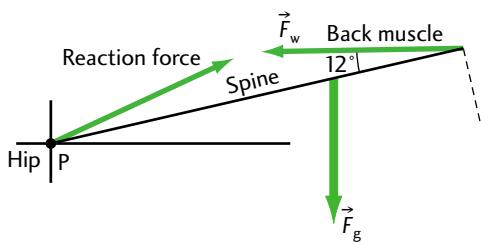


Fig.3.33b



Foot

Fig.3.34a

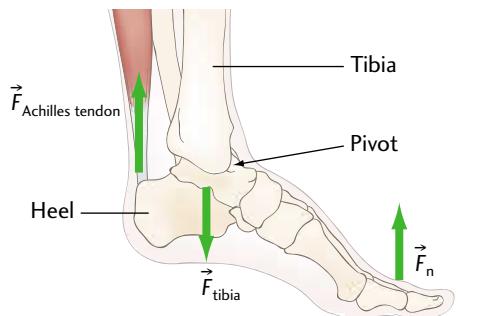
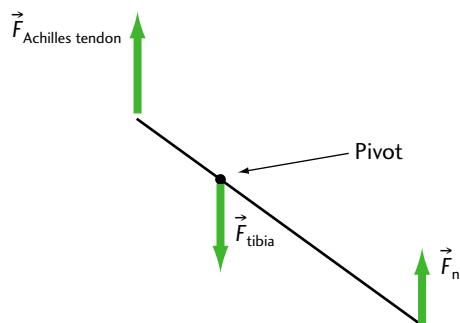


Fig.3.34b



Shoulder

Fig.3.35a

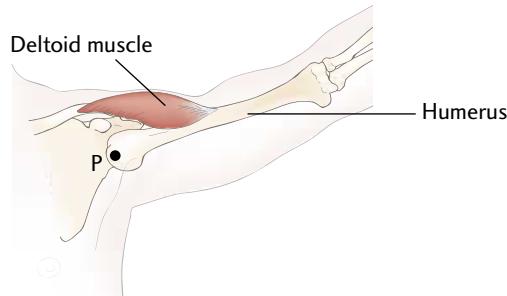
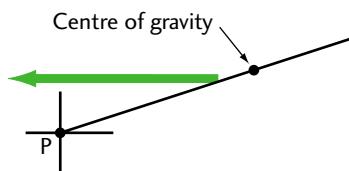


Fig.3.35b



EXAMPLE 8

Torque and weightlifting

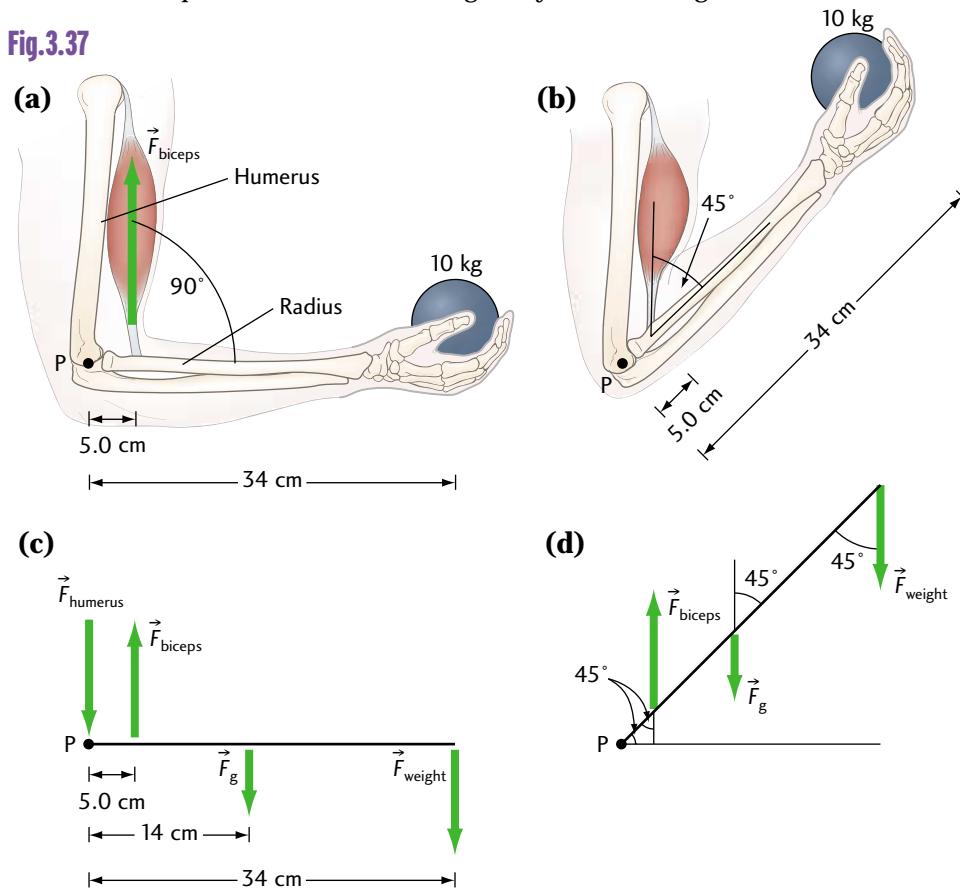
Fig.3.36



“Curls” (Figure 3.36) are a form of weight training that targets the biceps muscle. Figure 3.37a and Figure 3.37b show two different positions of an arm curling a 10-kg mass. The athlete’s forearm has a mass of 2.3 kg and a length of 34 cm from the elbow joint to the palm. The biceps muscle is attached to the bone vertically at a point 5.0 cm from the elbow joint. The centre of mass of the forearm is 14 cm from the elbow.

- Calculate the effort required (force of tension from the biceps muscle) for each orientation of the forearm (Figures 3.37a and b).
- What is the reaction force of the humerus on the radius at the elbow joint when the weight is held in the horizontal position?
- How do the tension on the biceps muscle and the reaction force at the elbow compare with the force of gravity on the weight?

Fig.3.37



Solution and Connection to Theory

Given

$$L_{\text{arm}} = 34 \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 3.4 \times 10^{-1} \text{ m} \quad m_{\text{arm}} = 2.3 \text{ kg}$$

$$m_{\text{weight}} = 10 \text{ kg} \quad r_{\text{biceps}} = 5.0 \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 5.0 \times 10^{-2} \text{ m}$$

$$r_{\text{arm}} = 1.4 \times 10^{-1} \text{ m} \quad r_{\text{weight}} = 34 \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 3.4 \times 10^{-1} \text{ m} \quad \theta = 90^\circ$$

- a) From Figure 3.37c, all torques are balanced around the pivot point (i.e., the elbow):

$$\vec{\tau}_{\text{net}} = \vec{\tau}_{\text{humerus}} + \vec{\tau}_{\text{biceps}} + \vec{\tau}_{\text{arm}} + \vec{\tau}_{\text{weight}} = 0$$

But $\tau_{\text{humerus}} = 0$ because the distance r_{humerus} from the pivot point is zero.

$$\text{So } \vec{\tau}_{\text{net}} = \vec{\tau}_{\text{biceps}} + \vec{\tau}_{\text{arm}} + \vec{\tau}_{\text{weight}} = 0$$

Let's designate the clockwise direction as positive.

$$\tau_{\text{biceps}} + \tau_{\text{arm}} + \tau_{\text{weight}} = 0$$

$$r_{\text{biceps}} F_{\text{biceps}} \sin \theta + r_{\text{arm}} F_{\text{arm}} \sin \theta + r_{\text{weight}} F_{\text{weight}} \sin \theta = 0$$

$$F_{\text{biceps}} = \frac{(-r_{\text{arm}} F_{\text{arm}} \sin \theta - r_{\text{weight}} F_{\text{weight}} \sin \theta)}{r_{\text{biceps}} \sin \theta}$$

But $\sin 90^\circ = 1$; therefore,

$$F_{\text{biceps}} = \frac{-(r_{\text{arm}} m_{\text{arm}} g + r_{\text{weight}} m_{\text{weight}} g)}{r_{\text{biceps}}}$$

$$F_{\text{biceps}} = \frac{-[(1.4 \times 10^{-1} \text{ m})(2.3 \text{ kg})(9.8 \text{ N/kg}) + (3.4 \times 10^{-1} \text{ m})(10 \text{ kg})(9.8 \text{ N/kg})]}{5.0 \times 10^{-2} \text{ m}}$$

$$F_{\text{biceps}} = -730 \text{ N or } -7.3 \times 10^2 \text{ N}$$

The force that the biceps muscle applies is $7.3 \times 10^2 \text{ N}$ counter-clockwise or up.

Similarly, if the weight is held up at $\theta = 45^\circ$, then

$$F_{\text{biceps}} = \frac{(-r_{\text{arm}} F_{\text{arm}} \sin \theta - r_{\text{weight}} F_{\text{weight}} \sin \theta)}{r_{\text{biceps}} \sin \theta}$$

But $\sin 45^\circ = 0.7071$; therefore,

$$F_{\text{biceps}} = \frac{-\cancel{\sin \theta}(r_{\text{arm}} m_{\text{arm}} g + r_{\text{weight}} m_{\text{weight}} g)}{r_{\text{biceps}} \cancel{\sin \theta}}$$

$$F_{\text{biceps}} = \frac{-(r_{\text{arm}} m_{\text{arm}} g + r_{\text{weight}} m_{\text{weight}} g)}{r_{\text{biceps}}}$$

This expression is identical to the expression for the horizontal case ($\theta = 90^\circ$); therefore, the tension, F_{biceps} , is independent of the angle of the curl.

- b)** The reaction force of the humerus on the radius at the elbow is found by balancing all the vertical forces:

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_{\text{humerus}} + \vec{F}_{\text{biceps}} + \vec{F}_{g(\text{arm})} + \vec{F}_{g(\text{weight})} = 0 \\ F_{\text{humerus}} &= -F_{\text{biceps}} - F_{g(\text{arm})} - F_{g(\text{weight})} \\ F_{\text{humerus}} &= -(F_{\text{biceps}} + m_{\text{arm}} g + m_{\text{weight}} g) \\ F_{\text{humerus}} &= -[-7.3 \times 10^2 \text{ N} + (2.3 \text{ kg})(9.8 \text{ N/kg}) + (10 \text{ kg})(9.8 \text{ N/kg})] \\ F_{\text{humerus}} &= +609 \text{ N or } 6.1 \times 10^2 \text{ N [down]}\end{aligned}$$

The humerus pushes down on the radius at the elbow with a force of $6.1 \times 10^2 \text{ N}$.

- c)** The force of gravity on the weight is $(10 \text{ kg})(9.8 \text{ N/kg}) = 98 \text{ N}$. The tension in the biceps muscle is $\frac{7.3 \times 10^2 \text{ N}}{98 \text{ N}} = 7.4$ times the force of the weight alone. The reaction force is $\frac{6.1 \times 10^2 \text{ N}}{98 \text{ N}} = 6.2$ times the force of the weight alone. No wonder that even mild physical activity can cause muscle damage!

EXAMPLE 9 Torque on outstretched arms

Just holding our arms straight out at our sides from our shoulders can provide a good isometric workout for our shoulders (deltoid muscles). A ballerina holds her 3.9-kg arm out horizontally such that her arm's centre of mass is 34 cm from her shoulder joint. Her deltoid muscle is attached to her arm 14 cm from the joint and pulls her arm upward 17° above the horizontal.

- a)** What is the tension in her deltoid muscle?
b) What is the reaction force (magnitude and direction) of her shoulder on her humerus (arm bone)?

Solution and Connection to Theory

Given

$$m_{\text{arm}} = 3.9 \text{ kg} \quad r_{\text{arm}} = 0.34 \text{ m} \quad r_{\text{deltoid}} = 0.14 \text{ m} \quad \theta_{\text{deltoid}} = 17^\circ \quad \theta_{\text{arm}} = 90^\circ$$

- a)** Once again, we use all the parameters to create a free-body diagram of the humerus (Figure 3.38).

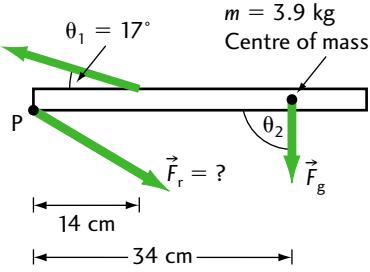
We then balance all the torques around the pivot point (shoulder):

$$\vec{\tau}_{\text{net}} = \vec{\tau}_{\text{shoulder}} + \vec{\tau}_{\text{deltoid}} + \vec{\tau}_{\text{arm}} = 0$$

But $\tau_{\text{shoulder}} = 0$ because the distance r_{shoulder} from the pivot point is zero. Therefore,

$$\vec{\tau}_{\text{net}} = \vec{\tau}_{\text{deltoid}} + \vec{\tau}_{\text{arm}} = 0$$

Fig. 3.38



Let's designate the clockwise direction as positive rotation. If the deltoid rotates clockwise and the weight of the arm rotates in the opposite direction, then

$$\tau_{\text{arm}} - \tau_{\text{deltoid}} = 0$$

$$r_{\text{arm}} F_{\text{arm}} \sin \theta_2 - r_{\text{deltoid}} F_{\text{deltoid}} \sin \theta_1 = 0$$

$$F_{\text{deltoid}} = \frac{r_{\text{arm}} F_{\text{arm}} \sin \theta_2}{r_{\text{deltoid}} \sin \theta_1}$$

$$F_{\text{deltoid}} = \frac{(3.4 \times 10^{-1} \text{ m})(3.9 \text{ kg})(9.8 \text{ N/kg})(\sin 90^\circ)}{(1.4 \times 10^{-1} \text{ m})(\sin 17^\circ)}$$

$$\vec{F}_{\text{deltoid}} = 317 \text{ N or } 3.2 \times 10^2 \text{ N [left } 17^\circ \text{ up]}$$

The tension on the deltoid muscle is $3.2 \times 10^2 \text{ N}$. This force is equivalent to the force required to lift a 30-kg mass directly, even though the mass of the arm is only about $\frac{1}{10}$ of this mass.

- b)** To find the reaction force of the shoulder on the arm (\vec{F}_r), we must balance the forces in the vertical and horizontal directions. In the vertical direction, let's designate up as positive. Then,

$$\vec{F}_{\text{net}} = \vec{F}_{r(v)} + \vec{F}_{\text{deltoid}} + \vec{F}_{g(\text{arm})} = 0$$

$$F_{r(v)} = -F_{\text{deltoid}(v)} + F_{g(\text{arm})}$$

$$F_{r(v)} = -F_{\text{deltoid}} \sin \theta_{\text{deltoid}} + m_{\text{arm}} g \sin \theta_{\text{arm}}$$

$$F_{r(v)} = -(3.2 \times 10^2 \text{ N})(\sin 17^\circ) + (3.9 \text{ kg})(9.8 \text{ N/kg})$$

$$\vec{F}_{r(v)} = -55 \text{ N or } 55 \text{ N [down]}$$

In the horizontal direction, let's designate right as positive. Then,

$$\vec{F}_{\text{net}} = \vec{F}_{r(h)} + \vec{F}_{\text{deltoid}(h)} = 0$$

$$F_{r(h)} = F_{\text{deltoid}(h)}$$

$$F_{r(h)} = (F_{\text{deltoid}} \cos \theta_{\text{deltoid}})$$

$$F_{r(h)} = (3.2 \times 10^2 \text{ N})(\cos 17^\circ)$$

$$\vec{F}_{r(h)} = 306 \text{ N [right]}$$

Combining both vertical and horizontal components of force in Figure 3.39,

$$\vec{F}_r = \vec{F}_{\text{deltoid}(v)} + \vec{F}_{\text{deltoid}(h)}$$

$$\vec{F}_r = 3.1 \times 10^2 \text{ N [down } 80^\circ \text{ right]}$$

The reaction force in the ballerina's shoulder joint is $3.1 \times 10^2 \text{ N [down } 80^\circ \text{ right]}$.

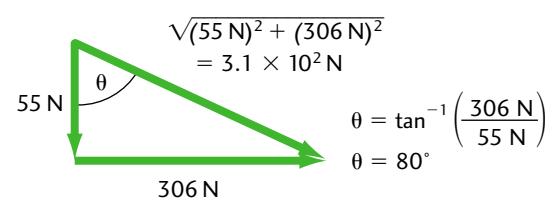


Fig.3.39 A component reminder

Fig.3.40 A component reminder

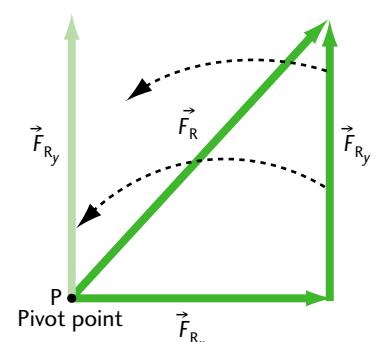
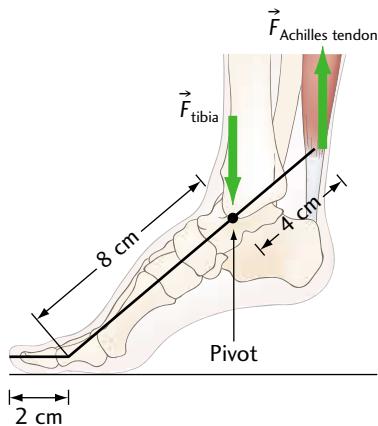




Fig. 3.41



1. A sprinter preparing for the Olympics is exercising her knee muscles by sitting in a chair and lifting weights (attached to her feet) from a hanging position to a complete horizontal position. The athlete's lower leg, of mass 5.0 kg and length 48 cm, has a centre of mass $\frac{1}{2}$ of the way down from the knee. What torque is exerted on the leg while holding a 10-kg extra mass at an angle of 45° from the vertical?

2. A small boy of mass 27 kg stands on his toes to peer over a fence. With each foot supporting half his mass in the foot orientation shown in Figure 3.41, what force must be exerted by the calf muscle through the Achilles tendon? Does the angle of the foot make any difference in how this question is answered?

Lifting Heavy Objects the Correct Way

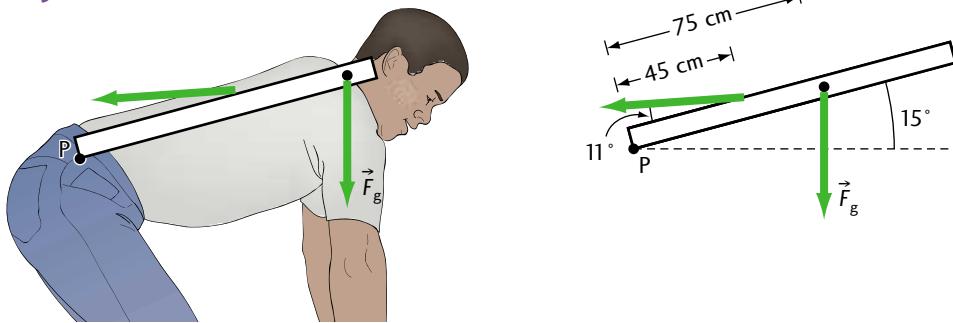
Health and safety officials have been publicizing for years that care should be taken to protect the muscles of the lower back during lifting. In Example 8, we discovered that the forces on muscles and joints are often many times larger than required to support the same weight by a simple cable or string. In fact, the forces become much larger if the angle between the muscle and the bone it acts on is very small. The muscles of the lower back are even more at risk because they must provide the lift force or torque for the load as well as for the upper body that acts as a lever.

Fig. 3.42



3. A teenager picks up his 19.0-kg little sister and holds her statically with a straight spine at an angle of 15° above the horizontal. Fifty-seven percent of his 85-kg body mass is located in his upper body, the centre of which acts through his shoulders, a distance of 75 cm from the pivot in his lower back. The muscles of his lower back are attached to the spine, 45 cm from the pivot at an angle of 11° to the spine (see Figure 3.43).
 - a) What is the effective tension in the lower back muscles and what reaction force is experienced by the lower vertebrae (pivot point)?
 - b) Use your knowledge of torque and static equilibrium to describe a technique for lifting heavy items.

Fig. 3.43



3.7 Stability and Equilibrium

Take a close look at the way a chair is designed. How many legs does it have? The difference between a four-, three-, or two-legged chair can be described by one word: **stability**. To be stable, an object must remain in static equilibrium when certain forces are applied to it, such as supporting someone's weight on a chair. There are three categories of static equilibrium: **stable**, **unstable**, and **neutral**. The difference between these types of equilibrium depends on what happens to the object's centre of mass when a force is applied. We will use the example of a baby playing with toys (Figure 3.44) to illustrate the difference between the three types of equilibrium.

Table 3.3 illustrates how a change in the position of the centre of mass and the resulting gravitational torque can determine the type of equilibrium of a static object.

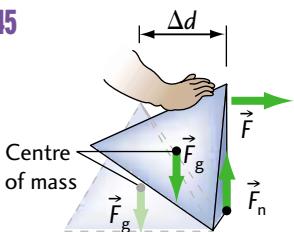
Fig.3.44



Table 3.3
Types of Equilibrium

Stable Equilibrium

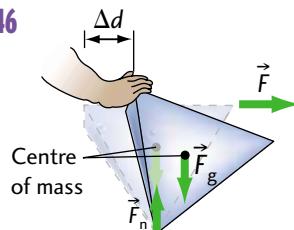
Fig.3.45



In **stable equilibrium**, a slight displacement causes the centre of mass to lift vertically and shift horizontally. Both of these changes provide a torque of $F_g \Delta d$ in a direction that lowers the centre of mass and returns the object to its original stable position. \vec{F}_g remains inside \vec{F}_n .

Unstable Equilibrium

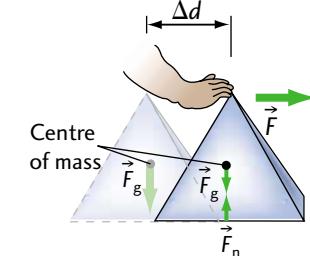
Fig.3.46



In **unstable equilibrium**, a slight displacement by a force causes a vertical drop and a horizontal shift of the centre of mass. Both of these changes provide a torque of $F_g \Delta d$ in a direction that continues the tipping motion and the block topples over. \vec{F}_g moves outside \vec{F}_n .

Neutral Equilibrium

Fig.3.47



If the disruptive force causes no vertical change in the centre of mass when it is pushed horizontally, then no new torque is produced and the object remains stable. This type of equilibrium is called **neutral equilibrium**. \vec{F}_g remains in line with \vec{F}_n .

An object that has all torques and forces balanced is in equilibrium. Equilibrium is *stable* as long as the vertical line from the centre of mass remains *inside* the area of the base of the body. If a disruption moves the vertical line from the centre of mass outside of the base, then the equilibrium is *unstable*. *Neutral* equilibrium exists when any disruptive force acts horizontally but the vertical height of the centre of mass remains unchanged. The object remains in equilibrium because the centre of mass doesn't move with respect to the object's base.

The stability of any free-standing object requires a large base of support with a low centre of mass. In sports, participants must stand in such a way to maintain their stability so as to have a positive influence in the game. The two karate stances depicted in Figures 3.48a and b illustrate how a well-trained person can apply the laws of physics to improve the stability of a stance.

Fig. 3.48 The low, wide stance makes the karateka stable. In Figure 3.48a, the karateka is in a horse stance, which provides stability against an attack from the side. The front stance in Figure 3.48b is more stable against a frontal attack.

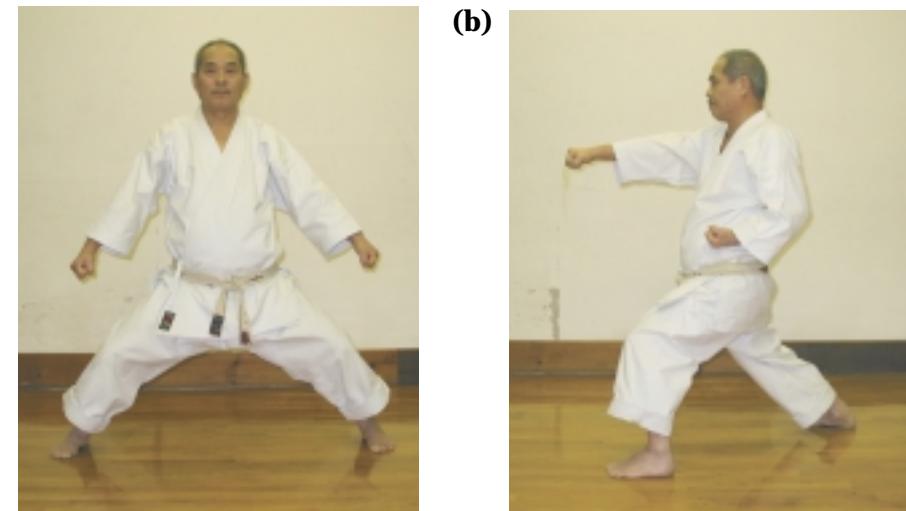


Fig. 3.48c The low rings place the centre of mass of this figure below the point of contact. The net result is that the figure hangs from the support instead of resting above it.



An object remains stable if the centre of mass remains low and inside the area of the base of the structure, sometimes called the **footprint**. Racecars need to be able to withstand great reaction forces when turning corners at high speed. That's why racecars have a lower centre of mass and a wider base than standard passenger cars. Large reaction forces that would topple taller passenger cars are less effective on low and wide racecars because it's more difficult to lift and shift the centre of mass outside the racecar's footprint.

The central character of the family game Tip-it by Mattel (see Figure 3.48c) can balance in stable equilibrium because the lower protrusions holding the rings place the centre of mass/gravity at or below the point of contact. Like a pendulum, the figure is always stable. Milton-Bradley has another game called Jenga (Figure 3.48d) that tests the players' understanding of stability. As the players remove the lower blocks and place them on the top, they are affecting the base and raising the centre of mass of the tower until it collapses.

Taller people are generally less stable than shorter people because their higher centre of mass can be more easily pushed outside their footprint. A tall person can improve his or her stability by forming a wide two-footed stance to increase the footprint. Shorter and longer dogs, such as the dachshund (Figure 3.48e), are much more stable than taller and shorter ones because of their wider base and lower centre of gravity. The three-legged dog shown in Figure 3.48f has had to shift her centre of mass by leaning to compensate for the missing limb. The dog increases her stability by moving her centre of mass to the centre of her smaller, three-legged footprint.

Fig.3.48e The low centre of mass and wide support stance of this dog make it very stable



Fig.3.48f The dog has shifted her centre of mass to a location over the remaining three-legged support system

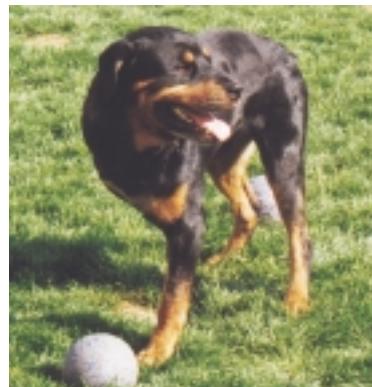


Fig.3.48d In this game, the structure becomes unstable when the support is removed and new mass is distributed above it, changing the position of the centre of mass



The geometry of objects allows us to use the concept of stability to calculate the **critical tipping angle**; that is, the angle at which an object is tipped when its centre of gravity is directly over the outer edge of its support base. At this point, \vec{F}_g and \vec{F}_n are along the same vertical line. If we know the position of the centre of gravity, we can calculate the tipping angle by applying the principles of static equilibrium.

EXAMPLE 10 The tipping angle of a mailbox

A 23.0-kg mailbox is 156.0 cm tall and 60.0 cm wide (see Figure 3.49a). Calculate its tipping angle. Assume its centre of mass is at its geometric centre.

Fig.3.49a

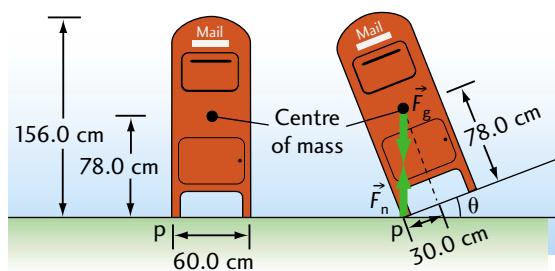
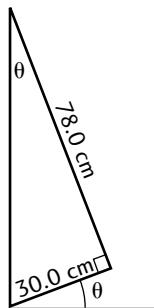
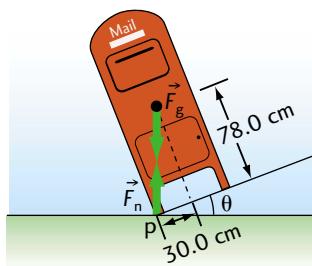


Fig.3.49b



Solution and Connection to Theory

Given

$$\text{length} = 156.0 \text{ cm} = 1.560 \text{ m} \quad L_{\text{cm}} = \frac{560 \text{ m}}{2} = 0.780 \text{ m}$$

$$\text{width} = 60.0 \text{ cm} = 0.600 \text{ m} \quad W_{\text{cm}} = \frac{0.600 \text{ m}}{2} = 0.300 \text{ m}$$

From Figure 3.49b, the critical tipping angle is the angle between the line drawn from the centre of mass to the base of the mailbox (the midline) and the line from the centre of mass to the horizontal, just beyond the corner of the mailbox. These lines form a triangle that we can use to calculate the critical tipping angle. Using the tangent function,

$$\tan \theta = \frac{W_{\text{cm}}}{L_{\text{cm}}}$$

$$\tan \theta = \frac{0.300 \text{ m}}{0.780 \text{ m}}$$

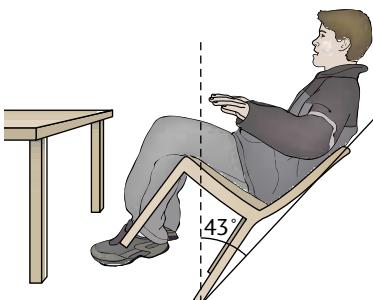
$$\tan \theta = 0.3846$$

$$\theta = 21.0^\circ$$

The critical tipping angle for this mailbox is 21.0° .



Fig.3.50



1. A physics student has placed his chair in unstable equilibrium by tipping it back and balancing on the back legs at an angle of 43° , as shown in Figure 3.50. The seat of the chair is $34.0 \text{ cm} \times 34.0 \text{ cm}$ and the legs are 40 cm long. The centre of mass of the chair and student lies along the midline of the chair.

- a) How high above the two lower legs is the centre of mass of the chair and student when the chair is tilted back?
- b) How high above the ground is the centre of mass of the chair and student when the student is sitting straight?

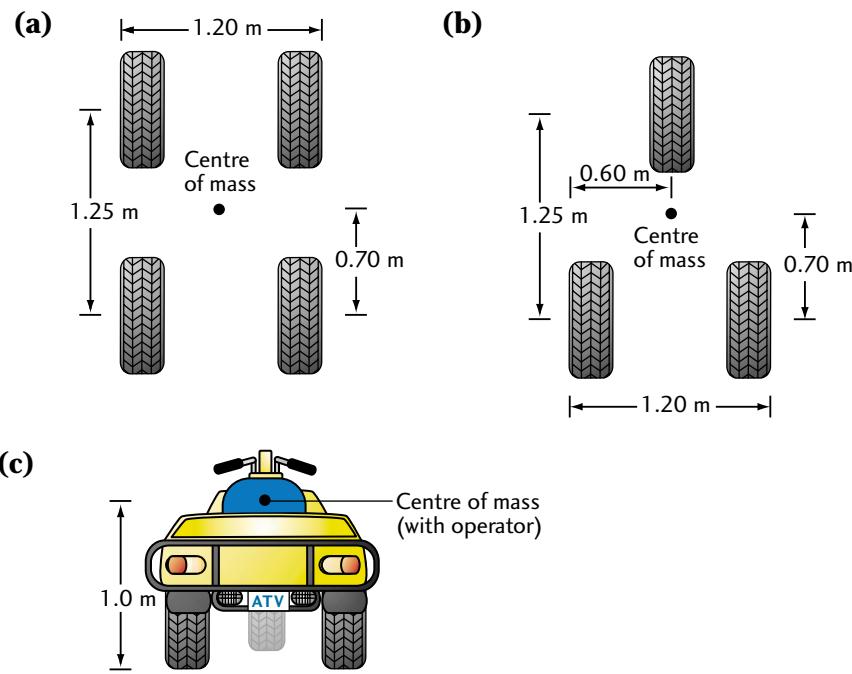
All-terrain Vehicles

All-terrain vehicles (ATVs) are growing in popularity for work and leisure. Small, manoeuvrable, yet still powerful, these vehicles are used for access and load carrying to remote sandy, marshy, or rocky areas, and often replace more expensive four-wheel-drive trucks that may be too large to pass through thick vegetation and trees. ATVs are also used for motorized hiking instead of less stable motorized dirt bikes or mountain bikes. One initial design for ATVs included only three wheels (a tricycle) (see Figure 3.51a) instead of the more

stable four-wheel model (Figure 3.51b). The three-wheel models became controversial because so many accidents were reported due to tipping forward, left, or right on either side of the front wheel. Many jurisdictions have banned the manufacture, sale, and grandfathered use of previously purchased three-wheel models.

- 2. a)** Use the wheel base and centre of gravity information given in Figure 3.52 to calculate the critical tipping angle for the three- and four-wheel ATVs.

Fig.3.52



- b)** Research the current laws for these vehicles in other jurisdictions around the world. Are three-wheel ATVs still manufactured or sold anywhere? If so, why is it permitted?

Fig.3.51 The three-wheel ATV, known for its instability, has been replaced by the wider wheel base design of the four-wheel model

(a)



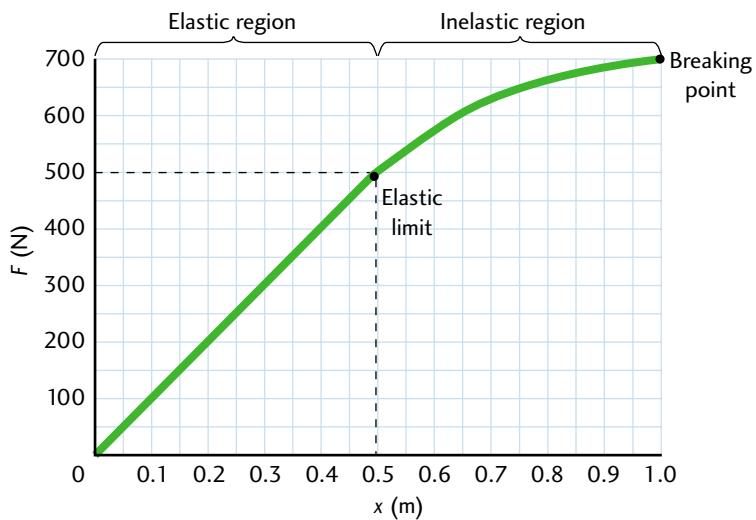
(b)



3.8 Elasticity: Hooke's Law

When a force is applied to an elastic medium, like a rubber band or a spring, it changes the object's length without affecting its other dimensions. The law that describes the simplest relationship between the applied force and the amount of length change or deformation in an elastic medium is called Hooke's law. This law describes a direct relationship between the restoring force and deformation of the elastic medium (see Figure 3.53).

Fig.3.53 The linear elastic region shows that the amount of deformation of an elastic material is proportional to the forces applied to deform it



The straight-line slope of k for this relationship implies that the restoring force and the deformation are directly related:

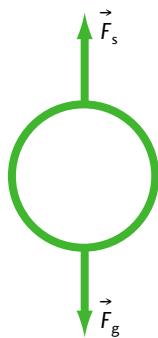
$$F = kx$$

where F is the restoring force exerted by the spring, in newtons, x is the deformation in the spring, in metres, and k is the constant of proportionality (spring constant) in newtons per kilogram.

EXAMPLE 11 The force to deform a spring

A 6.5-kg baby is placed into a Jolly Jumper that is suspended by one spring with a spring constant of 7.8×10^2 N/m. How far will the spring stretch when the baby is gently lowered into the chair?

Fig.3.54



Solution and Connection to Theory

Given

$$m = 6.5 \text{ kg} \quad k = 7.8 \times 10^2 \text{ N/m} \quad x = ?$$

$$\text{From Figure 3.54, } F = kx \text{ so } x = \frac{F}{k}$$

But $F = F_g = mg$; therefore,

$$x = \frac{mg}{k}$$

$$x = \frac{(6.5 \text{ kg})(9.8 \text{ N/m})}{7.8 \times 10^2 \text{ N/m}}$$

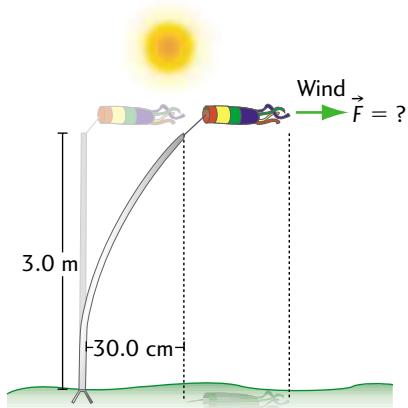
$$x = 8.2 \times 10^{-2} \text{ m}$$

The spring will stretch 8.2 cm when the baby is placed in the Jolly Jumper.

- A student stretches an elastic band with a spring constant of 16.0 N/m by 30.0 cm to launch a paper airplane.
 - What average force does the student apply to launch the plane?
 - What is the acceleration of the plane if its mass is 2.7 g and it experiences no friction or air resistance?
- A 67.5-kg mechanic sits on a truck suspension spring and notices that it only compresses by 1.0 cm . A spring is installed at each of the four equally spaced wheels of the truck with a body mass of $2.15 \times 10^3 \text{ kg}$. How far will each spring compress when the full truck body is lowered onto them if the truck's weight is equally distributed?
- A windsock helps helicopter pilots judge which way the wind is blowing and its strength. During a heavy wind, the aluminum pole that supports the 3.0-m-tall windsock is bent sideways by the wind, as shown in Figure 3.55. A shadow cast from the noonday Sun shows the top of the windsock pushed sideways a distance of 30.0 cm . If the spring constant for the pole is 120 N/m , what is the force of the wind on the windsock?



Fig.3.55



3.9 Stress and Strain — Cause and Effect

Thus far in this chapter, we have learned that when forces are applied to objects, they cause objects to translate, rotate, or remain in static equilibrium. Forces can also cause changes *within* an elastic material, making it longer or shorter, or distorting its shape. In this section, we will examine what happens within more rigid, less-elastic objects that experience reaction and torque forces. Structures, such as a post that supports the weight of a heavy beam in a house, are affected by the heavy load while keeping it in static equilibrium. The force of gravity pulling on the beam induces a change in the beam and the post. This force on the post gives rise to *stress*. The measurable changes in the post as a result of stress are called *strain*. In this section, we will quantify the concepts of stress and strain and study their cause-and-effect relationship.

Stress: The Cause of Strain

Stress is caused by a force applied on the *surface* of objects, not at one particular point. **Stress** is the force applied per unit area.

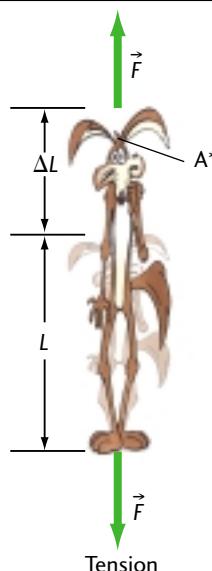
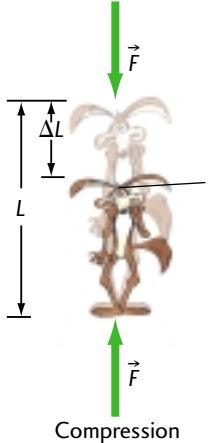
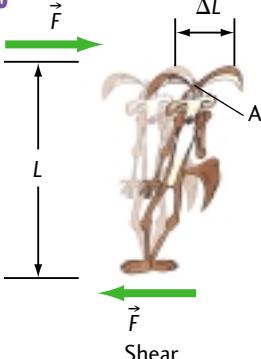
$$\text{Stress} = \frac{F}{A}$$

where F is the force applied (in newtons) over a surface area, A (in square metres).

Pressure is also the force per unit area on an object. Stress and pressure can both have the same units. To avoid confusion, in this text, pressure is expressed in pascals (Pa), and stress is expressed in newtons per square metre (N/m^2).

There are three different types of stress, classified according to *how* the stress is applied to the object (see Table 3.4).

Table 3.4
The Different Types of Stress

Type of Stress	Diagram/Example
Tensile stress occurs when outward forces along the length of an object cause the object to <i>increase in length</i> because the atoms are pulled farther apart.	Fig.3.56  <p>A diagram of a vertical object, resembling a plant or a stick, with a cross-sectional area labeled A^*. Two green arrows, each labeled \vec{F}, point away from the object's center along its length. A vertical double-headed arrow between the top and bottom of the object indicates its original length L. A shorter double-headed arrow above the top of the object indicates the increase in length, labeled ΔL. The word "Tension" is written at the bottom right.</p>
Compressive stress is the opposite of tensile stress: inward forces along the object's length tend to <i>decrease the length</i> of the object because the atoms are pushed closer together.	Fig.3.57  <p>A diagram of a vertical object, resembling a plant or a stick, with a cross-sectional area labeled A^*. Two green arrows, one pointing down labeled \vec{F} and one pointing up labeled \vec{F}, act along the length of the object. A vertical double-headed arrow between the top and bottom of the object indicates its original length L. A shorter double-headed arrow above the top of the object indicates the decrease in length, labeled ΔL. The word "Compression" is written at the bottom right.</p>
Shear stress changes the shape of an object but not its dimensions. It occurs when different forces act across the object at different points, warping or deforming the object as the atoms are forced sideways.	Fig.3.58  <p>A diagram of a vertical object, resembling a plant or a stick, with a cross-sectional area labeled A^*. Two green arrows, one pointing right labeled \vec{F} and one pointing left labeled \vec{F}, act on the object at different heights. A vertical double-headed arrow between the top and bottom of the object indicates its original length L. A horizontal double-headed arrow to the right of the object indicates the lateral displacement, labeled ΔL. The word "Shear" is written at the bottom right.</p>

* A = cross-sectional area

Strain: The Effect of Stress

Stress (the cause) has the effect of changing the dimensions of an object. The distortion of the dimensions of an object due to stress is called *strain*. We measure strain on an object by comparing the change in dimensions to the initial dimensions before the stress was applied.

$$\text{Strain} = \frac{\Delta L}{L}$$

where ΔL is the change in length of the entire object of initial length L .

What factors affect the amount of distortion in an object? Different materials respond differently to similar forces.

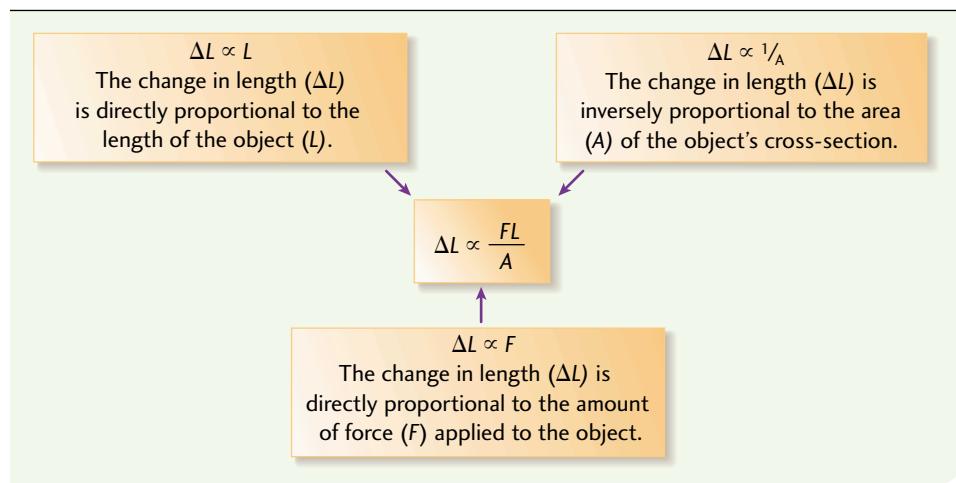


Fig.3.59 Soft, wet concrete is a much more pliable material than cured concrete. The more pliable the material, the more easily it is deformed.

In Figure 3.59, the force of a human foot on wet concrete produces different results than on dry concrete. The amount of distortion of a material depends on the type of material and can only be quantified by a constant of proportionality k (similar to the spring constant, k , in Hooke's law). An object's length, L , also affects the amount of distortion the object can undergo. The change in length is related to the overall length of the material ($\Delta L \propto L$). Compare the stretch of a long versus a short piece of fishing line. The long piece has more elasticity and will stretch more than the short piece, even though both pieces are made from the same material. In contrast, a wide object with a large cross-sectional area, A , is distorted less by the same force than an object with a small cross-sectional area ($\Delta L \propto \frac{1}{A}$). A thin elastic band can be stretched more than a thick band of the same material. Finally, the greater the force applied to the object, the greater the distortion of the object ($\Delta L \propto F$). Figure 3.60 illustrates the relationships among the parameters of strain.



Fig.3.60 The Relationships among the Parameters of Strain



Combining all the proportionality statements from Figure 3.60, we come up with the relationship

$$\Delta L \propto \frac{FL}{A}$$

To change this proportionality statement into an equation (see Appendix D), we add constant of proportionality k that quantifies the type of material.

$$\Delta L = \frac{kFL}{A}$$

In terms of cause and effect, the cause or *stress* on the object is the force applied to a specific surface area, $\frac{F}{A}$. The effect or *strain* on an object represents the change in an object's length with respect to its entire length, $\frac{\Delta L}{L}$. When we rearrange the equation to isolate the stress and the strain components from the constant, we obtain

$$\frac{1}{k} = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)}$$

This inverse constant, $\frac{1}{k}$, is called **Young's modulus** or the **elastic modulus**, E . It must be determined empirically for each material through experimentation.

$$E = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)} = \frac{\text{Stress}}{\text{Strain}}$$

where F is the force applied, in newtons, to a surface area A (in metres squared), ΔL is the change in the object's length (in metres), and L is the object's initial length (in metres).

This equation is much more useful than Hooke's law because it includes all the variables that affect strain when a stress is applied.

Young's modulus (E) is the ratio of stress to strain on an object and is therefore a useful measure of the cause and effect of forces on a particular material. The equation for Young's modulus, E , describes how tensile and compressive stresses are related to the strain on an object because it is concerned with change in the object's length. Other forms of stress, such as shear stress and pressure (compressive stress on a fluid), can also be examined in a similar fashion. Table 3.5 outlines the different ways in which stress can be applied and its effect on a material in terms of a specific constant of proportionality, called a **modulus**.

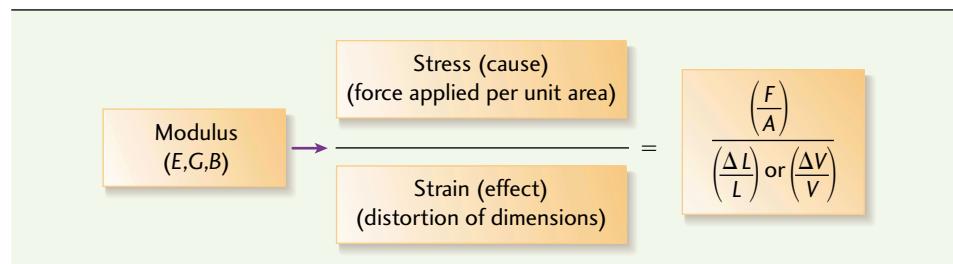
Young's modulus was named after the British scientist, Thomas Young (1773 – 1829). We will study more of Young's work in Unit D.

Table 3.5
Types of Stress

Type of Stress	Equation	Description
Tensile/Compressive	$E = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)} = \frac{\text{Stress}}{\text{Strain}}$	E is Young's modulus , measured in N/m ² .
Shear	$G = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)} = \frac{\text{Stress}}{\text{Strain}}$	G is called the shear modulus (also in N/m ²). It is usually smaller than E because a shear sideways flex is much easier to achieve than a compression or stretch.
Pressure (fluid)	$B = -\frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta V}{V}\right)} = \frac{-(\Delta P)}{\left(\frac{\Delta V}{V}\right)} = \frac{\text{Stress}}{\text{Strain}}$	B is the bulk modulus . It relates the pressure (stress) to the change in volume (strain) of a fluid as described by Boyle's law. The negative sign in this equation ensures that the bulk modulus is always a positive number.

Figure 3.61 summarizes how a modulus is calculated.

Fig.3.61 The Modulus — The Cause-to-effect Ratio



The moduli discussed in this section are constants of proportionality that are determined empirically by experimentation. Table 3.6 lists these constants for various construction or support materials as well as some liquids and gases.

Moduli for Various Substances			
Material	Young's elastic modulus (tensile/compressive stress) E (N/m ²)	Shear modulus (shear stress) G (N/m ²)	Bulk modulus (pressure on fluid) B (N/m ²)
Solids			
Cast Iron	100×10^9	40×10^9	90×10^9
Steel	200×10^9	80×10^9	140×10^9
Aluminum	70×10^9	25×10^9	70×10^9
Brass	100×10^9	35×10^9	80×10^9
Copper	110×10^9	38×10^9	120×10^9
Lead	16×10^9	5.6×10^9	7.7×10^9
Concrete	20×10^9		
Brick	14×10^9		
Marble	50×10^9	20×10^9	70×10^9
Glass	57×10^9	24×10^9	40×10^9
Granite	45×10^9	20×10^9	45×10^9
Wood (pine, parallel grain)	10×10^9		
Wood (pine, perpendicular grain)	1×10^9		
Nylon	5×10^9		
Bone (limb)	15×10^9	80×10^9	
Liquids			
Water			20×10^9
Alcohol			10×10^9
Kerosene			13×10^9
Glycerine			45×10^9
Mercury			26×10^9
Gases (isothermal at normal atmospheric pressure)			
Most gases (Air, H ₂ , He, CO ₂)			1.01×10^5

The equations and moduli studied in this section help us to identify the similarities and differences in the stress and strain of different materials. A few examples of how these equations and constants are used will help to implant them “statically” in our minds.

EXAMPLE 12 Stress on a steel cable

During building construction, a large bucket of concrete is lifted by a single crane to the upper floors, as described by the free-body diagram in Figure 3.62.

- What mass of concrete (and metallic bucket) can be lifted by a steel cable of diameter 1.5 cm such that the cable will only stretch by a maximum of 0.15%?
- What maximum mass can this cable hold statically if its maximum strength is $5.0 \times 10^8 \text{ N/m}^2$?
- How much would this maximum load stretch a cable with an initial length of 35 m?

Fig.3.62



Solution and Connection to Theory

Given

$$d = 1.5 \text{ cm} = 0.015 \text{ m} \quad \frac{\Delta L}{L} = 0.15\% = 1.5 \times 10^{-3}$$

$$E = 20 \times 10^{10} \text{ N/m}^2 \quad m = ?$$

- To find the circular cross-sectional area of the cable,

$$A = \pi r^2 = \pi \left(\frac{d}{2} \right)^2$$

We use Young's modulus equation because the cable is experiencing tensile stress:

$$E = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)}$$

To calculate the mass, we first isolate the force:

$$F = \frac{E \Delta L A}{L}$$

$$F = E \left(\frac{\Delta L}{L} \right) \pi \left(\frac{d}{2} \right)^2$$

$$F = (20 \times 10^{10} \text{ N/m}^2) (1.5 \times 10^{-3}) \pi \left(\frac{0.015 \text{ m}}{2} \right)^2$$

$$F = 5.3 \times 10^4 \text{ N}$$

The force is a lifting force, $F_g = mg$; therefore,

$$m = \frac{F_g}{g}$$

$$m = \frac{5.3 \times 10^4 \text{ N}}{9.8 \text{ N/kg}}$$

$$m = 5.4 \times 10^3 \text{ kg}$$

The total mass for a 0.15% increase in cable length is $5.4 \times 10^3 \text{ kg}$ or 5.4 metric tonnes.

- b)** The maximum stress or force per unit area on the cable can't exceed $5.0 \times 10^8 \text{ N/m}^2$; therefore,

$$\frac{F_g}{A} \leq 5.0 \times 10^8 \text{ N/m}^2$$

$$\frac{mg}{A} \leq 5.0 \times 10^8 \text{ N/m}^2$$

$$m \leq 5.0 \times 10^8 \text{ N/m}^2 \left(\frac{A}{g} \right)$$

$$m \leq 5.0 \times 10^8 \text{ N/m}^2 \left(\frac{\pi \left(\frac{0.015 \text{ m}}{2} \right)^2}{9.8 \text{ N/kg}} \right)$$

$$m \leq 9.0 \times 10^3 \text{ kg}$$

The maximum mass that the cable can hold statically is $9.0 \times 10^3 \text{ kg}$.

- c) Given**

$$L = 35 \text{ m} \quad m = 9.0 \times 10^3 \text{ kg} \quad \Delta L = ?$$

To calculate the stretch of the cable, we can isolate ΔL in Young's modulus equation and solve:

$$\Delta L = \frac{FL}{EA}$$

$$\Delta L = \frac{mgL}{E\pi\left(\frac{d}{2}\right)^2}$$

$$\Delta L = \frac{(9.0 \times 10^3 \text{ kg})(9.8 \text{ N/kg})(35 \text{ m})}{(20 \times 10^{10} \text{ N/m}^2)\pi\left(\frac{0.015 \text{ m}}{2}\right)^2}$$

$$\Delta L = 8.75 \times 10^{-2} \text{ m}$$

The load would cause the cable to stretch 8.8 cm.

The maximum strength of various materials is of particular interest to engineers and architects when designing support structures of new buildings. Table 3.7 lists the maximum strengths of various support materials.

Material	Strengths of Support Materials		
	Tensile strength (N/m ²)	Compressive strength (N/m ²)	Shear strength (N/m ²)
Cast iron	17×10^7	55×10^7	17×10^7
Steel	50×10^7	50×10^7	25×10^7
Aluminum	20×10^7	20×10^7	20×10^7
Brass	25×10^7	25×10^7	20×10^7
Concrete	0.2×10^7	2.0×10^7	0.2×10^7
Brick		3.5×10^7	
Marble		8×10^7	
Granite		17×10^7	
Wood (pine, parallel grain)	4.0×10^7	3.5×10^7	0.5×10^7
Wood (pine, perpendicular grain)		1.0×10^7	
Nylon	50×10^7		
Bone (limb)	13×10^7	17×10^7	

- A steel guitar string of diameter 0.29 mm and original length 0.90 m is elongated by 0.22 mm when a tensile force is applied. Calculate the diameter of a nylon string that has the same extension and tension.
- A marble column of cross-sectional area 3.0 m^2 supports a mass of $3.0 \times 10^4 \text{ kg}$.
 - What is the stress on the column?
 - What is the strain on the column?
 - By how much is the height of the 15-m-high column decreased under the load?

Stress and Strain on the Human Body

The human body is made up of living tissue supported by bone tissue. Muscle fibres provide tension forces through **tendons**, which connect bones to muscles, thereby giving us mobility. The large compressive strength of bone tissue ($1.7 \times 10^8 \text{ N/m}^2$) is attributed to the presence of **hydroxyapatite** crystals containing calcium. Long collagen fibres along the length of the bone also give it a great tensile strength ($1.3 \times 10^8 \text{ N/m}^2$). Bones are hollow, which makes them strong yet light. Their centre is filled with **bone marrow**, which produces red blood cells in the body. The spine is of particular interest in the study of forces on the human body. It is a column composed of a series of smaller bones called vertebrae that provide the structural support for the upper body and protection for the



Fig. 3.63 The structure of the human spine

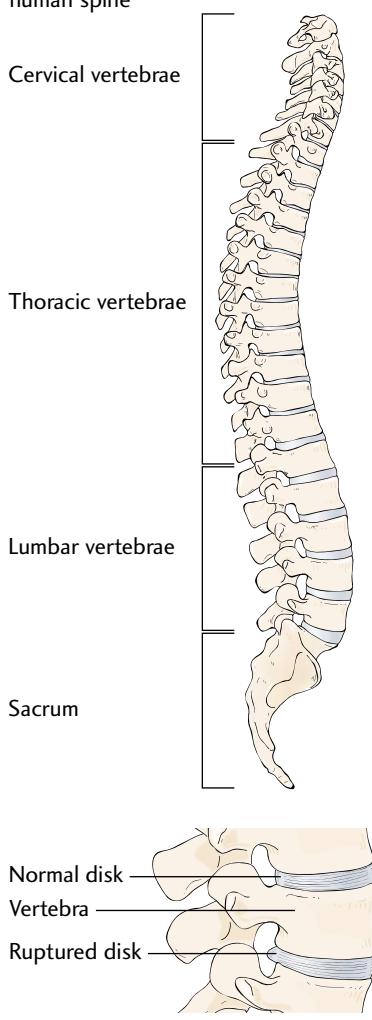
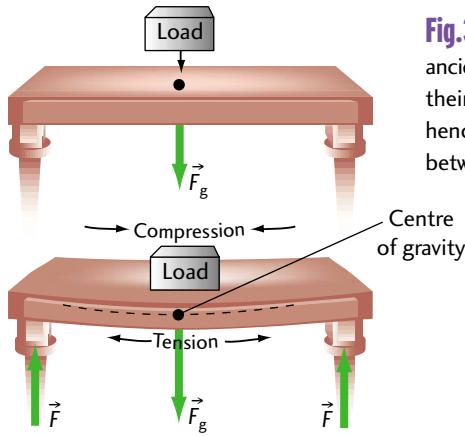


Fig. 3.64a The weight applied to the beam tenses the lower surface and compresses the top surface to transmit the load to the posts. A table is an example of a simple beam-and-post construction.



spinal nerves connected to the brain while providing mobility. Figure 3.63 illustrates the fluid-filled discs in between the vertebrae, which transmit force along the length of the spine, protect the vertebrae from stress damage, and prevent friction between vertebrae.

3. Olympic weightlifters lift masses in excess of 200 kg over their heads. The maximum compressive strength of a human bone such as the femur (thigh bone) of diameter 4.0 cm is given in Table 3.7.

- a) If the weight was supported by a weightlifter's two thigh bones only, what maximum mass could the athlete lift?
- b) After considering the article above and your own assumptions of the human body structure, what other parts of the body would be a more realistic limit to the “world record” mass lifted in a competition?

3.10 Stress and Strain in Construction

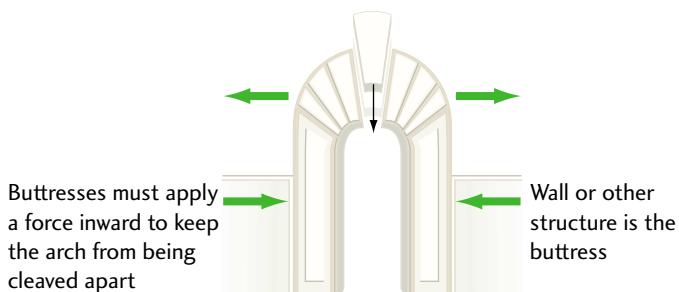
One of the major applications of statics is in the design and construction of buildings. All buildings, ancient and modern, contain vertical and horizontal support structures called **posts** and **beams**, respectively. Figure 3.64a outlines how posts and beams work to transmit forces to the ground. Like a flat tabletop, a beam flexes, creating tension in the lower portion of the material and compression in the upper layers. If the material in the beam can withstand these forces, the force of gravity of the supported object is transmitted through the beams to the ground by posts, which are like the legs of a table.

The ancients built some of their buildings out of stone, which has great compressive strength but poor tensile strength. The beams of ancient stone buildings are therefore very short, as illustrated by the close spacings between the posts (columns) in Figure 3.64b. To compensate, Roman architects began using the arch as a combined beam and post. The wedge-shaped pieces of stone in the arch experience compressive forces when supporting weight (see Figure 3.64c).

Fig. 3.64b The beams of ancient buildings transmitted their loads to posts poorly; hence the close spacing between posts.



Fig. 3.64c In an arch, the inward horizontal forces of the load are transferred to the vertical posts by compression only. The outward reaction forces are so large that the posts must be supported by buttresses.



The outward horizontal forces produced by an arch require a **buttress** to hold the arch together.

Eventually, stronger tensile materials such as steel, invented by Henry Bessemer in 1830, improved the effective span of the beam. Design improvements led to the development of a specialized beam called a **truss**. Trusses, like those shown in Figure 3.64d, use individual bars called **members** that direct the internal forces on the truss very efficiently while using a minimum amount of material. The lowest section of the Eiffel Tower (Figure 3.64e) is a combination of one giant arch constructed from a truss. The buttressing for this arch is provided through the four sloping legs of the tower.

Today, in modern housing construction, new materials are used to make longer spanning beams that minimize the number of unsightly and costly posts. Try finishing off a basement family room that has several support posts right where you want to put the billiard table. The composite wooden I-beam (Figure 3.64f), used as a floor-support beam, is made of pieces of fast-growth wood such as poplar that is compressed and glued together to achieve maximum compressive and tensile strength. The use of smaller pieces of wood for I-beams is a more inexpensive solution and helps to preserve old-growth forests while still allowing longer beam lengths in residential applications.

In high-rise concrete structures, steel bars called re-bars are placed inside concrete forms before the wet concrete is poured to improve the concrete's tensile strength. Wooden trusses are often reinforced by running metal cables or bars across the lower truss member, which is then tensioned and anchored at both ends. Concrete may be **pre-stressed** or tensioned by stretching steel cables or rods suspended in the wet concrete before it has cured and had a load applied to it. Any stress that would normally be created by a load is absorbed until the pre-stress amount has been reached. Concrete can also be **post-stressed**, where the tensioning is done after the concrete has set.

Fig. 3.64d A typical roof truss transfers forces efficiently through the many small parts that comprise its overall structure



Fig. 3.64e The Eiffel Tower in Paris, France looks like one huge truss. The lowest part is constructed as an arch because of the large distance between the vertical supports.



Fig. 3.64f A composite wood beam can span a greater distance, eliminating the need for many vertical support posts





The Ultimate Effect of Stress on a Structure

Fig.STSE.3.1 Damage done to the Versailles Hall in Jerusalem was caused by insufficient support in the floor area



Fig.STSE.3.2 The twin towers of the World Trade Center just before their collapse on September 11, 2001



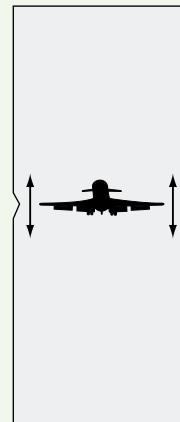
Web link: See <www.irwinpublishing.com/students> for links to Web sites on structural design as well as news sites on these disasters.

An expanding population centralized in growing urban centres has prompted the design of structures that expand upward instead of outward. The study of statics is applied directly to the design and construction of buildings and shelters, a basic human necessity. On May 25, 2001, the top floor of the Versailles Hall, a multi-storey building in the industrial section of Jerusalem, collapsed under the stress of the weight of about 650 people attending a wedding reception (see Figure STSE.3.1). The dance floor collapsed, taking wedding guests with it as it fell three stories through two vacant floors to the ground level below. Shaul Nevo, an engineer and a reserve army major who was helping in the disaster rescue, said that the use of thin concrete layers in the building's construction was probably one of the causes of the collapse. Concrete, a material that has great compressive strength but poor tensile strength, must be pre- or post-stressed and secondarily supported with steel cable or beams. Prior renovation of the lower floors in the Versailles Hall changed the structural support for the floors above. The weight of the structure above and of several hundred patrons on thin beams (thin concrete layers) over an increased span without structural support was most likely the cause of the collapse.

The same principle of great weight over an extreme beam span caused the eventual collapse of both towers of the World Trade Center in New York City on September 11, 2001 (see Figure STSE.3.2) after a heinous terrorist attack. The impact of a large aircraft removed key structural support for the weight of the building above the point of impact, as shown in the simplified diagram in Figure STSE.3.3. The World Trade Center was designed so that it could have withstood the impact of a Boeing 707. However, the damage was done by a larger 767 airplane. The structural degradation by the crash and residual fire caused the final horrific collapse.

As buildings get taller, much more work has to be done to the design of these structures to ensure structural safety. Disasters such as those in Jerusalem and New York City also serve as a reminder to architects and engineers of the importance of means of evacuation in building design.

Fig.STSE.3.3 The aircraft damage removed key structural support elements for the upper floors



Design a Study of Societal Impact

Many buildings in cities such as New York or Toronto exceed 75 floors and can at any time hold tens of thousands of occupants. Many design considerations must be taken into account to bring amenities to the upper floors of tall buildings. Research how services such as electricity, water, sewage, and telephone are supplied to the CN Tower in Toronto. Research the specific safety features that are part of the design of the CN Tower as a result of its size and shape.

Design an Activity to Evaluate

Analyze an animated cartoon of your choice (with teacher approval) for misconceptions in statics, kinematics, and dynamics. Make a list of all the plausible impossible or simply impossible situations you see and describe how they violate the laws of physics. For example, the base of a catapult shouldn't rotate up over the coyote; instead, the catapult arm should rotate to release the coyote. Write a short reflection stating your opinion about the role of accurate or inaccurate physics in cartoons. If you are artistically inclined, draw a cartoon strip that includes inaccurate physics.

With a camera (digital or 35-mm), create a slide show of natural and human-made structures (beams, posts, and arches) that are in static equilibrium. Superimpose free-body diagrams on your photos (scanned if using a 35-mm camera) using drawing software to illustrate stresses and strains in the structures. Present your slide show to the class.

Build a Structure

Design and construct a pegboard, force-board, or torque balance for use in the two labs at the end of this chapter. Build your board or balance using a series of pulleys (plastic thimbles with protractors) that can be mounted on a pegboard and kept for future use in class. Strings, protractors, and masses can be used to create various situations for verifying the necessary conditions for static equilibrium.

Design competition: Build a model bridge or other structure out of uncooked spaghetti or drinking straws such that it supports the most weight possible.

You should be able to*Understand Basic Concepts:*

- Define the concept of centre of mass and describe how it relates to the centre of gravity.
- Relate the concepts of Newton's laws of motion to situations of static equilibrium.
- Describe torque in qualitative and quantitative terms and use it to solve simple problems.
- State the two conditions necessary for static equilibrium and use them qualitatively and quantitatively to solve problems.
- Apply the two conditions of static equilibrium to quantitatively analyze situations involving the human body.
- Relate the concept of centre of mass/gravity and torques to the stability of certain structures, including the calculation of a tipping angle.
- Define and describe the quantities of stress and strain and relate them to the force (stress) that causes a deformation (strain) of structural materials.
- Compare and contrast the types of strain on an object by describing and illustrating the source and direction of the stress applied.
- Use Hooke's law to quantitatively describe how stress applied deforms construction materials.

Develop Skills of Inquiry and Communication:

- Conduct an experiment to determine the net force on an object in static equilibrium. Verify the first condition of static equilibrium, the balance of forces, and analyze discrepancies between the theoretical and empirical values.
- Conduct an experiment to verify the second condition for static equilibrium.
- Design and perform an experiment to evaluate the relationship of stress and strain on a metallic wire. Include a system of measuring small changes in dimensions, such as a vernier scale.

Relate Science to Technology, Society, and the Environment:

- Apply the concepts of centre of mass/gravity and static equilibrium to structural stability and stress and strain on the human body.
- Evaluate the social and economic impact of locating base sites of financial or government institutions and high-density housing in large urban centres. Describe instances where developments in construction patterns, intended to improve the quality of life, have degraded it.
- Relate the occurrence of disasters to the improvement of building design, construction, and emergency evacuation plans.

Equations

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = 0$$

$$\tau = rF_{\text{app}} \sin \theta$$

$$\vec{\tau}_{\text{net}} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \dots + \vec{\tau}_n = 0$$

$$F = kx$$

$$\text{Stress} = \frac{F}{A}$$

$$\text{Strain} = \frac{\Delta L}{L}$$

$$E = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)} = \frac{\text{Stress}}{\text{Strain}}$$

$$G = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)} = \frac{\text{Stress}}{\text{Strain}}$$

$$B = \frac{-\left(\frac{\Delta F}{A}\right)}{\left(\frac{\Delta V}{V}\right)} = \frac{-(\Delta P)}{\left(\frac{\Delta V}{V}\right)} = \frac{\text{Stress}}{\text{Strain}}$$

EXERCISES

Conceptual Questions

1. Why can't a Hydro line or telephone cable be run completely horizontally?
2. A ladder rests against a frictionless wall.
 - a) In which direction is the ladder pushing against the wall?
 - b) How is the force that the ladder exerts on the ground related to the weight of the ladder? Why?
3. In terms of static equilibrium and stability, what is the difference between standing with your feet together and apart? Explain your answer.
4. Why does wearing high-heeled shoes sometimes cause lower back pain?
5. Electricity or telephone line installers purposely allow lines to sag or droop, especially in areas where freezing rain and ice build-up are frequent. Why?
6. How can a wrench handle be adapted to better loosen a rusty bolt?
7. A ladder is placed against a wall at an angle of 65° to the ground. Will the ladder be more likely to slide down if a person stands on a lower rung or on a higher rung? Why?
8. A person on a bicycle presses down on each pedal half-way through each rotation. In which position is the torque at zero? at a maximum?
9. The benefits of weight training are maximized when muscles are exercised through their full range of motion. Is there any benefit for a weight trainer to do curls all the way to the highest weight position? (See Example 8.)
10. What type of equilibrium is your textbook in when it is sitting flat on your desk? What about if you balanced it on its corner?

11. Many people require the use of a cane to walk comfortably. What is the purpose of the cane in terms of stability?
12. When standing up from a sitting position, why must we first lean forward?
13. Office chairs with casters used to be supported by a base with four legs. Now the base consists of five legs. Why?
14. Why do tall fluted champagne glasses usually have a wide base?
15. Ships empty of cargo are often loaded with extra water or other weights such as rocks. Why?
16. The balancing toy shown in Figure 3.65 won't fall from its stable position even when pushed gently. Why is this figure so stable?

Fig.3.65



17. Two bones of equal radius but different lengths are subjected to equal twisting torques. Which bone will fracture first?
18. Why is a piece of $2'' \times 4''$ lumber placed with its wide side vertical when used to support long spans of fence?
19. A cantilever is a board that projects beyond its support, such as a diving board. Would you use concrete to create a large cantilevered structure? Why or why not?

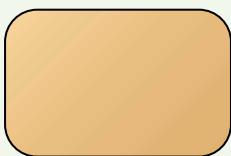
Problems

3.2 The Centre of Mass — The Gravity Spot!

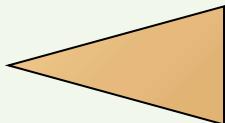
20. a) Copy the following shapes (Figure 3.66) into your notebook and mark the centre of mass on each.

Fig.3.66

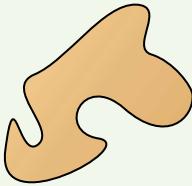
(a)



(b)



(c)



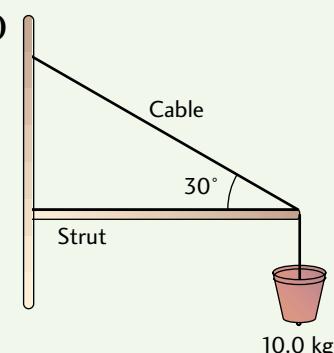
- b) For each shape, explain your choice of position for the centre of mass.
c) Trace these onto a piece of paper and use the technique described in Section 3.2 to verify the actual position of the centre of mass.

3.3 Balancing Forces ... Again!

21. A 10.0-kg flowerpot is suspended from the end of a horizontal strut by a cable attached at 30° above the horizontal, as shown in Figure 3.67. If the strut has no mass, find the tension in the cable.

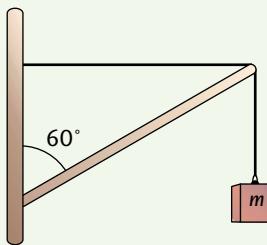
Fig.3.67

(a)



22. In problem 21, how much horizontal force must the strut provide?
23. A 100.0-kg mass is suspended from two ropes, each at an angle of 30° to the vertical. What is the tension in each rope?
24. What maximum mass, m , can be supported by the strut-and-cable arrangement in Figure 3.68 if the maximum force on the strut is 2500 N?

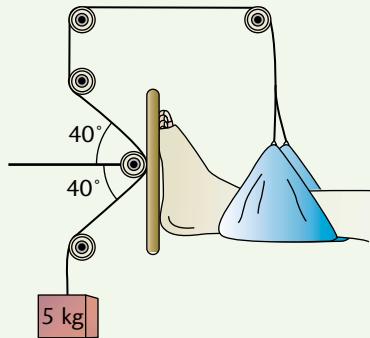
Fig.3.68



25. A 500-kg load of drywall is lifted up the side of a building by a crane. When the load is pulled to the side by a horizontal rope, the support cable of the crane makes an angle of 12° to the vertical. What is the tension in the support cable? What is the tension in the horizontal rope?
26. A 250-kg crate is being unloaded from a cargo ship by a crane with a cable 10 m long. The load must be pushed horizontally onto an awaiting wooden skid by a worker of mass 100 kg.
a) If the coefficient of friction between the worker's shoes and the floor is 0.63, what maximum horizontal force can he apply before his shoes begin to slip?
b) What is the crate's maximum horizontal displacement from rest?
27. A car is stuck in a snow bank, but the driver is very knowledgeable about physics. She ties a rope from her car to a tree 25.0 m away and then pulls sideways on the rope at the midpoint. If she applies a force of 425 N and draws the rope over a horizontal distance of 1.5 m, how much force is applied to the car?

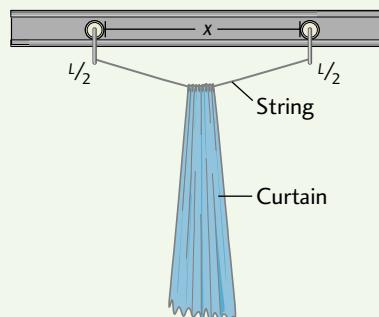
- 28.** A bird lands on a telephone wire midway between two poles 18 m apart. The wire (assumed weightless) sags by 52 cm. If the tension in the wire is 90 N, what is the mass of the bird?
- 29.** When a person's thighbone (femur) is broken, the muscles draw the broken parts so tightly together that the length of healed leg is slightly shorter than its original length. In the past, a traction device (see Figure 3.69) was used to oppose the natural muscle tension, allowing the bone to heal properly. What is the magnitude and direction of the tension force applied to the femur if the mass of the leg is 3.75 kg?

Fig. 3.69



- 30.** A string of length L is connected to two pulleys on an I-beam curtain rod, as shown in Figure 3.70. A curtain of mass m , is hung from the midpoint of the string and the pulleys are drawn as far apart as possible. The coefficient of static friction between the pulleys and the rod is μ . Find the maximum distance, x , in terms of L and μ , between the pulleys before they begin to roll toward each other.

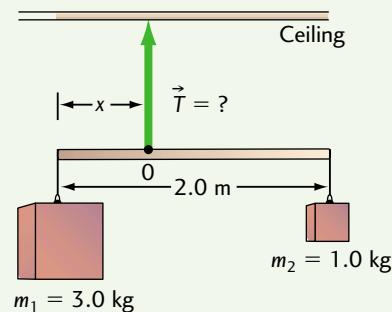
Fig. 3.70



3.4 Balancing Torques

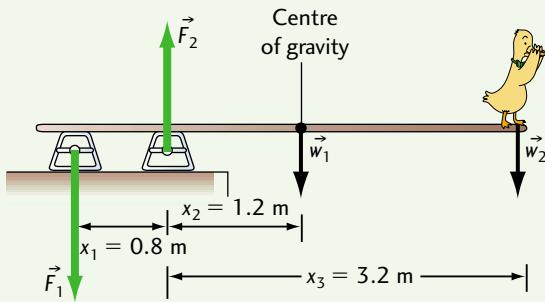
- 31.** Figure 3.71 shows a 2.0-m-long rod with a 1.0-kg mass at one end and a 3.0-kg mass at the other end.
- If from the heavy end, the mass of the rod is negligible, where is the centre of gravity of the system?
 - What is the tension in the single support cable?

Fig. 3.71



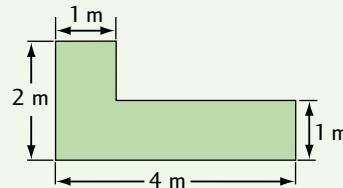
- 32.** Find the forces exerted by the two supports of a 4.0-m, 50-kg uniform cantilever (diving board) when a 8.5-kg duck stands at the opposite end, as shown in Figure 3.72.

Fig. 3.72



- 33.** Find the centre of mass of the L-shaped steel plate shown in Figure 3.73. (Hint: Think of the L as two separate figures, each with its own centre of mass. Use the total centre of mass in two dimensions as the coefficients of the two-dimensional centre of mass.)

Fig. 3.73

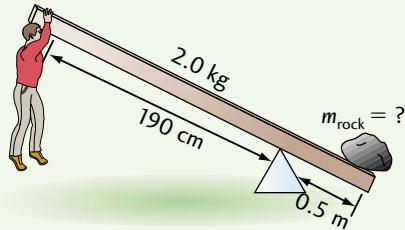


3.5 Static Equilibrium: Balancing Forces and Torque

- 34.** Three people carry an extension ladder of length 5.0 m in the horizontal position. The lead person holds the ladder's front end, and the other two people are side by side on opposite sides of the ladder a distance x from its back end. Calculate x if the two people in the rear each support one-third of the ladder's weight.

- 35.** An 86-kg man is trying to pry up a rock by hanging from the end of a class-one lever (a uniform piece of lumber of mass 2.0 kg), as illustrated in Figure 3.74. What is the maximum mass of the rock if it can be just lifted?

Fig. 3.74



- 36.** Two children of masses 17 kg and 27 kg sit at opposite ends of a 3.8-m teeter-totter that is pivoted at the centre. Where should a third child of mass 20 kg sit in order to balance the ride? Does the mass of the teeter-totter matter?
- 37.** A 5.0-kg bag of cement is placed on a 2.5-m-long plank at 1.5 m from its end. The 2.0-kg plank is picked up by two men, one at each end. How much weight does each man support?
- 38.** The centre of mass of a 30-kg dog standing on all fours is located 70 cm from her hind legs and 30 cm from her front legs. Find the force of the ground on each of her legs.

- 39.** The hinges of a 20-kg door, 2.4 m high and 0.8 m wide, are placed at the top and bottom of the door's vertical edge. The door is supported by the upper hinge.

- a)** What is the magnitude and direction of the force that the door exerts on the upper hinge?
b) What is the magnitude and direction of the force that the lower hinge exerts on the door?

- 40.** A weightless ladder 7.0 m long rests against a frictionless wall at an angle of 65° above the horizontal. A 72-kg person is 1.2 m from the top of the ladder. What horizontal force at the bottom of the ladder is required to keep it from slipping?

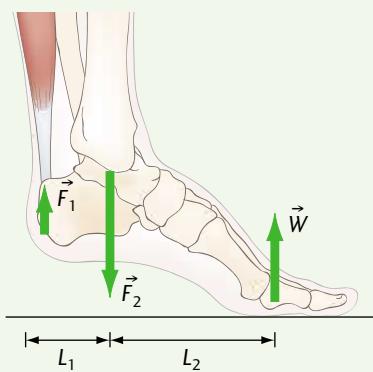
- 41.** A box of total mass 75 kg rests on a floor with a coefficient of static friction of 0.42. The box is 1.6 m high, 1.0 m deep, and has uniform weight distribution.
a) What is the minimum horizontal force required to start the box sliding across the floor?
b) What is the maximum height at which this force can be applied without tipping the crate?

3.6 Static Equilibrium and the Human Body

- 42.** Pierre holds a 10-kg bucket of water with his upper arm at his side and his forearm horizontal (90° at the elbow). The palm of his hand is 35 cm from the elbow, and his upper arm (shoulder to elbow) is 30 cm long. His biceps muscle is attached to the forearm 5.0 cm from the elbow. If the centre of mass of his 3.0-kg forearm is 16 cm from his elbow, what force does the biceps muscle exert to support both the arm and the bucket?

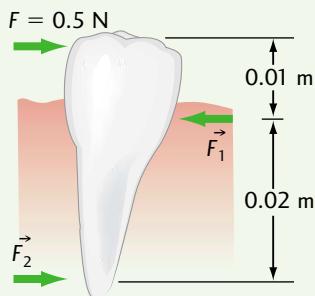
- 43.** The hand, forearm, and upper arm of a gymnast have masses of 0.4 kg, 1.2 kg, and 1.9 kg, respectively, and their respective centres of mass are 0.60 m, 0.40 m, and 0.15 m from her shoulder joint. Find the centre of mass of her unbent arm as it is held horizontally from her shoulder.
- 44.** When you stand on the ball of your foot, the reaction force upward on the ball of your foot is equivalent to your weight. To raise your heel as shown in Figure 3.75, you must apply an upward force, \vec{F}_1 , through your Achilles tendon so that the downward reaction force, \vec{F}_2 , is greater than your weight. Calculate \vec{F}_1 and \vec{F}_2 for a 65-kg woman with foot dimensions $L_1 = 4.0$ cm and $L_2 = 12$ cm. If L_1 was greater than L_2 , how would the force exerted by the Achilles tendon, \vec{F}_1 , be affected?

Fig.3.75



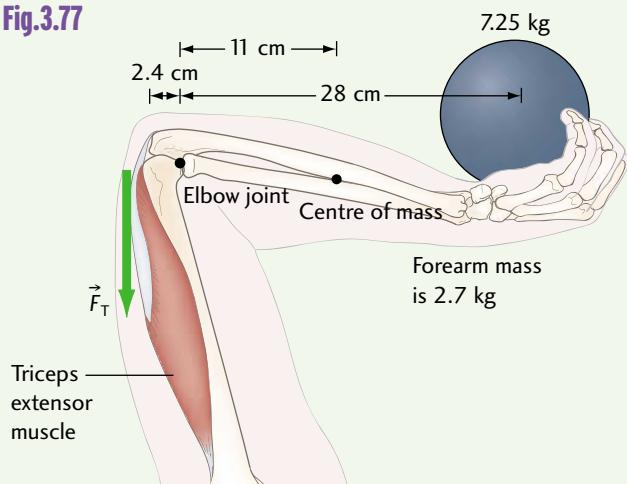
- 45.** In Figure 3.76, what are the forces \vec{F}_1 and \vec{F}_2 if the tooth remains in static equilibrium?

Fig.3.76



- 46.** An Olympic athlete is holding a 7.25-kg shot-put, as shown in Figure 3.77. Her forearm is 28.0 cm long and has a mass of 2.7 kg with centre of mass 11 cm for the elbow. The attachment of the triceps extensor muscle is 2.4 cm on the short end of the pivot and acts at 90° to the bone. What force must the triceps exert in order to hold the shot-put in static equilibrium?

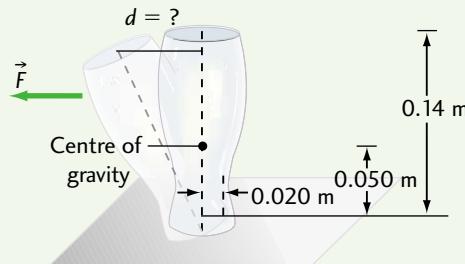
Fig.3.77



3.7 Stability and Equilibrium

- 47.** A square table is 0.6 m long with a centre of mass 0.6 m above the ground. What is its tipping angle?
- 48.** A square-based box of length 1.00 m and uniform weight distribution tips when tilted past 30° . How tall is the box?
- 49.** The centre of mass of a 0.14-m-tall drinking glass is 0.050 m from the bottom, which is a circle with radius 0.020 m, as shown in Figure 3.78. How far can the top of the glass be tipped without toppling it?

Fig.3.78

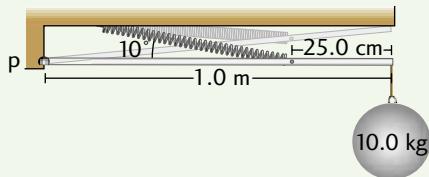


- 50.** A transport truck 4.3 m tall and 2.5 m wide has a centre of mass 2.5 m high along the midline. How steep a side slope can the truck be parked on without tipping over sideways?

3.8 Elasticity: Hooke's Law

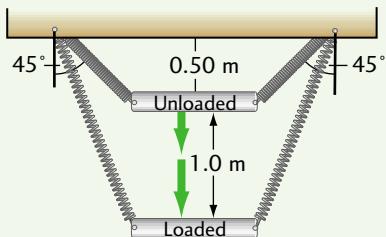
- 51.** A spring scale is used to measure the weight of nails in a hardware store. Nails of mass 3.0 kg cause the spring to stretch 1.8 cm. What is the spring constant?
- 52.** The spring in Figure 3.79 must support the lever in static equilibrium when a mass is placed on its end. What is the spring constant if the stretched spring is only 4.0 cm longer than its rest position?

Fig. 3.79



- 53.** A heavy steel bar is hung from the ceiling by two springs attached at each end, then released, as shown in Figure 3.80. What is the mass of the steel bar?

Fig. 3.80



3.9 Stress and Strain — Cause and Effect

- 54.** An aluminum wire is 20 m long and has a radius of 2.0 mm. The linear limit of force for aluminum is $6.0 \times 10^7 \text{ N/m}^2$.
- a)** What tension must be applied to reach this limit?
- b)** How much will the wire stretch when this force is applied?

- 55.** A 100-kg mass is suspended from the end of a vertical 2-m-long cast-iron post with a cross-sectional area of 0.1 m^2 .
- a)** What are the stress and strain on the post?
- b)** How much does this post stretch?
- c)** What is the maximum mass that can be suspended from this post?

- 56.** What tension load will cause a femur to fracture if the minimum cross-sectional area of this leg bone is $6.40 \times 10^{-4} \text{ m}^2$?
- 57.** What is the “spring constant” for a human femur under a compression force of 200 N if it has an average cross-sectional area of 10^{-3} m^2 with a length of 0.38 m?
- 58.** A cylindrical steel rod 2.0 m long has a radius of 0.01 m. If a load causes it to bend elastically with a radius of curvature of 20 m, what is the torque on the rod?

- 59.** A freight elevator and its contents have a mass of $1.00 \times 10^4 \text{ kg}$ and are at rest. The steel cable supporting them has a stress equal to 10% of its maximum tension.
- a)** What is the radius of the cable’s cross-section?
- b)** What is the strain on the cable when the elevator is accelerating upward at 2.0 m/s^2 ?

3.10 Stress and Strain in Construction

- 60.** A pine post with dimensions 10 cm by 15 cm by 3 m supports a load of 1000 N along its length.
- a)** What are the stress and strain on the post?
- b)** What is the change in length of the post while supporting this load?
- 61.** In ancient Rome, marble columns were used to support heavy structures. In one application, a cylindrical column 1.00 m in diameter and 22.0 m long was used to support a mass of $2.5 \times 10^4 \text{ kg}$. What length of unloaded column, 0.80 m in diameter, must be used to support the same mass at the same height when loaded?



Equilibrium in Forces

Purpose

To examine the first condition for static equilibrium: the balance of forces

Equipment

Force table or peg force board (see Chapter 3 STSE)

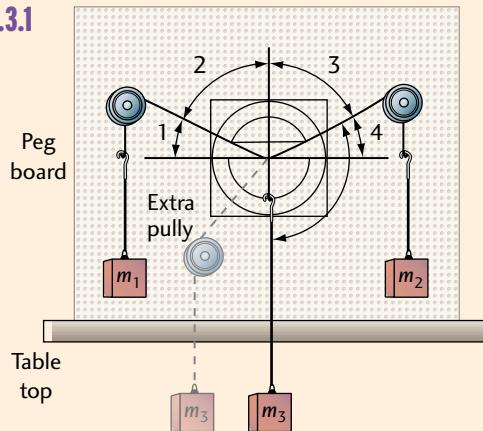
Protractor (360°) mounted on square background
3 pulleys

Masses (with hangers)

Small builder's level

String or fishing line

Fig. Lab.3.1



Procedure

- Set up a data table in your notebook similar to Table Lab.3.1.
- Set up the force board as shown in Figure Lab.3.1 with the two pulleys about 30 cm apart.
- Place one string across the two pulleys and fasten a mass, m_1 and m_2 , on each end such that they balance. The pulleys can be placed at any level to achieve balance.

- Tie a second piece of string to the horizontal string between the two pulleys and hang a third mass, creating a Y shape. Adjust the masses and the pulley positions such that the system reaches static equilibrium.
- Move the circular protractor to a point behind the strings. Measure the angles between the strings, as shown in Figure Lab.3.1. Use the builder's level to ensure that the top of the protractor is horizontal.
- Record the masses of m_1 , m_2 , and m_3 in the data table.
- Change masses m_1 and m_2 only. Reset the apparatus to static equilibrium and record all of the masses and angles in the data table.
- Apply a third pulley to the string attached to the middle mass, as shown in Figure Lab.3.1. Hold the pulley in place by hand and measure the angles (including the new one in the middle string) and masses being used when in static equilibrium. Record all your observations in the data table.

Uncertainty

Assign your angle measurements an instrumental uncertainty of $\pm 1^\circ$. The uncertainty of the masses will vary depending on the precision of the balance you used. For instructions on how to perform an uncertainty analysis, see Appendix C.

Table Lab.3.1

Masses (g)	Angles (degrees)					Forces (left/right) by masses (N)				Sum of all forces (N)	Force (middle mass)				
m_1	m_2	m_3	θ_1	θ_2	θ_3	θ_4	θ_5	$F_{1(v)}$	$F_{1(h)}$	$F_{2(v)}$	$F_{2(h)}$	$F_{t(v)}$	$F_{t(h)}$	$F_{3(v)}$	$F_{3(h)}$

Analysis

1. Calculate the sum of the vertical and horizontal components of the force supplied by the outer two masses (m_1 and m_2) for this lab. Record your answers in your data table.
2. Calculate the force of gravity on the middle mass (m_3) for steps 2–7, then calculate the horizontal and vertical components of this force for step 8. Record them in your data table.

Discussion

1. In each case, how does the sum of the vertical components relate to the force of gravity on the middle mass, within experimental uncertainty?

2. The sum of the horizontal components for steps 2–7 should equal zero. Is your sum approximately zero, considering the uncertainties?
3. How does the sum of the vertical and horizontal forces from the outer two masses compare with the vertical and horizontal forces of gravity on the middle mass in step 8?
4. Draw a free-body diagram that illustrates the first condition for static equilibrium.

Conclusion

Summarize how your results prove or disprove the first condition for static equilibrium.



Balancing Torque

Purpose

To examine the second condition for static equilibrium: the balance of torques

Equipment

Peg force/torque board (see Chapter 3 STSE)

Metre stick with hole drilled in centre

Newton spring scales

Masses with hangers

String

Pulleys (mounted on pegboard)

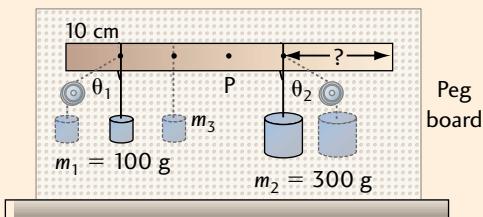
Procedure

1. Place the metre stick onto a peg on the pegboard such that it is horizontally balanced but can freely rotate.

Part A: Forces at 90°

2. Place a 100-g mass (m_1) at the 10-cm mark at the left end of the metre stick. Hang a 300-g mass (m_2) on the opposite end of the metre stick such that the metre stick is balanced horizontally, as illustrated in Figure Lab.3.2. Record all masses, angles, and positions in a data table in your notebook.

Fig.Lab.3.2

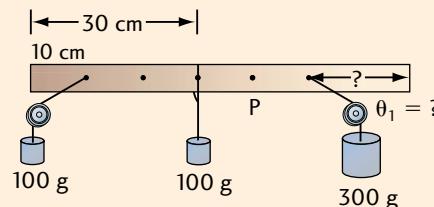


3. Add another 100-g mass, m_3 , at the 30-cm mark on the left side of the metre stick, beside m_1 . Move m_2 to the right along the metre stick until the metre stick is balanced, as shown in Figure Lab.3.2. Record all masses, angles, and positions of all three masses in your data table.

Part B: Forces at an Angle

4. Hold onto the metre stick and place a peg with pulley into the board two spaces to the left of the string from m_1 below the metre stick. Grasp the string from mass m_1 and drape it over the peg, as illustrated in Figure Lab.3.3.

Fig.Lab.3.3



5. Before releasing the metre stick, drape m_2 over a peg in a similar way that m_1 was draped but toward the right of the apparatus. Slide m_3 back and forth until the metre stick is balanced when released.
6. Record all masses, angles, and positions from the pivot in your data table.

Table Lab.3.2

Masses (kg)			Angles			Distance from pivot (m)			Torques (N·m)			Sum of all torques (N·m)
m_1 (g)	m_2 (g)	m_3 (g)	θ_1	θ_2	θ_3	r_1	r_2	r_3	τ_1	τ_2	τ_3	τ_{total}
Part A: Forces at 90°												
100	300	---	90°	90°	---				---			---
100	300	100	90°	90°	90°							
Part B: Forces at an angle												

Uncertainty

Assign instrumental and experimental uncertainties to your measurements. When working with uncertainties and trigonometric functions such as the sine function, a high- and low-range calculation may be performed (see Appendix C).

An uncertainty of $\pm 1^\circ$ leads to an interesting result when used in torque calculations.

$$\text{Example: } (100 \text{ N})(1 \text{ m})\sin 26^\circ = 43.8 \text{ N}\cdot\text{m}$$

$$(100 \text{ N})(1 \text{ m})\sin 25^\circ = 42.3 \text{ N}\cdot\text{m}$$

$$(100 \text{ N})(1 \text{ m})\sin 24^\circ = 40.7 \text{ N}\cdot\text{m}$$

The difference between the high and middle marks is $1.5 \text{ N}\cdot\text{m}$, but the difference between the lower two marks is 1.6 because of the nature of the sine function. In this case, the larger of the two uncertainties is usually used.

Analysis

Calculate the torque applied by each of the masses in Parts A and B, then find the sum of all the torques. Place all your results and uncertainties in the data table. Be sure to assign a positive or negative to the clockwise or counterclockwise rotational direction.

Discussion

1. The sum of all of the torques should be zero if the metre stick did not rotate. Is your sum close to zero, considering all of the uncertainties? Give reasons for any discrepancy.
2. Friction within the main pivot as well as with the pulleys in Part B may cause the second condition for static equilibrium to not be witnessed in this lab, even when considering experimental uncertainties. Describe how friction in the pivot would affect your results.

Conclusion

Summarize how the results of your own lab prove or disprove the second condition for static equilibrium.

UNIT

B

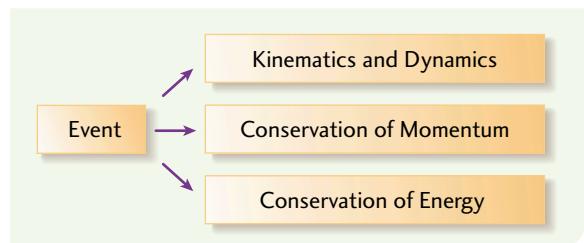
Energy and Momentum

- 4 Linear Momentum
- 5 Energy and Interactions
- 6 Energy Transfer
- 7 Angular Motion



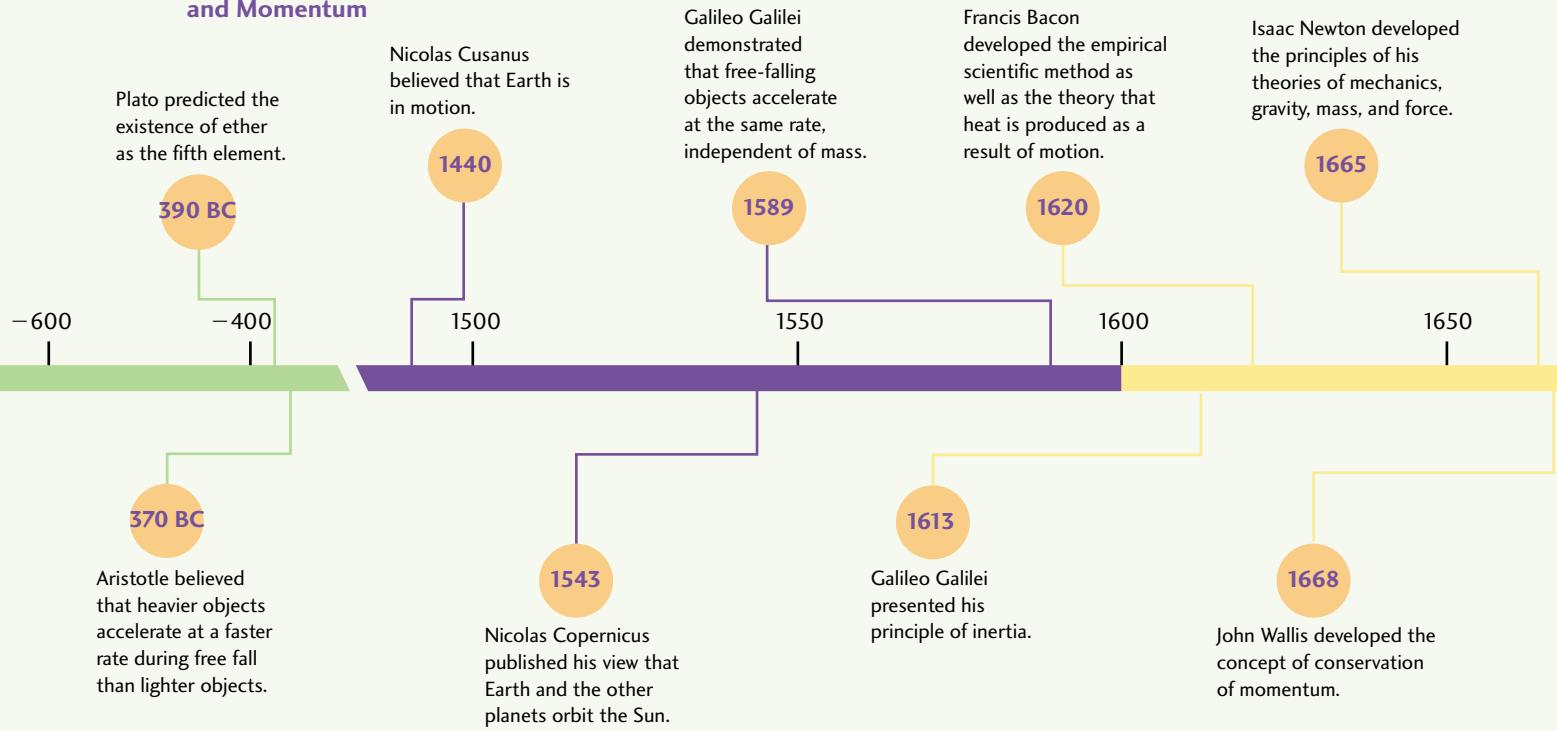
What does the expression “Energy cannot be created or destroyed” really mean? How does it relate to events such as a billiard ball collision, a car crash, or an arrow passing through an apple? In the *Principia*, Newton considered the quantity of motion as arising from “velocity and mass conjointly.” This fundamental quantity is known as *momentum*. What role does momentum play in the examples just mentioned?

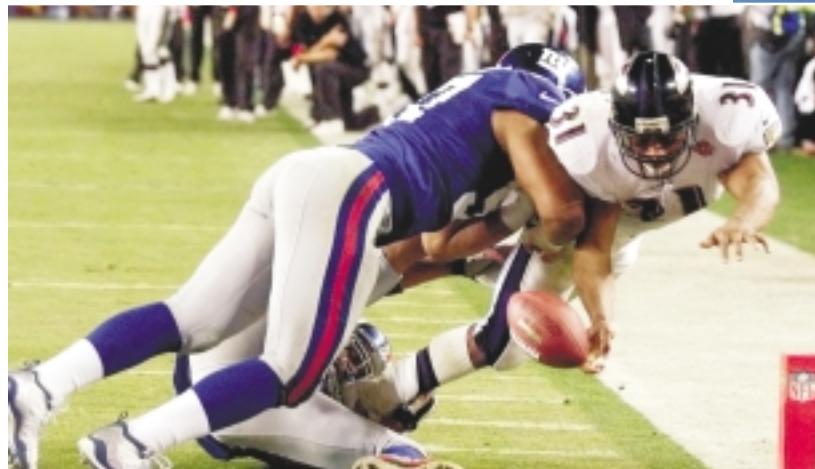
In this unit, we will introduce the concepts of energy and momentum and their interrelationship, along with work and power. These concepts lead to two fundamental laws: the law of conservation of energy and the law of conservation of momentum. Using these laws, we will investigate the interaction of objects in collisions and the relationship between objects and forces. The laws of conservation of energy and of momentum can be used to describe how shock absorbers cushion the ride of a car and how running shoes preserve our knee joints. The interaction of objects in motion with other objects causes the transfer of energy and momentum.



The study of everyday interactions of objects in terms of energy and momentum complements our explanations of motion in terms of dynamics in the previous unit and helps us obtain a more complete understanding of ordinary events.

Timeline: The History of Energy and Momentum





Robert Hooke developed the laws of springs and elasticity.

1676

1700

1750

Benjamin Thompson realized that the heat generated by motion equals the amount of work done.

1798

1800

1850

1900

Isaac Newton published the inverse square law for gravitational attraction.

1684

Gottfried Leibniz published the concept of conservation of energy.

1714

William Thomson (Lord Kelvin) developed the concept of absolute zero and the absolute temperature scale.

4

Linear Momentum

Chapter Outline

- 4.1 Introduction to Linear Momentum
- 4.2 Linear Momentum
- 4.3 Linear Momentum and Impulse
- 4.4 Conservation of Linear Momentum in One Dimension
- 4.5 Conservation of Linear Momentum in Two Dimensions
- 4.6 Linear Momentum and Centre of Mass

 Recreational Vehicle Safety and Collisions

LAB 4.1 Linear Momentum in One Dimension:
Dynamic Laboratory Carts

LAB 4.2 Linear Momentum in Two Dimensions:
Air Pucks (Spark Timers)

LAB 4.3 Linear Momentum in Two Dimensions:
Ramp and Ball



By the end of this chapter, you will be able to

- solve problems involving linear momentum and impulse
- use the law of conservation of linear momentum to solve momentum problems in one and two dimensions
- apply the concept of linear momentum to everyday situations

4.1 Introduction to Linear Momentum

Have you ever seen a car accident? Was it a head-on collision? Was one car travelling faster than the other? Have you ever been in-line skating, cycling, or skateboarding and had to execute a tight turn in order to avoid an object at rest? Would you have reacted differently if the object in your path was a soccer ball as opposed to a parked car? In either case, the masses of the objects involved and their velocities play a role in a collision.

Momentum and energy are two very important concepts in the study of physics. In this chapter, we will look at the effects of mass and velocity on objects involved in collisions and how these quantities are related in momentum. We will also study the conservation of linear momentum in one-dimensional and two-dimensional systems. The concept of centre of mass in relation to linear momentum will also be discussed.

4.2 Linear Momentum

The concept of momentum was first developed by Sir Isaac Newton, who thought that a change in momentum was caused by a force. He called **linear momentum** “the quantity of motion” and combined a moving object’s mass and velocity in the following way:

$$\vec{p} = m\vec{v}$$

where \vec{p} is the object’s linear momentum, in kilogram metres per second ($\text{kg}\cdot\text{m/s}$, the SI unit for momentum), m is the mass of the object, in kilograms (kg), and \vec{v} is the velocity of the object, in metres per second (m/s).

Linear momentum is a vector quantity that has the same direction as the velocity of the object. If a direction is not given in a problem, assume that the object is moving in the positive direction.

Figure 4.1 shows an in-line skater travelling east along a path. The skater’s momentum is also directed east.

Figure 4.2 shows a five-pin and a ten-pin bowling ball. Which ball has more momentum?

This question cannot be answered because the momentum depends on both the mass and velocity of the balls. Since the velocities are unknown, the momentum is unknown. However, a fast-moving ball will have greater momentum than a slow-moving ball of the same mass.

Fig.4.1 The in-line skater’s velocity and linear momentum are both in the same easterly direction. The direction of the linear momentum is always the same as the direction of the velocity.

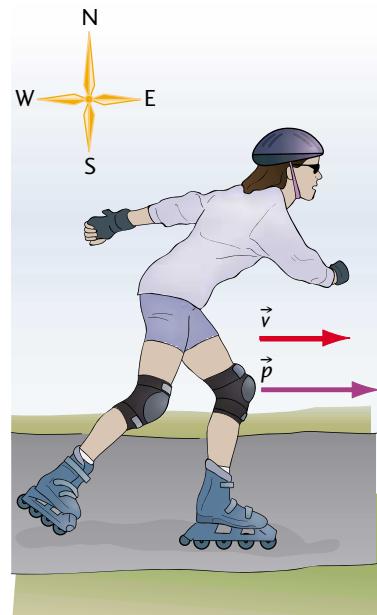
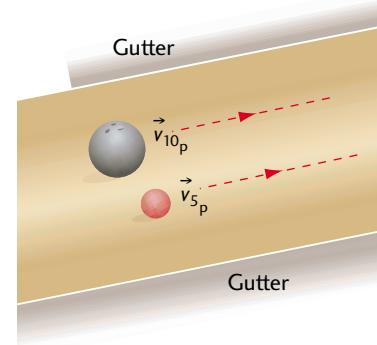


Fig.4.2 The linear momentum of an object varies directly as the mass and the velocity of the object



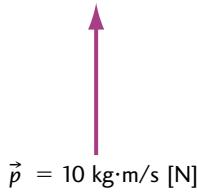
EXAMPLE 1

Calculating linear momentum

Calculate the momentum of a 50-g bullet travelling at 200 m/s [N].

Fig.4.3 Momentum is a vector. Its direction is indicated by an arrow, and its magnitude is indicated by the arrow's length.

Scale: 1 cm = 5 kg·m/s



Solution and Connection to Theory

Given

$$m = 50 \cancel{\text{g}} \left(\frac{1 \text{ kg}}{1000 \cancel{\text{g}}} \right) = 0.050 \text{ kg} \quad \vec{v} = 200 \text{ m/s} [\text{N}]$$

Using the momentum equation and assuming north to be the positive direction,

$$\vec{p} = m\vec{v}$$

$$p = (0.050 \text{ kg})(200 \text{ m/s})$$

$$p = 10 \text{ kg}\cdot\text{m/s}$$

Therefore, the momentum of the bullet is 10 kg·m/s [N].

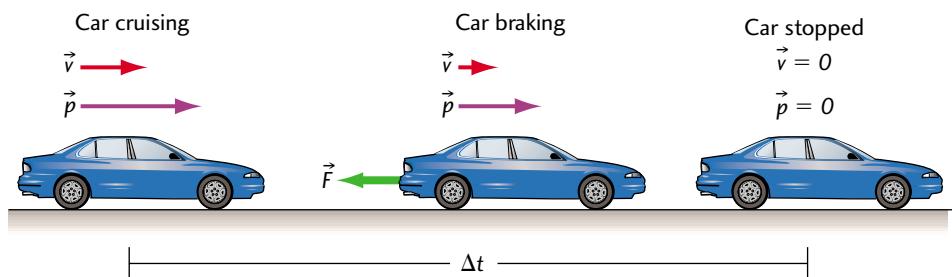


- What is the momentum of an 8.0-kg shot-put moving at 16 m/s [W20°N]? Draw a scale vector diagram to represent this situation.
- Determine the mass of a car that is travelling eastbound at 72 km/h with a momentum of $9.0 \times 10^4 \text{ kg}\cdot\text{m/s}$ [E].
- Draw a vector diagram to show
 - a 0.5-kg baseball travelling south toward home plate at 32 m/s.
 - a 0.5-kg baseball travelling north away from home plate at 45 m/s.
 - Calculate the change in momentum of the ball if its motion changes from a) to b). Draw a scale diagram.

4.3 Linear Momentum and Impulse

In order to change an object's momentum, we must change either its velocity or its mass. To change velocity, we need to apply a net force on the object. Newton suggested that the rate of change of momentum in an object is directly proportional to the net force applied to it.

Fig.4.4 A vehicle changes its speed as a force is applied during a time interval, Δt , thereby changing the linear momentum of the car



A driver approaching a red light must apply the brakes in order to reduce the vehicle's velocity. The force of friction between the brake pads and disks, and between the rubber tires and the road, allows the driver to slow down and to eventually stop. The force causes the momentum to change to zero in a time interval Δt . Mathematically, this change in momentum with respect to time can be written as $\Delta \vec{p}$ and

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

where \vec{F} is the force applied, measured in newtons (N), $\Delta \vec{p}$ is the change in momentum, measured in kilogram metres per second ($\text{kg} \cdot \text{m/s}$), and Δt is the change in time, measured in seconds (s).

The force applied and the change in momentum are both vector quantities. The direction of the force applied is the same as the direction of the change in momentum. The change in momentum can also be written as

$$\Delta \vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}}$$

where \vec{p}_{final} and \vec{p}_{initial} are the final momentum and initial momentum, respectively. To simplify this statement, we can use the subscript o to represent the initial or original momentum and the subscript f to represent the final momentum. The equation for the change in momentum is written as

$$\Delta \vec{p} = \vec{p}_f - \vec{p}_o$$

The change in linear momentum is called **impulse**, \vec{J} . Mathematically,

$$\vec{J} = \Delta \vec{p}$$

Isolating $\Delta \vec{p}$ in $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$ gives

$$\vec{J} = \vec{F} \Delta t$$

where \vec{J} is measured in newton-seconds ($\text{N} \cdot \text{s}$), \vec{F} is the net force, measured in newtons (N), and Δt is the time interval during which the force was applied, measured in seconds (s).

The impulse, the net force, and the change in momentum are all vector quantities with the same direction. Combining the two equations for the impulse, $\vec{J} = \Delta \vec{p}$ and $\vec{J} = \vec{F} \Delta t$, we obtain

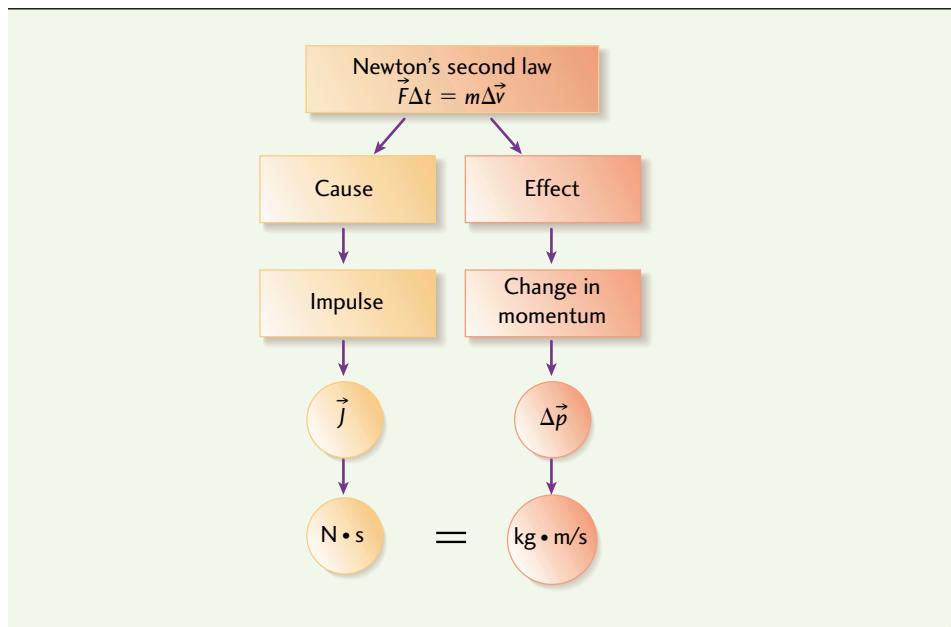
$$\begin{aligned}\vec{F} \Delta t &= \Delta \vec{p} \\ \vec{F} \Delta t &= \vec{p}_f - \vec{p}_o \\ \vec{F} \Delta t &= m \vec{v}_f - m \vec{v}_o \\ \vec{F} \Delta t &= m(\vec{v}_f - \vec{v}_o) \\ \vec{F} \Delta t &= m \Delta \vec{v}\end{aligned}$$

$$\begin{aligned}\vec{F} \Delta t &= m \Delta \vec{v} \\ \vec{F} &= \frac{m \Delta \vec{v}}{\Delta t} \\ \text{But } \vec{a} &= \frac{\Delta \vec{v}}{\Delta t}; \text{ therefore,} \\ \vec{F} &= m \vec{a} \text{ (Newton's second law)}\end{aligned}$$

which is the equation for Newton's second law of motion.

Figure 4.5 summarizes when to use the equation for impulse and when to use the equation for momentum.

Fig.4.5 Cause and Effect from Newton's Second Law



EXAMPLE 2

The impulse on a shell

The average accelerating force exerted on a 2.00-kg shell in a gun barrel is 1.00×10^4 N, and the muzzle velocity is 200 m/s. Calculate

- a) the impulse on the shell.
- b) the length of time it takes for the shell to exit the heavy gun barrel.

Solution and Connection to Theory

Muzzle velocity is the shell's velocity as it exits the gun barrel. In this case, the shell starts from rest and attains a velocity of 200 m/s. We will consider the shell's direction along the barrel of the gun as positive. Also associated with firearms is the **recoil velocity**. When a rifle, handgun, or cannon is fired, the device kicks backwards because of the force of the explosion.

Given

$$m = 2.00 \text{ kg} \quad F = 1.00 \times 10^4 \text{ N} \quad v_0 = 0 \quad v_f = 200 \text{ m/s}$$

- a)** Assuming the bullet goes in the positive direction, from the impulse equation,

$$\vec{J} = \Delta \vec{p}$$

$$\vec{J} = \vec{p}_f - \vec{p}_o$$

$$\vec{J} = m\vec{v}_f - m\vec{v}_o, \text{ where } \vec{v}_o = 0$$

$$J = (2.00 \text{ kg})(200 \text{ m/s})$$

$$J = 400 \text{ kg·m/s}$$

The impulse on the shell is 400 N·s along the barrel of the gun.

- b)** To find the length of time it took the shell to exit the gun barrel, we can use our result from part a):

$$\vec{J} = \vec{F}\Delta t$$

$$\Delta t = \frac{\vec{J}}{\vec{F}}$$

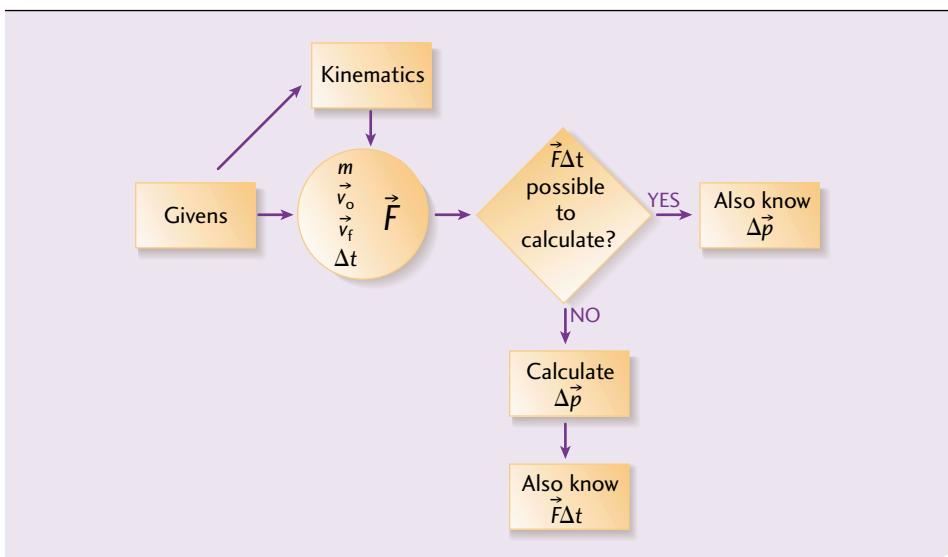
$$\Delta t = \frac{400 \text{ kg·m/s}}{1.00 \times 10^4 \text{ N}}$$

$$\Delta t = 0.0400 \text{ s}$$

The time it takes for the shell to leave the gun is $4.00 \times 10^{-2} \text{ s}$.

Figure 4.6 summarizes the steps to follow when solving impulse and linear momentum problems.

Fig.4.6 Calculating Impulse and Momentum



EXAMPLE 3

Solving impulse and momentum problems

UNIT ANALYSIS

Show that $\text{N}\cdot\text{s}/\text{kg} = \text{m/s}$

Starting with

$\text{N}\cdot\text{s}/\text{kg}$, where $1 \text{ N} = 1 \text{ kg}\cdot\text{m/s}^2$

$\text{N}\cdot\text{s}/\text{kg} = \text{kg}\cdot\text{m/s}^2\cdot\text{s/kg}$

Cancelling the common factors,
we are left with

$\text{N}\cdot\text{s}/\text{kg} = \text{m/s}$

What velocity will a 300-kg snowmobile acquire if pushed from rest by a force of 6240 N [E] for 1.25 s? What average force will stop this snowmobile from moving at this speed in 1.25 s?

Solution and Connection to Theory**Given**

$$m = 300 \text{ kg} \quad \vec{F} = 6240 \text{ N [E]} \quad \Delta t = 1.25 \text{ s} \quad \vec{v}_0 = 0$$

Let's assume that east is positive.

Using $\vec{F}\Delta t = m\Delta\vec{v}$,

$$\vec{F}\Delta t = m(\vec{v}_f - \vec{v}_0), \text{ where } \vec{v}_0 = 0.$$

$$\vec{v}_f = \frac{(\vec{F}\Delta t)}{m}$$

$$v_f = \frac{(6240 \text{ N})(1.25 \text{ s})}{300 \text{ kg}}$$

$$v_f = 26.0 \text{ m/s}$$

Therefore, the velocity of the snowmobile after the constant force was applied is 26.0 m/s [E].

In order to find the force required to stop the snowmobile,

$$\vec{F}\Delta t = m\Delta\vec{v}$$

$$\vec{F}\Delta t = m(\vec{v}_f - \vec{v}_0), \text{ where } \vec{v}_f = 0 \text{ because the snowmobile comes to a stop.}$$

Therefore,

$$\vec{F}\Delta t = -m\vec{v}_0$$

$$\vec{F} = \frac{-m\vec{v}_0}{\Delta t}$$

$$F = \frac{[-(300 \text{ kg})(26.0 \text{ m/s})]}{1.25 \text{ s}}$$

$$F = -6240 \text{ kg}\cdot\text{m/s}^2$$

Therefore, the force required to stop the snowmobile in 1.25 s is $-6.24 \times 10^3 \text{ N [E]}$ or $6.24 \times 10^3 \text{ N [W]}$. This force has the same magnitude but the opposite direction of the initial force.

Force-versus-Time Graphs

Another way to represent impulse is by graphing the force applied versus time. We will consider three different situations: a constant force, a uniformly varying force, and a non-uniformly varying force.

Fig. 4.7a A racecar accelerates at a uniform rate as a result of a constant applied force ($\vec{a} = \frac{\vec{F}}{m}$)



Fig. 4.7b A constant force of 400 N over 5 s allows the racecar to accelerate from rest

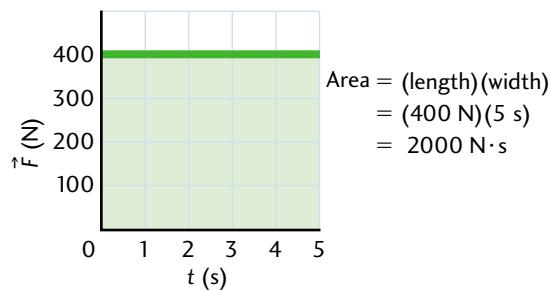


Figure 4.7b represents a constant force of 400 N [forward], applied over a given time period on a racecar accelerating from rest. Since the force is constant, we can calculate the impulse at any time interval. For instance, the impulse for the first 5 s is

$$\begin{aligned} \vec{J} &= \vec{F}\Delta t \\ J &= (400 \text{ N})(5 \text{ s}) \\ J &= 2000 \text{ N}\cdot\text{s} \end{aligned}$$

Notice that the area under the force-versus-time graph also represents impulse. What if the force is not constant? The impulse, \vec{J} is still the area under the force-versus-time graph. $\vec{J} = \vec{F}\Delta t$ is an algebraic description of the impulse only when the force is constant.

Fig. 4.8 Impulse in sports



E X A M P L E 4**A graphical representation of impulse**

Calculate the impulse for the time interval shown in each of the following graphs.

- a) Figure 4.9a represents a decreasing force over a time period, such as a stretched elastic band on a slingshot that is released.

Fig.4.9a A stretched elastic band applies more force than a limp elastic band. The applied force is decreasing over time as the stretched band returns to its relaxed state.

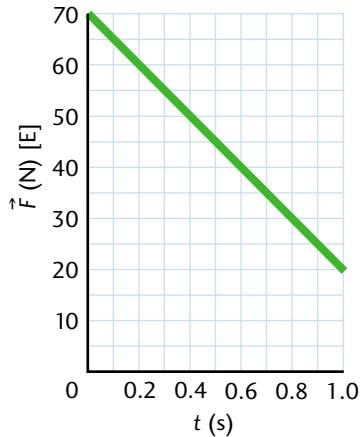
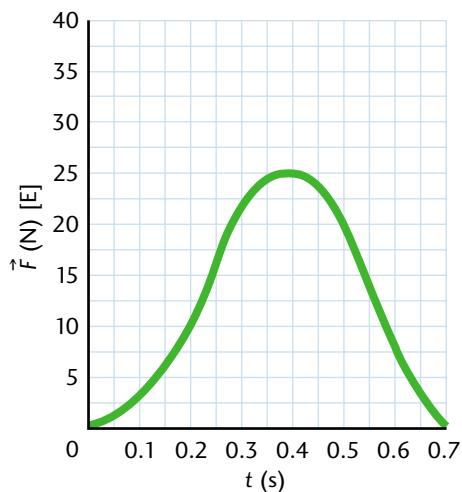


Fig.4.9c



Fig.4.9b The varying force of a tennis racket on a tennis ball. The force increases, then decreases over the time interval.



- b) Figure 4.9b represents a varying force over a short time period. Varying forces are found in sports, such as a golf driver coming into contact with a golf ball, or a tennis racket coming into contact with a tennis ball. The force on the tennis ball increases with the greater indentation of the racket, as illustrated in Figure 4.9c.

Solution and Connection to Theory

The impulse in Figure 4.9a is found by calculating the area under the graph (see Figure 4.10a). The shape under the graph is a trapezoid. The area of a trapezoid is given by

$$A = \frac{1}{2}(a + b)h, \text{ where } A = \vec{J} \text{ and } a = \vec{F}_o, b = \vec{F}_f, \text{ and } h = \Delta t$$

$$\vec{J} = \frac{1}{2}(\vec{F}_o + \vec{F}_f)\Delta t$$

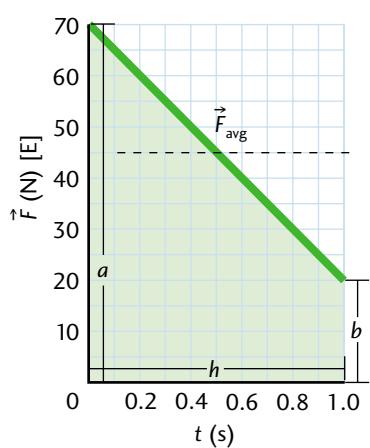
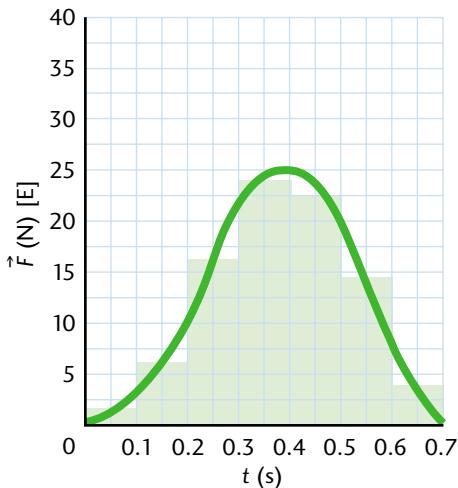
$$J = \frac{1}{2}(70 \text{ N} + 20 \text{ N})(1.0 \text{ s})$$

$$J = 45 \text{ N}\cdot\text{s}$$

The impulse applied in 1.0 s is 45 N·s [E].

Figure 4.10a also shows a horizontal line at \vec{F}_{avg} . The area under this line, $\vec{F}_{avg}\Delta t$, also represents the impulse for this uniformly changing force.

We can't use $\vec{J} = \vec{F}\Delta t$ to find impulse because the forces are not constant.

Fig.4.10a**Fig.4.10b**

- c) The force applied in Figure 4.9b is non-uniformly changing. Since we don't know the average force, we can't use the equation $\vec{F}_{\text{avg}}\Delta t$. Our only recourse is to determine the area under the graph. As illustrated in Figure 4.10b, we can only estimate its area by counting the total number of squares and multiplying that number by the length and width of each square. There are approximately 72 squares, each with an area of $(2.5 \text{ N})(0.05 \text{ s}) = 0.125 \text{ N}\cdot\text{s}$. The total area is therefore approximately $(72)(0.125 \text{ N}\cdot\text{s}) = 9 \text{ N}\cdot\text{s} [\text{E}]$.

ALTERNATIVE SOLUTION

Divide the graph time interval into n sub-intervals, each of duration $\frac{\Delta t}{n}$. Assuming the force is constant for each interval at F_{avg} , the total area (impulse) is the sum of the areas of the sub-intervals:

$$\vec{J} = \vec{F}_1\Delta t + \vec{F}_2\Delta t + \vec{F}_3\Delta t \dots + \vec{F}_n\Delta t$$

If we divide the 0.7-s interval into seven $\frac{1}{10}$ -s intervals as in Figure 4.10b, then the total impulse is

$$\vec{J} = \vec{F}_1\Delta t + \vec{F}_2\Delta t + \vec{F}_3\Delta t \dots + \vec{F}_n\Delta t$$

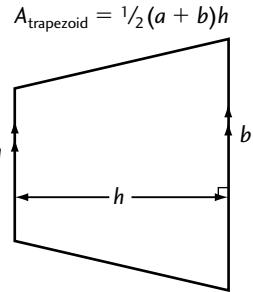
$$J = (2 \text{ N})(0.1 \text{ s}) + (7 \text{ N})(0.1 \text{ s}) + (16 \text{ N})(0.1 \text{ s}) + (24 \text{ N})(0.1 \text{ s}) + (23 \text{ N})(0.1 \text{ s}) + (14 \text{ N})(0.1 \text{ s}) + (4 \text{ N})(0.1 \text{ s})$$

$$J = 9 \text{ N}\cdot\text{s}$$

The impulse is $9 \text{ N}\cdot\text{s} [\text{E}]$.

In calculus, the method for finding the area under a curve is integration. The equation for calculating impulse is written as

$$J = \int_{t_1}^{t_2} F \cdot dt$$

Fig.4.11 The area of a trapezoid

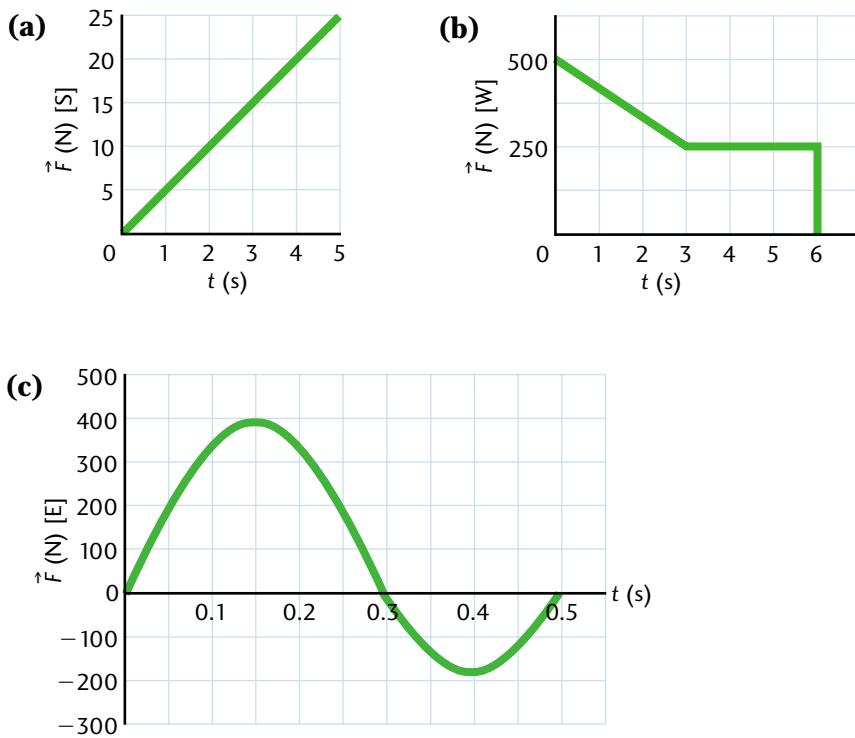
The expression $\frac{1}{2}(\vec{F}_o + \vec{F}_f)$ represents the average force applied, written as \vec{F}_{avg} .

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t$$



1. Calculate the impulse on each of the following objects.
 - a) A force of 3257 N [forward] is applied to a 2000-kg car for 1.3 s.
 - b) A 30.0-g bullet is fired from a gun. The bullet's speed increases from 0 m/s to 200 m/s in 0.05 s.
 - c) A 500-g ball falls vertically for 3 s.
2. A 54-kg truck tire strikes the pavement with a speed of 25 m/s [down] and rebounds with a speed of 20 m/s [up]. Ignoring any effects due to air resistance, determine the change in the tire's momentum.
3. The impulse on a human cannon ball is $2.5 \times 10^3 \text{ N}\cdot\text{s}$. The cannon ball has a mass of 65 kg.
 - a) What force does the cannon exert on the human cannon ball if it takes 0.2 s for the human cannon ball to leave the cannon?
 - b) How long is the barrel of the cannon if the cannon ball leaves the cannon at 120 km/h?
4. Calculate the impulse for each situation in Figure 4.12.

Fig. 4.12



4.4 Conservation of Linear Momentum in One Dimension

The **law of conservation of energy** states that energy cannot be created or destroyed; it can only change from one form to another. The same concept holds true for momentum. In the 17th century, Sir Isaac Newton recognized that momentum is conserved in a collision. The total momentum of a system before a collision is equal to the total momentum of the system after collision. This statement is known as the **law of conservation of linear momentum**. It applies to all collisions as long as the net external force acting on the system is zero.

A **system** represents all the objects involved in a collision. If the net external force on all objects as a group is zero, we say that the system is an isolated or a **closed system**. For instance, a ball rolling along a frictionless horizontal surface with no external forces acting on the ball is a closed system. On the other hand, a ball thrown upwards is not a closed system because the external force of Earth's gravity is pulling on it.

The conservation of momentum is written as

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$

$$\vec{p}_{T_0} = \vec{p}_{T_f}$$

Open, closed, and isolated systems will be further discussed in Chapter 5.

For two solid objects colliding, the conservation of momentum can be written as

$$m_1 \vec{v}_{1_0} + m_2 \vec{v}_{2_0} = m_1 \vec{v}_{1_f} + m_2 \vec{v}_{2_f}$$

The subscripts 1 and 2 refer to the different objects involved in a collision. The first part of the equation, $m_1 \vec{v}_{1_0} + m_2 \vec{v}_{2_0}$, represents the initial momentum (i.e., the momentum before the collision). The second half of the equation, $m_1 \vec{v}_{1_f} + m_2 \vec{v}_{2_f}$, represents the final momentum (i.e., the momentum after the collision) if the masses remain intact and their velocities change.

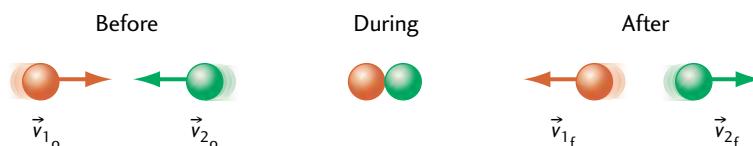


Fig.4.13 Momentum is conserved when two billiard balls collide head on

The law of conservation of momentum is very useful in predicting what will happen in a collision. Many real-life situations involve collisions in some form or another, from gas molecules in chemistry, police investigations of car accidents, and ballistics, to the study of subatomic particles and light by physicists.

EXAMPLE 5

Conservation of linear momentum
when firing a cannon

A shell of mass 7.0 kg leaves the muzzle of a cannon with a horizontal velocity of 490 m/s [right]. Find the recoil velocity of the cannon if its mass is 700 kg.

Fig.4.14 As a cannon fires, the cannon itself recoils or moves backward



THRUST

A phenomenon similar to muzzle velocity occurs in rocket propulsion. The rocket ejects gases from its tail at a high velocity, just as a rifle ejects bullets from its barrel. A rocket's mass isn't constant because the fuel it contains is constantly decreasing. The resultant force is called the **thrust**:

$$\vec{F}_{\text{thrust}} = \vec{v}_{\text{gas}} \left(\frac{\Delta m}{\Delta t} \right)$$

A simplification:

$$\vec{p}_{T_0} = 0 \quad (\vec{v}_{s_0} = \vec{v}_{c_0} = 0 \text{ m/s})$$

$$\text{so } \vec{p}_{T_f} = 0$$

Therefore,

$$m_s \vec{v}_{s_f} = -m_c \vec{v}_{c_f}$$

and

$$\vec{v}_{c_f} = \frac{-m_s \vec{v}_{s_f}}{m_c}$$

$$v_{c_f} = \frac{-(7.0 \text{ kg})(490 \text{ m/s})}{700 \text{ kg}}$$

$$v_{c_f} = -4.9 \text{ m/s}$$

Solution and Connection to Theory

Let the subscripts *s* and *c* represent the shell and the cannon, respectively. Right is the positive direction.

Before firing

$$m_s = 7.0 \text{ kg}$$

$$m_c = 700 \text{ kg}$$

$$\vec{v}_{s_0} = 0 \text{ m/s} \text{ (starts from rest)}$$

$$\vec{v}_{c_0} = 0 \text{ m/s} \text{ (starts from rest)}$$

After firing

$$m_s = 7.0 \text{ kg}$$

$$m_c = 700 \text{ kg}$$

$$\vec{v}_{s_f} = 490 \text{ m/s [right]} = +490 \text{ m/s}$$

$$\vec{v}_{c_f} = ?$$

Because the cannon is recoiling, we know that the direction of its velocity is to the left, or negative. Using the law of conservation of linear momentum,

$$\vec{p}_{T_0} = \vec{p}_{T_f}$$

$$m_s \vec{v}_{s_0} + m_c \vec{v}_{c_0} = m_s \vec{v}_{s_f} + m_c \vec{v}_{c_f}$$

$$\vec{v}_{c_f} = \frac{m_s \vec{v}_{s_0} + m_c \vec{v}_{c_0} - m_s \vec{v}_{s_f}}{m_c}$$

$$v_{c_f} = \frac{(7.0 \text{ kg})(0 \text{ m/s}) + (700 \text{ kg})(0 \text{ m/s}) - (7.0 \text{ kg})(+490 \text{ m/s})}{700 \text{ kg}}$$

$$v_{c_f} = -4.9 \text{ m/s}$$

The velocity of the cannon after collision is -4.9 m/s or 4.9 m/s [left] .

The velocities of the cannon and cannon ball are both measured with respect to Earth. With respect to the cannon, the speed of the cannon ball is 494.9 m/s ($490 \text{ m/s} + 4.9 \text{ m/s}$). In the case of a rifle and bullet, this speed would represent the **muzzle velocity**.

EXAMPLE 6 A collision where the masses stick together

An arrow flying at 60 m/s strikes and imbeds itself in a 300-g apple at rest. After impact, the apple and arrow move horizontally at 12 m/s. What is the mass of the arrow?

Solution and Connection to Theory

Assume that forward motion is positive and that the arrow is travelling forward.

Before collision

$$m_{\text{apple}} = 300 \text{ g} = 0.300 \text{ kg}$$

$$m_{\text{arrow}} = ?$$

$$v_{\text{apple}_0} = 0 \text{ m/s}$$

$$v_{\text{arrow}_0} = 60 \text{ m/s}$$

After collision

Since the arrow is embedded in the apple, we can write the mass and velocity after the collision as

$$m_{\text{total}} = m_{\text{apple}} + m_{\text{arrow}} \text{ and } \vec{v}_f = 12 \text{ m/s [forward]}$$

Using the law of conservation of linear momentum,

$$\vec{p}_{T_0} = \vec{p}_{T_f}$$

$$m_{\text{apple}} \vec{v}_{\text{apple}_0} + m_{\text{arrow}} \vec{v}_{\text{arrow}_0} = (m_{\text{apple}} + m_{\text{arrow}}) \vec{v}_f$$

Since $\vec{v}_{\text{apple}_0} = 0$,

$$m_{\text{arrow}} \vec{v}_{\text{arrow}_0} = (m_{\text{apple}} + m_{\text{arrow}}) \vec{v}_f$$

$$m_{\text{arrow}} \vec{v}_{\text{arrow}_0} = m_{\text{apple}} \vec{v}_f + m_{\text{arrow}} \vec{v}_f$$

$$m_{\text{arrow}} = \frac{m_{\text{apple}} \vec{v}_f}{\vec{v}_{\text{arrow}_0} - \vec{v}_f}$$

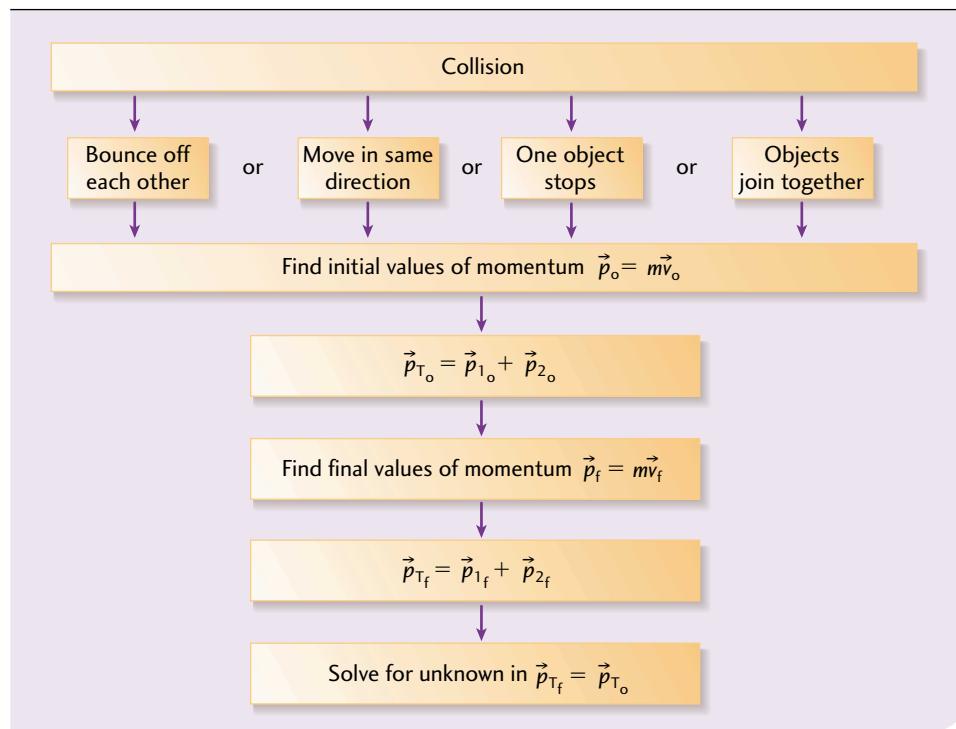
$$m_{\text{arrow}} = \frac{(0.300 \text{ kg})(12 \text{ m/s})}{60 \text{ m/s} - 12 \text{ m/s}}$$

$$m_{\text{arrow}} = 0.075 \text{ kg}$$

The mass of the arrow is 75 g.

Figure 4.15 summarizes how to solve problems involving the conservation of linear momentum.

Fig.4.15 Momentum is Conserved in All Situations

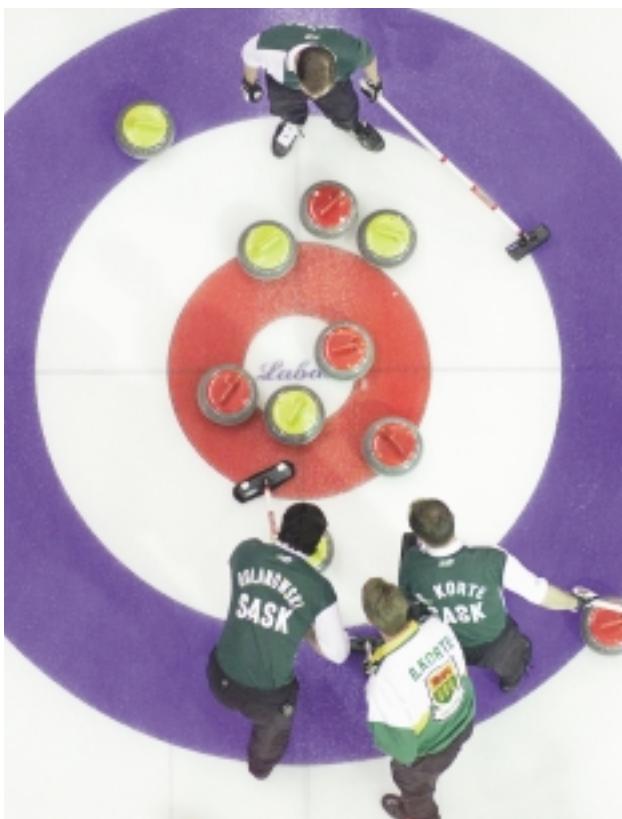


1. A lab cart of mass 1.2 kg and a velocity of 6.4 m/s [forward] collides with a stationary lab cart of mass 3.6 kg. Calculate the velocity of the second cart if the first cart rebounds with a velocity of 1.2 m/s [backward].
2. Find the recoil velocity of a 1.9-kg rifle if a 30-g bullet has a velocity of 750 m/s after firing.
3. A cue ball of mass 400 g hits the stationary eight ball of the same mass head on with a top spin. If the velocity of the cue ball is 3.0 m/s [forward] before collision and 1.0 m/s [forward] after collision, determine the velocity of the eight ball after collision.
4. An unstable atom of momentum 7.9×10^{-17} kg·m/s [left] disintegrates into two particles, one of which has a mass 80 times that of the other. If the larger particle moves to the right at 4.5×10^3 m/s and the smaller particle moves to the left with a speed of 1.5×10^6 m/s, what is the mass of the smaller particle?
5. Five coupled freight cars, each of mass m , are travelling at a constant speed, v , on a straight and level track. They collide with two coupled stationary cars, each of mass $2m$. If all the cars are coupled together after the collision, what is their common speed?

4.5 Conservation of Linear Momentum in Two Dimensions

During a **glancing collision**, the objects involved are deflected in more than one dimension. Typically, in a curling shot (Figure 4.16), the stones that collide move away at various angles because the collision was not a head-on collision. In this section, we will look at two-dimensional collisions and apply the law of conservation of momentum using vector addition.

Fig. 4.16 The game of curling often involves glancing collisions



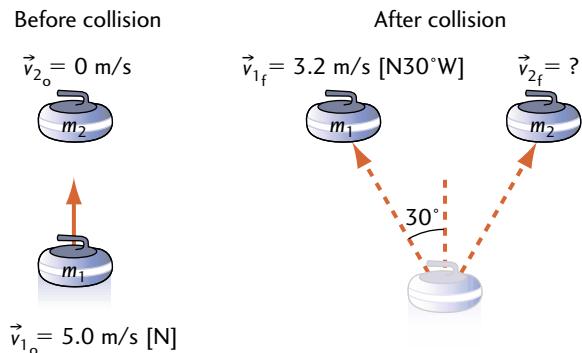
EXAMPLE 7

Solving momentum problems in two dimensions involving equal masses

Two identical curling stones of mass 19.5 kg collide, as shown in Figure 4.17. The first stone hits the stationary second stone with a velocity of 5.0 m/s [N]. If the velocity of the first stone is 3.2 m/s [N30°W] after collision, find the velocity of the second stone after collision. Omit any effects due to friction.

Solution and Connection to Theory

Fig.4.17 When solving momentum problems, we always draw two diagrams: one to represent the momentum before the collision, and one to represent the momentum after the collision



Method 1: Components

Let the subscripts 1 and 2 represent the first and second curling stones.

Given

$$m_1 = m_2 = 19.5 \text{ kg}$$

Before collision

$$\vec{v}_{1o} = 5.0 \text{ m/s [N]}$$

$$\vec{v}_{2o} = 0$$

After collision

$$\vec{v}_{1f} = 3.2 \text{ m/s [N30°W]}$$

$$\vec{v}_{2f} = ?$$

$$\vec{p}_{1o} = (19.5 \text{ kg})(5.0 \text{ m/s [N]}) = 97.5 \text{ kg}\cdot\text{m/s [N]}$$

$$\vec{p}_{2o} = 0$$

$$\vec{p}_{1f} = (19.5 \text{ kg})(3.2 \text{ m/s [N30°W]}) = 62.4 \text{ kg}\cdot\text{m/s [N30°W]}$$

$$\vec{p}_{2f} = ?$$

Let's assume that north and east are positive. Since momentum is always conserved in any collision,

$$\vec{p}_{1o} = \vec{p}_{1f}$$

$$\vec{p}_{1o} + \vec{p}_{2o} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$\vec{p}_{2f} = \vec{p}_{1o} - \vec{p}_{1f}$$

$$\vec{p}_{2f} = 97.5 \text{ kg}\cdot\text{m/s [N]} - 62.4 \text{ kg}\cdot\text{m/s [N30°W]}$$

Fig.4.19a

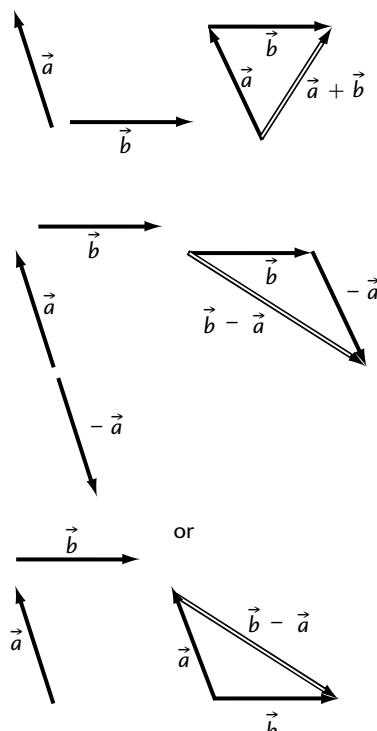
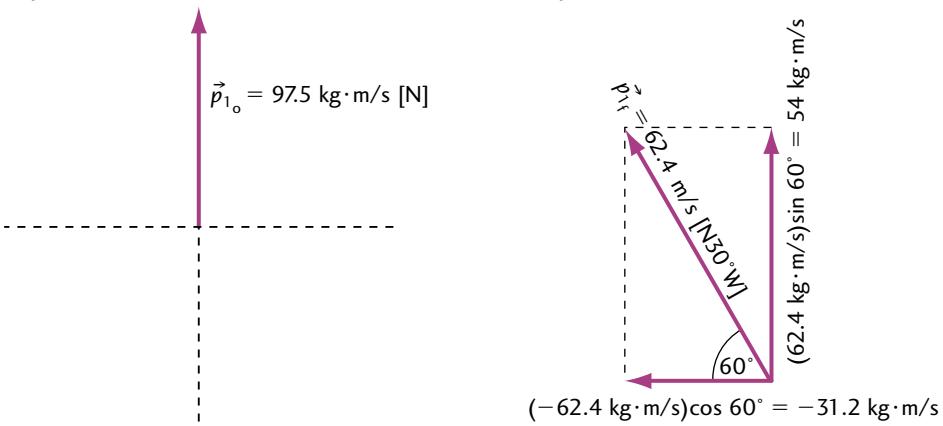


Fig.4.19b



From Figure 4.19b, for the vertical components,

$$\vec{p}_{1_{o(v)}} + \vec{p}_{2_{o(v)}} = \vec{p}_{1_{f(v)}} + \vec{p}_{2_{f(v)}}$$

$$\vec{p}_{2_{f(v)}} = \vec{p}_{1_{o(v)}} - \vec{p}_{1_{f(v)}}$$

$$p_{2_{f(v)}} = 97.5 \text{ kg} \cdot \text{m/s} - (62.4 \text{ kg} \cdot \text{m/s}) \sin 60^\circ$$

$$p_{2_{f(v)}} = 97.5 \text{ kg} \cdot \text{m/s} - 54.0 \text{ kg} \cdot \text{m/s}$$

$$p_{2_{f(v)}} = 43.5 \text{ kg} \cdot \text{m/s}$$

For the horizontal components,

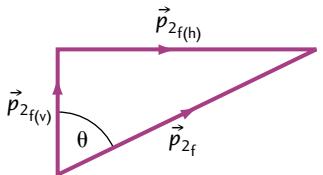
$$\vec{p}_{1_{o(h)}} + \vec{p}_{2_{o(h)}} = \vec{p}_{1_{f(h)}} + \vec{p}_{2_{f(h)}}$$

$$\vec{p}_{2_{f(h)}} = -\vec{p}_{1_{f(h)}}$$

$$p_{2_{f(h)}} = 0 - (-62.4 \text{ kg} \cdot \text{m/s}) \cos 60^\circ$$

$$p_{2_{f(h)}} = 31.2 \text{ kg} \cdot \text{m/s}$$

Fig.4.19c



$$p_{2_f} = \sqrt{(43.5 \text{ kg} \cdot \text{m/s})^2 + (31.2 \text{ kg} \cdot \text{m/s})^2}$$

$$p_{2_f} = 53.5 \text{ kg} \cdot \text{m/s}$$

$$\tan \theta = \frac{31.2 \text{ kg} \cdot \text{m/s}}{43.5 \text{ kg} \cdot \text{m/s}}$$

$$\theta = 35.6^\circ$$

From Figure 4.19c, the momentum direction is north and east; therefore,

$$\vec{p}_{2_f} = 53.5 \text{ kg} \cdot \text{m/s} [\text{N}35.6^\circ \text{E}]$$

To find the velocity of the second stone,

$$\vec{v}_{2_f} = \frac{53.5 \text{ kg} \cdot \text{m/s} [\text{N}35.6^\circ \text{E}]}{19.5 \text{ kg}}$$

$$\vec{v}_{2_f} = 2.7 \text{ m/s} [\text{N}35.6^\circ \text{E}]$$

As a check, we can draw a scale diagram (see Figure 4.19d) to verify the magnitude and direction of \vec{p}_{2_f}

Method 2: Trigonometry

Alternatively, we can use the sine and cosine laws to find the length of the vector and its direction.

Fig.4.19d

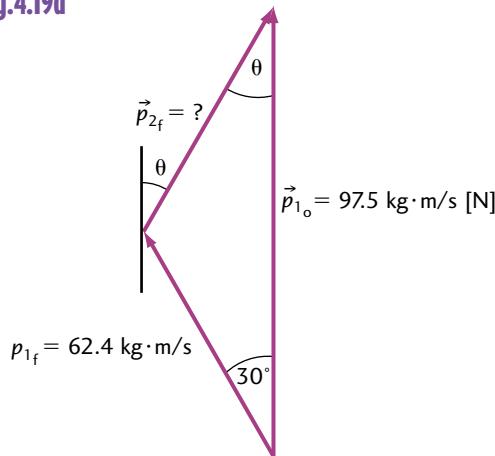
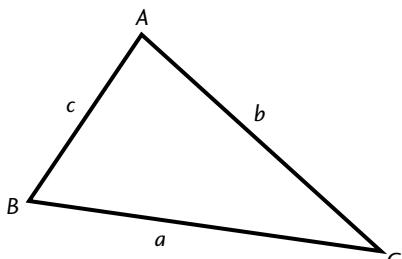


Fig.4.20 Sine law and cosine law for oblique triangles



$$\text{Sine law: } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine law:

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ b^2 &= a^2 + c^2 - 2ac \cos B \\ c^2 &= a^2 + b^2 - 2ab \cos C \end{aligned}$$

From Figure 4.19d, the angle between \vec{p}_{1o} and \vec{p}_{1f} is 30° . Using the cosine law, we can find the magnitude of \vec{p}_{2f} :

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\begin{aligned} (p_{2f})^2 &= (97.5 \text{ kg}\cdot\text{m/s})^2 + (62.4 \text{ kg}\cdot\text{m/s})^2 \\ &\quad - 2(97.5 \text{ kg}\cdot\text{m/s})(62.4 \text{ kg}\cdot\text{m/s}) \cos 30^\circ \end{aligned}$$

$$p_{2f} = 53.5 \text{ kg}\cdot\text{m/s}$$

The magnitude of the second stone's momentum after collision is $53.5 \text{ kg}\cdot\text{m/s}$. Now we use the sine law to determine the angle between \vec{p}_{1o} and \vec{p}_{2f} . We can call this angle θ .

$$\frac{\sin \theta}{62.4} = \frac{\sin 30^\circ}{53.5}$$

$$\theta = 35.6^\circ$$

Therefore, $\vec{p}_{2f} = 53.5 \text{ kg}\cdot\text{m/s} [\text{N}35.6^\circ\text{E}]$

$$m_2 v_{2f} = 53.5 \text{ kg}\cdot\text{m/s}$$

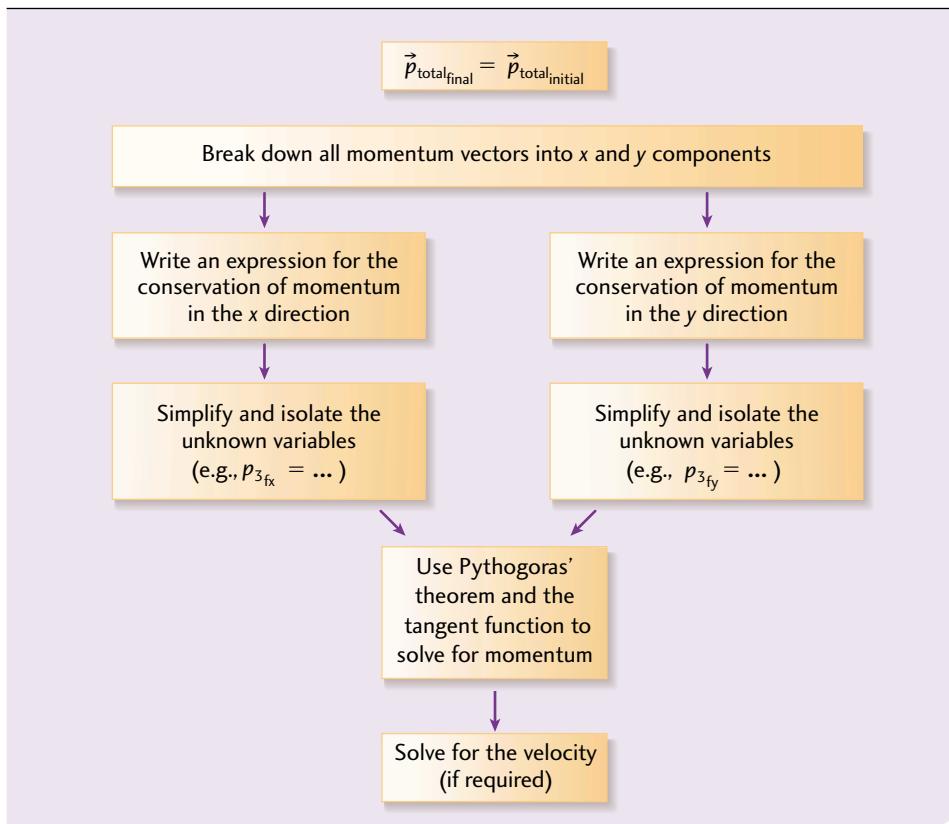
$$v_{2f} = \frac{53.5 \text{ kg}\cdot\text{m/s}}{19.5 \text{ kg}}$$

$$v_{2f} = 2.7 \text{ m/s}$$

Therefore, $\vec{v}_{2f} = 2.7 \text{ m/s} [\text{N}35.6^\circ\text{E}]$. The direction of the final velocity of the second stone is found by looking at the vector diagram in Figure 4.19d.

Figure 4.21 summarizes the method for solving linear momentum problems using the component method.

Fig.4.21 Component Method for Addition of Momentum Vectors



EXAMPLE 8

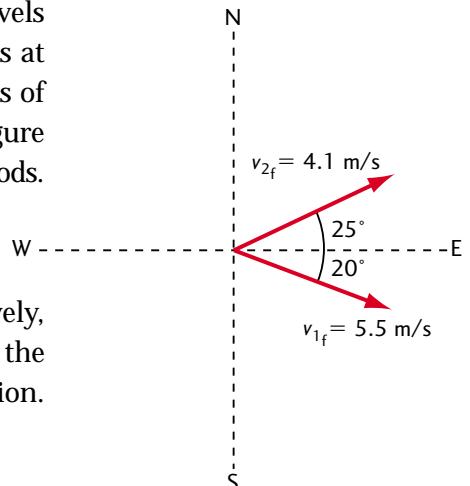
Solving momentum problems in two dimensions involving unequal masses

A 5.0-kg bomb at rest explodes into three pieces, each of which travels parallel to the ground. The first piece, with a mass of 1.2 kg, travels at 5.5 m/s at an angle of 20° south of east. The second piece has a mass of 2.5 kg and travels 4.1 m/s at an angle of 25° north of east (see Figure 4.22). Determine the velocity of the third piece. Use two different methods.

Solution and Connection to Theory

Let m and \vec{v}_0 represent the mass and velocity of the bomb, respectively, before the explosion. Let m_1 , m_2 , and m_3 , and \vec{v}_{1f} , \vec{v}_{2f} , and \vec{v}_{3f} represent the masses and velocities of the three pieces, respectively, after the explosion.

Fig.4.22



Given**Before explosion**

$$m = 5.0 \text{ kg}$$

After explosion

$$m_1 = 1.2 \text{ kg} \quad m_2 = 2.5 \text{ kg}$$

$$m_3 = 5 \text{ kg} - (1.2 \text{ kg} + 2.5 \text{ kg}) = 1.3 \text{ kg}$$

$$\vec{v}_o = 0 \text{ m/s}$$

$$\vec{v}_{1f} = 5.5 \text{ m/s [E}20^\circ\text{S]}$$

$$\vec{v}_{2f} = 4.1 \text{ m/s [E}25^\circ\text{N]}$$

$$\vec{v}_{3f} = ?$$

$$\vec{p}_o = 0$$

$$\vec{p}_{1f} = (1.2 \text{ kg})(5.5 \text{ m/s [E}20^\circ\text{S}]) = 6.6 \text{ kg}\cdot\text{m/s [E}20^\circ\text{S}]$$

$$\vec{p}_{2f} = (2.5 \text{ kg})(4.1 \text{ m/s [E}25^\circ\text{S}]) = 10.25 \text{ kg}\cdot\text{m/s [E}25^\circ\text{N}]$$

$$\vec{p}_{3f} = (1.3 \text{ kg})(\vec{v}_{3f})$$

From the law of conservation of momentum,

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$

$$\vec{p}_{T_o} = \vec{p}_{1f} + \vec{p}_{2f} + \vec{p}_{3f} = 0$$

$$\vec{p}_{3f} = -\vec{p}_{1f} + (-\vec{p}_{2f})$$

Method 1: Components

Since $p_o = 0$, the sum of the x and y components of momentum for the three pieces must be zero.

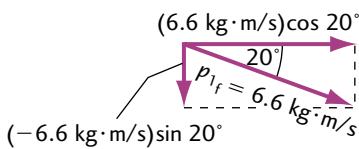
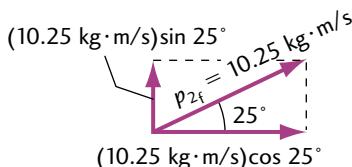
Adding the east–west components (Figures 4.23a and 4.23b),

$$0 = \vec{p}_{1fx} + \vec{p}_{2fx} + \vec{p}_{3fx}$$

$$\vec{p}_{3fx} = -\vec{p}_{1fx} - \vec{p}_{2fx}$$

$$p_{3fx} = -(6.6 \text{ kg}\cdot\text{m/s})\cos 20^\circ - (10.25 \text{ kg}\cdot\text{m/s})\cos 25^\circ$$

$$p_{3fx} = -15.5 \text{ kg}\cdot\text{m/s}$$

Fig.4.23a**Fig.4.23b**

Adding the north–south components (Figures 4.23a and 4.23b),

$$0 = \vec{p}_{1fy} + \vec{p}_{2fy} + \vec{p}_{3fy}$$

$$\vec{p}_{3fy} = -\vec{p}_{1fy} - \vec{p}_{2fy}$$

$$p_{3fy} = -(-6.6 \text{ kg}\cdot\text{m/s})\sin 20^\circ - (10.25 \text{ kg}\cdot\text{m/s})\sin 25^\circ$$

$$p_{3fy} = +2.26 \text{ kg}\cdot\text{m/s} - 4.33 \text{ kg}\cdot\text{m/s}$$

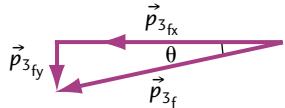
$$p_{3fy} = -2.07 \text{ kg}\cdot\text{m/s}$$

To find the magnitude of the momentum of the third piece,

$$p_{3f} = \sqrt{(p_{3fx})^2 + (p_{3fy})^2}$$

$$p_{3f} = 15.6 \text{ m/s}$$

Fig.4.23c



For the angle,

$$\tan \theta = \frac{p_{3fy}}{p_{3fx}}$$

$$\tan \theta = \frac{2.07 \text{ kg} \cdot \text{m/s}}{15.5 \text{ kg} \cdot \text{m/s}}$$

$$\theta = 7.6^\circ$$

Therefore, $\vec{p}_3f = 15.6 \text{ kg} \cdot \text{m/s} [\text{W}7.6^\circ\text{S}]$

$$m_3 v_3 = 15.6 \text{ kg} \cdot \text{m/s}$$

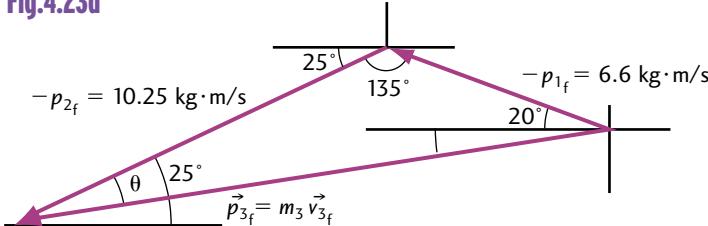
$$v_3 = \frac{15.6 \text{ kg} \cdot \text{m/s}}{1.3 \text{ kg}}$$

$$\vec{v}_3 = 12 \text{ m/s} [\text{W}7.6^\circ\text{S}]$$

Therefore, the final velocity of the third piece is 12 m/s [W7.6° S].

Method 2: Trigonometry

Fig.4.23d



Using the cosine law and Figure 4.23d,

$$(p_{3f})^2 = (6.6 \text{ kg} \cdot \text{m/s})^2 + (10.25 \text{ kg} \cdot \text{m/s})^2 - 2(6.6 \text{ kg} \cdot \text{m/s})(10.25 \text{ kg} \cdot \text{m/s})\cos 135^\circ$$

$$(p_{3f})^2 = 244.3 \text{ (kg} \cdot \text{m/s)}^2$$

$$p_{3f} = 15.63 \text{ kg} \cdot \text{m/s}$$

Substituting 1.3 kg for m_3 ,

$$v_{3f} = \frac{15.6 \text{ kg} \cdot \text{m/s}}{1.3 \text{ kg}}$$

$$v_{3f} = 12 \text{ m/s}$$

Therefore, the magnitude of the velocity of the third piece is 12 m/s.

We can determine its direction using the sine law.

From Figure 4.23d, solving for the angle θ between p_{3_f} and p_{2_f} , we obtain

$$\frac{\sin 135^\circ}{15.6 \text{ kg} \cdot \text{m/s}} = \frac{\sin \theta}{6.6 \text{ kg} \cdot \text{m/s}}$$

$$\theta = 17.4^\circ$$

We need the angle from the horizontal. From Figure 4.23d,

$$25^\circ - 17.4^\circ = 7.6^\circ$$

From Figure 4.23d, the directions of the angle are west and south; therefore, the direction of the third piece is [W7.6°S] and its velocity is 12 m/s [W7.6°S].



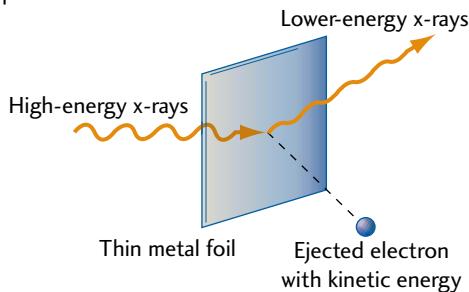
1. A 2.0-kg steel ball rolling at 5.0 m/s [W] strikes a second steel ball of equal mass at rest. After a glancing collision, the first ball is deflected [N35°W] at 3.0 m/s. Determine the velocity of the second ball.
2. A hockey player of mass 85 kg, travelling at 15 m/s [N], collides with another hockey player of mass 70 kg travelling at 5.0 m/s [E]. If the two players lock skates during the collision and are held together, find the resultant velocity of the pair. (Assume there is no friction.)
3. A 0.5-kg grenade explodes horizontally into three pieces. The first piece has a velocity of 10 m/s [N] and a mass of 0.10 kg. The second piece has a velocity of 5.0 m/s [S10°E] and a mass of 0.20 kg. Find the velocity of the third piece.
4. A billiard ball of mass 0.50 kg, moving with a velocity 2.0 m/s [forward], strikes a second ball of mass 0.30 kg, initially at rest. A glancing collision causes the first ball to be deflected at an angle of 30° to the left of its original direction with a speed of 1.5 m/s. Determine the velocity of the second ball after collision.

Linear Momentum and the Compton Effect

“Every great discovery I ever made, I gambled that the truth was there and then I acted on it in faith until I could prove its existence.” Arthur Compton (1892–1962), Nobel laureate in physics, 1927

The conservation of linear momentum applies even at the atomic level. The **Compton effect** (Figure 4.24) describes the conservation of momentum when x-ray photons collide with electrons. The momentum of an x-ray photon before a collision with an electron is equal to the momentum of the ejected electron and of the x-ray photon after collision with an electron. We will discuss the Compton effect in greater detail in Chapter 12.

Fig.4.24 The Compton effect



4.6 Linear Momentum and Centre of Mass

Recall from Chapter 3 that the *centre of mass* (*cm*) of a solid, homogeneous object is the point at which a body's entire mass may be considered to be concentrated for analyzing its motion (see Figure 4.25). The centre of mass for a *system* consisting of *two identical objects*, such as two billiard balls, is the point midway between the objects. Figure 4.26b represents the glancing collision of two identical objects. The dotted line represents the centre of mass of the two objects at every instant of the collision. Unlike the centre of mass of two individual objects, the path of the centre of mass between two objects doesn't deviate; that is, it is always midway between the objects. The *momentum* of the centre of mass is conserved. We can calculate the *momentum* of the centre of mass as follows:

$$\vec{p}_{\text{totalinitial}} = \vec{p}_{\text{totalfinal}} = \vec{p}_{\text{cm}}$$

Fig.4.25 For the human balancing act shown here, the acrobats' centre of mass lies in a vertical line somewhere above the feet of the supporting acrobat.



Fig.4.26a The centre of mass of a system of two identical masses in one dimension

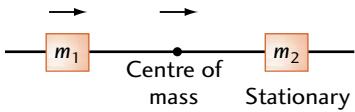
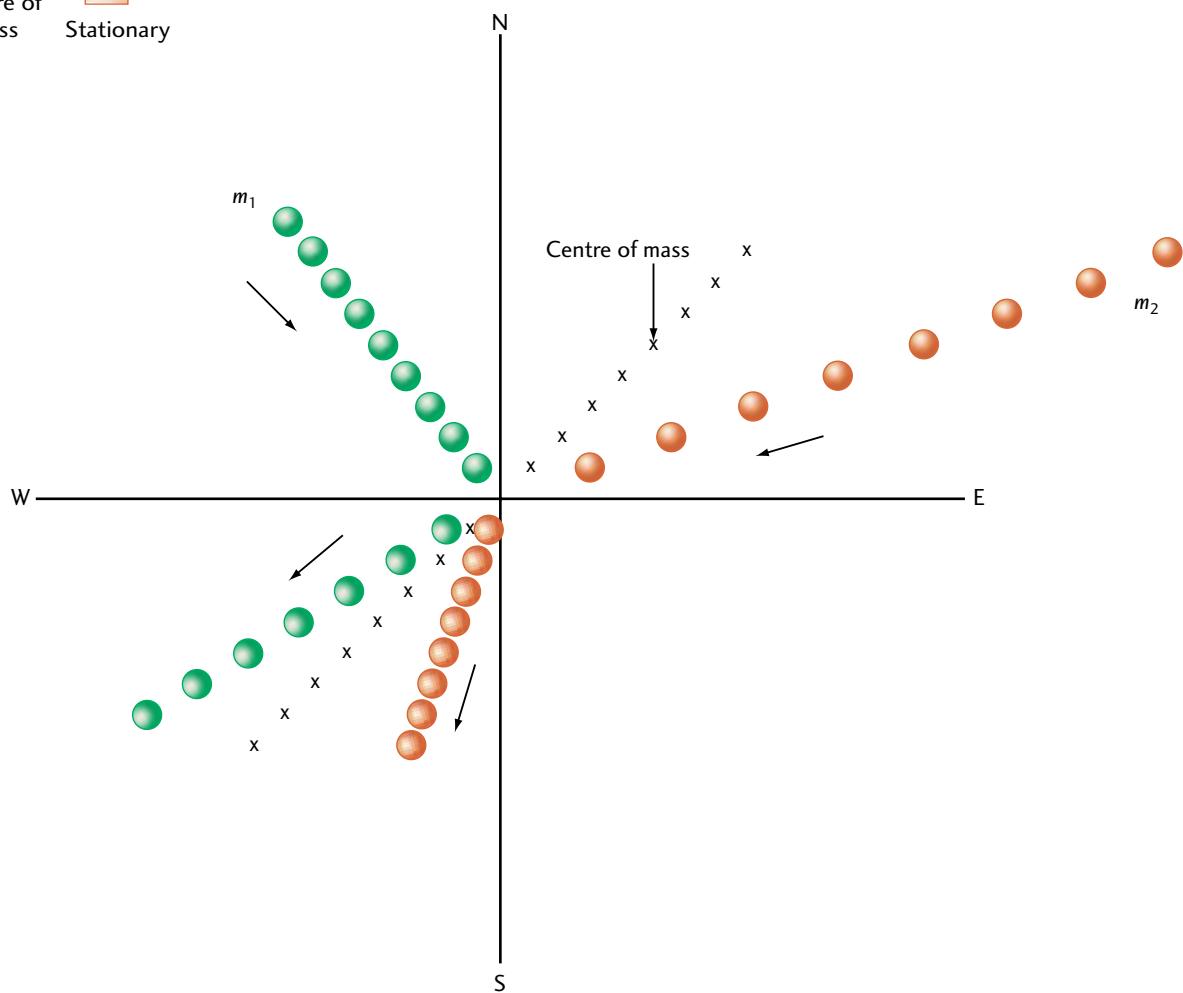
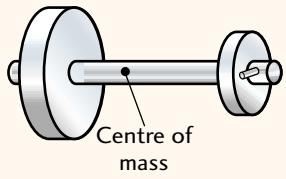


Fig.4.26b The centre of mass of a system of two identical masses in two dimensions. Note that the momentum of the centre of mass doesn't change direction after the collision. The time between consecutive balls is 0.1 s.



The centre of mass of two unequal masses is like the balance point of an unequal barbell.

Fig.4.27



For two objects of unequal mass, the centre of mass is found along the straight line between their centres. The point representing the centre of mass divides the line into two parts *in an inverse ratio* to the masses of the objects; that is, the centre of mass is always closer to the more massive object. For instance, for a system with a 2.0-kg steel ball and a 1.0-kg steel ball, the centre of mass is on a point $\frac{1}{3}$ the distance away from the 2.0-kg ball. Then we can consider the system as one 3.0-kg ball moving along at the location of the centre of mass, as shown in Figure 4.28.

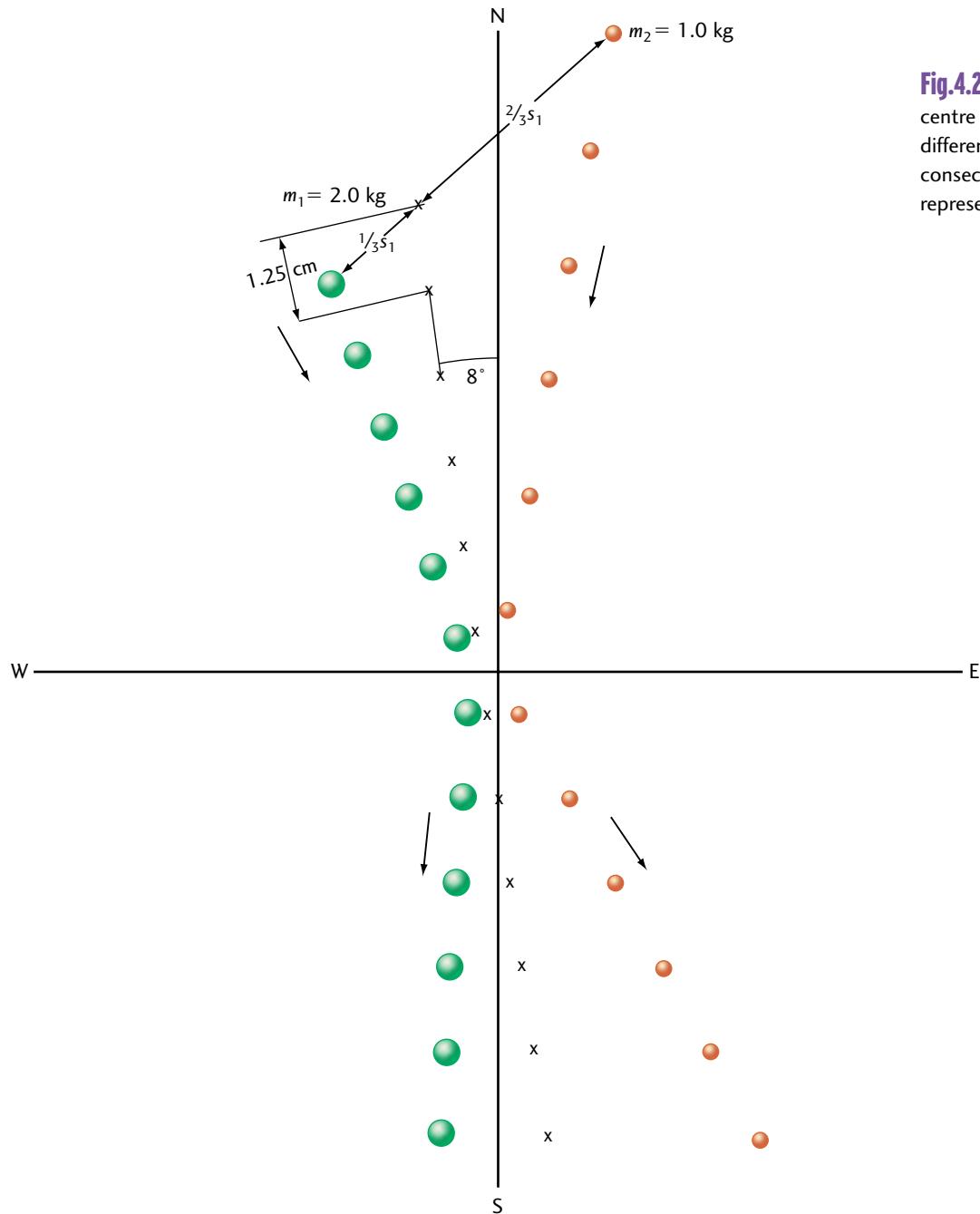


Fig. 4.28 The momentum of the centre of mass of two objects with different masses. The time between consecutive images is 0.1 s. “x” represents the centre of mass.

1. Determine the centre of mass of
 - a) two identical objects 3.0 m apart.
 - b) a 5.0-kg ball and a 2.0-kg ball 60 cm apart.
 - c) a 400-kg satellite and a 200-kg satellite 20 km apart.
2. Refer to the collision illustrated in Figure 4.28.
 - a) Using a ruler and a protractor, determine the momentum before and after the collision for the first ball ($m_1 = 2.0 \text{ kg}$), the second ball ($m_2 = 1.0 \text{ kg}$), and the centre of mass ($m_{\text{cm}} = 3.0 \text{ kg}$).
 - b) Draw a vector diagram to represent the total momentum
 - i) before collision.
 - ii) after collision.
 - c) How does the total momentum before and after collision compare to the momentum of the centre of mass?





Science—Technology—Society— Environmental Interrelationships

Fig.STSE.4.1 Snowmobiles are becoming more popular as a form of recreation. They are also becoming more massive and powerful.



Fig.STSE.4.2 Drivers of open recreational vehicles like these scooters are exposed to injury in the event of a mishap



Fig.STSE.4.3 Mopeds (motorized bicycles) were once regulated as regular bicycles before stricter licensing came into effect



Recreational Vehicle Safety and Collisions

Recreational vehicles such as all-terrain vehicles (ATVs), snowmobiles, motorized scooters, or motorcycles (Figures STSE.4.1 and 4.2) are a popular form of entertainment in many parts of Canada. Some of these vehicles, such as the snowmobile, had practical beginnings when they were the only type of vehicle able to access areas isolated by heavy winter snowfall. Now capable of achieving greater speeds, the open nature of the driving compartment of these vehicles means that the vehicle operators are exposed to and unprotected from the surroundings. It is impractical and most likely unsafe to install any form of passenger restraint systems, such as seat belts or air bags, to improve crash survivability. The operation of these vehicles is restricted by provincial and federal laws as well as by municipal bylaws in an attempt to ensure driver safety. Mopeds, a form of a motor-assisted bicycle (see Figure STSE.4.3), were considered to be a bicycle when they first became popular for their low fuel consumption during the “energy crisis” in the early 1970s. By 1973, they required the use of helmets and needed to be licensed as a motorized vehicle.

The collision dynamics of smaller vehicles is different than the collision dynamics of automobiles because the ratio of the passenger’s mass to the vehicle’s mass is greater. Without significant tethering, the passengers of smaller vehicles run the risk of becoming projectiles, which increases the chances of serious injury and death.

Design a Study of Societal Impact

Insurance companies base their premium rates on risk analysis, and injury and death rate statistics. Even your generic life insurance premiums may be different if you are licensed to drive a motorcycle or other small vehicle.

- a) Examine the relative safety of various vehicles, including recreational vehicles, by researching insurance rates for these vehicles. Many insurance companies provide online premium quote engines on their Web sites (see <www.irwinpublishing.com/students>). Find out what other factors affect your insurance rate.
- b) Research vehicle safety equipment such as helmets, extra padding, or other design changes that could improve vehicle safety. Write a short cost–benefit analysis paper on how safety equipment, although necessary, might detract from the enjoyment of riding the vehicle.
- c) What local laws or restrictions improve vehicle safety?

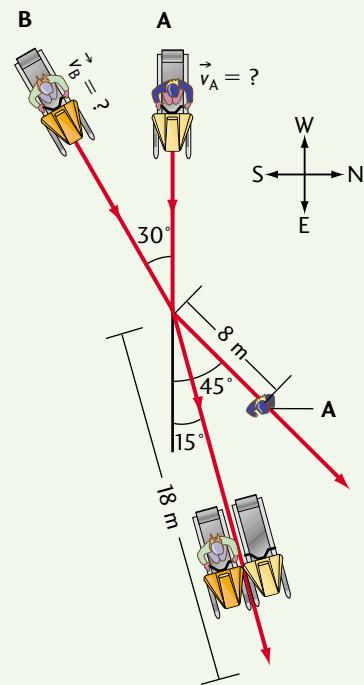
Design an Activity to Evaluate

Figure STSE.4.4 shows a snowmobile accident on Ramsey Lake in Northern Ontario.

A police officer arrives at the scene of the collision of the two snowmobiles (Figure STSE.4.4) to find both drivers unconscious. When the two vehicles collided, their skis became entangled and the two snowmobiles remained locked together as they skidded to a stop. One driver was thrown clear of the mishap, but the other driver remained in the driver's seat. The posted speed limit for snowmobiles in this cottage area is 60 km/h. The information the police officer obtained from eye-witness accounts and collision scene measurements are provided in Table STSE.4.1. One witness described how driver A was thrown horizontally at a constant speed from his seat (0.5 m above the snow surface) to his final resting position.

- Use the physics of kinematics, projectiles, conservation of momentum, and metric conversions to estimate the pre-collision speed of both vehicles.
- What assumptions did you make in your calculations?
- Which, if either, of the two vehicles was speeding?
- How would you respond if asked how confident you were of the results of your calculations? Could you be so sure that vehicle B was speeding that you would recommend the officer charge the driver?

Fig. STSE.4.4 Two snowmobiles collide. Conservation of linear momentum is applied in the police investigation.



Build a Structure

- Use the physics simulation software Interactive Physics™, Exploration of Physics™, or even a program of your own design to simulate the collision of the two snowmobiles.
- Create a simulation of the snowmobile collision on a standard two-dimensional spark-timer air table. Simulate the driver that is thrown during the collision by placing a loose marble on one of the air-table pucks. The landing position of the loose marble will be marked by impact on tracing paper with carbon paper underneath.

Table STSE.4.1

Mass of driver A	80 kg
Mass of driver B	90 kg
Mass of vehicle A	270 kg
Mass of vehicle B	310 kg
Direction of vehicle A before collision	[E]
Direction of vehicle B before collision	[E30°N]
Direction of entangled vehicles A and B after collision	[E15°N]
Length of final skid	18 m
Displacement of driver A from point of impact	8 m
Time from impact to end of skid	2.5 s

You should be able to*Understand Basic Concepts:*

- Define and describe the concepts and units related to momentum and impulse.
- Analyze with the aid of vector diagrams the linear momentum of a collection of objects, and apply quantitatively the laws of conservation of linear momentum.

Develop Skills of Inquiry and Communication:

- Investigate the conservation of momentum in one and two dimensions by carrying out experiments or simulations and the necessary analytical procedures.
- Compile, organize, and interpret data using appropriate formats and treatments, including tables, flowcharts, graphs, and diagrams.
- Select and use appropriate numeric, symbolic, graphical, and linguistic modes of representation to communicate scientific ideas, plans, and experimental results.
- Communicate the procedures and results of investigation and research.

Relate Science to Technology, Society, and the Environment:

- Analyze and describe, using the concepts and laws of momentum, some practical applications of momentum conservation.
- Identify and analyze social issues that relate to the development of vehicles.
- Identify careers related to momentum.

Equations

$$\vec{p} = m\vec{v}$$

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{m\Delta \vec{v}}{\Delta t} = m\vec{a}$$

$$\vec{J} = \Delta \vec{p}$$

$$\vec{J} = \vec{F}\Delta t$$

$$\vec{F}\Delta t = m\Delta \vec{v}$$

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t$$

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}}$$

$$\Delta \vec{p}_{\text{total}} = 0$$

$$\vec{p}_{\text{total initial}} = \vec{p}_{\text{total final}} = \vec{p}_{\text{cm}}$$

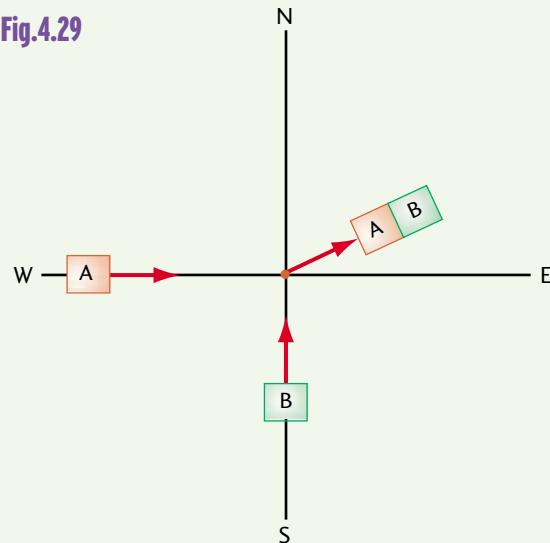
EXERCISES

Conceptual Questions

1. Describe momentum. Explain why momentum is a vector quantity.
2. Explain what is meant by a closed system and by an open isolated system.
3. What force is used in the calculation of impulse, the applied force or the net force?
4. How is impulse related to momentum?
5. Explain why the change in momentum is zero in an isolated system.
6. State the law of conservation of momentum in two ways.
7. Does a ball thrown upward lose momentum as it rises? Explain.
8. A grenade thrown upward explodes into 45 pieces. Determine the sum of the momentum vectors after the explosion.
9. A Canadian astronaut is at a space station working on the Canadarm while wearing his tool belt containing a left-handed monkey wrench. He loses his grip on the space station and begins to float in space. Explain how he could use the law of conservation of momentum to return to the space station.
10. Use the terms “momentum” and “impulse” to describe how a rocket can change its course in space.
11. Two balls of equal mass and speed are heading for each other along a horizontal surface. Write the general equation for the total momentum before and after collision.
12. An open-top freight car is coasting along a railway track at a constant speed. Suddenly, it begins to rain. Describe and explain the changes that will occur in the train’s motion.

13. In Figure 4.29, the objects are held together after the collision. If both objects have the same mass, which object is moving faster, A or B? Explain your answer.

Fig. 4.29



14. In what type of momentum problem would the component method be preferred over the trigonometric method for solving?
15. a) Why do grocery clerks lean back when carrying heavy boxes?
b) Explain what is meant by “centre of mass” and how this concept can be applied to simplify momentum problems.

Problems

- 4.1 Introduction to Linear Momentum
16. Calculate the momentum of a 7500-kg plane flying at 120 m/s.
 17. Determine the momentum of a 25-g butterfly flying at 3.0 m/s.
 18. What is the momentum of a 25-g ball moving at 90 km/h?
 19. An airplane with a speed of 500 km/h has a momentum of 23 000 kg· m/s. Calculate the mass of the plane.

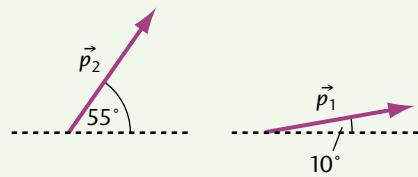
- 20.** The mass of a proton is 1.6726×10^{-27} kg. What is the speed of the proton if it has a momentum of 1.00 kg·m/s. Is your answer reasonable?
- 21.** Draw a vector diagram to show the momentum of a 50-g egg falling at a rate of 10 m/s.
- 22.** Draw a vector diagram representing the momentum of a 6000-kg plane flying northwest at 300 km/h.

4.3 Linear Momentum and Impulse

- 23.** A boy pulls a 50-kg wagon from rest horizontally with a force of 250 N [forward] for 3.0 s. What is the final speed of the wagon if there is no friction acting on it?
- 24.** A 150-kg go-cart accelerates from rest at a rate of 2.0 m/s^2 for 4.0 s.
- What is the go-cart's momentum after 4.0 s?
 - What was the impulse exerted on the go-cart?
- 25.** A loose 1.5-kg brick at the top of a 17-m wall falls to the ground.
- Calculate the time it takes to fall.
 - Calculate the force acting on the brick as it falls.
 - What is the impulse of the brick just before it hits the ground?
- 26.** A tennis player hits a tennis ball with a force of 700 N. The racquet is in contact with the ball for 0.095 s.
- What is the impulse received by the ball?
 - What is the tennis ball's change in momentum?
- 27.** A 0.20-kg rubber ball, initially at rest, is dropped from the window of a building. It strikes the sidewalk with a speed of 25 m/s and rebounds with a speed of 20 m/s. Ignoring any effect due to air resistance, calculate the magnitude and direction of the change in momentum of the ball as a result of its impact with the sidewalk.

- 28.** Using dimensional unit analysis, show that $F\Delta t = m\Delta v$.
- 29.** For the following momentum vectors (Figure 4.30), representing \vec{p}_1 and \vec{p}_2 , draw the vector representing the change of momentum, $\vec{p}_2 - \vec{p}_1$.

Fig. 4.30



- 30.** The average accelerating force exerted on a 3.0-kg shell in a gun barrel is 2.0×10^4 N. If the muzzle velocity is 250 m/s, calculate
- the impulse on the shell.
 - the length of time it takes the shell to exit the gun barrel.
- 31.** A 7000-kg transport truck passes a sports car at 110 km/h. The truck suddenly loses a wheel, which causes the driver to lose control of the truck. The truck hits a concrete barrier and comes to rest in 0.40 s.
- Calculate the average force acting on the truck.
 - What would be the magnitude of the average force if the truck driver had managed to drive onto the soft shoulder of the road and stop in 8.0 s?
- 32.** A police investigator doing some ballistics testing in his laboratory fires a 30-g bullet with a velocity of 360 m/s into a lead paperweight placed against a concrete wall. Because of a constant resistance force, the bullet penetrates 5.0 cm into the paperweight before coming to a stop. Calculate
- the initial momentum of the bullet before the collision.
 - the acceleration of the bullet in the paperweight.
 - the average force exerted on the bullet.
 - the time required to stop the bullet in the paperweight.

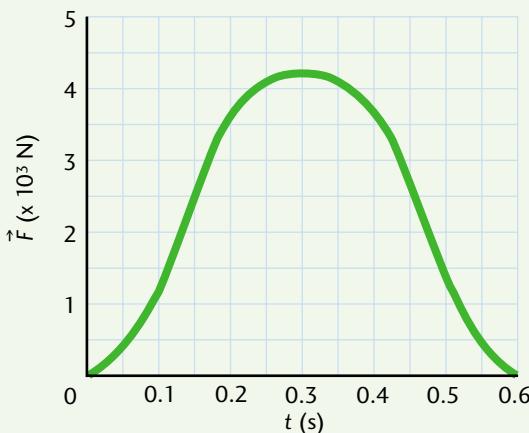
- e) the impulse.
- f) Draw a force-versus-time graph for this situation.
33. A rocket increases its upward force uniformly from 5.0×10^6 N to 8.0×10^6 N over 15 s.
- a) Draw a force-versus-time graph for the rocket.
- b) Calculate the impulse on the rocket.
34. Calculate the impulse for the first 0.9 s in Figure 4.31.

Fig.4.31



35. Calculate the impulse on a hockey puck represented by the graph in Figure 4.32.
36. If the hockey puck in Figure 4.32 has a mass of 250 g, determine the speed of the puck just after it is struck by a hockey stick.

Fig.4.32

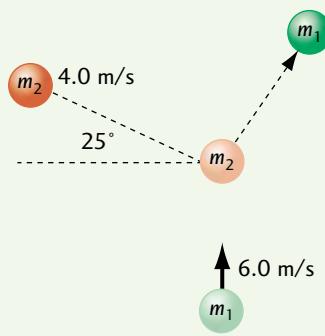


4.4 Conservation of Linear Momentum in One Dimension

37. A 5000-kg train moving at 5.0 m/s [S] collides with another train of equal mass at rest and the two trains become coupled. Calculate the speed of the coupled trains after collision.
38. To impress his friends, a 45-kg skateboarder runs at 5.0 m/s and jumps on his 2.0-kg skateboard, initially at rest. Find the combined speed of the skateboarder and the skateboard.
39. A 65-kg adult skier, skiing at 15 m/s, collides head on with another skier ($m = 100$ kg) moving toward her at 5.0 m/s. If the 65-kg skier slows to $\frac{1}{3}$ her initial velocity after the collision, calculate the velocity of the larger skier if all the velocities are horizontal and any effects due to friction are negligible.
40. A soccer ball of mass 0.50 kg is kicked with a horizontal speed of 20 m/s. If a 30-kg goalie jumps up and catches the ball in mid-air, what is the goalie's horizontal speed just after she catches the ball?
41. A billiard ball of mass 0.20 kg moving at 3.0 m/s [right] strikes an identical ball moving in the opposite direction at 1.0 m/s. If the velocity of the second ball after collision is 2.0 m/s [right], what is the velocity of the first ball after collision?
42. A train loaded with steel, moving at 90 km/h, collides head on with a stationary train of mass 6000 kg. If the trains couple after collision and move forward with a velocity of 80 km/h, find the mass of the train loaded with steel.
43. Beginning with Newton's third law ($F_1 = -F_2$), derive a statement for the conservation of momentum ($\Delta\vec{p} = 0$).

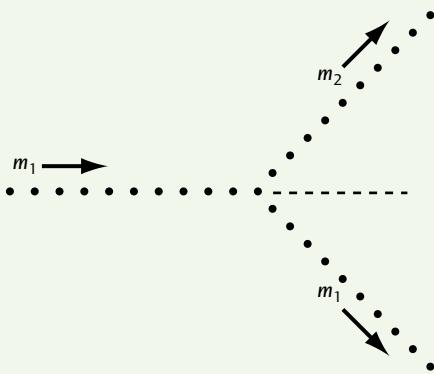
- 44.** A particle accelerator accelerates a stationary proton from rest to 2.2×10^7 m/s. The accelerated proton then strikes a stationary alpha particle (two protons and two neutrons). The proton combines with the alpha particle to form a new particle. Assuming that the mass of the proton is the same as the mass of the neutron, calculate the velocity of the new particle after collision.
- 45.** Three coupled freight cars, each of mass m , are travelling with a constant speed v on a straight and level track. They collide with two coupled stationary cars, each of mass $2m$. If all five cars are coupled together after collision, what is their common speed?
- 46.** A stationary 50-kg miser carrying a bag of gold bars (50 kg) is stranded on a frozen pond 200 m from shore. The miser decides to throw one of his 500-g shoes toward the opposite shore at 20 m/s. How long will it take the miser to reach shore? Omit any effects due to friction.
- 4.5 Conservation of Linear Momentum in Two Dimensions**
- 47.** A hockey player with a momentum of 375 kg·m/s [E] collides with another hockey player with a momentum of 450 kg·m/s [N45°E]. The hockey players grab on to each other's jerseys when they collide. Omit all friction between the skates and the ice.
- a)** Draw a vector diagram to represent the total momentum before collision.
- b)** Determine the total momentum after collision.
- 48.** A 3.2-kg hawk soaring at 20 m/s [N] collides with a 0.50-kg sparrow flying at 5.0 m/s [W]. If both the hawk and sparrow are on the same horizontal plane, find their velocity if the hawk hangs on to the sparrow after collision.
- 49.** A 3000-kg car travelling at 20 m/s [N] collides with a 5000-kg truck moving east on an icy road. The bumpers of the two vehicles become entangled and the vehicles remain joined after the collision. Calculate the initial speed of the truck if both vehicles after collision go [E30°N].
- 50.** Radioactivity is the result of atoms that decay or break apart spontaneously. A stationary parent nucleus of mass 1.2×10^{-24} kg decays into three particles. One particle of mass 3.0×10^{-25} kg moves away with a velocity of 2.0×10^7 m/s [E]. Another particle of mass 2.3×10^{-25} kg moves north at a speed of 4.2×10^7 m/s. Calculate the mass and velocity of the third particle.
- 51.** A curling stone thrown by the skip takes 4.8 s to travel 60 m. The stone collides with another stone. The collision is a glancing one. If the second stone is deflected 25° and travels 1.5 m/s, calculate the angle of deflection of the first stone after collision. Omit any effects due to friction.
- 52.** A 10 000-kg space shuttle moving east at 3000 km/h wishes to change its course by 10° . It does so by ejecting an object at a speed of 5000 km/h [S]. Calculate the mass of the ejected object.
- 53.** From Figure 4.33, determine the final velocity of the first ball after collision if $m_1 = m_2$.

Fig.4.33



- 54.** From Figure 4.33, determine the final velocity of the first ball after collision if $m_2 = 2m_1$.
- 55.** An air table with a spark timer produces the pattern shown in Figure 4.34. The mass of each puck is 0.30 kg. Using a ruler and a protractor, determine
- the speed of the pucks before and after collision, if the time between each dot is 0.10 s.
 - the velocity of the pucks before and after collision. The line of travel of the first puck before collision is 0° .
 - the total momentum of the pucks before and after collision. Use a vector diagram.
 - the components of the momentum of each puck before and after collision.
 - Is momentum conserved in this collision?

Fig.4.34



- 56.** A 1.0-kg grenade explodes into four pieces, all moving parallel to the ground. The first piece of mass 0.20 kg moves east at 24 m/s. The second piece of mass 0.30 kg flies north at 18 m/s. A third piece of mass 0.25 kg is directed west at 30 m/s. What is the velocity of the fourth piece?

4.6 Linear Momentum and Centre of Mass

- 57.** A system is made up of two trucks 400 m apart. One truck has a mass of 5000 kg and the other truck has a mass of 10 000 kg.
- What is the total mass of the system?
 - Where is the centre of mass located?
- 58.** A satellite of mass 2000 kg is moving at 200 m/s [E]. Another satellite of mass 1000 kg is moving at 200 m/s [S30° E]. Draw a vector diagram to indicate
- the momentum of the first satellite before collision.
 - the momentum of the second satellite before collision.
 - the momentum of the centre of mass before collision.
 - the momentum of the centre of mass after collision.
- 59.** From Figure 4.34, determine the momentum of the centre of mass before and after collision. Assume 0.10 s between consecutive balls.

Linear Momentum in One Dimension: Dynamic Laboratory Carts

Purpose

To verify the conservation of momentum in a simple lab-cart collision.

Equipment

2 laboratory carts

Timing equipment: tickertape and spark timer. An alternative method of timing may be used, such as a photo gates, video camera, motion sensors (CBR), or a stopwatch.

Metre stick

Pins (or nails) and corks (or rubber stoppers)

Newton scale or balance

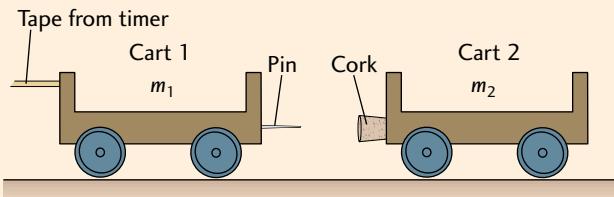
Various weights

Spring-loaded cart (optional)

Procedure

Part A: Head-on Collision (Equal Masses)

Fig. Lab.4.1



- Set up two lab carts: one with a pin and another with a cork, as shown in Figure Lab.4.1.
- Measure the mass of each cart. Add weights as needed so that both carts have about the same mass.
- Attach the tickertape to the first cart.
- On a smooth horizontal surface with the second cart stationary, collide the first cart with the second cart so that the pin sticks into the

cork during collision. Collect the data on the ticker timer, or with CRB motion sensors, for the velocity of cart 1 before the collision and the combined carts after the collision.

Part B: Head-on Collision (Unequal Masses)

- Prepare two lab carts such that the first cart is about twice the mass of the second cart. You can either add weights or stack two carts on top of each other.
- Measure the mass of each cart.
- Follow steps 3 to 5 in Part A.

Data

Assign appropriate instrumental uncertainties for all measurements and complete a table similar to Table Lab.4.1 for Parts A and B. Place all measurements of mass and velocity in the table with uncertainty.

Analysis

- Calculate the momentum for m_1 and m_2 before and the combined carts after collision, including uncertainty.
- Record your results in the data table.

Discussion

- Is the momentum in Part A conserved?
- Is the momentum in Part B conserved?
- What are some possible reasons conservation of momentum may not be observed here?
- Draw a vector diagram for the total momentum before and after collision for Part A only.

Conclusion

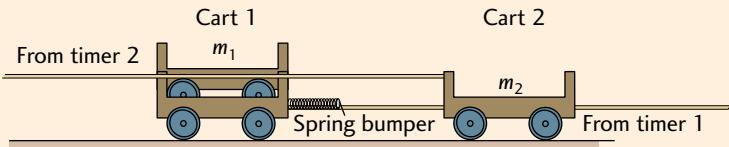
Make a concluding statement that summarizes the success of the lab, with experimental uncertainty.

Table Lab.4.1

Part	Cart 1					Cart 2					Combined carts after		
	m_1	v_{1_o}	v_{1_f}	p_{1_o}	p_{1_f}	m_2	v_{2_o}	v_{2_f}	p_{2_o}	p_{2_f}	m_{T_o}	v_T	p_T
	kg	m/s		kg · m/s		kg	m/s		kg · m/s		kg	m/s	kg · m/s
A													
B													
C													

Part C: Exploding Carts

1. Measure the masses of the spring-loaded cart and another cart of different mass.
2. Set up the spring-loaded cart and another cart as shown in Figure Lab.4.2. You will need two pieces of tickertape, one for each direction. Before the explosion, both carts will start at rest with the spring compressed.
3. Start the timing devices and release the spring. Save both tickertapes.
4. Analyze both tapes for final velocity.
5. Complete the analysis and discussion questions as in Parts A and B.
6. Is the momentum conserved in the explosion?

Fig. Lab.4.2



Linear Momentum in Two Dimensions: Air Pucks (Spark Timers)

Purpose

To investigate a two-dimensional glancing collision

Safety Consideration

Do not touch the air table when the spark timer is activated!

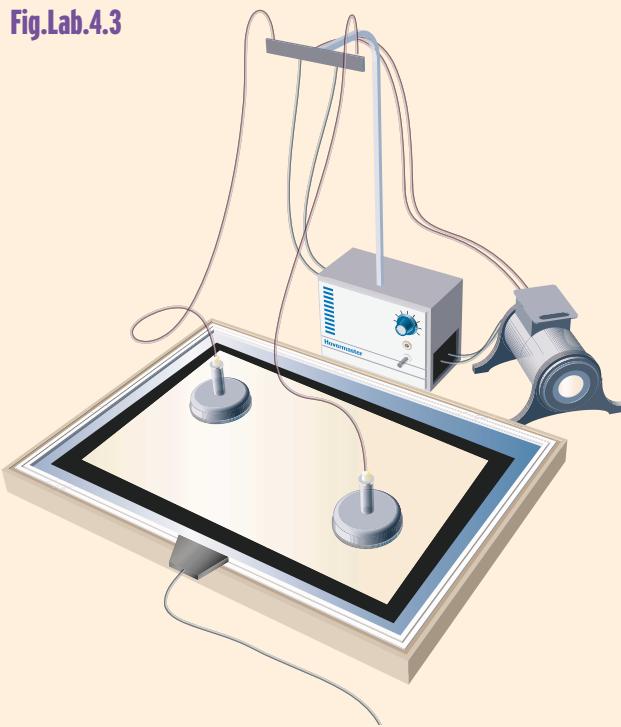
Equipment

Air table
Spark timer
Magnetic and non-magnetic steel pucks
Vacuum pump
Carbon paper
Metre stick
Tape

Procedure

- Set up the equipment as shown in Figure Lab.4.3.

Fig.Lab.4.3



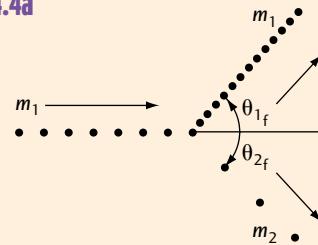
- Be careful with the electrical connections and take note of any safety features outlined by your teacher.
- Measure the mass of each puck, including uncertainty.

- Make sure that the air table is level. Adjust the legs of the table so that a puck sitting in the centre of the table will not slide in any direction with the air table turned on.

Part A: A Glancing Collision with the Second Puck Starting from Rest

- Place a blank sheet of paper on the carbon paper.
- Turn on the air pump.
- Place one of the pucks in the centre of the table. Call this puck the second puck or mass 2.
- Take a few practice shots by sliding a puck (mass 1) into the edge of mass 2 so that the collision is a glancing one, as shown in Figure Lab.4.4a.

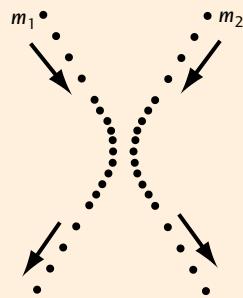
Fig.Lab.4.4a



- Have your partner turn on the spark timer as you release mass 1 into mass 2 so you can record the collision on the paper. Remove the paper from the table. On this paper, clearly label with a pencil the locations of mass 1 and mass 2 after the collision.
- Record the frequency on the spark timer.

Part B: A Glancing Collision with Both Pucks Moving Toward Each Other

- Place a blank sheet of paper on top of the carbon paper.
- With one hand on mass 1 and the other hand on mass 2, push the pucks toward each other such that the resulting collision is a glancing one, as shown in Figure Lab.4.4b. Practise this step several times before turning on the spark timer.

Fig.Lab.4.4b

3. Remove the paper from the air table. Clearly label the paths of mass 1 and mass 2 on your sheet.
4. Record the frequency of the spark timer.

Data

Part A

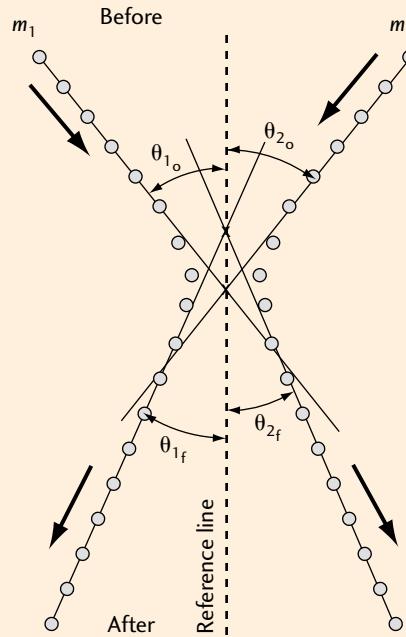
1. Prepare a data table similar to Table Lab.4.2. With a metre stick, draw a straight line following the path of the puck, m_1 , you released. Extend this line well beyond the point of collision. Now draw a line of best fit along the path of both pucks after collision. Measure the angle that both these lines make relative to the first line drawn. Record these angles on your paper, as shown in Figure 4.4a.
2. Measure the distance travelled by each puck before and after the collision in a specified time interval. To determine the time, count the dots. For instance, if the spark timer is set at 60 Hz, then 60 dots represent 1 s and 6 dots represent 0.1 s.

3. Record your results in a table like the one in Table Lab.4.2.

4. Retain your data for further analysis in the next chapter.

Part B

1. With a metre stick, draw four lines along the straight part of both curves, as illustrated in Figure Lab.4.5 (X-analysis). Draw a line through the points of intersection, as shown in Figure Lab.4.5. Measure the angle of the pucks before and after collision, as shown. Label these angles on your diagram.

Fig.Lab.4.5**Table Lab.4.2**

Part	m_1 (g)	m_2 (g)	$\Delta d_{1,o}$ cm	$\Delta t_{1,o}$ (s)	$\Delta d_{2,o}$ (cm)	$\Delta t_{2,o}$ (s)	$\Delta d_{1,f}$ (cm)	$\Delta t_{1,f}$ (s)	$\Delta d_{2,f}$ (cm)	$\Delta t_{2,f}$ (s)	$\Theta_{1,o}$ deg	$\Theta_{2,o}$ deg	$\Theta_{1,f}$ deg	$\Theta_{2,f}$ deg
A				—	—						0	—		
B														

Table Lab.4.3											
m_1	m_2	\vec{v}_{1_o}	\vec{v}_{2_o}	\vec{v}_{1_f}	\vec{v}_{2_f}	\vec{p}_{1_o}	\vec{p}_{2_o}	\vec{p}_{1_f}	\vec{p}_{2_f}	\vec{p}_{T_o}	\vec{p}_{T_f}
(g)	(g)	cm/s	cm/s	cm/s	cm/s	g·cm/s	g·cm/s	g·cm/s	g·cm/s	g·cm/s	g·cm/s

- Measure the distance travelled for each puck in a specified time interval and the angles of the pucks' path.
- Record your results in a table similar to Table Lab.4.2.
- Retain all your results and data for Chapter 5, when you will investigate the conservation of energy.

Analysis:

Calculate all velocity (\vec{v}) and momentum (\vec{p}) values, including the associated uncertainties, and record them in a table like Table Lab.4.3.

Graphical Vectors

Part A

- Draw a vector diagram of \vec{p}_{1_o} , \vec{p}_{1_f} , and \vec{p}_{2_f} .
- Draw the resultant for the total momentum after collision ($\vec{p}_{\text{total final}} = \vec{p}_{1_f} + \vec{p}_{2_f}$).

Part B

- Draw a vector diagram of \vec{p}_{1_o} , \vec{p}_{2_o} , \vec{p}_{1_f} , and \vec{p}_{2_f} .
- Draw the resultant for the total momentum before collision ($\vec{p}_{\text{total initial}} = \vec{p}_{1_o} + \vec{p}_{2_o}$).
- Draw the resultant for the total momentum after collision ($\vec{p}_{\text{total final}} = \vec{p}_{1_f} + \vec{p}_{2_f}$).

Component Method

- For Part A, calculate the total momentum before and after collision using the component method, including uncertainty.
- For Part B, calculate the total momentum before and after collision using the component method, including uncertainty. Use the prepared spreadsheet on <www.irwinpublishing.com/students> .

Discussion

- From your answers obtained using the graphical method, calculate the percent difference between the magnitude of the momenta before and after the collisions.
- Considering the percent differences, was momentum conserved in parts A and B?
- What are some possible reasons why momentum may not have been conserved?
- Was momentum conserved, within experimental uncertainty, using the component method?

Conclusion

Summarize your conclusions regarding the success of this lab with respect to the method of analysis.

Extension

- Write (or key) all your calculations, including those for uncertainties, and display them neatly on the original recording paper.
- Laminate or mount the page as a poster for future reference.
- Prepare a simulation of the collision using Interactive Physics™ or other software.
- Present the results of your lab to the class, using the poster and software as visual aids.



Linear Momentum in Two Dimensions: Ramp and Ball

Purpose

To investigate the two-dimensional collision of two solid balls using a ramp

Equipment

Ramp (2-D collision apparatus)

C-clamp

Plumb line

2 steel balls and 2 other solid balls (e.g., glass marbles)

Carbon paper

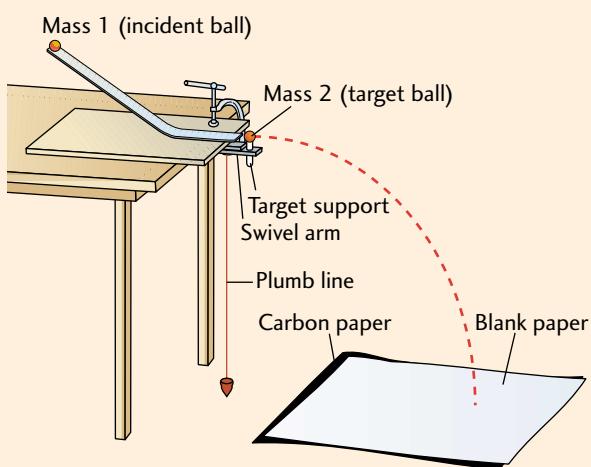
Masking tape

Paper and pencil

Procedure

- Set up the equipment as shown in Figure Lab.4.6.

Fig. Lab.4.6 A two-dimensional collision apparatus



- Take the two steel balls, mass 1 and mass 2. Hold mass 1 at the top of the ramp and balance mass 2 on the swivel at the bottom of the ramp. Both balls should have the same mass.

- Adjust the swivel so that mass 1 (the incident ball) just touches mass 2 (the target ball) at the point of collision at the bottom of the ramp. Adjust the swivel to ensure that mass 1 doesn't make contact with the swivel. Check your adjustment by releasing mass 1 without mass 2 in place and carefully listening for any sound as mass 1 clears the swivel without touching it. Adjust the swivel until no sound is heard.
- Place the carbon paper on the floor with the ink side up. Cover it with a large blank sheet of paper. Use masking tape to keep all the papers in place.
- Use a plumb line to locate the point on the floor directly below the point of collision. Mark this point with an X on your paper.
- Release mass 1, allowing it to roll down the ramp, just clearing the swivel arm. The ball should land on the paper and leave a mark on it.
- Repeat step 6 five times, always releasing mass 1 from the same height, to gather a cluster of points.
- Place mass 2 on the target support. Release mass 1 from the same height so that it collides with mass 2. Clearly label the points where both masses make the initial contact with the paper. Label this set of points "Trial 1."
- Adjust the swivel arm slightly to change the position of the target ball to produce a second set of points on the same sheet of paper. Label these points "Trial 2."
- Repeat step 9 for a third trial. Label these points "Trial 3."

Data

Label all impact and reference points on the large sheet of paper used to mark the impact points.

Analysis

1. Draw a line from X, the plumb-line mark, to the middle of the cluster of points in step 7 above. The length of this line represents the initial momentum vector for mass 1.
2. For Trials 1, 2, and 3, draw a line joining X with the point of contact of mass 1. Draw another line joining X with the point of contact of mass 2. The length of these two lines represents the final momentum vectors of mass 1 and mass 2 because mass 1 = mass 2 and $\vec{p} \propto \vec{v}$ and $\vec{v} \propto \Delta\vec{d}$ for this activity.
3. Measure all angles between these momentum lines. Draw a scale vector diagram for each trial to represent the sum of the two final momentum vectors. For addition of vectors, join them head to tail.
4. Find the resultant momentum for each trial.

Discussion

1. Do both balls hit the ground at the same time? Explain.
2. Why does the distance from X to the centre of the clustered points represent the initial momentum of mass 1?
3. Compare the vector sum of the final momentums for each trial with the initial momentum of mass 1.
4. Is momentum conserved in a two-dimensional collision? Explain.
5. What are some reasons why momentum was not conserved?
6. How can this experiment be improved?

Conclusion

State a conclusion based on your results.

Extension

Repeat the experiment using a steel incident ball and a glass target ball. Measure the mass of each ball and adjust the momentum vectors on your page accordingly ($\vec{p}_{\text{glass}} = \left(\frac{m_{\text{glass}}}{m_{\text{steel}}}\right)\vec{V}_{\text{glass}}$). Is momentum conserved?

Energy and Interactions



Chapter Outline

- 5.1 Introduction to Energy
- 5.2 Work
- 5.3 Kinetic Energy
- 5.4 Gravitational Potential Energy
- 5.5 Elastic Potential Energy and Hooke's Law
- 5.6 Power
- 5.7 Elastic and Inelastic Collisions
-  The Physics Equation — The Basis of Simulation
- LAB** 5.1 Conservation of Energy Exhibited by Projectile Motion
- LAB** 5.2 Hooke's Law
- LAB** 5.3 Inelastic Collisions (Dry Lab)
- LAB** 5.4 Conservation of Kinetic Energy

By the end of this chapter, you will be able to

- describe energy transfer from one form to another as a result of doing work on an object
- apply the concept of conservation of energy to solve energy problems
- solve elastic and inelastic collision problems

5.1 Introduction to Energy

Converting km/h/min to $\frac{\text{m}}{\text{s}^2}$

$$\begin{aligned}1600 \text{ km/h/min} \\&= 1600 \frac{\text{km}}{\text{h}\cdot\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ h}}{3600 \text{ s}} \\&\quad \times \frac{1000 \text{ m}}{1 \text{ km}} \\&= 7.4 \frac{\text{m}}{\text{s}^2}\end{aligned}$$

The countdown to takeoff is a very exiting moment for the thousands of people who visit Cape Canaveral to witness a shuttle launch. The sheer power and energy required for liftoff is an amazing sight! The space shuttle ignites its main engines, burning thousands of tons of solid and liquid fuel. The burning of fuel propels the shuttle upward while at the same time decreasing its mass. The energy in the fuel is converted to energy of motion of the rocket, causing it to accelerate at an average rate of about 1600 km/h/min in the first 60 seconds.

We are surrounded by numerous energy interactions in our daily lives. When we eat breakfast in the morning before going to school, our bodies convert food energy into kinetic energy as we walk to school, and to potential energy as we climb the stairs to the second-floor physics lab. When we start a car, the chemical potential energy stored in fuel is converted to electrical energy stored in the battery and used to power the radio, windshield wipers, and headlights. The fuel is also converted into mechanical energy of the car's motion. As the car does work to drive up a hill, the energy from the fuel is converted into mechanical energy to turn the wheels. As the car descends the hill, the gravitational potential energy it possesses at the top of the hill is converted to kinetic energy of motion.

In this chapter, we will explore how work and energy are related. We will expand on Chapter 4 to include collisions, interactions, and energy and momentum transfers. We will define various forms of energy, such as kinetic energy and elastic potential energy. We will also investigate devices where energy interactions are common, such as shock absorbers, clocks, and safety equipment used in sports.

Isolation and Systems

To undertake the study of energy and interactions, we first need to define two basic concepts: open and closed systems and isolated and non-isolated systems. If a system doesn't lose or gain particles during the time of measurement, it's said to be a **closed system**, such as the one shown in Figure 5.1a. Any other system is considered an **open system** (Figure 5.1b). If a system doesn't exchange energy with any object or circumstance outside of its own boundary, it is an **isolated system**. Systems that exchange energy with other systems are **non-isolated systems**.

Fig.5.1a The non-porous boundary of this box prevents movement of molecules between the box and the outer environment. The inside of the box is therefore a closed system.

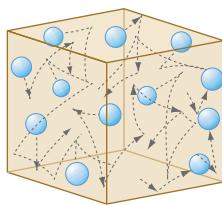
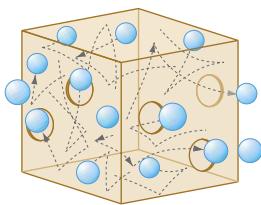


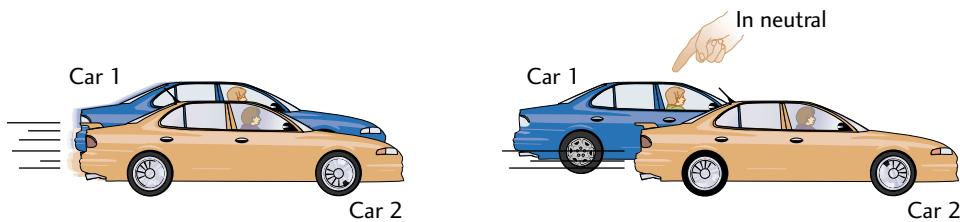
Fig.5.1b With a porous boundary, the molecules of gas in the box can escape while air molecules from outside the box can move in. In an open system, there is no way to ensure that the original contents of the box will remain the same.



A *system is closed if the amount of energy contained in it is constant*. In practice, a closed system is difficult to obtain. Nature adds and subtracts energy from a system in many ways, and most of these processes are not immediately obvious. But many systems lose only a small amount of their total energy, which allows us to accurately analyze energy transfer.

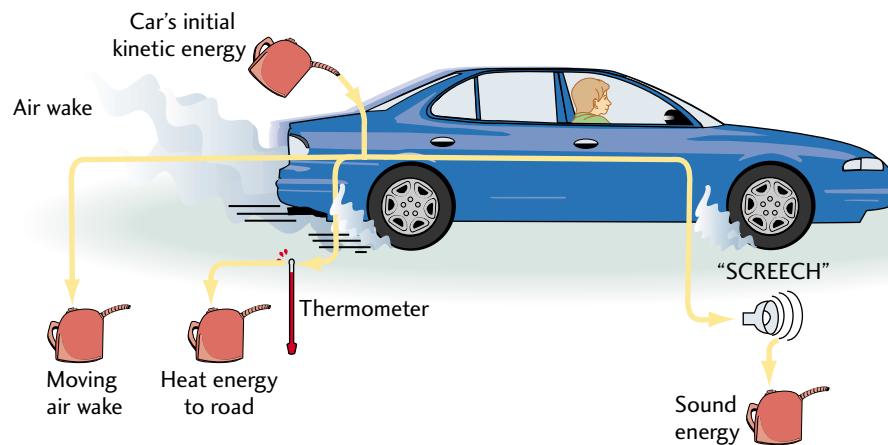
We can illustrate the difference between open and closed systems using the example of two cars (Figure 5.2). Both cars are initially moving at the same speed. Car 1 puts its transmission into neutral and slowly rolls to a stop. Car 2 continues with its transmission properly engaged and its speed unchanged. What is the difference between these two cars from an energy standpoint?

Fig.5.2 Which car is an open system?



For car 1, the energy removed from the initial kinetic energy of the car is lost to the environment in different forms. The most common type of energy loss is due to heat, but energy is also lost in the form of sound and other types of mechanical energy such as air drag, mechanical friction, and tire deformation as the car rolls. Ultimately, these forms of energy are also heat loss.

Fig.5.3 A car's initial kinetic energy is lost to the environment in various forms



In a closed system, energy is transferred from one form to another, but there is no energy transfer from a source that wasn't originally in the system. This constraint keeps the total energy constant. Therefore, for car 1, if we consider the car and the natural environment, then the system is closed since no energy is coming from a source outside our defined boundary.

For car 2, energy is still being fed into it by its engine. If the system boundaries we have defined are the same, then the system is open because chemical energy is being transferred from the fuel tank. However, if we broaden our system to include the energy in the fuel tank, then both systems (car 1 and car 2) are closed! The system would be open if the cars were being fuelled. Thus, we must be careful in defining our system. The definition of whether a system is closed or open is a relative one.



1. A ball rolls down a hill and stops at the bottom. Describe the conditions necessary for it to be
 - a closed system.
 - an isolated system.
 - an open system.
 - a non-isolated system.

Flight Data Recorders

Figure 5.4 shows a flight data recorder. These devices are installed on all commercial aircraft and are designed to withstand the horrific energies released during a crash. If a crash or other mishap occurs, investigators can obtain very important data on a number of aircraft systems prior to the mishap. Also used on large aircraft is the cockpit voice recorder, which records everything said in the cockpit until the time of impact. These devices help investigators piece together the possible human events that occurred prior to the mishap. They also reveal the amount of energy the plane had prior to impact. The information gained from

data and voice recorders has also helped to improve aircraft design and flight crew procedures.

Fig.5.4 A flight data recorder



2. Read reliable newspapers and magazines to find three examples of events such as accidents and other mechanical failures. Examine the evidence as described in the article and photographs.

- a)** As an accident-site investigator, what questions would you ask the survivors? Why?
- b)** What other evidence would you want from the scene? Explain how this evidence would contribute to a full knowledge of the energy budget.

5.2 Work

Work is the transfer of energy. Work is done when a force acts on an object, causing the object to move in the direction of the force. Mathematically, work is the dot product of the force applied and the displacement:

$$W = \vec{F} \cdot \Delta \vec{d}$$

where W is the work done on an object, measured in joules (J), \vec{F} is the force applied on the object, measured in newtons (N), and $\Delta \vec{d}$ is the displacement of the object, in metres (m).

In Figure 5.5, two construction workers are applying a force to the right. Worker A is pushing a wall that isn't moving, while worker B is pushing a wheelbarrow that's moving. Because work depends on displacement as well as force, only worker B is doing work. Worker A isn't displacing the wall; therefore, no work is being done on the wall. Worker B is displacing the wheelbarrow with his applied force; therefore, he is doing work on the wheelbarrow.

The SI unit for work is the joule (J) and it is defined as the force of one newton applied to an object to move it one metre:

$$1 \text{ J} = 1 \text{ N} \cdot \text{m}$$

Work is a scalar quantity because it doesn't have a direction. If the direction of motion is in the direction of the force, then $W = F\Delta d$.

Fig.5.5 Work is done only when an applied force displaces an object

(a)

No work

$$\vec{F}_{\text{app}}$$

$$\Delta \vec{d} = 0$$

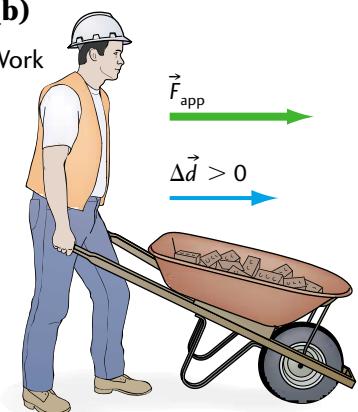


(b)

Work

$$\vec{F}_{\text{app}}$$

$$\Delta \vec{d} > 0$$



EXAMPLE 1

Calculating the work done

Calculate the work done by

- applying a force of 830 N [forward] on a 3000-kg car to displace it 25.0 m forward.
- applying a force of 20 N [right] to a 0.4-kg puck as it slides along a frictionless surface from rest to 10 m/s in 0.2 s.
- lifting a 57-kg outboard motor a distance of 1.4 m from the ground up to the box of a pickup truck.

Solution and Connection to Theory

- a) Let's assume that forward is positive.

Given

$$\vec{F} = 830 \text{ N [forward]} \quad m = 3000 \text{ kg} \quad \Delta\vec{d} = 25 \text{ m [forward]}$$

$$W = \vec{F} \cdot \Delta\vec{d} = F\Delta d$$

$$W = (830 \text{ N})(25 \text{ m})$$

$$W = 20\,750 \text{ N} \cdot \text{m}$$

$$W = 20\,750 \text{ J}$$

Fig.5.6a

$$\vec{F} = 830 \text{ N [forward]}$$

$$\Delta\vec{d} = 25 \text{ m [forward]}$$

The work required to move the car 25 m with a force of 830 N is 20 750 J.



- b) Given

$$F = 20 \text{ N} \quad m = 0.4 \text{ kg} \quad v_1 = 0 \text{ m/s} \quad v_2 = 10 \text{ m/s} \quad \Delta t = 0.2 \text{ s}$$

First, we calculate the displacement using the equation

$$\Delta d = \frac{1}{2}(v_1 + v_2)\Delta t$$

$$\Delta d = \frac{1}{2}(0 + 10 \text{ m/s})(0.2 \text{ s})$$

$$\Delta d = 1 \text{ m}$$

For work,

$$W = \vec{F} \cdot \Delta\vec{d} = F\Delta d$$

$$W = (20 \text{ N})(1 \text{ m})$$

$$W = 20 \text{ N} \cdot \text{m}$$

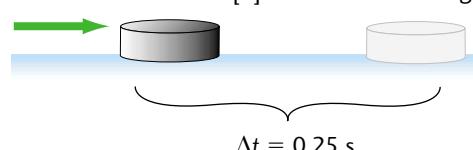
Fig.5.6b

$$\vec{v}_1 = 0$$

$$\vec{v}_2 = 10 \text{ m/s [R]}$$

$$\vec{F} = 20 \text{ N [R]}$$

$$m = 0.4 \text{ kg}$$



The work required to move the puck is 20 J.

Unit analysis for $W = F\Delta d$

Left side	Right side
W	$F \Delta d$
J	(N) (m)
	$\frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{m}$
	$\frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$
	J

c) Let's assume that up is positive.

Given

$$m = 57 \text{ kg} \quad \Delta d = 1.4 \text{ m}$$

In order to lift the motor, the minimum force required must be equal in magnitude but opposite in direction to the force of gravity acting on the motor. Therefore,

$$F = mg$$

$$F = (57 \text{ kg})(9.8 \text{ N/kg})$$

$$F = 558.6 \text{ N}$$

$$W = \vec{F} \cdot \vec{\Delta d} = F\Delta d$$

$$W = (558.6 \text{ N})(1.4 \text{ m})$$

$$W = 782 \text{ N}\cdot\text{m}$$

$$W = 782 \text{ J}$$

Fig.5.6c

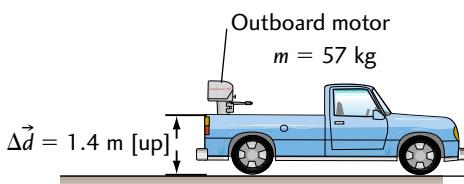
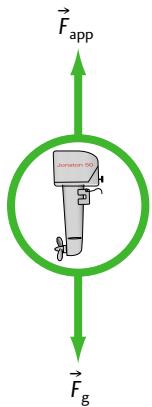


Fig.5.6d



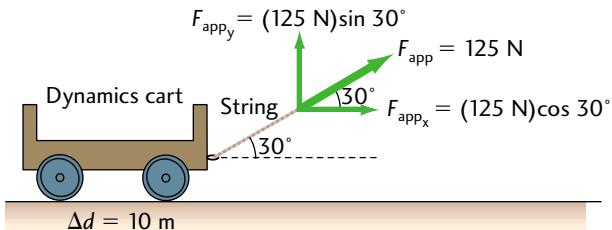
Therefore, the work required to lift the outboard motor onto the box of the pickup truck is 782 J.

The expression $\vec{F} \cdot \vec{\Delta d}$ is called a vector dot product with magnitude

$$W = F\Delta d \cos \theta$$

where θ is the angle between the direction of the force, \vec{F} , and the displacement, $\vec{\Delta d}$. If a force is applied to an object, the object may undergo a displacement in the direction of a *component* of the force. In Figure 5.7, even though the cart is pulled with a force, \vec{F}_{app} , of 125 N [R30°U], the cart's displacement is to the right; that is, horizontal, because it is being pulled along a horizontal surface. The force doing the work on the cart is the horizontal component of \vec{F}_{app} = 125 N [R30°U]; that is, $F_{app_x} = (125 \text{ N})\cos 30^\circ$.

Fig.5.7



To calculate the work, the force and the component of the displacement must be in the same direction

E X A M P L E 2**Calculating work using the equation**

$$W = F\Delta d \cos \theta$$

- a) A newspaper carrier pulls a wagon with a force of 275 N at an angle of 45° to the horizontal. How much work is required to move the wagon 8.00 m? Omit any friction with the road.
- b) Calculate the work done on a cyclist if a braking force of 40 N [backward] slows the cyclist from 20 m/s to 15 m/s in 2.0 s.

Solution and Connection to Theory

Let's assume that right is positive.

a) Given

$$\vec{F} = 275 \text{ N} [\text{U}45^\circ \text{ R}] \quad \Delta\vec{d} = 8.00 \text{ m} [\text{R}]$$

Since the force and the displacement are not in the same direction but 45° apart, the magnitude of the work done in the direction of the displacement is

$$W = F\Delta d \cos \theta$$

$$W = (275 \text{ N})(8.00 \text{ m})\cos 45^\circ$$

$$W = 1560 \text{ N}\cdot\text{m}$$

$$W = 1560 \text{ J}$$

The work done to move the wagon horizontally a distance of 8.00 m is 1560 J.

b) Given

$$\vec{F} = 40 \text{ N} [\text{L}] \quad \vec{v}_1 = 20 \text{ m/s} [\text{R}] \quad \vec{v}_2 = 15 \text{ m/s} [\text{R}] \quad \Delta t = 2.0 \text{ s}$$

We can calculate the displacement using the kinematics equation

$$\Delta d = \frac{1}{2}(v_1 + v_2)\Delta t$$

$$\Delta d = \frac{1}{2}(20 \text{ m/s} + 15 \text{ m/s})(2.0 \text{ s})$$

$$\Delta d = 35 \text{ m}$$

Because the cyclist is braking, she is applying a force that is in the opposite direction to her displacement. Therefore, the angle between her force and her displacement is 180° . To calculate the work done,

$$W = F\Delta d \cos \theta$$

$$W = (40 \text{ N})(35 \text{ m})\cos 180^\circ$$

$$W = (40 \text{ N})(35 \text{ m})(-1)$$

$$W = -1400 \text{ J}$$

The work done on the cyclist is -1400 J .

$$F_h = (275 \text{ N})\cos 45^\circ$$

$$F_h = 195 \text{ N}$$

Therefore, the work done in the direction of the displacement (horizontal) is

$$W = \vec{F}_h \cdot \Delta\vec{d}$$

$$W = (275 \text{ N})\cos 45^\circ(8.00 \text{ m})$$

$$W = 1560 \text{ N}\cdot\text{m}$$

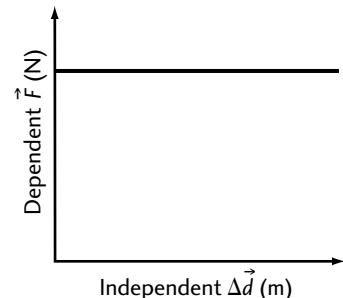
$$W = 1560 \text{ J}$$

The value for work is negative because the force and the displacement are in opposite directions. Negative work represents a flow or transfer of energy out of the object or system.

Work from an \vec{F} -versus- $\vec{\Delta d}$ Graph

Work can also be determined by finding the area under a force-versus-displacement graph. (If the displacement is the same as the distance, then the work can be calculated using the area under a force-versus-distance graph.) In Figure 5.8, a constant force is the dependent variable and the displacement is the independent variable. The work done is $\vec{F}\vec{\Delta d}$.

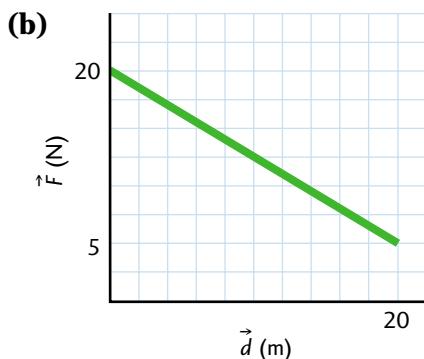
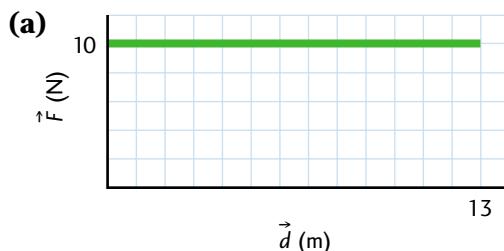
The dependent variable is located on the vertical axis while the independent variable is on the horizontal axis.



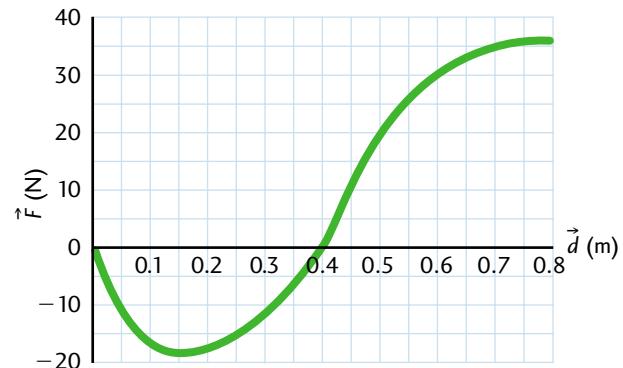
EXAMPLE 3 Calculating work given a \vec{F} -versus- $\vec{\Delta d}$ graph

Calculate the work done in each of the two cases represented in Figure 5.8.

Fig.5.8

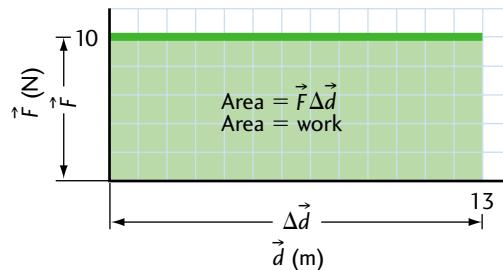


How would you calculate the work done for this graph?



Solution and Connection to Theory

Fig.5.9a



a) Given

$$F = 10 \text{ N} \quad \Delta d = 13 \text{ m}$$

$$W = F\Delta d$$

$$W = (10 \text{ N})(13 \text{ m})$$

$$W = 130 \text{ J}$$

The work done when a constant force of 10 N is applied over a distance of 13 m is 130 J.

b) Given

$$F_1 = 20 \text{ N} \quad F_2 = 5.0 \text{ N} \quad \Delta d = 20 \text{ m}$$

Figure 5.9b shows a uniformly decreasing force. We can calculate work by finding the average force,

$$W = F_{\text{avg}}\Delta d, \text{ where}$$

$$F_{\text{avg}} = \frac{1}{2}(F_1 + F_2)$$

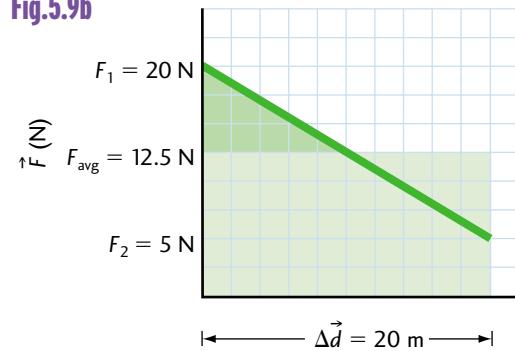
$$W = \frac{1}{2}(F_1 + F_2)\Delta d$$

$$W = \frac{1}{2}(20 \text{ N} + 5 \text{ N})(20 \text{ m})$$

$$W = \frac{1}{2}(25 \text{ N})(20 \text{ m})$$

$$W = 250 \text{ J}$$

The work done is 250 J.

Fig. 5.9b

The area of the trapezoid is equal to the area of the rectangle

W = area of trapezoid

$$W = \frac{1}{2}h(a + b)$$

$$W = \frac{1}{2}(20 \text{ m})(5 \text{ N} + 20 \text{ N})$$

$$W = 250 \text{ J}$$

An alternative solution would be to count the number of squares above the displacement axis (positive work) and below the displacement axis (negative work), and multiply the number of squares by the area of one square.

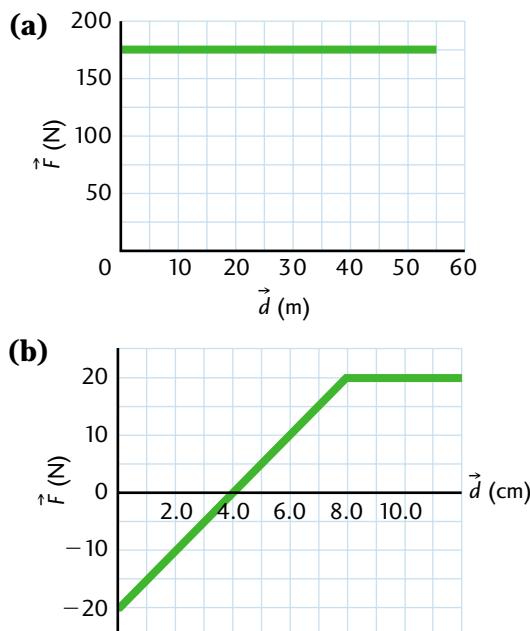


1. Determine the work done in each of the following cases:

- a) Kicking a soccer ball forward with a force of 40 N over a distance of 15 cm
- b) Lifting a 50-kg barbell straight up 1.95 m
- c) Pulling a sled with a force of 120 N at an angle of 25° to the horizontal if the sled is displaced 4.0 m forward

- A tow truck pulls a 3000-kg car from rest with a horizontal force of 5000 N. The truck and car accelerate at 2.5 m/s^2 for 5.0 s to reach the speed limit of 45 km/h. How much work is done by the tow truck?
- A wheelbarrow is pushed by a force of 78 N [U35° R] over a distance of 10 m. Determine the work done to move the wheelbarrow along the ground.
- A 52 000-kg train slows from 25 m/s to 14 m/s in 5.0 s. Calculate the work done on the train.
- Calculate the work done in each of the following graphs (Figure 5.10).

Fig.5.10



- Determine the height from which a 3-kg axe must be dropped so that it does 480 J of work to split a log resting on the ground.

5.3 Kinetic Energy

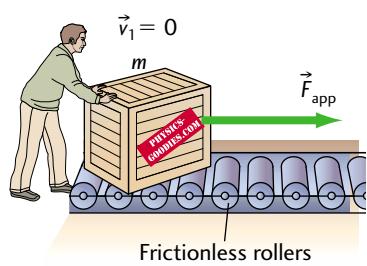
In Figure 5.11, a force is applied to a crate of mass m , initially at rest on some rollers. Because of the force \vec{F} , over time Δt , the crate undergoes a displacement of $\Delta\vec{d}$. Because of the work done on the crate, its velocity increases from zero to \vec{v}_2 .

Using our knowledge of kinematics and dynamics studied in Chapters 1 and 2, we can develop an equation for the work done on the crate.

$$W = E = F\Delta d = ma\Delta d$$

$$\text{where } a = \frac{(v_2 - v_1)}{\Delta t} \quad \text{and} \quad \Delta d = \left(\frac{v_1 + v_2}{2} \right) \Delta t$$

Fig.5.11



Since the force and the displacement are in the same direction along a horizontal line, we can omit the vector notation.

$$E = m \left(\frac{v_2 - v_1}{\Delta t} \right) \left(\frac{v_1 + v_2}{2} \right) \Delta t$$

Since $v_1 = 0$,

$$E = \frac{1}{2}mv_2^2$$

Unit analysis for $E_k = \frac{1}{2}mv^2$

Left side	Right side
E_k	$\frac{1}{2}mv^2$
(J)	$(kg)(m/s)^2$
	$(kg)m^2/s^2$
	$kg \cdot m/s^2 \cdot m$
	$\frac{kg \cdot m^2}{s^2}$
	J

If we let $v_2 = v$, then

$$E = \frac{1}{2}mv^2$$

The expression $\frac{1}{2}mv^2$ is the term for kinetic energy. **Kinetic energy** is the energy of motion when work is done on an object. Kinetic energy is a scalar quantity and its SI unit is the joule (J). (Work and kinetic energy both have the same unit.)

$$E_k = \frac{1}{2}mv^2$$

where m is the mass of the moving object, measured in kg, v is the object's velocity, measured in m/s, and E_k is the kinetic energy, measured in joules.

Since the initial velocity of the crate in Figure 5.11 is zero, the crate has no initial kinetic energy. Once work is done on the crate, its velocity increases and it now has kinetic energy. The change in kinetic energy is caused by the work done on the crate. Mathematically,

$$\begin{aligned} W &= \Delta E_k \\ W &= E_{k_2} - E_{k_1} \end{aligned}$$

where E_{k_2} and E_{k_1} represent the final and initial kinetic energies, respectively.

The relationship $W = E_{k_2} - E_{k_1}$ is called the work–energy theorem. The **work–energy theorem** states that if the speed of an object increases, the work done on the object is greater than zero:

If $v_2 > v_1$, then $W > 0$.

Conversely, if the work done is less than zero, then the object is doing work on the agent exerting the force:

If $v_1 > v_2$, then $W < 0$.

The greater the velocity, the greater the kinetic energy of the object, which has constant mass. The kinetic energy varies directly as the mass and the square of the velocity; that is,

$$E_k \propto m \quad \text{and} \quad E_k \propto v^2$$

In other words, for a moving object, if the mass is doubled, the kinetic energy is also doubled. If the velocity is doubled, the kinetic energy is quadrupled.

EXAMPLE 4 | The work–energy theorem

- a) How much kinetic energy does a 50.0-kg crate have if its velocity is 5.0 m/s?
- b) How much work is required to increase the crate's velocity to 7.0 m/s?

Solution and Connection to Theory

a) Given

$$m = 50.0 \text{ kg} \quad v = 5.0 \text{ m/s}$$

$$E_k = \frac{1}{2}mv^2$$

$$E_k = \frac{1}{2}(50.0 \text{ kg})(5.0 \text{ m/s})^2$$

$$E_k = 625 \text{ J}$$

Therefore, the kinetic energy of the crate is about 630 J.

b) Given

$$m = 50.0 \text{ kg} \quad v_1 = 5.0 \text{ m/s} \quad v_2 = 7.0 \text{ m/s}$$

According to the work–energy theorem, the amount of work required to increase the velocity is the change in kinetic energy:

$$W = \Delta E_k = E_{k_2} - E_{k_1}$$

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$W = \frac{1}{2}(50.0 \text{ kg})(7.0 \text{ m/s})^2 - \frac{1}{2}(50.0 \text{ kg})(5.0 \text{ m/s})^2$$

$$W = 1225 \text{ J} - 625 \text{ J}$$

$$W = 600 \text{ J}$$

The work required to increase the velocity of the crate by 2.0 m/s is 600 J.

From $p = mv$

$$v = \frac{p}{m}$$

Substituting this equation into the equation for kinetic energy,

$$E_k = \left(\frac{1}{2}m\right)\left(\frac{p}{m}\right)^2$$

$$E_k = \left(\frac{1}{2}m\right)\left(\frac{p^2}{m^2}\right)$$

$$E_k = \frac{p^2}{2m} \quad \text{or}$$

$$p = \sqrt{2mE_k}$$

Kinetic Energy and Momentum

Both kinetic energy and momentum contain mass and velocity variables in their equations. By manipulating the equations $p = mv$ and $E_k = \frac{1}{2}mv^2$, we can form an equation to relate momentum, p , and kinetic energy, E_k :

$$p = \sqrt{2mE_k}$$

EXAMPLE 5

Kinetic energy and momentum

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

Determine the momentum and speed of a proton that has a kinetic energy of 4274.7 eV. The mass of a proton is $1.67 \times 10^{-27} \text{ kg}$.

Solution and Connection to Theory

Given

$$E_k = 4274.7 \text{ eV} \quad m = 1.67 \times 10^{-27} \text{ kg} \quad 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

Converting the units for kinetic energy into SI units,

$$\begin{aligned} E_k &= (4274.7 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV}) \\ E_k &= 6.848 \times 10^{-16} \text{ J} \end{aligned}$$

To find the momentum of the proton,

$$\begin{aligned} p &= \sqrt{2mE_k} \\ p &= \sqrt{2(1.67 \times 10^{-27} \text{ kg})(6.848 \times 10^{-16} \text{ J})} \\ p &= 1.51 \times 10^{-21} \text{ kg} \cdot \text{m/s} \end{aligned}$$

We can use either the momentum or the kinetic energy equation to find the speed. Using the momentum equation,

$$\begin{aligned} v &= \frac{p}{m} \\ v &= \frac{1.51 \times 10^{-21} \text{ kg} \cdot \text{m/s}}{1.67 \times 10^{-27} \text{ kg}} \\ v &= 9.06 \times 10^5 \text{ m/s} \end{aligned}$$

The momentum of the proton is approximately $1.51 \times 10^{-21} \text{ kg} \cdot \text{m/s}$ and its speed is about $9.06 \times 10^5 \text{ m/s}$.



1. Calculate the kinetic energy in each of the following cases:
 - a) A 20 000-kg space shuttle moves at an orbital speed of 7.5 km/s.
 - b) A 1.0-kg eagle flies at 20 km/h.
 - c) A 30-g bullet moves at 400 m/s.
2. Determine the speed of a 245-kg boat if it possesses 3.9 kJ of kinetic energy.
3. Determine the mass of a ball that has a speed and kinetic energy of 15 m/s and 729 J, respectively.
4. What is the momentum of an electron with a kinetic energy of 6 keV in a particle accelerator?

- Calculate the change in kinetic energy when a 60.0-kg skateboarder slows down from 14.0 m/s to 5.0 m/s. Where does this energy go?
- An arrow of mass 350.0 g, travelling at 25.0 m/s, strikes a stationary wood fence post and penetrates it to a depth of 2.4 cm. Calculate
 - the kinetic energy of the arrow as it strikes the post.
 - the work done by the post on the arrow.
 - the average force of the wood on the arrow to stop the arrow.

5.4 Gravitational Potential Energy

When we see a book teetering on the edge of an overhead shelf, or when we walk under a ladder that has a can of paint hooked to the top rung, we can imagine the consequences of the objects falling. They could potentially damage any obstacles at ground level, including your toe. The more massive the object and the higher off the ground it is, the greater the possibility for damage. The ability of gravity to do work on an object by causing it to fall is known as **gravitational potential energy (E_g)**.

Figure 5.12 shows the variations in a brick's gravitational potential energy depending on its height. In Figure 5.12b, some work had to be done in order to elevate the brick from ground level (Figure 5.12a). The work done to lift the brick becomes the brick's gravitational potential energy. If the brick were to fall, its potential energy would decrease as it fell. According to the law of conservation of energy, the total amount of energy of a system must remain constant.

(a) Ground level

$$\Delta h = 0$$

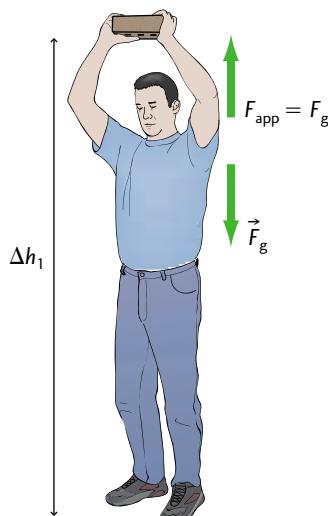
$$E_g = 0$$



Using the ground as a reference point,
 $E_g = 0$

(b) Lifting the brick

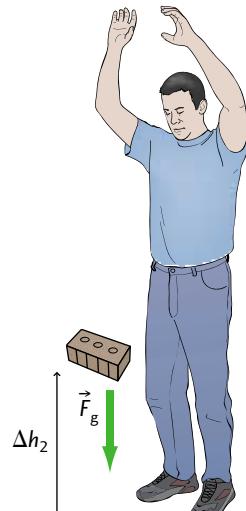
$$E_g = mg\Delta h$$



The brick's E_g increases the higher it is raised.

(c) A falling brick

$$E_T = E_g + E_k$$



As the brick falls, E_g decreases and E_k increases.

Fig.5.12

Therefore, if the brick's gravitational potential energy decreases when it falls, its kinetic energy must correspondingly increase. The brick accelerates as it falls, pulled by Earth's gravitational force.

We can derive an equation for gravitational potential energy from the equation for work,

$$W = F\Delta d$$

But $E_g = W$ and $\Delta h = \Delta d$; therefore,

$$W = F_g\Delta h$$

But $F_g = mg$. Therefore,

$$W = mg\Delta h$$

Using the ground as our reference point for measuring the change in height, Δh , the expression $mg\Delta h$ represents the change in potential energy from ground level 0 to a height of h . At ground level, $E_g = 0$.

$$\Delta E_g = mg\Delta h$$

From Chapter 1,

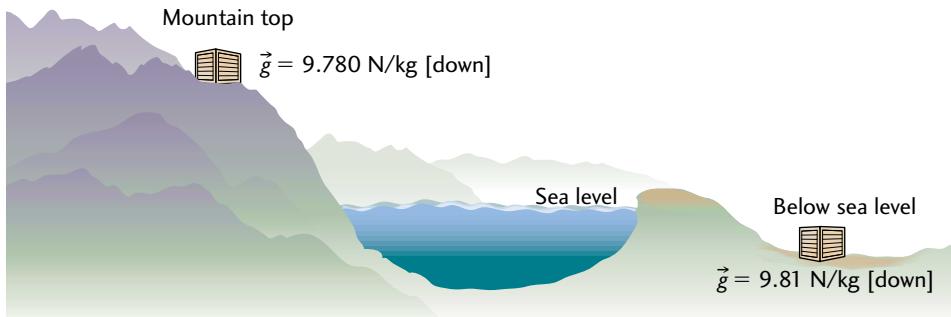
$$g = \frac{Gm_E}{r_E^2}$$

$$g = 9.8 \text{ N/kg}$$

where m is the mass of the object, measured in kilograms (kg), g is the gravitational constant, measured in newtons per metre (N/m), and Δh is the change in height of the object, measured in metres (m).

Near Earth's surface, the gravitational constant, g , barely changes. For simplicity's sake, we will use $g = 9.8 \text{ N/m}$ when dealing with objects near Earth's surface.

Fig. 5.13 The value of g varies slightly from location to location. The average value for g is 9.8 N/kg.



EXAMPLE 6 Solving potential-energy problems

A 2.0-kg planter is dangling from a balcony 8.0 m above the sidewalk.

- a) How much gravitational potential energy does the planter have with respect to the ground?
- b) A wind blows the planter off the balcony, causing it to fall straight down. With what speed does the planter hit the ground?

Solution and Connection to Theory

a) Given

$$m = 2.0 \text{ kg} \quad h = 8.0 \text{ m} \quad g = 9.8 \text{ N/kg}$$

$$E_g = mgh$$

$$E_g = (2.0 \text{ kg})(9.8 \text{ N/kg})(8.0 \text{ m})$$

$$E_g = 156.8 \text{ N}\cdot\text{m}$$

$$E_g = 1.6 \times 10^2 \text{ J}$$

The planter possesses about 160 J of gravitational potential energy relative to the sidewalk. Relative to any other point, the planter would have a different value for its gravitational potential energy. For instance, relative to a point 2 m below the sidewalk, the planter's gravitational potential energy would be $E_g = (2 \text{ kg})(9.8 \text{ N/m})(8.0 \text{ m} + 2 \text{ m}) = 196 \text{ J}$. Typically, gravitational potential energy is calculated relative to the ground or to any other useful reference point.

- b)** While on the balcony, the planter possesses gravitational potential energy only. When the planter falls, this energy is converted to kinetic energy. The moment the planter reaches the sidewalk, all its original gravitational potential energy has been converted to kinetic energy.

$$E_{g(\text{balcony})} = E_{k(\text{sidewalk})}$$

$$mgh = \frac{1}{2}mv^2$$

Dividing both sides of the equation by m , we obtain

$$gh = \frac{1}{2}v^2$$

$$v = \pm\sqrt{2gh}$$

$$v = \pm\sqrt{2(9.8 \text{ N/kg})(8.0 \text{ m})}$$

$$v = \pm 12.5 \text{ m/s}$$

The speed of the planter is 12.5 m/s. Since we know it is going down, its velocity is

$$\vec{v} = 12.5 \text{ m/s [down].}$$

The mass of the planter was not required for the speed calculation. The mass is irrelevant because all masses on Earth are attracted to Earth by a constant gravitational field (9.8 N/kg), which causes a uniform acceleration of 9.8 m/s².

Alternative Solution for part b) using Kinematics

Since the motion is vertical, the velocity can be calculated using the kinematics equations from Chapter 1.

Given

$$v_i = 0, \Delta d = -8 \text{ m}, a = -9.8 \text{ m/s}^2$$

$$v_f^2 = v_i^2 + 2a\Delta d$$

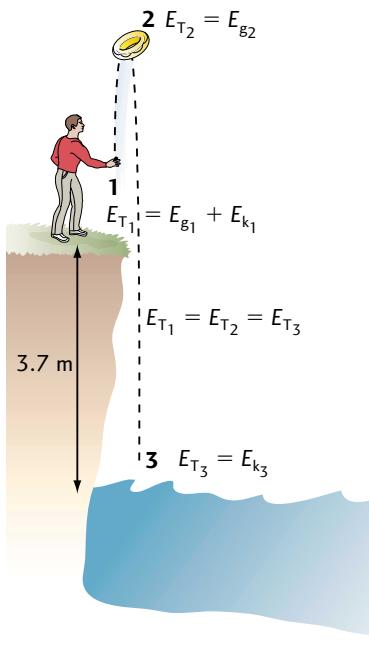
$$v_f = \pm\sqrt{2(-9.8 \text{ m/s}^2)(-8 \text{ m})}$$

$$\vec{v}_f = -12.5 \text{ m/s or } 12.5 \text{ m/s [down]}$$

EXAMPLE 7

Solving potential-energy problems

Fig. 5.14



Bounder of Adventure is standing on the edge of a 3.7-m-high cliff overlooking a lake. He throws a 2.5-kg life preserver upward with a speed of 12 m/s. The life preserver eventually falls into the water, as shown in Figure 5.14. If energy is conserved,

- what is the maximum height of the life preserver?
- with what velocity does the life preserver hit the water?
- with what average force did Bounder of Adventure throw the life preserver if he pushes it upward over a distance of 80 cm?

Solution and Connection to Theory

Let's use the lake's surface as our reference point because it is the lowest point in the problem. Let's use the subscripts 1, 2, and 3 to represent the three main points of the life preserver's trajectory; that is, the starting point, the highest point, and the lowest point, respectively, as shown in Figure 5.14.

a) Given

$$h_1 = 3.7 \text{ m} \quad m = 2.5 \text{ kg} \quad v_1 = 12 \text{ m/s}$$

Let's assume that the system consisting of Bounder of Adventure, the cliff, the life preserver, and the lake is closed. Therefore, according to the law of conservation of energy, the total amount of energy throughout the problem is constant. When Bounder of Adventure first throws the life preserver, it has some gravitational potential energy and some kinetic energy. At the top of its trajectory (maximum height), the life preserver possesses gravitational potential energy only, and its velocity is zero. Once the life preserver hits the water, all of its gravitational potential energy has been transferred to kinetic energy.

$$E_{\text{total}1} = E_{\text{total}2} = E_{\text{total}3}$$

To determine the maximum height of the life preserver, we are only concerned with the energy transfer between point 1 and point 2, where

$$E_{\text{total}1} = E_{\text{total}2}$$

$$E_{k_1} + E_{g_1} = E_{k_2} + E_{g_2}$$

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

But at maximum height, $v_2 = 0$; therefore,

$$\frac{1}{2}mv_1^2 + mgh_1 = mgh_2$$

$$\frac{1}{2}v_1^2 + gh_1 = gh_2$$

$$\frac{1}{2}(12 \text{ m/s})^2 + (9.8 \text{ N/kg})(3.7 \text{ m}) = (9.8 \text{ N/kg})(h_2)$$

$$72 \text{ m}^2/\text{s}^2 + 36.26 \text{ m}^2/\text{s}^2 = (9.8 \text{ m/s}^2)(h_2)$$

$$108.26 \text{ m}^2/\text{s}^2 = (9.8 \text{ m/s}^2)(h_2)$$

$$h_2 = 11.05 \text{ m} = 11 \text{ m}$$

This height represents the height above the water's surface. To find the maximum height of the life preserver above the top of the cliff, we subtract the height of the cliff:

$$11 \text{ m} - 3.7 \text{ m} = 7.3 \text{ m}$$

- b)** To calculate the velocity of the life preserver, we use the law of conservation of energy, which states that the total energy at the top of the life preserver's trajectory is equal to its total energy at the end of its trajectory; that is,

$$E_{\text{total}2} = E_{\text{total}3}$$

$$E_{\text{total}2} = mgh = (2.5 \text{ kg})(9.8 \text{ m/s}^2)(11 \text{ m})$$

$$E_{\text{total}2} = 270 \text{ J}$$

$$270 \text{ J} = \frac{1}{2}mv_3^2$$

$$270 \text{ J} = \frac{1}{2}(2.5 \text{ kg})v_3^2$$

$$v_3^2 = 216 \frac{\text{kg} \cdot \text{m}^2/\text{s}^2}{\text{kg}}$$

$$v_3 = \pm 14.7 \text{ m/s}$$

The speed of the life preserver is 14.7 m/s.

The velocity of the life preserver just as it hits the water is approximately 14.7 m/s [down].

- c)** To calculate the average force with which Bounder of Adventure threw the life preserver, recall that according to the work-energy theorem, the amount of work required to throw the life preserver upward is equivalent to the change in the life preserver's energy. The work done is equal to its increase in kinetic and potential energies.

Given

$$\Delta d = \Delta h = 0.80 \text{ m} \quad \Delta v = 12 \text{ m/s}$$

$$F\Delta d = mg\Delta h + \frac{1}{2}mv_1^2$$

$$F = mg + \frac{m\Delta v^2}{2\Delta d}$$

$$F = (2.5 \text{ kg})(9.8 \text{ m/s}^2) + \frac{(2.5 \text{ kg})(12 \text{ m/s})^2}{2(0.80 \text{ m})}$$

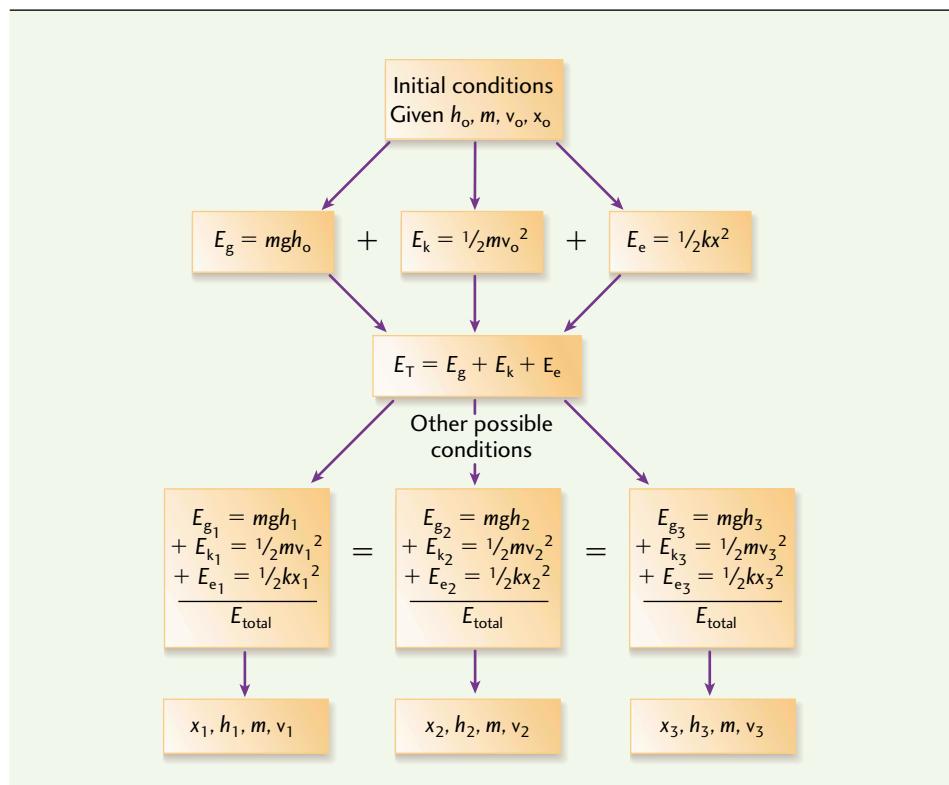
$$F = 24.5 \text{ N} + 225 \text{ N}$$

$$F = 250 \text{ N}$$

The average force required to throw the life preserver is 250 N [up].

Figure 5.15 summarizes the equations for the conservation of mechanical energy.

Fig.5.15 Summary of the Conservation of Mechanical Energy



1. Calculate the gravitational potential energy of each of the following:
 - A 3.5-kg bowling ball held 1.2 m above the ground by your fingers
 - A 2000-kg piano resting on the floor
 - The same 2000-kg piano with respect to the basement floor, 1.9 m below
2. A 65-kg stunt diver dives from a height of 27 m with no initial velocity.
 - Calculate the diver's velocity as she hits the water.
 - A second diver jumps up with a velocity of 3.0 m/s from the same platform as the first diver and just clears the board on her way down. Calculate the second diver's velocity as she hits the water.

- 3.** A 3.0-kg rocket is launched from a pad that is 5.0 m above the ground. The rocket's total energy at the top of its flight is 5460 J.
- What was the rocket's initial speed?
 - What height did the rocket achieve above the launch pad?
 - What is the potential energy and the kinetic energy of the rocket 2.0 s into its flight?
- 4.** A 5000-kg truck drives over a pothole in the road that causes the truck to push down on all four springs a distance of 4.0 cm. If energy is conserved and the springs obey Hooke's law, calculate the spring constant for each spring.

5.5 Elastic Potential Energy and Hooke's Law

Potential energy is often referred to as stored energy. As we saw in Section 5.2, an object in motion possesses kinetic energy and has the ability to do work. But an object need not necessarily be moving in order to have the ability to do work. Take the spring in a windup toy, for instance (see Figure 5.16). If the spring isn't wound, it has no energy and can't do any work to move the toy. If we give the spring some energy by winding it up, we give it the *potential* to do work to move the toy. Through a series of gears, the potential energy from the spring is transferred into kinetic energy in the toy. We say that the spring has the ability to do work; therefore, it has potential energy. This energy is somehow stored in the mechanism of the spring when it is wound. As the spring regains its natural (unwound) form, its potential energy decreases. We can consider a spring to be in **equilibrium** when it is in its normal, unwound state.

Fig. 5.16

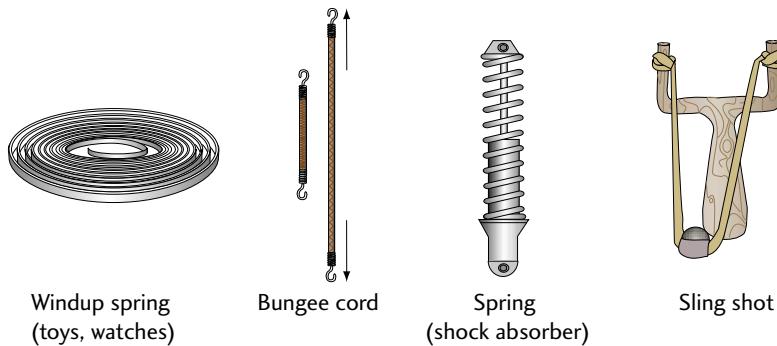
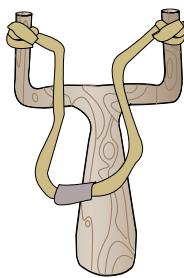


Fig.5.17



A loose slingshot possesses no elastic potential energy



A pulled slingshot possesses elastic potential energy

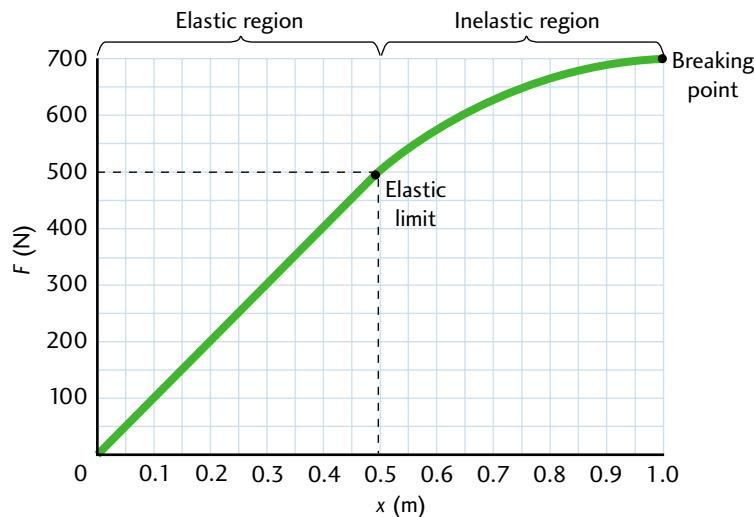
An object is considered *elastic* if it can be deformed by a force in order to store energy, and transfer its stored energy by returning to its normal state. Examples of elastic objects are watch springs, windup toys, car bumper mounts, trampolines, elastic slingshots (Figure 5.17), and leaf or coil springs.

Robert Hooke (1635–1703), a British scientist, was one of the first scientists to study the elasticity of matter.

Hooke's law states that the deformation of an elastic object is proportional to the force applied to deform it.

When an elastic object is deformed, the amount of deformation can be determined by measuring the length of the object's stretch or compression. The graph in Figure 5.18 represents a coiled spring to which an increasing force is applied until the spring breaks. If the force is not too great, the stretched spring can return to its normal length. If too much force is applied, the spring may become permanently deformed, or it may break. Deforming the spring by an excessive force destroys the elasticity of the spring, and we say that the spring becomes **inelastic**.

Fig.5.18



For an elastic spring, the graph of the force applied (F) versus the amount of deformation (x) is a straight line. Hooke's law states that

$$F \propto x$$

Changing the proportionality statement to an equation gives

$$F = kx$$

where F is the applied force to stretch or compress an elastic object, measured in newtons (N), x is the amount of deformation, measured in metres (m), and k is the spring constant of the elastic object, measured in newtons per metre (N/m).

The quantities F and x are both scalars. A simple sign convention is used to represent the force, F , and the amount of deformation, x . Positive (+) numbers are used for F and x when a spring is stretched, and negative (−) numbers are used when a spring is compressed. *The spring constant, k , can be found by calculating the slope of the straight line on the F -versus- x graph.*

EXAMPLE 8 Calculating the spring constant

Determine the spring constant for the spring in Figure 5.18.

Solution and Connection to Theory

The spring constant, k , is found by calculating the slope of the straight line on the graph. Let's use the points (0,0) and (0.5 m, 500 N).

$$k = \frac{\Delta F}{\Delta x}$$

$$k = \frac{F_2 - F_1}{x_2 - x_1}$$

$$k = \frac{500 \text{ N} - 0 \text{ N}}{0.5 \text{ m} - 0 \text{ m}}$$

$$k = 1.0 \times 10^3 \text{ N/m}$$

The spring constant is $1.0 \times 10^3 \text{ N/m}$.

Every spring has its own spring constant, which is a measure of the stiffness of a spring. The larger the value for k , the stiffer the spring.

We saw in Section 5.2 that work done is represented by the area under the force-versus-distance graph. The work done to deform the spring in Figure 5.18 is the same as the amount of stored energy in the spring. This stored energy has the potential to do work and is referred to as **elastic potential energy (E_e)**. The elastic potential energy has the same units as work, namely, the joule (J). A general expression for elastic potential energy can be derived using the fact that the area under the F -versus- x graph of any elastic object that obeys Hooke's law is always in the shape of a triangle, as shown in Figure 5.19.

Unit Analysis for $E_e = \frac{1}{2}kx^2$

Left side	Right side
E_e	$\frac{1}{2}kx^2$
(J)	$(\frac{N}{m})(m)^2$
	$N \cdot m$
	J

E_e = area under the F -versus- x graph

$$E_e = \frac{1}{2}bh$$

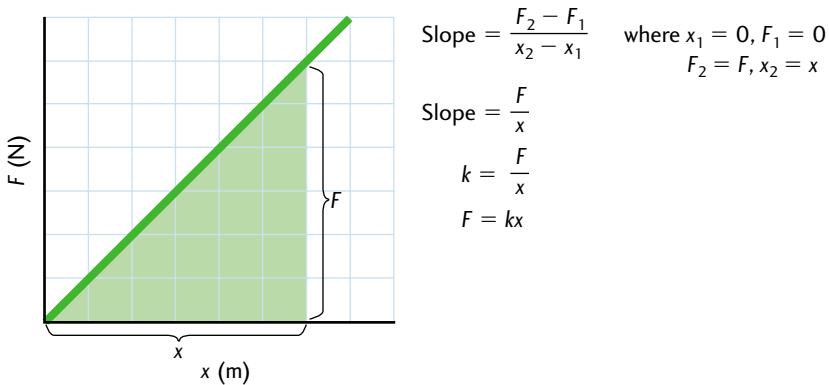
$$E_e = \frac{1}{2}(x)(F), \text{ where } F = kx$$

$$E_e = \frac{1}{2}(x)(kx)$$

$$E_e = \frac{1}{2}kx^2$$

where E_e is the elastic potential energy, measured in joules (J), k is the spring constant or force constant, measured in newtons per metre (N/m), and x is the amount of deformation, in metres (m).

Fig.5.19



EXAMPLE 9 Solving a typical spring problem

A spring with a force constant of 240 N/m has a 0.80-kg mass suspended from it. What is the extension of the spring and how much potential energy does it have once the mass is suspended?

Solution and Connection to Theory

Given

$$k = 240 \text{ N/m} \quad m = 0.80 \text{ kg}$$

The stretch of the spring depends on the force of gravity pulling on the mass.

$$F_e = F_g$$

$$kx = mg$$

$$x = \frac{mg}{k}$$

$$x = \frac{(0.80 \text{ kg})(9.8 \text{ N/kg})}{240 \text{ N/m}}$$

$$x = 0.033 \text{ m}$$

Therefore, the amount of stretch or the extension of the spring is 3.3 cm.

To calculate the potential energy,

$$E_e = \frac{1}{2}kx^2$$

$$E_e = \frac{1}{2}(240 \text{ N/m})(0.033 \text{ m})^2$$

$$E_e = 0.13 \text{ N}\cdot\text{m}$$

The amount of energy this spring possesses as a result of the weight suspended is 0.13 J. This value also means that 0.13 J of work was needed to stretch the spring.

Conservation of Energy

In Example 9, we learned that the work done on a spring is the same as the amount of potential energy stored in that spring. If the mass were removed from the suspended spring, the spring would recoil, thereby releasing stored energy as kinetic energy. This interaction between various forms of energy leads us to one of the fundamental laws in physics: the **law of conservation of energy**. Three possible ways to state this law are:

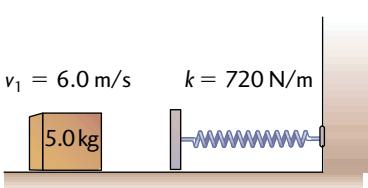
- 1) Energy cannot be created or destroyed, but only transferred from one object to another or transformed from one form to another without any loss.
- 2) In any closed system, the total energy remains constant.
- 3) Energy can change from one form to another, but the total amount of energy remains the same as long as the system being considered is a closed one.

Consider a moving laboratory cart colliding with a stationary cart. In the short time that the carts are in contact, the moving cart exerts a force on the stationary cart, doing work on it and transferring kinetic energy to it. At the same time, the moving cart experiences a force exerted on it by the stationary cart (Newton's third law), and its kinetic energy is decreased.

EXAMPLE 10

Using the law of conservation of energy to find x

Fig. 5.20



Another way to write that the kinetic energy lost by the block is the same as the elastic potential energy gained by the spring is

$$-\Delta E_{k(\text{block})} = \Delta E_{e(\text{spring bumper})}$$

$-\Delta E_{k(\text{block})}$ refers to the loss of kinetic energy by the block, while $\Delta E_{e(\text{spring bumper})}$ refers to the gain in elastic potential energy by the spring. A negative value for energy change indicates that energy has been *lost*.

In Figure 5.20, a frictionless metal block of mass 5.0 kg slides at a speed of 6.0 m/s into a fixed spring bumper with a spring constant of 720 N/m. If the block comes to rest, how much does the spring compress?

Solution and Connection to Theory

According to the law of conservation of energy, the kinetic energy lost by the block is the same as the elastic potential energy gained by the spring bumper. Therefore,

$$E_{k(\text{block})} = E_{e(\text{spring bumper})}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$mv^2 = kx^2$$

$$x = \pm \sqrt{\frac{mv^2}{k}}$$

$$x = \sqrt{\frac{(5.0 \text{ kg})(6.0 \text{ m/s})^2}{720 \text{ N/m}}}$$

$$x = \pm 0.5 \text{ m}$$

We choose the negative value for x because the spring is being compressed. Therefore,

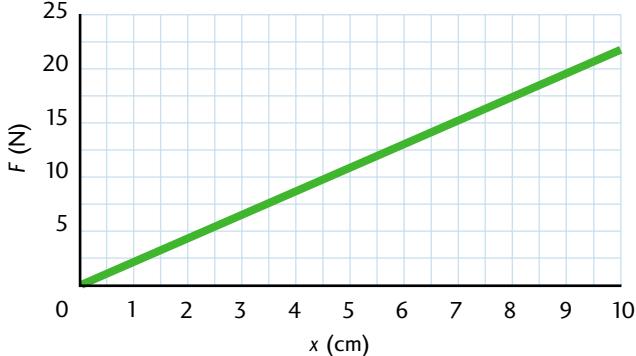
$$x = -0.5 \text{ m.}$$

The spring bumper was compressed 50 cm before coming to a full stop.



- Figure 5.21 is a graph of F versus x for an elastic spring. Determine
 - the spring constant.
 - the spring's maximum amount of elastic potential energy.
 - the change in elastic potential energy when the spring extends from 3 cm to 4 cm.

Fig. 5.21



2. A spring attached to a ceiling has a mass of 500 g suspended from it such that the spring stretches 4.0 cm. Calculate the spring constant.
3. How much work must be done to
 - a) compress a spring 4.0 cm if the spring constant is 55 N/m?
 - b) stretch a spring 8.0 cm if the spring constant is 85 N/m?
4. A slingshot with a spring constant of 200 N/m is pulled back 8.0 cm. A 20-g pea is launched by the slingshot horizontally. At what speed does the pea leave the slingshot?
5. The bumper of a 2000-kg car has a spring constant of 5×10^6 N/m. The car is moving at 4.5 m/s horizontally when it crashes into a solid brick wall. How much will the car's bumper be compressed if the car comes to a complete stop?
6. A 1.2-kg spring laboratory cart is held against a wall. The spring constant is 65.0 N/m. The spring is compressed 8.0 cm when held against the wall. What is the compression of the spring when the cart's velocity is 42.0 cm/s?

5.6 Power

So far in this chapter, our calculations concerned amounts of energy transfer. We must also consider how quickly this transfer occurs. **Power** is the *rate* of energy transfer; that is,

$$P = \frac{E}{\Delta t}$$

The unit of power is the watt, W, equal to J/s.

Fig. 5.22a Light bulbs are rated with a certain number of watts. Many light fixtures have limits on the size of light bulb that can be used.



Fig. 5.22b Stereo speakers are rated according to their power output in watts



Fig. 5.22c Increasing the size of an engine increases its power output



Power on the ski slopes**Fig.5.23**

- a)** What power is required for a ski-hill chair lift that transports 500 people (average mass 65 kg) per hour to an increased elevation of 1200 m?
- b)** What is the power of a high-speed chair that transports 25% more people up the same hill in half the time?

Solution and Connection to Theory**Given**

$$m = (500)65 \text{ kg} \quad g = 9.8 \text{ m/s}^2 \quad h = 1200 \text{ m} \quad \Delta t = 3600 \text{ s}$$

a) $P = \frac{E}{\Delta t}$

$$P = \frac{500(65 \text{ kg})(9.8 \text{ m/s}^2)(1200 \text{ m})}{3600 \text{ s}}$$

$$P = 1.06 \times 10^5 \text{ W}$$

The power required for the lift is $1.06 \times 10^5 \text{ W}$.

b) Since $P \propto \frac{m}{\Delta t}$,

$$\frac{P_1}{P_2} = \frac{m_1 \Delta t_2}{m_2 \Delta t_1}$$

$$P_2 = \frac{(P_1)(1.25m_2)\Delta t_1}{m_1(0.5\Delta t_1)} = 2.5P_1$$

$$P_2 = 2.5(1.06 \times 10^5 \text{ W}) = 2.65 \times 10^5 \text{ W}$$

The high-speed chair requires 2.5 times the power or $2.65 \times 10^5 \text{ W}$.

We can also derive an equation for the amount of power required to maintain an object in motion at constant speed. The concepts of work and power are related via energy. If

$$W = E = \vec{F} \cdot \vec{d}$$

then

$$P = \vec{F} \cdot \frac{\vec{d}}{t} = \vec{F} \cdot \vec{v}$$

If motion and force are in the same direction, then we can simplify the dot product to

$$P = Fv$$

EXAMPLE 12**A car engine's power**

A car travels at a constant speed of 20 m/s. The car's engine provides a force of 1800 N at the wheels to overcome air drag. What is its power?

Solution and Connection to Theory**Given**

$$v = 20 \text{ m/s} \quad F = 5000 \text{ N}$$

Since F and v are both in the same direction,

$$P = Fv$$

$$P = (1800 \text{ N})(20 \text{ m/s})$$

$$P = 36\,000 \text{ W} = 36 \text{ kW}$$

Therefore, the car engine's power is 36 kW.

ELECTRICAL POWER UNITS

We often use the word "power company" to describe an electrical utility, but generating stations really provide energy. The distribution system permits rates of energy transfer (power!) that are reasonable for the average house and company. A generating station has generating units that are rated according to how much power they possess. Typically, nuclear power stations have units rated from 500 MW to 1000 MW.

The electrical power meter outside your house is in units of kilowatt hours (kWh). To convert kWh to kilojoules, we multiply by 3600 kJ/kWh. The kWh is a more practical unit than the kilojoule for calculating cost.

The following example illustrates how a car's power must increase in order to climb a hill at a constant speed.

EXAMPLE 13**Power at constant speed**

A car climbs a hill inclined at 6° at a constant speed of 20 m/s.

- If the car's mass is 1000 kg and it uses 36 kW of power to overcome air drag, what is its total power?
- If 65% of the power generated by the engine is transferred to the car, what is the engine's power output?

Solution and Connection to Theory**Given**

$$v = 20 \text{ m/s} \quad m = 1000 \text{ kg} \quad \theta = 6^\circ \quad P_{\text{air drag}} = 36 \text{ kW}$$

Fig.5.24



We can calculate the change in height per second and convert it to the change in potential energy. Since the car's speed up the incline is 20 m/s, in 1.0 s, it travels 20 m.

$$h = d \sin 6^\circ$$

$$h = (20 \text{ m})(0.105)$$

$$h = 2.09 \text{ m}$$

In 1.0 s, the car climbs 2.09 m. The increase in its potential energy is

$$E_p = mgh = (1000 \text{ kg})(9.8 \text{ m/s}^2)(2.09 \text{ m})$$

$$E_p = 20\,482 \text{ J}$$

In 1.0 s,

$$P = \frac{20\,482 \text{ J}}{1.0 \text{ s}} = 20.5 \text{ kW}$$

Since 36 kW is required to overcome air drag, the total power is
 $P_T = 36 \text{ kW} + 20.5 \text{ kW} = 56.5 \text{ kW}$

Therefore, the car's power output increases to 56 kW.

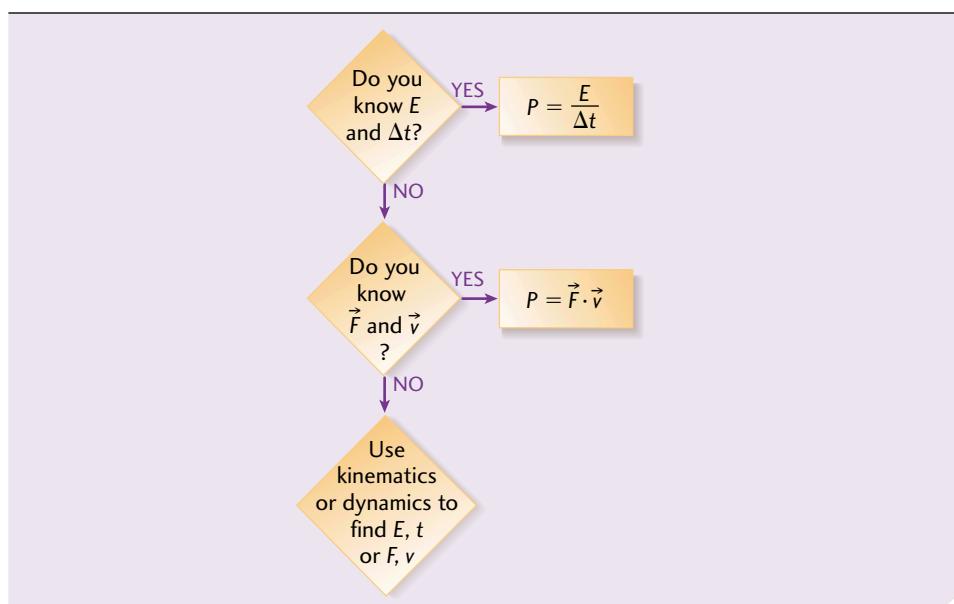
b) If 56.5 kW represents 65% of the engine's power, then the total power of the engine is

$$P = \frac{56.5 \text{ kW}}{0.65} = 87 \text{ kW}$$

We often hear the power of a car engine expressed in horsepower. One horsepower equals 746 W. In Example 13, the engine horsepower for the car to climb the hill at constant speed is $\frac{87\,000 \text{ W}}{746 \text{ W/hp}} = 117 \text{ hp} = 120 \text{ hp}$.

Figure 5.25 summarizes how to solve power problems.

Fig. 5.25 Solving Power Problems



1. An electric stove has a power rating of 1 kW on its large burner. If the stove heats 2.3 L of water for 10 minutes from 10°C to 65°C , how much energy is lost to the environment? (Assume that water requires $4.2 \times 10^3 \text{ J}/^{\circ}\text{C/L.}$)
2. Tarzan runs up a flight of stairs to a height of 13.0 m above his starting point in 18.0 s. If his mass is 83.0 kg,
 - a) what is his average power?
 - b) what is the total amount of energy transferred?
 - c) Is the value you found in b) accurate? Why or why not?
3. The Sun radiates $3.9 \times 10^{26} \text{ W}$ of power. If Earth's diameter is 13 740 km and its mean distance from the Sun is $1.49 \times 10^{11} \text{ m}$, how much of this radiation is intercepted by Earth each second?

Power and the Human Body

Humans dissipate or consume about 100 W of power in just staying alive. Our brains alone require about 15 W of power. This value is loosely proportional to our weight. As we begin to move around, we consume more energy and our power increases to the 200-W level. Somewhere around 200 W, respiration and heart rate increase. The body uses more chemical energy than its standard respiration rate can supply, so respiration increases to increase cell oxygen intake. As power consumption surpasses 200 W, the body sweats to release excess heat. With more exercise, respiration and heart rate increase further. Eventually, we require sustenance in the form of food and water. The human body has limited ability to **metabolize** its food and convert it to energy.

Fig.5.26



4. Calculate the total energy consumed by a hockey player during three 20-minute periods if he is on the ice 25% of the time consuming energy at a rate of 215 W. (The remainder of the time, he rests on the bench.)

5.7 Elastic and Inelastic Collisions

There are numerous energy interactions in everyday collisions. The different forms of energy involved in a collision may include kinetic energy, heat energy due to friction, the energy used to produce a sound (such as a crash or a bounce), gravitational potential energy, and elastic potential energy. Physics is concerned with the interactions between matter and the interactions involving energy. In all collisions, momentum is conserved according to the *law of conservation of momentum*, studied in Chapter 4. According to the *law of conservation of energy*, the total amount of energy involved in a collision is also conserved. In some collisions, the total amount of kinetic energy is conserved. This type of collision is referred to as an **elastic collision**. If the total final kinetic energy is different than the total initial kinetic energy in a collision, then the collision is said to be an **inelastic collision**. Momentum is conserved for both cases. In inelastic collisions, kinetic energy is lost to other forms of energy, such as heat and light.

Equations for One-dimensional Elastic Collisions

The following derivation leads to a shortcut for calculating the final velocities for two objects involved in an elastic collision where the second object is initially at rest.

Two objects of masses m_1 and m_2 are involved in an elastic collision. The initial velocity of the first mass is v_{1_0} . Since the second mass is initially at rest, $v_{2_0} = 0$. Using the law of conservation of momentum,

$$\begin{aligned}\vec{p}_{\text{total initial}} &= \vec{p}_{\text{total final}} \\ \vec{p}_{1_0} + \vec{p}_{2_0} &= \vec{p}_{1_f} + \vec{p}_{2_f} \\ m_1 \vec{v}_{1_0} + m_2 \vec{v}_{2_0} &= m_1 \vec{v}_{1_f} + m_2 \vec{v}_{2_f}\end{aligned}$$

In a linear system, we can let + and – indicate direction and omit the vector arrows.

But $v_{2_0} = 0$. Therefore,

$$\begin{aligned}m_1 v_{1_0} &= m_1 v_{1_f} + m_2 v_{2_f} \\ m_1 v_{1_0} - m_1 v_{1_f} &= m_2 v_{2_f} \\ m_1 (v_{1_0} - v_{1_f}) &= m_2 v_{2_f} \quad (\text{eq. 1})\end{aligned}$$

Using the law of conservation of kinetic energy for an elastic collision,

$$\begin{aligned}E_{k \text{ total initial}} &= E_{k \text{ total final}} \\ \frac{1}{2} m_1 v_{1_0}^2 + \frac{1}{2} m_2 v_{2_0}^2 &= \frac{1}{2} m_1 v_{1_f}^2 + \frac{1}{2} m_2 v_{2_f}^2\end{aligned}$$

But $v_{2_0} = 0$. Dividing both sides of the equation by $\frac{1}{2}$,

$$m_1 v_{1_0}^2 = m_1 v_{1_f}^2 + m_2 v_{2_f}^2$$

Moving the like terms (m_1) to one side of the equation, we obtain

$$\begin{aligned} m_1 v_{1_0}^2 - m_1 v_{1_f}^2 &= m_2 v_{2_f}^2 \\ m_1(v_{1_0}^2 - v_{1_f}^2) &= m_2 v_{2_f}^2 \quad (\text{eq. 2}) \end{aligned}$$

Dividing equation 2 by equation 1, we obtain

$$\frac{m_1(v_{1_0}^2 - v_{1_f}^2)}{m_1(v_{1_0} - v_{1_f})} = \frac{m_2 v_{2_f}^2}{m_2 v_{2_f}}$$

$$v_{1_0} + v_{1_f} = v_{2_f} \quad (\text{eq. 3})$$

Substituting equation 3 into equation 1,

$$\begin{aligned} m_1(v_{1_0} - v_{1_f}) &= m_2(v_{1_0} + v_{1_f}) \\ m_1 v_{1_0} - m_1 v_{1_f} &= m_2 v_{1_0} + m_2 v_{1_f} \end{aligned}$$

Isolating the v_{1_0} terms,

$$\begin{aligned} m_1 v_{1_0} - m_2 v_{1_0} &= m_1 v_{1_f} + m_2 v_{1_f} \\ v_{1_0}(m_1 - m_2) &= v_{1_f}(m_1 + m_2) \\ v_{1_f} &= v_{1_0} \left(\frac{m_1 - m_2}{m_1 + m_2} \right) \quad (\text{eq. 4}) \end{aligned}$$

where v_{1_f} is the final velocity of the first mass, in m/s, v_{1_0} is the initial velocity of the first mass, in m/s, and m_1 and m_2 are the first and second masses, respectively, measured in kilograms.

Similarly, we can also show that

$$v_{2_f} = v_{1_0} \left(\frac{2m_1}{m_1 + m_2} \right) \quad (\text{eq. 5})$$

where v_{2_f} is the velocity of the second mass after collision.

Equations 4 and 5 are used to find the final velocities of two objects involved in a linear elastic collision. You will have the opportunity to derive equation 5 in the Applying the Concepts section at the end of this section.

We can draw several conclusions based on an analysis of equations 4 and 5. Let's consider three cases:

$$v_{1f} = v_{1o} \left(\frac{m_1 - m_2}{m_1 + m_2} \right) \quad (\text{eq. 4})$$

$$v_{2f} = v_{1o} \left(\frac{2m_1}{m_1 + m_2} \right) \quad (\text{eq. 5})$$

Case 1: $m_1 = m_2$

In equation 4, the numerator ($m_1 - m_2$) becomes zero; therefore, $v_{1f} = 0$. The first object comes to rest when it collides with the second object at rest.

In equation 5, the numerator and denominator are equal to 1; therefore,

$$v_{2f} = v_{1o}$$

Case 2: $m_1 >> m_2$

In equation 4, the numerator ($m_1 - m_2$) and denominator ($m_1 + m_2$) both approach m_1 and cancel each other out; therefore, $v_{1f} = v_{1o}$.

In equation 5, the denominator approaches m_1 ; therefore, $v_{2f} = 2v_{1o}$.

Case 3: $m_1 << m_2$

In equation 4, the term $\frac{m_1 - m_2}{m_1 + m_2}$ approaches -1 ; therefore, $v_{1f} = -v_{1o}$ (the final velocity of the first object has the same magnitude but the opposite direction of its initial velocity).

In equation 5, the term $\frac{2m_1}{m_1 + m_2}$ approaches zero; therefore, $v_{2f} = 0$.

EXAMPLE 14 A linear elastic collision problem, $v_{2o} = 0$

A 300-g toy train and a 600-g toy train are involved in an elastic collision on a straight section of a model rail. The 300-g train, travelling at 2 m/s, strikes the 600-g train at rest. Determine the velocities of both trains after the collision.

Solution and Connection to Theory

Let's designate the forward direction as positive to simplify the vectors.

Given

$$m_1 = 300 \text{ g} = 0.3 \text{ kg} \quad m_2 = 600 \text{ g} = 0.6 \text{ kg} \quad v_{1o} = 2 \text{ m/s} \quad v_{2o} = 0$$

Since the collision is one-dimensional and elastic, we can use equations 4 and 5.

$$v_{1f} = v_{1o} \left(\frac{m_1 - m_2}{m_1 + m_2} \right) = (2 \text{ m/s}) \left(\frac{0.3 \text{ kg} - 0.6 \text{ kg}}{0.3 \text{ kg} + 0.6 \text{ kg}} \right) = -0.7 \text{ m/s}$$

$$v_{2f} = v_{1o} \left(\frac{2m_1}{m_1 + m_2} \right) = (2 \text{ m/s}) \left(\frac{2(0.3 \text{ kg})}{0.3 \text{ kg} + 0.6 \text{ kg}} \right) = 1.3 \text{ m/s}$$

The final velocity of the first train is -0.7 m/s (i.e., moving at 0.7 m/s in the opposite direction to its original path) and the final velocity of the second train is 1.3 m/s (i.e., moving in the same direction as the original path of the first train).

EXAMPLE 15 A linear elastic collision problem, $v_{2_0} \neq 0$

Two balls of equal mass are involved in an elastic head-on collision. If the first ball (red) was travelling at 4 m/s [E] and the second ball (yellow) was travelling at 2 m/s [W], determine the velocity of each ball after collision.

Fig.5.27



Solution and Connection to Theory

In order to use equations 4 and 5, we need to adjust the problem slightly so that the initial velocity of one of the balls equals zero. We can do so by changing our frame of reference to that of m_2 such that $v_{2_0} = 0$. *From the perspective of m_2 , both balls travelling toward each other at 4 m/s and 2 m/s, respectively, is the same as m_1 travelling toward m_2 at 6 m/s; that is, the speed of m_1 relative to m_2 is $4 \text{ m/s} + 2 \text{ m/s} = 6 \text{ m/s}$. The velocity of m_1 is therefore 6 m/s [E].*

Given

$$m_1 = m_2 = m \quad v_{1_0} = 6 \text{ m/s} \quad v_{2_0} = 0$$

$$v_{1_f} = v_{1_0} \left(\frac{m_1 - m_2}{m_1 + m_2} \right) = (6 \text{ m/s}) \left(\frac{m - m}{m + m} \right) = 0 \text{ m/s}$$

$$v_{2_f} = v_{1_0} \left(\frac{2m_1}{m_1 + m_2} \right) = (6 \text{ m/s}) \left(\frac{2m}{2m} \right) = 6 \text{ m/s}$$

The final velocities of m_1 and m_2 from the frame of reference of m_2 are 0 m/s and 6 m/s, respectively.

In order to complete the problem, we must return to our original frame of reference (in which both balls are initially in motion) and determine the final velocity of each ball. We can do so by subtracting the initial velocity of m_2 that was given in the problem (2 m/s [W] or -2 m/s) from the final velocities we obtained using equations 4 and 5.

$$v_{1_f} = 0 \text{ m/s} - 2 \text{ m/s} = -2 \text{ m/s} \text{ and}$$

$$v_{2_f} = 6 \text{ m/s} - 2 \text{ m/s} = 4 \text{ m/s}$$

The final velocities for the first and second ball are -2 m/s and 4 m/s , respectively.

Table 5.1
A Graphical Representation of an Elastic Collision

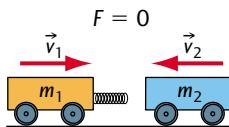
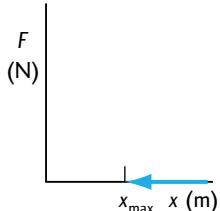
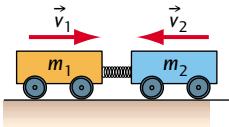
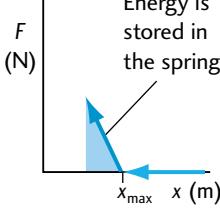
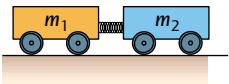
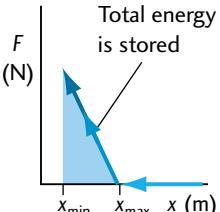
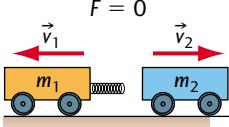
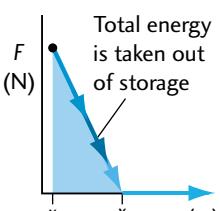
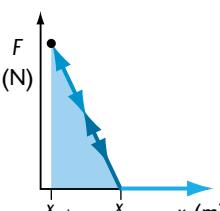
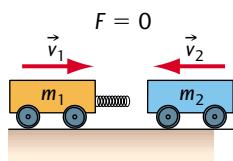
Snapshot of the collision	F versus x graph	Description
Fig. 5.29a 	Fig. 5.29b 	The carts are approaching each other ($F = 0$) until the spring touches the second cart. The separation at this point is labeled x_{\max} .
Fig. 5.30a 	Fig. 5.30b 	The carts are doing work on each other by compressing the spring between them. Kinetic energy is being transferred into elastic potential energy as the spring is being compressed.
Fig. 5.31a  Both carts stopped	Fig. 5.31b 	The spring has reached its maximum compression and the two carts are at a minimum separation, x_{\min} . In the general case, both carts reach the same velocity.
Fig. 5.32a  Entire collision	Fig. 5.32b 	The carts are coming out of the collision. As the spring returns to its equilibrium state, it expands, returning the stored potential energy of the spring into kinetic energy for both masses. Once the spring has returned to equilibrium, the carts are no longer affected by each other ($F = 0$) at x_{\max} .
	Fig. 5.33 	In an elastic collision, the total amount of energy stored is returned to kinetic energy.

Table 5.2
A Graphical Representation of an Inelastic Collision

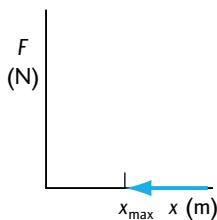
Snapshot of the collision

Fig. 5.34a



F versus x graph

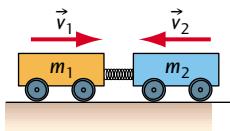
Fig. 5.34b



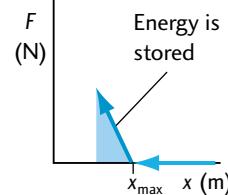
Description

The carts are approaching each other ($F_{\text{net}} = 0$). At the moment the spring touches m_2 , the separation distance is x_{max} .

Fig. 5.35a



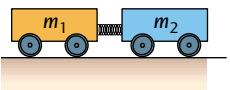
F versus x graph



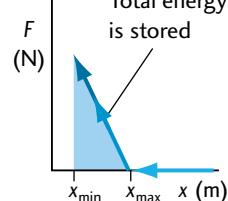
The carts are doing work on each other and kinetic energy is being transferred or stored in the collision mechanism.

Fig. 5.36a

Both carts stopped

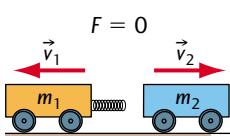


F versus x graph



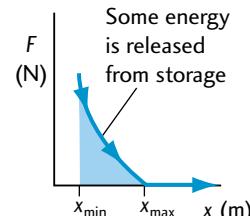
The spring has reached its maximum compression and all the kinetic energy is stored in the collision mechanism. The distance between the two carts is x_{min} . In the general case, both carts reach the same velocity.

Fig. 5.37a



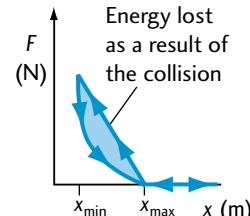
Entire collision

F versus x graph



As the carts come out of the collision, only some energy is taken out of storage. Energy may have been dissipated as heat so that the carts rebound with less kinetic energy than they initially had.

Fig. 5.38



The shaded area represents the energy that was put into storage but not taken out. In an inelastic collision, some energy is lost during the collision.

Graphical Representations of Elastic and Inelastic Collisions

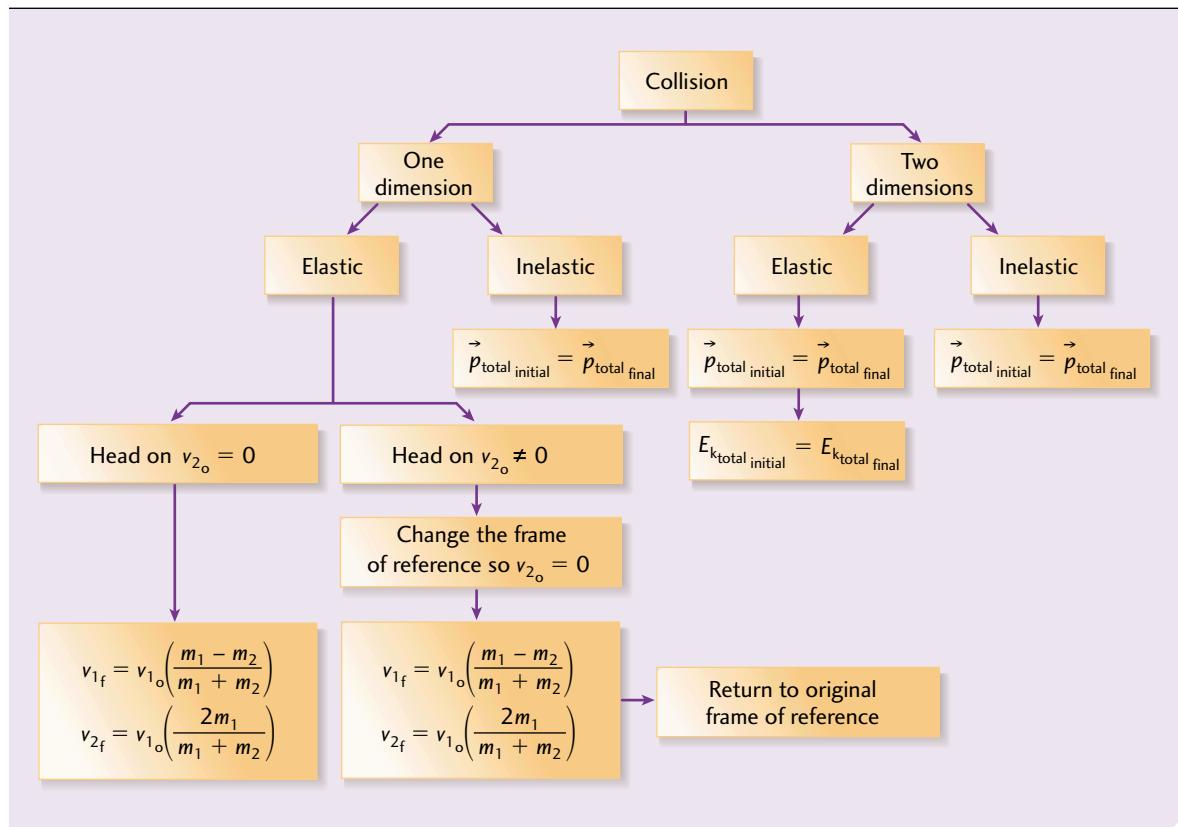
In most collisions, the objects colliding are in contact for a short time interval, during which various energy transfers can take place. In an elastic collision, some energy may be stored as potential energy during the collision and reappear later as kinetic energy after the collision. For instance, consider a physics laboratory spring cart colliding elastically against another cart with equal and opposite momentum (see Table 5.1).

Recall that the area under the force-versus-separation graph represents the energy stored during a collision. If the collision is elastic, the area under the before-collision portion of the graph is equal to the area under the after-collision portion of the graph. That way, all the energy stored during the collision as the objects are approaching is completely used up to separate the objects after the collision.

During an inelastic collision, some potential energy is transferred into another form of energy, such as heat, sound, or light. Table 5.2 represents an inelastic collision where the difference in the areas represents the energy lost.

Figure 5.28 summarizes the method of solving collision problems in one and two dimensions.

Fig.5.28 Solving Collision Problems in One and Two Dimensions



Safety During Collisions

In order to reduce the amount of impact during a vehicle collision, various safety features are used to dissipate the energy of the collision over a longer period of time. By using materials such as padding in helmets, bumpers on cars, or shock absorbers on bicycles, the kinetic energy of a collision can be transferred to elastic potential energy, which can then be converted back into kinetic energy.

Take, for example, a spring shock absorber on a mountain bike (Figure 5.39a). When the bike's front wheel rolls over a protruding tree root on the trail, the spring in the shock absorber absorbs some energy as it compresses, and returns that energy to kinetic energy of motion when it springs back into equilibrium. The spring absorbs the energy instead of the rider. If the spring wasn't there, the rider would feel the bicycle moving up over the root in his arms, legs, and back. Without the shock absorber, the rider's inertia would be interrupted abruptly by the tree root.

Seat belts and air bags (Figure 5.39b) in cars also provide a mechanism for transferring energy to prevent or reduce injury. In a head-on collision, the vehicle stops abruptly, but the driver still possesses kinetic energy because of Newton's first law of motion (an object in motion continues in motion unless acted on by an unbalanced force). The seat belt and the deployed air bag allow the kinetic energy of the driver's motion to be transferred to the air bag and seat belt. The fibres in the seat belt expand, and the deployed air bag compresses, absorbing some of the driver's energy. Some energy may also be dissipated as sound and heat.

A snowboarder doing aerials (Figure 5.39c) converts potential energy into some other form of energy as she falls through the air. Typically, her potential energy is converted into kinetic energy and then into other forms of energy when she touches down on the ground. Some of her kinetic energy is absorbed by her body when she bends her knees and flexes her muscles upon landing. Some kinetic energy is transferred to elastic potential energy in the deformation of the snowboard, and some of her energy is transferred to compress and heat the snow, and to create sound energy. In case they crash, snowboarders wear helmets that absorb their kinetic energy by deforming the plastic covering and permanently compressing the foam inside the helmet.

1. Should seat belts and air bags be mandatory in Canada? Research and write a short paragraph explaining your position.
2. Bicycle helmets are now mandatory equipment for cyclists in most municipalities. Explain how energy is transferred during a collision where a cyclist wearing a helmet collides headfirst with the pavement during an accident.

Fig.5.39a

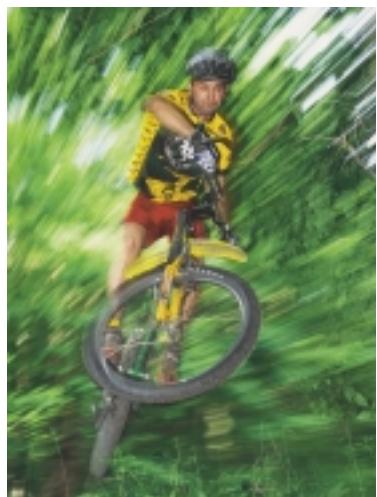


Fig.5.39b

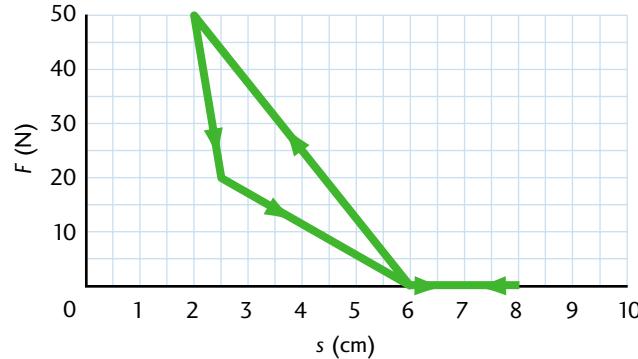


Fig.5.39c



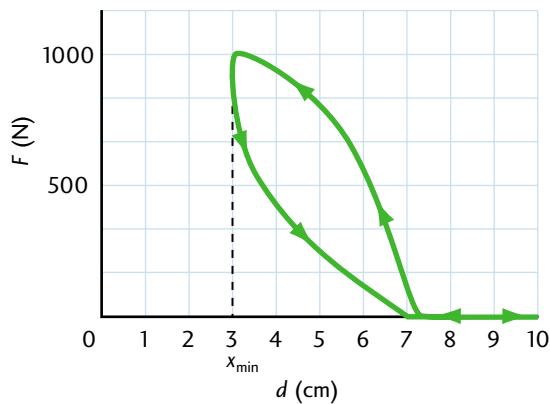
- 3.** A 3.0-tonne truck (1 tonne = 1000 kg) moving at 20 m/s [W] collides with a 1.0-tonne car, initially at rest. The collision slows the truck to 10 m/s [W].
- What is the speed of the car after the collision?
 - Is this collision elastic or inelastic? Explain.
 - How much work is done by the truck as it collides with the car?
- 4.** A 500-g hockey puck is travelling head-on at 33.0 m/s toward a goalie's pads. The goalie, initially at rest and completely padded, has a mass of 75 kg. The puck causes the goalie's pad to be compressed by 3 cm and pushes her backward at 0.30 m/s.
- Calculate the momentum and kinetic energy of the puck and the goalie before collision.
 - Calculate the velocity of the puck after collision.
 - Calculate the kinetic energy of the puck and the goalie after collision.
 - Is this collision elastic or inelastic? Explain.
- 5.** An elastic collision occurs between a 10-g marble and a 50-g marble. The smaller marble is travelling north at 5 m/s in a head-on collision course with the other marble, initially at rest. Determine the final velocities of both marbles.
- 6.** Two metal gliders are floating along a horizontal air track and collide elastically. The first glider (200 g) is moving at 32 cm/s while the second glider (300 g) is moving in the opposite direction at 52 cm/s. What is the velocity of each glider after impact?
- 7.** A metal sphere rolls toward another sphere of the same mass, initially at rest. They collide head-on and elastically. Using the laws of conservation of momentum and energy, show that there are two possible answers for the velocity of the metal sphere. What situations do the two solutions represent?
- 8.** Given the force-versus-separation graph for a collision in Figure 5.40,
- calculate the amount of energy stored before the collision.
 - calculate the amount of energy lost in the collision.
 - Is the collision elastic? Explain.

Fig.5.40



- 9.** The force-versus-separation graph in Figure 5.41 represents a shock absorber on a bike involved in a collision.
- Calculate the amount of energy stored before the collision.
 - Determine the amount of energy released during the collision.
 - Calculate the percentage of energy lost during the collision.
 - Is the collision elastic? Explain.
 - Where did the energy lost during this collision go?

Fig. 5.41





The Physics Equation — The Basis of Simulation

Fig.STSE.5.1 “Pokemon Pinball” models the motion of a ball in a pinball game, including the rolling of the ball up and down the incline and the striking action of a simulated paddle



In physics, we try to understand the behaviour of systems in the natural world. To help us achieve our goal, we make use of mathematical models in the form of equations. We come up with mathematical models by designing experiments in which we manipulate one variable and observe the effects on other variables. Appendix D describes the mathematical techniques physicists use to examine the relationships between variables in order to derive an equation that will model physical behaviour and help us predict how a system will behave. A single equation is a very simple simulation of an idealized natural event because during experiments, we manipulate only one variable at a time. Computers allow us to model highly complex situations because they can perform extensive calculations almost instantaneously, using multiple equations, and display the results graphically. Computers are used to simulate the performance of technological designs before they are built, which saves time and money, and is much safer. Prior to the development of computers, physical models of proposed designs had to be built and tested at great financial and human cost. Many video games, such as “Pokemon Pinball” (Figure STSE.5.1), are computer simulations of natural events.

To get an idea of how computers simulate natural events by combining equations, let’s consider the example of launching a spring. In this unit, we have studied the kinematics and dynamics of projectile motion, including the various energy transformations required to propel an object through space. The three equations that pertain to launching a spring are listed in Table STSE.5.1.

Table STSE.5.1

Equation	Description
$E_p = \frac{1}{2} kx^2$	Potential energy stored in a spring, where k is the spring constant and x is the deformation (stretch) of the spring
$E_k = \frac{1}{2} mv^2$	Kinetic energy of any object where m is the mass and v is the object’s speed
$R = \frac{v^2 \sin 2\theta}{g}$	Range, where v is the launch speed and θ is the launch angle of a projectile that takes off and lands at the same level

All three of these equations can be combined to create a single equation describing the range of a spring projectile in terms of its initial stretch and launch angle:

$$x = \sqrt{\frac{mRg}{k \sin \theta}}$$

where x is the stretch required for the spring to launch itself, m is the mass of the spring, R is the range it travels, g is the acceleration due to gravity, k is the spring constant, and θ is the launch angle.

We can use a computer program to simulate the flight of our spring given a specific set of parameters. Figure STSE.5.2 shows the output of a projectile simulation program.

The following activities will give us an opportunity to test the accuracy of our modelled behaviour of a spring projectile.

Design a Study of Societal Impact

Before the advent of computer processing power, intensive scientific research could be extremely expensive. Often, the financial resources necessary for such research were limited to government departments such as the armed forces. In fact, much of our understanding of kinematics, dynamics, and kinetic energy comes from the study of ballistics and projectile weapons such as the catapult, illustrated in Figure STSE.5.3.

Compile a list of significant technological inventions (such as the telephone, the television, and the nuclear reactor). Research the development costs of your chosen inventions and categorize them as high, medium, or low cost. Choose at least one item from each category and research the history of its development. Do any of these items have military applications? If so, have they been put to any significant non-military use? Will underfunding of military research due to public sentiment be detrimental to scientific discovery?

Design an Activity to Evaluate

Use the equations in Table STSE.5.1 to derive the projectile equation for a spring. Measure the mass of a spring (distributed by your teacher) and design and perform a simple activity that will evaluate the spring constant, k .

Build a Structure

Design and build a spring launcher that can propel a stretched spring at a defined angle toward a simple target (such as a wastebasket). See Figure STSE.5.4 for a simple design. This launcher should allow you to stretch the spring a set amount and launch it on an adjustable incline. Hold a design competition to see how well the physics equation predicts the range of the spring projectile.

Extension: Use the equation in spreadsheet software to model the behaviour of the projectile while in flight. See <www.irwinpublishing.com/students> for a copy of the projectile simulation software in Figure STSE.5.2.

Fig. STSE.5.2 The visual display from a projectile simulation program

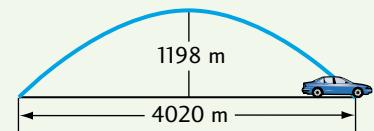


Fig. STSE.5.3 A counterweight-powered catapult, called a trebuchet, helped medieval “physicists” study projectile motion

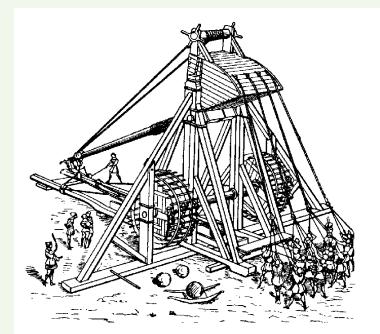
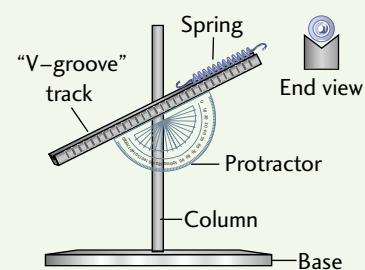


Fig. STSE.5.4 An example of a spring launcher design



You should be able to*Understand Basic Concepts:*

- Define and describe the concepts and units related to momentum and energy, including the work–energy theorem, gravitational potential energy, elastic potential energy, elastic collisions, and inelastic collisions.
- Analyze situations involving the concepts of mechanical energy and the laws of conservation of momentum and of energy.
- Distinguish between elastic and inelastic collisions.
- Analyze and explain common situations involving work and energy using the work–energy theorem.
- State Hooke’s law and analyze it in quantitative terms.

Develop Skills of Inquiry and Communication:

- Investigate the laws of conservation of momentum and of energy in one or two dimensions by carrying out experiments or simulations and the necessary analytical procedures.
- Design and conduct an experiment to verify the law of conservation of energy in a system involving kinetic energy, gravitational energy, and elastic potential energy.
- Conduct an experiment to verify Hooke’s law.

Relate Science to Technology, Society, and the Environment:

- Analyze and describe, using the concepts and laws of conservation of energy and of momentum, practical applications of energy transformations and momentum conservation.
- Analyze and describe the operation of a spring bumper, seat belts, protective equipment used in sports, and the workings of a clock.

Equations

$$\vec{p} = m\vec{v}$$

$$\Delta E_g = mg\Delta h$$

$$F = kx$$

$$P = \frac{E}{t}$$

$$W = F \cdot \Delta d = F \Delta d \cos \theta$$

$$E_k = \frac{1}{2}mv^2$$

$$P = FV$$

$$E_e = \frac{1}{2}kx^2$$

$$v_{1f} = v_{1o} \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$$

$$E_k = \frac{p^2}{2m}$$

$$v_{2f} = v_{1o} \left(\frac{2m_1}{m_1 + m_2} \right)$$

$$E_{\text{total}} = E_p + E_k + E_e$$

EXERCISES

Conceptual Questions

1. Holding your physics book steady in your outstretched arm seems like a lot of work. Explain why it is not considered work in physics.
2. A golf ball and a football have the same kinetic energy. Which ball has the greater momentum? Explain.
3. What does a negative area under a force-versus-displacement graph represent?
4. How is an object different as a result of having work done on it?
5. Explain how energy is transferred when a diver jumps on a spring diving board, then dives into the pool.
6. Use unit analysis to show that the units work out in the equation $E_k = \frac{1}{2}mv^2$.
7. Explain what is meant by $-\Delta E_e = \Delta E_k$.
8. Can an object have different amounts of gravitational potential energy if it remains at the same elevation?
9. Explain the difference between an elastic collision and an inelastic collision. Give an example of each.
10. Can an object have momentum without having any kinetic energy? Is the reverse possible? Explain.

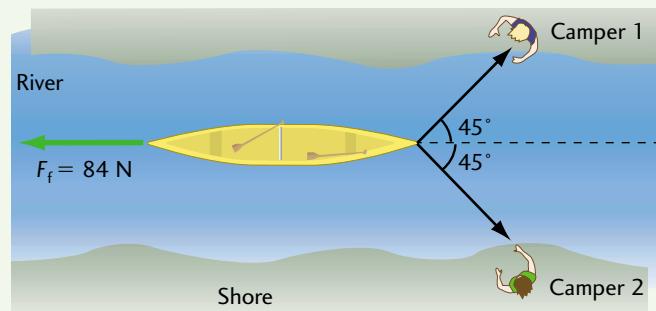
Problems

5.2 Work

11. How much work is required to
 - a) tow a boat with a force of 4000 N for 5.0 m?
 - b) kick a football with a force of 570 N over a distance of 8.0 cm?
 - c) accelerate an electron from rest to 1.6×10^8 m/s ($m_e = 9.1 \times 10^{-31}$ kg)?

12. How much work is done pushing a wheelbarrow full of cement 5.3 m [forward] if a force of 500 N is applied
 - a) horizontally?
 - b) 20° above the horizontal?
 - c) 20° from the vertical?
13. Jake the deliveryman pushes a box up a ramp, exerting a force of 350 N. He walks on the ramp, pushing the box for 25.0 m. If the box has a mass of 50.0 kg, what is the height of the ramp and the angle it makes with the horizontal? Ignore friction.
14. A snowplough diverts snow from the road to the ditch (an average movement of 5.0 m of snow at an average speed of 10.0 m/s). The density of the fresh snow is 254 kg/m^3 and the average depth is 35.0 cm. Assume that a lane of traffic is 4.0 m wide. If the snowplough clears a road that is 8.0 km long, how much work did it do on the snow?
15. Two campers pull a canoe, as illustrated in Figure 5.42. If the force of friction on the canoe is 84 N, how much work must each camper do to keep the canoe in the middle of the river for a displacement of 50 m?

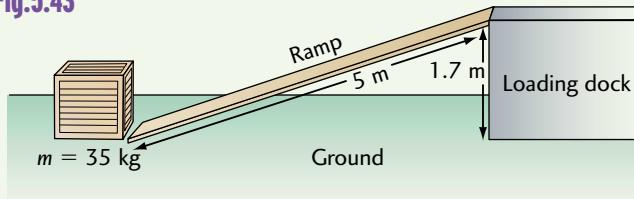
Fig.5.42



16. Calculate the amount of work done by a hammer thrower if an 8.0-kg hammer attached to a 1.3-m-long rope is rotated horizontally above the thrower's head. The hammer thrower holds the rope with a tension of 300 N.

- 17.** Explain what is meant by -350 J . Give an example.

Fig.5.43

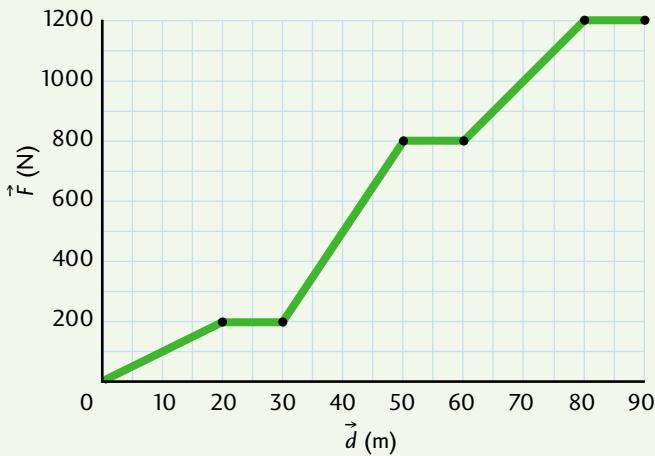


- 18.** A 35-kg box needs to be lifted to the top of a loading dock, which is also accessible by ramp. The ramp is 5.0 m long and has a vertical height of 1.7 m.

- a) What minimum force is required to lift the box straight up onto the loading dock?
- b) What minimum amount of work is required to lift the crate straight up onto the loading dock?
- c) What force is required to push the crate up the ramp such that the amount of work is the same as in b)? Assume no friction.

- 19.** Calculate the work done on a 50-kg wakeboard enthusiast who experiences the horizontal force indicated on the graph in Figure 5.44.

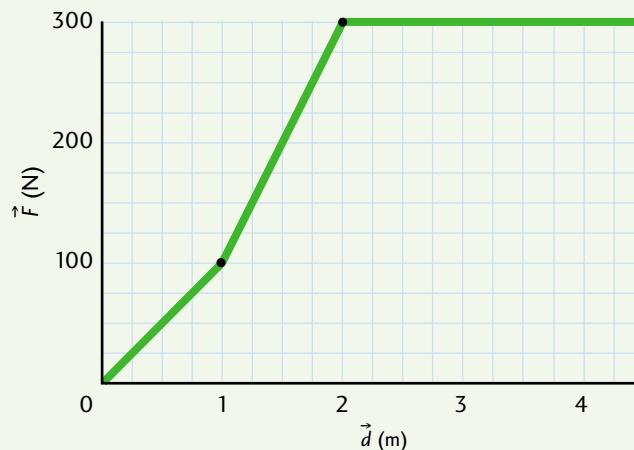
Fig.5.44



- 20.** A lawn tractor pulls a 120-kg wagon along a frictionless surface with a horizontal force given by the graph in Figure 5.45.

- a) How much work is done in moving the wagon 4.0 m?
- b) How is the wagon different as a result of the force applied?
- c) Calculate the speed of the wagon at a distance of 4.0 m from the start.

Fig.5.45



5.3 Kinetic Energy

- 21.** Calculate the kinetic energy of
 - a) a 45-kg sprinter running at 10 m/s.
 - b) a 2.0-g fly buzzing around your head every second. (Assume your head has a radius of 10 cm.)
 - c) a 15 000-kg army tank charging forward at 100 km/h.
- 22.** A fish swimming horizontally and nibbling the end of your barbless hook has a kinetic energy of 450 J. You notice that 5.0 m of line is released every 2.0 s. Calculate the mass of the fish.
- 23.** Calculate the velocity of a 1.2-kg falling star (meteorite) with $5.5 \times 10^8 \text{ J}$ of energy.
- 24.** A 15-kg mass is released from rest at a height of 200 m. If air resistance is negligible, determine the kinetic energy of the mass after it has fallen 199 m.
- 25.** Using unit analysis, show that $p = \sqrt{2mE_k}$ is correct.

- 26.** What percentage of the speed of light is the speed of an electron with 5.0 keV of kinetic energy? ($m_e = 9.1 \times 10^{-31}$ kg, $1 \text{ eV} = 1.6 \times 10^{-19}$ J)
- 27.** A 15-g bullet strikes a metal plate on an armoured car at a speed of 350 m/s. The bullet penetrates the armour 3.3 mm before coming to a stop.
- Calculate the average net force acting on the bullet while it is in the metal.
 - Calculate the average force exerted on the metal by the bullet.
- 28.** Figure 5.46 represents the horizontal force on a 1.5-kg trolley as it moves 3.0 m along a straight and level path. If the trolley starts from rest, calculate its kinetic energy and velocity after each metre of its motion.

Fig.5.46



- 29.** Calculate the momentum of a 5.0-kg briefcase with a kinetic energy of 3.0×10^2 J.
- 30.** A thin 200-g arrow moving horizontally at 125 m/s strikes a 1.0-kg apple, initially at rest. The arrow pierces the apple in a negligible time, emerging from it with a velocity of 100 m/s, and causing the apple to slide forward 3.0 m before coming to a standstill.

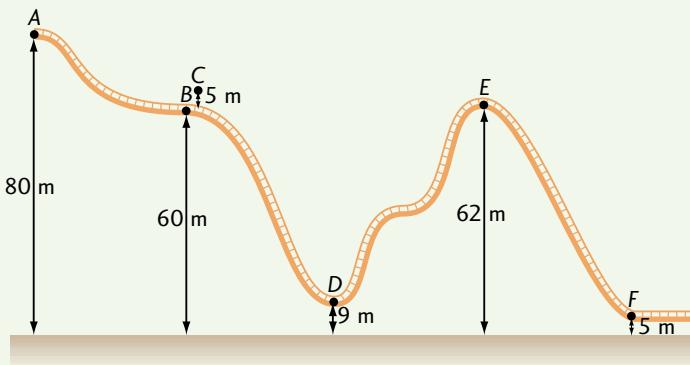
- What is the apple's velocity just after the arrow exits?
- What is the maximum kinetic energy of the apple?
- Is this collision elastic? Explain.
- What is the average frictional force stopping the apple?

5.4 Gravitational Potential Energy

- 31.** Calculate the gravitational potential energy of
- a 2.0-kg physics textbook sitting on your desk 1.3 m above the floor.
 - a 50-g egg dropped from the top of a 3.0-m-high chicken coup.
 - a 200-kg air glider flying 469 m above the ground.
 - a 5000-kg car parked on the road.
- 32.** A forklift requires a force of 4410 N to lift a roll of steel 3.5 m.
- What is the mass of the steel?
 - How much work is required to lift the steel?
- 33.** A rodeo rider is 1.8 m off the ground when on a bull. The bull suddenly throws the rider straight up at a velocity of 4.7 m/s. With what velocity will the rider land on the ground?
- 34.** A 3.0-kg ball is dropped from a height of 0.80 m onto a vertical spring with a force constant of 1200 N/m. What is the maximum compression of the spring?
- 35.** A ping-pong ball with a mass of 5.0 g is dropped from a height of 2.0 m. The ball loses 20% of its kinetic energy with each bounce. How many bounces would it take for the ping-pong ball to lose just over half of its original height?

- 36.** A 1000-kg roller coaster car starts from rest at point A on a frictionless track, shown in Figure 5.47.

Fig.5.47



- a) At which point on the track is the car's gravitational potential energy the greatest? the least?
- b) What is the car's maximum speed?
- c) What is the speed of the roller coaster car at point E?
- d) What constant braking force would have to be applied to bring the coaster car at point F to a stop in 5.0 m?

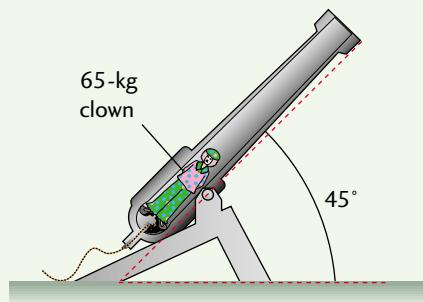
- 37.** A slingshot with a force constant of 890 N/m is used to propel a primitive 10 005-kg starship in deep space by releasing a 5.0-kg block of ice into space. How much should the slingshot be pulled in order to increase the ship's speed by 5.0 m/s?

- 38.** A toy rifle shoots a spring of mass 0.008 kg and with a spring constant of 350 N/m. You wish to hit a target horizontally a distance of 15 m away by pointing the rifle 45° above the horizontal. How far should you extend the spring in order to reach the target?

- 39.** Newton was lying down in an apple orchard when an apple struck his stomach. It then bounced straight back up, having lost 15% of its kinetic energy in the collision. How high did it rise on the first bounce if it dropped from a branch 2.0 m high?

- 40.** A human cannon (Figure 5.48) has a spring constant of 35 000 N/m. The spring can be extended up to 4.5 m. How far (horizontally) would a 65-kg clown be fired if the cannon is pointed upward at 45° to the horizontal?

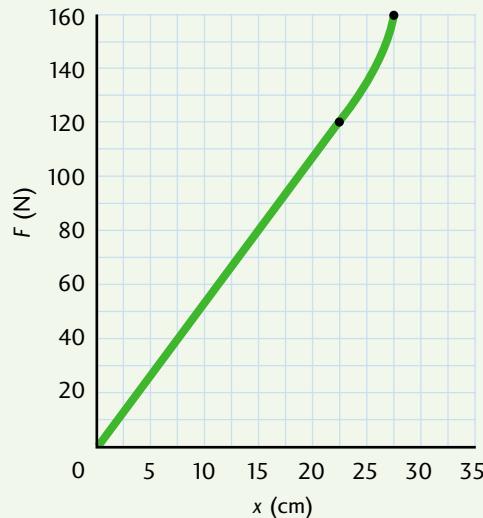
Fig.5.48



5.5 Elastic Potential Energy and Hooke's Law

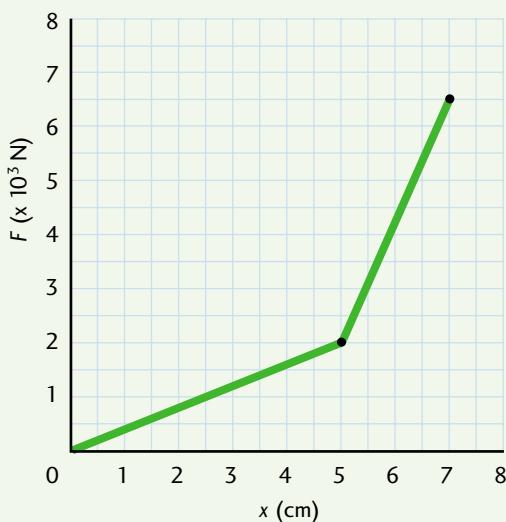
- 41.** Determine the spring constant for the elastic band represented in Figure 5.49.

Fig.5.49



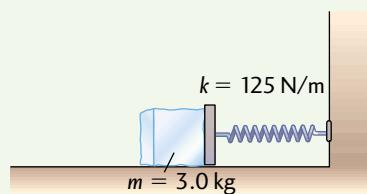
- 42.** A spring that obeys Hooke's law has the following F -versus- x graph (Figure 5.50). How much work is required to stretch the spring
- a) 5.0 cm?
 - b) 7.0 cm?

Fig.5.50



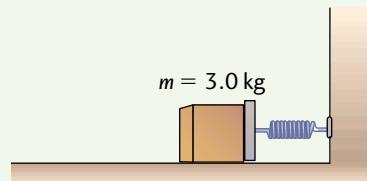
- 43.** A toy gun has its spring compressed 3.0 cm by a 50-g projectile. The spring constant was measured at 400 N/m. Calculate the velocity of the projectile if it is launched horizontally.
- 44.** A large bungee cord is used to propel a jet of mass 2.5×10^3 kg horizontally off an aircraft carrier. The rubber band is pulled back 35 m and released such that the jet takes off at 95 m/s. What is the spring constant of the rubber band?
- 45.** A small truck is equipped with a rear bumper that has a spring constant of 8×10^5 N/m. The bumper can be compressed up to 15 cm without causing damage to the truck. What is the maximum velocity with which a solid 1000-kg car can collide with the bumper without causing damage to the truck?
- 46.** Figure 5.51 shows a 3.0-kg block of ice held against a spring with a force constant of 125 N/m. The spring is compressed by 12 cm. The ice is released across a horizontal plank with a coefficient of friction of 0.10.
- a)** Calculate the velocity of the ice just as it leaves the spring. Assume the friction between the plank and the ice is negligible until the moment when the ice leaves the spring.
- b)** Determine the distance the ice travels after it leaves the spring.

Fig.5.51



- 47.** A spring with a force constant of 350 N/m (Figure 5.52) is compressed 12 cm by a 3.0-kg mass. How fast is the mass moving after only 10 cm of the spring is released?

Fig.5.52



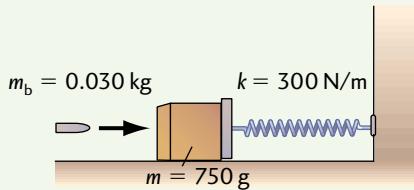
- 48.** What minimum force will compress a spring 15 cm if the spring constant is 4000 N/m?
- 49.** A mattress manufacturer estimates that 20 springs are required to comfortably support a 100-kg person. When supporting the person, the 20 springs are compressed 3.5 cm. Calculate the spring constant for one spring.
- 50.** A bungee cord needs to transfer 2.0×10^6 J of energy. A 10-kg mass extends the bungee cord 1.3 m. What is the maximum extension of the bungee cord?
- ## 5.6 Power
- 51.** A 60-W light bulb is left on for 3 days, 8 hours, and 15 minutes. How much energy is used? Express this value in kWh as well.
- 52.** **a)** A crane lifts a 3500-kg crate from the ground to a height of 13.4 m. If the lift takes 23 s and the crane's mechanical systems are 46% efficient, then what power must the engine provide?
- b)** Convert the value in part a) to horsepower.

- 53.** Suppose that your home uses 9.4 kWh of power in one day and you would like to replace that energy by riding a bicycle generator for 4 h. Using energy transfer theory, explain the physical condition (comfortable, exhausted, dead, etc.) you would be in at the end of the ride.
- 54.** An elevator in a large hotel has a mass of 4400 kg. The maximum passenger load is 2200 kg. Suppose the speed of the elevator is 2.4 m/s.
- What is the average power required of the lifting device?
 - Compare your answer in a) with some other power value in your daily experience.
 - What would be the power consequence of counterweighing the elevator with a steel mass of 4400 kg?
- 55.** A cyclist coasts down a 7.2° hill at a steady speed of 10.0 km/h. If the total mass of the bike and rider is 75.0 kg, what power output must the rider have to climb the hill at the same speed?
- 5.7 Elastic and Inelastic Collisions**
- 56.** Derive the expression $v_{2f} = v_{1o} \left(\frac{2m_1}{m_1 + m_2} \right)$ for two objects involved in an elastic collision, where the first object is initially at rest.
- 57.** A 15-kg object, moving at 3.0 m/s, collides elastically (head-on) with a 3.0-kg object, initially at rest.
- What is the velocity of each object after collision?
 - How much energy was transferred to the smaller object?
- 58.** A 35-g sparrow, travelling at 8.0 m/s with its beak open, swallows a 2.0-g mosquito, travelling in the opposite direction at 12 m/s. Calculate the velocity of the sparrow and mosquito just after collision.
- 59.** A 3.2-kg dynamics cart, moving to the right at 2.2 m/s, has a spring attached to one end. The cart collides with another cart of equal mass, initially at rest. After the collision, the first cart continues to move in the same direction at 1.1 m/s.
- Calculate the total momentum and the total kinetic energy before the collision.
 - Find the final velocity of the second cart.
 - Calculate the total amount of kinetic energy after the collision.
 - Is this collision elastic? Explain.
- 60.** A 15-g bullet travelling at 375 m/s penetrates a 2.5-kg stationary block of wood sitting on a frictionless surface. If the bullet emerges at 300 m/s, find the final velocity of the block.
- 61.** A totally elastic, head-on collision occurs between object A, with a mass of $6m$, and object B, with a mass of $10m$. Object A is moving at 5 m/s [E], while object B is moving at 3 m/s [W]. Calculate the velocity of each mass after the collision.
- 62.** An elastic collision occurs on an air track between a moving mass m_1 and a stationary mass m_2 . If the initial velocity of m_1 is 5 m/s and $m_1 = 3m_2$,
- calculate the velocity of the first mass after the interaction.
 - calculate the velocity of the second mass after the interaction.

- 63.** A 750-g block of wood is attached to a spring with $k = 300 \text{ N/m}$, as shown in Figure 5.53. A 0.030-kg bullet is fired into the block, and the spring compresses 10.2 cm.

- a) Calculate the velocity of the bullet before the collision.
 b) Is this collision elastic or inelastic? Explain.

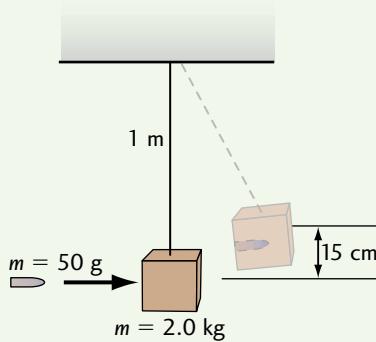
Fig.5.53



- 64.** As part of a forensic experiment, a 50-g bullet is fired horizontally into a 2.0-kg wooden pendulum, as illustrated in Figure 5.54. The pendulum with the bullet imbedded in it rises 15 cm vertically from its initial position.

- a) Calculate the velocity of the block and bullet just after the collision.
 b) What is the velocity of the bullet just before impact?

Fig.5.54



- 65.** A glancing elastic collision occurs between a cue ball and the eight ball. Both balls have the same mass and the eight ball is initially at rest. Prove that the angle between the velocity of the cue ball and the eight ball after collision is 90° .



Conservation of Energy Exhibited by Projectile Motion

Purpose

To calculate the height from which a ball must be released from a curved ramp in order to land at a specified point

Equipment

Using a curved ramp (where a ball is released horizontally) and a solid steel ball as a starting point, state any other apparatus needed for your experiment. Draw a fully labeled diagram of your set-up.

Procedure

1. Calculate the theoretical height from which a metal ball must be released such that the ball will land 30.0 cm (horizontally) from the edge of the ramp. You must measure and record the drop height first before performing the actual experiment.
2. Write a procedure, including all steps required, to collect the data necessary to calculate the height from which a solid ball must be released to land 30.0 cm from the base of the ramp.
3. Using the theoretical height you calculated in step 1, carry out the procedure in step 2 to determine the horizontal distance.
4. Using a systematic trial-and-error method, determine the drop height to achieve the distance of 30.0 cm. Record all of your results; you will use them later.

Data

Organize your data in a neat and organized fashion.

Analysis

1. Determine the amount of energy lost or gained when you compare the experimental versus the theoretical values.
2. Write the difference as a percentage loss or gain.

Discussion

1. Does it matter how you record the height?
2. Of the methods you used, which was the best one for determining the horizontal distance?
3. Describe where the loss or gain in energy went.
4. Was the energy lost or gained a direct relation to the distance that the metal ball travelled along the ramp? Using all of the data obtained, supply facts to back up your opinion. Graph your results.

Conclusion

State your conclusion in relation to the original purpose of this lab.

Extension

1. Does the mass of the ball affect the theoretical calculation? Check and record your results experimentally.
2. Determine an absolute error for the theoretical height in your experiment.



Hooke's Law

Purpose

To determine a relationship between the extension of a spring and the force applied

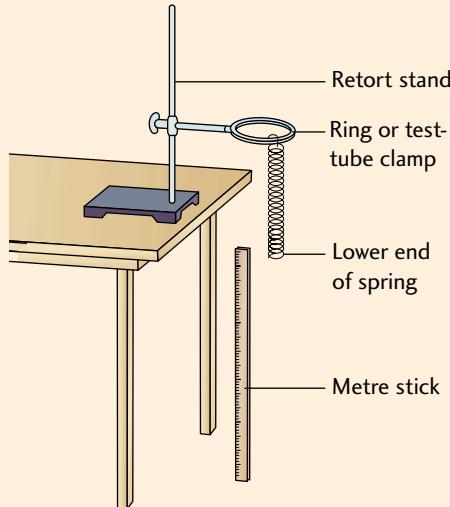
Equipment

Retort stand
Test-tube clamp
Spring
1 metre stick
Various masses

Procedure

- Attach the spring to a clamp, as illustrated in Figure Lab.5.1. Adjust the height of the spring so that the lower end of the spring is level with the zero mark on a metre stick.

Fig. Lab.5.1



- Set up a data table like the one in Table Lab.5.1.
- Add a small mass (100 g) to the spring. Measure the extension from the zero point and record your result in the data table. The size and number of the masses you add will depend on the rigidity of the spring.
- Repeat step 3 roughly 10 times, increasing the mass each time without damaging the spring.

Table Lab.5.1

Attached mass (kg)	Force (N)	Extension (m)
0	0	0
0.20	1.96	

Data

Calculate the force of gravity on each mass and record your result in your data table.

Analysis

- Draw a graph of force (F) versus extension (x) of the spring.
- From the shape of the graph, write a proportionality statement relating F and x .
- Write the proportionality statement as an equation, using the constant k .
- Determine the value of k , the spring constant, by finding the slope of the line of best fit on your graph. Rewrite your equation relating F and x .
- Use an equation to calculate the extension on the spring if
 - a 50-g mass is suspended from it.
 - a 2.0-kg mass is suspended from it.
- Using the graph, calculate the energy stored in the spring when the largest mass you used was suspended from it.
- Using the equation, calculate the energy stored in the spring when the largest mass you used was suspended from it.

Discussion

Is the energy the same in steps 6 and 7? Explain any differences.

Conclusion

State a conclusion for your spring.

Extension

- How would the graph of F versus x be different for a stiffer spring?
- Repeat the experiment with two different springs attached to each other.



Inelastic Collisions (Dry Lab)

Purpose

To study the effects of a head-on inelastic collision

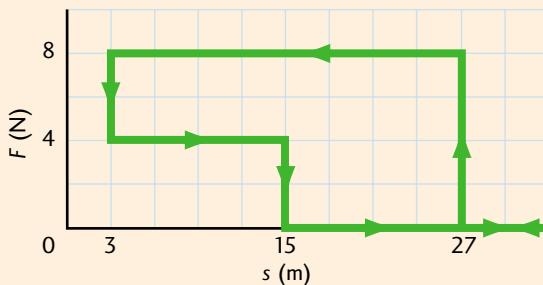
Equipment

None

Procedure

A head-on collision between mass A and mass B occurs as described in the F -versus- s graph below.

Fig. Lab.5.2



Mass A = 1.0 kg, mass B = 2.0 kg

1. Answer the questions in the analysis part.
2. Complete the table in the data section.

Data

Table Lab.5.2
A Detailed Study of a Head-On Inelastic Collision

t (s)	\bar{x}_A (m)[E]	\bar{x}_B (m)[E]	s (m)	\bar{v}_A (m/s)[E]	\bar{v}_B (m/s)[E]	\bar{p}_A $(\frac{\text{kg}\cdot\text{m}}{\text{s}})$	\bar{p}_B $(\frac{\text{kg}\cdot\text{m}}{\text{s}})$	\bar{p}_T $(\frac{\text{kg}\cdot\text{m}}{\text{s}})$	E_{k_A} (J)	E_{k_B} (J)	E_{kt3} (J)	\bar{x}_{cm} (m)	\bar{v}_{cm} (m/s)
0	0	75		24	0								
1.0													
2.0													
3.0													
4.0													
5.0													
6.0													
7.0													
8.0													

Analysis

1. Complete the first five columns of the data table, being careful to apply the correct equation. Write out the equation that applies to each object at each stage of the collision.
2. Complete the next six columns to determine the momenta and kinetic energies of each object and of the system.
3. Complete the last two columns to determine the position and velocity of the centre of mass.

Discussion

1. How can we be certain from the information given that this collision will be inelastic?
2. a) Outline the four stages of the collision.
b) How will we know when one stage ends and the next stage begins?
3. Was momentum conserved at every moment during the collision?
4. Was kinetic energy conserved at each moment of the collision?

5.
 - a) Was the velocity of the centre of mass constant?
 - b) Determine the momentum of the centre of mass and compare it to the total momentum of the system.
 - c) Determine the kinetic energy of the centre of mass.
6.
 - a) What was the minimum separation of the two objects?
 - b) What notable events occurred at minimum separation?
7.
 - a) On one grid, draw position-versus-time graphs for objects A and B and the centre of mass.
 - b) On one grid, draw velocity-versus-time graphs for objects A and B and the centre of mass.
 - c) On one grid, draw momentum-versus-time graphs for objects A and B and the system.
 - d) On one grid, draw kinetic-energy-versus-time graphs for objects A and B and the system.

Conclusion

State a conclusion based on your analysis and discussion of the collision.



Conservation of Kinetic Energy

Purpose

To determine if kinetic energy is conserved in a glancing collision

Equipment

See Lab 4.2, Linear Momentum in Two Dimensions: Air Pucks (Spark Timers).

Procedure

See Lab 4.2, Linear Momentum in Two Dimensions: Air Pucks (Spark Timers).

Data

Use the data collected during Lab 4.2, Linear Momentum in Two Dimensions: Air Pucks (Spark Timers).

Analysis

For the data in Part A and Part B, calculate

1. the total amount of kinetic energy before collision.

2. the total amount of kinetic energy after collision.
3. the change in kinetic energy.

Discussion

For the calculation in Part A and Part B,

1. is the kinetic energy conserved in your collision?
2. calculate the percent difference.
3. explain some possible reasons for the difference.
4. What are some things you can do to improve the experiment?

Conclusion

State a conclusion based on your results.

Note

You may access the Irwin Web site at <www.irwinpublishing.com/students> to verify your calculations and to check your uncertainties. Follow the steps outlined in the program to input your observations.

6

Energy Transfer

Chapter Outline

6.1 Gravity and Energy

6.2 Orbits

6.3 Simple Harmonic Motion —
An Energy Introduction

6.4 Damped Simple Harmonic Motion

 The International Space Station

LAB 6.1 The Pendulum



By the end of this chapter, you will be able to

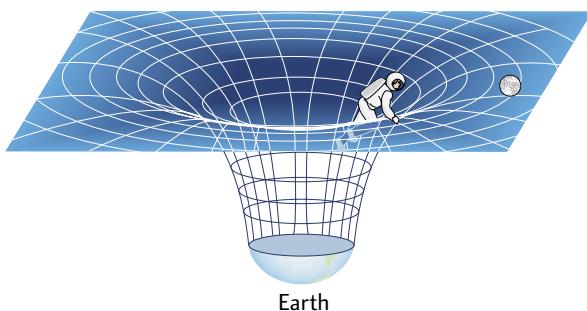
- describe planetary and satellite motion in terms of energy and energy transformations
- calculate the energy and speed required for a rocket to escape Earth's gravitational field
- find the gravitational potential energy of a satellite in a stable circular orbit
- describe the forms of energy involved in simple harmonic motion
- understand how damping effects simple harmonic motion

6.1 Gravity and Energy

According to current theory, **gravity** is a bending of space and time. The force of gravity can also be explained by field theory. The attractive nature of gravity can be represented by an **energy well**. When an object is in a **gravitational well** of another mass, energy is required to move it out of the well because the mass attracts it. (This type of relationship is also shared by electric force fields. In electrical theory, repulsion between two like charges produces energy “hills” and attraction between opposite charges produces energy “wells.” The forces of gravity and electrostatics will be compared in greater detail in Chapter 8.)

A well representation of gravity is similar to a golf ball in a hole. If the green is considered to be the zero-potential-energy level, then positive kinetic energy is required to raise the ball out of the hole. Objects on the surface of Earth are in a type of hole. Energy is required to remove them from the influence of Earth’s gravitational pull. When we escape Earth’s gravitational pull, we leave the hole (see Figure 6.1).

Fig. 6.1 Earth is at the bottom of an energy well



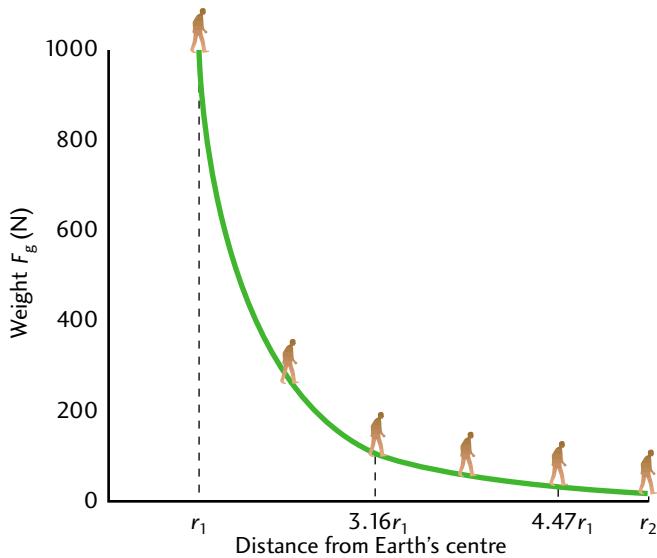
On Earth’s surface, elevations vary only a few kilometres from the mean sea level. These variations are a very small fraction of the distance to Earth’s centre. From Chapter 1, we know that using the law of universal gravitation is clumsy for applications involving gravity near Earth’s surface. If we assume that the distance from Earth’s surface to its centre is fixed, we can divide both sides of the universal gravitation law by the mass of the object on Earth and obtain the mean acceleration of gravity for Earth’s surface. This value is 9.8 m/s^2 . However, when astronomical distances are involved, we must use the universal gravitation equation,

$$F = \frac{GMm}{r^2}$$

One of the differences between the electrostatic force and the gravitational force is that the electrostatic force has a repulsive component. The repulsive component of gravity is *antigravity*. To date, antigravity has not been discovered.

$$F_g = mg = \frac{Gm_E m}{r^2}$$
$$g = \frac{GM_E}{r^2}$$

Fig.6.2 The force of gravity on an object varies inversely as the distance from Earth's centre



Checking units for the work done by gravity, we obtain N·m or joules, the unit for area under a force-versus-distance graph.

The area under the graph in Figure 6.2 represents the *work done*, or the *change in potential energy*, when the distance between M and m is changed. In other words, as we increase the distance from Earth's centre, we must do work against gravity.

Because the function in the graph is a curve, it is more difficult to calculate its area. Using integral calculus,

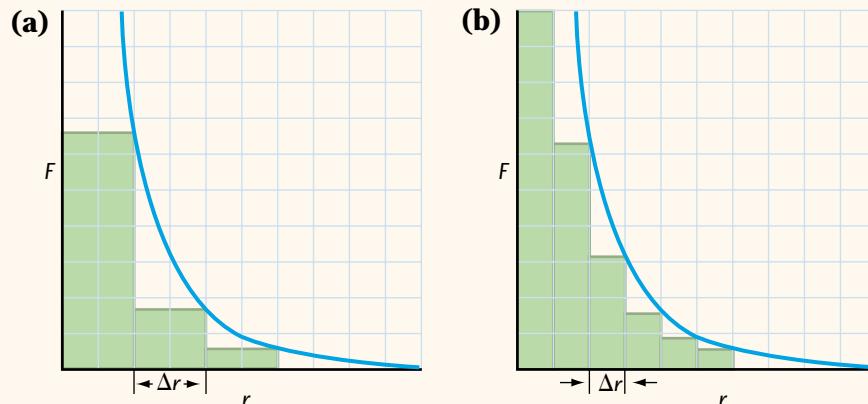
$$\text{Area} = \text{work} = \int F \, dr$$

We substitute $\frac{GMm}{r^2}$ for F to obtain

$$W = \int \left(\frac{GMm}{r^2} \right) dr$$

Integrating to find the area under the force-versus-distance graph is like adding a set of rectangular areas with bases Δr times heights F . Delta (Δ) is replaced by dr when the bases, Δr , shrink to a very small value. Thus, $\sum F \Delta r$ becomes $\int F \, dr$ when Δr becomes infinitely small. Our step-like approximation becomes a continuous curve once Δr shrinks to dr (see Figure 6.3).

Fig.6.3



The area under the $\frac{1}{r^2}$ graph. As Δr becomes very small, the rectangles fit the curve exactly, and the sum of the areas equals the area under the curve.

The GMm does not depend on the area under the curve, so

$$W = GMm \int \frac{1}{r^2} dr$$

Using calculus, the area under the curve $\frac{1}{r^2}$ is

$$\int \frac{1}{r^2} dr = -\frac{1}{r}$$

From this result, we arrive at our final equation for the work done by gravity:

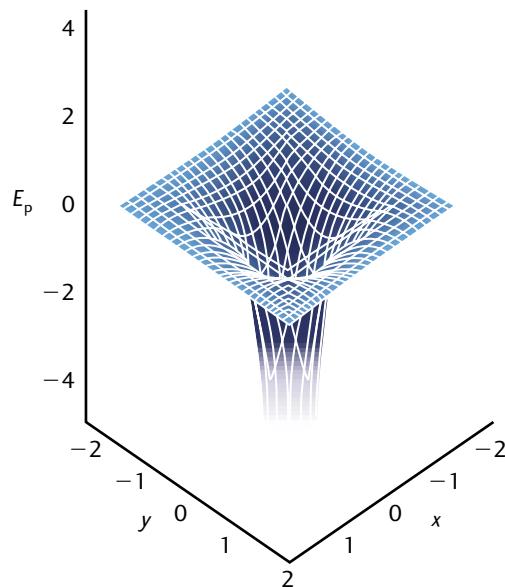
$$W = \frac{-GMm}{r}$$

But if an object is lifted from Earth's surface, it gains potential energy; thus,

$$E_p = \frac{-GMm}{r}$$

The negative sign indicates that while we are in a gravitational field, like standing on Earth's surface, energy must be *added* in order to move us farther away from the centre of the energy well we are in. Consider the graph of E_p versus r (Figure 6.4). In three dimensions, it looks like a well. In fact, it's an "energy well." A rocket on Earth's surface (a distance r_E from Earth's centre) is as close as it can get to Earth's centre. It is located at the bottom of Earth's gravitational energy well. If its gravitational energy is increased by $\frac{GMm}{r_E}$, it will escape from Earth's gravitational well, reaching a level of zero potential energy.

Fig. 6.4 An E_p -versus- r graph (where $r = \sqrt{x^2 + y^2}$ and r is the distance from the vertical line passing through $(0, 0, 0)$) is an "energy well"



$$W = GMm \int \frac{1}{r^2} dr$$

$$W = \frac{-GMm}{r} + \text{constant}$$

$$W = -GMm \int_r^\infty \frac{1}{r^2} dr$$

$$W = -GMm \left(\frac{-1}{r} \right) \Big|_r^\infty$$

$$W = -GMm \left(\frac{-1}{\infty} - \left(\frac{-1}{r} \right) \right)$$

Since $\frac{1}{\infty} = 0$, we are left with

$$W = \frac{-GMm}{r}$$

E X A M P L E 1**A rocket moving upward from Earth**

How much work is done moving a 1000-kg spacecraft from Earth's surface to a height of 200 km above Earth's surface?

Solution and Connection to Theory**Given**

$$M = 5.98 \times 10^{24} \text{ kg} \quad m = 1000 \text{ kg} \quad r_1 = r_{\text{Earth}} = 6.38 \times 10^6 \text{ m}$$

$$d = 200 \text{ km} = 2 \times 10^5 \text{ m}$$

$$r_2 = r_1 + d = 6.38 \times 10^6 \text{ m} + 2 \times 10^5 \text{ m} = 6.58 \times 10^6 \text{ m}$$

$$W = \Delta E = E_{p2} - E_{p1}$$

$$W = \frac{-GMm}{r_2} - \left(\frac{-GMm}{r_1} \right)$$

$$W = -GMm \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

$$W = -(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(1000 \text{ kg}) \left(\frac{1}{6.58 \times 10^6 \text{ m}} - \frac{1}{6.38 \times 10^6 \text{ m}} \right)$$

$$W = 1.90 \times 10^9 \text{ J}$$

The work done to move the spacecraft is $1.90 \times 10^9 \text{ J}$.

E X A M P L E 2**The potential energy of the Moon**

Fig.6.5 The Moon



Calculate the potential energy of the Moon ($m_{\text{Moon}} = 7.35 \times 10^{22} \text{ kg}$) relative to Earth ($m_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg}$) given that the distance between their centres is $3.94 \times 10^5 \text{ km}$.

Solution and Connection to Theory**Given**

$$m_{\text{Moon}} = 7.35 \times 10^{22} \text{ kg} \quad m_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg}$$

$$r_{\text{E/M}} = 3.94 \times 10^5 \text{ km} = 3.94 \times 10^8 \text{ m}$$

For potential energy,

$$E_p = \frac{-GMm}{r}$$

$$E_p = \frac{-(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(7.35 \times 10^{22} \text{ kg})}{3.94 \times 10^8 \text{ m}}$$

$$E_p = -7.4 \times 10^{28} \text{ J}$$

The potential energy of the Moon relative to Earth is $-7.4 \times 10^{28} \text{ J}$.

A Comparison of $\Delta E_p = mg\Delta h$ and $E_p = \frac{-GMm}{r}$

In Chapter 5, we learned that the potential energy near Earth's surface is described by the equation

$$\Delta E_p = mg\Delta h$$

where Δh is the change in height. Using this equation, the change in potential energy is a positive value. How does this equation relate to the equation we derived earlier in this section?

Imagine that you have fallen down a well and are rescued by firemen hoisting a rope to pull you out. They have to add energy to pull you to the surface. If we assume that your potential energy is zero at Earth's surface, then your potential energy in the well is negative until you come out. At the bottom of the well, you have the most negative value of potential energy; for example, -1000 J. Nearer the top, it becomes less negative (approaching zero), for example, -200 J, so

$$\Delta E_p = E_{p_2} - E_{p_1} = -200 \text{ J} - (-1000 \text{ J}) = +800 \text{ J}$$

The *change* in potential energy is positive. Similarly, we obtain a positive value when we use the equation $E_p = mg\Delta h$ because we set an arbitrary $E_p = 0$ and consider the change in E_p relative to this reference point.

EXAMPLE 3

Finding the energy change on Earth in two different ways

What is the energy change in moving a 1.00-kg mass from Earth's surface to a distance two times the radius of Earth? Use both equations for gravitational potential energy.

Solution and Connection to Theory

Given

$$r_{\text{Earth}} = 6.38 \times 10^6 \text{ m} = r_1 \quad r_2 = 2r_{\text{Earth}} = 1.28 \times 10^7 \text{ m}$$
$$M_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg} \quad m = 1.00 \text{ kg}$$

Using $E_p = \frac{-GMm}{r}$:

$$\Delta E_p = E_{p_2} - E_{p_1}$$

$$\Delta E_p = \frac{-GMm}{r_2} - \left(\frac{-GMm}{r_1} \right)$$

Deriving $\Delta E = mg\Delta h$ from the general case of $E_p = \frac{-GMm}{r}$

$$E_p = \frac{-GMm}{r}$$

If we move a height Δh above Earth's surface, then r becomes $r + \Delta h$. Thus,

$$\Delta E_p = E_{p_2} - E_{p_1} = \frac{-GMm}{r} + \Delta h - \left(\frac{-GMm}{r + \Delta h} \right)$$

$$\Delta E_p = GMm \left(\frac{-1}{r + \Delta h} + \frac{1}{r} \right)$$

$$\Delta E_p = GMm \left(\frac{-r + r + \Delta h}{r(r + \Delta h)} \right)$$

$$\Delta E_p = \frac{GMm\Delta h}{r(r + \Delta h)}$$

For small values of h (1 km or less), $r + \Delta h \approx r$ (where r is Earth's radius, 6.38×10^6 m) because $r \gg \Delta h$.

Therefore,

$$\Delta E_p = \frac{GMm\Delta h}{r^2} \quad (\text{eq. 1})$$

But $F = \frac{GMm}{r^2}$ and $\frac{F}{m} = a = g$ (the acceleration due to gravity)

$$\text{So, } g = \frac{\frac{GMm}{r^2}}{m} = \frac{GM}{r^2}$$

Substituting g for $\frac{GM}{r^2}$ in equation 1, we obtain

$$\Delta E_p = gm\Delta h = mg\Delta h$$

$$\Delta E_p = \frac{-(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(1.00 \text{ kg})}{1.28 \times 10^7 \text{ m}} - \frac{(-6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(1.00 \text{ kg})}{6.38 \times 10^6 \text{ m}}$$

$$\Delta E_p = -3.12 \times 10^7 \text{ J} + 6.25 \times 10^7 \text{ J}$$

$$\Delta E_p = 3.13 \times 10^7 \text{ J}$$

Using $\Delta E_p = mg\Delta h$:

If we assume that $g = 9.8 \text{ m/s}^2$, then

$$E_p = mg\Delta h = (1.00 \text{ kg})(9.8 \text{ m/s}^2)(6.38 \times 10^6 \text{ m}) = 6.25 \times 10^7 \text{ J}$$

We have used $6.38 \times 10^6 \text{ m}$ for Δh because this distance is the distance above Earth's surface. Notice that both values are positive. In the first calculation, the object's gravitational potential energy became less negative due to the change in height.

The different answers arrived at using the two methods are due to the difference in the values of g at this distance. This difference becomes noticeable at about 400 000 m above Earth's surface, where the difference between the surface value of g (9.8 m/s^2) and the local value of g is roughly 10%.

Kinetic Energy Considerations

Potential energy is only part of the energy equation. From Chapter 5, we know that in a closed system, the total energy must be constant at all times. We also know that kinetic energy is given by the equation

$$E_k = \frac{1}{2}mv^2$$

Thus, in general,

$$E_T = E_k + E_p$$

But

$$E_p = \frac{-GMm}{r}$$

Therefore,

$$E_T = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

EXAMPLE 4

Ballistic trajectory

A 2300-kg rocket shuts off its engines (burns out) 494 km above Earth's surface. Its velocity at burnout is 3.0 km/s directly upward. Ignoring air resistance, what maximum height will the rocket reach?

Solution and Connection to Theory**Given**

$$m = 2300 \text{ kg} \quad v_1 = 3.0 \text{ km/s} = 3000 \text{ m/s} \quad r_E = 6.38 \times 10^6 \text{ m}$$

$$r_1 = 494 \text{ km} + r_E = (0.494 + 6.38) \times 10^6 \text{ m} = 6.87 \times 10^6 \text{ m} \quad r_2 = ?$$

Since we know the rocket's mass and speed at burnout, we can calculate its kinetic energy. We also know that at maximum height, $v = 0$; therefore, the kinetic energy is also zero, meaning that all the kinetic energy from the launch has been transferred to gravitational potential energy, E_p . We need to calculate the difference in gravitational potential energy at launch and at the peak of the ballistic arc as E_k , then solve for the unknown distance.

$$E_k = \Delta E_p$$

$$E_k = E_2 - E_1$$

$$\frac{1}{2}mv^2 = \frac{-GMm}{r_2} - \left(\frac{-GMm}{r_1} \right)$$

The only unknown variable is r_2 . We can solve the equation algebraically before substituting any values into it. First, we can cancel m and multiply by 2 to eliminate the fraction.

$$v^2 - \frac{2GM}{r_1} = \frac{-2GM}{r_2}$$

$$r_2 = \frac{-2GMr_1}{v^2r_1 - 2GM}$$

Substituting values for the variables,

$$r_2 = \frac{-2(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(6.87 \times 10^6 \text{ m})}{(3000 \text{ m/s})^2(6.87 \times 10^6 \text{ m}) - 2(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}$$

$$r_2 = 7.45 \times 10^6 \text{ m}$$

If Earth's mean radius is 6378 km, then the height of the rocket at burnout is

$$h = r_2 - r_E = 7450 \text{ km} - 6378 \text{ km} = 1072 \text{ km} = 1100 \text{ km}$$

This rocket's maximum height is 1100 km.

Fig.6.6 Liftoff of a *Saturn V* Moon rocket



Escape Energy and Escape Speed

How much energy must we give a rocket at Earth's surface in order for it to escape entirely from Earth's gravitational pull?

We know that the gravitational potential energy at Earth's surface is given by

$$E_p = -\frac{GMm}{r_E}$$

We also know that to escape Earth's gravity, the sum of the kinetic and potential energies of the rocket (i.e., the rocket's **total mechanical energy**) must equal zero. If we wish our rocket to just escape Earth's gravitational field, we must give it an initial kinetic energy of

$$E_k = +\frac{GMm}{r_E}$$

Escape speed is the minimum speed required for an object to escape the gravitational pull of another object at a given distance. Solving for the escape speed, v_{esc} ,

$$\frac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{r} = 0$$

The rocket mass cancels:

$$\frac{1}{2}v_{\text{esc}}^2 - \frac{GM}{r} = 0$$

Therefore,

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

EXAMPLE 5

Escape speed from Earth

Find the escape speed required to leave Earth.

Fig.6.7 Liftoff of a space shuttle



Solution and Connection to Theory

Given

$$m_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg}$$

$$r_{\text{Earth}} = 6.38 \times 10^6 \text{ m}$$

$$E_T = 0 \text{ for escape; therefore, } \frac{1}{2}mv^2 = \frac{GMm}{r^2}$$

The mass of the craft cancels out:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r_E}}$$

$$v_{\text{esc}} = \sqrt{\frac{2(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{6.38 \times 10^6 \text{ m}}}$$

$$v_{\text{esc}} = 1.1 \times 10^4 \text{ m/s} = 11 \text{ km/s}$$

Therefore, the escape speed required to leave Earth is 11 km/s.

Escape speed is a simplification that describes a one-time velocity that would be required to escape Earth's gravitational pull without any subsequent work being done by the rocket engine. In practice, only probes that leave Earth to explore other planets must reach this speed.

Implications of Escape Speed

Case 1: $v_{\text{probe}} > v_{\text{esc}}$

If $v_{\text{probe}} > v_{\text{esc}}$, then $E_T > 0$. Thus, when the craft reaches an infinite distance, it will not stop but keep going.

Case 2: $v_{\text{probe}} < v_{\text{esc}}$

Conversely, if $v_{\text{probe}} < v_{\text{esc}}$, then $E_T < 0$. If v_{probe} is close to v_{esc} , ideally, the probe will return, but gravitational forces in the universe will do work on the probe and may change its course, preventing its return.

Case 3: $v_{\text{probe}} = v_{\text{esc}}$

The craft has just enough energy to escape a body's gravitational pull and will not return. We can calculate the minimum speed required to escape a gravitational field using the equation

$$\frac{1}{2}mv^2 = \frac{GMm}{r} \text{ or } v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

Fig. 6.8 Pioneer 10 was the first human-made satellite to leave our solar system (1983). It had an escape speed greater than the Sun's escape speed.



EXAMPLE 6 Leaving the Moon

Apollo astronauts had to fire their rocket engine in order to return to Earth after the Moon landings. If their spacecraft had an altitude of 100 km above the lunar surface, what was the escape speed from the Moon if its mass is $7.35 \times 10^{22} \text{ kg}$? (Assume that the Moon's diameter is 3476 km.)

Solution and Connection to Theory

Given

$$m_{\text{Moon}} = 7.35 \times 10^{22} \text{ kg} \quad d_{\text{Moon}} = 3476 \text{ km} = 3.476 \times 10^6 \text{ m}$$

$$h = 100 \text{ km} = 1 \times 10^5 \text{ m} \quad v_{\text{esc}} = ?$$

First, we need to determine the distance from the spacecraft to the centre of the Moon:

$$r = \frac{d_{\text{Moon}}}{2} + h$$

$$r = \frac{3.476 \times 10^6 \text{ m}}{2} + (1 \times 10^5 \text{ m})$$

$$r = 1.838 \times 10^6 \text{ m}$$

For escape speed,

$$v_{\text{esc}} = \sqrt{\frac{2Gm_{\text{Moon}}}{r}}$$

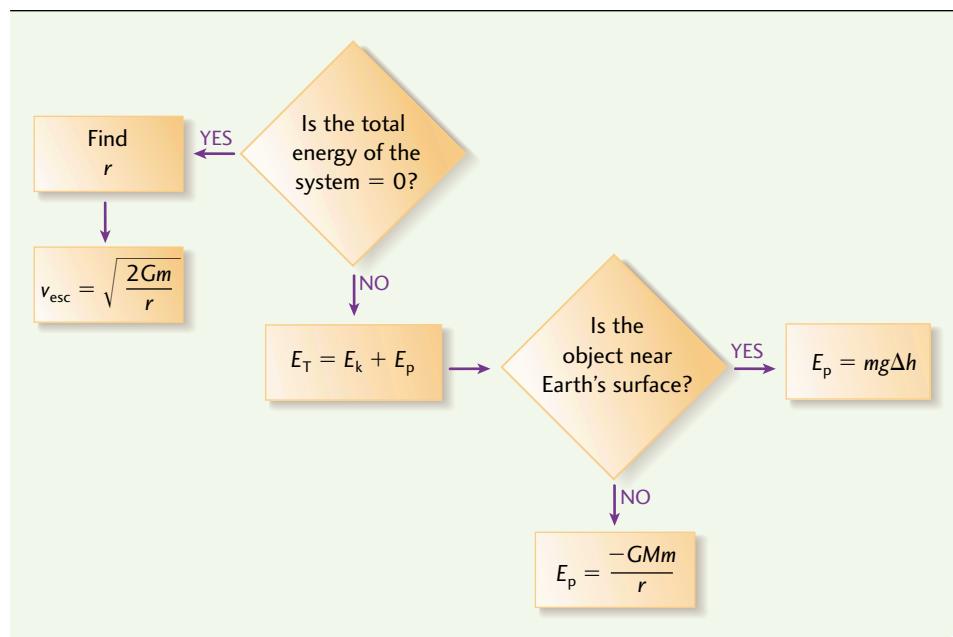
$$v_{\text{esc}} = \sqrt{\frac{2(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(7.35 \times 10^{22} \text{ kg})}{1.838 \times 10^6 \text{ m}}}$$

$$v_{\text{esc}} = 2.309 \times 10^3 \text{ m/s}$$

Therefore, the required speed to escape lunar gravity at this altitude is 2309 m/s.

Figure 6.9 summarizes the steps in solving energy-transfer problems.

Fig.6.9 Escape Speed and Energy Transfer





1. Assume Earth is orbiting the Sun in a circular orbit. Look up Earth's mass and mean distance to the Sun and calculate
 - a) its kinetic energy.
 - b) its potential energy relative to the Sun.
 - c) the total energy of the orbit.
2. What is the effective value of g 1000 km above Earth's surface? Earth has a mass of 5.98×10^{24} kg and a radius of 6.38×10^6 m.
3. A 1000-kg rocket fired from Earth has a speed of 6.0 km/s when it reaches a height of 1000 km above Earth's surface.
 - a) If the rocket has used all of its fuel, will it escape from Earth?
 - b) If it does not escape, what maximum height above Earth will it reach?

6.2 Orbits

If we throw a baseball, it comes back to Earth very quickly. If we fire an artillery shell, the projectile travels possibly 40 km over the surface before coming back to Earth. Some experimental rockets travel part way around the world before landing. Figure 6.10 illustrates that if we can make an object travel fast enough, then it will go all the way around the world (ignoring the effects of the atmosphere's drag).

In Chapter 2, we learned that objects moving in a circular path are acted upon by an unbalanced force inward, called the *centripetal force*. An object orbiting Earth or any other body has an unbalanced force of gravity acting 90° to its velocity. Thus

$$F_{\text{net}} = F_g$$

For circular motion,

$$F_{\text{net}} = F_c$$

Therefore,

$$F_c = \frac{GMm}{r^2}$$

But

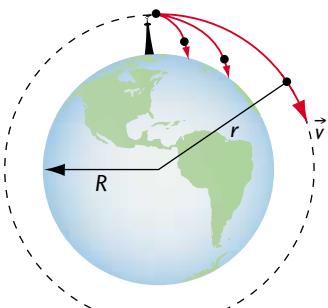
$$F_c = \frac{mv^2}{r}$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

where m is the mass of the orbiting object. Canceling the common factors, we obtain the **equation for the orbital speed**:

$$v = \sqrt{\frac{GM}{r}}$$

Fig.6.10 With sufficient velocity, a horizontal projectile could circle Earth



E X A M P L E 7**Calculating orbital speed**

What speed is required for an object to stay in orbit just above Earth's surface?

Solution and Connection to Theory**Given**

$$M = 5.98 \times 10^{24} \text{ kg} \quad r_E = 6378 \text{ km}$$

To find the orbital speed,

$$v^2 = \frac{GM}{r}$$

$$v^2 = \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.378 \times 10^6 \text{ m})}$$

$$v^2 = 6.253 \times 10^7 \text{ m}^2/\text{s}^2$$

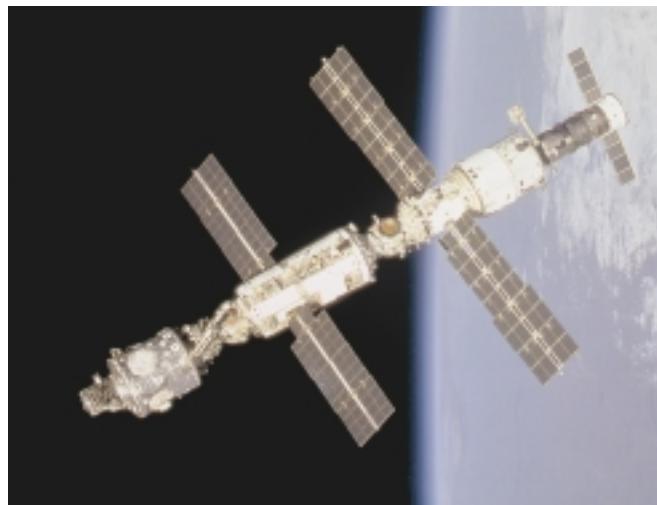
$$v = 7908 \text{ m/s} = 7900 \text{ m/s}$$

The object must travel at a speed of 7900 m/s. This speed is the reason why jetliners that fly at speeds approaching 1000 km/h don't escape Earth, nor do military aircraft, which fly a lot faster. Spacecraft not only have to fly fast, but also above the atmosphere. Since the effects of atmospheric drag are a function of the speed squared, the heat produced by an object travelling at this speed through the atmosphere would cause it to burn up.

E X A M P L E 8**The orbital speed of a space station**

What is the speed of the International Space Station (ISS) in its typical orbit of 340 km above Earth's surface?

Fig.6.11 The International Space Station



Solution and Connection to Theory

Given

$$M = 5.98 \times 10^{24} \text{ kg} \quad h = 340 \text{ km} = 3.4 \times 10^5 \text{ m}$$

The value of r in the equation is the distance from the orbiting object to the centre of Earth. So,

$$r = r_E + h = 6.378 \times 10^6 \text{ m} + 3.4 \times 10^5 \text{ m} = 6.718 \times 10^6 \text{ m}$$

For orbital speed,

$$v^2 = \frac{GM}{r}$$

$$v^2 = \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{6.718 \times 10^6 \text{ m}}$$

$$v^2 = 5.94 \times 10^7 \text{ m}^2/\text{s}^2$$

$$v = 7705 \text{ m/s} = 7700 \text{ m/s}$$

Therefore, the ISS is travelling at an orbital speed of 7700 m/s.

If we consider a satellite travelling in a circular orbit around Earth, we know that the required centripetal force is supplied by gravity such that

$$F_c = \frac{mv^2}{r_{\text{orbit}}} = \frac{GMm}{r_{\text{orbit}}^2}$$

$$mv^2 = \frac{GMm}{r_{\text{orbit}}}$$

Dividing by 2,

$$\frac{1}{2}mv^2 = \frac{GMm}{2r_{\text{orbit}}}$$

Therefore, for a circular orbit

$$E_k = \frac{GMm}{2r_{\text{orbit}}}$$

Since the energy required to escape Earth's gravitational pull is $\frac{GMm}{r}$, a satellite in orbit has one-half the required energy to escape. Therefore, the additional energy it needs to escape Earth's gravitational pull is

$$\frac{GMm}{2r}$$

E X A M P L E 9**The total mechanical energy of Earth's orbit**

What is the total mechanical energy of Earth's orbit around the Sun if the mean Earth–Sun distance is 1.5×10^{11} m, the mass of Earth is 5.98×10^{24} kg, and the mass of the Sun is 2.0×10^{30} kg?

Solution and Connection to Theory**Given**

Total mechanical energy of Earth's orbit

$$= \frac{GMm}{2r}$$

$$= \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(2.0 \times 10^{30} \text{ kg})}{2(1.5 \times 10^{11} \text{ m})}$$

$$= 2.7 \times 10^{33} \text{ J}$$

The total mechanical energy of Earth's orbit around the Sun is 2.7×10^{33} J, a huge amount of energy!

Kepler's Laws of Planetary Motion

Johannes Kepler (1571–1630) struggled with the problem of planetary motion. The accepted view of the time was Copernicus's solar-centred (heliocentric) universe consisting of circular orbits for the planets due to their geometric perfection. But observations revealed that the orbits of the planets weren't perfect circles. The motion of Mars was especially puzzling because it made loops in the sky every two years or so (known as **retrograde motion** — see Figure 6.12). Other planets did so as well, but Mars was the most obvious. Kepler believed that if he could solve the problem of Mars' motion, then the motions of the other planets would also be explained.

Fig.6.12 Martian retrograde motion

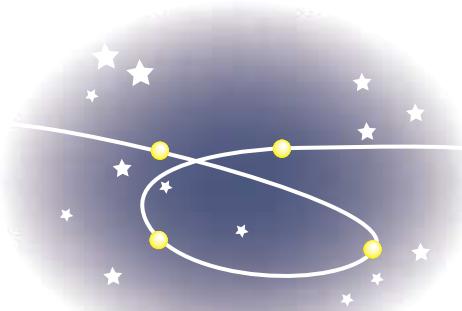


Fig.6.13a

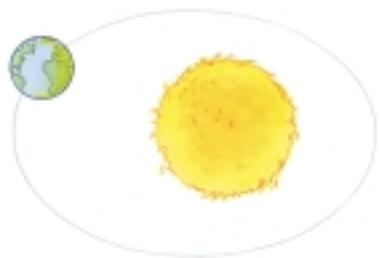


Fig.6.13b

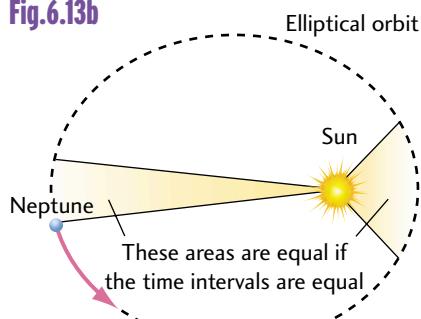
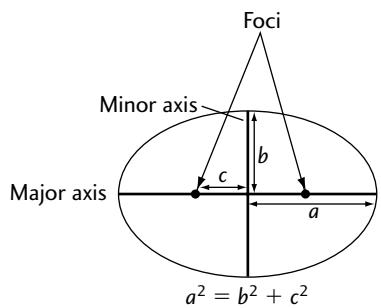


Fig.6.13c The ellipse



Kepler tried various geometric solutions. When he tried elliptical paths, the predictions agreed remarkably well with observations. From his work, Kepler formulated three laws of planetary motion.

Law 1: The path of each planet is an ellipse with the Sun at one focus (see Figure 6.13a).

Law 2: A body in orbit around another body sweeps out equal areas in equal times (see Figure 6.13b). The body moves fastest at its closest approach and slowest when farthest away, which allows us to determine the orbiting object's location at any chosen time.

Law 3: The ratio of the radius of the orbit cubed divided by the square of the period is equal to the same constant for all planets:

$$\frac{r^3}{T^2} = K$$

If a satellite of mass m is in a circular orbit about Earth, then the centripetal force is supplied by gravity:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = \frac{m4\pi^2r}{T^2}$$
 (Recall the

three equations for centripetal force from Section 2.8.)

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

But $\frac{GM}{4\pi^2} = K$, a constant; therefore,

$$\frac{r^3}{T^2} = K$$

Kepler's third law can also be expressed in terms of a , the average radius of an elliptical orbit. This radius is approximately the sum of the **apogee** and **perigee** distances. The period of the orbit (T) and the semimajor axis of the orbit are related by the equation

$$T^2 = ka^3.$$

In this case,

$$k = \frac{4\pi^2}{Gm}$$

EXAMPLE 10 A year on Venus

Fig.6.14 Venus



If the distance from Venus to the Sun is 0.723 times the distance from Earth to the Sun, how many Earth days are there in a year on Venus?

Solution and Connection to Theory

Given

$$r_V = 0.723r_E \quad T_E = 365 \text{ days}$$

Using Kepler's third law,

$$\frac{r_V^3}{T_V^2} = \frac{r_E^3}{T_E^2}$$

$$T_V^2 = \frac{r_V^3 T_E^2}{r_E^3}$$

$$T_V^2 = \frac{(0.723r_E)^3 (365 \text{ days})^2}{r_E^3}$$

$$T_V^2 = (0.723)^3 (365 \text{ days})^2 = 50\,350 \text{ days}^2$$

$$T_V = \sqrt{50\,350}$$

$$T_V = 224 \text{ days}$$

A year on Venus is 224 Earth days long.

Kepler's Third Law for Similar Masses

The constant K in Kepler's equation assumes that the orbiting mass is infinitesimally small. If the mass of the orbiting object is a significant fraction of the larger mass, then the equation $KT^2 = r^3$ is invalid because the objects orbit about the *centre of mass* of the two objects, known as the **barycentre**, not about the larger mass. In the case of Earth and the Moon, the centre of mass of the Earth–Moon system is about 1600 km below Earth's surface. Thus, Earth wobbles while the Moon orbits. For similar masses, the equation for K must be modified in the following manner:

$$K = \frac{G(M + m)}{4\pi^2}$$

This equation is valid when m is a significant fraction of M . In our solar system, Pluto and its moon Charon are an example of two large masses orbiting a barycentre. Charon is about 20% of the mass of Pluto, so both objects orbit around a point in space between them approximately every six days.

Binary stars have similar orbital paths. Sometimes, a star is observed to undergo a small amount of movement or “wobble” due to an unseen companion. Given the period of this wobble, we can calculate the distance between the unseen companion and the star along with its mass relative to the star. This wobble period is used to find black hole candidates and extrasolar planets.

Extension: Orbital Parameters

There are five main parameters in celestial mechanics that describe the shape of orbits in space and help locate orbiting objects in their trajectories.

The **orbital period**, T , is the time required to complete one orbit from an inertial frame of reference.

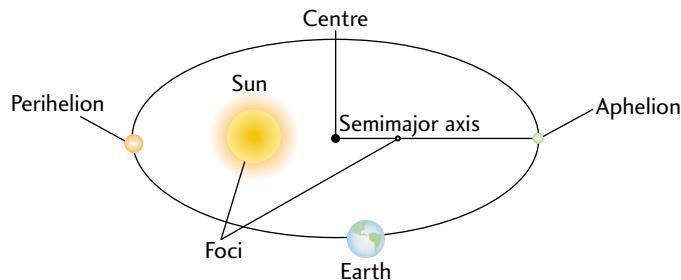
The **eccentricity**, e , is the flatness of the ellipse.

The **semimajor axis**, a , is the distance along the major (longest) axis, from the geometric centre of the ellipse to its edge.

The **pericynthion** is the point of closest approach to the large central mass.

The **apocynthion** is the point of farthest approach to the large central mass.

Fig.6.15 Orbital parameters for our solar system



The Total Energy of an Elliptical Orbit

The total energy of a circular orbit is given by the equation

$$E_T = \frac{-GMm}{2r}$$

For an elliptical orbit, we replace r with the semimajor axis, a , which is the average radius, r_{avg} . Thus,

$$E_T = \frac{-GMm}{2a}$$

The *shape* of an orbit and its orientation in space doesn't depend on its total energy. If Earth's orbit was highly elongated but had the same semimajor axis, it would have *exactly the same energy and orbital period* (see Figure 6.16).

Table 6.1
Extreme Points of Passage in Orbits

Central mass object	Trajectory extrema suffix
Earth	gee
Sun	helion
Moon	lune
Jupiter	jove
Star	astron

Fig.6.16 The length of the semimajor axis determines the energy and orbital period of a body

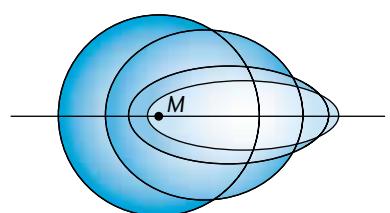
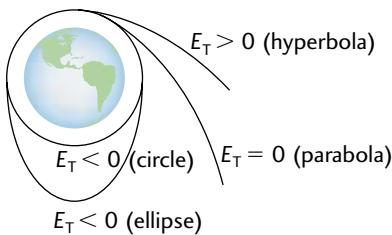


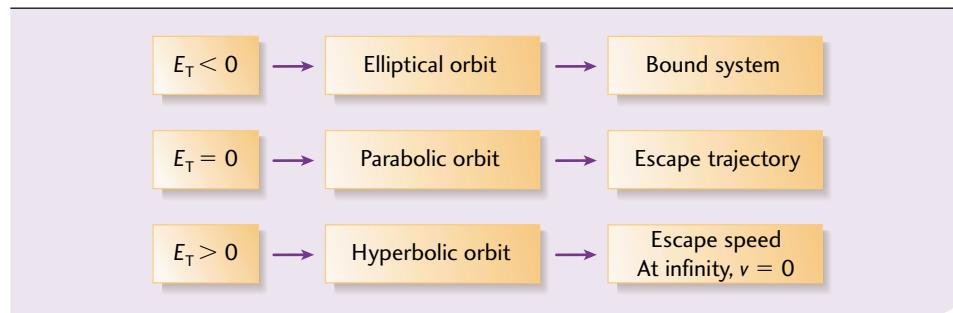
Fig.6.17 The total energy of the orbit determines the orbital shape



There are three possibilities for the total energy of an orbit: $E_T < 0$, $E_T > 0$, and $E_T = 0$. When $E_T < 0$, the orbit is periodic and is called a **bound system**. For example, the Moon is bound to Earth, and is not about to go visit Mars. When $E_T > 0$, the energy is positive and the object is no longer bound. Since potential energy is always negative and is a function of position, the kinetic energy must increase to make $E_T > 0$. This type of orbit is a hyperbola, such as a satellite launched from Earth and leaving the solar system. When $E_T = 0$, the orbit is a parabola and is called an **escape trajectory**; that is, it has just enough energy to overcome the gravitational pull of the mass it orbits (see Figure 6.17). Any force acting on the orbiting body changes its path and may also change the type of orbit.

Figure 6.18 summarizes the three orbital shapes and their energies.

Fig.6.18 Summary of Orbital Shapes



1. Comet Halley has an orbital period of 76.1 years.
 - a) What is its semimajor axis?
 - b) What is the eccentricity of its orbit? (Look it up in a resource.)
 - c) How fast is this comet moving at perihelion?
2. What is the escape speed of an object 10 000 km above Jupiter's "surface"?
3. a) How much speed would have to be added to the Moon for it to leave Earth's influence?
 - b) How much energy would it take to achieve your answer in a)?
 - c) Compare your answer in b) with an everyday energy value.
4. Recall from Chapter 2 that some communications satellites are in *geostationary Earth orbit*. What are the special circumstances of such satellites? Where are they located? (Hint: They are maintained in position by their orbit and gravity, *not* by continually firing rocket engines.)

The Elliptical Path of a Projectile

Kepler's laws allow us to revisit one of the concepts covered in Chapter 2. The range of a ballistic projectile is given by the equation

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

This equation is based on the assumption that the particle starts and leaves at the same height above ground. The path travelled by the object is parabolic. This equation is also based on the assumption that Earth is flat because no change in the direction of g is given: g always points to Earth's centre.

Technically, the direction of g changes slightly on a projectile's flight path. The amount of the change in the direction of g is illustrated by the Verrazano Narrows Bridge in New York City. Opened in 1964 and with a span of 1280 m, it was the world's longest bridge until the 1990s. The two support towers are about 15 cm farther apart at the top of the tower than at the bottom due to Earth's curvature.

Earth's curvature changes the path of a projectile from a parabola to an ellipse. Nevertheless, we can still apply the equation $R = \frac{v_0^2 \sin 2\theta}{g}$ since the error is insignificant on short-range projectiles. According to Kepler, the projectile is actually in orbit about Earth's centre. It would continue orbiting Earth in an elliptical trajectory, but the ground gets in the way.

5. Calculate the orbital period of a baseball thrown at a speed of 25 m/s parallel to Earth's surface if the baseball were to pass through Earth. Assume that all of Earth's mass is concentrated at a point at Earth's centre.

Fig.6.19 The Verrazano Narrows Bridge in New York City



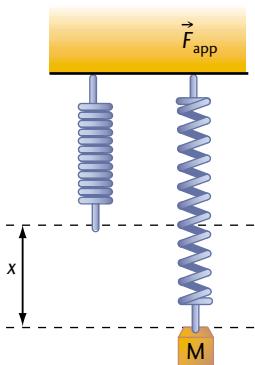
6.3 Simple Harmonic Motion — An Energy Introduction

Fig.6.20 This 40-kg brass pendulum at the Smithsonian Institution in Washington, DC, swings to and fro all day. It's an example of repetitive motion, where the force on the object is proportional to the distance from the equilibrium point.



If the spring in Figure 6.21 is supported vertically, the mass will stretch the spring until an equilibrium point is found. Suppose that the mass is pulled down and released. In its lower position, the spring exerts a force that accelerates the mass upward toward the equilibrium point. At the equilibrium point, the mass has very little force on it since the spring is no longer stretched, and continues past the equilibrium point due to its momentum. The spring then begins to compress to a point ideally the same distance from the equilibrium point. Once the spring is compressed enough that it has stored all the kinetic energy possessed by the mass when it passed the equilibrium point, it then begins to fall back down past the equilibrium point, to the point from which it was released.

Fig.6.21



Actually, the mass will not reach the exact same point because there is friction in the coiling of the spring that will slowly damp the oscillation. The air, acting as a fluid, also exerts a small amount of drag on the spring and mass. Thus, *all oscillating motion is damped* and will slowly come to rest after a number of oscillations.

Hooke's Law

Recall from Chapter 5 that the restoring force of a spring system is proportional to the displacement from the equilibrium position, defined by the equation

$$F = -kx$$

where k is the spring constant and x is the displacement from the equilibrium point.

Notice that the force operates in the opposite direction to the displacement vector, which is why it is called a restoring force. Expanding on Hooke's law and applying Newton's second law, we can derive the equation for the acceleration of the mass:

$$\vec{F}_{\text{net}} = \vec{F}_{\text{spring}}$$

$$ma = -kx$$

or

$$a = -\left(\frac{k}{m}\right)x$$

EXAMPLE 11 The acceleration of a mass on a spring

A 500-g puck is connected to the side of an air table by a spring. A force of 1.4 N is applied to pull the puck 8.0 cm to the right. Then, the puck is released.

- What is the maximum acceleration of the puck?
- What is the acceleration of the puck as it passes its original rest position?

Solution and Connection to Theory

Given

$$m = 500 \text{ g} = 0.50 \text{ kg} \quad F = 1.4 \text{ N} \quad x = 8.0 \text{ cm} = 0.08 \text{ m}$$

a) $k = \frac{F}{x} = \frac{1.4 \text{ N}}{0.08 \text{ m}} = 17.5 \text{ N/m}$

$$a = -\left(\frac{k}{m}\right)x$$

The acceleration is greatest when $x = 0.08 \text{ m}$.

$$a = -\left(\frac{17.5 \text{ N/m}}{0.50 \text{ kg}}\right)0.08 \text{ m} = -2.8 \text{ m/s}^2$$

The acceleration of the puck is 2.8 m/s^2 to the left.

- As the puck passes the original rest position, the puck's displacement is zero, so the acceleration at that point is also zero.

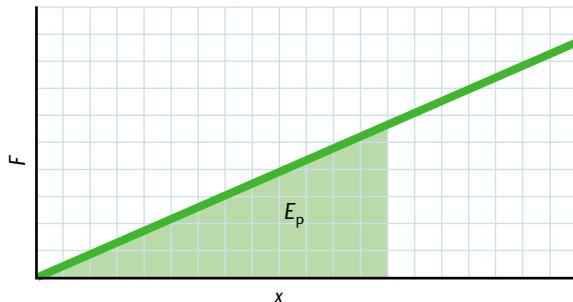
We ignore the negative sign in Hooke's law because Hooke's law specifically considers the action of a **restoring force** *against* an applied force. Since we are only concerned with the energy stored in an ideal spring being compressed or expanded, the defining equation for force becomes $F = kx$.

Motion that obeys Hooke's law is called **simple harmonic motion (SHM)** (see also Chapter 10). To determine some of the properties of the mass–spring system, we can study its energy. The equation for kinetic energy is

$$E_k = \frac{1}{2}mv^2$$

We know that the work done to compress the spring equals its potential energy because the system is closed and isolated. Hooke's law gives us the following graph:

Fig.6.22



The energy stored in the spring is given by the area under the graph in Figure 6.22, which is $A = \frac{1}{2}bh$ or $E = \frac{1}{2}Fx$. Combining the equations $E = \frac{1}{2}Fx$ and $F = kx$, we obtain

$$E = \frac{1}{2}(kx)x$$

$$E = \frac{1}{2}kx^2$$

Therefore, the total energy of the mass–spring system is

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

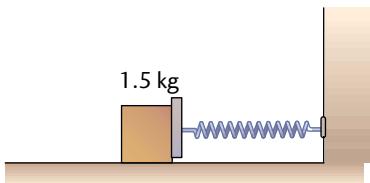
We can determine the total energy of the system from the spring's maximum compression. At maximum compression, the kinetic energy of the system is zero because the mass is motionless. Similarly, we can also determine the total energy of the system from the speed of the mass as it passes the equilibrium point. At this point, there is no potential energy stored in the spring because $x = 0$. Thus,

$$E_T = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

EXAMPLE 12 The speed of an oscillating mass

A spring with a spring constant of 80.0 N/m has a 1.5-kg block attached to its free end. If the block is pulled out 50.0 cm from its rest position and released, what is its speed when it returns to the equilibrium position? Assume there is no friction.

Fig.6.23



Solution and Connection to Theory

Given

$$k = 80.0 \text{ N/m} \quad m = 1.5 \text{ kg} \quad x = 50.0 \text{ cm} = 0.500 \text{ m}$$

At the equilibrium position, $x = 0$ and all the initial potential energy of the spring is converted to kinetic energy.

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$v = \sqrt{\frac{kx^2}{m}} = \sqrt{\frac{(80 \text{ N/m})(0.500 \text{ m})^2}{1.5 \text{ kg}}}$$

$$v = 3.7 \text{ m/s}$$

The block's speed at the equilibrium position is 3.7 m/s.

At maximum compression (displacement) from equilibrium (zero kinetic energy), the amplitude of the system's motion is A . The total energy of the system is

$$E_T = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

which becomes

$$kA^2 = mv^2 + kx^2$$

EXAMPLE 13

Calculating the energy and speed of a pendulum

Determine the total energy and maximum speed of a pendulum with a mass of 0.200 kg, a maximum amplitude of 0.500 m, and a spring constant of 1.00 N/m.

Solution and Connection to Theory

Given

$$k = 1.00 \text{ N/m} \quad m = 0.200 \text{ kg} \quad A = 0.500 \text{ m}$$

We can calculate the *total energy* of the system from the amplitude:

$$E_T = \frac{1}{2}kA^2$$

$$E_T = \frac{1}{2}(1.00 \text{ N/m})(0.500 \text{ m})^2$$

$$E_T = 0.125 \text{ J}$$

From this result, we can determine the *maximum speed* of this mass and spring by using the equation

$$E = \frac{1}{2}mv_{\max}^2$$

$$v_{\max}^2 = \frac{2E}{m}$$

$$v_{\max}^2 = \frac{2(0.125 \text{ J})}{0.200 \text{ kg}}$$

$$v_{\max} = 1.12 \text{ m/s}$$

The pendulum has a total energy of 0.125 J and a maximum speed of 1.12 m/s.

- 1.** Given a mass–spring system with a bob of mass 0.485 kg, a spring constant of 33 N/m, and an initial displacement of 0.23 m, determine
 - a)** the kinetic energy of the bob as it passes the equilibrium point.
 - b)** the change in energy when the bob passes the equilibrium point in the opposite direction.
 - c)** the bob's speed as it passes the equilibrium point.
- 2.** From the data given in problem 1 above, determine
 - a)** the period.
 - b)** the bob's speed when the displacement is 0.16 m.
 - c)** the bob's kinetic energy at the 0.16-m point.
- 3.** Table 6.2 lists data for the position and speed of a mass–spring system. Calculate the time and plot a graph of position versus time. What is the shape of this graph? Explain.



Table 6.2
Position and Speed Data for a Mass–Spring System

x (m)	v (m/s)	Δt (s)	Total time (s)
0.5	0	0	0
0.4	0.6708	0.298	0.298
0.3	0.8944	0.1278	0.4258
0.2	1.0247	0.1042	0.53
0.1	1.09545	0.09433	0.6243
0	1.118	0.0904	0.71473
-0.1	1.095	0.0904	0.80513
-0.2	1.0247	0.09433	0.89946
-0.3	0.8944	0.1042	1.00366
-0.4	0.6708	0.1278	1.13146
-0.5	0	0.298	1.4294

6.4 Damped Simple Harmonic Motion

All motion experiences friction. The effect of friction on SHM is called **damping**. If SHM is damped, it may stop in less than one oscillation, or it may oscillate any number of times before finally stopping. Damping affects amplitude and makes its value much more time-dependent.

Three Types of Damping

The three types of damping and the corresponding graph of the motion are given below.

- 1) **Overdamping:** Oscillation ceases and the mass slowly returns to equilibrium position (Figure 6.24).
- 2) **Critical damping:** Oscillation ceases and the mass moves back to equilibrium position *as fast as theoretically possible without incurring further oscillations*. This special point is never perfectly reached in nature (Figure 6.25).
- 3) **Underdamping:** Oscillation is continually reduced in amplitude (Figure 6.26).

Fig.6.24 Overdamping

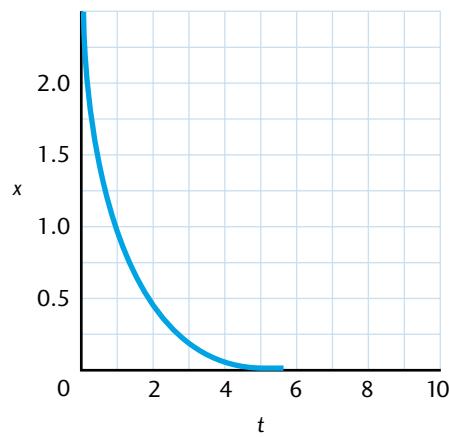


Fig.6.25 Critical damping

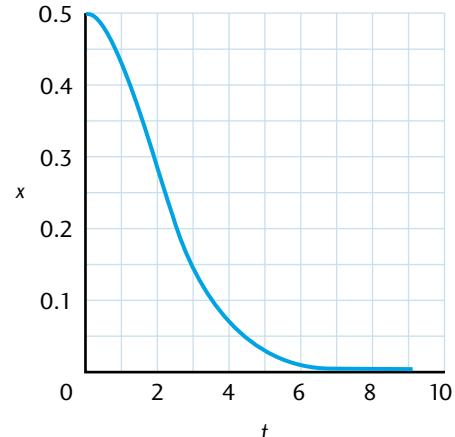
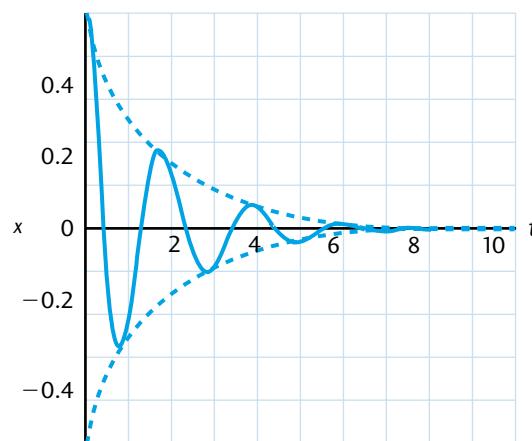


Fig.6.26 Underdamping



Applications of Damping

Door Closers

Fig.6.27a Ouch!

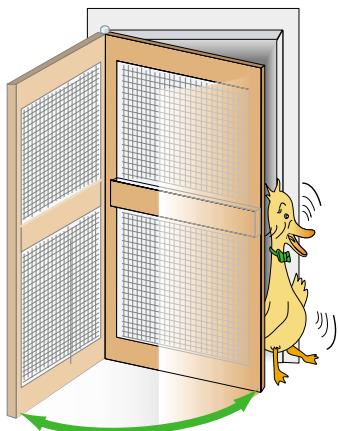
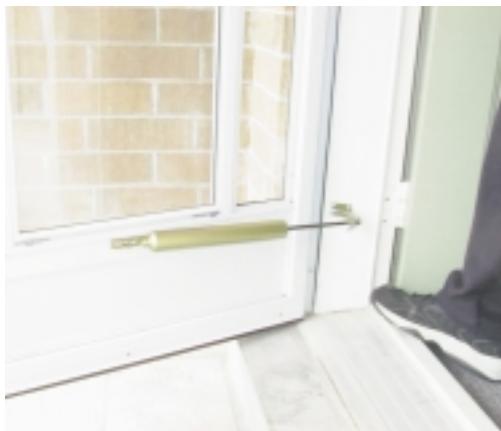


Fig.6.27b A door-closing mechanism



Spring-loaded storm door closers would slam the door closed if they were not damped (see Figures 6.27a and b). The closer contains a spring, a piston, and a valve. The valve allows air into the cylinder when opening the door pulls the piston back. As the door closes, the valve lets the air out slowly, damping the motion. The motion is slightly overdamped, which means that the door stops before it is completely closed. Then the spring slowly closes it the rest of the way.

Shock Absorbers

The suspension system of all cars is made up of a spring-loaded wheel mount and a strut or shock absorber to damp the motion so that the car does not continue to bounce up and down in SHM every time you go over a bump. Shocks may be either gas filled or oil filled. A piston inside the shock is forced up and down as the car's suspension moves up and down. As the piston moves up or down, the fluid inside is forced to flow through a small hole in the piston, which restricts its motion. The car's motion is slightly underdamped. If you push down on the front corner of your car, it should spring back slightly past the rest position and stop when it returns to equilibrium if your shock absorbers are working properly.

Fig.6.28 Shock absorbers on vehicles act as dampers



1. Explain three examples of damped SHM not given in this text.





The International Space Station

Fig.STSE.6.1 An artist's rendition of the completed International Space Station (ISS)



One way to save energy during shuttle launch is to take advantage of Earth's rotation. When the shuttle launches, it quickly pitches and rolls to fly up and east. At a distance of two Earth radii from Earth's centre, the shuttle (and all of us, for that matter) moves eastward at a speed of one Earth circumference per day, or $2\pi(6.38 \times 10^6 \text{ m})/24 \text{ h}$, or about 464 m/s. The shuttle can use the kinetic energy from this speed to begin its journey.

Objects in the ISS are normal-forceless rather than weightless; that is, they are still acted upon by the force of gravity, but they are in free fall toward Earth. The circular motion of their orbit also means that as they fall, they also move tangentially to Earth's surface. The net result is that the objects have no normal force holding them in place and are therefore constantly accelerating toward Earth's surface.

Space travel is always in the media — whether we are exploring a new world with robotic spacecraft or sending astronauts into space aboard the space shuttle. Flying in space is tremendously expensive. The energy required to reach orbital speeds is phenomenal. Outside Earth's atmospheric envelope, there is nothing to support life. The region is a vacuum that is 20 times deeper than that easily obtained on Earth. Radiation from the Sun, normally stopped by the atmosphere, creates hazards for astronauts and their equipment. Without the surrounding atmosphere to moderate temperatures by dispersing heat, sunlit regions of a spacecraft are about 200°C hotter than their shaded counterparts a few metres away. If we go into space, we need to bring everything. Given that space travel is neither cheap nor easy, why is the ISS a priority for the international scientific community?

Very expensive laboratories around the world are designed to simulate any environment possible to perform research on a wide array of topics, but none of them can suspend the effects of gravity. In a micro-gravity environment, many chemical and physical reactions change their nature. For example, when heating water in a sealed pot on an electric stove, only the water near the stove element will heat by conduction. The "boiling" that is typically seen in a pot of boiling water is due to convection, which occurs because the density of water decreases with heat, so the hotter water moves higher, forced up by the cooler, denser water that takes its place. In a micro-gravity environment, although density changes also occur, convection is impossible because there is no unbalanced force of gravity. The water farthest from the burner remains at the same cool temperature, while the water at the bottom boils. The entire pot of water eventually heats, but at a much slower rate, and only by conduction. In space, agitation of fluids is necessary to ensure thermodynamic stability.

The ISS is currently being constructed in Earth orbit by the Russians and the US space-shuttle fleet. The construction project is a consortium of 16 nations, including Canada. Canada's prime contribution to the station is the Space Station Remote Manipulator System (SSRMS), otherwise known as *Canadarm 2*, which will move about outside the station, performing many tasks that would otherwise require space walks. The station will be completed around 2005.

Experiments are already being conducted on the ISS. Current experiments include protein crystal experiments, where scientists expect to learn how proteins actually work and how to grow them; cell and tissue growth experiments for research in cancer, diabetes, and AIDS; and a number of experiments to measure human adaptability to the micro-gravity environment. Once the ISS is complete, scientists will be able to perform more complicated experiments.

The ISS may be able to accommodate more than 10 people at a time, so the chances of being able to work in this unique environment will be much greater than in the past when spacecraft were limited to only two or three individuals. If you are curious about the kind of background required to fly in space, examine the NASA astronaut biography Web site to see what some of these amazing people did before they flew in space (see <www.irwinpublishing.com/students>).

Design a Study of Societal Impact

Research current experiments on the ISS as well as those that are planned in the next few years. Explain what benefits these experiments will have for humanity, and why these experiments must be conducted in space.

The space station will cost about US\$75 billion. Explain the value of this price tag for humanity. Compare this cost to what Canadians typically spend per year on items such as recreation, music, cosmetics, alcohol, and cigarettes.

Design an Activity to Evaluate

Research the mass of a typical space shuttle, including payload and the orbit parameters for a typical mission. Calculate the gravitational potential energy of the shuttle and payload in typical orbit. What percentage of the total launch energy was provided by the rotational kinetic energy described in this STSE? How much fuel must be burned to send the shuttle into space?

Investigate the cost of possible commercial transportation ventures to the ISS. What is the likelihood that one of us will visit it?

Build a Structure

Space hardware must be strong yet lightweight and fit into a small cargo area. Design an object that is made of lightweight materials such as toothpicks and paper. Use paper to represent a solar panel. The device must fit into a 10-cm³ cube when stowed, and deploy to a length of 1 m without external support.

SUMMARY | SPECIFIC EXPECTATIONS

You should be able to

Understand Basic Concepts:

- Analyze planetary and satellite motion.
- Analyze the factors affecting the motion of isolated celestial objects.
- Explain how astronauts can be “weightless” while a large gravitational force is exerted on them.
- Describe the energy transformations that take place when a spaceship escapes Earth’s gravitational field.
- State Kepler’s three laws.
- Explain the mathematical model for SHM.
- Understand how energy is transferred during SHM.
- Explain damping and give a practical example not given in this text.
- Solve SHM-type problems.
- Explain the negative signs in Newton’s law of universal gravitation and in the gravitational potential energy equation.

Develop Skills of Inquiry and Communication:

- Calculate the gravitational potential energy of isolated celestial objects.
- Calculate the energy and speed required to propel a spacecraft from Earth’s surface out of Earth’s gravitational field.
- Calculate the kinetic and gravitational potential energies of a satellite that is in stable circular orbit around a planet.
- Perform experiments with a pendulum, including analysis, to verify the theoretical relationships between its period, mass, initial angle, and string length.

Relate Science to Technology, Society, and the Environment:

- Understand the current developments with respect to the International Space Station and the reason why essential research needs to be performed in space and not on the ground.
- Gain experience in constructing models of lightweight, deployable devices for space-type applications.
- Gain an understanding of the skills and background required of an astronaut or scientist.
- Identify examples of SHM and damped SHM in the natural world.

Equations

$$F = \frac{GMm}{r^2}$$

$$E_p = \frac{-GMm}{r}$$

$$E_k = \frac{1}{2}mv^2$$

$$E_T = E_k + E_p$$

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

$$E_T = \frac{-GMm}{2r} \text{ (orbit)}$$

$$\frac{r^3}{T^2} = K$$

$$a = -\left(\frac{k}{m}\right)x$$

$$F = -kx$$

$$E_p = \frac{1}{2}kx^2$$

$$E_T = \frac{1}{2}kA^2$$

$$kA^2 = mv^2 + kx^2$$

EXERCISES

Conceptual Questions

1. Why do we not require the more general form of Newton's law of universal gravitation when we are calculating the force of gravity in our classroom?
2. Explain the difference in launch requirements if a spacecraft was launched westward instead of eastward. Assume that it will achieve the same orbit.
3. Why does only one side of the Moon face Earth at all times?
4. Explain the relationship between the force of gravity and gravitational potential energy.
5. If a spacecraft jettisoned a large piece of itself into space, making its new mass about 26% of its original mass, what happens to the orbit of the smaller section?
6. During a space rendezvous, two spacecraft have to match orbits very carefully before one of them can move in to dock. If an astronaut in the shuttle simply points at the other docking craft in the distance and then rockets toward it, the docking spacecraft will move farther away! Why?
7. In a Jules Verne novel, a spacecraft is "shot" to the Moon in a large cannon. Suppose the barrel is 80 m long. If the spacecraft experienced constant acceleration for the entire 80 m, determine
 - a) if this mission would be survivable. Explain the reasons carefully.
 - b) the force of the cannon's recoil.
8. Assume that our knees can absorb the impact from a fall of 2 m without damage. If we attach springs to our feet that have 500-N/m spring constants and 0.45 m of travel, from what maximum height could we survive a fall?

9. Using the same "spring boots" as in question 8, if the spring completely compresses and our knees absorb their maximum amount of energy, would we bounce off the ground? Explain.
10. Identify three SHMs in your daily experience. Explain how you are convinced that the motion is in fact typical of SHM.
11. Find three examples of damping in oscillatory systems (it need not be exactly SHM). Is the damping desired or undesired? Why?
12. Design a device that could be used for measuring the mass of Canadian astronaut Roberta Lynn Bondar while in orbit. Due to continual free fall, she appears weightless, thus rendering conventional scales useless.

Problems

6.1 Gravity and Energy

13. If a 100 000-kg shuttle enters Earth's atmosphere at 4 km/s and lands at 80 m/s, how much energy has it released to the atmosphere? What is its change in height if its initial height was 100 km?
14. A 920-kg satellite is projected vertically upward from Earth's surface with an initial kinetic energy of 7.0×10^9 J. Find
 - a) its maximum height.
 - b) the initial kinetic energy it would have needed to keep going indefinitely.
 - c) the initial speed it would have needed to keep going indefinitely.
15. A 550-kg satellite projected upward from Earth's surface reaches a maximum height of 6000 km. Find
 - a) its change in gravitational potential energy.
 - b) its initial kinetic energy.

- 16.** A 20 000-kg meteorite from outer space is headed directly toward Earth with a speed of 3.0 km/s. Find its speed when it is 200 km above Earth's surface.
- 17.** The escape speed at the **event horizon** of a black hole is defined as the speed of light, c . What would the size of Earth have to be for it to be compressed to a black hole?
- 18.** At what location from Earth are the gravitational fields of Earth and the Moon balanced?
- 19.** Find the energy per kilogram required to move a payload from Earth's surface to the Moon's surface.

6.2 Orbit

- 20.** Find the speed of an Earth satellite in orbit 400 km above Earth's surface. What is the period of the orbit?
- 21.** Find the altitude of a communications satellite that is in geostationary Earth orbit above the equator.
- 22.** When the space shuttle delivers a crew to the International Space Station, it usually boosts the orbit of the station from about 320 km to 350 km. How much energy does the shuttle add to the station's orbit?
- 23. a)** Show that speed decreases as the radius of a satellite's orbit increases.
b) What effect does increasing an orbit's radius have on the period of the satellite?
- 24.** Calculate the Moon's energy in its orbit around Earth.
- 25.** Saturn has a mass of 5.7×10^{26} kg and a radius of 6.0×10^7 m. What is the minimum speed of a satellite orbiting Saturn?
- 26.** The *Apollo* astronauts were typically in an orbit 100 km above the lunar surface. What is the escape speed from the Moon at this altitude?

- 27.** Given the orbit height in problem 26, how long would it take for the *Apollo* spacecraft to complete one orbit around the Moon?
- 28. a)** Calculate the speed of Mars as it moves about the Sun. Its mean distance from the Sun is 2.28×10^{11} m, its radius is 3.43×10^6 m, and its mass is 6.37×10^{23} kg
b) Calculate the speed required to orbit Mars at an altitude of 80 km.
- 29.** Calculate the escape speed of a spacecraft leaving the Moon's surface.

6.3 Simple Harmonic Motion — An Energy Introduction

- 30.** Three waves pass the end of a pier in every 12 s. If there is 2.4 m between the wave crests, what is the frequency?
- 31.** If a spring with $k = 12$ N/m is connected to a mass of 230 g and set in motion with an amplitude of 26 cm, calculate the speed of the mass as it passes the equilibrium point.
- 32.** A 2.0-kg mass on a spring is extended 0.30 m from the equilibrium position and released. The spring constant is 65 N/m.
a) What is the initial potential energy of the spring?
b) What maximum speed does the mass reach?
c) Find the speed of the mass when the displacement is 0.20 m.
- 33.** For the mass in problem 32, find
a) its maximum acceleration.
b) its acceleration when the displacement is 0.20 m.
- 34. a)** The Minas Basin in the Bay of Fundy has the largest tides in the world (around 15 m — see Figure 7.43a). Suppose that a device is stretched across the mouth of the bay for 10 km. As the floats inside it rise and fall with the tide, their motion is converted to electricity. Suppose that the mechanical

linkages of this proposed system are 29% efficient. What power would be produced if the period of the tide is 12 hours and 32 minutes?

- b)** Compare the value you obtained in a) to the output of a reactor at Ontario's Darlington Nuclear Station (900 MW).
- 35.** A 100-kg mass is dropped from 12 m onto a spring with 0.64 cm of recoil. What is the spring constant?
- 36.** Consider a spring of $k = 16$ N/m connected to a block mass (m) and having an amplitude of motion of 3.7 cm. What is the total energy of this system?
- 37.** A 5-g bullet is discharged at 350 m/s into a mass–spring system. If the mass is 10 kg and the spring constant is 150 N/m, how far will the spring be compressed if the bullet stays in the mass?
- 6.4 Damped Simple Harmonic Motion**
- 38.** Damped oscillators are complicated. For simplified cases, the amplitude of the damped oscillator decreases exponentially according to the equation $x = x_0 e^{\frac{-bt}{2m}}$, where x_0 is the

maximum amplitude at the start of the oscillation, t is the time at which you calculate the amplitude, m is the mass on the spring, and b is a damping constant in kg/s. If $b = 0.080$ kg/s, $m = 0.30$ kg, and the starting amplitude is 8.5 cm, calculate the value for x at the following times:

- a)** 0.1 s
b) 1.5 s
c) 15.5 s
d) 3.0 min
e) 5.2 h
- 39.** For problem 38, how long does it take for the amplitude to reach one-half its initial value?
- 40.** **a)** Mechanical energy also decreases exponentially according to the equation $E = \frac{1}{2}kx_0^2 e^{\frac{-bt}{m}}$. For the oscillator in problem 38, calculate the time it takes for the mechanical energy to drop to one-half its initial value if $k = 100$ N/m.
- b)** Calculate the energy of the oscillator at
i) 0.1 s.
ii) 22.3 s.
iii) 2.5 min.
iv) 5.6 a.



The Pendulum

Purpose

To analyze the motion of a pendulum

Equipment

1 retort stand

1 test-tube clamp (rubber grips work best to hold the string)

1-m length of string

1 hook mass set

1 stopwatch or sonic rangers

Procedure

1. Set up the pendulum so that the string is in a V shape. This shape will permit the pendulum to swing in one plane only. Tie one end of the string to a clamp near the post and grip the other end with the clamp to permit quick changes of the string length.
2. Determine how to measure the release angle of the pendulum and how much amplitude is being attenuated by air and mechanical friction. To accurately determine the period, let the pendulum swing 10 or 20 oscillations if the damping is not too high, then divide the time you measure by the number of oscillations.
3. Determine the mass dependence on the period by swinging three or four masses.
4. Determine the dependence of the period on the length of the string. Try at least five different lengths.

Data

Organize your data in chart form. Use a different chart for steps 3 and 4 of the procedure.

Analysis

1. Plot a graph of string length versus period.
2. Perform a logarithmic transformation on the data to obtain a straight line (see Appendix D).
3. Determine the equation of this line, then transform it back to a curve so that you have a relationship between L and T .

Discussion

1. Was the period of the pendulum dependent on the mass? Was it dependent on the string length?
2. Find the theoretical equation for a pendulum. Compare your equation with this equation and determine if the constants, such as π and g , are reasonably correct.
3. In light of your values from question 2, discuss where uncertainties occurred in this lab and how they directly affected your results.
4. What other possible factors affecting the pendulum's period could you check?

Conclusion

Summarize your results in a concluding statement.

Angular Motion



Chapter Outline

- 7.1 Introduction
- 7.2 A Primer on Radian Measure
- 7.3 Angular Velocity and Acceleration
- 7.4 The Five Angular Equations of Motion
- 7.5 Moment of Inertia
- 7.6 Rotational Energy
- 7.7 Rotational Kinetic Energy
- 7.8 The Conservation of Energy
- 7.9 Angular Momentum
- 7.10 The Conservation of Angular Momentum
- 7.11 The Yo-yo
-  Gyroscopic Action — A Case of Angular Momentum
- LAB** 7.1 Rotational Motion: Finding the Moment of Inertia

By the end of this chapter, you will be able to

- compare angular motion to linear motion
- calculate various aspects of angular motion, such as angular displacement, velocity, and acceleration
- calculate and discuss aspects of rotational forces, energy, and momentum
- explain everyday phenomena in terms of angular variables

7.1 Introduction

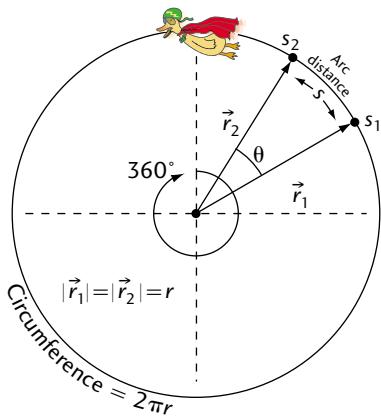
So far in this text, we have, for the most part, studied linear motion. We have used Newton's laws to describe how objects behave at rest, and in uniform and accelerating motion. Chapter 2 briefly touched on aspects of circular motion and introduced the concept of centripetal acceleration and force. In this chapter, we will study the angular equivalents to displacement, velocity, acceleration, force, mass, and momentum and derive the corresponding angular equations. We will also learn why tops continue to spin if no external forces act on them, why skaters spin faster when they tuck their arms in, and why gyroscopes are used in missile guidance systems.

Fig. 7.1 Objects undergoing angular motion



1. Make a list of toys that rely on angular motion to function. Do any of them combine different angular components?
2. List kitchen aids, both manual and electric, that rely on angular motion to perform their function.
3. List workshop tools, both manual and electric, that rely on angular motion to perform their tasks.

Fig. 7.2a Projecto moves $2\pi r$ or 360° to complete one cycle



7.2 A Primer on Radian Measure

Consider Projecto, the great circus duck, moving in a circular path of radius r , as shown in Figure 7.2a. Projecto completes one cycle when the angle through which he moves is 360° . The distance he travels is the circumference, $C = 2\pi r$. Any part of Projecto's travel along the circular path is called an **arc length**, s , or **angular displacement**, θ . Radian measure is the angle through which Projecto moves in traversing an arc length, s . If we know the arc length and the radius, we can determine the radian measure, θ .

Radian measure equals $\frac{\text{arc length}}{\text{radius}}$ or

$$\theta = \frac{s}{r}$$

where θ is the angle measured in radians (rad), s is the part of the path subtended by the angle θ , and r is the radius.

If we travel one complete cycle, then $\theta = 360^\circ$ and $s = C = 2\pi r$. Therefore, $\frac{s}{r} = \frac{2\pi r}{r} = 2\pi$. Remember that $\frac{s}{r} = \theta$, so a 360° angular move results in an equivalent move of 2π radians.

To convert radians to degrees,

$$2\pi \text{ rad} = 360^\circ$$

so

$$1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

Therefore, to convert an angle measured in radians to an angle measured in degrees, we multiply the number of radians by $57.3^\circ/\text{rad}$.

The **equation for arc length** travelled is

$$s = r\theta$$

where the angle θ is measured in radians.

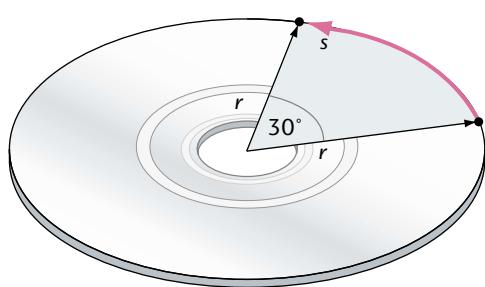
The link between angular and linear measurement is the radius, r . We will see it again in equations for angular velocity, acceleration, and torque.

EXAMPLE 1

Using radian measure

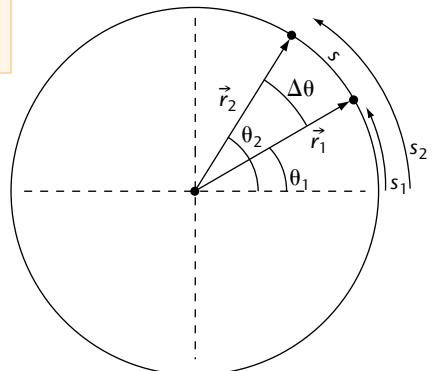
A CD, 12.0 cm in diameter, rotates 30° counterclockwise. How far does a point on the outer rim of the CD move, in centimetres and radians?

Fig.7.4



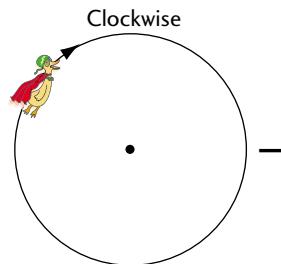
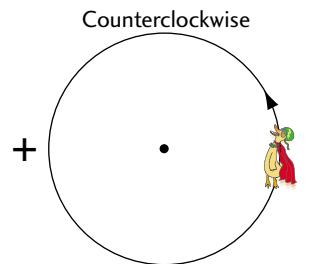
The Parameters of Angular Displacement

Fig.7.2b \vec{r} = position vector, $\Delta\theta$ = radian measure, s = arc length



By convention, angular displacement is *positive* if the motion is *counterclockwise*, and *negative* if the motion is *clockwise*.

Fig.7.3 Angular motion conventions



The radian (rad) is a unitless quantity and can be omitted in the final answer. It is similar to the term "cycles" in cycles/s. The term "cycles" is dropped and the unit becomes 1/s or Hz.

Solution and Connection to Theory

Given

$$d = 12.0 \text{ cm} \quad \theta = 30^\circ \quad s = ? \quad r = ?$$

The radius of the CD is $\frac{12.0 \text{ cm}}{2} = 6.0 \text{ cm}$.

In radians, $30^\circ = \frac{30^\circ}{57.3^\circ/\text{rad}} = 0.52 \text{ rad}$.

Gradians

Some calculators come with a *grad* mode. In Europe, gradians are sometimes used to measure angles. There are 400 gradians in one complete cycle.

To find the distance (arc length) travelled, we substitute into the equation $s = r\theta$, where the angle is measured in radians and r is the radius:

$$s = (6.0 \text{ cm})(0.52 \text{ rad})$$

$$s = 3.1 \text{ cm}$$

The point on the outer rim of the CD travels 3.1 cm (equivalent to a 30° turn). A positive value means that the CD is rotating in a counterclockwise direction. Notice also that the rad unit is omitted from the final answer for the length the point moves along the rim.

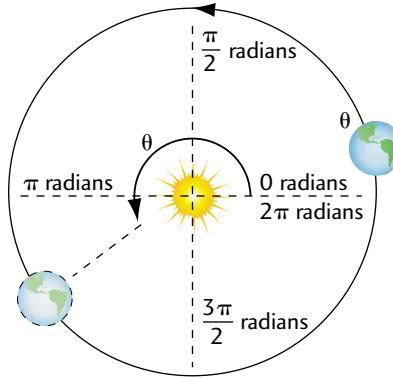
EXAMPLE 2

Orbital motion using angular measure

Earth's average orbital radius around the Sun is $1.49 \times 10^{11} \text{ m}$. If it travels $\frac{3\pi}{4}$ radians, calculate

- this angle in degrees.
- the distance Earth has travelled in orbit.

Fig.7.5



Solution and Connection to Theory

Given

$$r = 1.49 \times 10^{11} \text{ m} \quad \theta = \frac{3\pi}{4} \text{ rad} \quad s = ?$$

- a)** We don't need to substitute the numeric value for π in $\frac{3\pi}{4}$ rad because it will cancel out in the calculation:

$$2\pi \text{ rad} = 360^\circ; \text{ therefore,}$$

$$\frac{\frac{3\pi}{4} \text{ rad}}{\theta} = \frac{2\pi \text{ rad}}{360^\circ}$$

$$\theta = 360^\circ \cdot \frac{\left(\frac{3\pi}{4} \text{ rad}\right)}{(2\pi \text{ rad})}$$

$$\theta = 135^\circ$$

Therefore, Earth has travelled 135° .

- b)** To calculate the distance (arc length) travelled,

$$s = r\theta$$

$$s = (1.49 \times 10^{11} \text{ m}) \left(\frac{3\pi}{4} \text{ rad} \right)$$

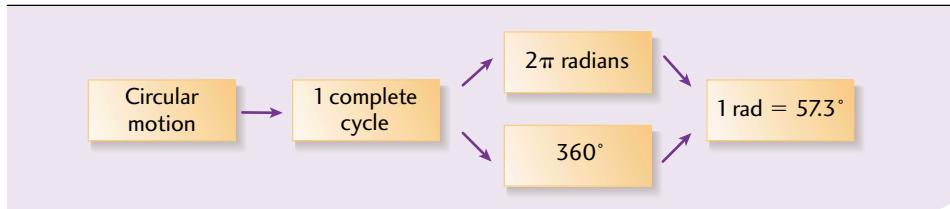
$$s = 3.51 \times 10^{11} \text{ m}$$

Earth has travelled 3.51×10^{11} m when it moves 135° in its orbit.

We can also convert to degrees by multiplying the radians by $57.3^\circ/\text{rad}$:
 $\frac{3\pi}{4} \text{ rad} \times 57.3^\circ/\text{rad} = 135^\circ$

Figure 7.6 summarizes the relationship between radians and degrees.

Fig. 7.6 The Radian–Degree Connection



- Convert the following angles to radian measure.
 - 10°
 - 60°
 - 90°
 - 176°
 - 256°
- Convert the following angles in radian measure to degrees.
 - π rad
 - $\frac{\pi}{4}$ rad
 - 3.75π rad
 - 11.15 rad
 - 40 rad
- How many radians are the following quantities?
 - Earth's rotation in 6.0 h
 - Earth's orbit in 265 d
 - The second hand of a clock moving 25 s
 - A long-distance runner doing 25.6 laps of a track

7.3 Angular Velocity and Acceleration

Angular Velocity

In linear motion,

$$\frac{\Delta d}{\Delta t} = v_{\text{avg}}$$

where Δd represents the distance travelled over a period of time Δt . For angular motion, this equation becomes

$$\frac{s}{\Delta t} = v_{\text{avg}}$$

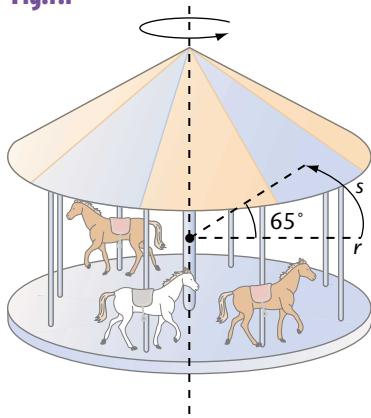
where s is the arc length. Similarly, **angular velocity** is the change in angular position of an object over a time period. When an object moves from θ_1 to θ_2 in a time period Δt , $\frac{\Delta\theta}{\Delta t}$ equals the angular velocity, given the symbol ω (omega), with units rad/s.

The **equation for angular velocity** is given by

$$\omega = \frac{\Delta\theta}{\Delta t}$$

EXAMPLE 3 Calculating angular velocity

Fig.7.7



A merry-go-round turns through 65.0° in 3.5 s from rest. Find the average angular speed of the ride.

Solution and Connection to Theory

Given

$$\theta_1 = 0^\circ \quad \theta_2 = 65.0^\circ \quad \Delta t = 3.5 \text{ s} \quad \omega = ?$$

We will assume a counterclockwise rotation so that the values of ω are positive.

We first convert the angles to radian measure:

$$65.0^\circ \times \frac{1 \text{ rad}}{57.3^\circ} = 1.13 \text{ rad}$$

Substituting into the angular velocity equation,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{\Delta t}$$

$$\omega = \frac{1.13 \text{ rad} - 0 \text{ rad}}{3.5 \text{ s}}$$

$$\omega = 0.32 \text{ rad/s}$$

The average angular speed of the ride is 0.32 rad/s.

Note that in Example 3, we calculated the *average angular speed* for the merry-go-round. It starts from rest and accelerates to a final angular speed. We will calculate final angular speed in the next section.

Relating Angular Variables to Linear Ones

In this section, we will derive a set of fundamental relationships between distance, speed, acceleration, and their angular equivalents. From the definition for arc length,

$$s = r\theta$$

we divide both sides of the equation by Δt :

$$\frac{s}{\Delta t} = \frac{r\Delta\theta}{\Delta t}$$

But

$$\frac{s}{\Delta t} = v_{\text{avg}} \quad \text{and} \quad \frac{r\Delta\theta}{\Delta t} = \omega_{\text{avg}}$$

When we substitute into our first equation, we obtain the relationship

$$v = r\omega$$

We can omit the subscript *avg*. For small time periods, this equation relates the instantaneous speed to the equivalent angular speed.

Similarly, if we let

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

where α is the **angular acceleration** measured in rad/s^2 , then

$$a = r\alpha$$

where a is the linear acceleration in m/s^2 .

From the circular motion section in Chapter 2, we found that a point moving in an arc is really trying to move in a straight line while experiencing an external force acting perpendicular to its direction of motion (see Figure 7.8a).

Derivation of $a = r\alpha$

If a rotating object's speed changes from v_1 to v_2 , then the object's angular velocity changes from ω_1 to ω_2 . We can thus write $\Delta v = r\Delta\omega$. Dividing both sides by Δt , we obtain

$$\frac{\Delta v}{\Delta t} = \frac{r\Delta\omega}{\Delta t}$$

We know that $\frac{\Delta v}{\Delta t} = a$, the acceleration. We define $\frac{\Delta\omega}{\Delta t}$ as the angular acceleration, α ; that is, the change in angular velocity per unit time. Thus, $a = r\alpha$, where $\alpha = \frac{\Delta\omega}{\Delta t}$.

Fig.7.8a Tangential velocity

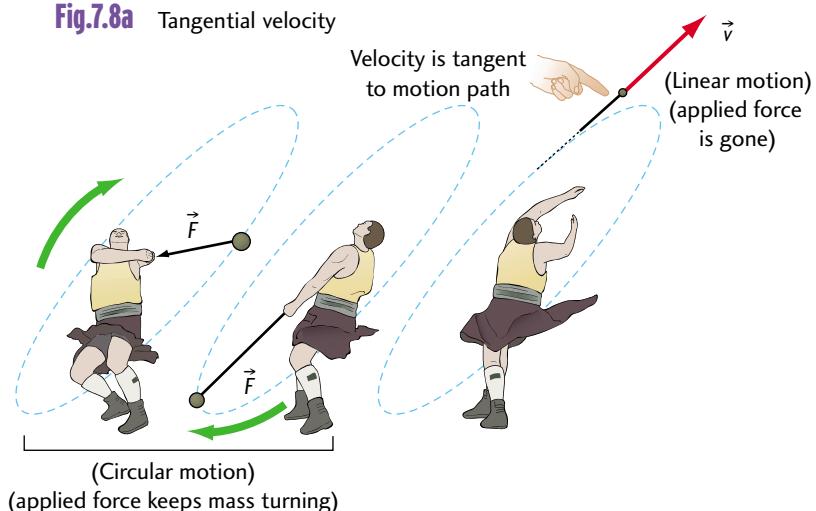
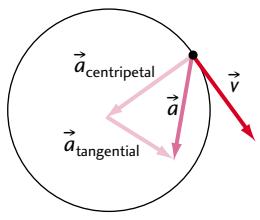


Fig. 7.8b



$$\vec{a} = \vec{a}_{\text{centripetal}} + \vec{a}_{\text{tangential}}$$

The instantaneous velocity (\vec{v}) is tangent to the path at a given point. If the object is speeding up or slowing down while moving in a circular path, there is also a linear acceleration, which is also tangent to the path. It lies in the same direction as the velocity vector (see Figure 7.8b).

From now on, we will refer to linear velocity (\vec{v}) and linear acceleration (\vec{a}) as **tangential velocity** and **tangential acceleration**, respectively.

EXAMPLE 4 Finding angular acceleration

At low speed, a fan blade is turning at 80 rad/s clockwise. The fan is turned up a notch to rotate at 125 rad/s clockwise. If the time to change speeds is 0.73 s, find the angular acceleration of the fan blades.

Solution and Connection to Theory

Given

$$\omega_1 = -80 \text{ rad/s} \quad \omega_2 = -125 \text{ rad/s} \quad \Delta t = 0.73 \text{ s} \quad \alpha = ?$$

$$\alpha = \frac{\omega_2 - \omega_1}{\Delta t}$$

$$\alpha = \frac{-125 \text{ rad/s} - (-80 \text{ rad/s})}{0.73 \text{ s}}$$

$$\alpha = -61.6 \text{ rad/s}^2 = -62 \text{ rad/s}^2$$

The negative sign means that the fan is accelerating clockwise. The angular acceleration is therefore 62 rad/s^2 [clockwise].

EXAMPLE 5 Relating angular variables to tangential ones

A sprinkler with two arms of length 20 cm rotates at 15 rad/s. If the arms have an angular acceleration of 6.5 rad/s^2 , find the initial tangential velocity and acceleration for the tip of the sprinkler arm.

Solution and Connection to Theory

Given

$$\omega_1 = 15 \text{ rad/s} \quad \alpha = 6.5 \text{ rad/s}^2 \quad r = 20 \text{ cm} = 0.20 \text{ m}$$

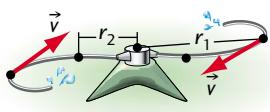
The tangential velocity is

$$v_1 = r\omega_1$$

$$v_1 = (0.20 \text{ m})(15 \text{ rad/s})$$

$$v_1 = 3.0 \text{ m/s}$$

Fig. 7.9



The tangential acceleration is

$$a = r\alpha$$

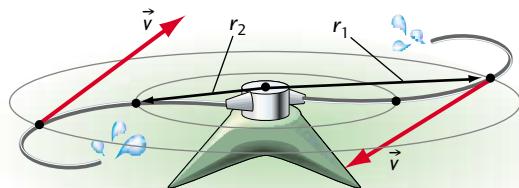
$$a = (0.20 \text{ m})(6.5 \text{ rad/s}^2)$$

$$a = 1.3 \text{ m/s}^2$$

Notice how the rad unit doesn't appear in the answer. We have assumed positive values for the angular measurements, meaning that the sprinkler turns counterclockwise. Since there is an angular acceleration, the sprinkler speeds up.

Note that points along the arm of the sprinkler in Example 5 all travel at the same *angular* velocity, but at different *tangential* velocities. If we chose to analyze a point halfway down the sprinkler arm (see Figure 7.10), its effective radius of turn becomes 10 cm. Its tangential velocity is $v = r\omega$, or $(0.10 \text{ m})(15 \text{ rad/s}) = 1.5 \text{ m/s}$. Although the angular speed is constant, the point farther down the sprinkler arm travels at a slower tangential speed and hence covers a smaller distance.

Fig. 7.10 Tangential speeds depend on the radius of turn, whereas the angular speed remains constant



More About Centripetal Acceleration

In Chapter 2, we were introduced to the concept of centripetal or centre-seeking acceleration, where the motion was circular and the object moving with a constant speed. The instantaneous acceleration always pointed to the *centre* of the circle. The equation we derived was $a = \frac{v^2}{r}$, where v is the tangential velocity of a point moving in a radius r .

Earlier in this section, we learned that $v = r\omega$. Substituting this equation into the equation for centripetal acceleration, we obtain

$$a_c = \frac{(r\omega)^2}{r} = r\omega^2$$

where a_c is the centripetal acceleration in m/s^2 , r is the radius of rotation in metres, and ω is the angular velocity in rad/s .

In everyday life, we don't usually refer to cyclic values in radians. We usually express objects undergoing circular motion in terms of revolutions per second (rps), or their SI equivalent, hertz (Hz). An object moves through 2π radians in one complete cycle. So, to convert rps to radians, we multiply by 2π . To convert radians to rps, we divide by 2π . In Example 5, the sprinkler's angular velocity was 15 rad/s , which equals $\frac{15 \text{ rad/s}}{2\pi \text{ rad/rev}} = 2.4 \text{ rev/s}$, or 2.4 rps , or 2.4 Hz .

EXAMPLE 6**Finding centripetal acceleration from angular variables****Fig.7.11**

In the Olympic hammer-throw event, the athlete swings the hammer in a circular arc. Assuming that the speed is constant at 1.91 rps and that the end of the hammer moves in an arc of radius 1.32 m, find the centripetal acceleration of the hammer head.

Solution and Connection to Theory**Given**

$$\omega = 1.91 \text{ rev/s} \quad r = 1.32 \text{ m} \quad a_c = ?$$

First, we need to convert the angular velocity to radian measure:

$$\omega = (1.91 \text{ rev/s})(2\pi \text{ rad/rev})$$

$$\omega = 12 \text{ rad/s}$$

Then, we can substitute the given values into the equation for centripetal acceleration:

$$a_c = r\omega^2$$

$$a_c = (1.32 \text{ m})(12 \text{ rad/s})^2$$

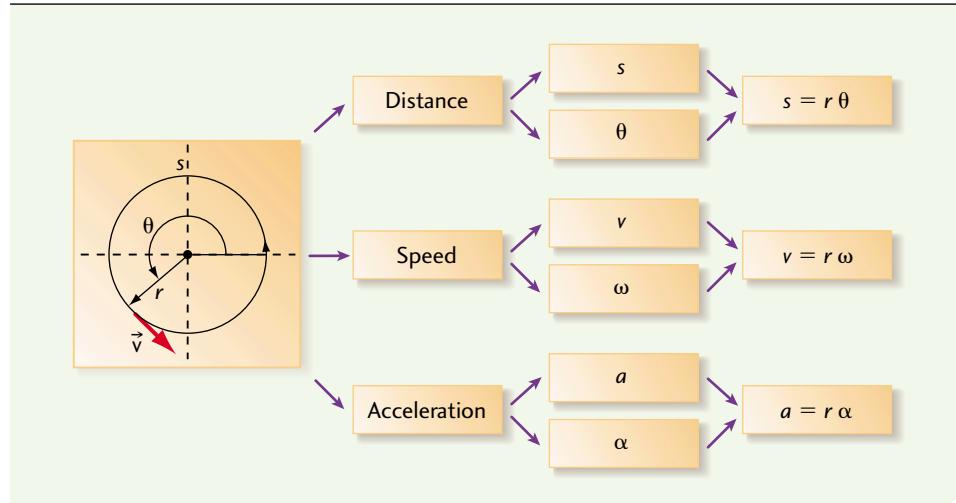
$$a_c = 190 \text{ m/s}^2$$

The centripetal acceleration is 190 m/s^2 .

We can also find the tangential velocity of the hammerhead from the equation $a = \frac{v^2}{r}$:

$$v = \sqrt{ra} = 15.8 \text{ m/s} = 16 \text{ m/s}$$

Figure 7.12 summarizes the variables and equations for angular motion.

Fig.7.12 Summary of Angular and Linear Variables

Artificial Gravity

In the film *2001: A Space Odyssey*, directed by Stanley Kubrick, the space station generated artificial gravity by rotating with a uniform circular motion. In the future, the strength of the artificial gravitational pull of a space station, like the one shown in Figure 7.13, will be determined by the space station's size and speed of rotation. Astronauts and cosmonauts require an artificial gravitational force because the human body is built to live in an environment where the force of gravity constantly acts on it. In space, the human body quickly loses bone strength because in a zero-gravity environment, the skeletal support structure becomes unnecessary. The artificial gravity of the space station is generated by the centripetal force (the normal force of the space station acting on the astronaut).

1. When an astronaut stands in a rotating space station like the one in Figure 7.13, which is essentially a hollow tube, where does the astronaut think the floor is? Is it outside or inside the tube?
2. a) For a station of radius 1200 m, what tangential speed must the station have in order for the astronaut to experience an acceleration of 9.8 m/s^2 ?
b) Convert the acceleration and speed in part a) to angular values.
3. a) If a space station rotates at 1.2 rpm (revolutions per minute), what is its angular velocity?
b) If the station has a radius of 1500 m, calculate the centripetal acceleration of the astronaut.
c) Convert the acceleration obtained in b) to an expression with angular variables.
d) How much larger or smaller is the artificial gravity experienced by astronauts in a space station than the gravity we experience on Earth?

Fig. 7.13 Space stations will generate artificial gravity by centripetal force



7.4 The Five Angular Equations of Motion

In Chapter 1, we used the following two equations of motion:

$$v_{\text{avg}} = \frac{\Delta d}{\Delta t} \quad \text{and} \quad \Delta d = \frac{1}{2}(v_1 + v_2)\Delta t$$

Also,

$$a = \frac{\Delta v}{\Delta t} \quad \text{or} \quad a = \frac{v_2 - v_1}{\Delta t}$$

We then obtained three more kinematics equations by isolating a variable in one equation and substituting it into another equation. The five kinematics equations for linear motion are

$$\Delta d = \frac{1}{2}(v_2 + v_1)\Delta t$$

$$a = \frac{v_2 - v_1}{\Delta t}$$

$$\Delta d = v_1\Delta t + \frac{1}{2}a\Delta t^2$$

$$\Delta d = v_2\Delta t - \frac{1}{2}a\Delta t^2$$

$$v_2^2 = v_1^2 + 2a\Delta d$$

Derivation of $\Delta\theta = \omega_1\Delta t + \frac{1}{2}\alpha\Delta t^2$

From $\alpha = \frac{\omega_2 - \omega_1}{\Delta t}$, we obtain

$$\omega_2 = \omega_1 + \alpha\Delta t$$

We substitute into

$\Delta\theta = \frac{1}{2}(\omega_2 + \omega_1)\Delta t$ for ω_2 to obtain

$$\Delta\theta = \frac{1}{2}(\omega_1 + \alpha\Delta t + \omega_1)\Delta t$$

$$\Delta\theta = \omega_1\Delta t + \frac{1}{2}\alpha\Delta t^2$$

We can derive the five equations for angular motion in a similar fashion.

From $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$, we obtain $\Delta\theta = \left(\frac{\omega_2 + \omega_1}{2}\right)\Delta t$

Our two basic angular motion equations are:

$$\Delta\theta = \frac{1}{2}(\omega_2 + \omega_1)\Delta t \quad \text{and} \quad \alpha = \frac{\omega_2 - \omega_1}{\Delta t}$$

Combining the first two equations yields the following three angular motion equations:

$$\Delta\theta = \omega_1\Delta t + \frac{1}{2}\alpha\Delta t^2$$

$$\Delta\theta = \omega_2\Delta t - \frac{1}{2}\alpha\Delta t^2$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\Delta\theta$$

The derivation of these equations is summarized in Figure 7.14.

Fig.7.14 Deriving the Equations for Angular Motion

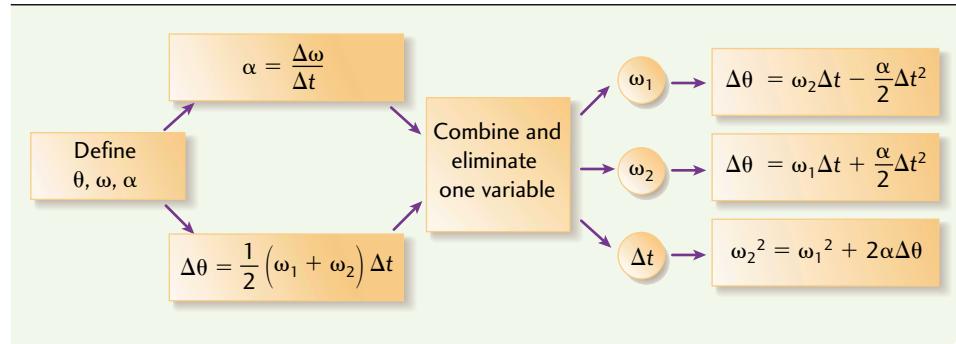
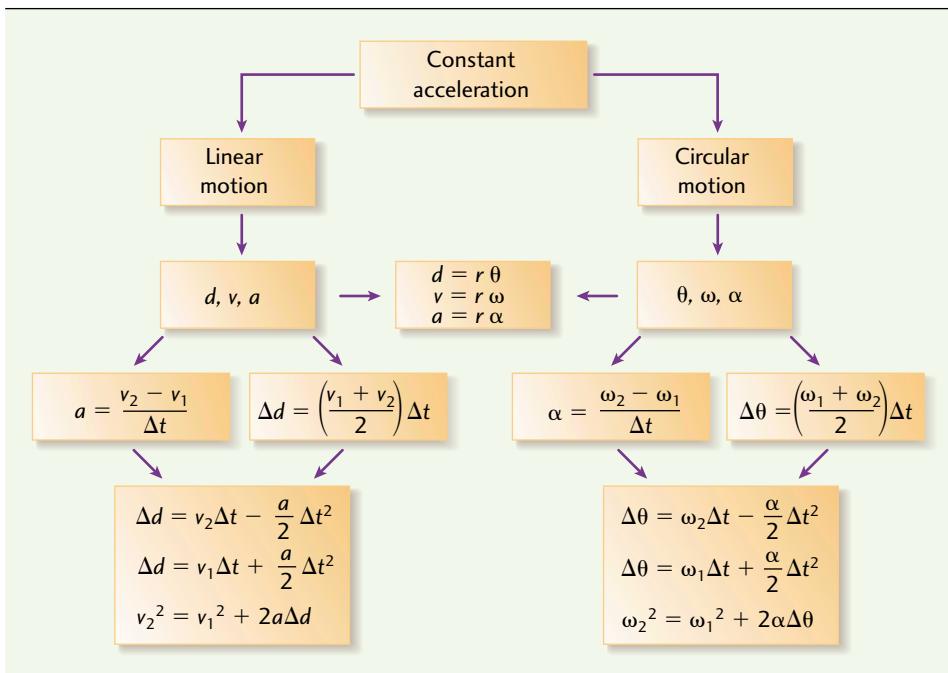


Figure 7.15 shows how the linear motion equations are related to the angular motion equations.

Fig.7.15 Relating Linear Motion to Angular Motion



EXAMPLE 7

Finding the final angular velocity of our merry-go-round

Repeat Example 3 — A merry-go-round turns through 65.0° in 3.5 s from rest. Find the final angular speed of the ride — using the equations for angular motion.

Solution and Connection to Theory

Given

$$\theta_1 = 0^\circ \quad \theta_2 = 65.0^\circ \quad \Delta t = 3.5 \text{ s} \quad \omega_1 = 0 \quad \omega_2 = ?$$

In Example 3, we converted the angles to radians ($\Delta\theta = 1.13 \text{ rad}$) to find the final angular speed. To find the final angular speed (ω_2), we use the equation

$$\Delta\theta = \frac{1}{2}(\omega_1 + \omega_2)\Delta t$$

Rearranging the equation for ω_2 and substituting the given values, we obtain

$$\omega_2 = \frac{2\Delta\theta}{\Delta t}$$

$$\omega_2 = \frac{2(1.13 \text{ rad})}{3.5 \text{ s}}$$

$$\omega_2 = 0.65 \text{ rad/s}$$

The final angular speed of the merry-go-round is 0.65 rad/s. To express the value in SI units, we can divide the answer by 2π rad/cycle to obtain 0.10 Hz.

EXAMPLE 8

Something a little more complicated

A blender turning counterclockwise at 400 rad/s is switched to high power. If the blender accelerates at 1280 rad/s² for 8.0 revolutions, find its final angular speed.

Fig. 7.16



Solution and Connection to Theory

Given

$$\Delta\theta = ? \text{ in radians (but we know it's 8.0 revolutions)}$$
$$\omega_1 = 400 \text{ rad/s} \quad \alpha = 1280 \text{ rad/s}^2 \quad \omega_2 = ?$$

First we convert 8.0 revolutions to radian measure:

$$\Delta\theta = (8.0 \text{ rev})(2\pi \text{ rad/rev})$$
$$\Delta\theta = 50 \text{ rad}$$

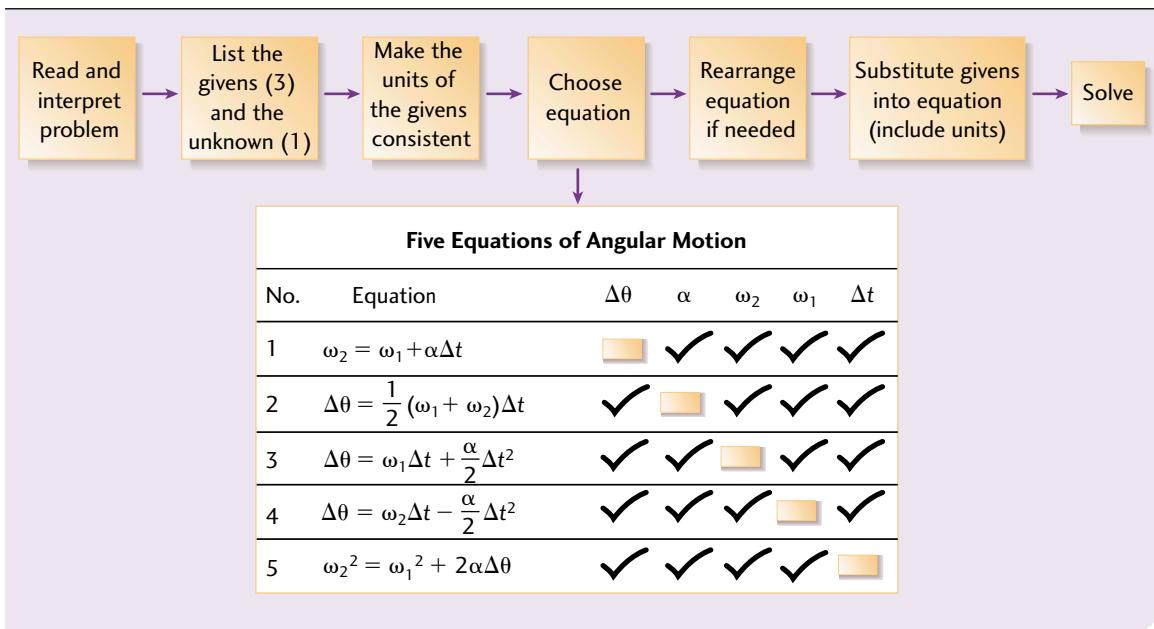
For the givens and unknown in the problem, the appropriate equation is

$$\omega_2^2 = \omega_1^2 + 2\alpha\Delta\theta$$
$$\omega_2^2 = (400 \text{ rad/s})^2 + 2(1280 \text{ rad/s}^2)(50 \text{ rad})$$
$$\omega_2^2 = 2.88 \times 10^5 \text{ rad}^2/\text{s}^2$$
$$\omega_2 = 537 \text{ rad/s}$$

We use the positive square root only because the blades don't reverse their rotation. Therefore, the final angular speed of the blender is 537 rad/s.

Figure 7.17 summarizes how to solve problems using equations.

Fig.7.17 Problem Solving using Angular Equations



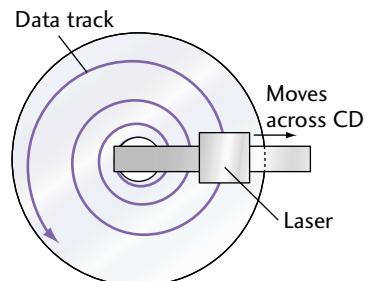
CD Players

A compact disc (CD) has spiral tracks that contain the encoded information read by the laser (see Figure 7.18). The laser, mounted on an arm running across the radius of the disc, moves from the centre of the disc to the outer rim along the arm. The laser reads the music at a constant tangential speed at any point on the laser disc; therefore, the angular speed of the disc must change in order for the information to be read at a constant rate. From the equation $v = r\omega$, we see that if v is constant, the angular speed must increase as information toward the inner part of the disc is being read.

1. a) Calculate the number of radians a CD turns in the length of time it takes to play a song 2 minutes 50 seconds long given that the CD turns on average 3.35 rev/s.
- b) Calculate the average acceleration of the CD turntable given that it takes 0.5 s for the table to reach an angular speed of 22.0 rad/s from rest.
2. a) A roulette wheel moving at 1.75 rad/s slows to a stop with an acceleration of 0.21 rad/s². Find the time it took to stop.
- b) How many radians did the roulette wheel travel in this time period?
- c) Convert the answer for b) into cycles.
- d) How much time did it take for the wheel to turn through half the number of cycles?



Fig.7.18 A spiral data track on a CD

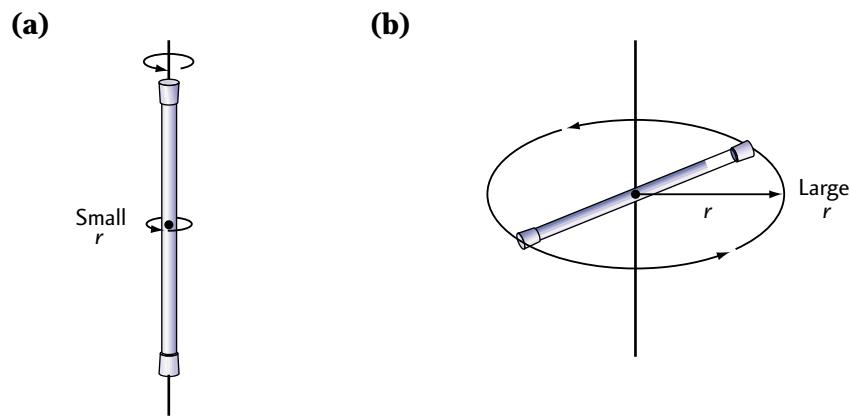


- 3. a)** A football spinning through the air in a tight spiral, with an angular speed of 16.1 rad/s, slows to 14.5 rad/s for an angular displacement of 92.2 rad. Calculate the time it takes to slow down.
b) Calculate the average acceleration of the football.

7.5 Moment of Inertia

Newton's laws also apply to spinning or rotating objects, but the concept of inertia becomes slightly more complicated. Recall that mass is a measure of an object's inertia. The more massive the object, the greater the force required to accelerate it. For a spinning object, not only is its mass important, but also its *mass distribution* about its rotational axis. The spinning batons in Figures 7.19a and 7.19b have different spin axes. The baton in Figure 7.19a spins about the axis along its length, so it has a small radius of rotation with less mass distributed about the rotation axis. The baton in Figure 7.19b spins about its centre (with the axis perpendicular to its length). It has a large radius of rotation and therefore a greater amount of mass outside the rotation axis. In both cases, the baton will continue to spin unless acted upon by an external unbalanced force (Newton's first law).

Fig. 7.19 Different axes of rotation of a baton



We now define the **moment of inertia**, I , as the angular equivalent to mass, with units $\text{kg}\cdot\text{m}^2$. Moment of inertia depends on the object's mass and its rotation axis. For a hoop rotating about a central axis,

$$I = mr^2$$

where m is the mass of the hoop and r is its radius of rotation.

Table 7.1 illustrates the moments of inertia and the corresponding equations for various common shapes. These equations were derived using calculus.

Table 7.1
Moments of Inertia of Common Shapes

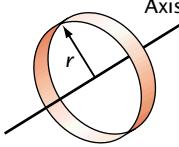
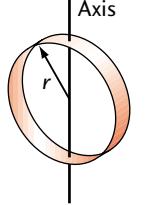
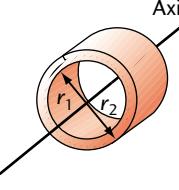
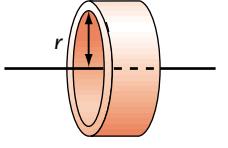
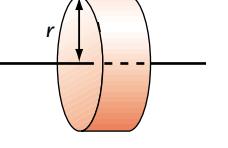
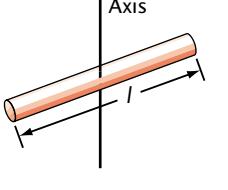
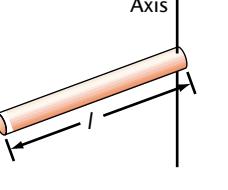
Shape	Description	Equation
Fig.7.20a 	Hoop about central axis	$I = mr^2$
Fig.7.20b 	Hoop about any diameter	$I = \frac{1}{2}mr^2$
Fig.7.20c 	Hollow cylinder (or ring) about central axis	$I = \frac{1}{2}m(r_1^2 + r_2^2)$
Fig.7.20d 	Thin-walled hollow cylinder or hoop	$I = mr^2$
Fig.7.20e 	Solid cylinder or disk	$I = \frac{1}{2}mr^2$
Fig.7.20f 	Thin rod about axis through centre perpendicular to length	$I = \frac{1}{12}ml^2$
Fig.7.20g 	Thin rod about axis through one end perpendicular to length	$I = \frac{1}{3}ml^2$

Table 7.1 (cont'd)
Moments of Inertia of Common Shapes

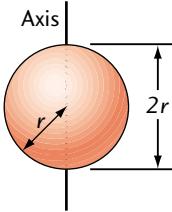
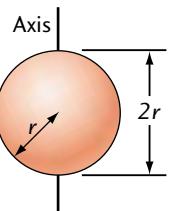
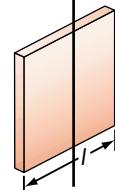
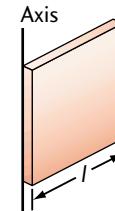
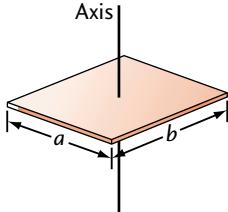
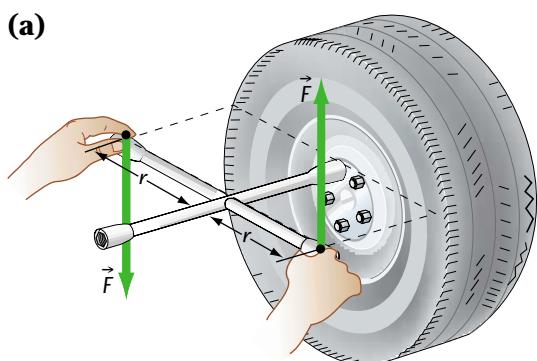
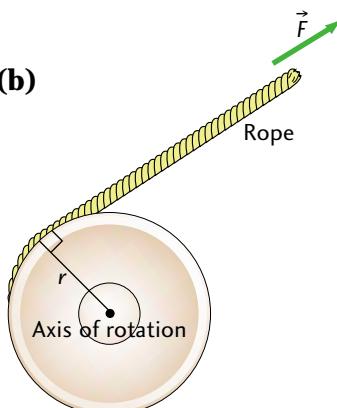
Shape	Description	Equation
Fig.7.20h 	Solid sphere about any diameter	$I = \frac{2}{5}mr^2$
Fig.7.20i 	Thin spherical shell about any diameter	$I = \frac{2}{3}mr^2$
Fig.7.20j 	Thin rectangular sheet, axis parallel to one edge and passing through centre of other edge	$I = \frac{1}{12}ml^2$
Fig.7.20k 	Thin rectangular sheet, axis along one edge	$I = \frac{1}{3}ml^2$
Fig.7.20l 	Thin rectangular sheet, perpendicular axis through centre	$I = \frac{1}{12}m(a^2 + b^2)$

Fig.7.21

(a)



(b)



$$\tau = r \times F = rF \sin \theta$$

$$\theta = 90^\circ$$

$$\text{so } \tau = rF$$

Newton's second law of motion is $\vec{F} = m\vec{a}$. In angular motion, the equivalent of force is **torque**, τ . In Chapter 3, we learned that *torque* is a turning action on a body caused by a force applied through a point relative to the object's rotation axis. Mathematically,

$$\tau = rF$$

From Section 7.3, we also know that

$$a = r\alpha$$

Substituting $\frac{\tau}{r}$ for F and $r\alpha$ for a in $F = ma$, we obtain

$$\tau = mr^2\alpha$$

But mr^2 is just the moment of inertia, I , for a hoop. The angular motion equivalent for Newton's second law becomes

$$\tau = I\alpha$$

where τ represents the net torque applied. Just as net force in Newton's second law of motion, a net torque produces a net angular acceleration.

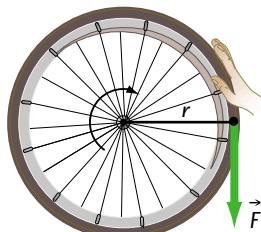
EXAMPLE 9

Calculating the net torque and moment of inertia of a bicycle wheel

A rider pushes down on the rim of her 0.60-m-diameter bicycle wheel with a force of 30 N. Find the torque applied and the moment of inertia if the wheel experiences an angular acceleration of 26.5 rad/s^2 .

The units for torque are N·m, the same units as for energy. While the torque unit remains N·m, the unit for energy becomes the joule (J).

Fig.7.22



Solution and Connection to Theory

Given

$$F = 30 \text{ N} \quad d = 0.60 \text{ m}; \text{ therefore, } r = 0.30 \text{ m} \quad \alpha = 26.5 \text{ rad/s}^2$$
$$\tau = ? \quad I = ?$$

We know $\tau = \vec{r} \times \vec{F} = rF \sin \theta$. When the angle between the force and the rotation axis is 90° , $\sin \theta = 1$; therefore,

$$\tau = (0.30 \text{ m})(30 \text{ N})$$

$$\tau = 9.0 \text{ N}\cdot\text{m}$$

$\tau = I\alpha$, so

$$I = \frac{\tau}{\alpha}$$

$$I = \frac{9.0 \text{ N}\cdot\text{m}}{26.5 \text{ rad/s}^2}$$

$$I = 0.34 \text{ kg}\cdot\text{m}^2$$

Unit Analysis for Moment of Inertia

$$\begin{aligned} \frac{\text{N}\cdot\text{m}}{\frac{\text{rad}}{\text{s}^2}} &= \frac{\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)\text{m}}{\frac{\text{rad}}{\text{s}^2}} \\ &= \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \times \frac{\text{s}^2}{\text{rad}} = \text{kg}\cdot\text{m}^2 \end{aligned}$$

The torque applied is 9.0 N·m and the moment of inertia is 0.34 kg·m².

Bonus! Since we know the moment of inertia, we can also find the mass of the wheel. If we consider the wheel's mass to be concentrated at the rim, we can assume it has the same moment of inertia as a hollow cylinder, which is mr^2 (see Table 7.1). If the wheel's radius is 0.30 m, then

$$I = mr^2$$

$$m = \frac{I}{r^2}$$

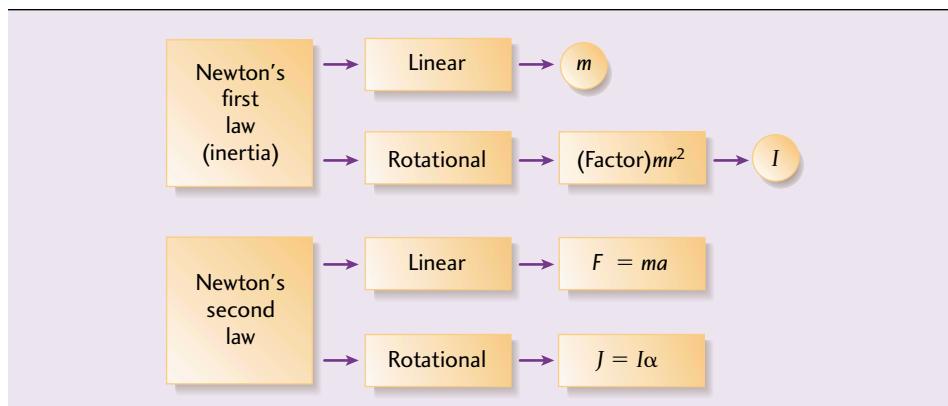
$$m = \frac{0.34 \text{ kg}\cdot\text{m}^2}{(0.30 \text{ m})^2}$$

$$m = 3.8 \text{ kg}$$

The wheel's mass is 3.8 kg.

Figure 7.23 summarizes the relationship between linear and angular variables in Newton's first and second laws of motion.

Fig.7.23 The Rotational Equivalents of Newton's First and Second Laws



Extension: The Parallel-axis Theorem

In Chapter 3, we defined the *centre of mass* (*cm*) as a point at which the entire mass of the system or body may be considered to be concentrated for the purposes of analyzing its motion. In this chapter, we may consider the centre of mass as a balance point around which an object's entire mass is equally distributed.

We learned earlier in this section that an object's moment of inertia depends on the location of its rotational axis. Thus far, all the rotational axes in the examples and problems in this chapter have passed through the object's centre of mass (see Figure 7.24).

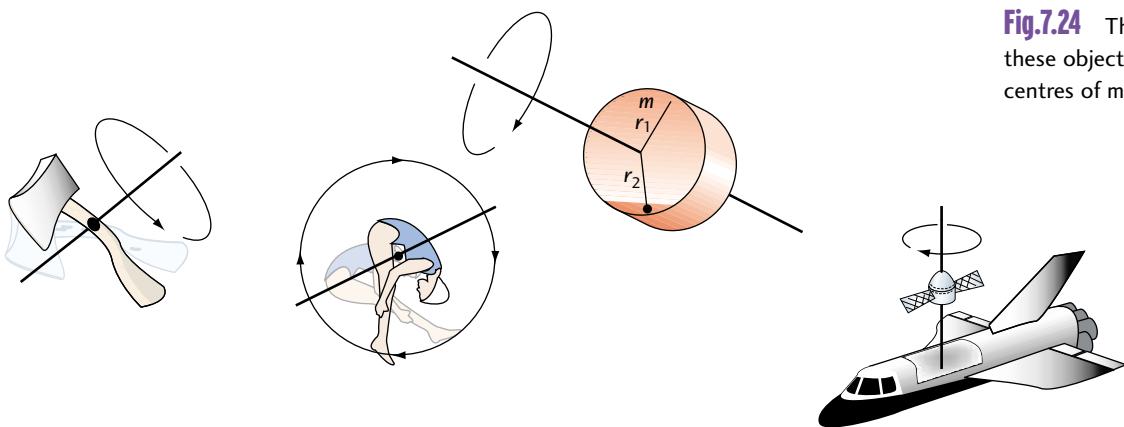


Fig.7.24 The rotational axes of these objects all pass through their centres of mass

If an object rotates about an axis that doesn't pass through its centre of mass, then we can find its **rotational inertia** or **total moment of inertia** by using the **parallel-axis theorem**:

$$I_{\text{total}} = I_{\text{cm}} + ml^2$$

where the subscript *cm* stands for the centre of mass, *m* is the mass in kilograms, and *l* is the perpendicular distance between the rotation axes and the centre of mass (see Figures 7.25a and b).

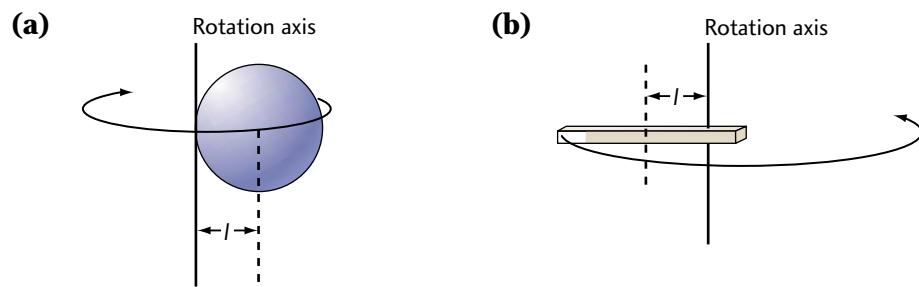


Fig.7.25 For each figure, *l* is measured from the centre of mass of each object

From Table 7.1, the moment of inertia through the centre of mass of a solid sphere is

$$I_{\text{cm}} = \frac{2}{5}mr^2$$

If the rotation axis is shifted, then the moment of inertia becomes

$$I_{\text{total}} = \frac{2}{5}mr^2 + ml^2$$

For the sphere in Figure 7.25a, the distance from the edge of the sphere to its centre is its radius, r . But this distance is also l , the distance between the centre of mass and the rotation axis; therefore, $l = r$. The final equation for the moment of inertia of this sphere is

$$I_{\text{total}} = \frac{7}{5}mr^2$$



1. State Newton's first two laws of motion, as you learned them earlier in your physics courses. Now restate them to apply to angular motion. Provide an example of each law.
2. **a)** Before the invention of CD and cassette players, there were vinyl record players, which rotated at speeds of 78 rpm, 45 rpm, and $33\frac{1}{3}$ rpm (Figure 7.26). If the moment of inertia of a turntable is $0.045 \text{ kg}\cdot\text{m}^2$, find the torque required to provide an average acceleration of -1.90 rad/s^2 .

Fig.7.26 A turntable playing a record (from the olden days)



- b)** For each angular speed, how many turns does it take for the turntable to reach its maximum speed starting from rest?
3. Find the moment of inertia of an object experiencing an angular acceleration of 12.2 rad/s^2 due to a net torque of $8.45 \text{ N}\cdot\text{m}$ applied to it.
4. A disk of radius 1.22 m and mass 5.55 kg rotates about an axle passing through its centre. If a force of 15.1 N was applied to the outside of the disk to cause it to turn, find
 - a)** the moment of inertia of the disk.
 - b)** the torque applied.
 - c)** the angular acceleration of the disk.

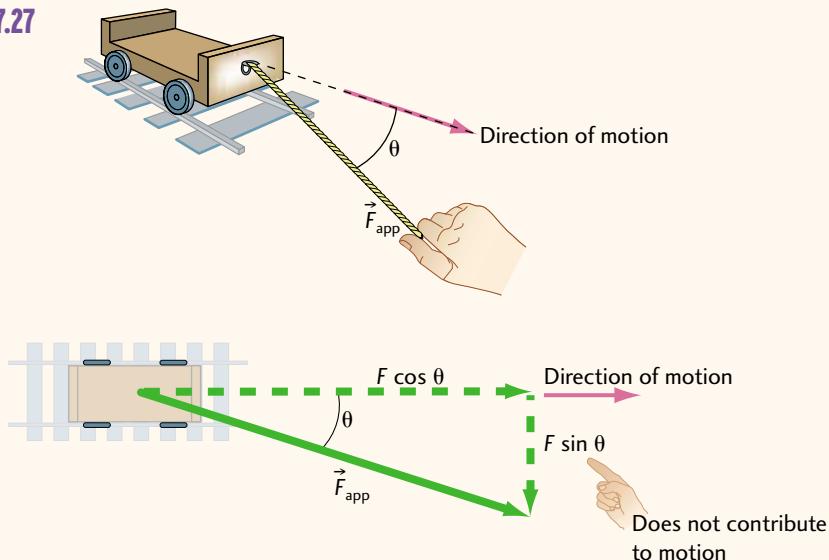
7.6 Rotational Energy

In Chapter 5, we learned that work is $W = Fd \cos \theta$. In Section 7.5, we found that the equivalent of force in angular motion is torque.

WORK

Work is done when an applied force causes an object to *move* in the direction of the force. The definition of work is $W = \vec{F} \cdot \vec{d}$. But the only component of the force that we need to consider is the component that is in the same direction as the applied force; therefore, $W = Fd \cos \theta$.

Fig. 7.27



To derive an **equation for angular work**, we start with the definition for work, $W = \vec{F} \cdot \vec{d}$. We now substitute the angular equivalents to force and distance and obtain

$$W = \left(\frac{\tau}{r}\right)(r\theta), \text{ which simplifies to}$$

$$W_R = \tau\theta$$

where W is measured in joules and θ is measured in radians.

Turning an object requires a torque. If the object rotates through an angle, the product of torque and the angle through which the object turns is the angular work done on the object. As with linear work, if the applied torque produces no turning action, then the net rotational work is zero. Thus, when you strain against that rusted nut on a wheel (Figure 7.28a) and it doesn't move, you may sweat a lot but do no work!

Fig. 7.28a The applied force cannot overcome friction and the nut doesn't turn, so no work is done

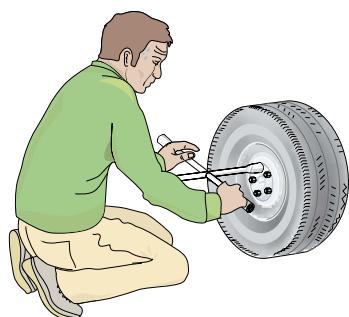
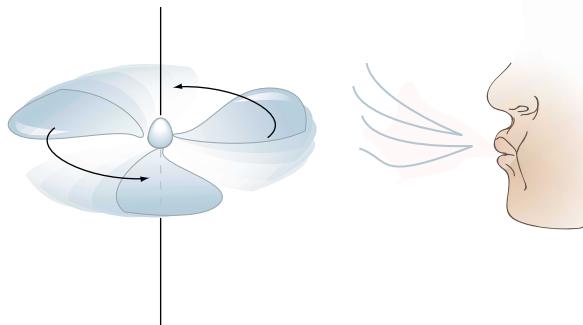


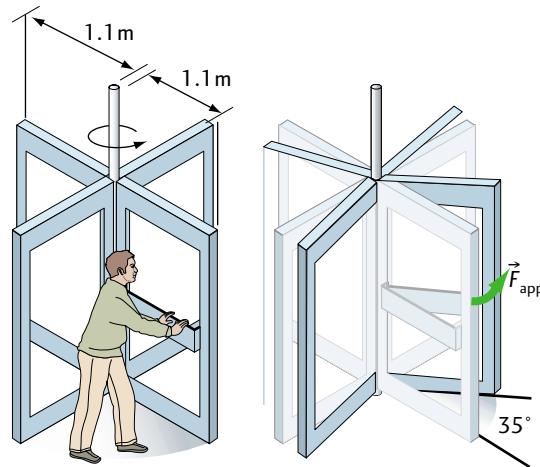
Fig.7.28b The applied force causes the blades to turn, so work is being done



EXAMPLE 10 Work done in pushing a revolving door

A revolving door of mass 360 kg with four rectangular panels is set in motion by a person pushing on one panel. If the width of one panel is 1.1 m (the distance from the centre post, which is the axis of rotation), calculate the torque on the revolving door and the work done if the door turns 35° with an angular acceleration of 0.45 rad/s^2 .

Fig.7.29



Solution and Connection to Theory

Given

$$m = 360 \text{ kg} \quad \omega = 1.1 \text{ m} \quad \alpha = 0.45 \text{ rad/s}^2$$

$$I = \frac{1}{3}m\omega^2 \text{ for one panel (see Table 7.1), so}$$

$$I = \frac{4}{3}m\omega^2 \text{ for all four panels.}$$

For a rotating object,

$$\tau = I\alpha$$

$$\tau = \frac{4}{3}(360 \text{ kg})(1.1 \text{ m})^2(0.45 \text{ rad/s}^2)$$

$$\tau = 260 \text{ N}\cdot\text{m}$$

The angle for the work done is

$$\theta = \frac{35^\circ}{57.3^\circ/\text{rad}} = 0.61 \text{ rad}$$

For angular work,

$$W_R = \tau\theta$$

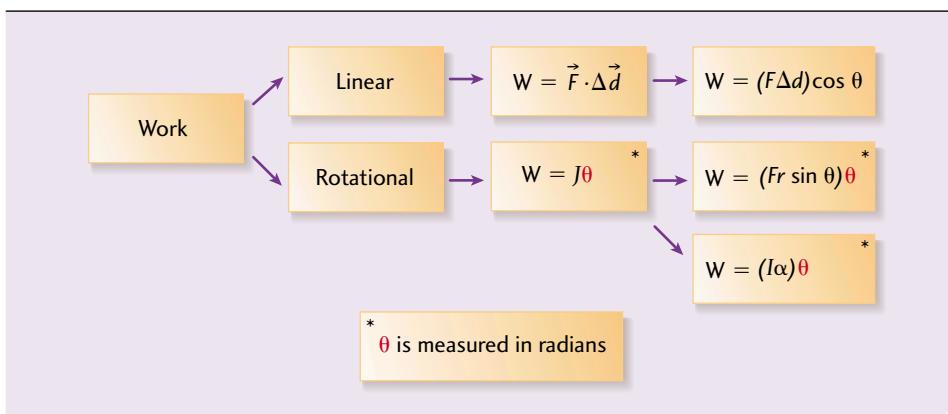
$$W_R = (260 \text{ N}\cdot\text{m})(0.61 \text{ rad})$$

$$W_R = 159 \text{ J}$$

The torque on the revolving door is 260 N·m and the work done is 159 J.

Figure 7.30 summarizes the equations for linear and rotational work.

Fig.7.30 Summary of Rotational Energy (Work)



1.
 - a) Calculate the work done in turning a nut off a wheel (one full turn) if a force of 23.1 N is applied 20 cm away from the nut.
 - b) How much work is done over 1.5 rad?
 - c) How much work is done if the nut is turned 95° ?
2.
 - a) Calculate the work done on a solid cylinder of mass 5.0 kg and radius 55.6 cm by an applied force of 12.2 N if it turns 45° .
 - b) By how much does the work in a) change if the object is a ring?



7.7 Rotational Kinetic Energy

If $E_k = \frac{1}{2}mv^2$ for linear motion, then for rotational motion,

$$E_{k_{\text{rotational}}} = \frac{1}{2}I\omega^2$$

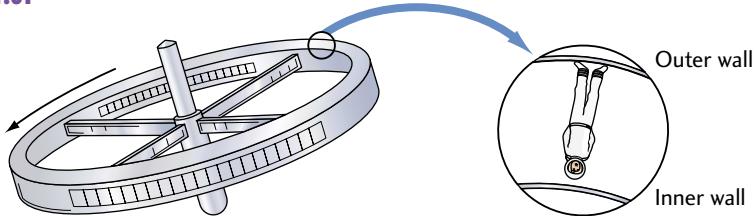
To derive the $E_{k_{\text{rotational}}}$ equation, substitute $r\omega$ for v and remember that $mr^2 = I$.

where I is the moment of inertia in $\text{kg}\cdot\text{m}^2$ and ω is the angular speed in rad/s. Rotational energy is measured in joules. *Be sure to use the correct equation for the moment of inertia for an object, which depends on its shape and rotational axis (Table 7.1)!*

EXAMPLE 11 Rotational energy in space stations

A 100-kg astronaut stands in the rim of a rotating ring-shaped space station. What is his rotational velocity if his kinetic energy is $4.51 \times 10^5 \text{ J}$ and the radius of the station is $1.5 \times 10^3 \text{ m}$?

Fig.7.31



Solution and Connection to Theory

Given

$$E = 4.51 \times 10^5 \text{ J} \quad r = 1.5 \times 10^3 \text{ m} \quad m = 100 \text{ kg}$$

Consider the astronaut to be a point mass rotating around the centre of the space station. For this case, the moment of inertia is given by

$$\begin{aligned} I &= mr^2 \\ I &= (100 \text{ kg})(1.5 \times 10^3 \text{ m})^2 \\ I &= 2.25 \times 10^8 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

We can now apply the rotational kinetic energy expression to solve for ω .

$$\begin{aligned} E_{k_{\text{rotational}}} &= \frac{1}{2}I\omega^2 \\ \omega &= \sqrt{\frac{2E}{I}} \\ \omega &= \sqrt{\frac{2(4.51 \times 10^5 \text{ J})}{2.25 \times 10^8 \text{ kg}\cdot\text{m}^2}} \\ \omega &= 0.063 \text{ rad/s} \end{aligned}$$

The astronaut's rotational velocity is 0.063 rad/s.

To get an idea of how fast the astronaut in Example 11 is moving, we use the equation $v = r\omega$:

$$v = (1.5 \times 10^3 \text{ m})(0.063 \text{ rad/s})$$

$$v = 95 \text{ m/s or } 340 \text{ km/h}$$

Compare this speed to a roller coaster ride!

The astronaut in Example 11 experiences artificial gravity due to the centripetal force of the rotating station. The normal force supplies the force that keeps the astronaut turning. The centripetal acceleration is $a_c = \frac{v^2}{r}$. If $v = 95 \text{ m/s}$ and $r = 1.5 \times 10^3 \text{ m}$, then

$$a_c = \frac{(95 \text{ m/s})^2}{1.5 \times 10^3 \text{ m}}$$

$$a_c = 6.0 \text{ m/s}^2$$

This acceleration is 61% of the acceleration due to Earth's gravity.

Fig.7.32

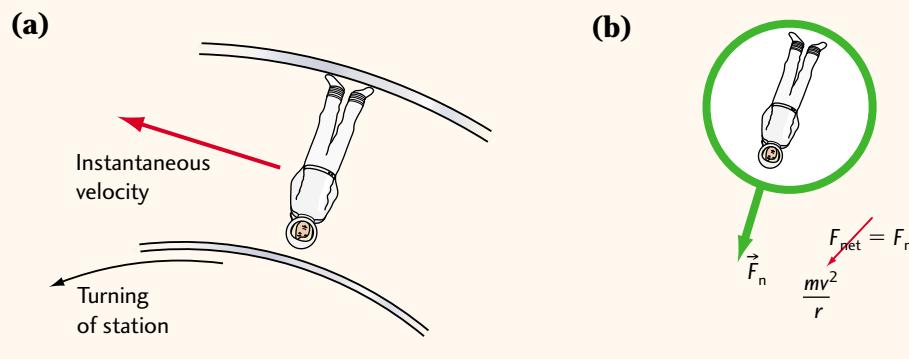
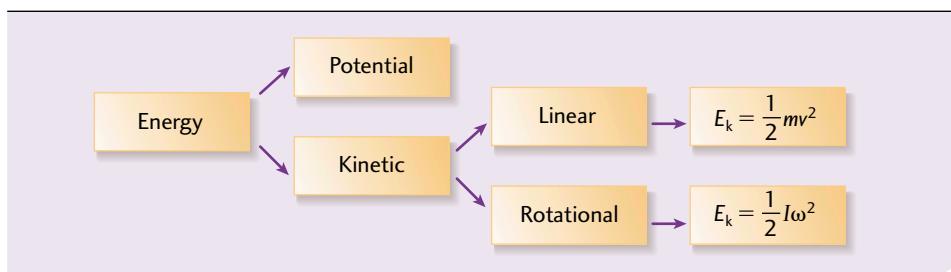


Figure 7.33 summarizes the linear and rotational equations for kinetic energy.

Fig.7.33 Summary of Linear and Rotational Kinetic Energies

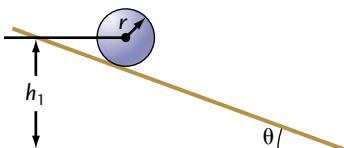




- Calculate the rotational kinetic energy of a 35.0-g ball of radius 3.5 cm rotating at 165 rad/s about its centre.
- a) Find the total rotational kinetic energy of four wheels on a car if the moment of inertia of each wheel is $0.900 \text{ kg}\cdot\text{m}^2$ and the radius of the wheel is 0.320 m. Assume each wheel turns 5.3 times per second.
b) Calculate the kinetic energy of the car if its mass is 1000 kg.

Fig. 7.34 The conservation of energy involving rotational and translational components

(a) Initially

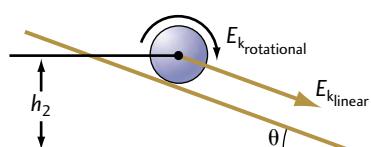


$$E_{k\text{linear}} = 0$$

$$E_{k\text{rotational}} = 0$$

$$E_p = mgh_1$$

(b) Later



$$E_{k\text{linear}} = \frac{1}{2}mv^2$$

$$E_{k\text{rotational}} = \frac{1}{2}I\omega^2$$

$$E_p = mgh_2$$

7.8 The Conservation of Energy

When an object both rotates and moves forward (translates), it possesses a combination of rotational and translational kinetic energy. These two types of energy must come from some source. If the system is closed and the object doesn't lose any energy to heat, sound, etc., then we can apply the law of conservation of energy to the system. Consider the case of a ball rolling down a hill (see Figure 7.34).

The ball at the top of the hill has no motion, but it possesses gravitational potential energy, calculated by $mg\Delta h$, which is its total energy, E_T . As the ball starts to roll down the hill, part of this energy transfers to translational kinetic energy ($\frac{1}{2}mv^2$) and part of it transfers to rotational kinetic energy ($\frac{1}{2}I\omega^2$). The total energy of the system remains the same:

$$E_{T_1} = E_{T_2}$$

where the subscripts 1 and 2 represent initial and final total energy, respectively. In the case of our ball,

Total energy (potential)		Kinetic translational energy
mgh_1	$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh_2$	
		Kinetic angular energy
		Residual potential energy

EXAMPLE 12 Rolling down a hill

A large, cylindrical duck rolls down a hill of height 8.5 m. If the speed of the duck is 10.5 m/s at the bottom of the hill, what is its angular speed there? The rather large duck has a mass of 25 kg and a body radius of 0.80 m.

Solution and Connection to Theory

Given

$$h = 8.5 \text{ m} \quad m = 25 \text{ kg} \quad v_{\text{bottom}} = v_2 = 10.5 \text{ m/s} \quad r = 0.80 \text{ m}$$

$$I = ? \quad \omega = ?$$

Fig.7.35



Using the law of conservation of energy and assuming no energy losses due to friction and air resistance,

$E_{T_1} = E_{T_2}$, which expands to

$$mgh_1 + \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgh_2$$

But $v_1 = 0$, $\omega_1 = 0$, and $h_2 = 0$; so

$$mgh_1 = \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2$$

$$\frac{1}{2}I\omega_2^2 = mgh_1 - \frac{1}{2}mv_2^2$$

From Table 7.1, for a solid cylinder,

$$I = \frac{1}{2}mr^2$$

$$I = \frac{1}{2}(25 \text{ kg})(0.80 \text{ m})^2$$

$$I = 8 \text{ kg}\cdot\text{m}^2$$

Rearranging $\frac{1}{2}I\omega_2^2 = mgh_1 - \frac{1}{2}mv_2^2$ for ω^2 and substituting,

$$\omega_2^2 = \frac{mgh_1 - \frac{1}{2}mv_2^2}{\frac{1}{2}I}$$

$$\omega_2^2 = \frac{(25 \text{ kg})(9.8 \text{ m/s}^2)(8.5 \text{ m}) - \frac{1}{2}(25 \text{ kg})(10.5 \text{ m/s})^2}{4 \text{ kg}\cdot\text{m}^2}$$

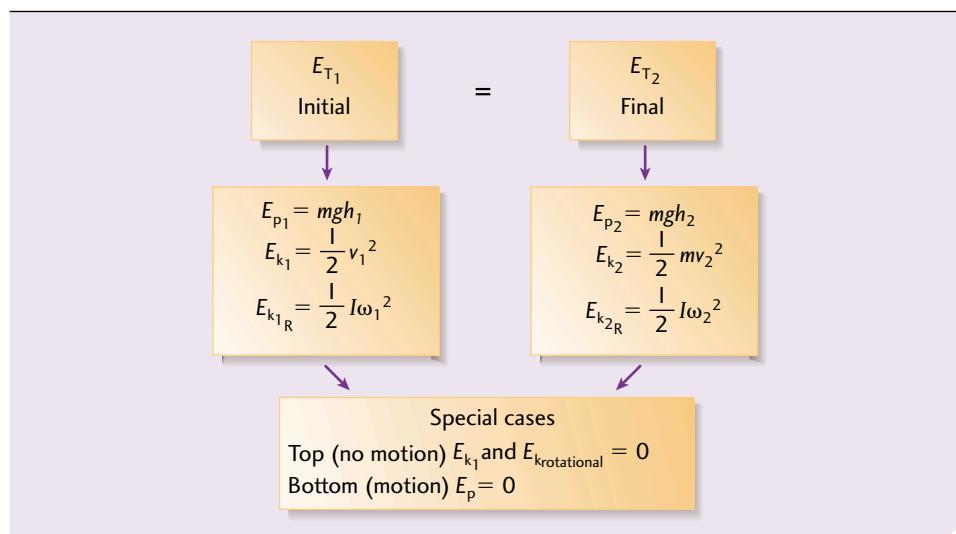
$$\omega_2 = 13.27 \text{ rad/s}$$

$$\omega_2 = 13 \text{ rad/s}$$

The duck is rotating at 13 rad/s, or 2.1 times per second. That's one dizzy duck!

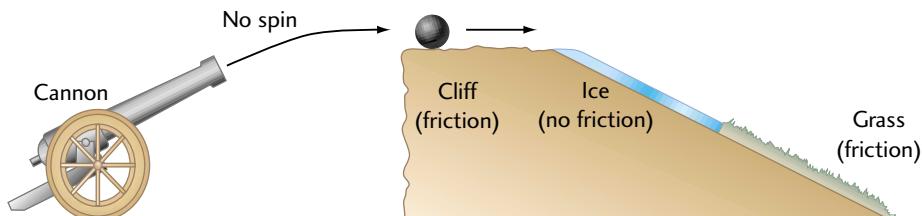
Figure 7.36 summarizes the equations for the conservation of energy for translational and rotational motion.

Fig.7.36 The Conservation of Energy



1. A car moving at 25 m/s has wheels of radius 0.320 m, each with a moment of inertia of $0.900 \text{ kg}\cdot\text{m}^2$.
 - a) Find the total rotational kinetic energy of the wheels.
 - b) Find the linear kinetic energy of the car if its mass is 1300 kg.
 - c) Find the total energy of the car.
2. A hollow cylinder, starting from rest, rolls down a 12.0-m-high incline. The cylinder has a mass of 2.2 kg and a radius of 5.6 cm. Find
 - a) the cylinder's total energy at the top of the incline.
 - b) the cylinder's gravitational potential energy halfway down the incline.
 - c) the cylinder's angular speed if its translational speed was 10.8 m/s.
3. In Figure 7.37, describe the ball's actions in terms of its various forms of energy.

Fig.7.37



7.9 Angular Momentum

In Chapter 4, we defined momentum as

$$\vec{p} = m\vec{v}$$

For angular momentum,

$$L = I\omega$$

where L represents the angular momentum in units of $\text{kg}\cdot\text{m}^2/\text{s}$, I is the moment of inertia in $\text{kg}\cdot\text{m}^2$, and ω is the angular speed in rad/s . Once again, remember that the object's shape and axis of rotation determine the equation we use for the moment of inertia (Table 7.1).

EXAMPLE 13 Different shapes with their corresponding angular momenta

Compare the angular momenta of a solid cylinder and a hollow ring, each of mass 10 kg and radius 0.52 m, if they each rotate at 3.0 rad/s.

Solution and Connection to Theory

Given

$$\begin{aligned} m &= 10 \text{ kg} & r &= 0.52 \text{ m} & \omega &= 3.0 \text{ rad/s} \\ I_{\text{ring}} &= mr^2 & I_{\text{cylinder}} &= \frac{1}{2}mr^2 \end{aligned}$$

$$L_{\text{ring}} = I\omega$$

$$L_{\text{ring}} = mr^2\omega$$

$$L_{\text{ring}} = (10 \text{ kg})(0.52 \text{ m})^2(3.0 \text{ rad/s})$$

$$L_{\text{ring}} = 8.1 \text{ kg}\cdot\text{m}^2/\text{s}$$

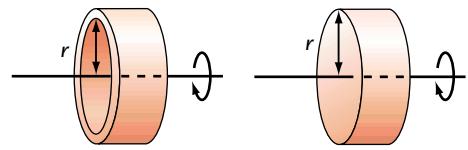
$$L_{\text{cylinder}} = I\omega = \frac{1}{2}mr^2\omega, \text{ or } \frac{1}{2}L_{\text{ring}}. \text{ Thus,}$$

$$L_{\text{cylinder}} = \frac{8.1 \text{ kg}\cdot\text{m}^2/\text{s}}{2}$$

$$L_{\text{cylinder}} = 4.1 \text{ kg}\cdot\text{m}^2/\text{s}$$

The angular momentum of the ring is greater than that of the cylinder because its mass is concentrated farther away from its axis of rotation. Thus, it takes more energy to overcome the inertia of a ring than of a cylinder.

Fig. 7.38

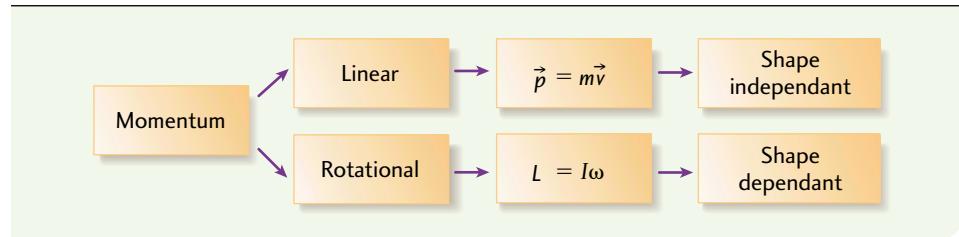


$$I = mr^2$$

$$I = \frac{1}{2}mr^2$$

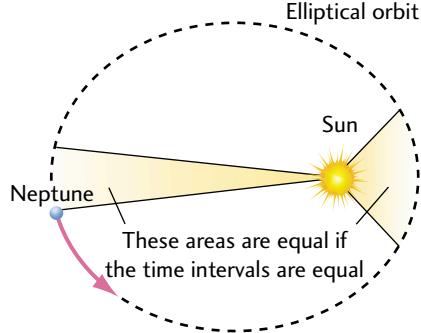
Figure 7.39 summarizes the differences between linear and angular momentum.

Fig.7.39 Momentum



1. Calculate the angular momentum of Earth rotating on its axis if its mass is 5.98×10^{24} kg and its radius is 6.38×10^6 m.
2. Calculate the angular momentum of a diver of mass 85 kg, rotating 4.5 times in 1.1 s, if his size in the tuck position is 1.8 m.
3. Neptune moves in an elliptical orbit about the Sun (see Figure 7.40). At the closest point to the Sun (the perihelion), it's moving at 5.4723 km/s at a radius of 4.4630×10^9 km. At its farthest point from the Sun (the aphelion), it's moving at 5.3833 km/s at a radius of 4.5368×10^9 km. Calculate its angular momentum at each point if its mass is 1.027×10^{26} kg.

Fig.7.40 Neptune's orbit
(from Kepler's second law)



7.10 The Conservation of Angular Momentum

In Chapter 4, we learned that linear momentum is conserved; that is, the total initial momentum equals the total final momentum. Mathematically,

$$\sum m_i v_i = \sum m_f v_f$$

where i represents the initial momenta and f represents the final momenta. Angular momentum, L , is also conserved such that

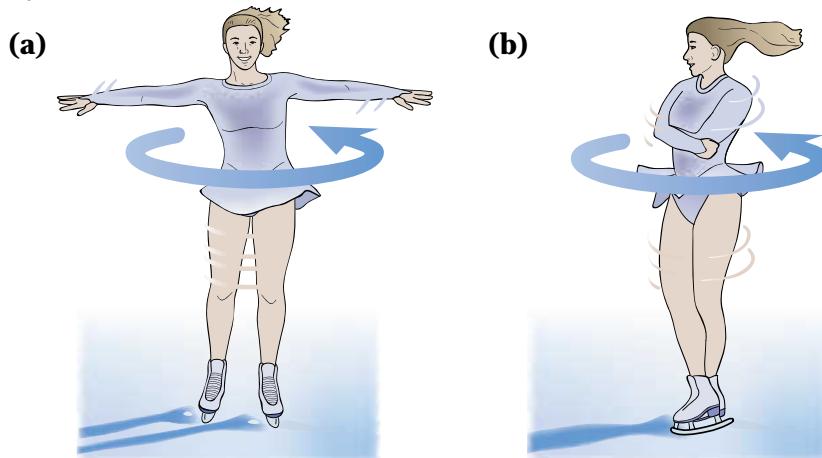
$$\sum I_i \omega_i = \sum I_f \omega_f$$

where I is the moment of inertia and ω is the angular velocity.

EXAMPLE 14 A spinning skater

A professional figure skater ends her program by spinning with her arms outstretched, then with her arms tucked in. If she is originally spinning at 1.5 rev/s with her arms outstretched, what is her angular speed after she tucks her arms in? Assume that the length of her outstretched arms from fingertip to fingertip is 2.2 m. When her arms are tucked in, the length is 50 cm.

Fig.7.41



Solution and Connection to Theory

Given

$2r_1 = 2.2 \text{ m}$ $2r_2 = 0.50 \text{ m}$; we now calculate half of the full length to obtain r .

$$r_1 = 1.1 \text{ m} \quad r_2 = 0.25 \text{ m} \quad m = ? \quad \omega_2 = ?$$

When the skater tucks her arms in, her total momentum doesn't change, but her moment of inertia does because her radius of spin has decreased. If we neglect losses of energy due to the friction of skates on ice as well as air resistance, we can write the following conservation of angular momentum statement:

$$I_1\omega_1 = I_2\omega_2$$

For the skater, $I = \Sigma mr^2$. Her mass remains constant, so as an approximation, we will let $I = mr^2$.

$$mr_1^2\omega_1 = mr_2^2\omega_2$$

Converting ω_1 to rad/s,

$$\omega_1 = (1.5 \text{ rev/s})(2\pi \text{ rad/rev})$$

$$\omega_1 = 9.4 \text{ rad/s}$$

Solving for ω_2 ,

$$\omega_2 = \frac{mr_1^2\omega_1}{mr_2^2}$$

$$\omega_2 = \frac{r_1^2\omega_1}{r_2^2}$$

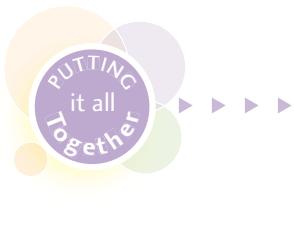
$$\omega_2 = \frac{(1.1 \text{ m})^2(9.4 \text{ rad/s})}{(0.25 \text{ m})^2}$$

$$\omega_2 = 182 \text{ rad/s}$$

$$\frac{182 \text{ rad/s}}{(2\pi \text{ rad/rev})} = 29 \text{ rev/s!}$$

The skater spins faster by bringing her arms into her body. Even though this calculation is a simplification, it clearly shows the effect of reducing the moment of inertia on the angular speed.

Fig. 7.42 A Comparison of Linear and Angular Momenta



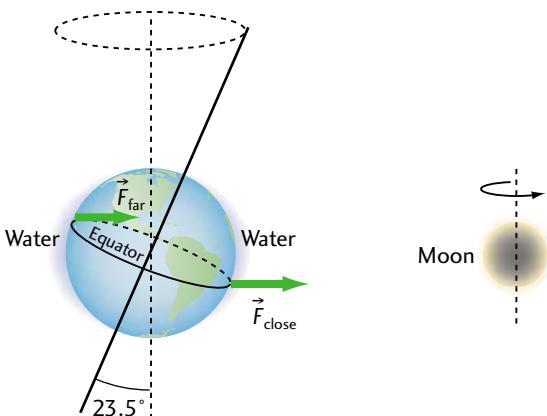
Tides and Day Length

Tides are the diurnal rising and falling of the sea. Tidal levels vary according to region. The greatest difference in water level in the world occurs in the Minas Basin of the Bay of Fundy in New Brunswick, with an impressive 15-m difference between high and low tide (see Figure 7.43a)!

Fig. 7.43a High and low tide at the Bay of Fundy



Fig. 7.43b The precession of Earth's axis due to unequal forces acting on it



Tides are caused primarily by the Moon's gravitational pull on Earth. (The Sun also contributes to this effect, but to a lesser extent.) Earth's rotation about its own axis is faster than the Moon's rotation about Earth. As Earth rotates, it takes its water with it. Because of the fluid properties of water and the effect of frictional forces between the ocean floor and the water, the bulges of water on either side of Earth become slightly asymmetric (see Figure 7.43b). The difference in mass on either side of Earth's axis due to the asymmetric distribution of water causes a net torque on the Moon, which increases the Moon's angular momentum.

1. **a)** Use the law of conservation of momentum in the Earth–Moon system to explain why the Earth day decreases because of the gravitational attraction between Earth and the Moon.
- b)** Explain what happens to the length of a month because of the effect in a). (Hint: The Moon's angular speed is increasing.)
- c)** It has been found that the length of our day is decreasing at a rate of about 20 ms per year. Dinosaurs roamed Earth about 230 million years ago. Find the length of a day in hours during their time on Earth.
2. A star's size changes over time. Our Sun spins on its axis with a period of 2.14×10^6 s and has an average radius of 6.95×10^8 m. If we hypothetically shrink the Sun's radius to 5.5 km (the size of a neutron star), calculate the new angular speed and period of rotation. (Note: This situation is purely hypothetical: the Sun doesn't possess enough mass to become a neutron star.)
3. Kepler's second law of planetary motion, which states that a planet sweeps out equal areas in equal time periods, is another example of the conservation of angular momentum. Calculate Earth's apogee speed by applying Kepler's second law to the planetary data for Earth ($m = 5.98 \times 10^{24}$ kg; **perigee** distance = 1.47×10^8 km; **apogee** distance = 1.52×10^8 km).

7.11 The Yo-yo

Fig.7.44a



Fig.7.44b Sleeping yo-yos



Energy Analysis

Before you drop the yo-yo, it has a potential energy due to gravity. When you drop the yo-yo, it converts this energy to translational kinetic and rotational kinetic energies. According to the law of conservation of energy,

$$E_{T_1} = E_{T_2}$$

$$mgh_1 = mgh_2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

At the bottom of the drop, the translational velocity reaches zero because the yo-yo stops falling. All the yo-yo's energy is angular kinetic energy:

$$mgh_1 = \frac{1}{2}I\omega^2$$

If you give the yo-yo an initial “snap” (with your wrist) instead of just letting it drop, the yo-yo spins and falls faster because you add extra energy in the form of kinetic translational and kinetic rotational energies. The conservation of energy equation then becomes

$$mgh_1 + \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 = mgh_2 + \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2$$

Force Analysis

Figure 7.45b is a free-body diagram of the yo-yo, with outer radius R and axle radius r .

From Figure 7.45b,

$$\vec{F}_{\text{net}} = \vec{F}_{\text{T}} + \vec{F}_{\text{g}}$$

So

$$F_{\text{net}} = F_T - F_g$$

$$ma = F_T - mg$$

The torque statement equivalent to the F_{net} statement is

$$\tau_{\text{net}} = F_T r$$

so

$$I\alpha = F_T r$$

The tangential and angular accelerations are related by the equation

$$a = r\alpha$$

When we solve for α and substitute into the equation $I\alpha = F_T r$, we obtain

$$F_T = \frac{Ia}{r^2} \quad (\text{eq. 1})$$

From the FBD statement $ma = F_T - mg$,

$$F_T = ma + mg \quad (\text{eq. 2})$$

Combining equations 1 and 2, we obtain

$$ma + mg = \frac{Ia}{r^2}$$

We rearrange this equation for the yo-yo's acceleration and obtain

$$a \left(\frac{I}{mr^2} - 1 \right) = g \quad \text{or} \quad a = \frac{g}{\left(\frac{I}{mr^2} - 1 \right)}$$

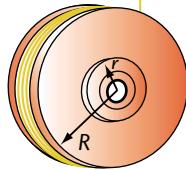
- From the equation $a = \frac{g}{\left(\frac{I}{mr^2} - 1 \right)}$, what conditions must be present

for the yo-yo to roll down the string with a large acceleration?

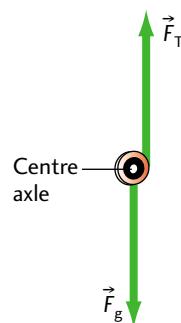
- Describe the types of energy involved when
 - you spin the yo-yo downward.
 - the yo-yo comes back to your hand.
 - you swing the yo-yo out and up and it comes back to your hand.
- Calculate the acceleration of a yo-yo with axle radius 7.0 mm and disk radius 4.0 cm.
- The yo-yo analysis has been simplified. List the approximations and how they qualitatively affect the yo-yo's motion.

Fig.7.45

(a)



(b)





Gyroscopic Action — A Case of Angular Momentum

What do a football spiraling down the field and the missile guidance system in a launched missile have in common? The answer lies in the **gyroscope**, or spinning mass (Figure STSE.7.2). The greater the mass of the gyroscope, the greater its angular or rotational inertia. A gyroscope maintains its angular orientation with respect to external coordinates. Instead of tipping over, it moves in a circle in a fixed direction given by its original spin axis.

Rotational inertia is the rotational counterpart to Newton's first law of motion: *an object will continue to spin in a given direction unless acted upon by an external unbalanced torque*. If a force is applied to the gyroscope, it will start to move at right angles to the applied force. This motion is called precession.

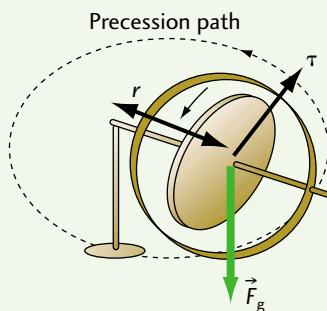
Fig.STSE.7.1 The gyroscopic action of a spiraling football and a launched missile



Fig.STSE.7.2a



Fig.STSE.7.2b If the gyroscope didn't spin, it would fall over



When a football flies through the air, the air hitting the surface of the ball would normally cause the football to tumble (rotate end over end), thus increasing the air resistance as the broad sides of the ball encounter the wall of air. If the thrower puts a spin in the ball (hence giving the ball angular momentum), the ball behaves much like a gyroscope. Instead of tumbling when encountering the resistive force of the air, it wobbles slightly, thus keeping its original trajectory. The tip of the ball remains pointed in the direction of motion, reducing air resistance and increasing the distance travelled.

The same principle is used in navigation. When a wheel spins at a high speed, its angular momentum and rotational inertia increase. By fixing the wheel in a set of circular frames, called **gimbals**, which themselves move around the spinning gyroscope, the orientation of the gyroscope remains fixed (no net torque). The airplane, missile, ship, submarine, or spacecraft can now orient relative to this unchanging axis (see Figure STSE.7.3).

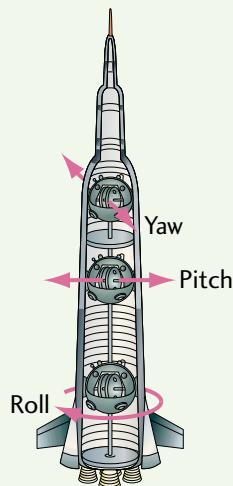
Gyrostabilizers are devices used in ships and planes to reduce the side-to-side rolling effect by creating a stable direction due to their rotation axis. If the ship pitches or rolls, the gyroscope feels a net force acting on it due to the change in position of its axis of rotation relative to Earth's gravity. However, because it is spinning, it resists this movement, causing the ship attached to it to resist the "urge" to roll.

Earth spinning on its axis also acts much like a gyroscope. The north axis continues to point to the North Star in spite of the gravitational pull of the Sun and Moon (although there is a slight precession due to the unequal distribution of Earth's mass).

Fig. STSE.7.3a A gyroscope from a ship's navigation system



Fig. STSE.7.3b The three gyroscopes provide reference frames for the three possible motions (yaw, pitch, and roll) of the rocket



Design a Study of Societal Impact

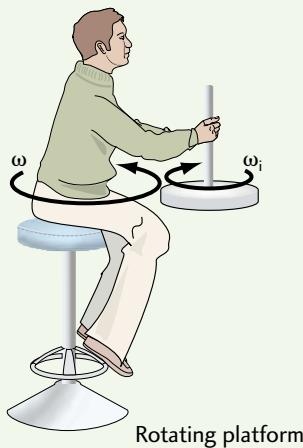
Gyroscopic action is behind today's high-tech missile guidance systems. Research the history of guidance systems, from the first early versions used in torpedoes in the 1890s, to the gyroscope invented by Hermann Anschultz Kampfe in 1908 and refined by Elmer A. Sperry in 1911, to the laser-guided gyros of today. Discuss how their invention has changed the way wars are fought. Also discuss the pros and cons of pilotless planes during war and peace.

Design an Activity to Evaluate

Attach a series of masses to the circumference of a bicycle wheel. See if you can design it in such a way as to allow for the possibility of changing the amount of mass you put on so that you can investigate the moment of inertia. Add a set of freely sliding masses to the spokes. Research the history of racing bicycle wheels and use your modified wheel to verify design changes made to the racing wheel. How is gyroscopic action exhibited when riding a bicycle?

Build a Structure

Fig. STSE.7.4 A rotating platform



Build a rotating turntable, large enough to support a person (see Figure STSE.7.4). The structure must minimize friction and be able to rotate freely. Try using recycled materials only, such as old Rollerblade wheels, broken Formica-top tables, or old desktops. Also, modify an old bicycle wheel so that it can be held with handles while spinning. Use this setup to study the moment of inertia and conservation of angular momentum. Relate your findings to gyroscopic properties.

You should be able to*Understand Basic Concepts:*

- Explain radian measure and relate it to degree measure.
- Describe in qualitative terms the angular equivalents to distance, speed, acceleration, mass, force, energy, and momentum.
- Write variables for angular distance, speed, acceleration, mass, force, energy, and momentum.
- Write equations for angular distance, speed, acceleration, mass, force, energy, and momentum.
- Calculate various aspects of angular motion using the angular equations.
- Relate tangential variables to angular ones qualitatively and quantitatively.
- Describe the equivalent angular laws of motion (Newton's three laws, conservation of energy, and momentum).
- Use the angular conservation laws to solve problems where you previously used linear variables (apogee/perigee of orbits, centripetal acceleration, and tangential speeds of spinning objects).
- Use conservation laws to solve problems involving mixed motion (ball rolling down a hill).

Develop Skills of Inquiry and Communication:

- Perform experiments to verify aspects of angular motion.
- Develop extensions to current labs and create demonstrations to verify the conservation laws.
- Discuss how problems can be solved in different ways, depending on your reference frame and associated variables.
- Describe the interconnection between linear and rotational variables.
- Describe everyday events in terms of angular variables.
- Describe the gyroscopic principle qualitatively and relate it to everyday events.

Relate Science to Technology, Society, and the Environment:

- Explain how gyroscopic principles are used in guidance systems.
- Explain how rotational principles are used in the toy industry.
- Analyze the motion of a toy, such as a yo-yo, in terms of linear and rotational motion.
- Describe aspects of appliances that involve rotational motion.
- Explain how a compact disc system reads information.
- Analyze how various sports use the principles of rotational motion (with and without equipment).

Equations

$$s = r\theta$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$v = r\omega$$

$$a = r\alpha$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$a = r\omega^2$$

$$\alpha = \frac{\omega_2 - \omega_1}{\Delta t}$$

$$\Delta\theta = \frac{1}{2}(\omega_1 + \omega_2)\Delta t$$

$$\Delta\theta = \omega_1\Delta t + \frac{1}{2}\alpha\Delta t^2$$

$$\Delta\theta = \omega_2\Delta t - \frac{1}{2}\alpha\Delta t^2$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\Delta\theta$$

$$\tau = \vec{r} \times \vec{F} = rF \sin \theta$$

$$\tau = I\alpha$$

$$I_{\text{total}} = I_{\text{cm}} + ml^2$$

$$W_R = \tau\theta$$

$$E_{k,\text{rotational}} = \frac{1}{2}I\omega^2$$

$$L = I\omega$$

$$\sum I_i\omega_i = \sum I_f\omega_f$$

Conceptual Questions

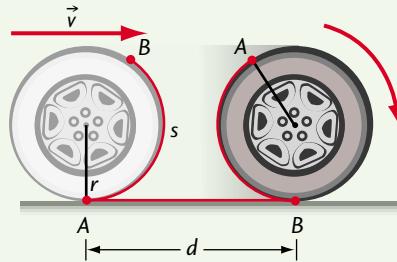
- On the surface of Earth, do all objects have the same angular velocity? Do they all have the same tangential velocity? Explain your answer.
- A differential on a car allows the wheels on the inside of a turn to rotate at a different angular speed than the wheels on the outside of a turn when rounding a corner. Why is this mechanism necessary?
- If the CN Tower was located at the equator, where would its tangential speed be the greatest? Explain your answer.
- Does tire size (radius) make a difference in the energy (the gas consumption rate) required to move a car?
- a)** If an odometer is calibrated to a certain tire size because the distance is converted from an angular to a linear measurement of the tire's motion, does changing the size of the tire affect the odometer reading?
b) Is the speedometer reading affected?
- Our definition for linear work is $F\Delta d$. Compare it to $W_{\text{rotational}} = \tau\Delta\theta$. For circular motion, one of these variables goes to zero. Compare the two in light of this statement.
- When a diver enters the water after performing a series of somersaults in a tuck position, she appears to enter the water straight (for a perfect dive). Does her entry position violate the law of conservation of angular momentum?
- In Figure 7.46, all the riders on the swing ride have the same angular velocity. Are the forces acting on all the riders the same given that their radii of turn are different? Explain your answer.

Fig.7.46



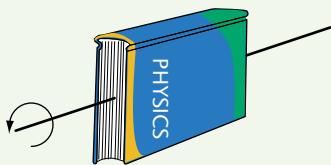
- In 1986, while flying by Uranus, *Voyager 2* set itself into an unwanted rotation each time the tape recorder turned on high speed. To counteract this effect, thrusters had to be fired each time. Use conservation of angular momentum to explain this effect.
- Which object reaches the ground first, starting at rest at the top of a frictionless incline:
 - a solid cylinder or a hollow one (same-sized objects with same masses)?
 - a solid cylinder or a rectangular solid?
 Does adding friction change the problem?
- Explain why rotational motion increases stability for projectiles such as rugby balls, footballs, and bullets.
- Given the wheel in Figure 7.47, what is the ratio between the distance a wheel moves linearly (as measured from the centre of the wheel) and the arc length along the outer part of the wheel?

Fig.7.47



- 13.** What are the various possible rotation axes for this textbook? Rank the moments of inertia.

Fig. 7.48 One possible rotation axis



- 14.** For a planet in orbit around the Sun, is there a torque exerted on the planet? Is angular momentum conserved?
- 15.** Why is it easier to balance on a moving bike than on a stationary one?
- 16.** Why do motorcycles rotate up in mid-air if the rear wheel is caused to turn faster?

Problems

7.2 A Primer on Radian Measure

- 17.** Convert the following angles to radians.

- a)** 1°
- b)** 90°
- c)** 220°
- d)** 459°
- e)** 1200°

- 18.** Convert the following measurements to radians.

- a)** 15.3 revolutions
- b)** $\frac{3}{4}$ of a turn
- c)** the motion of an hour hand in 4.4 h
- d)** Earth's rotation in 28.5 h

- 19.** Convert the following radians to degrees.

- a)** 0 rad
- b)** $\frac{2\pi}{3}$ rad
- c)** 20π rad
- d)** 466.6 rad

- 20.** How many cycles are in

- a)** 3.5 rad ?
- b)** $\pi \text{ rad}$?
- c)** 50° ?
- d)** 450° ?

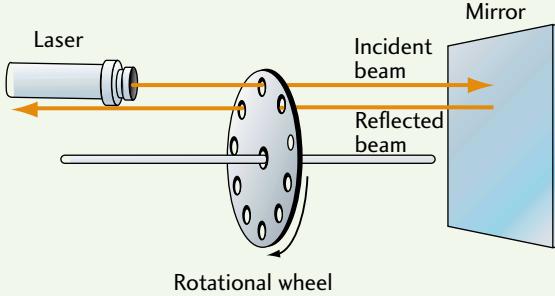
- 21.** How far does a person travel on a circular track of radius 40 m if he goes
- a)** 2π rad?
 - b)** 6.7π rad?
 - c)** 124° ?
 - d)** 560° ?

7.3 Angular Velocity and Acceleration

- 22.**
 - a)** An object rotates 15 times in 3.5 s counter-clockwise. Calculate the number of radians it rotated.
 - b)** Calculate the average angular speed.
 - c)** What happens to the answer in part b) if the object rotates in the other direction?
- 23.** A Ferris wheel completes four cycles in 26 s. What is its average angular speed in radians per second?
- 24.**
 - a)** A baseball pitcher can pitch a ball spinning at 1700 rev/min. Calculate the angular speed of the ball.
 - b)** If the ball travels for 0.56 s, find the number of radians that the ball turns.
- 25.**
 - a)** A centrifuge, used to condition astronauts to various g forces, speeds up from rest to 2.55 rad/s in 115 s. Find its angular acceleration.
 - b)** What is the frequency in hertz of the centrifuge?
- 26.** A turntable playing old 45s (vinyl singles) rotates at 45 rpm (revolutions per minute). If it slows down to a stop in 22.5 s, find its angular acceleration.
- 27.** A flywheel rotating at 18.0 rad/s slows to a stop in 22.0 s. Find
- a)** its angular acceleration.
 - b)** the number of radians it turns before stopping.
 - c)** the number of cycles it completes in this time.
 - d)** the angular speed after 8.7 s.

- 28.** A disk with angular acceleration 0.95 rad/s^2 starts with an angular velocity of -1.2 rad/s . How fast is it going after
- 0.30 s ?
 - 1.26 s ?
 - 13.5 s ?
- 29.** A fisherman uses a reel with a spool of diameter 5.6 cm to reel in a fish at 12 cm/s . What is the angular speed of the reel?
- 30.** Calculate the angular centripetal acceleration of a wheel that is 0.50 m in radius, rotating at a constant 3.5 rev/s .
- 31.** A space station, $2.50 \times 10^3 \text{ m}$ in diameter, produces an artificial gravity with an acceleration of 7.98 m/s^2 . Find
- the station's tangential speed.
 - the station's angular acceleration in rad/s^2 .
 - the number of revolutions that the station makes in 24 h .
 - the arc distance that a point on the station's rim travels in 45 minutes .
- 32. a)** A disk with evenly spaced holes along the edge turns such that light, which travels through one hole, reflects off a distant mirror and returns through the adjacent hole on the wheel. If the distance to the mirror is 10.0 km and the speed of light is $3.0 \times 10^8 \text{ m/s}$, find the angular speed of the disk that has 360 evenly spaced holes and a radius of 0.80 m (see Figure 7.49).
b) What is the tangential speed for a point on the edge of the wheel in Figure 7.49?

Fig. 7.49



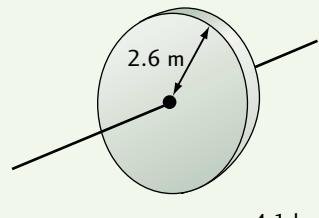
- 33.** Two cars are moving along a circular track of radius 40 m . If they start from opposite sides of the track and move in opposite directions, each with an angular speed of 0.13 rad/s , how long will it take for them to meet?
- 34.** How long will it take the two cars in problem 33 to meet if one car's angular speed is 1.6 times greater than the other car's?
- 7.4 The Five Angular Equations of Motion**
- 35.** A variable speed drill has an initial angular speed of 4.2 rad/s . By pressing the trigger, you accelerate the drill to a new speed. If the angular acceleration is 1.80 rad/s^2 and you held the trigger for 2.8 s , find
- the drill's final angular speed.
 - the angular displacement.
- 36.** The blades of a ceiling fan are spinning counterclockwise at 190 rad/s . If the blades' angular speed is changed to 80 rad/s clockwise in 6.4 s , find
- the angular acceleration.
 - the angular displacement in radians.
 - the angular displacement in degrees.
 - the time when the blades came to a momentary rest before rotating in the opposite direction.
- 37.** A wheel with a constant angular acceleration of 3.8 rad/s^2 in 3.5 s rotates through 110 rad . Find
- the wheel's initial angular speed.
 - the wheel's final angular speed.
 - Check the value of the acceleration by using another formula and the values you have calculated.
 - Check the value of the angular displacement using two different methods.
- 38.** A wheel rotating on an axis with friction present slows from an initial rate of 400 rev/min through 10 complete turns in 1.2 s . Find

- a)** the wheel's angular displacement in radians.
b) the wheel's final angular speed.
c) the angular rate of acceleration.
- 39.** A dentist's drill bit rotates through 2.0×10^4 rad. If the drill rotating at 3.5×10^3 rad/s increased its speed to 2.5×10^4 rad/s, find
a) the time it took to accelerate.
b) the angular acceleration.
- 40.** The dentist's drill rotates 2.0×10^4 rad while changing its angular speed from 3.5×10^3 rad/s to 2.5×10^4 rad/s. Find the time it takes for the drill to reach an angular speed of 3.5×10^4 rad/s from rest.
- 41.** A wheel accelerates at a rate of 2.3 rad/s^2 to a speed of 15 rad/s in 3.4 s. Find
a) the wheel's angular displacement.
b) its initial angular speed.
- 42.** The planets travel at slower tangential speeds as they move farther from the Sun. Find the time it would take Earth to catch Mars if they are separated by 30° , with Earth behind Mars. (Note: It takes Earth 3.16×10^7 s and Mars 5.94×10^7 s to go around the Sun once.)
- 43.** John, who is trying to catch up with Jane, is moving at 0.380 rad/s around a circular track. If John is 25° behind Jane, who is running at 0.400 rad/s, and he starts to accelerate at 0.080 rad/s^2 , how long will it take him to overtake her? (Hint: This problem is a two-body problem.)

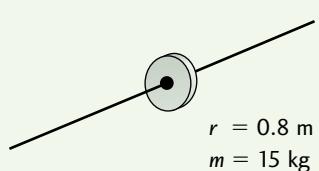
7.5 Moment of Inertia

- 44.** Rank the shapes in Figure 7.50 according to their rotational inertias. Assume rotation about the axle.

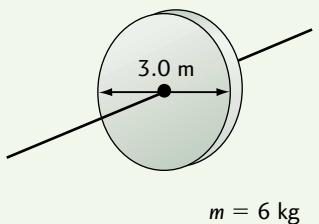
Fig.7.50



(a)



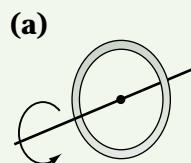
(b)



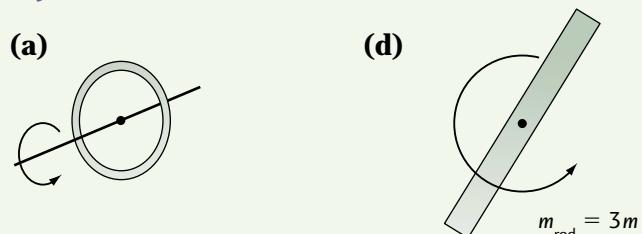
(c)

- 45.** Rank the shapes in Figure 7.51 according to their rotational inertias. Assume that all shapes have the same mass (m) and radius (r), unless otherwise stated.

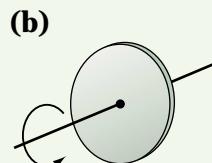
Fig.7.51



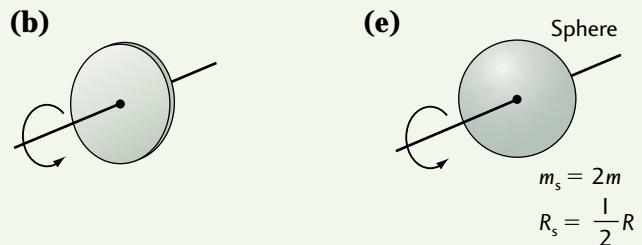
(a)



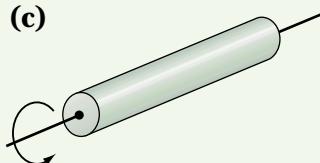
(d)



(b)



(e)



(c)

- 46.** A steel solid cylinder used in a steel mill has a mass of 4200 kg and a radius of 0.3 m. Find its moment of inertia.
- 47.** For a constant mass of 3.5 kg, find the moment of inertia for
- a hoop of radius 21 cm, rotating about an axis through the centre of the hoop, but not touching the hoop.
 - a solid cylinder of length 5.0 m and radius 21 cm, rotating about the cylinder axis.
 - a solid sphere of diameter 50 cm.
 - a hoop of radius 50 cm, rotating about a diameter.
- 48.** A wheel in the shape of a uniform disk with mass 1.4 kg has a radius of 12 cm. If it is rotating at 60 times a second, find
- its moment of inertia.
 - its angular velocity.
- 49.** A thick ring with a mass of 10.0 kg has an interior diameter of 54 cm. If the exterior diameter is 1.4 times larger, find its moment of inertia.
- 50.** A hollow sphere of radius 1.5 m and mass 2.0 kg is rotating at 200 rpm. Find
- its moment of inertia.
 - its angular speed.
 - its moment of inertia if the sphere is solid.

7.6–7.7 Rotational Energy and Rotational Kinetic Energy

- 51.** Calculate the work needed to stop a 20-kg wheel of radius 0.9 m rotating at 12.3 rev/s.
- 52.** A cylindrical satellite, launched from a space shuttle, is set spinning at 1.40 rad/s. Its mass is 1450 kg with a diameter of 1.35 m. Calculate
- the rotational inertia of the satellite.
 - its rotational kinetic energy.
 - the tangential speed of a point on the exterior of the satellite.
 - the number of turns it makes in 6.5 s.

- 53.**
 - Calculate Earth's rotational energy, assuming it's a perfect sphere of radius 6.38×10^6 m.
 - What is a person's tangential speed at the equator?
- 54.** An ion with a mass of 8.30×10^{-25} kg moves in a circular path in a cyclotron of radius 3.5 m. If it completes 1000 cycles in 1.0 s, find
- the moment of inertia.
 - the angular speed.
 - the kinetic energy.
- 55.** An electron of mass 9.11×10^{-31} kg moves in a circular orbit around a nucleus of mass 1.67×10^{-27} kg. If the radius of orbit is 5.0×10^{-11} m and the angular momentum is 1.05×10^{-34} kg·m²/s, find
- its moment of inertia.
 - its angular speed.
 - its angular kinetic energy.

7.8 The Conservation of Energy

- 56.** A solid cylinder of radius 20 cm is released from a 2.5-m-high incline. If it rolls down without losing any energy to friction, find
- the cylinder's velocity at the bottom of the incline.
 - the angular speed of the cylinder at the bottom of the incline.
- 57.** Repeat problem 56 for a hollow cylinder.
- 58.** For a sphere of mass m and radius r , derive the equation $v = \sqrt{\frac{10}{7}gh}$ for the speed of the sphere at the bottom of an incline of height h .
- 59.** A 2.8-m rod standing on its end is allowed to fall. The falling tip traces an arc. Find the speed of the tip when it hits the floor.

7.9 Angular Momentum

60. A bowling ball with mass 3.9 kg and radius of 13 cm rotates at 150 rad/s on an axis through the centre of the ball. Find
- the ball's moment of inertia.
 - the ball's angular momentum.
61. A disk with mass 2.4 kg and radius of 30 cm starts from rest and accelerates to 250 rad/s in 3.5 s. Find
- its moment of inertia.
 - its change in angular speed.
 - its change in angular momentum.
 - its angular acceleration.
 - the applied torque needed to cause this acceleration.
62. A knife thrown during a circus act completes an integral number of spins in flight before sticking into the wall behind the target person. If the knife makes 3.0 rotations over a distance of 4.5 m and has a moment of inertia of $1.50 \times 10^{-3} \text{ kg}\cdot\text{m}^2$, find
- the time it takes to reach the target if the knife was thrown with an initial speed of 17.0 m/s.
 - the angular speed of the knife.
 - the angular momentum of the knife.
63. A uniform rod of length 2.5 m, radius 1.0 cm, and mass 3.2 kg rotates 28 times in 13 s clockwise about an axis through its centre along its length. Find
- its moment of inertia.
 - its angular momentum.
64. Repeat problem 63 for a rod spinning counterclockwise about an axis through its centre and perpendicular to the rod.
65. For the data in problem 63, find the total moment of inertia (rotational inertia) for the rod if it was rotating about an axis 0.5 m from one end.

7.10 The Conservation of Angular Momentum

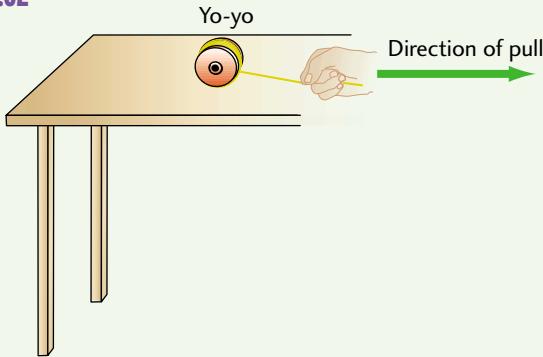
66. Superman is rotating on a turntable at 6.85 rad/s with his hands at his sides. If Superman extends his hands, his angular speed becomes 4.40 rad/s. Find the factor by which Superman's moment of inertia has changed.
67. If the value of the rotational inertia changes to $\frac{1}{2}$ of its original value, by how much does the final angular speed increase or decrease?
68. **a)** A satellite's orientation is altered using a motor mounted parallel to the probe's axis. If the motor's rotational inertia is $1.5 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ and the satellite's rotational inertia is $8.5 \text{ kg}\cdot\text{m}^2$, find the angular speed of the motor if it causes the satellite to rotate at 10 rad/s.
b) How many degrees does the satellite rotate in one second?
c) How many degrees does the motor rotate in one second?
d) How many times does the motor rotate to cause the satellite to rotate 45° ?
69. **a)** A toy railway track, mounted on a rotating platform of radius 4.3 m and mass 600 kg, rotates at 6.4 rad/s counterclockwise. If a train with cars is added to the platform from rest and has a mass of 35 kg, what is the final angular speed of the platform and train? Assume the train circles the rim of the platform.
b) If the train is running in the same direction as the platform with an angular speed of 3.1 rad/s, what is the final speed of the platform and train?
c) If the train is running at 6.4 rad/s in the opposite direction as the platform, find the final speed of the train and platform.

- 70.** A wheel of mass 30 kg and radius 1.5 m is rotating clockwise about a shaft at 12 rad/s. A second wheel of radius 1.0 m and mass 20 kg is suddenly coupled to the first wheel.
- Find the final angular speed of the combination of wheels.
 - If the second wheel rotates at 12 rad/s in the same direction as the first wheel, find the angular speed of the wheel combination.
 - If the second wheel rotates at 12 rad/s in the opposite direction as the first wheel, find the angular speed of the wheel combination.
 - Find the angular speed necessary to stop the whole system from turning.
- 71.** A 40-kg duck (wow!) walks from the outside to the inside of a rotating circular table of mass 100 kg. If the rotational inertia of the table is $250 \text{ kg}\cdot\text{m}^2$ and the duck moves from a radius of 2.5 m to 1.5 m, find the final angular speed of the table if it rotates at 2.0 rad/s at the moment the duck starts to move.

7.11 The Yo-yo

- 72.** A yo-yo rests on a level surface with friction. If you give the yo-yo a horizontal pull (see Figure 7.52), describe what happens in terms of its motion, forces, and torque.

Fig. 7.52



- 73.** A yo-yo has a rotational inertia of $8.50 \times 10^{-5} \text{ kg}\cdot\text{m}^2$ and a mass of 135 g. It has a central axis of radius 3.0 mm and a 110-cm-long string. If the yo-yo rolls from rest down to the bottom of the string, assuming the string has zero thickness, find
- the acceleration of the yo-yo.
 - the time taken to reach the bottom.
 - the linear speed at the bottom of the string.
 - the angular speed at the bottom of the string.
 - the linear kinetic energy at the bottom.
 - the angular kinetic energy at the bottom.
 - the total energy at the top of the string.
- 74.** Repeat problem 73 for a yo-yo with an initial speed down of 1.0 m/s.

Purpose

To find the moment of inertia by determining the angular acceleration and torque

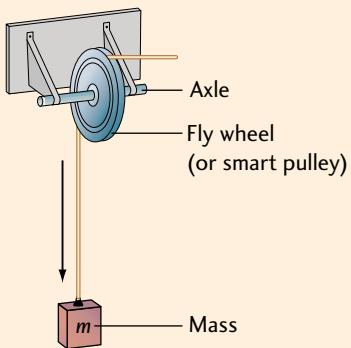
Equipment

Known mass with string

Tickertape apparatus or photo-gate pulley apparatus or programmable calculator sensor

Wheel or pulley

Fig. Lab.7.1 Use tickertape or photo gates to obtain speed



Procedure

Finding the Angular Acceleration

- Set up the wheel so that it can freely rotate on the side of the table, as illustrated in Figure Lab.7.1.
- Wrap a length of string around the wheel and attach a mass to the end of it.
- Set up the measuring system you are using.
 - Tickertape:** Attach the tickertape to the mass. Position the clacker so that the tape can move freely. You may wish to use large retort stands to hold the clacker vertically, or have a group member hold it.
 - Photo-gate pulley:** Set up the photo-gate pulley to produce a position-time graph for the falling mass.
 - Programmable calculator:** Place the sensor directly above the mass using a

retort stand clamped to the table. Make sure it's high enough so that the reading lies in the sensitive range of the instrument. Set the calculator up so that it records displacements and times.

- Drop the mass from the table. Make sure you are not recording data during or after the mass hits the floor.
- Repeat the drop 5 to 10 times.
- Measure the radius (r) of the wheel (centre to inner rim, where the string touches the wheel) and its mass.

Data

Use the following data, depending on the measuring system you used.

- Tickertape:** period of clacker; number of spaces between dots; length from first dot to last dot; radius of wheel; mass in kg.
- Photo-gate pulley:** printout of the $d-t$ graph; radius of wheel; mass in kg.
- Programmable calculator:** printout of the $d-t$ graph; radius of wheel; mass in kg.

Calculations

Part 1: Angular Acceleration

- Find the linear acceleration of the system.
 - Tickertape:** Use the equation $a = \frac{2\Delta d}{\Delta t^2}$, where $\Delta t = (\text{number of spaces})(\text{period of clacker})$
 - Photo-gate pulley** and
 - Programmable calculator:** Find the slope of the best-fit line on the $v-t$ graph generated by the system.
- Average your acceleration values. Find the standard deviation of the mean (see Appendix B).
- Multiply the value of the average acceleration you obtained by the radius of the wheel. This number is the angular acceleration of the wheel.

Part 2: Moment of Inertia

- Calculate the moment of inertia for each of the trials you did by using the equation $I = \frac{m(g - a)r^2}{a}$.
- Calculate the moment of inertia using the equation $I = mr^2$.
- Calculate the percent deviation between the two sets of values from steps 1 and 2.

Analysis

- Derive the equation $I = \frac{m(g - a)r^2}{a}$ from an FBD and F_{net} statement for the falling mass, and the torque applied to the wheel,

$$\tau = rF_T = I\alpha.$$

(Hint: From your F_{net} statement, set F_{net} equal to ma , then rearrange the equation to solve for F_T , the tension force. Use the definitions $rF_T = \tau = I\alpha$ and $a = r\alpha$.)

Discussion

- How well do the values for moment of inertia calculated from the experiment and from the equation $I = mr^2$ agree within the percent deviation?

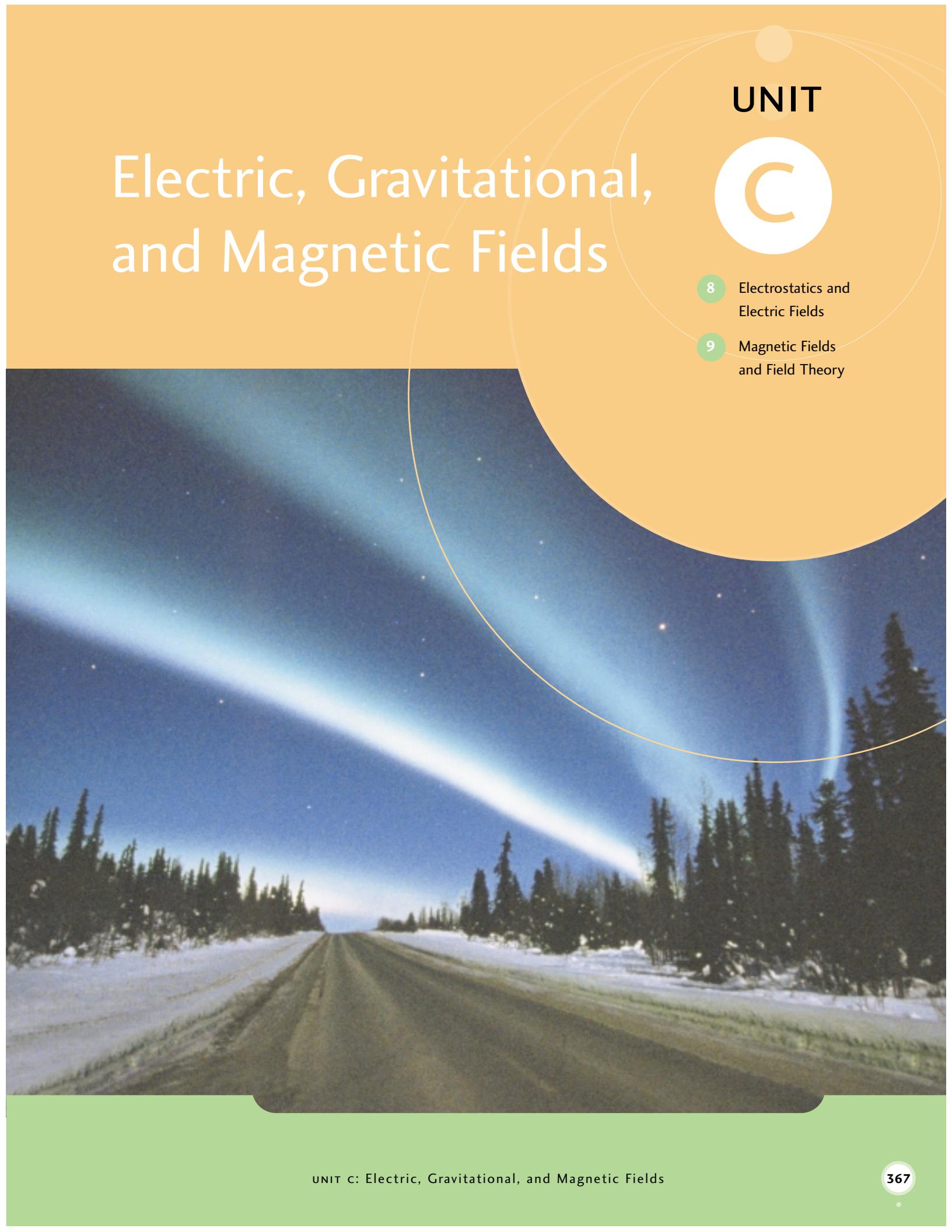
- What role does the mass of the string play in the experiment?
- What role does friction have on the value of the moment of inertia?
- What other factors affect your results?

Conclusion

State your results for the moment of inertia of a ring.

Extension

- Design and perform an experiment to determine the moment of inertia of different shapes (rings, solids, cylinders, etc.).
- Design and perform an experiment to determine the effect of radius on the moment of inertia.
- Design and perform an experiment to determine the effect of mass on the moment of inertia (the mass of a rotating object).
- Design and perform an experiment to determine how the moment of inertia affects the linear acceleration of a system.



UNIT

C

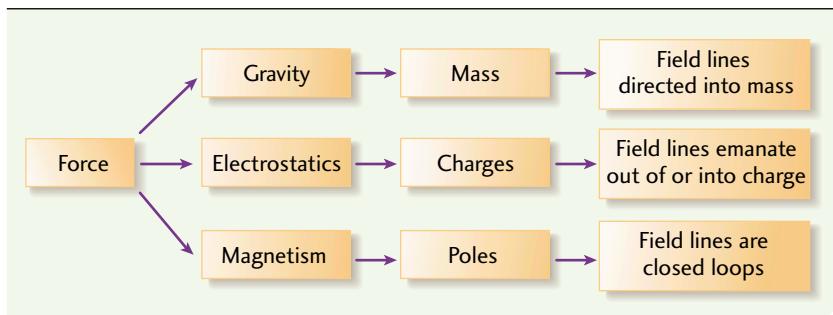
Electric, Gravitational, and Magnetic Fields

- 8 Electrostatics and Electric Fields
- 9 Magnetic Fields and Field Theory

Thus far in this text, we have explained events in terms of free-body diagrams, kinematics equations based on observation, and Newton's laws. In the momentum and energy unit, the interaction of an object and its environment was explained in terms of kinematics and dynamics. But the study of the interaction of forces with objects was omitted.

Contact forces, such as a pull or a push acting directly on an object, are easy to visualize. But gravitational, electric, and magnetic forces can influence objects without direct contact. These forces vary in strength with distance and only affect objects with specific traits that respond to these forces. For example, objects having mass can influence other objects having mass without contact.

An event such as diving off a cliff into a bay of water could not be explained properly until Michael Faraday (1791–1867) solved the riddle of how a force can influence an object over a distance. He introduced the idea of a *field*. Fields surround objects. Mass has an associated gravitational field, positive charges and negative charges have fields emanating outward and inward, respectively, and magnetic fields can be mapped using bar magnets and iron filings. In this unit, we will study how field-creating objects affect the motions of particles in their fields. We will compare and contrast the various types of fields, and investigate the impact of field theory on the development of new technologies and on the advancement of scientific theories.



Timeline: The History of Electric, Gravitational, and Magnetic Fields

Thales of Miletus studied magnets and attraction to rubbed amber.

Robert Norman used a magnetic compass needle to show that Earth is magnetic.

Stephen Gray studied the conduction of electricity.

Luigi Galvani discovered electricity in animals.

Hans Christian Oersted discovered that an electric current could deflect a magnetic needle.

James Joule and Hermann von Helmholtz discovered that electricity is a form of energy.

Michael Faraday established a relationship between electromagnetism and light.

580 BC

1581

1729

1820

1827

1840

1845

-600

-400

1550

1600

1700

1800

1900

2000

1600

John Mitchell published theories of magnetic induction and the inverse square law for magnetic fields.

Michael Faraday built the first electric motor.

1750

1821

1831

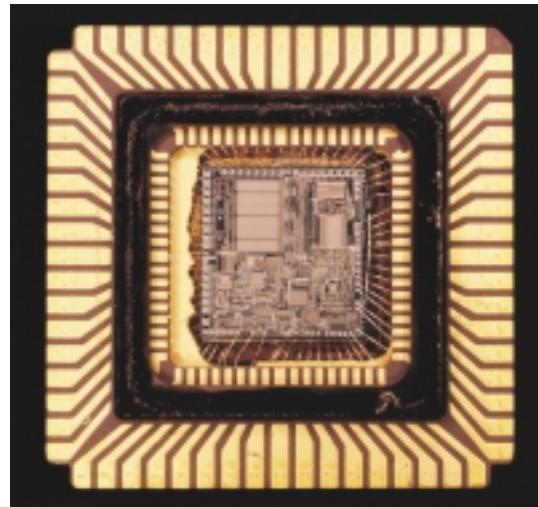
1840s

1864

William Gilbert studied static electricity and magnetism.

Gustav Kirchhoff formulated his voltage and current laws for electric circuits.

James Clerk Maxwell showed theoretically that light is a transverse electromagnetic wave. Derived an expression for the speed of light.



Heinrich Rudolf Hertz verified the existence of long-wave electromagnetic radiation.

1887

Robert Millikan measured the charge on an electron.

1909

Albert Einstein said, "God does not play dice with the universe" as an objection to the random behaviour of subatomic particles proposed by quantum theory.

1926

Ernest Rutherford measured radioactive half-life.

1900

Guglielmo Marconi received signals at Telegraph Hill in St. John's, Newfoundland, transmitted from Cornwall, England.

1901

Werner Heisenberg published the uncertainty principle.

1926

Albert Prebus and James Hillier, graduate students at the University of Toronto, designed and built the first North American electron microscope and used the wave nature of particles to extend the resolution of microscopes.

1937

James Chadwick measured the mass of a neutron.

Atomic bombs were dropped on Hiroshima and Nagasaki, Japan.

1945

The Manhattan Project began development of the atomic bomb.

2000

8

Electrostatics and Electric Fields

Chapter Outline

8.1 Electrostatic Forces and Force Fields

8.2 The Basis of Electric Charge — The Atom

8.3 Electric Charge Transfer

8.4 Coulomb's Law

8.5 Fields and Field-mapping Point Charges

8.6 Field Strength

8.7 Electric Potential and Electric Potential Energy

8.8 Movement of Charged Particles in a Field — The Conservation of Energy

8.9 The Electric Field Strength of a Parallel-plate Apparatus

 Electric Double-layer Capacitors

LAB 8.1 The Millikan Experiment

LAB 8.2 Mapping Electric Fields



By the end of this chapter, you will be able to

- define the law of electric charges and apply it to the mapping of electric fields around charge distributions
- apply Coulomb's law to various electric field situations, and compare and contrast this law with Newton's universal law of gravitation
- apply the law of conservation of energy to charged particles moving in electric fields, including that of a parallel plate apparatus

8.1 Electrostatic Forces and Force Fields

Do you realize that as you sit reading this book, your body isn't really touching the chair? There are strong forces at work, preventing the atoms and molecules that make up the chair and the clothes on your body from directly contacting one another. These forces are *repulsion* forces. When clothes just pulled from a clothes dryer stick together, similar forces are at work, causing your socks to cling. These forces are *attraction* forces. The force responsible for repulsion and attraction is called the electrostatic force. In biology and chemistry, **electrostatic forces** are responsible for the chemical bonds that link atoms and molecules in living and non-living matter.

Fig. 8.1 The force of attraction between charges



Like gravitational and magnetic forces, the electrostatic force is an example of a force that acts at a distance.

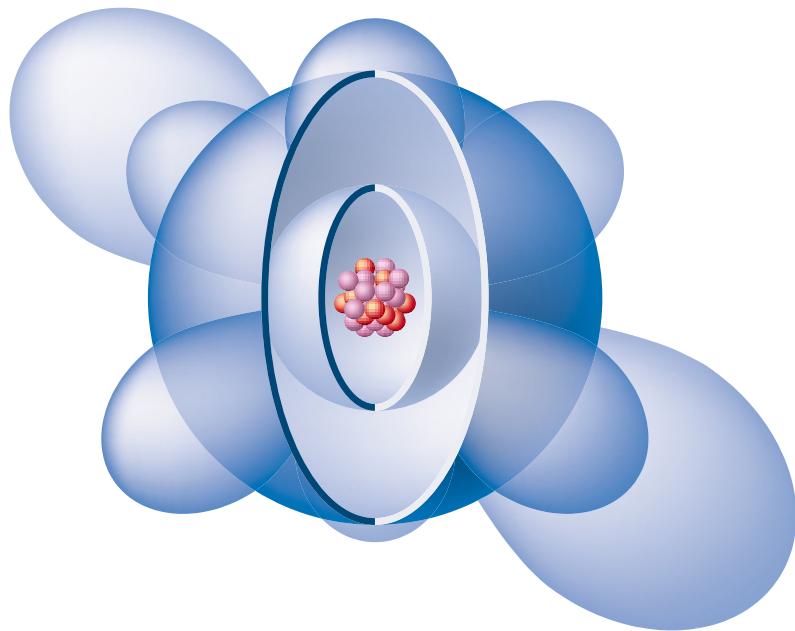
Even though the balloon and the wall in Figure 8.1 appear to be touching, they are in fact sitting a microscopic distance apart. In this chapter and the next, we will discuss the concept of *force at a distance* in our study of field theory. We will draw parallels between electrostatic, gravitational, and magnetic forces.

8.2 The Basis of Electric Charge — The Atom

All matter is made of many incredibly small particles called **atoms**. The Greek philosopher Democritus postulated the existence of atoms in the fourth century BC. He reasoned that if you cut an object into smaller and smaller pieces, you will eventually cut it down to the smallest possible piece. “Atom” means “uncuttable.”

Figure 8.2 illustrates a simplified version of the current model we have of an atom, which is based on a combination of the nuclear model proposed by Ernest Rutherford and a quantum-mechanical electron model.

Fig.8.2 The current **Bohr-Rutherford** model of the atom consists of protons and neutrons in the nucleus surrounded by probability clouds of electrons. The structure of the atom will be discussed in greater detail in Unit E.



The atom is made of three types of components, each with their own electric state or **charge**. The positively charged **protons** are clustered together in the atomic nucleus with neutral (no electric charge) particles called **neutrons**. The negatively charged **electrons** move in a region at some distance around the nucleus. The transfer of these mobile electrons between objects is the basis of electrostatic current.

Charge was observed in the fifth century BC by the Greek philosopher Thales of Miletus who noticed that amber rubbed with fur attracted pith and pieces of feathers. The Greek name for amber is “elektron.” Benjamin Franklin noted the oppositely charged natures of amber and fur and designated amber as negative. When charged objects are placed near each other, they experience forces of attraction or repulsion. These two different charges follow the **law of electric charges**.

Law of Electric Charges:

Opposite charges attract each other.

Like charges repel each other.

Charged objects attract some neutral objects.

An electroscope is a simple device that allows us to detect charge and its transfer. Figure 8.3 illustrates a typical electroscope and how it detects charge. The signs (+) and (−) are used to represent positive and negative

charge, respectively. When two opposite charges unite on the same object, they cancel each other's electric character. If one type of charge is in excess, then the object is left with an overall similar charge. For example, if positive charges are in excess, then the overall charge of the object is positive. The positively charged protons don't move from the atom's nucleus, so the amount of positive charge it carries remains constant. *Any change in the overall charge on an object is due to a deficit or excess of electrons.* As illustrated in Figure 8.4, excess electrons lead to a negative charge, a deficit of electrons results in a positive charge, and an equal number of protons and electrons leaves the object neutral.

Fig.8.4 The overall charge on an object is the arithmetic sum of positive and negative charges on it

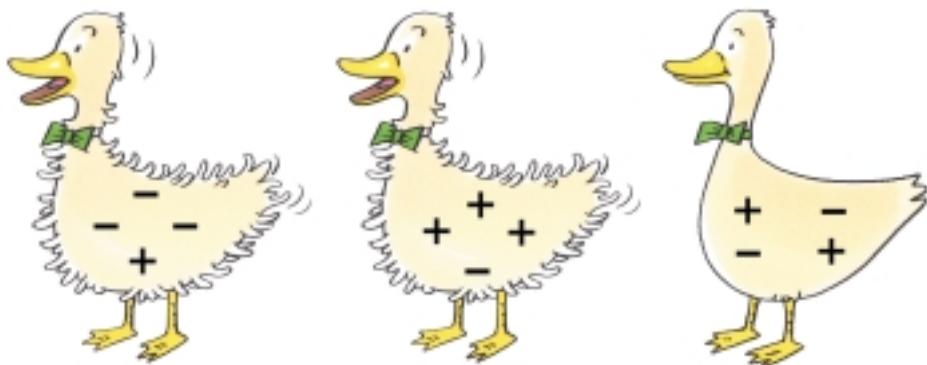
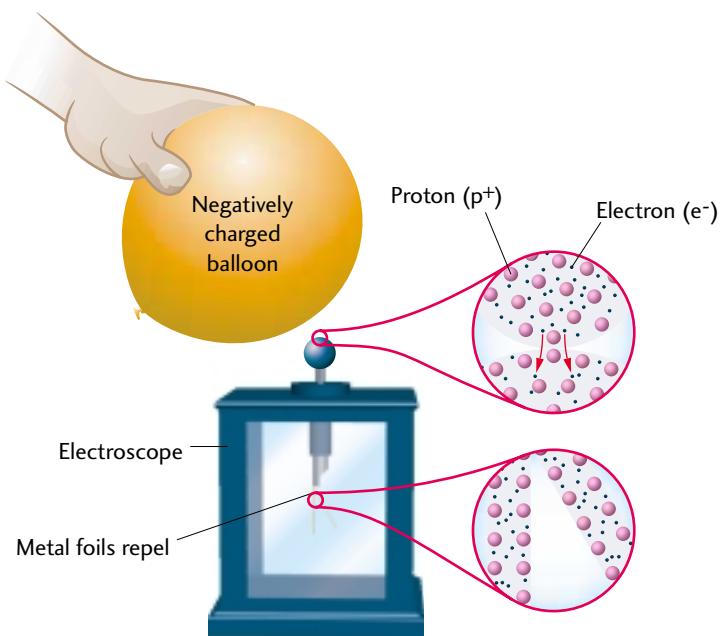


Fig.8.3 An electroscope detects the movement of negatively charged electrons as they are forced into or out of the lower leaves by a charged object held close to it



8.3 Electric Charge Transfer

The transfer of charge from one object to another is caused by a large difference in the number of unbalanced electrons in the two objects, and hence in their overall charge. When object A has excess electrons, the electrons experience a force of repulsion that pushes them as far away as possible from one another. A deficit of electrons in a nearby object B attracts the excess electrons from object A. Both conditions (an electron excess in one object and an electron deficit in another object) compound the forces that cause charge transfer. During a static electric shock or a lightning strike, electrical charges are transferred between oppositely charged regions.

Fig.8.5 Lightning is a rapid transfer of charge

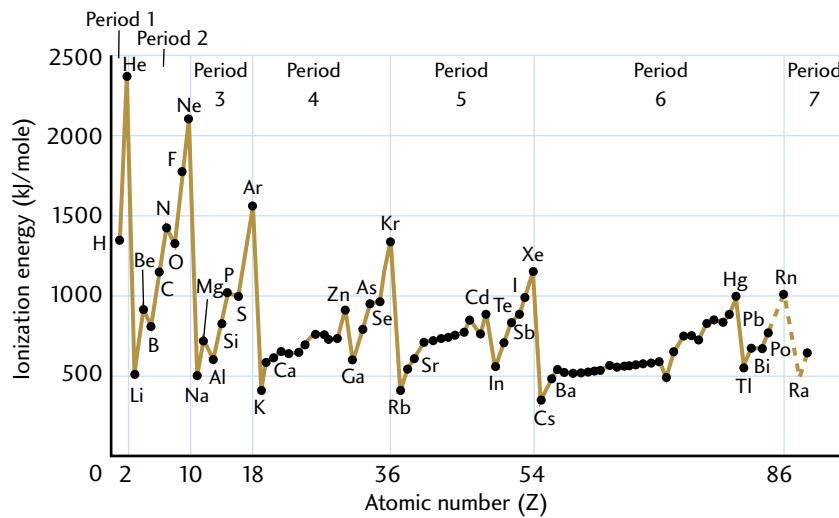


Objects are charged in three general ways: by **friction**, by **contact**, and by **induction**. Friction energy produces an initial charge from two neutral objects, creating the necessary repulsive or attractive forces to allow electrons to flow from one object to the other when the objects are in close proximity. Once an object has been given an initial charge by friction, it can be used to charge other objects, either by contact or by induction.

Charging by Friction

Friction is the simplest method of charge transfer between any two objects. The direction and amount of charge transfer depends on two complementary properties of the atoms in an object: ionization energy and electron affinity. Each atomic nucleus has a certain degree of attraction toward the mobile electrons in its outermost orbitals. The stronger the force of attraction

Fig.8.6 The first ionization energy for various elements



between a nucleus and its outer electrons, the greater the energy required to remove these electrons from the atom. When electrons are removed from an atom, the atom becomes **ionized**. The energy required to remove the outermost electron from an atom is called the **ionization energy**. Figure 8.6 shows the variations in ionization energy for some elements in the periodic table.

If an element has a high ionization energy, we can state that it has a high affinity for its electrons. **Electron affinity** is the measure of the degree of attraction of an atomic nucleus for its electrons. Table 8.1 compares the affinity of various materials to accept electrons.

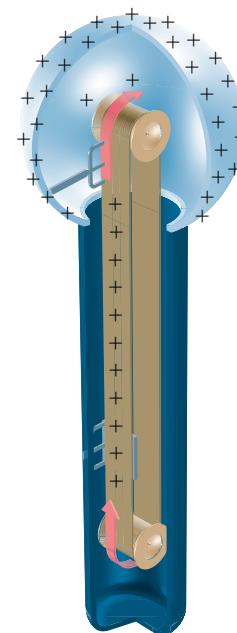
When any two substances in Table 8.1 are rubbed together, friction causes the more mobile outer electrons to transfer from the material with the lower electron affinity to the material with the higher electron affinity. Cat fur, for example, gives up electrons to a silk shirt, resulting in fur's characteristic but annoying attraction to many articles of clothing. Friction is also used to charge the Van de Graaff electrostatic generator (Figures 8.7a and b). Friction in the belt of this generator causes the great charge buildup.

Charge is carried on the *outer surface* of materials, regardless of whether they are **conductors** or **insulators** of electric charge, because repulsive forces push excess charges as far away from one another as possible.

Fig. 8.7a The hairdo is due to the excess charges experiencing mutual repulsion



Fig. 8.7b A Van de Graaff electrostatic generator



The Electrostatic Series	
Cat's fur	Low affinity for electrons
Acetate	
Glass	
Wool	
Lead	
Silk	
Wax	
Ebonite	
Copper	
Rubber	
Amber	
Sulphur	
Gold	High affinity for electrons

Conductor: A material, such as metal, with very loosely bound electrons that can easily transfer electrons between neighbouring atoms

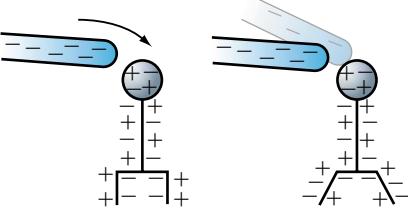
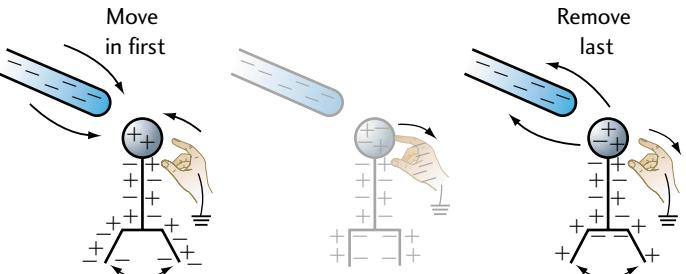
Insulator: A material, such as glass, with tightly bound electrons that cannot be easily transferred between neighbouring atoms

A Van de Graaff generator builds up a positive charge by removing electrons from the sphere. Some people prefer to envision the flow of electrons in the opposite direction.

Charging by Contact and Induction

As stated previously, once an object is charged by friction, it can then be used to transfer a charge to other objects by *contact* or *induction*. Figures 8.8a and b in Table 8.2 summarize the steps required to transfer charge by these two methods.

Table 8.2
Charging by Contact and Induction

Type of Charge Transfer	Description
Charging by Contact Fig.8.8a	 <p>When a charged object comes into contact with a neutral object, the excess or deficit of electrons in the charged object causes the transfer of charge to the previously neutral object. In Figure 8.8a, a negatively charged object transfers some of its excess electrons to the neutral object, thereby causing it to become negatively charged. A positively charged rod draws electrons out of a neutral object, leaving it with an overall positive charge.</p>
Charging by Induction Fig.8.8b	 <p>A charged object brought close to a neutral object <i>without contact</i> induces a movement of electrons in the neutral object. If the neutral object is attached to a grounding source such as Earth, which can give or receive electrons freely, the induced charge separation is momentarily overcome by charge flowing to or from the ground. Removal of the ground first and then of the charged object causes the remaining electrons (can be excess or deficit) to redistribute in the previously neutral object. In Figure 8.8b, a negatively charged rod induces a positive charge in the electroscope. Charging by induction always leaves the neutral object with the opposite charge of the charged object.</p>

The device in Figure 8.9 is a Whimshurst machine that uses the principle of induction to create a charge separation between the two spherical contacts. The metal foil pieces on each disk never touch each other. Instead, each piece induces a charge separation on its partner through the disks as they rotate past each other.

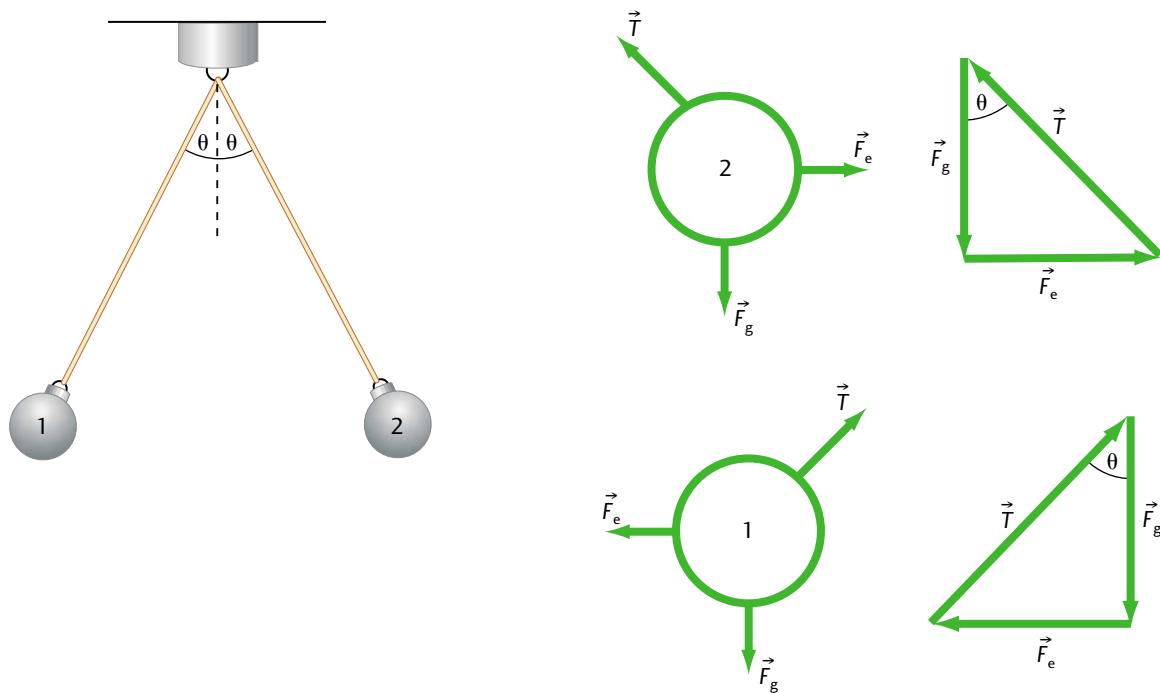
Fig.8.9 A Whimshurst machine charges by induction



8.4 Coulomb's Law

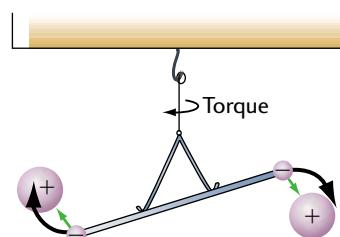
We have all experienced “static cling” where two objects with opposite electrostatic charges attract one another according to the law of electric charges. Charles-Augustin de Coulomb (1736–1806) experimented with the forces that exist between any two electric point charges. **Point charges** are extremely small particles (i.e., they have no measurable dimensions) that carry a charge. The magnitude of the force exerted between any two charges depends on the magnitude of each charge and the distance between them. Figure 8.10 illustrates how the magnitude of electrostatic forces can be studied in a lab.

Fig.8.10 Forces acting on charged spheres in static equilibrium



By charging spheres with different magnitudes and varying the distances between them, Coulomb determined the relationship between distance and magnitude of charge, and the electrostatic force. Coulomb modelled his experiment after one performed by another scientist studying gravity, Henri Cavendish (1731–1810), whom we mentioned in Chapter 1. Using Cavendish’s torsion balance (Figure 8.11), Coulomb was able to measure the torque and therefore the force applied between the charges placed specific distances apart.

Fig.8.11 A torsion balance for studying Coulomb’s law



In the next example, we will derive Coulomb's law.

EXAMPLE 1

Deriving Coulomb's law

Use proportionality techniques (see Appendix D) and the data for the electric force, F , and the charges, q_1 and q_2 , in Table 8.3 to derive a proportionality statement that summarizes the relationship between the magnitudes of the force and the two charges when the distance between them is constant.

Solution and Connection to Theory

Given

Table 8.3				
q_1 (C) ($\times 10^{-7}$)	q_2 (C) ($\times 10^{-7}$)	r (m)	k (N·m 2 /C 2)	F (N) $\times 10^{-1}$
$\frac{1}{2} \left(\begin{matrix} 80 \\ 40 \end{matrix} \right) \frac{1}{4}$	40	1	9.0×10^{-9}	$2.88 \left(\begin{matrix} 1 \\ 2 \end{matrix} \right)$
20	$\frac{1}{2} \left(\begin{matrix} 40 \\ 20 \end{matrix} \right) \frac{1}{4}$	1	9.0×10^{-9}	$1.44 \left(\begin{matrix} 1 \\ 2 \end{matrix} \right)$
20	10	1	9.0×10^{-9}	0.72 $\left(\begin{matrix} 1 \\ 2 \end{matrix} \right)$
20	10	2	9.0×10^{-9}	0.36
20	10	2	9.0×10^{-9}	0.18
20	10	3	9.0×10^{-9}	$0.045 \left(\begin{matrix} 1 \\ 4 \end{matrix} \right)$
k_{avg}			9.0×10^{-9}	0.02

Using the multiplier method of data analysis (as described in Appendix D) on the data in Table 8.3, we can determine that the electrostatic force is directly proportional to each of the two charges and therefore to the product of the magnitudes, q_1 and q_2 , of the two point charges:

$$F \propto q_1 q_2$$

Coulomb also found that the electrostatic force was inversely proportional to the square of the distance between the centres of the two spheres:

$$F \propto \frac{1}{r^2}$$

where r is the distance between the centres of the two charged spheres.

Combining both proportionality statements, we obtain

$$F \propto \frac{q_1 q_2}{r^2}$$

E X A M P L E 2**Using Coulomb's law comparatively**

The electrostatic force between two charges is known to be 3.0×10^{-5} N. What effect would each of the following changes have on the magnitude of the force if made independently of one another?

- a) The distance between the charges is tripled.
- b) One charge is quartered and the other is doubled.
- c) What would happen to the force if both changes were made simultaneously?

Solution and Connection to Theory**a) Given**

$$F_1 = 3.0 \times 10^{-5} \text{ N} \quad r_2 = 3(r_1) \quad F_2 = ?$$

From Coulomb's law, $F \propto \frac{1}{r^2}$, which means that F and $\frac{1}{r^2}$ are directly proportional. It then follows that forces and distances from two separate cases are related by the equation

$$\frac{F_1}{F_2} = \frac{(r_2)^2}{(r_1)^2}$$

$$F_2 = \frac{F_1(r_1)^2}{(r_2)^2}$$

$$F_2 = \frac{(3.0 \times 10^{-5} \text{ N})(r_1)^2}{(3r_1)^2}$$

$$F_2 = (3.0 \times 10^{-5} \text{ N})\left(\frac{1}{9}\right)$$

$$F_2 = 3.3 \times 10^{-6} \text{ N}$$

The resulting force is one-ninth that of the original or 3.3×10^{-6} N.

- b) In this example, we have two different charges, q_1 and q_2 , that change magnitudes. We will use q_1 and q_2 to represent the original charge magnitudes and q_1' and q_2' to represent the new charge magnitudes.

Given

$$F_1 = 3.0 \times 10^{-5} \text{ N} \quad q_1' = \frac{1}{4}q_1 \quad q_2' = 2q_2 \quad F_2 = ?$$

$F \propto q_1 q_2$; therefore,

$$\frac{F_1}{F_2} = \frac{q_1 q_2}{q_1' q_2'}$$

$$F_2 = \frac{F_1 q'_1 q'_2}{q_1 q_2}$$

$$F_2 = \frac{(3.0 \times 10^{-5} \text{ N}) \left(\frac{1}{4} q_1\right) (2 q_2)}{q_1 q_2}$$

$$F_2 = (3.0 \times 10^{-5} \text{ N}) \left(\frac{1}{4}(2)\right) = \frac{3.0 \times 10^{-5} \text{ N}}{2}$$

$$F_2 = 1.5 \times 10^{-5} \text{ N}$$

The force is one-half the initial force.

c) $F \propto \frac{q_1 q_2}{r^2}$; therefore,

$$\frac{F_1}{F_2} = \frac{(q_1 q_2) r_2^2}{(q'_1 q'_2) r_1^2}$$

$$F_2 = \frac{F_1 (q'_1 q'_2) r_1^2}{(q_1 q_2) r_2^2}$$

$$F_2 = \frac{(3.0 \times 10^{-5} \text{ N}) \left(\frac{1}{4} q_1\right) (2 q_2) (r_1)^2}{(q_1 q_2) (3r_1)^2}$$

$$F_2 = (3.0 \times 10^{-5} \text{ N}) \left(\frac{1}{9}\right) \left(\frac{1}{2}\right)$$

$$F_2 = 1.7 \times 10^{-6} \text{ N}$$

Tripling the distance between the charges and changing their magnitudes results in an electrostatic force of $1.7 \times 10^{-6} \text{ N}$.

 $1 \text{ C} = 6.242 \times 10^{18} e^-$ or
 $1 e^- = 1.602 \times 10^{-19} \text{ C}$
The overall charge on an object can be determined by the equation

$$q = Ne$$

where q is the amount of charge in coulombs, N is the total number of electrons in either deficit or excess, and e is the charge on an electron: $1.602 \times 10^{-19} \text{ C}$.

Early experiments in electrostatics required quantitative measurements of electric charge. Without the ability to define the basic unit of charge, scientists grouped them into “packages” containing a consistent and reproducible magnitude. This package of **charge**, **q** , was given the unit name of **coulomb** (**C**), after Coulomb himself. The idea of charge is analogous to consistently filling an egg carton with a dozen eggs but not knowing that there are 12 eggs in a dozen. In the early 1900s, Robert Millikan performed his famous oil-drop experiment in which he determined that one coulomb of charge equals 6.242×10^{18} electrons. We will study Millikan’s experiment in Section 8.9.

EXAMPLE 3**Calculating total charge**

Two substances transfer charge when rubbed together. If 3.7×10^{24} electrons are transferred between the two substances, what is the amount of charge on the negative item?

Solution and Connection to Theory**Given**

$$N = 3.7 \times 10^{24} \text{ electrons} \quad q = ?$$

The excess of negative charge (3.7×10^{24} electrons) on one item is equal to the deficit of negative charge on the other item.

$$q = Ne$$

$$q = (3.7 \times 10^{24} e)(1.602 \times 10^{-19} \text{ C/e})$$

$$q = 5.9 \times 10^5 \text{ C}$$

Therefore, the charge on both items is $\pm 5.9 \times 10^5 \text{ C}$.

When we combine all variables that affect the electrostatic force between charges, the general proportionality statement, $F \propto \frac{q_1 q_2}{r^2}$, is the basis of the equation known as **Coulomb's law of electric forces**. Adding a constant of proportionality, our proportionality statement becomes an equation.

$$F_e = \frac{k(q_1 q_2)}{r^2}$$

where F is the electric force, k is a constant of proportionality known as **Coulomb's constant**, q_1 and q_2 are the magnitudes of the charges in coulombs (C) on the two charged spheres, and r is the distance between their centres in metres (m).

If we rearrange the equation for the constant k , we can empirically determine its value. For example, if we use the data from our Coulomb's law experiment in Table 8.3, we can calculate the value of k :

$$k = \frac{F_e r^2}{q_1 q_2}$$

$$k = \frac{(2.88 \times 10^{-1} \text{ N})(1 \text{ m})^2}{(80 \times 10^{-7} \text{ C})(40 \times 10^{-7} \text{ C})}$$

$$k = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

$$k = \frac{1}{4\pi\epsilon_0}$$

where ϵ_0 is the permeability of free space.

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$$

Therefore,

$$F = \left(\frac{1}{4\pi\epsilon_0}\right) \frac{q_1 q_2}{r^2}$$

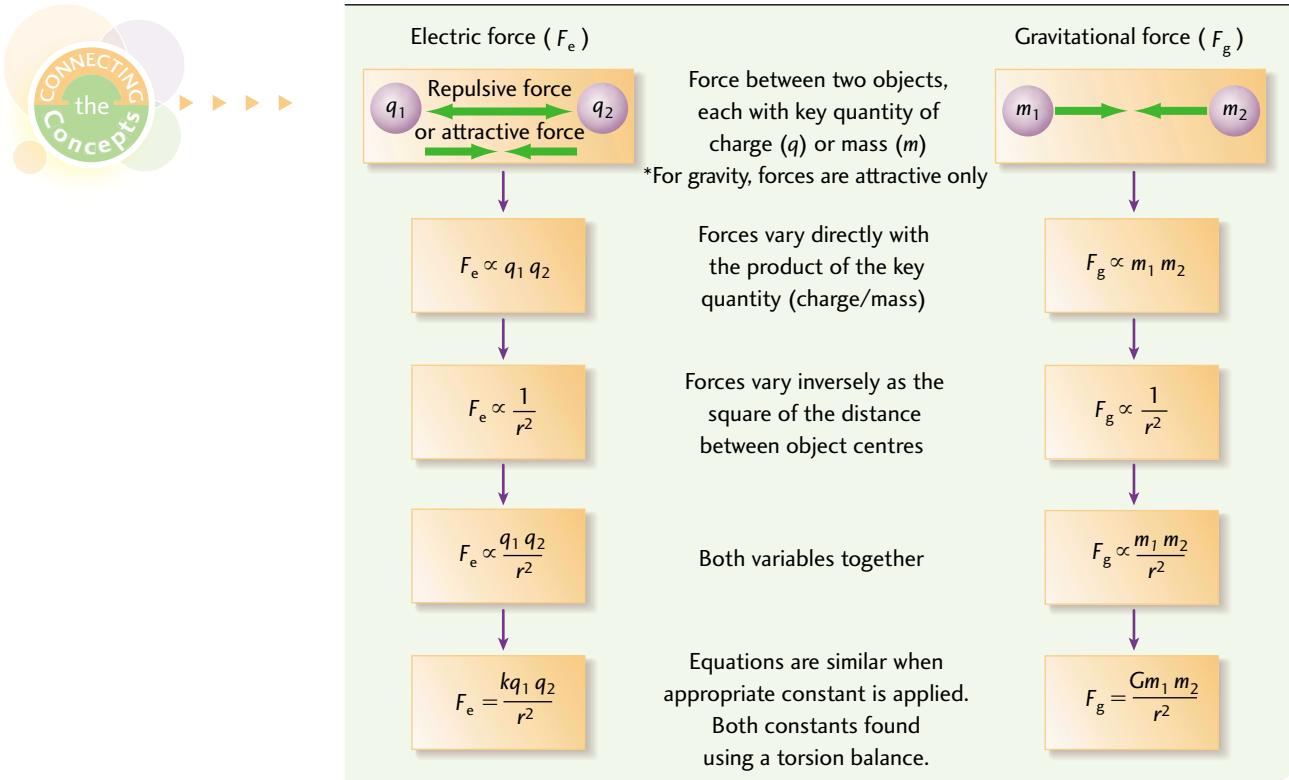
Déjà vu — Gravity

From Chapter 1, recall Newton's universal law of gravitation,

$$F_g = \frac{Gm_1 m_2}{r^2}.$$

The equation for Coulomb's law is very similar in form to Newton's universal law of gravitation equation. While the gravitational force depends on the masses of the objects, the electrostatic force depends on their charges. The constant of proportionality for each equation quantifies the difference between each type of force. In both equations, the force is inversely proportional to the square of the distance between the two bodies and directly proportional to the product of the *property* of the object governed by that law (i.e., charge or mass). Figure 8.12 compares the two laws.

Fig.8.12 A Comparison of Coulomb's Law and Newton's Law of Universal Gravitation



Now let's use the specific equation for Coulomb's law in a few examples.

EXAMPLE 4

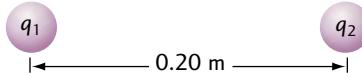
Using the Coulomb's law equation

Two point charges, $q_1 = 4.0 \times 10^{-6}$ C and $q_2 = 3.0 \times 10^{-6}$ C, are 0.20 m apart (Figure 8.13). What is the electrostatic force between them?

Fig.8.13 Charges exert a mutual force on each other

$$q_1 = 4.0 \times 10^{-6}$$
 C

$$q_2 = 3.0 \times 10^{-6}$$
 C



Solution and Connection to Theory

Given

$$q_1 = 4.0 \times 10^{-6} \text{ C} \quad q_2 = 3.0 \times 10^{-6} \text{ C} \quad r = 0.20 \text{ m}$$
$$k = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \quad F_e = ?$$

$$F_e = \frac{kq_1q_2}{r^2}$$

$$F_e = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(4.0 \times 10^{-6} \text{ C})(3.0 \times 10^{-6} \text{ C})}{(0.20 \text{ m})^2}$$

$$F_e = 2.7 \text{ N}$$

Because both charges are positive, the force that each charge exerts on the other is a repulsive force of 2.7 N. If one of these charges was negative, then the force would have the same magnitude but the opposite sign, and would therefore be an attractive force.

A positive (+) force represents a repulsion of two positive or two negative charges. A negative (−) force represents attraction between two opposite charges.

EXAMPLE 5 Solving Coulomb's law for a different variable

A small, negatively charged foam sphere is touched by a similar neutral sphere. The two spheres experience a repulsive force of 6.4 N when they are held 10 cm apart. What is the magnitude of the original charge on the foam sphere?

Solution and Connection to Theory

Given

$$F_e = 6.4 \text{ N} \quad r = 10 \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 0.10 \text{ m} \quad q = ?$$

The original negative charge on the first sphere must be shared between the two spheres after they come into contact. Therefore,

$$q_1 = q_2 = \frac{1}{2}q$$

$$F_e = \frac{kq_1q_2}{r^2}$$

$$F_e = \frac{k\left(\frac{1}{2}q\right)^2}{r^2}$$

$$q = \sqrt{\frac{4F_e r^2}{k}}$$

$$q = \sqrt{\frac{4(6.4 \text{ N})(0.10 \text{ m})^2}{9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}}$$

$$q = \pm 5.3 \times 10^{-6} \text{ C}$$

The original charge on the sphere was $-5.3 \times 10^{-6} \text{ C}$. If the original charge had been positive, its magnitude would have been the same.

The Vector Nature of Electric Forces between Charges

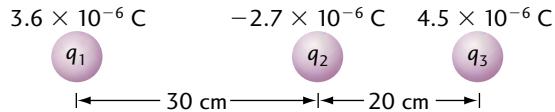
Coulomb's law only describes the force that exists between two point charges. For more than two charges, we must consider two charges at a time. Once we have calculated the forces between charge pairs, we can determine the overall force on any one charge by calculating the vector sum of all the forces. Let's do an example.

EXAMPLE 6

The total electric force of a charge distribution

Three point charges, $q_1 = 3.6 \times 10^{-6} \text{ C}$, $q_2 = -2.7 \times 10^{-6} \text{ C}$, and $q_3 = 4.5 \times 10^{-6} \text{ C}$, are arranged in a one-dimensional line, as shown in Figure 8.14. Find the total force on charge q_3 .

Fig.8.14



Solution and Connection to Theory

Given

$$q_1 = 3.6 \times 10^{-6} \text{ C} \quad q_2 = -2.7 \times 10^{-6} \text{ C} \quad q_3 = 4.5 \times 10^{-6} \text{ C}$$

In one-dimensional problems, the vector sum and the arithmetic sum are the same. Therefore, the total force on q_3 equals the sum of the forces between q_1 and q_3 , and q_2 and q_3 :

The notation r_{1-3} represents the distance, r , between the charges q_1 and q_3 .

$$_{\text{net}}\vec{F}_3 = {}_1\vec{F}_3 + {}_2\vec{F}_3$$

But \vec{F}_{2-3} is an attractive force [left].

$${}_{\text{net}}F_3 = \frac{kq_1q_3}{r_{1-3}^2} + \frac{kq_2q_3}{r_{2-3}^2}$$

$$\begin{aligned} {}_{\text{net}}F_3 &= \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.6 \times 10^{-6} \text{ C})(4.5 \times 10^{-6} \text{ C})}{(0.50 \text{ m})^2} \\ &\quad + \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-2.7 \times 10^{-6} \text{ C})(4.5 \times 10^{-6} \text{ C})}{(0.20 \text{ m})^2} \end{aligned}$$

$${}_{\text{net}}F_3 = 0.5832 \text{ N} - 2.734 \text{ N}$$

$${}_{\text{net}}F_3 = -2.2 \text{ N}$$

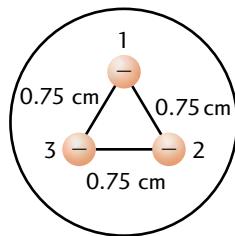
Therefore, the net force on q_3 is 2.2 N [left].

EXAMPLE 7

The distribution of charge in a symmetrical conductor

When a symmetrical conductor (such as the spherical ball on an electro-scope or a single wire) is given an excess charge, the particles that make up the excess charge exert a force on each other that repels them to the surface of the sphere so that they are as far away as possible from similar charges within the conductor. Figure 8.15a shows three representative excess electrons inside a conductor with a circular cross-section at one instant. Each electron is situated at the vertex of an equilateral triangle with side length 0.75 cm. Use vector addition and Coulomb's law to find the net force, including direction, on each electron.

Fig.8.15a



Solution and Connection to Theory

Given

$$q_1 = q_2 = q_3 = 1.602 \times 10^{-19} \text{ C} \text{ (the charge on one electron)}$$

$$r_{2-1} = r_{3-1} = r_{2-3} = 7.5 \times 10^{-3} \text{ m}$$

From Figure 8.15a, the net force, ${}_{\text{net}}F_1$, on the top charge, q_1 , is the vector sum of the force of q_2 on q_1 and the force of q_3 on q_1 ; that is,

$${}_2F_1 + {}_3F_1 = {}_{\text{net}}F_1$$

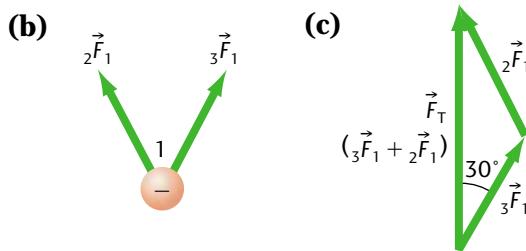
Because the magnitudes of the charges and the distances between them are the same, the force of q_2 on q_1 is the same as the force of q_3 on q_1 :

$${}_2F_1 = {}_3F_1 = \frac{kq_1q_2}{r_{2-1}^2}$$

$${}_2F_1 = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})^2}{(7.5 \times 10^{-3} \text{ m})^2}$$

$${}_2F_1 = 4.1 \times 10^{-24} \text{ N}$$

Fig.8.15



These forces act upward and outward, as shown in Figure 8.15b, along the line directly connecting charges q_1 to q_2 and q_1 to q_3 .

We can find the vector sum, ${}_2F_1 + {}_3F_1 = {}_T F_1$, by measuring the resultant vector from a scale diagram or by solving using the trigonometric method. We will use the trigonometric method.

$$\text{net } F_1^2 = {}_2F_1^2 + {}_3F_1^2 - 2({}_2F_1)({}_3F_1)\cos \theta$$

$$\begin{aligned}\text{net } F_1^2 &= (4.1 \times 10^{-24} \text{ N})^2 + (4.1 \times 10^{-24} \text{ N})^2 - 2(4.1 \times 10^{-24} \text{ N}) \\ &\quad (4.1 \times 10^{-24} \text{ N})\cos 120^\circ\end{aligned}$$

$$\text{net } F_1^2 = 5.0 \times 10^{-47} \text{ N}^2$$

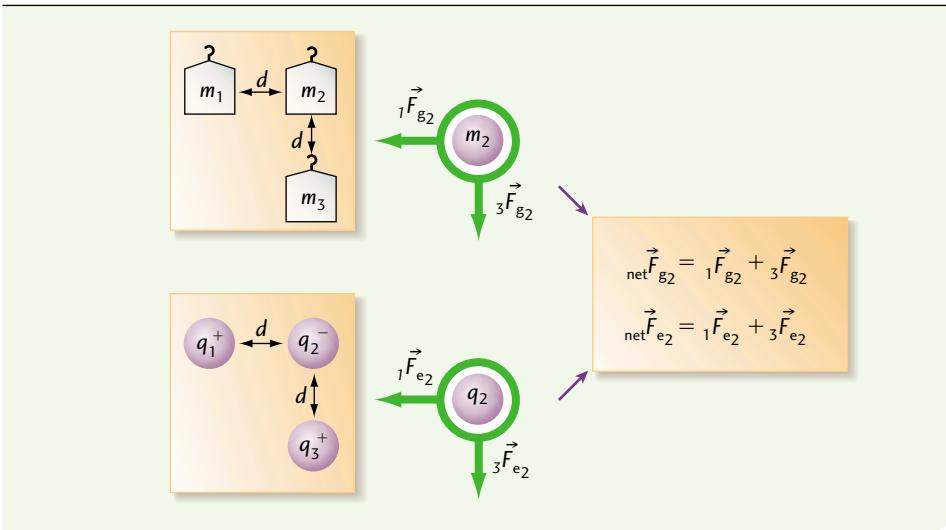
$$\text{net } F_1 = 7.1 \times 10^{-24} \text{ N}$$

Because the three electrons are equidistant, the two forces, ${}_2F_1$ and ${}_3F_1$, are symmetrical and at an angle of 30° from the vertical on each side as shown in Figure 8.15c. The direction of ${}_T F_1$ is directly up, toward the surface of the conductor. Similarly, the forces on the other two charges, q_2 and q_3 , also point outward, toward the surface of the conductor.

Déjà vu — Gravity

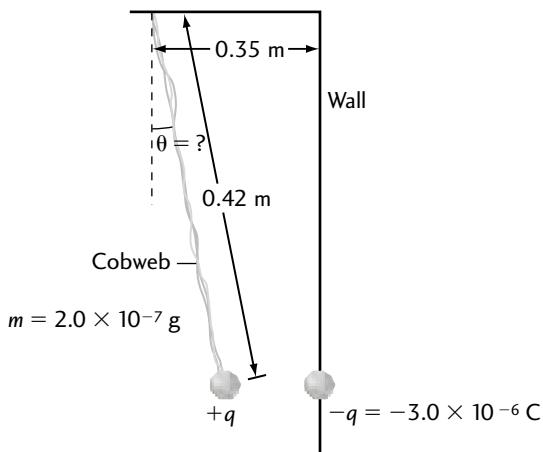
The force of gravity between three objects is calculated in the same way as we calculated the electrostatic force between three charges in Example 7. The force of gravity between two small masses is so tiny that it's negligible compared to the electrostatic force. Figure 8.16 compares the gravitational force and the electrostatic force acting on three masses and charges, respectively.

Fig.8.16 The Vector Nature of the Gravitational and Electrostatic Forces



1. What is the force between charges of $+3.7 \times 10^{-6} \text{ C}$ and $-3.7 \times 10^{-6} \text{ C}$ placed 5.0 cm apart?
2. How far apart would the same two charges in problem 1 have to be to experience a force that is twice as strong?
3. A dust cobweb is drawn from an initial vertical position toward a nearby wall by an electrostatic force. Assume the cobweb to be like a single dust ball of mass $2.0 \times 10^{-7} \text{ g}$ suspended on a massless string of length 0.42 m connected a horizontal distance of 0.35 m from the wall, as shown in Figure 8.17. The tethered dust ball is drawn to the wall by another similar dust ball of opposite charge, $-q = -3.0 \times 10^{-6} \text{ C}$, as shown.

Fig.8.17



- a) Draw a free-body diagram for the tethered dust ball in its final resting position.
- b) Transfer the force information from the free-body diagram to a triangle similar to Figure 8.15c.
- c) What final angle does the cobweb make to the vertical?

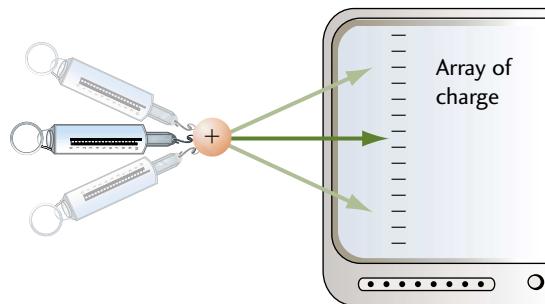
8.5 Fields and Field-mapping Point Charges

The picture on a TV screen is created when high-speed electrons strike the inside of a phosphorescent coated screen. We will learn more about TVs in Section 8.8.

Charges can be formed by an excess or deficit of a few electrons on any object. Dust build-up on your TV screen occurs because even the lightest charged particles of dust experience a significant attractive force near the TV screen, which can carry an electric charge. When charges are spread over a wide area called a **charge distribution**, it is impossible to calculate the total force acting on any one piece of dust. Instead of concerning ourselves with the individual force between a single charged dust speck and each charge on the screen, we consider how all of the charges distributed on the screen affect the position of the dust speck. The presence of an electrostatic charge or charges creates an electric field in the surrounding space. An **electric field** is a region of space created by a single charge or charge array that can produce an electrostatic force on any other charge introduced into the region.

A **field** is a region in three-dimensional space in which a property or quantity, such as a force, may be distributed.

Fig.8.18 The charges distributed on the TV screen produce an electric field in the space around the screen's surface. The charged dust speck verifies the existence of the field if a force is exerted on it when it is introduced into the electric field. The total force acting on the speck, as described hypothetically by the Newton spring scales, is due to the strength of the field.



Force at a Distance

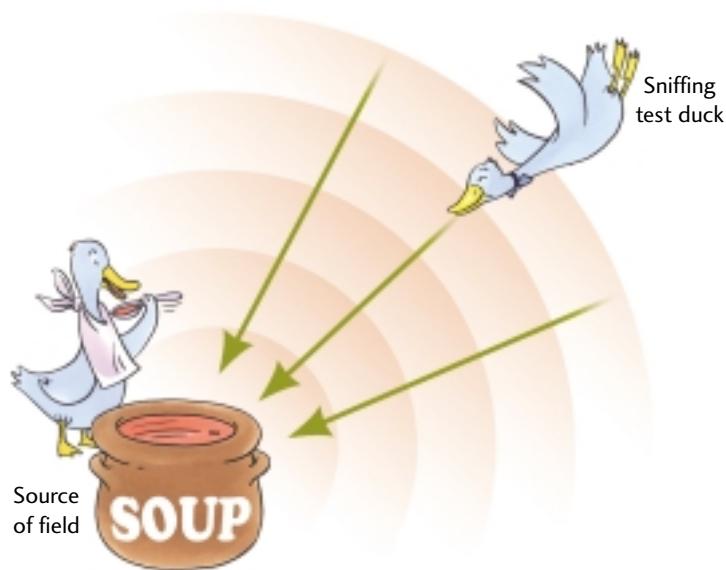
Other forces that act at a distance are magnetic forces and the force of gravity. These two force fields are mapped using a **test magnet** and a **test mass**, respectively, and will be described in greater detail in Chapter 9.

From Chapter 1, we know that a force is a push or pull on an object. As we learned in Section 8.1, charged objects don't need to be in contact with each other in order to experience forces of attraction or repulsion; therefore, we can say that the electrostatic force *acts at a distance*. According to Coulomb's law, the greater the distance between the charges, the weaker the force between them.

When we wish to determine our position in relation to other geographical areas, we use a map. A **field map** is used to describe the forces exerted on any charge placed in an electric field. If we place a **test charge** inside an existing electric field created by another single point charge, the two charges will experience a force of attraction or repulsion. Just as your road map helps you to determine the direction in which you should travel, an electric field map tells you both the relative strength and direction of an electrostatic force on a test charge. A force field is analogous to an aroma emanating from somewhere inside your home. In Figure 8.19, a “field of aroma” in the three-dimensional space around a source is detected by a test object some distance away.

Fig. 8.19 An “aroma field” is created by the soup.

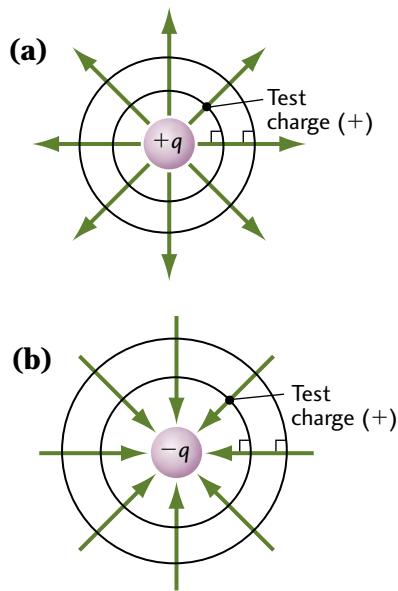
The field lines point in to show the direction in which the test duck is drawn.



The closer the test object gets to the source of the field (i.e., the soup), the stronger the field (i.e., the aroma) becomes. In an electric field map, the relative strength of the electric field is indicated by the distance between the field lines. The stronger the force field, the closer together the field lines. The arrows on the field lines show the direction of the force. By convention, the arrows on the field lines point in the direction of force on a positive (+) test charge at that spot in the electric field.

The electric field on the TV screen, represented by field lines, is created by a group of negative charges that are distributed across the screen. The **shape** of the field is mapped by taking a positive (+) test charge, like the dust speck, and placing it at various points near the screen to determine the direction of the electrostatic force on the test charge. Figure 8.20a shows the direction of the electrostatic force on any positive test charge at eight locations in a field created by a point charge.

Fig. 8.20 Field maps around charges with concentric equipotential lines at right angles to field lines



An **electric dipole** is a system of two separated point charges that have the same magnitude but the opposite charge.

Charges in an electric field possess electric potential energy (see Section 8.7). We can study electric fields by examining the places in the field where the force, and therefore the electric potential energy, is the same. These lines of similar potential are called **equipotential lines** (see Figures 8.20a and b). Equipotential lines and field lines are perpendicular to each other.

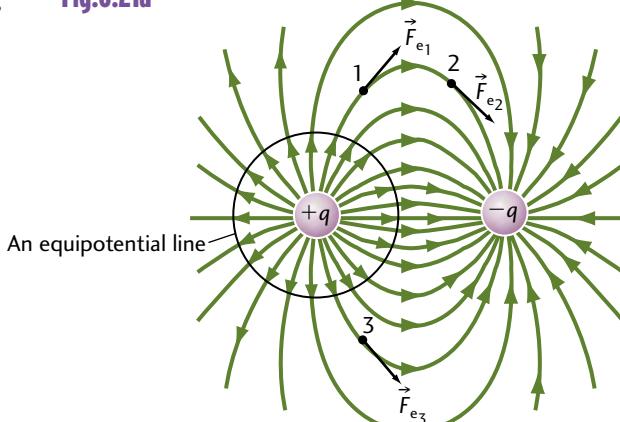
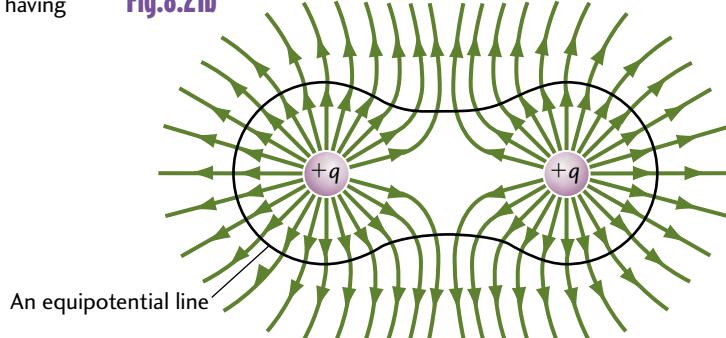
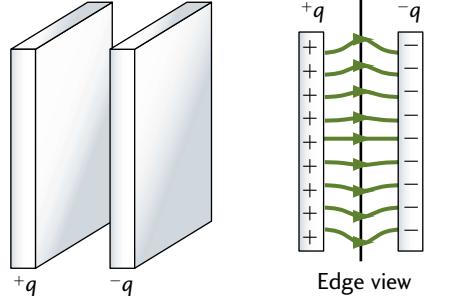
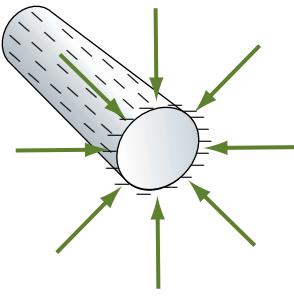
In Figure 8.20a, a positive test charge experiences a repulsive force along any of the eight field lines. We create a field map by moving our test charge to various points in the field, making note of the direction of the force with a small arrow. When we have accumulated enough small arrows on our map, we can connect all the arrows pointing tip to tail to form singular field lines. Each field line is like a road for the test charge; its direction is determined by the “terrain” created by the charges in the field. By analogy, the direction of a boulder tumbling down a rocky hillside is determined by the hill’s slope and the objects in the boulder’s path.

Figure 8.20b shows the field lines around a negative point charge. The field lines have the same shape as those around a positive point charge, but the opposite direction because the forces on the test charge (always positive) are now attractive forces.

Rules for Drawing Electric Field Lines

- 1) By convention, we use a positive test charge for field mapping; therefore, the *electric field lines always start at and point away from any positive charge producing the field*. In theory, we can draw an infinite number of field lines because there are an infinite number of places to put a test charge. Even though fields are three-dimensional, they are usually represented by a few lines drawn in a two-dimensional plane.
- 2) *The number of field lines emanating from a charge is proportional to the magnitude of that charge.* From Coulomb’s law, we know that stronger forces occur at closer distances. On a field map, the stronger field forces are represented by lines that are closer together, while weaker fields are represented by lines that are farther apart. Similarly, *the number of field lines per unit area passing at right angles through a surface is proportional to the strength of the electric field*.
- 3) Fields can also be mapped by first finding equipotential lines, where forces (and therefore electric potential) are all equal. Field lines are drawn at right angles to the equipotential lines.
- 4) Field maps can take on many shapes depending on the charge distribution that is creating them. The four basic charge distributions and their associated fields are summarized in Table 8.4.

Table 8.4
Electric Field Configurations

Configuration name	Field map	Description
Two point charges having opposite signs (dipole)	Fig.8.21a  <p>An equipotential line</p>	Curved field lines mean that the force experienced by a test charge differs depending on where it is placed in relation to the point charges that are creating the field. Notice that the electric force vector, \vec{F}_e , is always tangent to the electric field line at any point. Field lines cross equipotential lines at right angles.
Two point charges having the same sign	Fig.8.21b  <p>An equipotential line</p>	The curved field lines have a beginning but no visible end. If both charges were negative, the field line arrows would point the other way. Notice that there is no electric field in the area midway between the charges.
Parallel plates	Fig.8.21c  <p>An equipotential line</p> <p>Edge view</p>	The pairs of opposite charges are evenly distributed on opposing parallel plates. In the middle of the two plates, the field lines have uniform density and therefore exert the same force on any charge placed between them. At the edges of the plates, the field lines curve outwards, indicating that their density decreases, so a test charge would experience a lesser electric force.
Single conductor	Fig.8.21d 	Any excess charge resides on the surface of a conductor, so the field lines just outside the conductor are perpendicular to the surface. At the centre of the conductor, the field strength is zero because all the forces on a test charge are balanced.

Déjà vu — Gravity and Magnetism

Field lines are also used to represent the direction of the force of gravity on masses or forces between magnetic poles. As shown in Figure 8.22a, the gravitational field lines around a massive body like Earth are very similar to those around a single negative point charge (Figure 8.22b).

The magnetic field lines around the bar magnet in Figure 8.23a can be observed by the way iron filings distribute themselves in the field (Figure 8.23b). The key difference between a magnetic field and a gravitational or an electrostatic field is that the magnetic field requires *two distinct poles* of a magnet, called a **dipole**, and cannot be created by a single magnetic pole (a **monopole**). The result is that every magnetic field line is continuous with no real origin termination point. The direction of these field lines is defined by the direction of the north end of a compass needle that is experiencing the field. We will discuss magnetic forces in greater detail in Chapter 9.

Fig.8.22a The direction of Earth's gravitational field lines

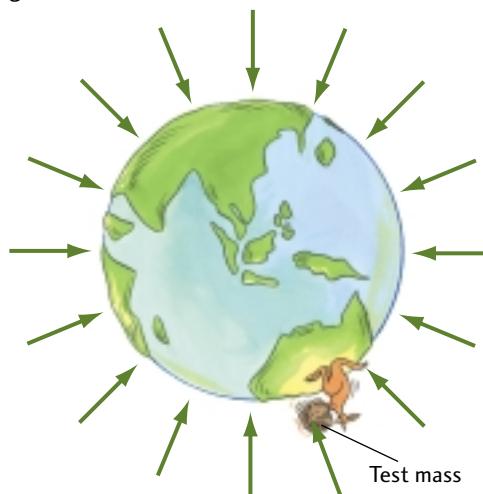


Fig.8.22b The direction of field lines around a single negative point charge

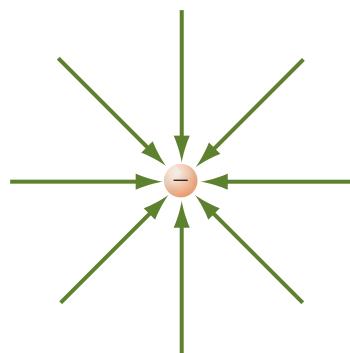


Fig.8.23 Magnetic field lines around a bar magnet

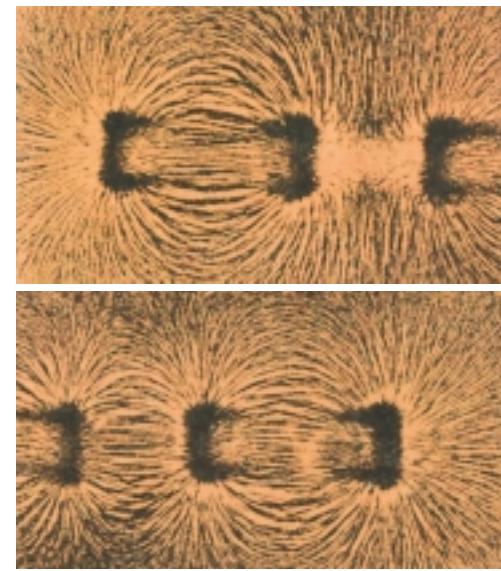
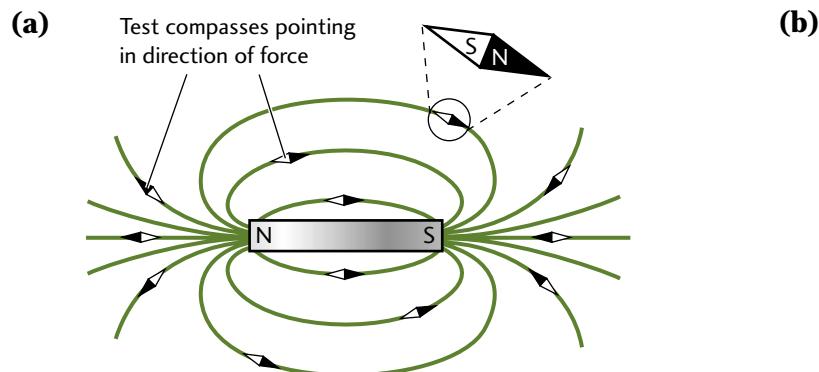
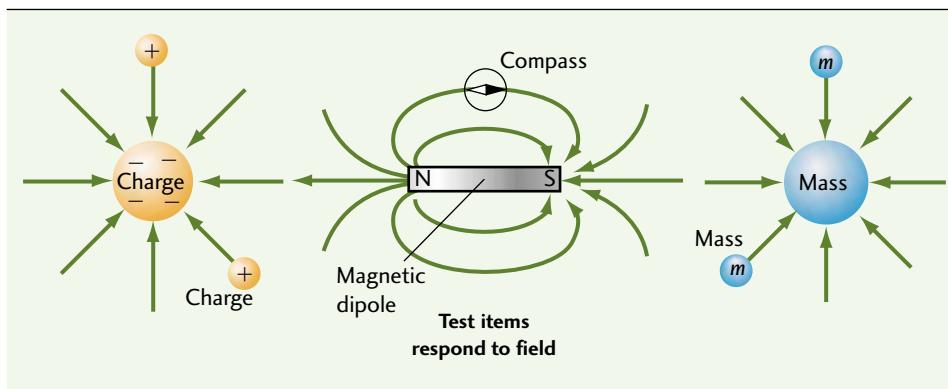


Figure 8.24 compares the field shapes of the electrostatic, magnetic, and gravitational fields.

Fig.8.24 A Comparison of Electric, Magnetic, and Gravitational Field Shapes



The strength of the force that a test charge experiences in an electric field created by a single point charge depends on three factors: 1) the magnitude of the test charge, 2) the magnitude of the source or point charge, and 3) the distance between them. When many charges create a field in a charge distribution, the last two items are difficult to quantify. Field theory helps to resolve this limitation by examining the field's influence on the test charge.

1. On a separate piece of blank paper in your notebook, sketch the charge distribution and its associated electric field for each of the following.
 - a) Three positive charges at the vertices of an equilateral triangle with sides measuring 5 cm each
 - b) A positive point charge at a 5-cm perpendicular distance from a 5-cm-long negatively charged plate
 - c) Create field maps for Figures 8.25a, b, and c using field-map simulation software (see <www.irwinpublishing.com/students>).

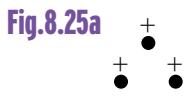
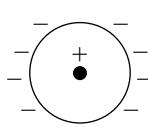


Fig.8.25b



Fig.8.25c



4. Figure 8.25c is a schematic of the coaxial cable shown in Figure 8.25d. How does the outer conductor protect the cable from stray electric fields?
2. Extremely close-range electrostatic forces, such as those between the pages of this textbook, are repulsive forces. What evidence supports this argument? Which aspect of current atomic theory supports the idea that the atoms comprising two bodies never come into contact, even during the most forceful collision?

Fig.8.25d



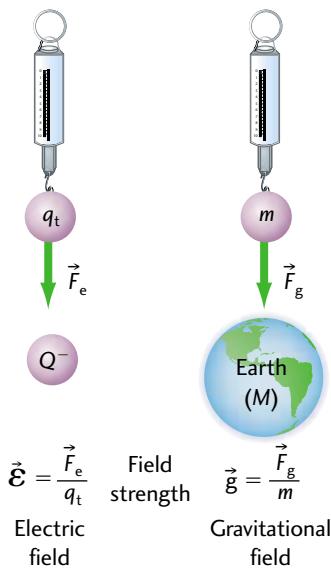
8.6 Field Strength

When we use field theory, we can consider the force on a single test charge, q_t , in a field (like that illustrated in Figure 8.26) to be dependent on two factors: the magnitude of the test charge and the field strength. **Field strength** is the force available to influence a test charge. At a particular point, the field strength is the result of all the charges in the region that are creating the field. The field in Figure 8.26 is created by only one charge.

Fig. 8.26 A test charge experiences a combined force from all charges creating the electric field



Fig. 8.27 Hypothetically, we can determine the field strength anywhere in an electric field by inserting a known charge, q_t , tethered to a Newton spring scale



The Direction of \vec{E}

The convention for the direction of the field strength depends on the direction of the electric force on a positive (+) test charge in the field at a particular point. A positive (+) \vec{E} represents the direction of the repulsive force on a positive (+) charge at that point in space. A negative (-) \vec{E} represents the direction of the attractive force on a positive (+) test charge.

We can simplify the multiple forces of a charge distribution by considering them to cause an electric field with field strength \vec{E} . The electric force created by the field, \vec{F}_e , is the product of the magnitude of the test charge, q_t , and the field strength. Therefore, $\vec{F}_e = q_t \vec{E}$. Both \vec{F}_e and \vec{E} are vector quantities. Rearranging this relationship for \vec{E} ,

$$\vec{E} = \frac{\vec{F}_e}{q_t}$$

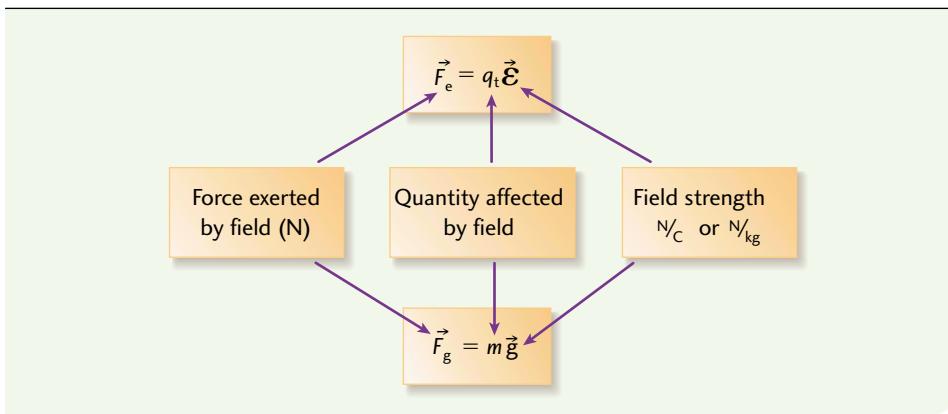
where \vec{F}_e is the electric force in newtons (N) at a particular point in the electric field, q_t is a charge in coulombs (C) experiencing the electric force, and \vec{E} is the field strength in newtons per coulomb (N/C). The field strength, \vec{E} , is the force, \vec{F}_e , experienced by a unit positive charge.

Déjà vu — Gravity

Recall that all objects gravitate toward each other, regardless of their mass. But the forces of attraction between small objects are negligible compared with the force they each experience toward the massive Earth. In electrostatics, the forces exerted on other charged objects are all significantly large. The charges occur in such quantity that they must be considered individually, or as a group in field theory. Electric fields are generally composed of many smaller fields created by point charges, each having a different strength and orientation. Electric field strength at a point must be determined specifically for that point because it depends on the magnitude and location of all the charges that create it. If q_t (magnitude of the test charge) is known and \vec{F}_e is measured (direction too), then \vec{E} can be determined by $\frac{\vec{F}_e}{q_t}$.

Figure 8.28 compares the parameters of the basic equations for gravitational and electric forces.

Fig.8.28 A Comparison of $\vec{F}_g = m\vec{g}$ and $\vec{F}_e = q_t \vec{\mathcal{E}}$



We have used a hypothetical Newton spring scale to measure the electric force, \vec{F}_e . In practice, dynamics may be used to find the electric force, as shown in Figure 8.29.

EXAMPLE 8 Calculating field strength

A small foam pith ball carrying a charge of $1.5 \times 10^{-6} \text{ C}$ experiences a force of 3.0 N to the left. What is the electric field strength at this point? Assume that left is positive.

Solution and Connection to Theory

Given

$$q = +1.5 \times 10^{-6} \text{ C} \quad \vec{F}_e = 3.0 \text{ N [left]} \quad \vec{\mathcal{E}} = ?$$

$$\vec{F}_e = q\vec{\mathcal{E}}; \text{ therefore, } \vec{\mathcal{E}} = \frac{\vec{F}_e}{q}$$

$$\mathcal{E} = \frac{3.0 \text{ N}}{+1.5 \times 10^{-6} \text{ C}}$$

$$\mathcal{E} = 2.0 \times 10^6 \text{ N/C}$$

Therefore, the electric field strength is $2.0 \times 10^6 \text{ N/C}$ to the left.

Fig.8.29 Determining the electric force

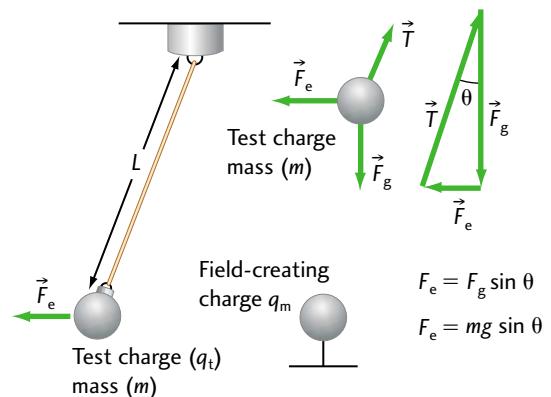
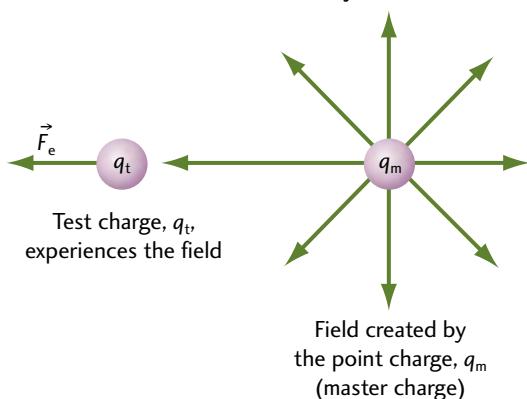


Fig.8.30 Relating Coulomb's law to field theory



Coulomb's Law Revisited

One way to think of Coulomb's law is to consider the electric force, \vec{F}_e , acting on a test charge, q_t , in a field created by a master point charge, q_m .

Coulomb's law becomes

$$F_e = \frac{kq_t q_m}{r^2}$$

But the force experienced by the charge q_t due to the field created by q_m is

$$F_e = q_t \mathbf{E}$$

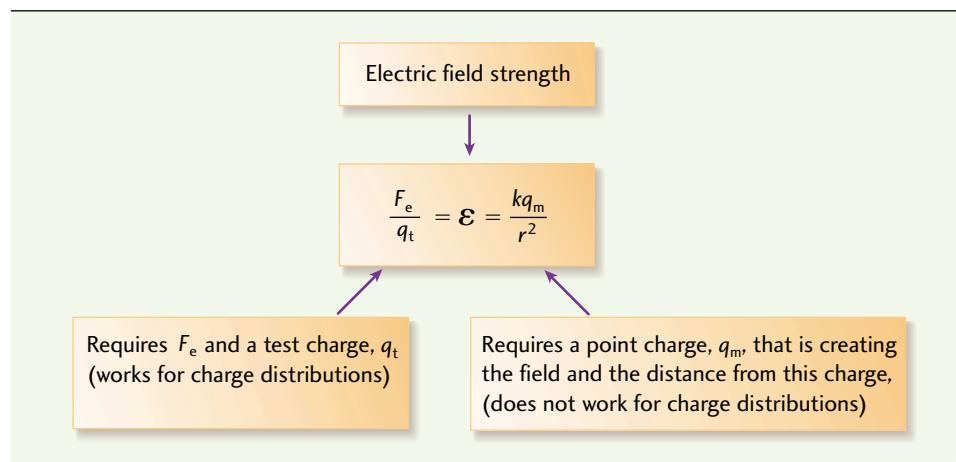
$$q_t \mathbf{E} = \frac{kq_t q_m}{r^2}$$

Therefore,

$$\mathbf{E} = \frac{kq_m}{r^2}$$

This equation also describes the field strength, but this time from the point of view of the point charge that is creating the field. Figure 8.31 shows the relationship between the two equations for magnitude of the electric field strength.

Fig.8.31 Field Strength from Two Points of View



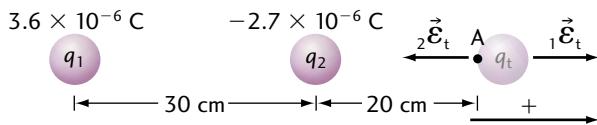
From Figure 8.31, we can see that the field strength, $\vec{\mathbf{E}}$, can be calculated in terms of the test charge, q_t , experiencing the force in the field as well as in terms of the charge, q_m , creating the field a distance, r , away from the charge creating the field. The second equation for field strength, $\mathbf{E} = \frac{kq_m}{r^2}$, can only be applied to fields created by single point charges. Like Coulomb's law, it can be applied to fields created by multiple charges in a charge distribution. Because electric field strength is a vector quantity, calculating the field strength for multiple charges requires vector addition.

EXAMPLE 9

The total electric field produced by a charge distribution

Two point charges, $q_1 = 3.6 \times 10^{-6}$ C and $q_2 = -2.7 \times 10^{-6}$ C, are arranged as shown in Figure 8.32.

Fig. 8.32



- a) Find the net electric field strength at point A due to the combined electric fields of both charges. (In one-dimensional problems, the vector sum and the arithmetic sum are the same.)
- b) What force is exerted on a charge of 4.5×10^{-6} C placed at point A?

Solution and Connection to Theory

Given

$$q_1 = 3.6 \times 10^{-6} \text{ C} \quad q_2 = -2.7 \times 10^{-6} \text{ C} \quad \vec{\mathcal{E}} = ?$$

- a) The net electric field strength at point A is the sum of the electric field strengths from each of the two charges at point A. Let's assign right to be the positive direction.

$${}_{\text{net}}\vec{\mathcal{E}}_A = {}_1\vec{\mathcal{E}}_A + {}_2\vec{\mathcal{E}}_A$$

$${}_{\text{net}}\mathcal{E}_A = {}_1\mathcal{E}_A + {}_2\mathcal{E}_A$$

$${}_{\text{net}}\mathcal{E}_A = \frac{kq_1}{r_{1-A}^2} + \frac{kq_2}{r_{2-A}^2}$$

$${}_{\text{net}}\mathcal{E}_A = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.6 \times 10^{-6} \text{ C})}{(0.50 \text{ m})^2} + \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-2.7 \times 10^{-6} \text{ C})}{(0.20 \text{ m})^2}$$

$${}_{\text{net}}\mathcal{E}_A = -4.8 \times 10^5 \text{ N/C}$$

Therefore, the total electric field strength at point A is -4.8×10^5 N/C. The negative sign is the direction of the field strength on a positive test charge in Figure 8.32, indicating a field strength pointing left.

b) $\vec{F}_e = q\vec{\mathcal{E}}$

$$F_e = (4.5 \times 10^{-6} \text{ C})(-4.8 \times 10^5 \text{ N/C})$$

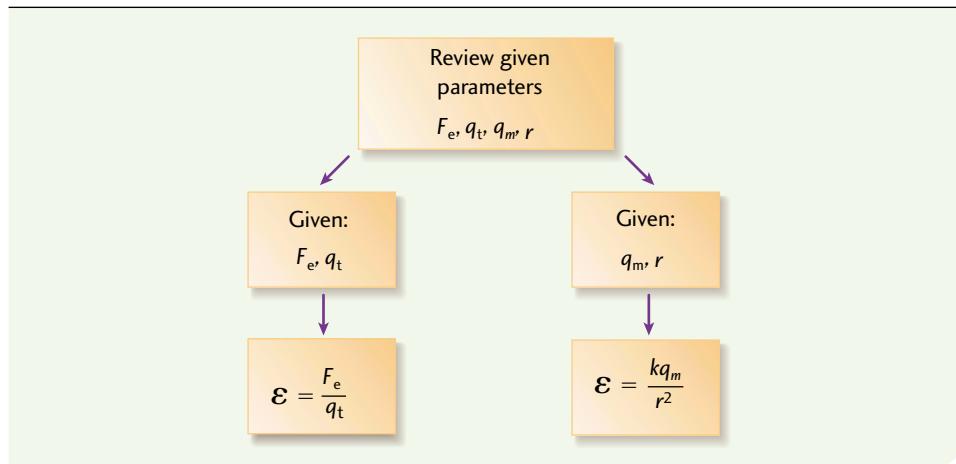
$$F_e = -2.2 \text{ N}$$

Therefore, the force on this charge is 2.2 N [left].

Example 9 is a duplicate of Example 6 (the total electric force of a charge distribution), only it is done using the vector addition of field strengths instead of the vector addition of forces. All problems involving point-charge distributions can be solved in a similar fashion.

Figure 8.33 summarizes when to use which equation to solve for the electric field strength.

Fig.8.33 Calculating Electric Field Strength

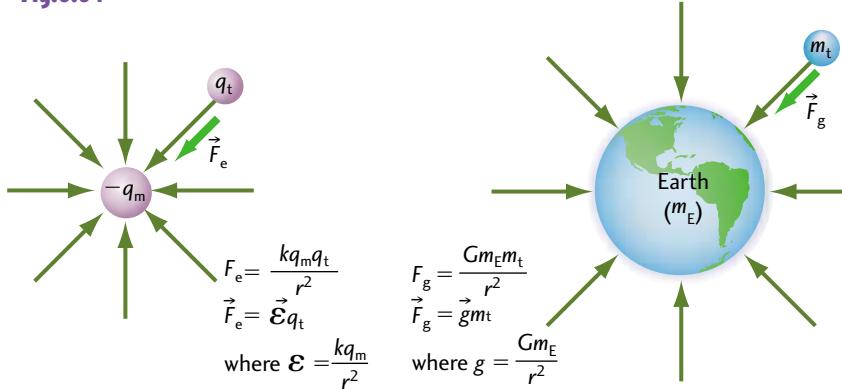


Electricity, Gravity, and Magnetism: Forces at a Distance and Field Theory

We learned in Section 8.1 that electric, magnetic, and gravitational forces all act at a distance. In Section 8.5, we explained how they do so in terms of field theory. We can classify the parameters of the three field types as either *quantities of matter* or *quantities of field*. Charge and mass are both considered quantities of matter because they are measurable properties of tangible objects. \vec{E} and \vec{g} , the electric and gravitational field strengths, are both quantities of field. In Chapter 9, we will study the nature and creation of magnetic fields. For electric and gravitational fields, the forces \vec{F}_e and \vec{F}_g are consequences of matter interacting with a field. Note the similarities between the equations for Coulomb's law and Newton's universal law of gravitation in terms of field strength (see Figure 8.34).

Even though gravitational fields are created by the presence of any matter in mass distributions, earthbound humans need only consider the field created by one single mass, Earth ($m_E = 5.98 \times 10^{24}$ kg). By comparison, the gravitational pull of all other masses on us is negligible.

Fig.8.34



For calculations, we consider Earth's mass to be concentrated at its centre. But humans live very far from Earth's centre, or one Earth radius ($1 r_E$), an average distance of 6.38×10^6 m. Therefore,

$$g = \frac{Gm_E}{r^2}$$

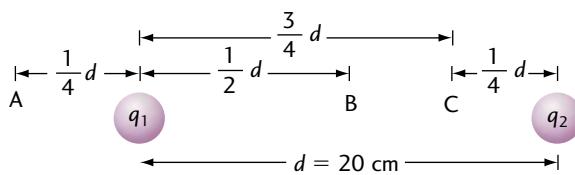
$$g = \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.38 \times 10^6 \text{ m})^2}$$

$$g = 9.80 \text{ N/kg at Earth's surface}$$



1. a) What force is exerted on a charge of -1.0×10^{-6} C in a field of strength 1.7×10^6 N/C [right]?
- b) If the field strength is doubled, what force is exerted on a charge of $+1.0 \times 10^{-6}$ C?
2. \vec{F}_e can be determined using an object such as a ping-pong ball that has a measurable mass. Referring to Figure 8.29, draw a diagram of a ping-pong ball on a string along with a free-body diagram to calculate the electrostatic force in problem 1.
3. A single point charge of $+3.0 \times 10^{-6}$ C creates an electric field with radiating field lines.
 - a) Draw a simple sketch of the electric field. Which way are the field lines pointing in relation to the charge?
 - b) What is the field strength 2.0 cm to the right of the field-creating charge? 4.0 cm away? 6.0 cm away?
 - c) What happens to the field strength when the distance from the master charge is doubled or tripled?
 - d) Write a proportionality statement describing how the field strength varies with distance r away from the source charge.
 - e) What force does a point charge of $+1.0 \times 10^{-6}$ C experience if placed 8.0 cm to the right of another point charge of 3.0×10^{-6} C?
4. Two charges of 1×10^{-6} C each are placed 20 cm apart, as shown in Figure 8.35.

Fig.8.35



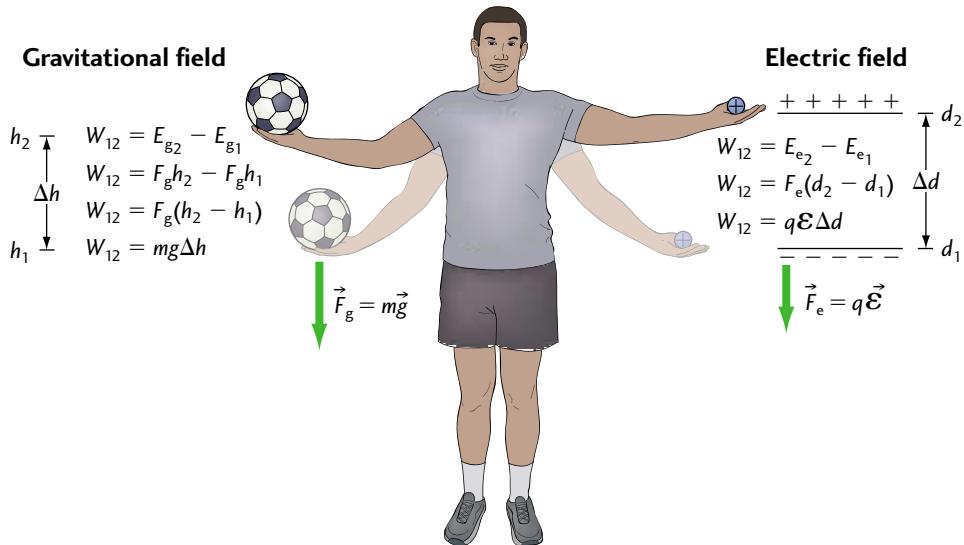
- a) What is the field strength at points A, B, and C?
- b) Explain your answer for the field strength at B.
- c) What conditions must exist in order for the effect in b) to occur?

Electric fields from separate charges may cancel one another at different points in the electric field.

8.7 Electric Potential and Electric Potential Energy

From our studies of dynamics, we have learned that forces may cause objects to move or even accelerate by doing work on them. Forces applied at a distance also do work on objects and are responsible for transferring energy. When we move a mass in a gravitational field by lifting it up, we are doing work on the mass by transferring gravitational potential energy to it (see Figure 8.36).

Fig. 8.36 The similarity between gravitational and electrostatic potential energy



Like the gravitational analogy, this simplification works only where the electric field strength, \mathcal{E} , is uniform, such as between two parallel plates (see Figure 8.21c). It does not apply to field strengths created by single point charges.

The concepts of work and potential energy also apply to electric forces. An electric force that displaces a charged particle from point A to point B does work on the particle. As a result, the charged particle has an increased *ability to do work*, or increased **electric potential energy**, E_e . The work done in pushing the charge against the electric field is equal to the difference in the electric potential energy between point A and point B.

$$W_{12} = E_{e2} - E_{e1}$$

$$W_{12} = F_e(d_2 - d_1)$$

$$W_{12} = q\mathcal{E}(d_2 - d_1)$$

By forcing the charge through a distance, work done increases the *electric potential energy* of the charge and creates an electric potential energy difference as the charge moves between its initial and final positions. This energy

difference depends on the magnitude of the charge, q , that is forced through the field: the greater the charge, the greater the difference in electric potential energy. On a work done *per unit charge* basis, the equation $W_{12} = E_{e2} - E_{e1}$ becomes

$$\frac{W_{12}}{q} = \frac{E_{e2}}{q} - \frac{E_{e1}}{q} = \frac{\Delta E_e}{q}$$

Electric potential energy *per unit charge* is referred to as *electric potential* or just *potential*.

The **electric potential**, V , at any given point in an electric field is the **electric potential energy**, E_e , of a point charge, q , at that point divided by the magnitude of the charge:

$$V = \frac{E_e}{q}$$

The concepts of electric potential energy and electric potential are closely related. *Electric potential energy*, E_e , is the energy associated with a charged object, whereas *electric potential*, V , is the amount of energy that any unit charge possesses at a point in an electric field. Therefore, electric potential energy, E_e , is measured in joules and potential, V , is measured in joules per coulomb of charge, or volts. If the magnitude of the potential in an electric field changes, then the potential difference can be determined as follows:

$$V_2 - V_1 = \frac{E_{e2}}{q} - \frac{E_{e1}}{q}$$

$$V_2 - V_1 = \frac{W_{12}}{q}$$

$$\Delta V = V_2 - V_1$$

$$\Delta V = \frac{\Delta E_e}{q} = \frac{W_{12}}{q}$$

We can determine the potential difference or voltage if we can measure the work done (force through a distance) in moving a charge from one point to another.

A **volt** is the electric potential at a point in an electric field if 1 J of energy is expended to bring 1 C of charge from infinity to that point.

The SI unit for potential, the volt (V), is derived from the electric energy per unit charge: 1 V = 1 J/C. The volt commemorates the scientist Alessandro Volta (1745–1827). The potential difference between two different positions in a field may also be referred to as voltage.

Electric potential energy can be considered to be a quantity of matter (the charge), whereas the potential is a quantity of field.

EXAMPLE 10

Electric potential versus potential energy

The work done on a test charge of magnitude $q = +1.0 \times 10^{-6} \text{ C}$ in moving it a distance Δd against an electric field is $2.5 \times 10^{-5} \text{ J}$.

- What is the change in electric potential energy of the charge for this displacement?
- What is the potential difference between these two positions?

Solution and Connection to Theory**Given**

$$q = +1.0 \times 10^{-6} \text{ C} \quad W = 2.5 \times 10^{-5} \text{ J} \quad \Delta E_e = ?$$

- The difference in electric potential energy is caused by the work done on the charge:

$$\Delta E_e = E_{e2} - E_{e1}$$

$$\Delta E_e = W_{12}$$

$$\Delta E_e = 2.5 \times 10^{-5} \text{ J}$$

The difference in electric potential energy of the test charge between its final and initial positions is $2.5 \times 10^{-5} \text{ J}$.

- $\Delta V = V_2 - V_1$

$$\Delta V = \frac{E_{e2} - E_{e1}}{q}$$

$$\Delta V = \frac{2.5 \times 10^{-5} \text{ J}}{1.0 \times 10^{-6} \text{ C}}$$

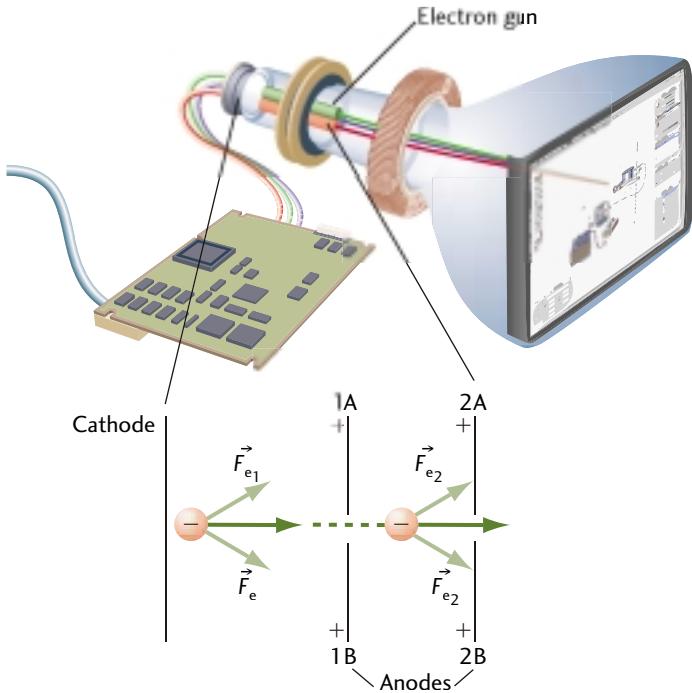
$$\Delta V = 25 \text{ V}$$

The electric field's potential is greater at its final position by 25 V.

In video display terminals, computer monitors (CRTs), and televisions, electrons are accelerated from the back projection of the picture tube through an electric field. These energized electrons strike the coloured red, green, and blue **phosphors** on the inside of the television screen, as shown in Figure 8.37. The path of the electron beam across the screen is controlled by magnetic fields (Chapter 9), but their energy is provided by an electric field.

From the basic definition of a volt, one joule of electric energy is sufficient to move one coulomb of charge across a potential difference of one volt. When dealing with the energy of small, discrete particles such as electrons, a more convenient unit for energy is used. The **electron volt, eV**,

Fig. 8.37 Electrons are accelerated from a cathode by positively charged anodes in an electron gun



If the soccer ball in Figure 8.36 is released from h_2 , its potential energy decreases as it falls. The falling object accelerates as it moves from an area of higher potential energy to an area of lower potential energy. The same occurs for any charge, q . Positively charged particles like protons will accelerate from an area of high electric potential energy (high potential) to an area of low electric potential energy (low potential). Conversely, negatively charged particles like electrons will accelerate from an area of low potential to an area of high potential.

is the energy of one electron after it has been accelerated through a potential difference of one volt. If potential energy is $q(\Delta V)$, then the energy of 1 eV is

$$1 \text{ eV} = (1.602 \times 10^{-19} \text{ C})(1.00 \text{ V}) = 1.602 \times 10^{-19} \text{ VC} = 1.602 \times 10^{-19} \text{ J}$$

Recall that $1 \text{ V} = 1 \text{ J/C}$, so $1 \text{ VC} = 1 \frac{\text{J}}{\text{C}} = 1 \text{ J}$

1. A positive test charge of $1.5 \times 10^{-6} \text{ C}$ is placed in an electric field 10 cm from another charge of magnitude $-5.0 \times 10^{-6} \text{ C}$ that is anchored in place.
 - a) What is the electric potential energy of the test charge?
 - b) What is the electric potential 10 cm away from the negative charge?
 - c) What is the potential difference between the test charge's initial position and a point 5.0 cm closer to the negative charge?
2. Two masses (each $5.0 \times 10^{-9} \text{ g}$) with charge magnitude $q_1 = 4.0 \times 10^{-10} \text{ C}$ and $q_2 = 1.0 \times 10^{-10} \text{ C}$ are accelerated through the same 50-V potential difference in a vacuum.
 - a) How much work is done on each charged particle?
 - b) Find the ratio of the final velocities of the two masses ($\frac{v_1}{v_2}$) after the 50-V potential difference is completed.



Even though all charges create fields, for the sake of simplicity, we consider test charges as only experiencing fields created by other charges.

Intensive and Extensive Properties

Many physical properties that we observe in the physics lab depend on the size or magnitude of the sample being studied. **Extensive properties** are properties such as mass and volume that are proportional to the size or amount of the object being observed. **Intensive properties**, such as temperature or density, are independent of how much of the substance is present. We can illustrate the difference between extensive and intensive properties by visiting the grocery store. The price of an item, such as toilet paper, is an *extensive property*: the larger the package, the more it will cost. But what brand of toilet paper is the best value? To answer this question, we look at the product's *intensive property*, the unit price; that is, the price per sheet of paper. If all toilet paper was created equal, the best value would be the toilet paper with the lowest unit price. Customers of bulk grocery stores must decide between the extensive property of raw cost and the intensive property of unit cost. Should we buy the 20-L tub of ketchup just because of the low unit price?

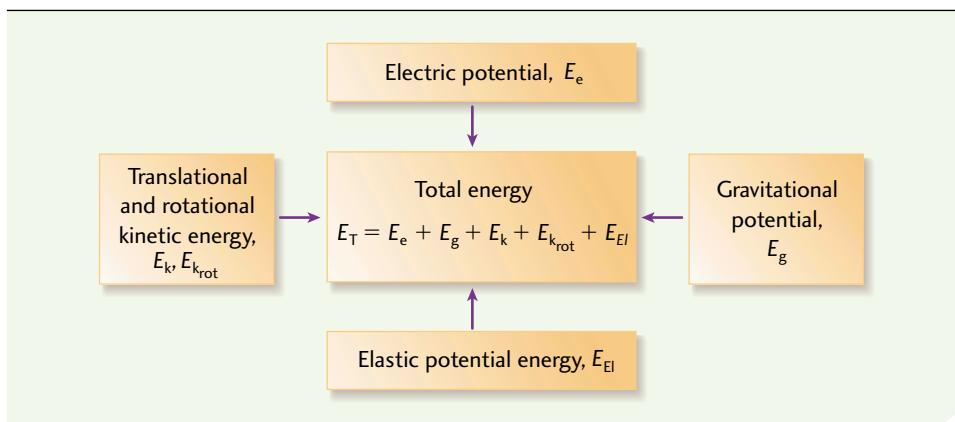
3. a) Identify the extensive and intensive properties of the following quantities: electric force, field strength, potential energy, and electric potential.
- b) For each extensive property in a), discuss the quantity that affects this property.
- c) Brainstorm examples of extensive and intensive properties in the scientific community or in your everyday life. List them in your notebook in a table such as the one below.

Extensive properties	Intensive properties

8.8 Movement of Charged Particles in a Field — The Conservation of Energy

The law of conservation of energy states that the total amount of energy in a closed system always remains the same. Once provided with some initial energy, the system expresses that energy in various combinations of the different types of energy shown in Figure 8.38. Like all the other forms of energy that we have studied, the energy of charged objects in an electric field is also conserved.

Fig.8.38 Total Energy



We learned in Section 8.7 that small charged particles can accelerate in the presence of an electric field. In doing so, electric potential energy is transferred to kinetic energy. If no energy is transferred to heat or light, then we can apply the law of conservation of energy to charged particles.

Most of the charges in this chapter are so small that they have no rotational kinetic energy.

EXAMPLE 11 Electric potential and electric potential energy

A small particle of mass 1.0×10^{-5} kg and charge $+1.5 \times 10^{-5}$ C is released from rest at position 1, which has a potential that is 12 V higher than the potential at position 2, as shown in Figure 8.39.

- a) What will happen to the particle upon release?
- b) What is the speed of the particle at position 2?

Solution and Connection to Theory

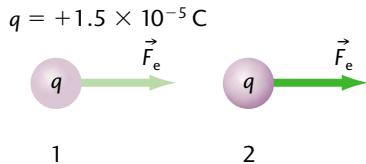
Given

$$m = 1.0 \times 10^{-5} \text{ kg} \quad q = +1.5 \times 10^{-5} \text{ C} \quad \Delta V = 12 \text{ V} \quad v_2 = ?$$

- a) The particle will accelerate from position 1 toward position 2 because positive charges always accelerate from an area of high potential to an area of low potential.
- b) As the particle accelerates, energy is transferred to it as work done by the electric field. The total amount of energy transferred is conserved; therefore, the total energy at position 1 is the same as the total energy at position 2, or

$$E_{T1} = E_{T2}$$

Fig.8.39



The particle's electric potential energy is transferred to kinetic energy of translation only. Our simplified expression becomes:

$$E_{k_1} + E_{e_1} = E_{k_2} + E_{e_2}$$

$$\frac{1}{2}mv_1^2 + qV_1 = \frac{1}{2}mv_2^2 + qV_2$$

But $v_1 = 0$ because the particle was released from rest; therefore,

$$\frac{1}{2}mv_2^2 = q(V_1 - V_2)$$

$$v_2 = \sqrt{\frac{2q(V_1 - V_2)}{m}}$$

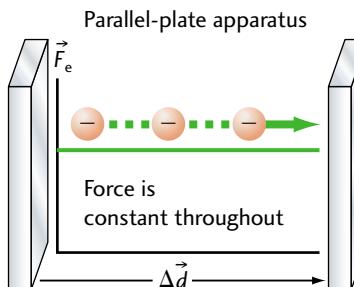
$V_1 - V_2 = +12$ V because the potential at position 1 is greater than the potential at position 2.

$$v_2 = \sqrt{\frac{2(+1.5 \times 10^{-5} \text{ C})(12 \text{ V})}{1.0 \times 10^{-5} \text{ kg}}}$$

$$v_2 = 6.0 \text{ m/s [right]}$$

At position 2, the speed of the particle is 6.0 m/s.

Fig. 8.40 Electric force versus charge position



Work is the dot product of force and displacement, or $W = F_e \Delta d \cos \theta$, where θ is the angle between the two vectors. If the displacement of the charge is always in the same direction as the applied force, then the angle between them is zero and $\cos 0^\circ = 1$. The dot product is reduced to the equation for the area of a rectangle, $W = F_e \Delta d$.

If our electric field is in a closed system (i.e., free from the effects of gravity, air resistance, and rotation), we can use the following simple relationship for the law of conservation of energy:

$$-\Delta E_e = \Delta E_k$$

where any decrease in the electric potential energy of the charged object is expressed as an increase in its kinetic energy.

So far, our calculations of electric potential using the law of conservation of energy have been quite simple because we have been considering electric fields located between parallel plates. Because these fields are uniform, the force experienced by a charge is independent of the charge's position. Work is the dot product of the applied force vector, \vec{F}_e , and the displacement vector, $\vec{\Delta d}$. The graph of the electric force applied to a charge located between parallel plates versus the charge's position (Figure 8.40) is therefore a horizontal straight line with a constant slope of zero. Work done in moving a charge between these plates equals the area underneath this graph; that is, the product of the length and width of the rectangle, or $F_e \Delta d$.

The other electric field configurations illustrated in Table 8.4, such as those created by point charges, have a non-uniform field strength. Therefore, the force on charges *and* the potential in the field vary depending on the position of the test charge. Figure 8.41a shows the force on a test charge at various distances away from a field-creating point charge.

Fig.8.41a Change in electric force with distance from master charge

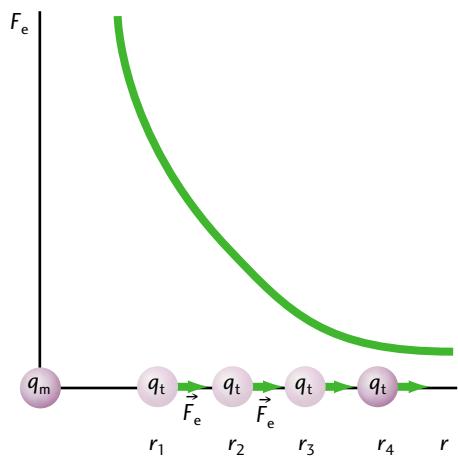
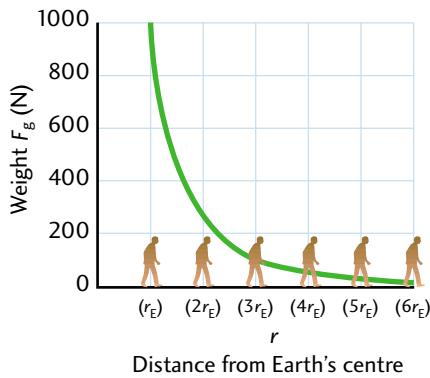


Fig.8.41b Change in force of gravity (weight) with distance from Earth's centre ($1r_E = 6.4 \times 10^6$ m)



The graph in Figure 8.41a is based on the force–distance relationship between point charges as described by Coulomb’s law. The shape of this graph is similar to that of the force–distance relationship between two masses in Newton’s universal law of gravitation (see Figure 8.41b). Work done to move the test charge from position 1 to position 2 is the area under the \vec{F}_e – \vec{r} graph, which can be found using integral calculus. The result is that the work done or the potential energy increase in moving the charge from position 1 (r_1 distance from the point charge) to position 2 (a distance r_2 away) is given by the equation

$$W_{12} = E_{e2} - E_{e1} = \frac{kq_1q_2}{r_2} - \frac{kq_1q_2}{r_1}$$

Factoring out all constants, the relationship becomes

$$E_{e2} - E_{e1} = kq_1q_2 \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

As r_2 moves farther and farther away from the master charge, approaching infinity, the term $\frac{1}{r_2}$ becomes zero. Therefore, E_{e2} also equals zero. Our equation now simplifies to

$$\cancel{E_{e2}} - \cancel{E_{e1}} = kq_1q_2 \left(\cancel{\frac{1}{r_2}} - \frac{1}{\cancel{r_1}} \right)$$

In calculus, the approximation for $\frac{1}{r_2}$ approaching the value of zero when r_2 approaches infinity is called the limit and is written as

$$\lim_{r_2 \rightarrow \infty} \frac{1}{r_2} = 0$$

$$E_e = \frac{kq_1q_2}{r}$$

where E_e is the potential energy stored between two point charges of magnitude q_1 and q_2 , r is the distance between them, and k is Coulomb’s constant, $9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{kg}^2$.

We can calculate the area under the $F_e - r$ curve for an attraction between oppositely charged particles using the geometric average for the electric force, $\sqrt{F_{e_1} F_{e_2}}$

$$\text{Area} = \sqrt{F_{e_1} F_{e_2}} (r_2 - r_1)$$

$$\text{Area} = \sqrt{\left(\frac{kq_1 q_2}{r_1^2}\right) \left(\frac{kq_1 q_2}{r_2^2}\right)} (r_2 - r_1)$$

$$\text{Area} = \left(\frac{kq_1 q_2}{r_1 r_2}\right) (r_2 - r_1)$$

$$\text{Area} = \left(\frac{kq_1 q_2}{r_1}\right) - \left(\frac{kq_1 q_2}{r_2}\right)$$

If we compare this equation to our equation for ΔE_e , we obtain

$$\Delta E_e = E_{e_2} - E_{e_1}$$

$$\Delta E_e = \left(\frac{kq_1 q_2}{r_1}\right) - \left(\frac{kq_1 q_2}{r_2}\right)$$

$$\Delta E_e = \left(\frac{-kq_1 q_2}{r_2}\right) - \left(\frac{-kq_1 q_2}{r_1}\right)$$

$$\text{If } \Delta E_e = \left(\frac{-kq_1 q_2}{r_2}\right) - \left(\frac{-kq_1 q_2}{r_1}\right)$$

then E_e between opposite charges at any separation distance r becomes

$$E_e = \frac{-kq_1 q_2}{r}$$

For like charges, the product $q_1 q_2$ is positive, so the equation for electric potential energy is

$$E_e = \frac{kq_1 q_2}{r}$$

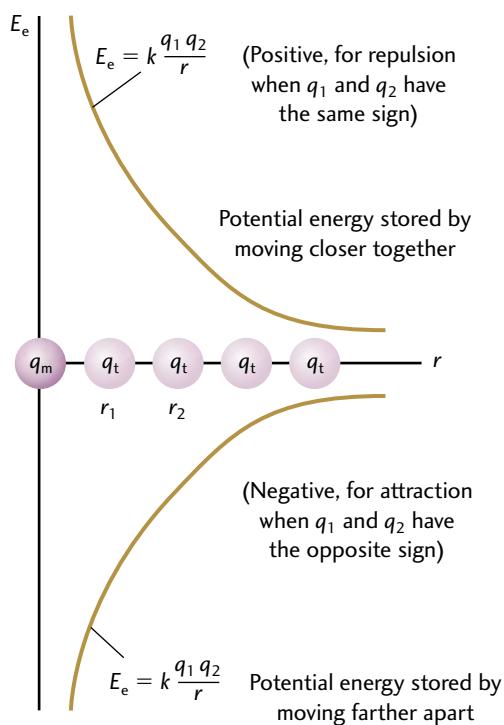
MASSES OF ATOMIC PARTICLES

Proton: 1.67×10^{-27} kg

Neutron: 1.67×10^{-27} kg

Electron: 9.11×10^{-31} kg

Fig.8.42 A graph of electric potential energy versus charge separation



The graph of electric potential energy with respect to position from a point charge is shown in Figure 8.42. Unlike gravity, the electric force can be either attractive or repulsive, depending on the sign of the product of the two charges involved (positive for repulsion and negative for attraction). In Figure 8.42, the graph in the lower quadrant represents attraction between two opposite charges, and the graph in the upper quadrant represents repulsion between two like charges.

EXAMPLE 12

Electric potential energy and point charges

An electron with an initial speed 10^3 m/s is aimed at an electron held stationary 1.0×10^{-3} m away. How close to the stationary electron will the moving electron approach before it comes to a stop and reverses its direction?

Solution and Connection to Theory

Given

$$v_e = 10^3 \text{ m/s} \quad r_1 = 1.0 \times 10^{-3} \text{ m} \quad q_1 = q_2 = e = 1.602 \times 10^{-19} \text{ C}$$

$$r_2 = ?$$

$$E_{e_1} = \frac{kq_1 q_2}{r_1} = \frac{ke^2}{r_1} \text{ and } E_{e_2} = \frac{ke^2}{r_2}$$

The change in the electron's potential energy equals the change in its kinetic energy.

$$\Delta E_k = -\Delta E_e$$

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = -\left(\frac{ke^2}{r_2} - \frac{ke^2}{r_1}\right)$$

But $v_2 = 0$; therefore,

$$-\frac{1}{2}mv_1^2 = -ke^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\frac{1}{2}mv_1^2 = ke^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Solving for r_2 ,

$$\frac{1}{r_2} = \frac{mv_1^2}{2ke^2} + \frac{1}{r_1}$$

$$r_2 = \left(\frac{mv_1^2}{2ke^2} + \frac{1}{r_1}\right)^{-1}$$

$$r_2 = \left(\frac{(9.11 \times 10^{-31} \text{ kg})(10^3 \text{ m/s})^2}{2(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})^2} + \frac{1}{1.0 \times 10^{-3} \text{ m}}\right)^{-1}$$

$$r_2 = 3.4 \times 10^{-4} \text{ m}$$

The closest these two electrons will approach each other is $3.4 \times 10^{-4} \text{ m}$ or 0.34 mm.

The Electric Potential around a Point Charge

Recall that the electric potential or voltage is the electric potential energy that any unit test charge possesses:

$$V = \frac{E_e}{q_t}$$

So, the electric potential at a separation distance r from a master charge creating a field is

$$V = \frac{E_e}{q_t} = \frac{\frac{kq_m q_t}{r}}{q_t} = \frac{kq_m}{r}$$

The **equation for electric potential** becomes

$$V = \frac{kq_m}{r}$$

where V is the electric potential in volts (V) at a distance r in metres from a point charge of magnitude q_m that is creating the charge.

Let's use this equation in some examples.

EXAMPLE 13

Calculating the electric potential around a point charge

What is the electric potential 4.0 cm from a point charge of $+3.20 \times 10^{-19}$ C?

Solution and Connection to Theory

Given

$$r = 4.0 \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 4.0 \times 10^{-2} \text{ m} \quad q_m = +3.20 \times 10^{-19} \text{ C} \quad V = ?$$

$$V = \frac{kq_m}{r}$$

$$V = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(+3.20 \times 10^{-19} \text{ C})}{4.0 \times 10^{-2} \text{ m}}$$

$$V = 7.2 \times 10^{-8} \text{ J/C or } 7.2 \times 10^{-8} \text{ V}$$

The potential at 4.0 cm from this charge is 7.2×10^{-8} V.

EXAMPLE 14

Relating work done on a charge through a potential difference

How much work must be done to increase the potential of a charge q (2.5×10^{-7} C) by 100 V?

Solution and Connection to Theory

Given

$$q = 2.5 \times 10^{-7} \text{ C} \quad \Delta V = 100 \text{ V} \text{ (the potential difference)} \quad W = ?$$

The work changes the electric potential energy of the charge.

$$W = \Delta E_e$$

$$W = q\Delta V$$

$$W = (2.5 \times 10^{-7} \text{ C})(1.00 \times 10^2 \text{ V})$$

$$W = 2.5 \times 10^{-5} \text{ J}$$

The work required to increase the potential energy of the charge is 2.5×10^{-5} J.

EXAMPLE 15**An alternative solution to Example 12**

Example 12 could have been completed using the concept of potential and potential difference. The first electron, with a speed of 10^3 m/s, is aimed at a stationary electron from a distance of 1.0×10^{-3} m. How close does the mobile electron come to the stationary electron before stopping and reversing direction?

Solution and Connection to Theory**Given**

$$v_e = 10^3 \text{ m/s} \quad r_1 = 1.0 \times 10^{-3} \text{ m} \quad q_1 = q_2 = e = 1.602 \times 10^{-19} \text{ C}$$
$$r_2 = ?$$

The mobile electron moves between two positions having two different potentials; therefore, it passes through a potential difference, ΔV , due to its decrease in kinetic energy.

$$\Delta V = V_{e_2} - V_{e_1} = \frac{\Delta E_k}{q}$$

$$V_{e_2} = \frac{\Delta E_k}{q} + V_{e_1}$$

$$V_{e_2} = \frac{1}{2} \frac{(9.11 \times 10^{-31} \text{ kg})(10^3 \text{ m/s})^2}{1.602 \times 10^{-19} \text{ C}} + \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})}{1.0 \times 10^{-3} \text{ m}}$$

$$V_{e_2} = 2.84 \times 10^{-6} \text{ V} + 1.44 \times 10^{-6} \text{ V}$$

$$V_{e_2} = 4.28 \times 10^{-6} \text{ V}$$

$$\text{But } V_{e_2} = V_{e_1} = \frac{kq_m}{r_2}$$

$$r_2 = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})}{4.28 \times 10^{-6} \text{ V}}$$

$$r_2 = 3.4 \times 10^{-4} \text{ m}$$

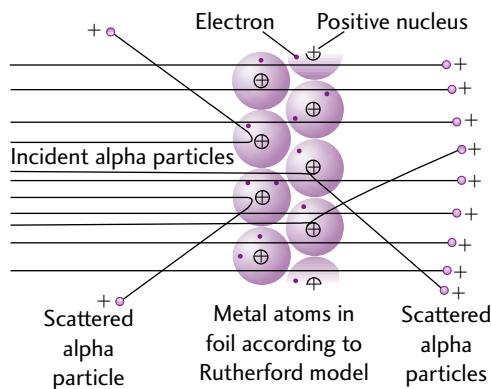
The closest distance between the two electrons is 3.4×10^{-4} m or 0.34 mm.



Rutherford's Gold-foil Experiment

One of the most famous experiments of all time used the concept of charges moving in an electric field. Ernest Rutherford's gold-foil experiment (1911–1913), depicted in Figure 8.43, used a positively charged particle to probe the structure of heavy gold atoms.

Fig. 8.43 A close-up of Rutherford's gold-foil experiment



The gold-foil experiment was performed by two other scientists, Hans Geiger and Ernest Marsden, in 1911 at Rutherford's suggestion. From 1898 to 1907, Rutherford worked at McGill University in Montreal, Canada. Geiger is more famous for his radiation detection apparatus, the Geiger counter.

In the gold-foil experiment, Rutherford fired positively charged alpha particles (helium nuclei) with kinetic energies of 7.7 MeV at a thin gold foil. Knowing the thin nature of the foil and the large kinetic energy of the particles, he expected most of the particles to pass through the gold foil. He observed, however, that some of the particles were deflected at wide angles and, in some cases, scattered backward. Rutherford commented that this result was analogous to a large artillery shell being fired at a piece of tissue paper, then rebounding. The only model of the atom that could account for these results was one that postulated a small but heavy nucleus at the centre of the atom having a net positive charge, with negatively charged electrons relatively far away from it. The current model of the atom (discussed in Chapter 14) is partly based on the results of this experiment.

1. In Rutherford's gold-foil experiment, alpha particles with charge $+2e$ and a kinetic energy of 7.7 MeV were beamed at gold foil. The nucleus of a gold atom contains 79 protons, giving it a charge of $+79e$. What is the closest distance that an alpha particle can get to a gold nucleus when it approaches head on?
2. Explain the results of Rutherford's gold-foil experiment in terms of the concepts you have learned in this section: charge, fields, electric forces, potential energy, potential difference, and the law of conservation of energy.
3. In Figure 8.39, what is the speed of a charge of $-1.5 \times 10^{-5} \text{ C}$ at position 1 if it is released from rest at position 2, keeping all other parameters the same?

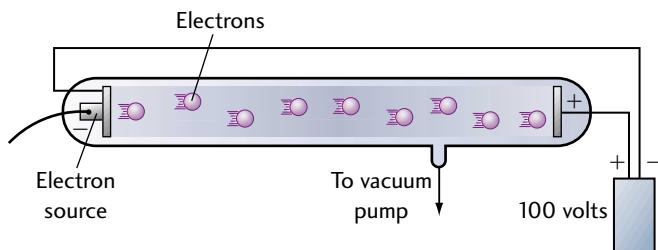
- 4.** A set of parallel plates with potential difference 1.5×10^3 V is used to accelerate alpha particles ($m_\alpha = 6.68 \times 10^{-27}$ kg, $q_\alpha = 2e$, where $e = 1.602 \times 10^{-19}$ C).

- What is the velocity of the alpha particles at the negative plate if they are released from rest at the positive plate? (Ignore the effects of gravity and air resistance.)
- What is the speed of the particles halfway between the two plates?
- How does the electric potential between two parallel plates vary with the position from one plate to the other?

The Cathode-ray Tube

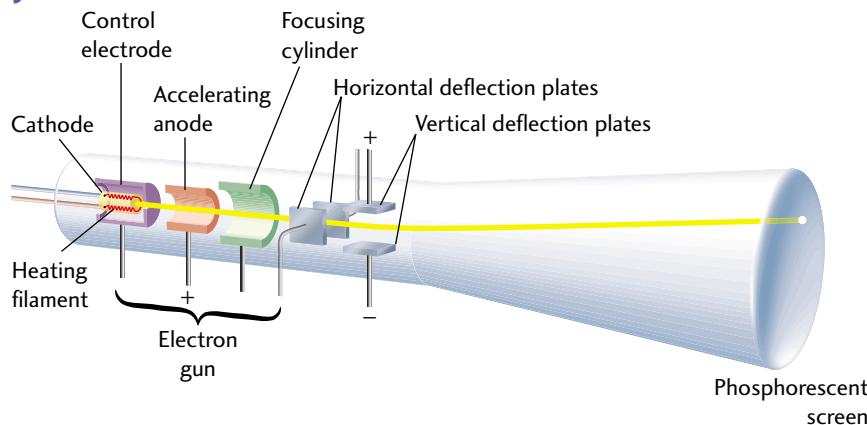
A **cathode-ray tube** (CRT) (Figure 8.44) was originally invented to study the nature of **cathode rays**, which we now know to be beams of electrons. In this device, cathode rays are created by accelerating electrons through a potential difference.

Fig. 8.44



The electron beam begins at a negative source plate, the **cathode**, and passes through an evacuated area (created with a vacuum pump) to a positive plate, the **anode**. An accelerating anode, a focusing cylinder, and horizontal and vertical deflection plates were later added to the CRT to direct and control the beam to a phosphorescent screen (Figure 8.45).

Fig. 8.45



When the electrons hit the phosphorescent screen, they produce light in the visible part of the spectrum. This technology is the basis for devices such as the simple oscilloscope (Figure 8.46) and the standard TV picture tube (Figure 8.47).

Fig.8.46 An oscilloscope



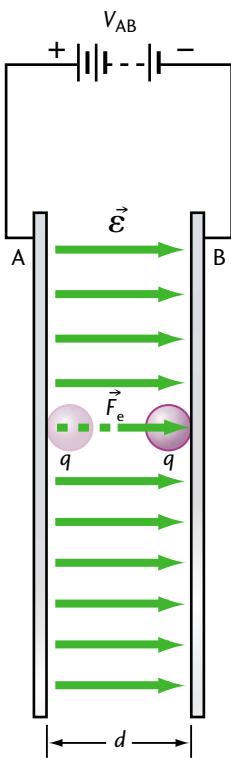
Fig.8.47 A TV



5. A potential difference of 20 kV is used to accelerate electrons in the electron gun of a cathode ray tube.
- How much kinetic energy do these electrons have when they leave the gun?
 - What is the speed of these electrons?

8.9 The Electric Field Strength of a Parallel-plate Apparatus

Fig.8.48 The movement of a charge in a parallel-plate apparatus



Recall from Section 8.6 that electric field strength can be determined from the electric force experienced by a charge, q , in an electric field. In Figure 8.48, a charge q is being forced against the electric field from the negative plate to the positive plate.

$$F_e = q\mathcal{E}$$

The work done to push this charge against the field is

$$W = F_e d = q\mathcal{E} d$$

The change in electrical potential energy is $\Delta E = qV = W$

Therefore,

$$q\mathcal{E} d = qV$$

and the magnitude of the uniform field strength anywhere within a parallel-plate apparatus is given by the equation

$$\mathcal{E} = \frac{V}{d}$$

where \mathcal{E} is the field strength in N/C, V is the potential difference applied across the two plates, in volts, and d is the distance between the plates, in metres. The direction of the field strength is from the positive plate to the negative plate.

This equation illustrates that the field strength is independent of the charge's position between the plates. The parallel-plate apparatus is used in situations requiring uniform field strength, such as an electrical microbalance, used to measure elementary charge.

From the equation $\mathbf{E} = \frac{V}{d}$, another possible unit for field strength in a parallel-plate capacitor is V/m.

Elementary Charge

Early researchers of electricity, like Benjamin Franklin and Charles Augustin de Coulomb, thought that objects obtained their charge by transferring particles to other objects. Franklin considered electricity to be the flow of positive charge through a conductor. The coulomb was the unit of charge assigned to represent a reproducible amount of elementary charge units and was widely adopted in the scientific community. But it wasn't until an experiment performed by Robert A. Millikan (1868–1953) that anyone had been able to verify the existence of elementary charges, let alone decide how many of them constituted one coulomb of charge. Millikan's experiment was performed over a seven-year period (1906–1913) and is a perfect example of how different aspects of physics can be combined to solve a problem.

Millikan's experiment consisted of two parts. In the first part, Millikan used an oil-drop apparatus (Figure 8.49) to determine the charge, in coulombs, on an oil droplet (a small charged particle). In the second part, Millikan tried to determine the smallest possible charge that an oil droplet could have. He did so by measuring the charges of a great many oil droplets to show that they were multiples of the smallest discrete unit of charge, called the **elementary charge**.

Figure 8.50 illustrates how the basic principle of the balance of forces lies at the heart of this experiment. Any charged particle experiences a downward force of gravity, F_g , due to its mass, and an electric force, F_e , when it exists inside an electric field. By applying a specific electric field, the upward electrostatic force can be adjusted to balance the downward gravitational force such that \vec{F}_e [up] = \vec{F}_g [down].

$$F_e = F_g$$

$$\text{But } F_e = q\mathbf{E} \text{ and } F_g = mg$$

Therefore,

$$q = \frac{mg}{\mathbf{E}}$$

From this equation and using the method summarized in Table 8.5, Millikan calculated the charge on a single oil droplet.

Fig.8.49 Millikan's oil-drop apparatus

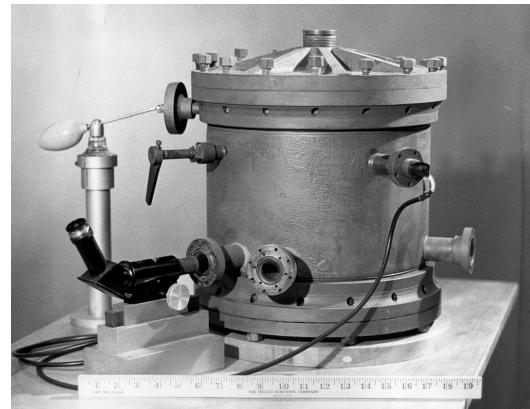


Fig.8.50 Millikan used a horizontally oriented parallel-plate capacitor to create the electric force because of its uniform electric field and the ease of controlling and calculating the electric field strength

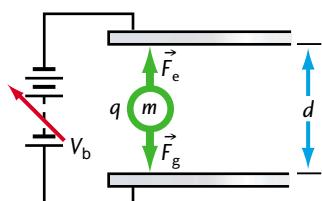


Table 8.5
Millikan's Method

Equation	Have	Need	Strategy
$q = \frac{mg}{\mathcal{E}}$	$g = 9.8 \text{ N/kg}$	m, \mathcal{E}	Find \mathcal{E} from parallel-plate apparatus. $\mathcal{E} = \frac{V}{d}$, where V is the potential difference between the plates and r is the plate separation.
$q = \frac{mgd}{V}$	g, V, d	m	<p>The mass of an oil droplet can be determined from its density (ρ_{oil}) and volume (V) (sphere):</p> $m = \rho_{\text{oil}}V$ <p>The volume of a sphere is determined from Stokes' law, which identifies the terminal velocity (v_t) of a small sphere in a fluid. $v_t = \left(\frac{2}{9}\right)\left(\frac{\rho_{\text{oil}} - \rho_{\text{air}}}{\eta}\right)\left(\frac{r^2 g}{\pi}\right)$ where r is the radius of the spherical oil droplet, g is the gravitational field strength, η is the viscosity of the fluid (air), and ρ_{oil} and ρ_{air} are the densities of oil and air, respectively. Stokes' law rearranged for radius is</p> $r = \sqrt{\frac{9(\eta v_t)}{2(g)}}(\rho_{\text{oil}} - \rho_{\text{air}})$ <p>The volume of a sphere is $V = \frac{4}{3}\pi r^3$, so the mass becomes</p> $m = \rho_{\text{oil}}V$ $m = \rho_{\text{oil}}\left(\frac{4}{3}\pi r^3\right)$ $m = \frac{4}{3}\pi\rho_{\text{oil}} \left[\frac{9}{2}\left(\frac{\eta v_t}{g}\right)(\rho_{\text{oil}} - \rho_{\text{air}})\right]^{\frac{3}{2}}$
$q = \frac{4}{3}\pi\rho_{\text{oil}} \left[\frac{9}{2}\left(\frac{\eta v_t}{g}\right)(\rho_{\text{oil}} - \rho_{\text{air}})\right]^{\frac{3}{2}} \frac{gd}{V}$	$g, V, d, \rho_{\text{oil}}, \rho_{\text{air}}, \pi, \eta$	v_t	To find the terminal velocity, v_t , of an oil droplet in free fall, Millikan devised a way of timing the free fall of the oil droplet through a specified distance:
$q = \frac{4}{3}\pi\rho_{\text{oil}} \left[\frac{9}{2}\left(\frac{\eta v_t}{g}\right)(\rho_{\text{oil}} - \rho_{\text{air}})\right]^{\frac{3}{2}} \frac{gd}{V}$			Knowing g , d , ρ_{oil} , ρ_{air} , π , η , Millikan balanced the forces on an oil droplet in his electrical microbalance and recorded the potential across the plates (V). Then he turned off the electric field and watched the oil droplet fall to determine its terminal velocity in air. Finally, he substituted all the parameters into the equation at left to solve for q , which enabled him to determine the charge on any oil droplet.

But the charge on an oil droplet isn't necessarily the smallest possible charge. The second part of Millikan's experiment consisted of determining the elementary charge from a statistical analysis of a series of charges. He looked for the smallest charge he had calculated, and theorized that all the other charges were integral multiples of this value. Table 8.6 illustrates a simplification of his analysis.

Based on Millikan's work, and other more precise experiments since then, the accepted value for the elementary charge, e , is

$$e = 1.602 \times 10^{-19} \text{ C}$$

Table 8.6
Millikan's Charge Analysis

Charge	Charge Proposed elementary charge	Integral number of charges
$8.0 \times 10^{-19} \text{ C}$	$\frac{8.0 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}}$	5
$1.4 \times 10^{-18} \text{ C}$	$\frac{1.4 \times 10^{-18} \text{ C}}{1.6 \times 10^{-19} \text{ C}}$	9
$4.8 \times 10^{-19} \text{ C}$	$\frac{4.8 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}}$	3
$1.6 \times 10^{-19} \text{ C}$	$\frac{1.6 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}}$	1
$6.4 \times 10^{-19} \text{ C}$	$\frac{6.4 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}}$	4
$3.2 \times 10^{-19} \text{ C}$	$\frac{3.2 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}}$	2
$1.6 \times 10^{-18} \text{ C}$	$\frac{1.6 \times 10^{-18} \text{ C}}{1.6 \times 10^{-19} \text{ C}}$	10

We can think of this number as a conversion factor between the charge, in coulombs, and the particle unit that carries charge, the electron. The conversion factor $1.602 \times 10^{-19} \text{ C}/e$ is a way of determining the number of elementary charge units, e , given any charge in coulombs.

The number of elementary charges on a charged object is given by the equation

$$N = \frac{q}{e}$$

where N is the number of elementary charges (protons or electrons), q is the charge in coulombs, and e is the elementary charge ($1.602 \times 10^{-19} \text{ C}$).

1. $2.4 \times 10^{-4} \text{ J}$ of work is required to move $6.5 \times 10^{-7} \text{ C}$ of charge between two points in an electric field. What is the potential difference between these two points?
2. What is the magnitude of the electric field strength between a parallel-plate apparatus of dimensions 0.75 cm with a potential difference of 350 V?
3. An oil droplet with a mass of $2.166 \times 10^{-15} \text{ kg}$ requires a potential difference of 530 V to just balance it against the force of gravity between two parallel plates 1.2 cm apart. What charge must the oil droplet have if the upper plate is negative?

The source of elementary charge, e , is either the electron ($-1e$) or the proton ($+1e$).

One electron has a charge of $-1.602 \times 10^{-19} \text{ C}$ and one proton has a charge of $+1.602 \times 10^{-19} \text{ C}$.

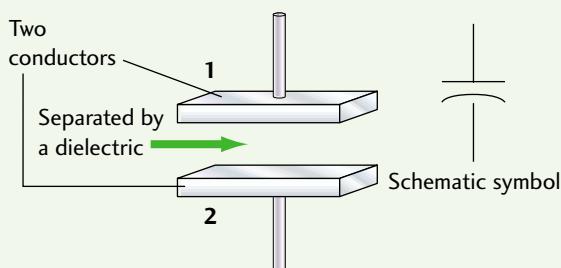




Electric Double-layer Capacitors

A capacitor is a device that is used to store electric charge. In its simplest form, it is made of two parallel metal plates separated by some dielectric material (an insulating material that transfers an electric field but not charge). (See Figure STSE.8.1.)

Fig. STSE.8.1 A capacitor consists of two parallel plates separated by a dielectric



As electrons pass onto plate 1 from the negative terminal of a direct-current (DC) power supply, the negative charge repels the electrons through the dielectric out of the opposite plate (2), making it positive. The positive plate, connected to the positive terminal of the power supply, also helps to extract electrons from itself as it attracts more electrons onto the original plate (1) through the dielectric, thereby storing charge.

Fig. STSE.8.2a A charging capacitor

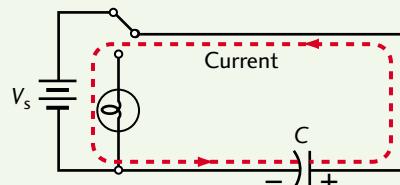
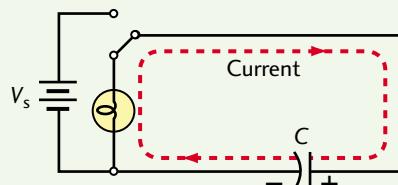


Fig. STSE.8.2b A discharging capacitor



Once the charge is stored by a mutual attraction of charge through the dielectric, it may be used to supply current to small applications, such as electronic camera flashes or small power supplies, as well as larger equipment requiring high-voltage and short-duration current pulses.

For a parallel-plate capacitor, capacitance, C , is calculated using the equation

$$C = \frac{kA}{d}$$

where k is the dielectric constant of proportionality, A is the area of the parallel charged plates, and d is the distance between them. Capacitance is measured in farads, F.

When an electric potential is applied across the conductive plates, capacitors store charge according to the equation

$$Q = CV$$

where Q is the charge and V is the voltage. Substituting C from the first equation into the second equation, we obtain

$$Q = \frac{kAV}{d}$$

The first capacitor was called a **Leyden jar**, named after a town in the Netherlands. The Leyden jar was invented in 1746 by Pieter Van Musschenbroek, a professor of mathematics. He placed de-ionized water (a dielectric) into a metal jar that acted as one electrode, covered the jar with a cork, and pushed a brass wire (a second electrode) through the cork into the water.

Electric double-layer capacitors (Figure STSE.8.3) store electrical energy as electric charge in the double layer formed in the phase boundary between the electrolyte and the electrodes. The use of new electrodes with a large surface area, such as activated carbon, has increased the energy storage capability of these capacitors so much that they are now used in electric vehicle design and electrochemistry. For example, a **flow-through capacitor** (Figure STSE.8.4) is now being used in water treatment applications. When water flows through the capacitor, the strong electric field draws the ionic materials (chemicals with an ionic charge) in the water, such as calcium and carbonate ions, toward the carbon electrodes. A short circuit then momentarily neutralizes the electrodes, allowing the contaminants to be released into a waste stream. The company Sabrex of Texas has designed and is marketing a device that uses a flow-through capacitor in electronic water purification (EWP) (see Figure STSE.8.5). This simple, low-power device ($\frac{1}{2}$ kWh of energy per US gallon (3.8 L)) can be used instead of other water purification systems like reverse-osmosis or ion-exchange systems. Ion-exchange systems remove ions from hard water, but end up softening the water by adding other ions such as sodium. This side effect is not evident with EWP systems.

Design a Study of Societal Impact

Research the health effects of drinking water softened using traditional salt-ion-exchange resin water softeners. Is soft water or other mineral-reduced water a healthy drinking alternative, or does it leach essential minerals out of the bodies of those who drink it? What beneficial effects does softened water have on the longevity of water heaters and piping, as well as on environmentally sensitive soaps and detergents?

Fig.STSE.8.3 A diagram of a double-layer capacitor

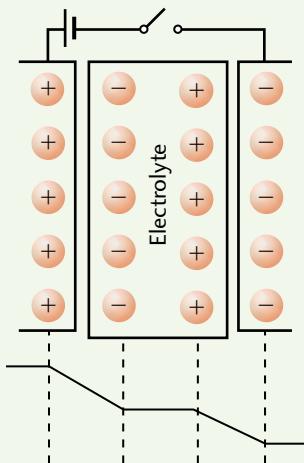


Fig.STSE.8.4 A flow-through capacitor

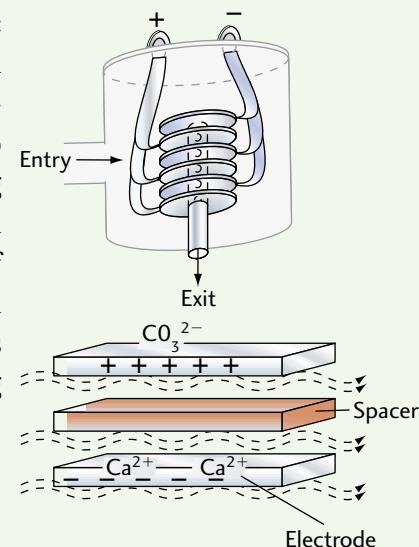


Fig.STSE.8.5 A Sabrex EWP device



Design an Activity to Evaluate

Evaluate commercially available capacitors for the amount of charge they can hold. Carefully dismantle an electronic camera flash from an inexpensive or disposable camera. Use the batteries supplied to charge the capacitor. Measure the electrical parameters of potential across the plates, as well as the capacitance, and determine the charge storage capacity of this capacitor.

Build a simple circuit that uses a capacitor. Use your circuit to explain how the capacitor works in a camera flash. Research capacitor circuits and experiment with circuit design to see how factors such as resistance affect the discharge time and current-generating capabilities of a capacitor.

Build a Structure

Build a capacitor using the Leyden jar or another type of capacitor as your model. Attempt to safely charge your capacitor. Build another device that is capable of producing a charge separation, such as a Van de Graaff electrostatic generator or a Whimshurst machine. You may use prepared electronic equipment such as hobby kits.

SUMMARY SPECIFIC EXPECTATIONS

You should be able to

Understanding Basic Concepts:

- Define and describe the concepts and units related to electric and gravitational fields, including electric and gravitational field strengths and potential energy.
- State Coulomb's law and Newton's universal law of gravitation qualitatively and compare them.
- Apply Coulomb's law and Newton's universal law of gravitation quantitatively in specific situations.
- Compare and contrast the properties of electric, gravitational, and magnetic fields by describing and illustrating the source and direction of the field in each case.
- Compare the characteristics of electric potential energy with those of gravitational potential energy, and apply the concept of electric potential energy in a variety of situations.
- Illustrate, using field and vector diagrams, the electric field and the electric forces produced by a single point charge, two point charges, and two oppositely charged parallel plates.
- Analyze, in quantitative terms, the electric force required to balance the gravitational force on an oil drop, or on latex spheres between parallel plates.
- Describe and explain, in qualitative terms, the electric field that exists inside and on the surface of a charged conductor, such as a coaxial cable.

Developing Skills of Inquiry and Communication:

- Demonstrate the balancing of electrostatic and gravitational forces on charged latex spheres, and collect, analyze, and interpret the quantitative data to demonstrate the presence of a smallest unit of charge.
- Explain the properties of electric fields and demonstrate how an understanding of these properties can be applied to control or alter the electric field around a conductor, such as a coaxial cable.

Relating Science to Technology, Society, and the Environment:

- Explain how the concept of a field developed into a general scientific model that could be used to explain force at a distance in electrostatic and gravitational situations.
- Describe how scientific theories, such as the structure of the atom or the existence of a minimum electric charge, evolved from experimentation involving many different scientific principles.
- Evaluate the social and economic impacts of new technologies, such as a flow-through capacitor.

Equations

$$F = \frac{kq_1q_2}{r^2}$$

$$\frac{F_1}{F_2} = \frac{(q_{1-1}q_{2-1})r_2^2}{(q_{1-2}q_{2-2})r_1^2}$$

$$\vec{F}_e = q\vec{\mathcal{E}}$$

$$\mathcal{E} = \frac{kq_m}{r^2}$$

$$\frac{W_{12}}{q} = \frac{E_{e2}}{q} - \frac{E_{e1}}{q} = \frac{\Delta E_e}{q}$$

$$\Delta V = \frac{\Delta E_e}{q} = \frac{W_{12}}{q}$$

$$\frac{1}{2}mv^2 = q(V_1 - V_2)$$

$$E_e = \frac{kq_1q_2}{r}$$

$$\text{Parallel plates: } \mathcal{E} = \frac{V}{d}$$

$$\text{Point charge: } V = \frac{kq}{r}$$

EXERCISES

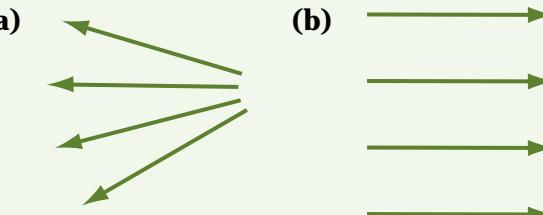
Conceptual Questions

1. Explain why a neutral object can be attracted to a charged object. Why can this neutral object not be repelled by a charged object?
2. What is the function of an electroscope?
3. When you rub a balloon against your hair on a dry day, you can stick the balloon to the ceiling. Explain what happens in terms of charge separation, using a diagram.
4. When two substances, such as acetate and silk, are rubbed together, electrons move from one substance to the other. Explain what happens in terms of basic atomic theory.
5. A new solid material is being tested for its electrostatic properties. Describe how you would test this material to determine its place in an updated electrostatic series.
6. A computer technician always touches the metal body of a computer before touching any of its electronic parts. Why? Explain using your knowledge of electrostatics.
7. Use the table below to compare and contrast Newton's universal law of gravitation and Coulomb's law.

Criterion	Newton's law of universal gravitation	Coulomb's law
Equation		
Constant of proportionality		
Type of force(s)		
Conditions for use		

8. Why can't electric field lines cross?
9. In which direction do charges always move in an electric field?
10. An insulating rod has a charge of $+q$ at one end and a charge of $-q$ at the other end. What will the rod tend to do when placed inside a uniform electric field oriented
 - a) perpendicular to the rod?
 - b) parallel to the rod?
11. Eight negative point charges of equal magnitude are distributed evenly around a circle. Sketch the electric field in the region around and within this charge distribution. Explain how this charge distribution can be used to model the electric field inside a coaxial cable.
12. If a test charge is moved from one area in an electric field to another area along an equipotential line, how much work is done on the charge? If a constant force is applied to move the test charge, what happens to the charge's speed?
13. Why do we use the term "point charge" when studying electric fields? How would our study be affected if we used charged bodies with large dimensions?
14. Figure 8.51 shows two sets of electric field lines. Use a table like the one below or another means of recording your answer to summarize the given true-or-false statements. Explain your reasoning in each case.

Fig. 8.51

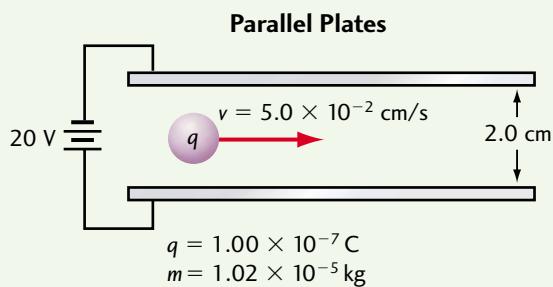


Statement	True?	False?	Reasoning
In each case the field gets stronger as you proceed from left to right.			
The field strength in (a) increases from left to right but in (b) it remains the same everywhere.			
Both fields could be created by a series of positive charges on the left and negative ones on the left.			
Both fields could be created by a single positive point charge placed on the right.			

- 15.** Although there are similarities between electric and gravitational fields, electric fields are more complicated to work with. Support this statement with evidence from the textbook.
- 16.** Describe the field shape around a single negative point charge.
- 17.** If you were to double the magnitude of a test charge used to map an electric field, what would happen to the strength of the electric field that you were mapping?
- 18.** How can you tell the difference between a weak electric field and a strong electric field?
- 19.** Compare and contrast the various aspects of an electric field and a gravitational field.
- 20.** What is the direction of an electric field between a positive and a negative charge?
- 21.** Explain why the electric potential energy between two like charges is greater than for two unlike charges the same distance apart.
- 22.** If a high-voltage wire falls onto a car, will the people inside be safe from electrocution? Under what conditions would electrocution not occur?
- 23.** When a parallel-plate apparatus is connected to a power supply, one plate becomes positively charged and the other plate becomes negatively charged. What is the net charge on the apparatus? Explain your answer.
- 24.** What would happen to the uniform field strength inside a parallel-plate capacitor if the following changes were made independently of each other?
- The distance between the plates is doubled.
 - The charge on each plate is doubled.
 - The plates are totally discharged and neutral.
- 25.** What point charges of similar magnitude should be placed side by side so that both the electric field strength and the potential are zero at the midpoint of the distance between them? Where would the field strength and potential be zero if one of the two charges was twice the magnitude of the other?
- 26.** No electric field means a field strength and a potential of zero. Use your discussion of question 25 to describe the conditions necessary for both the field strength and the potential to be zero at a point in the presence of electric fields.
- 27.** A proton and an electron are released from rest a distance apart and allowed to accelerate toward each other. Just before collision, which particle is travelling faster? Explain.

- 28.** A parallel-plate capacitor is mounted horizontally and a charge is released into it at a constant speed of 5.0×10^{-2} cm/s, as shown in Figure 8.52.

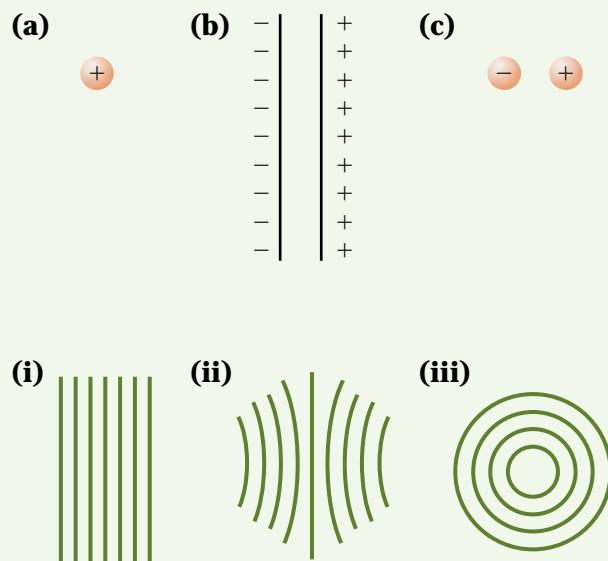
Fig.8.52



In your notebook, sketch the path of the moving charge as it passes between the plates. Where do we see this type of motion around us?

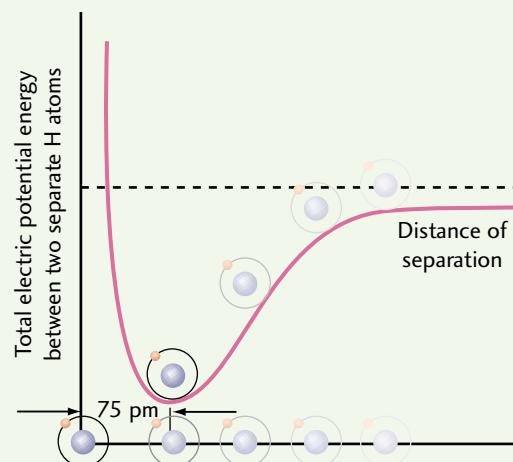
- 29.** Does a parallel-plate capacitor have uniform potential as well as field strength? If not, is there any path that a charge can take where the potential is uniform (does not change)? If so, what is the path called?
- 30.** Match each charge distribution in Figure 8.53 with the appropriate set of equipotential lines below. Use the equipotential lines as clues to drawing the field lines for each charge distribution.

Fig.8.53



- 31.** One of the simplest chemical bonds is a covalent bond between two hydrogen atoms to make the molecule H₂. Figure 8.54 illustrates the electric potential energy between two separate hydrogen atoms.

Fig.8.54



- a)** What electrostatic interactions cause the large increase in E_e when the two nuclei are brought very close together?
- b)** Why does this increase in E_e cause design problems for engineers in the nuclear energy industry?
- c)** Pushing these atoms together increases their potential energy, but so does pulling them apart. What electrostatic interactions cause this smaller increase in E_e ?
- d)** How would you use the concepts of potential energy and forces to explain why two hydrogen atoms can form a stable bond at a distance of 75 pm apart?
- 32.** A potential-energy curve is like a topographic map of a mountain or valley road that a charge could "roll" on. What topography would be analogous to a positive test charge moving along a line between two identical negative point charges? How would the topography change
- a)** if the two point charges were positive?
- b)** if a negative test charge was placed between these two charges?

Problems

8.2 The Basis of Electric Charge — The Atom

33. Which part of the atom is represented by positive signs? by negative signs?
34. What is the charge on each of the following?
- a) A neutral oxygen atom
 - b) An electron
 - c) A nucleus
 - d) A neutron
 - e) A proton

8.3 Electric Charge Transfer

35. State which of the two items listed below is left with an overall positive or negative charge:
- a) A piece of rubber rubbed with silk
 - b) The silk from part a)
 - c) An acetate sheet rubbed with cat's fur
 - d) Glass rubbed with wool
36. A piece of amber is rubbed with fur.
- a) What type of charge is on the amber?
 - b) What particles are transferred between the amber and the fur?
37. A suspended glass rod is rubbed with a piece of silk.
- a) What type of charge is on each material after rubbing?
 - b) What happens if the silk is brought close to the glass rod?
38. State whether each of the following is an electric conductor or an insulator. Give a reason for your answer.
- a) Plastic food wrap
 - b) A lightning rod
 - c) A plastic comb
 - d) A party balloon stuck to a wall
 - e) A car's tire during a lightning storm
 - f) The rubber belt on a Van de Graaff electrostatic generator

39. A silk shirt is removed from a clothes dryer along with several pairs of wool socks. If the shirt attracts loose dog hair, what is the charge on the dog hair?

40. A metal-leaf electroscope is touched by a positively charged rod.

- a) What is the charge on the electroscope? Explain how the electroscope got this charge. What phenomenon causes this process to occur?
- b) What happens to the leaf (or leaves) of the electroscope? Explain.
- c) What happens to the leaves of the electroscope if the system is grounded?

41. A wire passes a charge of 15.0 C . How many electrons pass through the wire?

42. Small charges are measured in microcoulombs (μC). A shock of $1.1 \mu\text{C}$ is passed from one student to another in a dry physics classroom. How many electrons were transferred?

43. What is the charge on an electroscope that has a deficit of 4.0×10^{11} electrons?

44. A metal ball with a charge of 5.4×10^8 electrons is touched to another metal ball so that all the excess electrons are shared equally. What is the final charge on the first ball?

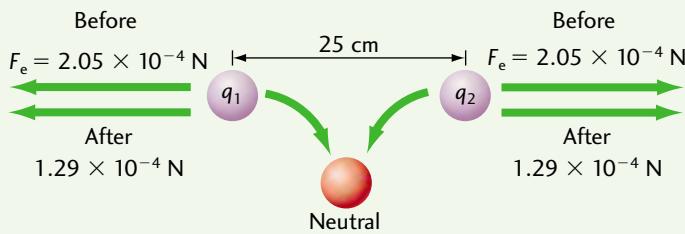
45. A nucleus has a charge of $+2.4 \times 10^{-12} \text{ C}$. How many electrons does this neutral atom have?

8.4 Coulomb's Law

46. Two small oppositely charged spheres experience a force of attraction of $1.4 \times 10^{-2} \text{ N}$. What would happen to this force if
- a) the distance between the charges is quadrupled?
 - b) the magnitude of the charge on each is doubled?
 - c) both (a) and (b) occurred simultaneously?

- 47.** Two small, similarly charged foam spheres experience a force F_{e1} when separated by a distance r_1 . Both spheres are touched with identical, electrically neutral spheres that are then removed. Where must these two spheres be moved in relation to each other in order to regain their initial force of repulsion?
- 48.** What force of repulsion exists between two electrons in a molecule that are 100 pm apart? (The charge on an electron is $1.602 \times 10^{-19} \text{ C}$.)
- 49.** Two small, identical foam spheres repel each other with a force of $2.05 \times 10^{-4} \text{ N}$ when they are 25.0 cm apart. Both spheres are forced to touch an identical, neutral third sphere that is then removed (see Figure 8.55). The two charged spheres now experience a force of $1.29 \times 10^{-4} \text{ N}$ when returned to their initial 25.0-cm separation.

Fig. 8.55



- a)** What is the charge on each sphere after contact with the neutral sphere?
- b)** What was the initial charge on each sphere before touching the neutral sphere? Does it matter if the charge is positive or negative?
- 50.** Two small, identical spheres, with an initial charge of $+q$ and $-3q$, respectively, attract each other with a force of F_{e1} when held a distance r apart. The two spheres are allowed to touch and are then drawn apart to the distance r . Now they repel with a force of F_{e2} . Find the ratio $\frac{F_{e2}}{F_{e1}}$ of the two forces. Describe what this ratio means in terms of magnitude and direction of the two forces, F_{e1} and F_{e2} .

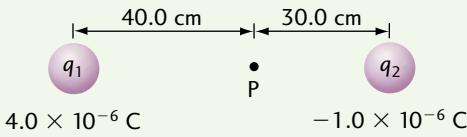
- 51.** A stationary proton holds an electron in suspension underneath it against the force of gravity ($m_{\text{electron}} = 9.1 \times 10^{-31} \text{ kg}$).
- a)** Draw a free-body diagram of this situation.
- b)** How far below the proton would the electron be suspended?
- 52.** A point charge of $+3.8 \times 10^{-6} \text{ C}$ is placed 0.20 m to the right of a charge of $-2.0 \times 10^{-6} \text{ C}$. What is the force on a third charge of $+2.3 \times 10^{-6} \text{ C}$ if it is placed
- a)** 0.10 m to the left of the first charge?
- b)** 0.10 m to the right of the second charge?
- c)** halfway between the first two charges?
- d)** Where would the third charge experience a net force of zero?
- 53.** Prove that a charge of $+q$ would come to rest with no net force on it $\frac{1}{3}$ of the way between two charges, $+q$ and $+4q$, that are held some distance apart.
- 54.** Three charges of $+1.0 \times 10^{-4} \text{ C}$ form an equilateral triangle with side length 40 cm. What is the magnitude and direction of the electric force on each charge?
- 55.** A square with side length 2.0 cm has a charge of $-1.0 \times 10^{-6} \text{ C}$ at every corner.
- a)** What is the magnitude and direction of the electric force on each charge? (Hint: Use the symmetry of the figure to simplify the problem.)
- b)** What is the force on a fifth charge placed in the centre of this square?
- c)** Does the sign of the fifth charge affect the magnitude or direction of force on it?
- 8.5 Fields and Field-mapping Point Charges**
- 56.** In your notebook, draw two small circles, about 5 cm apart, and label them with positive (+) signs. Use the concept of placing test charges on the page to map what the electric field around these charges would look like.

- 57.** How would the field map in problem 56 change if the charge on the left was tripled?
- 58.** In your notebook, draw two parallel lines representing metal plates, one positive and one negative, and a circle (negative) with a positive conductor in the centre (coaxial cable). Map the electric field around the two plate configurations.

8.6 Field Strength

- 59.** A positive charge of 2.2×10^{-6} C experiences a force of 0.40 N at a distance r from another charge, q_m . What is the field strength at this position?
- 60.** What is the magnitude of a test charge that experiences a force of 3.71 N in a field of strength 170 N/C?
- 61.** Two charges of $+4.0 \times 10^{-6}$ C and $+8.0 \times 10^{-6}$ C are placed 2.0 m apart. What is the field strength halfway between them?
- 62.** A point charge of 2.0×10^{-6} C experiences an electric force of 7.5 N to the left.
- What is the electric field strength at this point?
 - What force would be exerted on a -4.9×10^{-5} C charge placed at the same spot?
- 63.** What is the electric field strength (magnitude and direction) 0.5 m to the left of a point charge of 1.0×10^{-2} C?
- 64.** What is the electric field strength (magnitude and direction) at point P between the two charges in Figure 8.56?

Fig. 8.56



- 65.** In a hydrogen atom, the electron and the proton are separated by an average distance of 5.3×10^{-11} m. What is the field strength from the proton at the position of the electron?
- 66.** Two charges of $+1.5 \times 10^{-6}$ C and $+3.0 \times 10^{-6}$ C are 0.20 m apart. Where is the electric field between them equal to zero?
- 67.** Four charges of $+1.0 \times 10^{-6}$ C are at the corners of a square with sides of length 0.5 m. Find the electric field strength at the centre of the square.
- 68.** What is the electric field strength at the vertex of an equilateral triangle with sides of 0.50 m if the charges at the other vertices are $+2.0 \times 10^{-5}$ C?

8.7 Electric Potential and Electric Potential Energy

- 69.** A particle with a charge magnitude 0.50 C is accelerated through a potential difference of 12 V. How much work is done on the particle?
- 70.** A 6.0-V battery does 7.0×10^2 J of work while transferring charge to a circuit. How much charge does the circuit transfer?
- 71.** A charge of 1.50×10^{-2} C experiences a force of 7.50×10^3 N over a distance of 4.50 cm. What is the potential difference between the initial and final position of the charge?
- 72.** How much work is done by a system in which a field strength of 130 N/C provides a force of 65 N through a potential difference of 450 V?
- 73.** What is the electric potential 0.30 m from a point charge of $+6.4 \times 10^{-6}$ C?
- 74.** A small mobile test charge of magnitude -1.0×10^{-6} C is forced toward a stationary charge of -5.0×10^{-6} C.
- How much electric potential energy does the test charge have 0.25 m away from the stationary charge?

- b)** How much work was done on the charge to move it from an original distance of 1.00 m away?

8.8 Movement of Charged Particles in a Field — The Conservation of Energy

75. What is the electric potential two-fifths of the way through a parallel-plate apparatus (from the positive plate) if the plates have a total separation of 5.0 cm and a field strength of $5.0 \times 10^3 \text{ N/C}$?
76. A particle carrying a charge of 10^{-5} C starts moving from rest in a uniform electric field of intensity 50 N/C .
- What is the force applied to the particle?
 - How much kinetic energy does the particle have after it has moved 1.0 m?
 - If the particle's speed is $2.5 \times 10^4 \text{ m/s}$ at this point, what is its mass?
77. Two electrons are 10^{-9} m apart when they are released. What is their speed when they are 10^{-8} m apart?
78. How does doubling the accelerating voltage of the electron gun in a cathode ray tube affect the speed of the electrons that reach the screen?
79. The electron gun of a TV picture tube has an accelerating potential difference of 15 kV and a power rating of 27 W.
- How many electrons reach the screen per second?
 - What speed does each electron have?
80. The electrodes in a neon sign (Figure 8.57) are 1.2 m apart and the potential difference across them is $7.5 \times 10^3 \text{ V}$.
- What is the acceleration of charge ($+e$) of a neon ion of mass $3.3 \times 10^{-26} \text{ kg}$ in the field?
 - How much energy does the ion gain if it is released from a positive electrode and accelerates directly to the negative electrode?

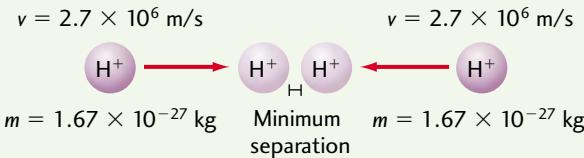
- c)** Do you think an ion would really gain this much energy? Explain your reasoning.

Fig.8.57



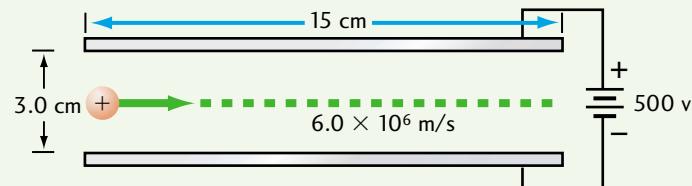
81. To start a nuclear fusion reaction, two hydrogen atoms of charge $+1e$ and mass $1.67 \times 10^{-27} \text{ kg}$ must be fired at each other. If each particle has an initial velocity of $2.7 \times 10^6 \text{ m/s}$ (Figure 8.58) when released, what is their minimum separation?

Fig.8.58



82. An alpha particle with a speed of $6.0 \times 10^6 \text{ m/s}$ enters a parallel-plate apparatus that is 15 cm long and 3.0 cm wide, with a potential difference of 500 V (see Figure 8.59).

Fig.8.59



- How close is the particle to the lower plate when it emerges from the other side?
- What is the magnitude of the velocity of the alpha particle as it leaves the plates? (Hint: Find the vertical and horizontal components of velocity first.)

8.9 The Electric Field Strength of a Parallel-plate Apparatus

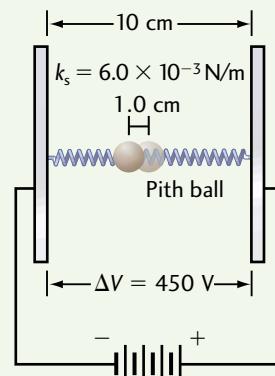
- 83.** A set of parallel plates, separated by a distance of 0.050 m, has a potential difference of 39.0 V. What is the field strength?
- 84.** The electric field strength between two plates 6.35 cm apart is 2.85×10^4 N/C. What is the potential difference between them?
- 85. a)** How strong an electric field is required to support an alpha particle (a $2+$ charged helium nucleus with two protons and two neutrons) against the force of gravity?
($m_{\text{proton}} \approx m_{\text{neutron}} = 1.67 \times 10^{-27}$ kg)
b) If the alpha particle is suspended between a set of parallel plates 3.0 cm apart, what potential must be provided across the plates?
- 86.** What is the electric field strength of a parallel-plate apparatus that has a plate separation of 0.12 m and a potential difference of 92 V?
- 87.** An electric field stronger than 3×10^6 N/C causes a spark in air. What maximum potential difference can be applied across two metal plates 1.0×10^{-3} m apart before sparking begins?
- 88.** A potential difference of 50 V is applied across two parallel plates, producing an electric field strength of 10^4 N/C. How far apart are the plates?
- 89.** The potential difference applied to an adjustable parallel-plate capacitor is 120 V. What is the plate separation if the field strength is 450 N/C?
- 90.** An oil droplet of mass 2.2×10^{-15} kg is suspended between two horizontal parallel plates that are 0.55 cm apart. If a potential difference of 280 V is applied,

- a)** what is the charge on the droplet?
b) how many electrons, in excess or deficit, does the droplet have?

- 91.** An electron is released from rest from the negative plate of a parallel-plate apparatus.
a) At what speed will the electron hit the positive plate if a 450-V potential difference is applied?
b) What is the electron's speed one-third of the way between the plates?
- 92.** A foam pith ball is supported by two small springs ($k_s = 6.0 \times 10^{-3}$ N/m) between two vertical parallel plates 10 cm apart, as shown in Figure 8.60. When the potential across the plates is 450 V, the pith ball moves 1.0 cm to the right before coming to rest. Ignoring any effects due to gravity and friction,

Fig. 8.60

Parallel plates



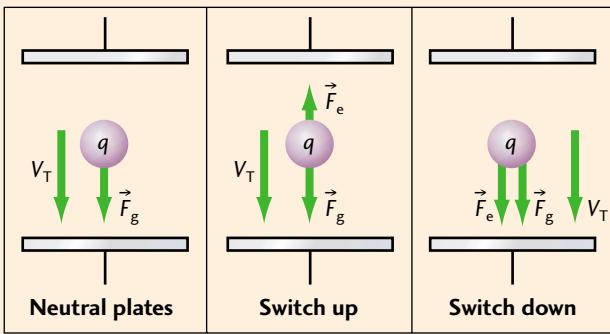
- a)** what is the field strength between the two plates?
b) what force changes the length of the two springs?
c) what is the magnitude of the force acting on the foam pith ball?
d) what is the charge on the pith ball?

LAB 8.1

The Millikan Experiment

Introduction

In this lab, we will study small, discrete units of charge and statistically analyze the number of elementary charges on small latex spheres. We will assume that small latex spheres reach terminal velocity quickly when moving through a fluid such as air, and that the terminal velocity of each sphere is directly proportional to the total force acting on the sphere. Figure Lab.8.1 shows the three different force situations that each sphere will experience.

Fig.Lab.8.1**Purpose**

To measure the smallest unit of electrical charge and to compare this experimental value with the one accepted by the scientific community

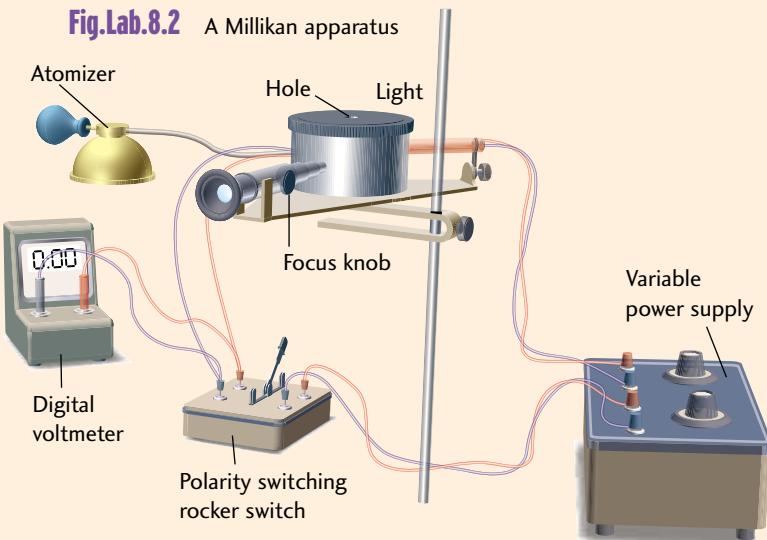
Equipment

A Millikan apparatus that uses latex spheres (available from most scientific supply companies) (Figure Lab.8.2)

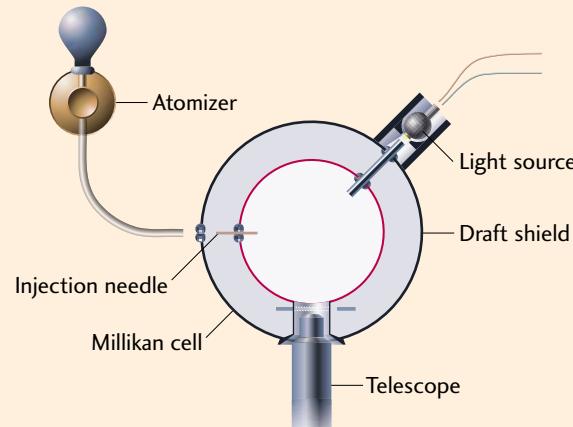
Supply of latex spheres of known diameter

High-voltage reversible power supply

Stopwatch

Fig.Lab.8.2 A Millikan apparatus**Safety Consideration**

This apparatus uses a high-voltage power supply (about 200 V). Be sure that the power supply is unplugged and in the “off” position before connecting the wires of the apparatus to it. Working with the power supply turned on could cause sparking.

Fig.Lab.8.3**Table Lab.8.1**

Trial #	Time _{g+e} (s)	v _{g+e} (spaces/s)	Time _{g+e} (s)	v _{g-e} (spaces/s)	v _{g+e} + v _{g-e} (spaces/s)	v _{g+e} - v _{g-e} (spaces/s)	Number of elementary charges
1 ↓ 20	↓	↓	↓	↓	↓	↓	↓

Procedure

- Set up the Millikan apparatus so that looking through the telescope for an extended period of time won't cause you discomfort. Be sure that your data table (like Table Lab.8.1), stopwatch, latex spheres, and the controls to the power supply are within reach.
- With the power supply turned off and unplugged, connect the low-voltage line of the power supply to the viewing light of the apparatus, and the two high-voltage lines to the proper terminals on the polarity-reversing switch of the Millikan apparatus. This switch controls whether the plates are neutral or charged. Fill the latex sphere reservoir with the properly diluted solution and verify all set-up parameters against the written instructions provided with your equipment.
- Turn on the light bulb and adjust the filament so that it sits vertically as it illuminates the field of view. With the light tilted slightly up over the apparatus, adjust it so that a sharp image of the filament forms on your finger if you hold it above and off-centre of the apparatus. Point the light back down. Bring the sphere injection needle into view while adjusting the final focus of the viewing telescope.
- With the stopwatch ready and the polarity switch in the centre position (neutral), squeeze the bulb of the latex sphere reservoir once very quickly to inject a cloud of latex spheres into the apparatus. The spheres should appear to rise because of the inverted image from the telescope. Select one sphere to watch consistently over the next few moments. Time this sphere *for one trial only* with the stopwatch as it falls under the influence of gravity over about 10 spaces.

Tip: Choose the spheres that are slow-moving because they have a very small charge on them. You can eliminate the highly charged spheres after injection by moving the polarity switch quickly from the neutral to the up position, then to the down position before returning it to the neutral setting.

- Place the switch in the up position and time the sphere over about 10 spaces. Turn the switch to the down position (reverse the plate polarity) and time it again over the same number of spaces.
- Squeeze the bulb again to inject another cloud of spheres. Choose another sphere and repeat the same two measurements. Repeat steps 5 and 6 for a minimum of 20 different spheres.

Data

- Time one of the spheres as it falls through 10 spaces under the influence of the force of gravity only (plates neutral).
- For each trial, measure and record the time it takes for each sphere to move about 10 spaces
 - up with the gravitational and electric forces in the same direction
(top plate negative).
 - up or down with the gravitational and electric forces in opposite directions
(bottom plate negative).

Record both times in the data table. Then calculate the speed for each motion, in spaces per second, using the equation $v = \frac{d}{t}$. For example, if the sphere travelled 10 spaces in 3.5 s, then $v = \frac{10 \text{ spaces}}{3.5 \text{ s}} = 2.8 \text{ spaces/s}$.

Uncertainty

The only uncertainty for this lab is the time recorded using the stopwatch. Assign an appropriate reflex and instrumental uncertainty for the time measurement. For this lab, all time measures should have the same uncertainty.

Analysis

- Calculate the sum and difference of the two sets of terminal velocities you measured for each trial sphere ($v_{g+e} + v_{g-e}$ and $v_{g+e} - v_{g-e}$), including the uncertainty. Using spreadsheet software will speed up your data analysis and graphing.
- Recall that the terminal velocities directly relate to the forces applied to the sphere.

$$v_{g+e} \propto F_g + F_e$$

$$v_{g-e} \propto F_g - F_e$$

Adding these two proportionality statements gives

$$v_{g+e} + v_{g-e} \propto (F_g + F_e) + (F_g - F_e)$$

Therefore,

$$v_{g+e} + v_{g-e} \propto 2F_g$$

The mass of each sphere, and therefore the force of gravity on it, should always be consistent. If a trial sphere has an inconsistent value for $v_{g+e} + v_{g-e}$, omit it from your analysis and mark it accordingly in your data table.

- $v_{g+e} - v_{g-e} \propto (F_g + F_e) - (F_g - F_e)$

Therefore,

$$v_{g+e} - v_{g-e} \propto 2F_e$$

F_e is dependent on charge, so $v_{g+e} - v_{g-e}$ can be used to measure the charge on the spheres. Rank the 20 (experimentally significant) trials from the lowest value of $v_{g+e} - v_{g-e}$ to the highest value of $v_{g+e} - v_{g-e}$.

- Draw a bar graph with the trial number along the horizontal axis, and scaled values of $v_{g+e} - v_{g-e}$ along the vertical axis.

Discussion

- You determined two terminal velocities for each trial sphere, one with the top plate negative and one with the top plate positive; but you measured the terminal velocity once only for the force of gravity (no potential, with the switch in neutral position) acting on the sphere. Why was only one measurement required for gravity?
- Why must a minimum of 20 trials be done for this lab?
- What evidence in this lab supports the existence of a small, discrete unit of charge?
- For the shortest bar on the bar graph, what is the smallest number of elementary charges that were measured within experimental uncertainty? For the tallest bar, what was the largest number of elementary charges?
- For every valid trial sphere, calculate the total number of elementary charges on each. Enter this number in your data table.

Conclusion

Summarize your findings in this lab. Were you able to produce evidence of a smallest discrete charge?



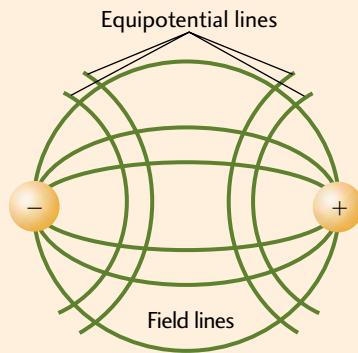
Mapping Electric Fields

Introduction

Unlike magnetic fields that are easy to demonstrate using iron filings, electric fields are very difficult to visualize or to represent. This lab uses conductive carbon paper and paint to set up and map electric fields. Instead of measuring forces on charges, an impossible task, we will examine the electric potential, which is measurable with a voltmeter at various places in the field.

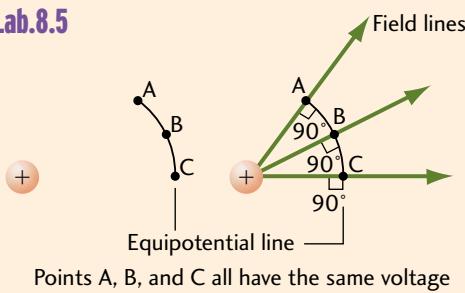
Electric field lines may be plotted indirectly by first examining the equipotential lines around the array of charges that are contributing to the electric field. Equipotential lines, like those illustrated in Figure Lab.8.4, are lines in three-dimensional space that mark positions where the electric potential is the same.

Fig. Lab.8.4



Field lines may be drawn at 90° (Figure Lab.8.5) to any equipotential line. To use an analogy, an equipotential line is like a wavefront and the field lines show the direction in which each part of the wave travels.

Fig. Lab.8.5



Purpose

To map out the shape and structure of electric fields created by different charge distributions

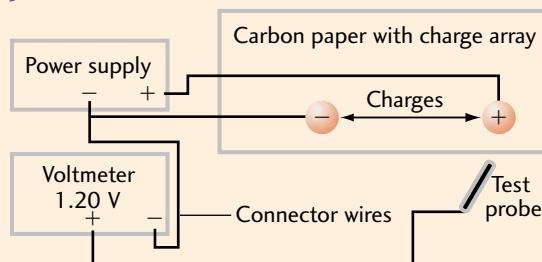
Safety Consideration

Keep the power supply in a safe location and at a low voltage setting. Ensure that all the wires are undamaged and fully insulated to prevent a short circuit.

Equipment

Variable- or constant-voltage power supply
2 alligator connectors
Carbon paper
Conductive ink
Metallic poster tacks
Digital voltmeter

Fig. Lab.8.6



Procedure

1. Use the conductive ink to paint different charge distributions on a piece of carbon paper, according to the manufacturer's instructions. Some suggestions are a two-point source, two parallel plates or a single point surrounded by a circle (coaxial cable). Your teacher may have some other suggestions for the shape of conductors or hand out pre-made carbon paper sheets.
2. Connect the power supply, wires, and voltmeter to the carbon paper, as shown in Figure Lab.8.6. Trace an image of the charge distribution on a blank worksheet.
3. Set the power supply voltage to 3–5 V for the remainder of the lab. This voltage can be verified with the voltmeter by touching the red test probe to the positive (+) terminal of the power supply at the point where it contacts the field map.

4. Map the field by touching the test probe to the carbon paper and then taking note of the potential difference on the voltmeter. Keep moving the test probe around the paper until you find the same potential at a different point. Find consistent places of equal potential all over the field map. Transfer an image of the equipotential line to your blank worksheet of the field map.
5. Start from one section of the field and work radially out to other charges. Find as many equipotential lines as possible and transfer them to your worksheet. **Note:** Trial and error is the key.
6. Connect all positions of equal potential by drawing best-fit curves on your worksheet. Trace at least ten equipotential lines in the centre of the field.
7. On each equipotential line, draw several short lines that cross at 90° .
8. Using your imagination and different colours than the ones you used to draw your field lines, draw field lines for the charge array such that they cross all the equipotential lines at 90° .

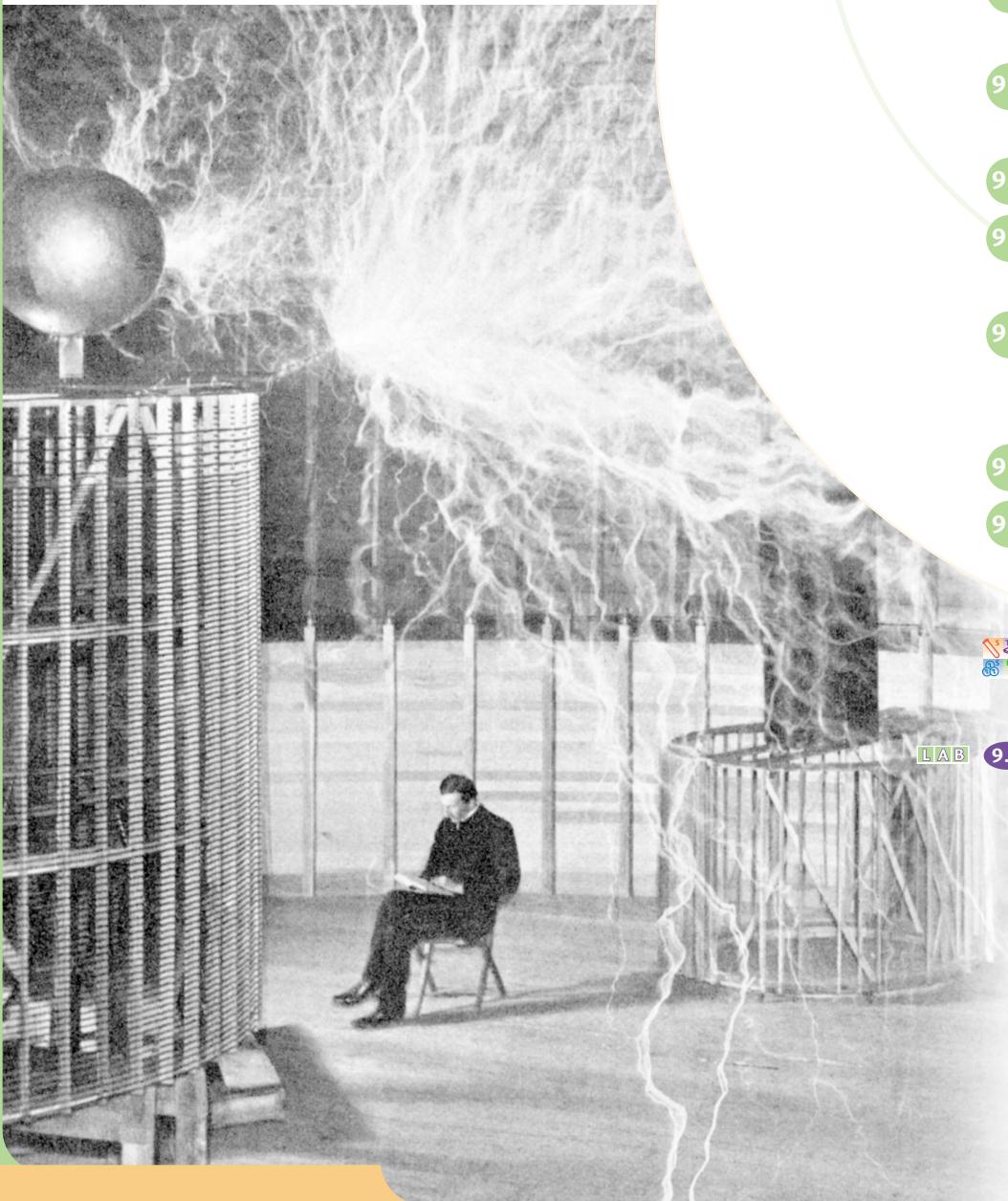
Questions

1. On your field map, indicate with a *W* or an *S* the two areas where the electric field is strongest (*S*) and weakest (*W*).
2. Show that your field map is correct by choosing any point on a field line and drawing an estimated free-body diagram of the force that a positive test charge would experience at that point.
3. Describe what would happen to the field diagram if the voltage of the power supply was increased.

Extension

Code the co-ordinates that represent your field shape into field-mapping software. This software is available to download from the Irwin Publishing Web site at <www.irwinpublishing.com/students> . Print out the field map generated by the computer and use it to complete your own field map.

Magnetic Fields and Field Theory



9.1 Magnetic Force — Another Force at a Distance

9.2 Magnetic Character — Domain Theory

9.3 Mapping Magnetic Fields

9.4 Artificial Magnetic Fields — Electromagnetism

9.5 Magnetic Forces on Conductors and Charges — The Motor Principle

9.6 Applying the Motor Principle

9.7 Electromagnetic Induction — From Electricity to Magnetism and Back Again

 Magnetic Resonance Imaging (MRI)

LAB 9.1 The Mass of an Electron

By the end of this chapter, you will be able to

- define the law of magnetic poles and apply it to mapping magnetic fields as another example of force at a distance
- quantitatively analyze the forces involved in the magnetic field of various magnets and electromagnets
- describe and apply the concepts described by scientists such as Oersted, Ampere, Faraday, and Maxwell in an attempt to unify the theories relating electricity, electromagnetism, and gravity

9.1 Magnetic Force — Another Force at a Distance

The term “magnetism” comes from Magnesia, a Greek province where naturally magnetic iron ore material is found. The ancient Greeks discovered magnetic forces when shepherds unknowingly magnetized the metallic tips of their shepherding staffs by contact with naturally occurring magnets in rock they called **lodestone**. Magnetic force, as illustrated by the floating magnets in Figure 9.1, is another example of a force that acts at a distance.

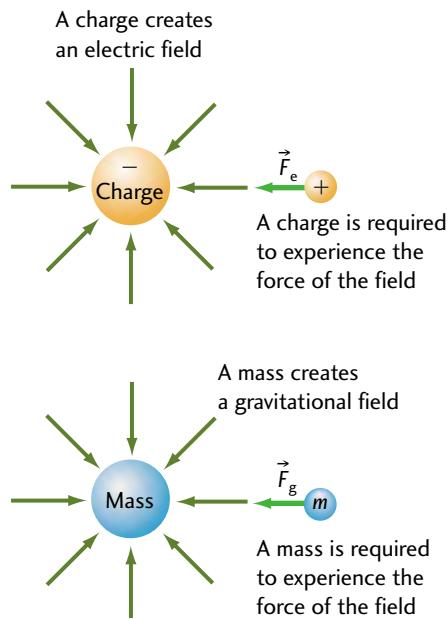
There are two characteristics that distinguish magnetic fields from electrostatic and gravitational fields. First, magnetic fields occur naturally in substances that have a magnetic character, such as iron, nickel, and cobalt. Magnetic character is created *from within matter* as a result of the internal make-up of a substance rather than by the mere *presence of matter*, as gravity is created by the presence of mass. Second, magnetic forces are more versatile than the other two types of forces because they can affect magnetic substances as well as electric charges.

In Chapter 8, we used field theory to describe and predict how forces at a distance act on objects. All force fields (gravitational, electrostatic, and magnetic) have two objects in common: an object that creates the field and another object that responds to the field. For electric fields, these objects are charges and for gravitational fields, they are masses. For magnetic fields, the objects are magnetic dipoles. Dipoles will be discussed in the next section. Figure 9.2 illustrates the connection between field creation and response for electrostatic and gravitational forces.

Fig.9.1 Repulsion by like magnetic poles makes the magnets float



Fig.9.2 Each field is created by an object, and an object of similar character experiences a force due to the field



9.2 Magnetic Character — Domain Theory

What gives a substance its magnetic character? Although this process is not entirely understood, it is related to a condition within the atoms that make up the magnetic material, which we will study in Section 9.4. We attribute the overt magnetic character of a substance to the presence of many smaller regions of magnetic character, called **domains**. As with positive and negative charges in electric fields, there are two opposite magnetic elements, neither of which can exist without the other. Domains are in turn made up of even smaller individual elements that are polar and exist as a unit called a dipole. By convention, the poles of the magnetic **dipole** are called *north* and *south* because of their opposite magnetic character (see Figure 9.3a).

Domain Theory: All large magnets are made up of many magnetic regions called *domains*. The magnetic character of domains comes from the presence of even smaller unit magnets called *dipoles*. Dipoles interact with their neighbouring dipoles. If they align with all the poles in one direction, then a larger **magnetic domain** is produced.

The north and south dipoles of large magnets are created by many microscopic dipoles, all acting in unison; that is, aligned in the same direction (see Figure 9.3b). Materials made of domains that can be readily aligned to create a larger object of magnetic character are called **ferromagnetic** materials, such as iron, nickel, and cobalt. Domains also experience a force in a magnetic field, described by the *law of magnetic forces*.

The Law of Magnetic Forces

Similar magnetic poles (north and north or south and south) *repel* one another with a force, even at a distance apart.

Dissimilar poles (north and south or south and north) *attract* one another with a force, even at a distance apart.

Figure 9.4a illustrates how magnets act according to the law of magnetic forces.

Fig.9.4a The law of magnetic forces

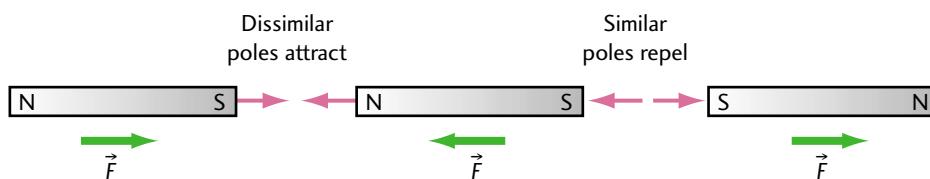
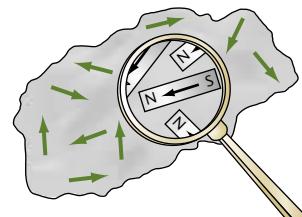


Fig.9.3a Large magnetic dipoles (domains) are created from smaller magnetic dipoles



A magnetic domain is an example of a “black box” explanation in science: we don’t really know how it works; we just know that it does and that it is useful in describing phenomena. Similarly, John Dalton’s atomic theory claimed that matter was made up of smaller forms of matter called atoms with very little explanation of what atoms were. In biology, cell theory states that the cell is the basic structural and functional unit of life, but it doesn’t explain how the individual cell functions.

Fig.9.3b A large magnetic dipole is created by the same orientation of its constituent dipoles

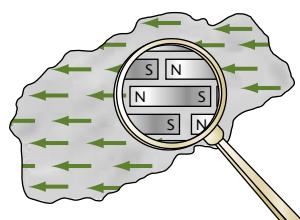
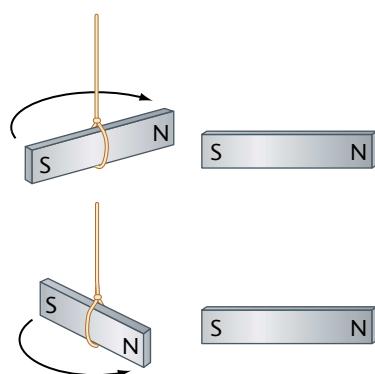


Fig.9.4b



A dipole not only creates a magnetic field, but also responds to the field by experiencing a magnetic force. Dipoles align by rotating with the magnetic force, as shown in Figure 9.4b.

Table 9.1 explains various magnetic phenomena in terms of domain theory.

Table 9.1
Magnetic Phenomena and Domain Theory

Observation	Explanation using domain theory
Magnetic induction: Ferromagnetic materials can be magnetized. Earth's magnetic field magnetizes railroad tracks and construction girders.	Domains pointing in random directions can be aligned if they are placed in a large magnetic field with a fixed direction.
Permanent and temporary magnetism: Some materials such as soft iron demagnetize very easily, but hard steel magnets maintain their magnetic character indefinitely.	Pure ferromagnetic material such as iron has domains that are easily manipulated, whereas the impurities of the harder steel alloys lock the domains, preventing them from changing position.
Demagnetization: Ferromagnetic materials can lose their magnetic strength.	Domains can lose their alignment and point in different directions, causing a dilution and overall weakening of the magnet.
Reverse magnetization: The polarity of magnets can be reversed.	A large external magnetic field pointing in the opposite direction of a particular magnet may cause all the domains to line up with the new field, reversing the magnet's overall polarity.
Breaking a large magnet: A large magnet can be broken into smaller active magnets.	Each piece of a broken magnet still possesses the aligned domains, allowing each domain to act as an independent magnet.
Maximum strength: There is a fundamental limit to how strong an individual magnet can be.	Once all or most of the domains are aligned, the magnet's strength cannot be increased any further.

Fig.9.5 The north end of this compass will point away from a north pole of a magnetic dipole that is creating the magnetic field

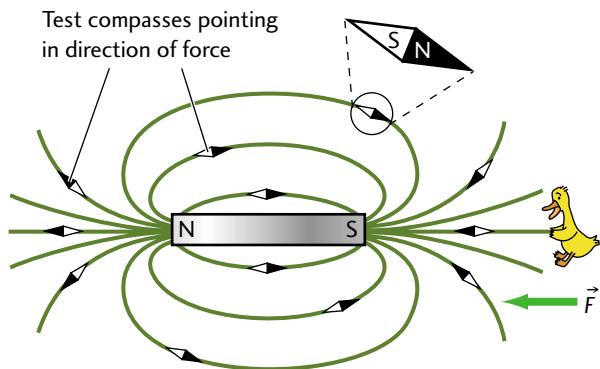


9.3 Mapping Magnetic Fields

Recall from Chapter 8 that to map an electric or gravitational field, we use a test charge or test mass, respectively. To map a magnetic field, we use a dipole that is large enough to respond to the magnetic field. A compass is a very small magnetic dipole that is allowed to freely rotate in a horizontal plane (Figure 9.5). It is used to detect the presence of a magnetic field by applying the law of magnetic forces. The *north end* of the compass is repelled by the *north pole* of any magnet creating the field.

In Figure 9.6, the **test compass** maps the field lines around a simple bar magnet by rotating its dipole to indicate the direction of force that its north end experiences in the magnetic field at that particular point in space. Magnetic field lines are drawn tangent to the compass needle at any point. The number of lines per unit area is proportional to the *magnitude* of the magnetic field. The *direction* of the magnetic field is defined as the direction

Fig.9.6 The field shows direction of force

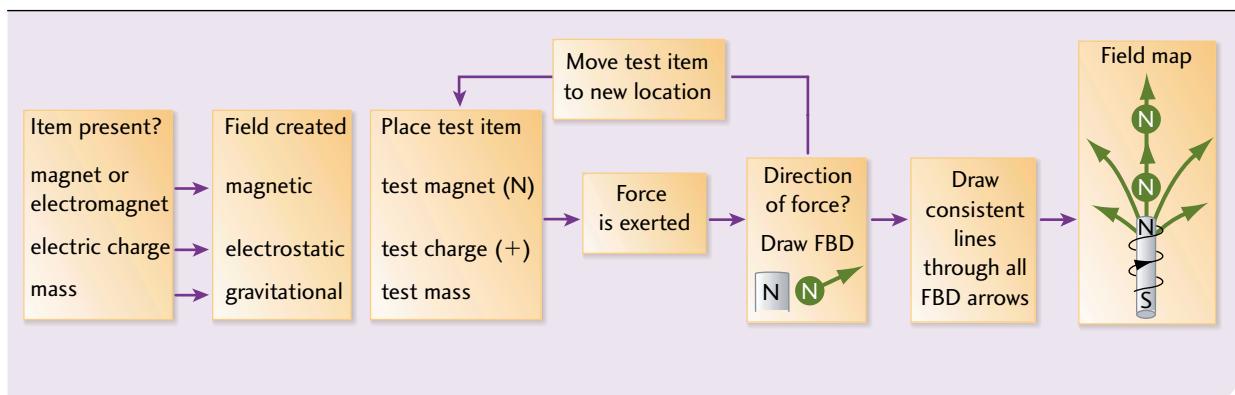


in which the north pole of a test magnet (compass) would point when placed at that location.

Figure 9.7 explains how to draw magnetic, electrostatic, and gravitational field maps.



Fig.9.7 Drawing Field Maps



Magnetic fields are so intricate that it is easier to map them by distributing some ferromagnetic material, such as iron filings, around the magnet. Using iron filings is equivalent to using many tiny compasses with undefined poles. Figures 9.8a and b show examples of magnetic fields, mapped using iron filings. Notice how the magnet is a dipole because each field line begins at one pole and flows to the corresponding point on the opposite pole. The law of magnetic forces dictates the convention that field lines flow from the north pole to the south pole of the field-creating magnet.

Fig.9.8a Magnetic field lines flow from one end of a dipole to the other end

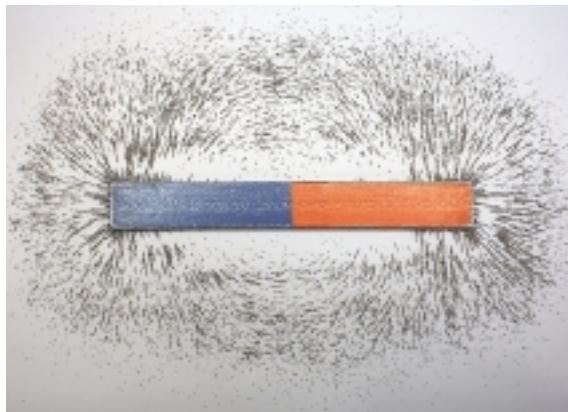


Fig.9.8b Field lines around a horseshoe magnet

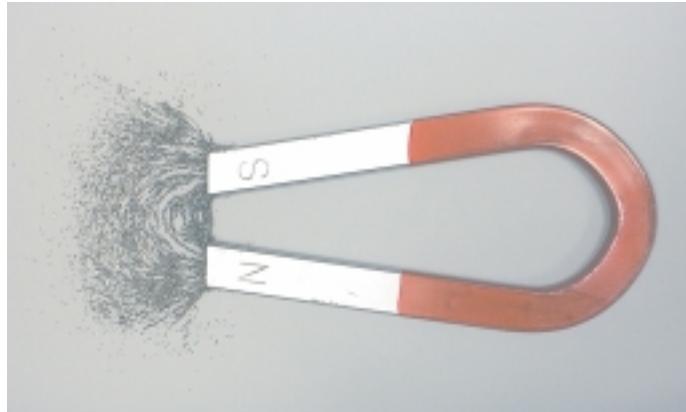


Fig.9.9a Earth's magnetic field

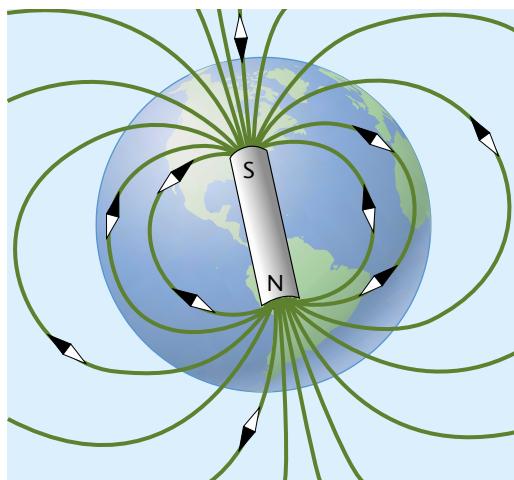
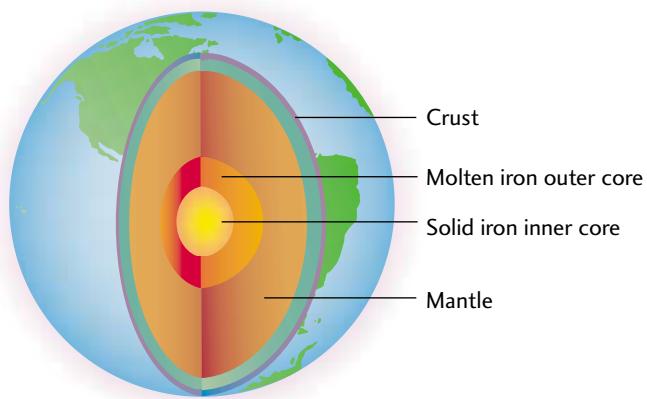


Fig.9.9b Earth's molten core creates Earth's magnetic field



By examining how solid iron from lava flow is magnetized, geologists have determined that the direction of Earth's magnetic field has changed in the past. Layers of Earth's crust from different eras show that the domain directions are in opposite directions. Earth's North Pole is really a magnetic south because the north end of a compass points to it.

Holding a compass horizontally away from any strong magnets allows the compass to align with Earth's magnetic field, as shown in Figure 9.9a. In 1600, English physicist Sir William Gilbert suggested that Earth's magnetic field is created by the flowing motion of hot liquid metals under Earth's crust, as shown in Figure 9.9b. This notion led scientists to a better understanding of the cause of magnetic character at the atomic level (see Section 9.4). The interaction between a compass (a free-spinning permanent magnet) and Earth's magnetic field has been of great importance for navigators around the world. Early exploration and cartography of Earth was made possible by using the compass to locate the cardinal directions: north, south, east, and west.

9.4 Artificial Magnetic Fields — Electromagnetism

In Section 9.1, we learned that magnets and their associated fields can be created by aligning naturally occurring magnetic dipoles (domain theory). In this section, we will study the atomic origin of the magnetic dipole.

The similarities of force at a distance between electrostatics and magnetism directed the research of many scientists such as William Gilbert (1540–1603) and Hans Christian Oersted (1777–1851). They spent much of their time trying to link these two forces together. Oersted is credited with the accidental discovery of the link between electricity and magnetism. During a lecture at the University of Copenhagen, he discovered that moving electric charges in a straight conductor create a magnetic field in the region around the conductor, as shown in Figures 9.10a and b.

Oersted's Principle: The Magnetic Field around a Straight Conductor

Charge moving through a straight conductor produces a circular magnetic field around the conductor. The field is represented by concentric rings around the conductor.

Figures 9.11a and b illustrate the shape and direction of the magnetic field around a straight conductor, as described by Oersted's principle.

Fig. 9.10 The compass aligns with the magnetic field around a current-carrying conductor

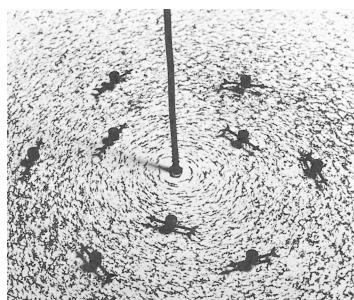
(a)



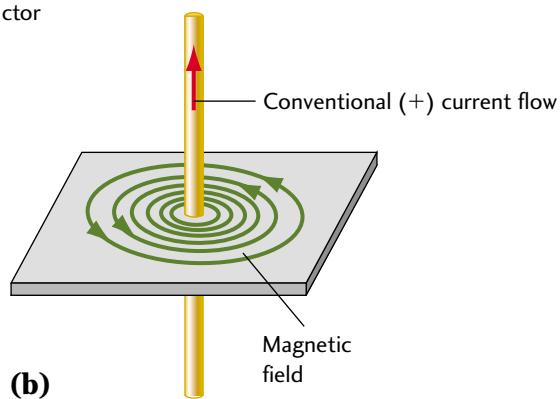
(b)



Fig. 9.11 Oersted's principle: The shape and direction of the magnetic field around a straight conductor



(a)

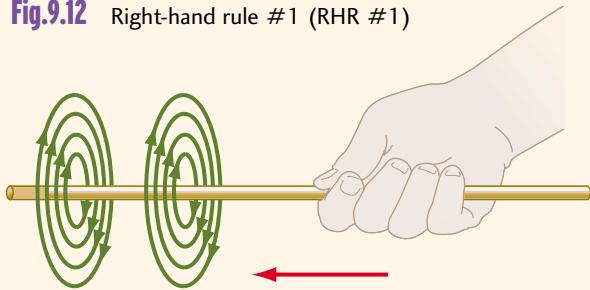


(b)

We now know that magnetic fields can be created in other ways besides the presence of a magnet with aligned dipoles and regional domains. Charge moving through a conductor produces a circular magnetic field around the conductor even in the absence of dipoles. Experimentation by Oersted led to the development of a series of hand signs that help us predict the directions of magnetic fields and forces. They are called **right-hand rules** because they require use of the right hand. Figure 9.12 summarizes the **first right-hand rule** for a straight current-carrying conductor.

Left-hand rules are applied when electron current is used. Right-hand rules assume conventional (positive, +) current flow.

Fig.9.12 Right-hand rule #1 (RHR #1)



Right-hand rule #1 (RHR #1): Grasp the conductor with the thumb of the *right hand* pointing in the direction of conventional, or positive (+) current flow. The curved fingers point in the direction of the magnetic field around the conductor.

At the time of Gilbert, Oersted, and Faraday, electric current was still considered to be the result of the flow of positive electric charge, as suggested by Benjamin Franklin. In materials such as solid metallic conductors, the positive entities (protons) are locked in a stationary lattice while the negatively charged electrons flow. The early chemistry of voltaic cells (batteries—the first known sources of current) involved liquid electrolytes in which both positive and negative charges flowed. Today, engineers and scientists still use the convention of positive charge flow, even when working with charge flow in a solid metallic conductor. It is equally correct to assume that positive charges flow from the positive terminal to the negative terminal of a power supply.

Magnetic Character Revisited

Oersted made the connection between moving charge and the creation of a magnetic field. We have deliberately avoided the discussion of the real basis for the existence of magnetic fields, the magnetic dipole, until we reviewed Oersted's principle. Even Earth's magnetic field can be explained in terms of Oersted's principle of charge flow creating magnetic dipoles. The dipoles created deep within Earth act like a giant electromagnet because of the flow of molten metals below its surface; the turning motion of the molten material produces a charge flow, similar to current passing through a coil of wire to produce the magnetic field.

Today, scientists believe that magnetic character (magnetic dipole) in solid ferromagnetic material is related to Oersted's principle and the movement of charge. A magnetic dipole is created by the movement (spinning) of electrons in individual atoms. According to quantum theory (see Chapter 12), electrons spin about their central axis very much like a spinning top, as shown in Figure 9.13. When an electron spins, it gives rise to a changing electric field that acts like a small electron current. An electron can spin in only one of two directions: clockwise and counterclockwise. An electron spinning by itself in an orbital sub-shell creates a magnetic field around the atom, causing the atom to become a *magnetic dipole*. Materials with unpaired electrons exhibiting a small magnetic field are called **paramagnetic**.

Fig.9.13 A spinning electron creates an electron current

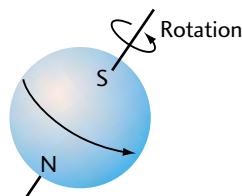


Fig.9.14 Oxygen is paramagnetic and responds to an external magnetic field at low temperatures. At higher temperatures, it doesn't respond to a magnetic field because the dipole attraction is very weak.



Paramagnetic materials such as iron, nickel, and cobalt are attracted to a magnetic field. *Naturally occurring dipoles are actually atoms of ferromagnetic (paramagnetic) materials.* The alignment of dipoles and their associated domains is responsible for the overt magnetic character of larger magnets.

For an *electron pair*, the second electron must spin in the opposite direction of the first electron in order to cancel out the overall magnetic character of the atom. **Diamagnetic** materials have paired electrons that produce no measurable magnetic field because of opposite spin cancellation. These materials are weakly repelled by a magnetic field.

A Magnetic Field around a Coiled Conductor (a Solenoid)

When the shape of a straight conductor is changed to form a coil (solenoid), the resulting magnetic field also changes shape, as shown in Figures 9.15a and b.

Fig. 9.15a The magnetic field around a conductor that has been looped to form a coil

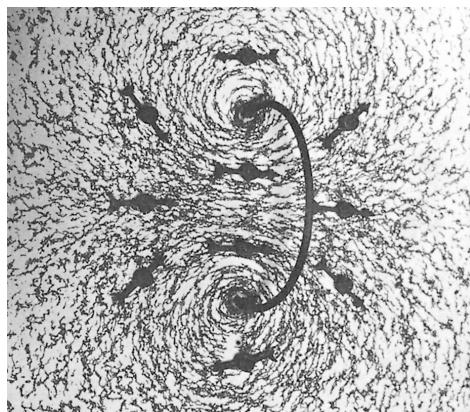
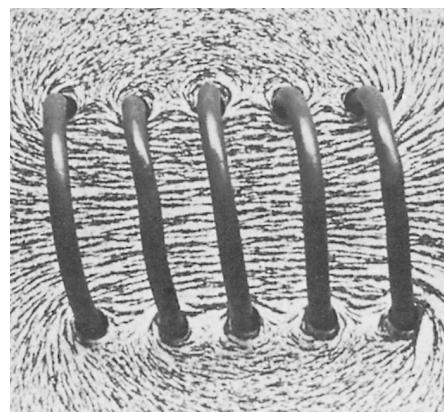


Fig. 9.15b The magnetic field around a large solenoid (coil) as mapped using iron filings



If the wire is coiled, the individual circular field lines from each looped conductor interact as shown in Figure 9.15c. In Figure 9.15c, the opposite magnetic fields between the loops mutually cancel, but inside and outside the coil, the fields are aligned. Outside the coil, the field is circular (around the coil). Inside the coil, the alignment of circular fields around each loop is so prevalent that the internal magnetic field is strong and linear. The pattern of the field looks very similar to that produced by a simple bar magnet.

A current-carrying coiled conductor is called an **electromagnet**. But which end of the electromagnet acts like the north or south end? Figure 9.16 summarizes a **second right-hand rule** that predicts the relationship between the direction of charge flow in a coil and the direction of the magnetic field emerging from the ends of the electromagnet.

Fig. 9.15c A coil produces a magnetic field similar to that of a simple bar magnet

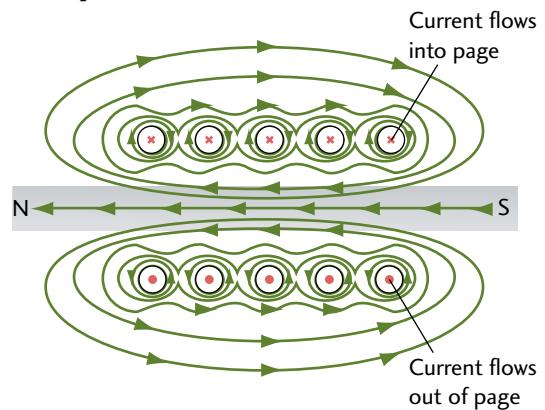
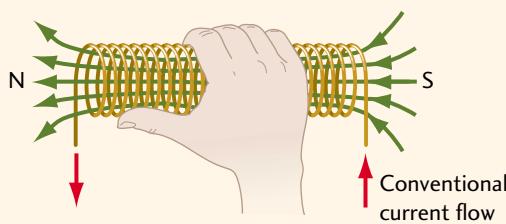


Fig.9.16 Right-hand rule #2 (RHR #2)

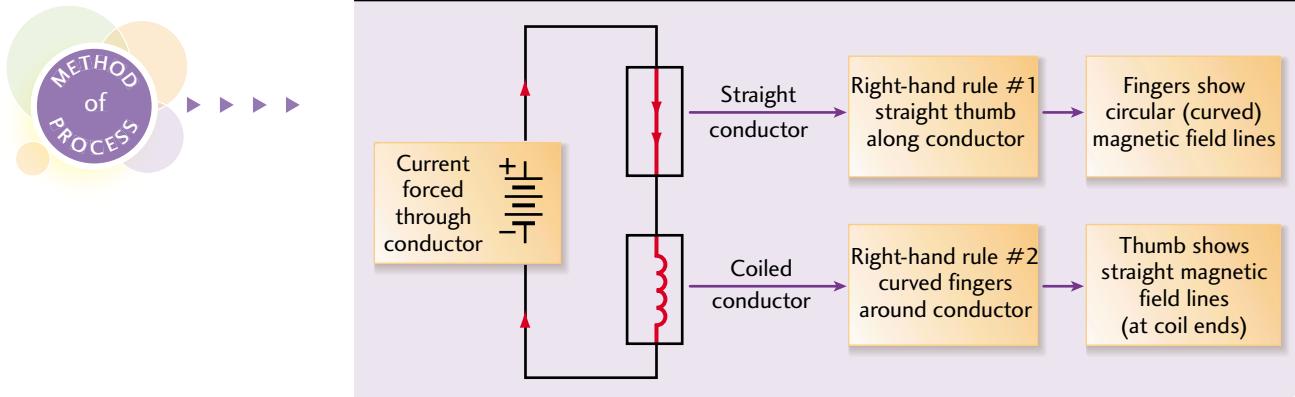


Right-hand rule #2 (RHR #2) for conventional current flow

Right-hand rule #2 (RHR #2) for conventional current flow: Grasp the coiled conductor with the *right hand* such that the curved fingers point in the direction of conventional, or positive (+), current flow. The thumb points in the direction of the magnetic field within the coil. Outside the coil, the thumb represents the north (N) end of the electromagnet produced by the coil.

Figure 9.17 summarizes the first and second right-hand rules for conductors.

Fig.9.17 Creation of Magnetic Fields around Current-carrying Conductors



The strength of the magnetic field created by an electromagnet depends on four factors that are summarized in Table 9.2.

Table 9.2 Factors that Determine the Strength of an Electromagnet	
Factor	Description
Current in the coil	The greater the current flow, the greater the field strength. Strength varies directly as the current in the coil.
Number of turns in the coil	The greater the number of coils, the greater the field strength. Strength varies directly as the number of turns in the coil if the current is constant.
Size of coil	The smaller the diameter of the coil, the stronger the magnetic field.
Type of material in the coil's centre	The more ferromagnetic the material within the coil, the greater the magnet's strength. Iron is one of the better materials to use. Strength varies directly as the measure of the ferromagnetic properties (magnetic permeability, μ) of the core material.

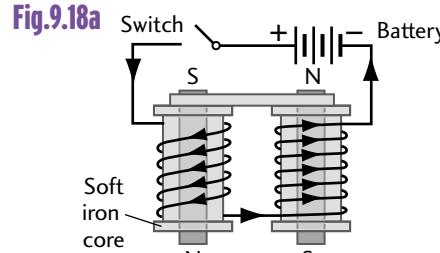
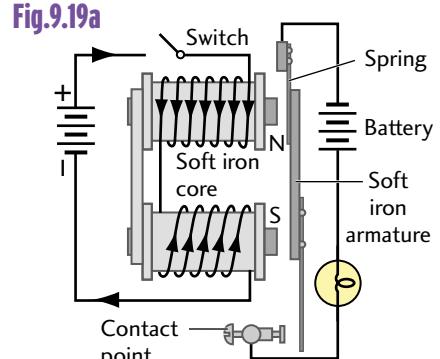
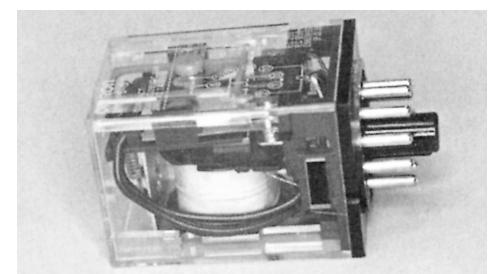
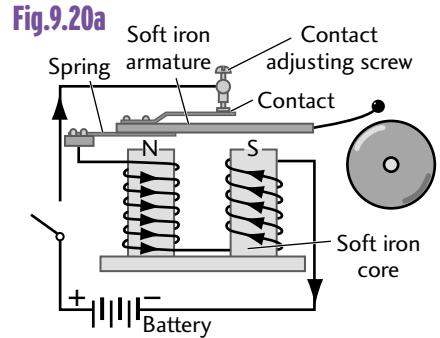
Factor	Description
Current in the coil	The greater the current flow, the greater the field strength. Strength varies directly as the current in the coil.
Number of turns in the coil	The greater the number of coils, the greater the field strength. Strength varies directly as the number of turns in the coil if the current is constant.
Size of coil	The smaller the diameter of the coil, the stronger the magnetic field.
Type of material in the coil's centre	The more ferromagnetic the material within the coil, the greater the magnet's strength. Iron is one of the better materials to use. Strength varies directly as the measure of the ferromagnetic properties (magnetic permeability, μ) of the core material.

If the coil is wrapped around a ferromagnetic core such as iron, the magnetic field can be made much stronger because the ferromagnetic domains of the core align when current is applied. Different ferromagnetic substances have the ability to alter the strength of magnetic fields when used as a core material in an electromagnet. The **magnetic permeability**, μ , of a material is the ratio of the magnetic field strength of the electromagnet with the core present to the field strength of the coil only. Table 9.3 lists the magnetic permeabilities of some common substances. Soft iron metal is usually the material of choice as an electromagnetic core because, unlike other materials, the domains randomize their direction when the current is shut off, causing the metal to lose its magnetic character.

Electromagnets have many practical applications, some of which are summarized in Table 9.4.

Table 9.3 Magnetic Permeability (μ)	
Permalloy	10 000
Iron	6100
Steel	2000
Nickel	1000
Cobalt	170
Aluminum	1.000 02
Oxygen	1.000 002
Vacuum	1.000 000
Water	0.999 999
Copper	0.999 99

Table 9.4
Applications of Electromagnets

Application and Description	Schematic diagram	Photo
Lifting electromagnets lift large ferromagnetic materials. Electromagnetic clutches are used to lower neutron-absorbing control rods into the calandria of a CANDU nuclear reactor.	Fig.9.18a 	Fig.9.18b 
In a relay , the electromagnet closes another switch that operates in some remote location. Turning on high-current circuits, such as a bank of lighting in the Sky Dome (Toronto, ON), with a single switch requires the use of relays.	Fig.9.19a 	Fig.9.19b 
An electric bell is a self-switching electromagnet. The design makes the magnet oscillate on and off, ringing the bell.	Fig.9.20a 	Fig.9.20b 



1. Copy the following images into your notebook. For each current-carrying conductor, sketch a view of the magnetic field, based on the direction of current flow shown.

a) Fig.9.21a



b) Fig.9.21b



c) Fig.9.21c



d) Fig.9.21d

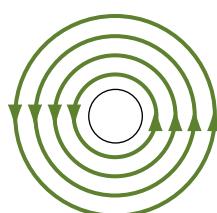


e) Fig.9.21e



2. Copy the following images into your notebook. For each current-carrying conductor, show the direction of current flow, based on the structure of the magnetic field shown.

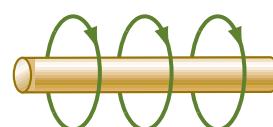
a) Fig.9.22a



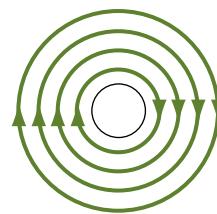
b) Fig.9.22b



c) Fig.9.22c



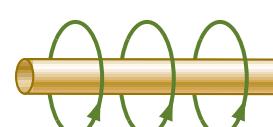
d) Fig.9.22d



e) Fig.9.22e

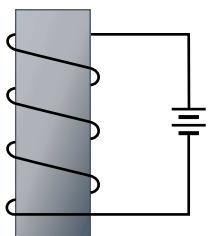


f) Fig.9.22f

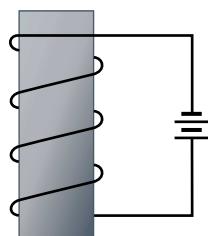


3. Copy the following images of a solenoid into your notebook. For each current-carrying coil, sketch a view of the magnetic field around the coil, based on the direction of current flow shown. On each, label the polarity (north and south) of the electromagnet.

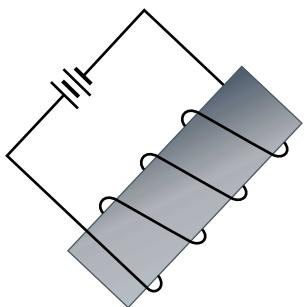
a) Fig.9.23a



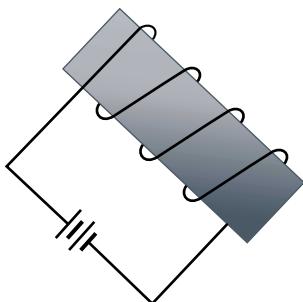
b) Fig.9.23b



c) Fig.9.23c

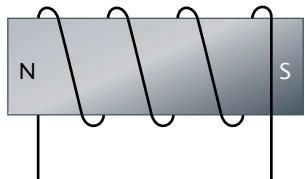


d) Fig.9.23d

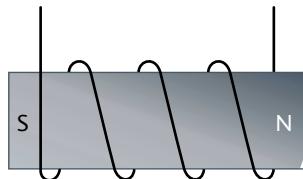


4. Copy the following images of solenoids into your notebook. For each coil, show the direction of current flow that would cause the labelled magnetic polarity.

a) Fig.9.24a



b) Fig.9.24b



9.5 Magnetic Forces on Conductors and Charges — The Motor Principle

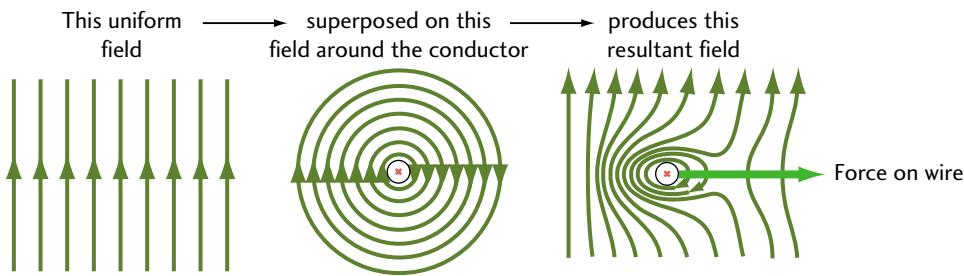
Electromagnets are extremely versatile because they provide a magnetic force that can be varied in strength and direction or even shut off when desired. The development of electromagnets provided the perfect opportunity for scientists such as Oersted and Michael Faraday to begin research on the forces exerted between two magnetic fields.

When two different magnets interact, a force of attraction or repulsion occurs (recall Figure 9.4a). The application of a magnetic force generated between two magnets is called the **motor principle**. An electric motor is a device designed to continuously provide a magnetic force in a particular direction.

The Motor Principle: When two magnetic fields interact, they produce a force. If a current-carrying conductor cuts through a uniform magnetic field, it experiences a force directed at 90° to both the direction of the charge flow and to the uniform external magnetic field (Figure 9.25). The strength of this force depends on the strength of the uniform external magnetic field and on the strength of the magnetic field around the conductor.

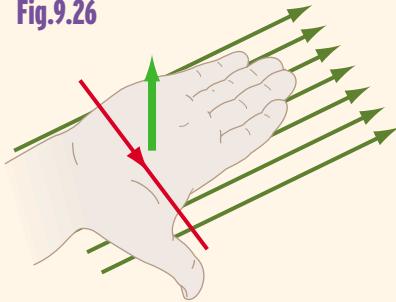
Figure 9.25 shows the interaction of two magnetic fields and the resulting force.

Fig.9.25 The interaction of two magnetic fields produces a force



The direction of the force on a current-carrying conductor is important to the application of the motor principle in practical devices. **Right-hand rule #3** allows us to predict the direction of the resulting magnetic force on the conductor if we know the direction of current flow in the conductor and the direction of the external uniform magnetic field (see Figure 9.26).

Fig.9.26



Right-hand rule #3 (RHR #3) for conventional current flow

The motor principle: Open the *right hand* so that the fingers point in the direction of the magnetic field, from north to south. Rotate the hand so that the thumb points in the direction of conventional, or positive (+), current flow. The orientation of the palm indicates the direction of the force produced.

The magnetic force (F) depends on the field strength around the conductor. Because the resulting field is not uniform, we relate it to the length L of the coiled conductor and to the current flowing through it.

What parameters affect the force resulting from the interaction of two magnetic fields? Like electrostatic and gravitational fields, a magnetic field has a field strength, B . The force, F , varies directly as the magnetic field strength (B), the length of the conductor (L) in the field, and the current (I) flowing through the conductor.

The magnetic force is at a *maximum* when the conductor is *perpendicular* to the external magnetic field, and *zero* when it is *parallel* to the external magnetic field. If θ is the angle between the conductor and the magnetic field, then the force is proportional to $\sin \theta$. Combining these variables, we obtain the proportionality statement

$$F \propto BIL \sin \theta$$

The initial equation for the magnetic force is

$$F = kBIL \sin \theta$$

Rearranging the magnetic force equation for the magnetic field strength,

$$B = \frac{F}{kIL \sin \theta}$$

If one ampere of current in a one-metre-long wire produces a maximum force of one newton, then $k = 1$ and the field strength is defined as one tesla (T), where

$$1 \text{ T} = 1 \frac{\text{N}}{\text{A} \cdot \text{m}}$$

The equation that describes the force experienced by a current-carrying conductor in a uniform external magnetic field then becomes

$$F = BIL \sin \theta$$

where F is the magnetic force in newtons (N), B is the magnetic field strength in tesla (T), I is the current in the conductor in amperes (A), L is the length of the conductor in the magnetic field in metres (m), and θ is the angle between the conductor and the magnetic field, in degrees.

A few examples will illustrate how to use the magnetic force equation.

EXAMPLE 1

Calculating the change in magnetic force using proportions

What happens to the strength of the magnetic force on a conductor if the current through the conductor and the length of wire exposed to the field are doubled while the conductor is rotated 30° from perpendicular to the field?

Solution and Connection to Theory

Given

$$I_2 = 2I_1 \quad L_2 = 2L_1 \quad \theta_1 = 90^\circ \quad \theta_2 = 90^\circ - 30^\circ = 60^\circ$$

Using the proportionality relationship for the parameters given,

$$\frac{F_2}{F_1} = \left(\frac{B_2}{B_1} \right) \left(\frac{I_2}{I_1} \right) \left(\frac{L_2}{L_1} \right) \left(\frac{\sin \theta_2}{\sin \theta_1} \right)$$

The unit for magnetic field strength is the tesla (T), named after American engineer Nikola Tesla (1856–1943).

Recall that a current of one ampere (1 A) is equivalent to one coulomb (1 C) of charge (6.25×10^{18} positive charges) passing through a point in a conductor every second. Some sources give the unit for the magnetic field, T, in terms of charge and charge speed:

$$1 \text{ T} = \frac{1 \text{ N}\cdot\text{s}}{\text{C}\cdot\text{m}} = \frac{\text{N}}{\text{A}\cdot\text{m}}$$

$$\vec{F} = I\vec{L} \times \vec{B} \text{ (cross product)} \\ = ILB \sin \theta$$

But $B_1 = B_2$; therefore,

$$F_2 = F_1 \left(\frac{I_2}{I_1} \right) \left(\frac{L_2}{L_1} \right) \left(\frac{\sin \theta_2}{\sin \theta_1} \right)$$

$$F_2 = F_1 \left(\frac{2I_1}{I_1} \right) \left(\frac{2L_1}{L_1} \right) \left(\frac{\sin 60^\circ}{\sin 90^\circ} \right)$$

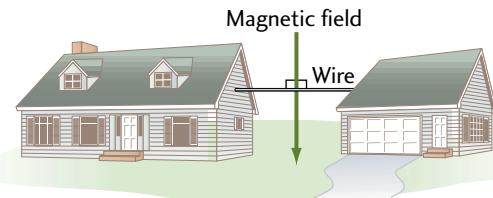
$$F_2 = 3.46F_1$$

Therefore, the new magnetic force is 3.46 times stronger than the initial force.

EXAMPLE 2 Calculating the magnetic force directly

A wire carrying a direct current of 10.0 A is suspended 5.0 m east between a house and a garage (see Figure 9.27) through Earth's magnetic field (5.0×10^{-5} T). What is the magnitude of the force that acts on the conductor? What is the direction of this force in relation to the horizontal wire?

Fig.9.27



Solution and Connection to Theory

Given

$$I = 10.0 \text{ A} \quad L = 5.0 \text{ m [E]} \quad B = 5.0 \times 10^{-5} \text{ T [D]} \quad F = ?$$

The conductor is perpendicular to the magnetic field because the east-west wire cuts the northbound magnetic field line at 90° .

$$F = BIL \sin \theta$$

$$F = (5.0 \times 10^{-5} \text{ T})(10.0 \text{ A})(5.0 \text{ m}) \sin 90^\circ$$

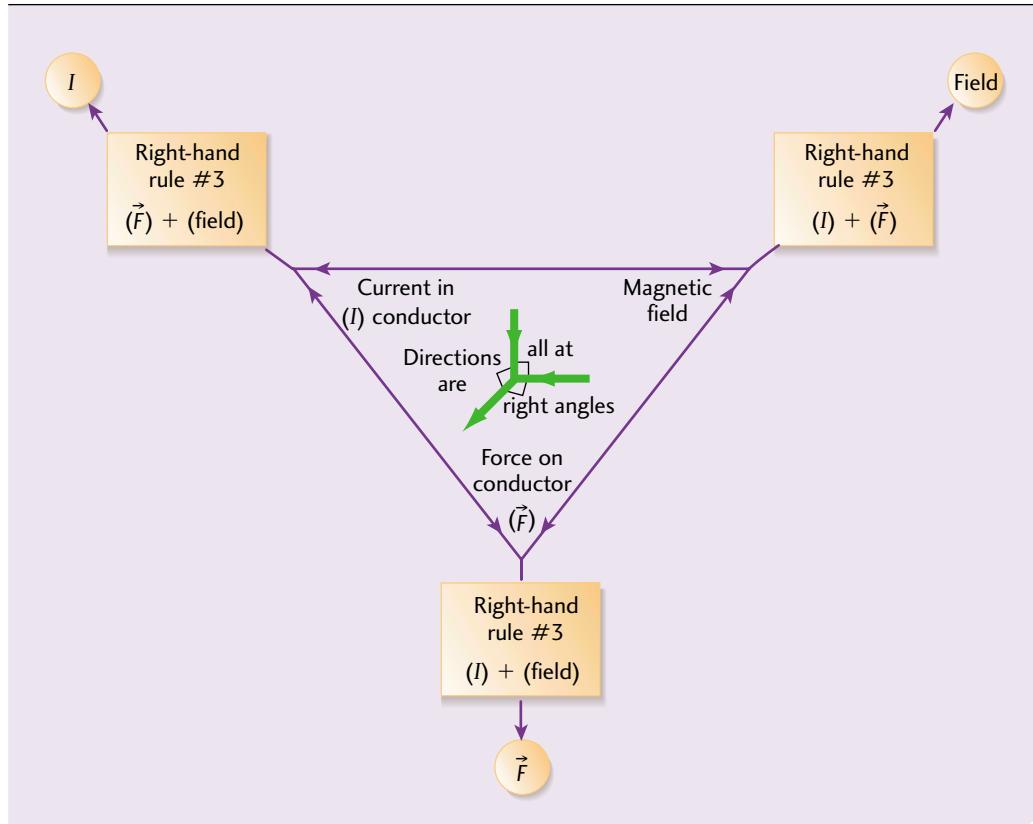
$$F = 2.5 \times 10^{-3} \text{ N}$$

The magnetic force on this conductor is 2.5×10^{-3} N. From the RHR #3, we know that the magnetic force acts 90° to the wire and to the direction of the magnetic field. The magnetic field would cross the horizontally running electrical line at the same angle as the angle at which it contacts Earth's surface.

The angle at which the magnetic field lines pass through Earth's surface at a particular point is called the **angle of magnetic inclination** or the **dip angle**.

Figure 9.28 summarizes the right-hand rule for the motor principle.

Fig.9.28 Right-hand Rule #3: The Motor Principle



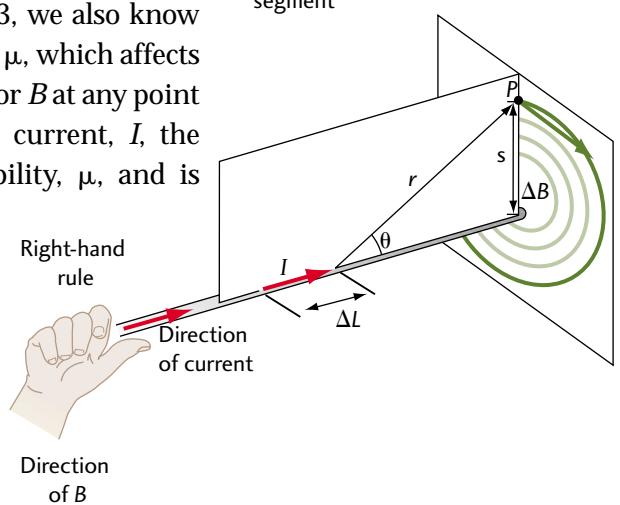
The Field Strength around a Current-carrying Conductor

In Section 9.4, we learned that moving electric charges create magnetic fields, and that the motor principle describes the force created when two magnetic fields interact. But what affects the strength of the magnetic field around a conductor? As with electrostatic and gravitational fields, the strength of a magnetic field around a field-creating conductor decreases with, or varies inversely as, the distance, r , from the source. From Table 9.3, we also know that different materials have different magnetic permeability, μ , which affects the overall field strength, B , of the electromagnet. The value for B at any point around a current segment is directly proportional to the current, I , the length of segment, ΔL , $\sin \theta$, and the magnetic permeability, μ , and is inversely proportional to the square of the distance away from the current segment (see Figure 9.29). This relationship is known as **Biot's law**: magnetic field strength changes (ΔB) with the conductor length (ΔL).

The mathematical expression for Biot's law is

$$\Delta B = \frac{\mu I \Delta L \sin \theta}{4\pi r^2}$$

Fig.9.29 The factors that affect the field strength around a current segment



This statement shows that the field strength changes with the current and with the length of the current segment. The 4π refers to the circular geometry of the field lines about the current segment and is part of the constant of proportionality.

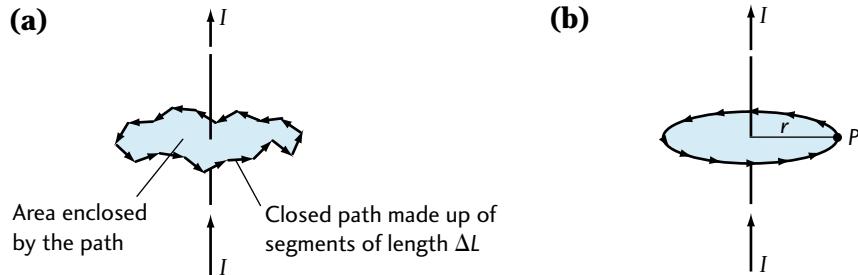
We can simplify the Biot's law equation by using Ampère's law, named after French scientist André Marie Ampère (1775–1836).

Ampère's Law: Consider any closed path around a current-carrying conductor that is made up of many short segments of length ΔL , as shown in Figure 9.30. If we add all the products of each line segment and the parallel component of B , the sum is μI , where μ is the magnetic permeability and I is the current through the enclosed path; that is,

$$(B\Delta L)_1 + (B\Delta L)_2 + (B\Delta L)_3 + \dots + (B\Delta L)_n = \mu I \text{ or}$$

$$\sum B\Delta L = \mu I$$

Fig. 9.30 The field strength at a point P along a straight conductor is related to I , μ , and r



If the magnetic field, B , around the straight conductor is constant, we can factor it out:

$$B(\Delta L_1 + \Delta L_2 + \Delta L_3 + \dots + \Delta L_n) = \mu I$$

The sum of all segments, $\sum \Delta L$, equals the circumference of the circle, $2\pi r$, as shown in Figures 9.30a and b, so

$$B(2\pi r) = \mu I$$

Rearranging the equation for B , we obtain the simplified equation for Biot's law,

$$B = \frac{\mu I}{2\pi r}$$

where B is the field strength in tesla (T), μ is the magnetic permeability of the substance in T·m/A, I is the current in amperes (A), and r is the perpendicular distance away from the conductor in metres (m).

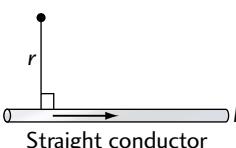
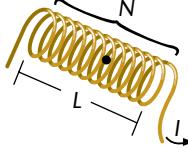
We can also simplify the equation for Biot's law using integral calculus. If the perpendicular distance is r and $\sin \theta = 1$, then

$$B = \frac{\mu I}{4\pi} \int_{L_1}^{L_2} \frac{\sin \theta}{r^2} dL$$

$$B = \frac{\mu I}{2\pi r}$$

The field strengths for conductors having more complex shapes can also be derived from Ampère's law, but their derivations are beyond the scope of this course. The equations for the magnetic field strengths of loops and coils of current-carrying conductors are summarized in Table 9.5.

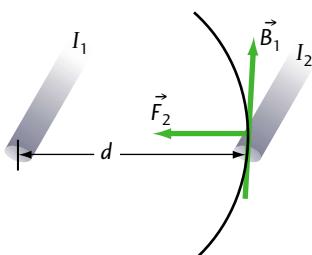
Table 9.5
Equations for Field Strength around Various Conductor Configurations

Solenoid	Equation	Parameters
Fig.9.31a  Straight conductor	Magnetic field strength around a straight conductor: $B = \frac{\mu I}{2\pi r}$	μ = magnetic permeability I = current r = the perpendicular distance from the single straight conductor
Fig.9.31b  Flat coiled conductor	Magnetic field strength in the centre of a flat loop of wire with N turns: $B = \frac{\mu NI}{2r}$	μ = magnetic permeability I = current N = the number of loops in a flat coil of radius r
Fig.9.31c 	Magnetic field strength in the centre of a long solenoid of length L with N turns: $B = \frac{\mu NI}{L}$	μ = magnetic permeability I = current N = the number of loops L = the length of the solenoid The radius of the coil isn't important as long as it is much smaller than its length.

The Unit for Electric Current (for Real this Time)

In previous studies of current electricity, we learned that current is the rate of charge flow through a conductor, measured in amperes, where one ampere is one coulomb of charge passing a point in a conductor every second. Although this unit is convenient for studying current electricity, it is easier to picture than to measure. It is very difficult to obtain a given amount of charge precisely, let alone its rate of flow through a conductor. The accepted definition of the ampere is actually based on the magnetic field created by current flowing in two straight parallel conductors. The force experienced by one conductor depends on the magnetic field, \vec{B} , created by the other conductor (see Figure 9.32).

Fig.9.32



The magnetic force on the first conductor is given by the equation

$$F = BLI \sin \theta$$

where $B = \frac{\mu I}{2\pi r}$ and $\sin \theta = 1$; therefore,

$$F = \frac{\mu I^2 L}{2\pi r}$$

where L is the length of the two conductors in metres (m), r is the distance between them in metres (m), and I is the current flowing through each wire (must be the same for both wires) in amperes (A). If $I = 1$ A, $L = 1$ m, and $\mu_{\text{air}} = 4\pi \times 10^{-7}$ T·m/A, then the force on every 1 m of wire is

$$F = \frac{\mu I^2}{2\pi r}$$

$$F = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1 \text{ A})^2}{2\pi(1 \text{ m})}$$

$$F = 2 \times 10^{-7} \text{ N/m}$$

From our new definition of the ampere, we can define one coulomb as the charge transported by a current of one ampere in one second:

$$1 \text{ C} = 1 \text{ A} \cdot \text{s}$$

One ampere (1 A) is the current flowing through two parallel conductors, placed one metre apart in air, that exert a force of 2×10^{-7} N/m on each other for each metre of their length.

EXAMPLE 3

Calculating the field strength around a straight conductor

Find the magnetic field strength in air 1.0 cm away from a straight conductor passing a current of 1.0 A.

Solution and Connection to Theory

Given

$$\begin{aligned} r &= 1.0 \text{ cm} = 1.0 \times 10^{-2} \text{ m} & I &= 1.0 \text{ A} \\ \mu_{\text{air}} &= 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} & B &=? \end{aligned}$$

$$B = \frac{\mu I}{2\pi r} \quad (\text{from Table 9.5})$$

$$B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.0 \text{ A})}{2\pi(1.0 \times 10^{-2} \text{ m})}$$

$$B = 2.0 \times 10^{-5} \text{ T}$$

The field strength 1.0 cm away from this conductor is therefore 2.0×10^{-5} T.

EXAMPLE 4**Calculating magnetic field strength around a single flat loop**

The conductor from Example 3 is coiled once into a single flat loop of radius 3.0 cm. What is the magnetic field strength at the centre of this loop?

Solution and Connection to Theory**Given**

$$r = 3.0 \text{ cm} = 3.0 \times 10^{-2} \text{ m} \quad I = 1.0 \text{ A} \quad N = 1$$
$$\mu_{\text{air}} = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \quad B = ?$$

$$B = \frac{\mu NI}{2r} \text{ (from Table 9.5)}$$

$$B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1)(1.0 \text{ A})}{2(3.0 \times 10^{-2} \text{ m})}$$

$$B = 2.1 \times 10^{-5} \text{ T}$$

The field strength at the centre of this flat loop is $2.1 \times 10^{-5} \text{ T}$.

EXAMPLE 5**The force between two conductors**

How far from a conductor carrying 3.0 A of current is a second wire with a current of 9.5 A if the force between the two wires is $4.0 \times 10^{-4} \text{ N/m}$?

Solution and Connection to Theory**Given**

$$I_1 = 3.0 \text{ A} \quad I_2 = 9.5 \text{ A} \quad F = 4.0 \times 10^{-4} \text{ N/m}$$

The current in these two parallel wires is not the same; therefore, the current term is the product of I_1 and I_2 instead of I^2 .

$$F = \frac{\mu I^2 L}{2\pi r} = \frac{\mu I_1 I_2 L}{2\pi r}$$

Isolating r ,

$$r = \frac{\mu I_1 I_2 L}{2\pi F}$$

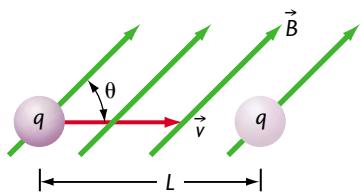
$$r = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(3.0 \text{ A})(9.5 \text{ A})(1 \text{ m})}{2\pi(4.0 \times 10^{-4} \text{ N})}$$

$$r = 1.4 \times 10^{-2} \text{ m}$$

The wires are $1.4 \times 10^{-2} \text{ m}$ apart.

Magnetic Force on Moving Charges

Fig. 9.33 A charge q cuts through a magnetic field \vec{B} at an angle θ while moving a distance L



In Section 9.4, we learned that the force experienced by a conductor in a magnetic field is due to the flow of charge through it. Without moving charge, there would be no magnetic force. But moving charges *need not be bound by a conductor* in order to experience a magnetic force as they move.

The charged particle q (see Figure 9.33) moving with a velocity \vec{v} at an angle θ to the magnetic field \vec{B} constitutes a current if n particles pass a certain point in a given time Δt . The current, I , of the charges is given by the equation

$$I = \frac{nq}{\Delta t}$$

The distance L (similar to the length of a conductor) is given by the equation

$$L = v\Delta t$$

so the equation for the magnetic force becomes

$$F = BIL \sin \theta$$

$$F = B\left(\frac{nq}{\Delta t}\right)(v\Delta t)\sin \theta$$

For a single charge, $n = 1$; therefore,

$$F = qvB \sin \theta$$

The magnetic force, \vec{F} , is the cross product $q\vec{v} \times \vec{B}$.

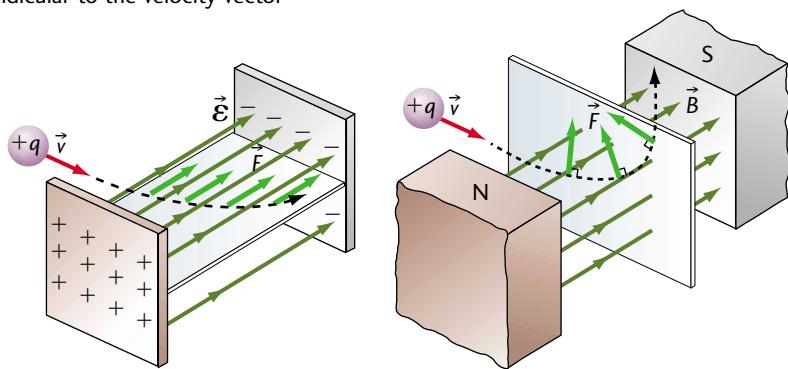
The **magnetic force on an individual moving charge** is given by the equation

$$F = qvB \sin \theta$$

where B is the magnetic field strength in tesla (T), q is the magnitude of charge in coulombs (C) that is moving at a velocity v in m/s, and θ is the angle between \vec{v} and \vec{B} .

As with charges moving in a conductor, the force on charges moving freely in a field is *strongest* when the current is *perpendicular* to the magnetic field and *weakest* (zero) when it is *parallel* to the magnetic field. We can use the **right-hand rule #3** for the motor principle to determine the direction of the force applied to each moving charge. Notice from Figure 9.34 that the magnetic force is always perpendicular to the velocity vector. In the case of electric fields, the force is parallel to the electric field lines and could at some point be parallel to the velocity vector.

Fig.9.34 The magnetic force is always perpendicular to the velocity vector



EXAMPLE 6

Calculating the force on an electron moving in a magnetic field

Figure 9.35 shows a magnetic field of strength 0.30 T emerging from the page (shown by the series of dots). An electron with a negative charge of 1.602×10^{-19} C enters this magnetic field at 6.0×10^6 m/s [right].

What is the force (magnitude and direction) on this electron at point A?

Solution and Connection to Theory

Given

$$\vec{B} = 0.30 \text{ T} \text{ [out]} \quad q = -1.6 \times 10^{-19} \text{ C} \quad \vec{v} = 6.0 \times 10^6 \text{ m/s [right]}$$

Because we're using the right-hand rule, the current direction must be that of a positive charge. We can consider our charge as positive as long as its direction of motion is considered in reverse; that is, $q = +1.602 \times 10^{-19}$ C as long as $\vec{v} = 6.0 \times 10^6$ m/s [left]. For the magnetic force,

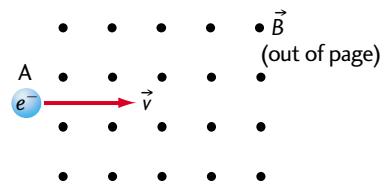
$$F = qvB \sin \theta$$

$$F = (1.602 \times 10^{-19} \text{ C})(6.0 \times 10^6 \text{ m/s})(0.30 \text{ T}) \sin 90^\circ$$

$$F = 2.9 \times 10^{-13} \text{ N}$$

The magnitude of the magnetic force is 2.9×10^{-13} N. Now we can apply the right-hand rule #3 to determine its direction. With fingers pointing out of the page in the direction of the magnetic field and the thumb pointing to the left (for conventional (+) current flow), the force (palm) is directed towards the top of the page. Therefore, $\vec{F} = 2.9 \times 10^{-13}$ N [up].

Fig.9.35



EXAMPLE 7

Calculating the magnetic field affecting a moving charge

Electrons are not the only charged particles affected by a magnetic field. A proton with a charge of $+e$ and a velocity of 1.0×10^6 m/s [down] enters a magnetic field and is pushed by a force of 4.5×10^{-14} N [right]. What is the magnitude and direction of the magnetic field experienced by this charge?

Solution and Connection to Theory**Given**

$$q = 1.602 \times 10^{-19} \text{ C} \quad \vec{v} = 1.0 \times 10^6 \text{ m/s [down]} \\ \vec{F} = 4.5 \times 10^{-14} \text{ N [right]}$$

$$B = \frac{F}{qv \sin \theta}$$

$$B = \frac{4.5 \times 10^{-14} \text{ N}}{(1.602 \times 10^{-19} \text{ C})(1.0 \times 10^6 \text{ m/s}) \sin 90^\circ}$$

$$B = 0.28 \text{ T}$$

Using the right-hand rule #3, the thumb (current) points down the page and the palm (force) faces right, so the fingers (magnetic field) point into the page. Therefore, the magnetic field strength $B = 0.28 \text{ T}$ [into the page].



Fig. 9.36 Power lines produce electromagnetic field lines



1. What force is experienced by a 30-cm wire carrying a 12-A current perpendicular to a uniform magnetic field of strength 0.25 T?
2. What current runs through a 0.15-m-long conductor that's at right angles to a magnetic field of strength 3.5×10^{-2} T if the magnetic force is 9.2×10^{-2} N?

Are electrical transmission lines safe?

Recently, there has been much controversy over the safety of electric and magnetic fields produced by above-ground electrical transmission lines. Through the wide-open space of a typical countryside, the fields produced are not cause for concern. Because these electricity lines feed urban centres, they must pass through more densely populated areas, as shown in Figure 9.36.

The open space under transmission lines may not be used for housing, but it may be a prime location for park green space or even school yards and sports playing fields. A report from the U.S. National Council on Radiation Protection, released in October, 1995, implied that even very low exposure to electromagnetic radiation has detrimental

long-term effects on health. The study also showed that extremely low-frequency (ELF) electromagnetic fields (EMFs) can disturb the production of the hormone melatonin (linked with sleep patterns), and might even be a factor in the occurrence of sudden infant death syndrome (SIDS). Children exposed to ELF EMFs may also be at a higher risk of leukemia. ELF EMFs may also cause increased estrogen levels in adults, which is linked with estrogen-sensitive cancers like breast cancer in both women and men.

Transmission lines pass an alternating current (AC) at voltages between 10 kV and 500 kV, so the field oscillates back and forth. Therefore, when we calculate field strength for electrical transmission lines, we will use direct current (DC) as a simplification.

- 3.** What is the magnetic field strength around an electrical transmission line suspended between two towers 50 m apart that carries a current of 100 A west? In Canada, the dip angle for the magnetic field is 45° and the net magnetic force on the wire is 0.25 N.
 - a)** What is the magnitude of Earth's magnetic field strength?
 - b)** Sketch the direction of the magnetic field with respect to the orientation of the power line. Use the right-hand rule for the motor principle to determine the direction of the 0.25-N force on the wire.
- 4.** A high-school student wants to construct a solenoid that will balance Earth's magnetic field (i.e., have equal magnitude but the opposite direction) of 3.0×10^{-5} T at its centre. If the student has just enough wire conductor to make the solenoid 20 cm long and 4.0 cm in diameter with 200 turns, what current must be passed through the coil?
- 5.** An electrical transmission line that carries a DC of 100 A west is suspended between two towers 50 m apart. The dip angle is 45° and the magnetic field strength is 3.0×10^{-5} T.
 - a)** How far from the high-voltage power lines do you have to be in order for the artificial magnetic field to balance Earth's magnetic field?
 - b)** If the transmission lines are 25 m above the ground and all the physical parameters (current, dip angle, and field strength) remain the same, calculate the exact location of this cancellation. (Hint: Find how far below the transmission lines and how far to the north or south you must stand.)
- 6. a)** The two wires in a typical household extension cord are 2.4 mm apart. What force per metre pushes them apart when 13.0 A of direct current flows to power a hair dryer? (Hint: Consider the rubber insulation to have the same permeability as that of air, where $\mu_0 = 4\pi \times 10^{-7}$ T·m/A.)
b) Does the fact that household current is AC make a difference for this problem? Explain your answer.

7. A bullet travelling at 400 m/s picks up a charge of 20 C. What is the maximum force exerted on the bullet by Earth's magnetic field (4.5×10^{-5} T)?
8. What is the magnitude and direction of the magnetic force on a proton moving vertically upward at 4.3×10^4 m/s in a 1.5-T magnetic field pointing horizontally to the west?

9.6 Applying the Motor Principle

The motor principle implies that a charge moving at a constant speed at right angles to a magnetic field will experience a force at right angles to the direction of motion and to the magnetic field; that is, the moving charge will constantly experience a force at 90° to its motion.

Fig. 9.37 *Yamato 1* is a ship that uses magnetohydrodynamics (MHD) for propulsion

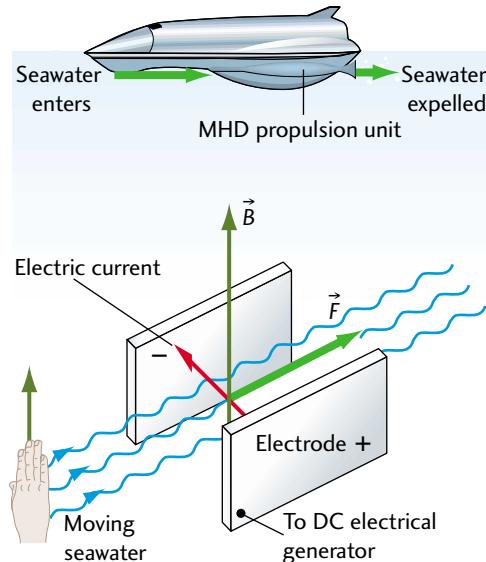
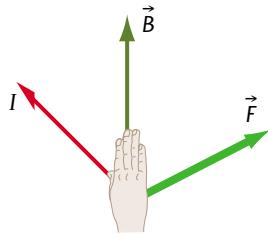


Magnetohydrodynamics

One practical application of the motor principle is **magnetohydrodynamics (MHD)**. MHD is the application of forces to charges (in this case, ions in seawater) by the application of a magnetic and an electric field. The *Yamato 1* (Figure 9.37) is a watercraft that uses the technology of MHD for propulsion.

A magnetohydrodynamic force is produced on the charged ions in seawater when an electric field is applied between two horizontally placed parallel plates that cut a magnetic field pointing upward at 90° . The force produced moves the boat, as described in Figure 9.38.

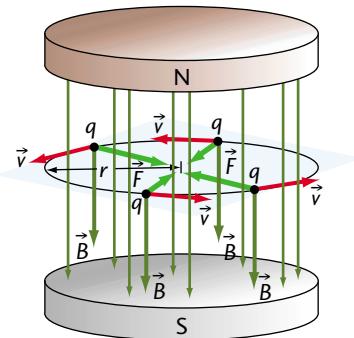
Fig. 9.38 The right-hand rule #3 (the motor principle) as it applies to MHD propulsion



Centripetal Magnetic Force

A particle moving at a constant speed and experiencing a constant magnetic force at 90° to its motion traces a circular path, as illustrated in Figure 9.39.

Fig. 9.39 The magnetic force always produces centripetal motion



According to the RHR #3, the magnetic force, always at right angles to the particle's motion, provides the centripetal force to keep the particle moving in a circle of radius r . If the mass m on a charged particle q is moving at a velocity \vec{v} at right angles to a magnetic field B , then

$$\vec{F}_\text{net} = \vec{F}_\text{magnetic}$$

$$F_\text{magnetic} = F_\text{centripetal}$$

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq}$$

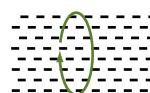
This equation allows us to calculate the radius of the circular path traced out by charged particles injected into magnetic fields. The aurora borealis is a breathtaking example of electrons spiraling in Earth's magnetic field (see Figure 9.40).

Fig. 9.40 The aurora borealis is due to spiraling electrons in Earth's magnetic field

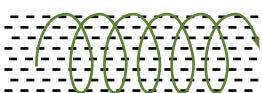
(a)



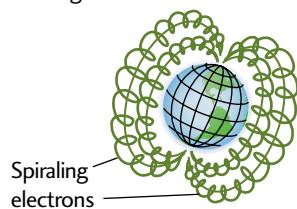
(b)



The electron has no velocity component parallel to the magnetic field



The electron has a velocity component parallel to the magnetic field



Spiraling electrons

EXAMPLE 8

Calculating the velocity of charged particles

- a) What is the velocity of an alpha particle moving in a circular path of radius 10.0 cm in a plane perpendicular to a 1.7-T magnetic field?
- b) If this alpha particle is accelerated by the application of an electric field over a set of parallel plates, what voltage is required to accelerate the alpha particle from rest?

Solution and Connection to Theory**a) Given**

$$r = 1.0 \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 1.0 \times 10^{-2} \text{ m} \quad B = 1.7 \text{ T}$$

$$q = 2(+1.602 \times 10^{-19} \text{ C}) = 3.204 \times 10^{-19} \text{ C} \quad m_{\alpha} = 6.68 \times 10^{-27} \text{ kg}$$

$$v = ?$$

$$v = \frac{rBq}{m}$$

$$v = \frac{(1.0 \times 10^{-2} \text{ m})(1.7 \text{ T})(3.204 \times 10^{-19} \text{ C})}{6.68 \times 10^{-27} \text{ kg}}$$

$$v = 8.1 \times 10^5 \text{ m/s}$$

The velocity of the alpha particle is $8.1 \times 10^5 \text{ m/s}$ in a circular path.

- b) From Chapter 8, recall the equation for the conservation of energy of charges moving in an electric field,

$$qV = \frac{1}{2}mv^2$$

$$V = \frac{mv^2}{2q}$$

$$V = \frac{(6.68 \times 10^{-27} \text{ kg})(8.1 \times 10^5 \text{ m/s})^2}{2(3.204 \times 10^{-19} \text{ C})}$$

$$V = 6.8 \times 10^3 \text{ V}$$

The voltage required to accelerate the alpha particle from rest is $6.8 \times 10^3 \text{ V}$.

The Mass of an Electron and a Proton

From the equation $r = \frac{mv}{Bq}$, we can see that if the charge (q) and velocity (\vec{v}) of a particle are kept constant in a magnetic field (B), then even the slightest difference in mass will result in a different radius of motion for the particle as it passes through the field. As we learned in Section 8.8, *cathode rays* were streams of electrons that were accelerated between two electrodes by a large potential difference. J. J. Thomson (1856–1940), a British scientist,

studied these cathode rays by subjecting them to electric and magnetic fields in a cathode-ray tube, like the one illustrated in Figure 9.41.

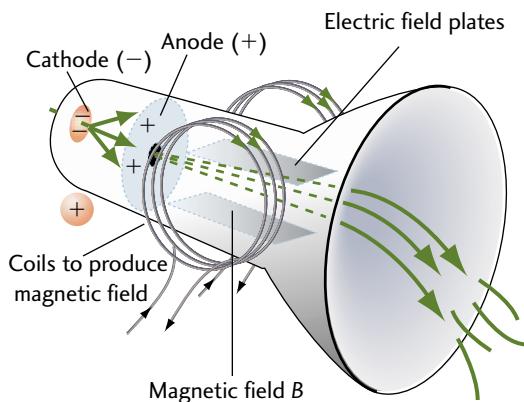


Fig. 9.41 A diagram of a cathode-ray tube (CRT) that was used to find the effects of electric and magnetic fields on moving electrons (cathode rays)

If we inject an electron that has been accelerated by an electric field into a magnetic field, then we can apply electric and magnetic field theory as well as uniform circular motion to find the electron's mass. Recall that the centripetal force on the charge is provided by the magnetic force:

$$\vec{F}_B = \vec{F}_c$$

$$qvB = \frac{mv^2}{r}$$

We can replace q with e to represent the charge of an electron:

$$evB = \frac{mv^2}{r}$$

where e and v are the charge and the velocity of the electron, respectively. When we isolate e/m (the charge-to-mass ratio of the electron), we obtain

$$e/m = \frac{v}{Br}$$

We can determine the strength of the magnetic field, B , from the current through and the configuration of the coils, and r can be measured experimentally. As we studied in Chapter 8, Thomson determined the speed of the electron, v , when he accelerated electrons through an electric field created by parallel plates (see Figure 9.41). In an electron tug-of-war, Thomson adjusted the electric field between the two parallel plates to cancel the effects of the magnetic field he was creating with the coils so that electrons would pass through to the screen in an unaltered path; therefore,

$$\vec{F}_{\text{magnetic}} = \vec{F}_{\text{electric}}$$

$$Bev = e\mathcal{E}$$

$$v = \frac{\mathcal{E}}{B}$$

where \mathcal{E} is the electric field strength between the two parallel plates.

From these equations, Thomson was able to express the charge-to-mass ratio for electrons.

Recall from Chapter 8 that \mathcal{E} can be calculated by the equation $\mathcal{E} = \frac{v}{d}$.

Substituting $v = \frac{\mathcal{E}}{B}$, the charge-to-mass ratio for electrons is given by

$$e/m = \frac{v}{Br} = \frac{\mathcal{E}}{B^2 r}$$

Thomson calculated the e/m value to be $1.76 \times 10^{11} \text{ C/kg}$.

As we learned in Chapter 8, in the early 1900s, Robert Millikan performed his oil-drop experiment in which he determined the charge e on an electron to be $1.602 \times 10^{-19} \text{ C}$. From Thomson's and Millikan's results, we can calculate the mass of a single electron.

To find the mass of a single electron,

$$e/m = 1.76 \times 10^{11} \text{ C/kg}$$

$$m = \frac{e}{1.76 \times 10^{11} \text{ C/kg}}$$

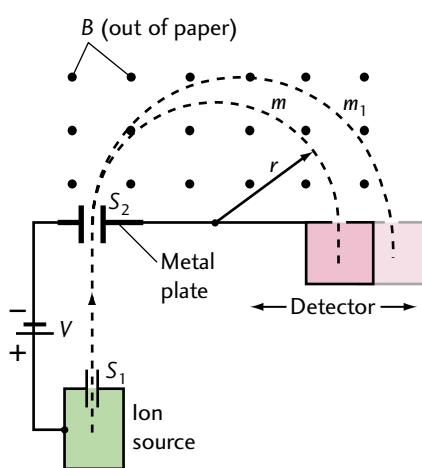
$$m = \frac{1.602 \times 10^{-19} \text{ C}}{1.76 \times 10^{11} \text{ C/kg}}$$

$$m = 9.11 \times 10^{-31} \text{ kg}$$

Fig. 9.42a A mass spectrometer



Fig. 9.42b A simplified diagram of a mass spectrometer



$$\frac{1}{2}mv^2 = qV$$

$$v = \sqrt{\frac{2qV}{m}} \quad (\text{eq. 1})$$

where v is the velocity at which the particles enter the magnetic field.

The magnetic force provides the centripetal force according to the equation

$$F_B = F_c$$

$$qvB = \frac{mv^2}{r}$$

so that

$$v = \frac{qBr}{m} \quad (\text{eq. 2})$$

Equating equations 1 and 2, we obtain

$$\frac{qBr}{m} = \sqrt{\frac{2qV}{m}}$$

$$\frac{q^2B^2r^2}{m^2} = \frac{2qV}{m}$$

$$m = \frac{qB^2r^2}{2V}$$

where m is the mass of the particle.

The more massive the ion, the larger the radius it traces in the magnetic field. Therefore, the mass spectrometer will detect a more massive ion at a different position than a smaller ion. Modern mass spectrometers are sensitive enough to detect mass differences between **isotopes** (different particles of the same substance that have the same number of protons and electrons but a different number of neutrons).

Larger voltages can impart a kinetic energy sufficient for some charges to reach relativistic speeds. The technique for compensating for relativistic effects is covered in Chapter 13.

EXAMPLE 9 Calculating the charge-to-mass ratio

A charged particle enters a uniform magnetic field of strength 3.2×10^{-2} T at a speed of 6.0×10^6 m/s. When the charge enters the field perpendicular to the field lines, it traces a circular path of radius 0.018 m. What is the charge-to-mass ratio for the particle?

Solution and Connection to Theory

Given

$$B = 3.2 \times 10^{-2} \text{ T} \quad v = 6.0 \times 10^6 \text{ m/s} \quad r = 0.018 \text{ m} \quad e/m = ?$$

$$e/m = \frac{v}{Br}$$

$$e/m = \frac{6.0 \times 10^6 \text{ m/s}}{(3.2 \times 10^{-2} \text{ T})(0.018 \text{ m})}$$

$$e/m = 1.0 \times 10^{10} \text{ C/kg}$$

The charge-to-mass ratio for this particle is 1.0×10^{10} C/kg.

EXAMPLE 10

**Calculating the velocity and circular path
of an accelerated plutonium ion**

A singly ionized $^{239}_{94}\text{Pu}$ ion of mass $4.0 \times 10^{-25} \text{ kg}$ is accelerated through a potential difference of $1.0 \times 10^5 \text{ V}$.

- What is the maximum speed of the ion?
- What is the radius of the ion's path if it is injected perpendicular to a uniform magnetic field of strength 0.12 T ?

Solution and Connection to Theory**Given**

$$m = 4.0 \times 10^{-25} \text{ kg} \quad V = 1.0 \times 10^5 \text{ V} \quad B = 0.12 \text{ T}$$

- a) $v = ?$

$$\frac{1}{2}mv^2 = qV$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$v = \sqrt{\frac{2(1.602 \times 10^{-19} \text{ C})(1.0 \times 10^5 \text{ V})}{4.0 \times 10^{-25} \text{ kg}}}$$

$$v = 2.8 \times 10^5 \text{ m/s}$$

The ion's maximum speed is $2.8 \times 10^5 \text{ m/s}$.

b) $r = \sqrt{\frac{2mV}{qB^2}}$

$$r = \sqrt{\frac{2(4.0 \times 10^{-25} \text{ kg})(1.0 \times 10^5 \text{ V})}{(1.602 \times 10^{-19} \text{ C})(0.12 \text{ T})^2}}$$

$$r = 5.9 \text{ m}$$

The radius of the ion's path in the magnetic field is 5.9 m.

9.7 Electromagnetic Induction — From Electricity to Magnetism and Back Again

In 1831, Michael Faraday discovered a concept that complimented Oersted's principle that moving charge produces a magnetic field.

Faraday's Law of Electromagnetic Induction:

A magnetic field that is moving or changing in intensity in the region around a conductor causes or *induces* charge to flow in the conductor.

Figure 9.43 illustrates how a changing magnetic field near a conductor causes a very faint current to flow in the conductor.

Figure 9.44 summarizes the relationship between Oersted's principle and Faraday's principle.

Fig.9.44 Oersted's and Faraday's Principles Complement Each Other

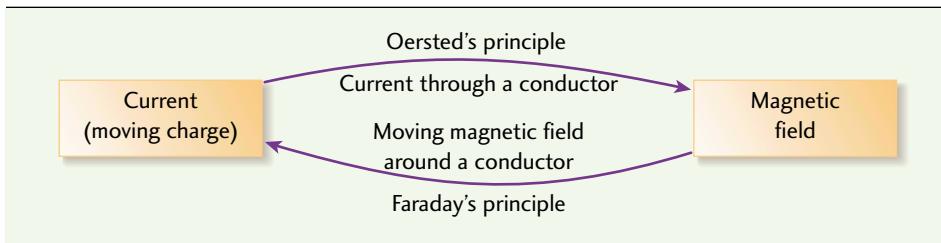
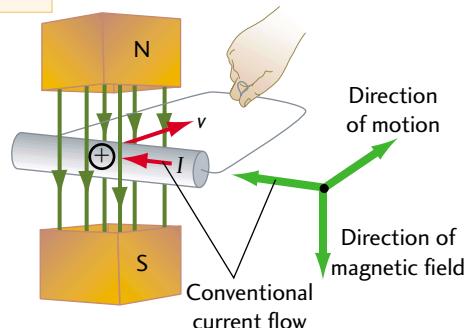


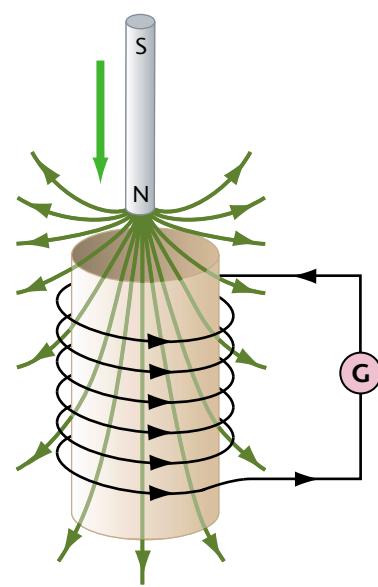
Fig.9.43 Current flows through a conductor in a changing magnetic field



Coiling the conductor into a helix with a smaller cross-sectional area makes a big difference in the amount of current produced in the presence of a magnetic field because the conductors cut the magnetic field closer to the magnet where the field is stronger. In Figure 9.45, when the magnet is plunged into or pulled out of a coil of wire, it causes current to flow. What determines the direction of current flow?

In 1835, German physicist Heinrich Lenz formally noted the relationship between the direction of movement of the inducing magnetic field and the direction of induced charge flow. Applying the law of conservation of energy to electromagnetic induction, Lenz considered that the electrical energy induced in a conductor must originate from the kinetic energy of the moving magnetic field. The *increase* in the *electric potential energy* of the charges in the induced current results in a *decrease* in the *kinetic energy* of the *moving magnetic field*. This loss in kinetic energy is felt as an *opposition* to the moving field. Lenz reasoned that the opposition to the motion of the external magnetic field is from an induced magnetic field created by the induced charge flow.

Fig.9.45 Coiling a conductor induces a larger current



Note the operative word in Lenz's law is *opposes*, not repels.

Lenz's Law: The direction of the induced current creates an induced magnetic field that *opposes* the motion of the inducing magnetic field.

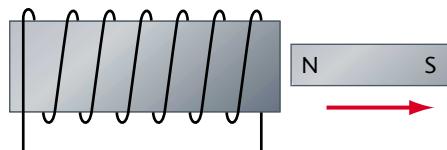
Lenz's law allows us to predict the direction of current flow by determining the direction of the induced magnetic field and then using the **right-hand rule #2** to predict the direction of conventional current flow. The electric potential or **electromotive force (EMF)** created as a result of the induced current depends on the speed and strength of the inducing magnetic field, and on the number of turns and the cross-sectional area of the induction coil.

EXAMPLE 11

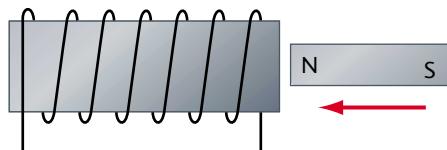
The direction of induced current flow

Use Lenz's law to predict the direction of induced conventional current flow in the coils in Figures 9.46a and 9.46b.

Fig.9.46



(a)

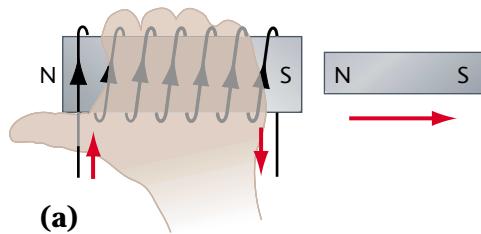


(b)

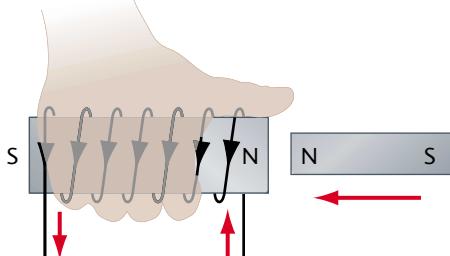
Solution and Connection to Theory

In Figure 9.46a, Lenz's law predicts that to oppose the motion of an outgoing north magnet, a south pole must be induced at the end of the coil. Applying the RHR #2 for solenoids, we grasp the coil with the thumb of the right hand pointing to the left. Conventional current flow is up the front of the coil, as shown in Figure 9.47a.

Fig.9.47



(a)

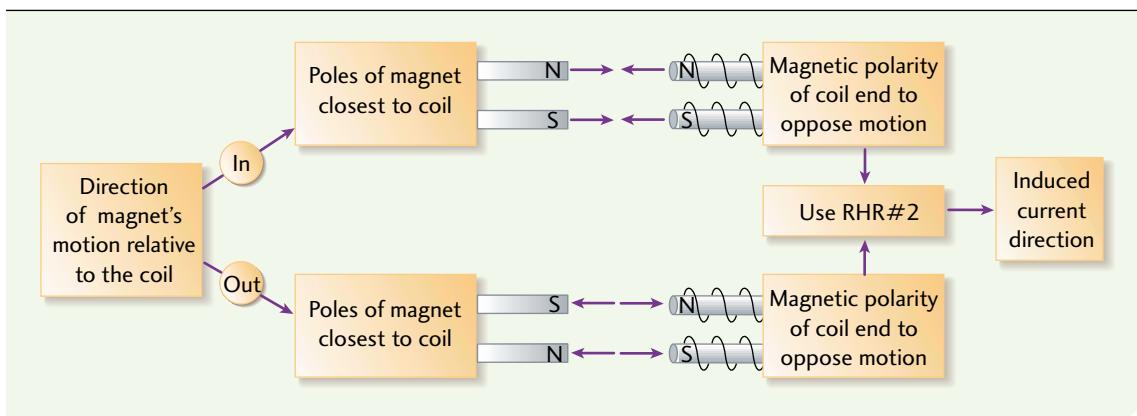


(b)

In Figure 9.46b, Lenz's law predicts that to oppose an incoming north magnet, a north pole must be induced at the end of the coil. Applying the RHR #2, we grasp the coil with the thumb of the right hand pointing to the right. Conventional current flow is down the front of the coil, as shown in Figure 9.47b. This direction is exactly opposite to that in Figure 9.47a.

Figure 9.48 Summarizes Lenz's law and the second right-hand rule for solenoids.

Fig.9.48 Lenz's Law and RHR #2



We have come almost full circle in our studies of electric and magnetic fields. Electrostatics, electric fields, current, and potential gave Oersted the means to study charge flow and to discover the link between current and magnetic fields. J.J. Thomson used magnetic and electric fields and circular dynamics to further study charge flow and the nature of the small particles that comprise it. Faraday completed the circle by discovering how simple conductors, forces, and moving magnetic fields can be used to create charge flow.

Sir Isaac Newton (1642–1727) stated, “If I have seen a little further, it is by standing on the shoulders of giants.” The work of the “giants” Oersted and Faraday was brought to fruition in 1864 by Scottish physicist James Clerk Maxwell (1831–1879) in his theories about electromagnetic fields. Originally known as **Maxwell’s equations of electromagnetism**, his theories involved four basic premises, the main concepts of which are captured in the following four statements.

The Four Premises of Maxwell’s Equations

- 1) The distribution of electric charges in space is dictated by the electric field that the charges produce.
- 2) Magnetic and electric field lines are similar except for the fact that magnetic field lines are continuous; they don’t begin or end the way electric field lines do on charges. Magnetic fields are dipolar, so there is no such thing as magnetic monopoles in the same way that a single electric field is created by charges.
- 3) Electric fields are created by changing magnetic fields.
- 4) Magnetic fields can be produced by changing electric fields or by moving electric charges (current).

The discovery that magnetic fields can produce electric fields and vice versa in free space led Maxwell to conclude that oscillating magnetic and electric fields could self-propagate through space as an electromagnetic wave (see Figure 9.49).

Fig. 9.49 A self-propagating electromagnetic field

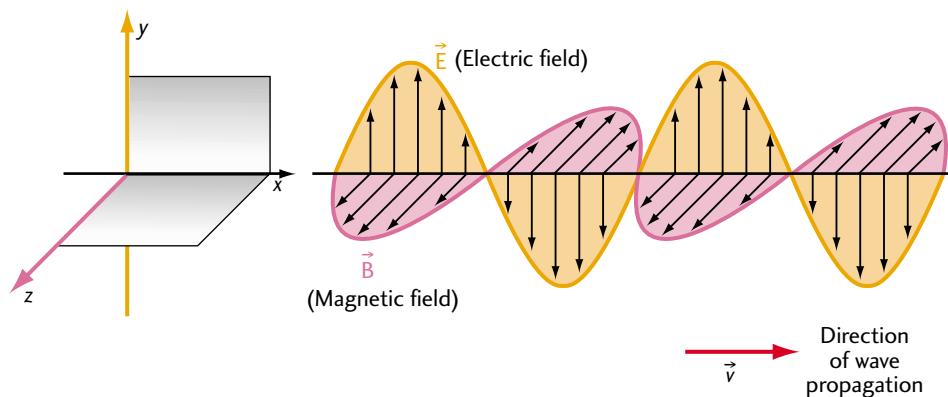
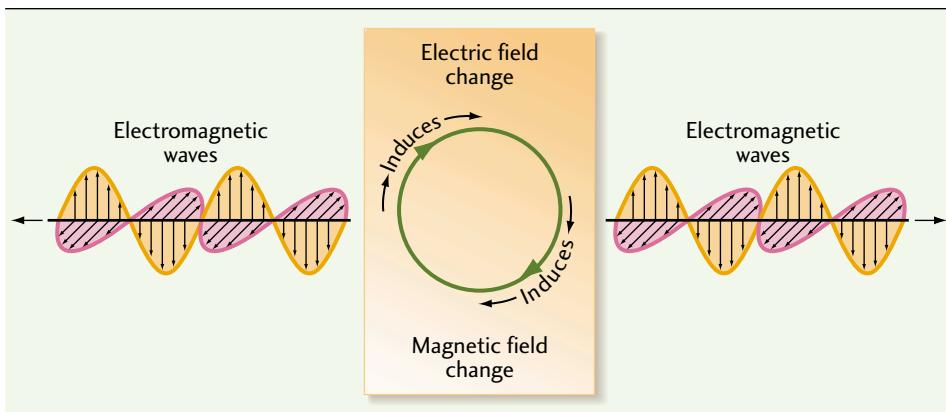


Figure 9.50 shows how the self-propagation of the electromagnetic wave in Figure 9.49 takes place.

Fig.9.50 The Self-propagation of Electromagnetic Waves



Maxwell succeeded in unifying the components of electric and magnetic fields with his electromagnetic field theory. Since 1955, scientists have been trying to come up with a way to unify all field theories; that is, the fundamental forces of nature (the weak force, the strong force, gravity, and electromagnetism). This theory is known as **unified field theory**. A theory that brings order to the seemingly chaotic and diverse aspects of physics might bring us closer to an understanding of the origin of the universe.



Magnetic Resonance Imaging (MRI)

Fig.STSE.9.1 An MRI machine



Fig.STSE.9.2 A precessing hydrogen nucleus

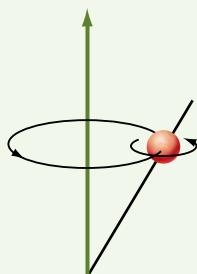


Fig.STSE.9.3 An MRI scan

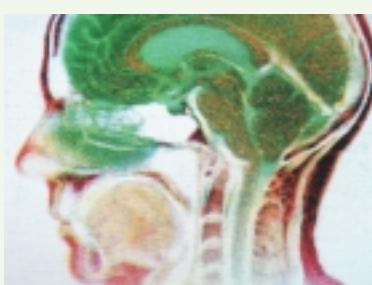
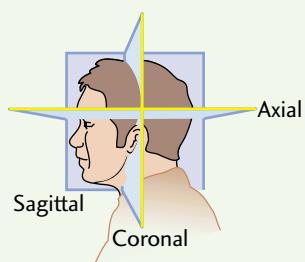


Fig.STSE.9.4 Different sections taken with an MRI



One beneficial and practical application of the magnets and related technology is **magnetic resonance imaging (MRI)** in medical diagnosis. MRI, shown in Figure STSE.9.1, a relatively new form of diagnostic imaging, is an improvement on standard x-rays or computed axial tomography (CAT) scans because it doesn't require the use of ionizing radiation, which damages human DNA.

An MRI works by placing the subject inside a very strong uniform magnetic field of strength up to two tesla. The nuclei of atoms in human tissue, especially hydrogen, respond to this field because of their magnetic dipole. Hydrogen nuclei, with their single positive proton, spin and produce their own magnetic field, according to Oersted's principle. These hydrogen dipoles line up with the external magnetic field until a radio-frequency (RF) electromagnetic wave is applied that specifically targets the hydrogen atoms. Like a radio receiver, these nuclear magnetic dipoles are tuned to the RF waves, absorbing some of their energy and causing them to spin off axis (Figure STSE.9.2) in a process called **precessing**. When the RF waves are turned off, the precessing nuclei return to their original aligned positions in the magnetic field and radiate some of their stored energy. This radiating electromagnetic energy is picked up by detectors, computer reconstructed, and displayed (Figure STSE.9.3) for analysis by the medical community.

An MRI provides incredibly detailed images of very small areas from any axis. A traditional CAT scan can only capture vertical sections of the subject, like slices from a loaf of bread. With its adjustable magnets, an MRI can capture sections from any of the axes shown in Figure STSE.9.4.

Used for almost any medical imaging, from infections and torn ligaments to tumours, cysts, or herniated discs, MRI nevertheless has some disadvantages. The strong magnetic field poses a safety hazard if even the smallest piece of loose ferromagnetic metal is present in the room. That's why all jewellery, watches, and metal-capped teeth need to be removed before the scan. Mops, buckets, paper clips, and fire extinguishers are examples of objects that have mistakenly been attracted into the bore of an MRI. Even metallic fragments inside an eye (not covered by scar tissue) left from long-standing injuries can be hazardous in this strong magnetic field because these fragments may damage eye tissue as they experience extreme forces. The fine resolution of an MRI requires that the patient remain perfectly still for the duration of the scan. An expensive scan needs to be redone if a patient moves even slightly at an inopportune time.

Design a Study of Societal Impact

Although no ionizing radiation is used for MRIs, they may not be routinely performed on pregnant women unless doctors decide that the benefit of performing the scan outweighs any risk to the fetus and the mother. An MRI is performed only after a case-by-case review of a patient's situation.

Research at least three types of medical diagnostic imaging techniques, such as x-ray, ultrasound, CAT scan, and MRI. Rank the types of imaging in order of most risk to least risk. Write a short paper on the following topic: "You are a family member of a young pregnant woman who has been hospitalized for a rare viral infection of the brain. Do you allow the use of an MRI?" What other information would you wish to have before making an informed decision?

Design an Activity to Evaluate

- a) Build a potential-energy hill or well using plaster or some other material. Use a mathematical relationship such as the inverse-square law or the potential-energy equation to dictate its shape. Use a marble rolling on the sculpted surface to model the behaviour of a charge moving in a magnetic or electric field. Build a marble launcher that can roll projectiles toward a potential-energy hill in order to model Rutherford's gold-foil experiment. Evaluate the shape of the field or surface that must be present to achieve marble scattering.
- b) Use a simple electromagnetic field strength monitor to measure the magnetic field strength in your classroom, other school area, or even your home. Organize your results in a table and try to find an explanation for the cause of these local fields.

Build a Structure

Some amateur physicists have taken up a hobby of building high-current pulsed electromagnets. See <www.irwinpublishing.com/students> for Web sites that describe powerful electromagnets able to sustain a huge current generated by way of energy stored in a parallel-plate capacitor (see the Chapter 8 STSE). These magnetic fields are able to crush cans and "shrink" coins with the forces they generate.

Build your own lifting electromagnet from a model-building kit or using everyday objects such as nails, wire, and batteries. Hold a competition to see which design can lift the greatest weight. Construct a transformer to determine the greatest alternating current that can be generated from standard household current.

Take appropriate safety precautions when working with high currents. Less than 1 A can be fatal.

You should be able to*Understand Basic Concepts:*

- Define and describe the concepts and units related to magnetic fields.
- State Oersted's principle of electromagnetism and apply the right-hand rules for straight conductors and coils to predict the direction of magnetic fields around electromagnets.
- Derive and apply the equations that relate the magnetic field strength to the forces that these fields apply to other magnets and electric charges.
- Use the concepts of electromagnetism to give an alternative definition of an ampere for electric current.
- Compare and contrast the properties of electric, gravitational, and magnetic fields by describing and illustrating the source and direction of the field in each case.
- Illustrate using field and vector diagrams, the magnetic fields and the magnetic forces produced by a single conductor, coiled conductors and other uniform magnetic fields.
- Analyze in quantitative terms the magnitude and the direction of the magnetic force applied to electric charges including ions in a uniform magnetic field.

Develop Skills of Inquiry and Communication:

- Demonstrate the technique of field mapping using iron filings and a bar magnet and compare the field characteristics to those of electric and gravitational fields.
- Perform an experiment that calculates the mass of a single charged particle such as that of an electron.

Relate Science to Technology, Society, and the Environment:

- Explain how the concept of a field developed into a general scientific model that could be used to explain force at a distance in both electrostatic and gravitational situations.
- Describe how scientific theories such as those of Oersted, Ampère, Faraday, and Lenz can be adapted or related to develop certain scientific principles and drive further research.
- Evaluate, using their own criteria, the social and economic impact of new technologies such as MRI for medical imaging and magnetohydrodynamics in propulsion.

Equations

$$F = BIL \sin \theta$$

$$B = \frac{\mu NI}{L}$$

$$B = \frac{\mu I}{2\pi r}$$

$$F = qvB \sin \theta$$

$$B = \frac{\mu NI}{2r}$$

$$m = \frac{qB^2r^2}{2V}$$

EXERCISES

Conceptual Questions

1. Summarize the law of magnetic forces.
2. Why do magnets attract other non-magnetic materials?
3. What do you call a material that is attracted to a magnet or that can be magnetized? Give at least two examples of this type of material. What is responsible for the magnetic character of this material?
4. In terms of domain theory, explain why magnets can lose their strength over time.
5. In terms of domain theory, explain what happens to a magnet when it is dropped or heated up.
6. Sketch the field lines around the cross-section of two parallel wires when the current in each wire flows
 - a) in the same direction.
 - b) in opposite directions.
7. When you're facing a computer screen, what is the direction of the magnetic field relative to the electron beam?
8. A current is running through a power line from west to east. What is the direction of the magnetic field on the north and south sides of the wire?
9. A current is passed through an insulated spring, creating a magnetic field of strength B . What happens to the field strength if the spring is compressed to one-half its original length?
10. A magnetic field is applied to a current-carrying conductor.
 - a) What angle should the wire make with the field for the force to be a maximum?
 - b) What should the angle be for the force to be a minimum?
11. A current is flowing east through a conductor when it enters a magnetic field pointing vertically down. What is the direction of the force on the conductor?
12. An electron is moving vertically down when it enters a magnetic field directed north. In what direction is the electron forced at that instant?
13. What is the direction of an electron along the axis of a current-carrying solenoid?
14. A cathode-ray tube aims electrons parallel to a nearby wire that carries current in the same direction. What will happen to the cathode rays in terms of deflection?
15. Would anything happen to the length of a helical spring when a current is passed through it? Explain.
16. If current is passed through a highly flexible wire loop, what shape does the loop assume? Why?
17. State Faraday's principle and describe at least three things that could be done to improve the electromotive force induced in a conductor.
18. What conditions must be met in order to induce current flow in a conductor?
19. Explain how the law of conservation of energy is related to Lenz's law.
20. Faraday's principle implies that an induced current in a coil (created by a moving magnet) creates an induced magnetic field. Explain why the induced magnetic field can't boost the induction process by moving the inducing magnet as in a "motor principle scenario."
21. One suggestion for a new automobile brake design is to use modified electromagnetic generators as brakes.
 - a) Explain how this type of brake might work in terms of the law of conservation of energy.

- b)** What would be the environmental or monetary benefits for using this type of brake in an electric car?

Problems

9.4 Artificial Magnetic Fields — Electromagnetism

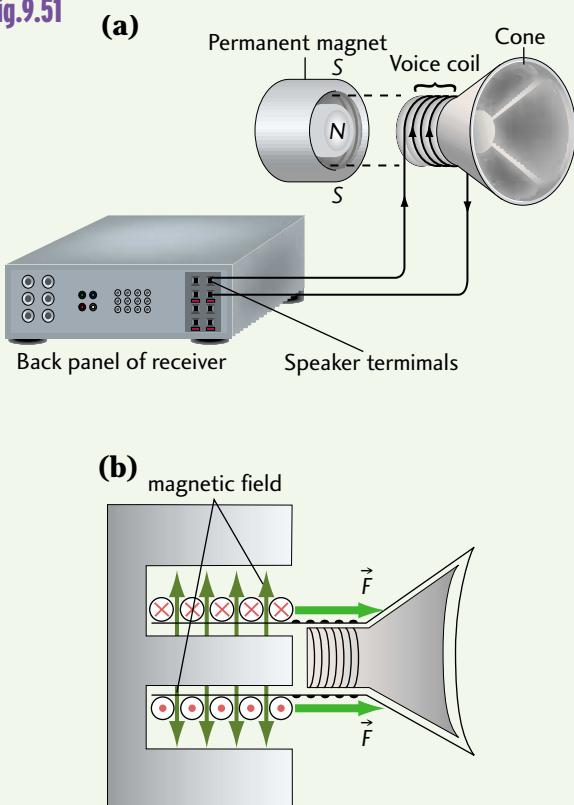
22. How far from a long conductor passing a current of 12.5 A is the magnetic field of strength 3.1×10^{-5} T?
23. Power lines 12 m from the ground carry 4.50×10^3 A of current across a farmer's field. What magnetic field strength do the cattle directly underneath experience?
24. A current of 8.0 A is passed through a single wire loop, producing a magnetic field of 1.2×10^{-3} T at the loop's centre. What is the radius of the loop?
25. A circular coil with 12 turns and a radius of 2.5 cm carries a current of 0.52 A. What is the magnetic field strength at the centre of this coil?
26. A long solenoid has 35 turns/cm. With a current of 4.0 A, what is the field strength at the core?
27. Two parallel conductors each carry 10 A of current in the same direction.
- a)** What is the magnetic field strength at the midpoint between these wires?
 - b)** What is the field strength at the same point if the current ran in opposite directions?
28. A student winds a 10-cm-long toilet paper tube with one layer of 400 turns of wire, then overlays it with a second layer of the same wire with 200 turns in the opposite direction. If the student applies a current of 0.1 A to the coil, what is the field strength in the interior of the tube?

29. A single wire loop of radius 2.0 cm is covered with another solenoid wound with 15 turns/cm, passing a current of 0.4 A. What current must be supplied to the inner loop in order to just cancel out the solenoid's field right at the centre?

9.5 Magnetic Forces on Conductors and Charges — The Motor Principle

30. A horizontal 6.0-m-long wire that runs from west to east is in a 0.03-T magnetic field with a direction that is northeast.
- a)** If a 4.5-A current flows east through the conductor, what is the magnitude and direction of the force on the wire?
 - b)** What is the magnitude and direction of the force if the current direction is reversed?
31. Copper metal wire has a linear density of 0.010 kg/m. A sample of this wire is stretched horizontally in an area where the horizontal component of Earth's magnetic field of strength 2.0×10^{-5} T passes through the wire at right angles.
- a)** What current must be applied to the wire if the weight of the entire wire is supported by the magnetic force?
 - b)** If this current is applied, what might happen to the wire?
32. The voice coil of a loudspeaker has a diameter of 2.2×10^{-2} m and contains 60 turns of wire in a 0.12-T magnetic field (see Figure 9.51). A current of 2.2 A is applied to the voice coil.
- a)** What is the force that acts on the cone and on the coil?
 - b)** What is the acceleration of the voice coil and cone if their combined mass is 0.025 kg?

Fig.9.51



33. An electron is injected into a magnetic field of strength 0.02 T at a speed of $1.5 \times 10^7 \text{ m/s}$ in a direction perpendicular to the field. What is the radius of the circle traversed by this electron?
34. What is the minimum radius of curvature for an alpha particle, ${}^4\text{He}^{2+}$, moving at $2 \times 10^6 \text{ m/s}$ in a magnetic field of $2.9 \times 10^{-5} \text{ T}$?
35. Some particles, such as electrons, are affected by both gravitational and magnetic fields. An electron in a television picture tube travels at $2.8 \times 10^7 \text{ m/s}$. Which force has more influence on the electron: the gravitational force or the magnetic force?
36. A charge of $1.5 \times 10^{-6} \text{ C}$ moves at 450 m/s along a path parallel to and 0.15 m away from a straight conductor. With a current of 1.5 A flowing in the same direction as the charge, what is the magnitude and direction of force on the charge?

37. An electron moves at $5 \times 10^7 \text{ m/s}$ at a distance of 5 cm from a long, straight wire carrying a current of 35 A . Find the magnitude and direction of the force on the electron when it is moving parallel to the wire

- in the opposite direction of the current.
- in the same direction as the current.

38. The Bohr model of the atom describes an electron circling a proton at a speed of $2.2 \times 10^6 \text{ m/s}$ in an orbit of radius $5.3 \times 10^{-11} \text{ m}$.

- What is the magnetic field strength at the proton?
- A scientist wishes to simulate the same electron orbit artificially by applying a magnetic field to the electron. What field strength must be applied to the electron to keep it in this orbit?

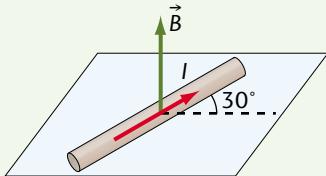
39. An electron moving through an electric field of 475 V/m and a magnetic field of 0.1 T experiences no force. If the electron's direction and the directions of the electric and magnetic fields are all mutually perpendicular, what is the speed of the electron?

40. The velocity selector of a mass spectrometer uses a magnetic field of strength $5.0 \times 10^{-2} \text{ T}$ and parallel plates that are 1.0 cm apart to produce a perpendicular magnetic field. What potential difference should be applied to the plates to permit singly charged ions only of speed $5 \times 10^6 \text{ m/s}$ to pass through the selector?

41. An electric power transmission line has two wires 3.5 m apart that carry a current of $1.5 \times 10^4 \text{ A}$. If towers are 190 m apart, how much force does each conductor exert on the other between the towers?

- 42.** Figure 9.52 shows conductors of length $L = 0.65\text{ m}$ and current $I = 12\text{ A}$ lying in a plane that's perpendicular to a magnetic field $B = 0.20$.

Fig.9.52



What is the magnetic force (both magnitude and direction) on the wire shown?

9.6 Applying the Motor Principle

- 43.** An electron moves at a speed of $5.0 \times 10^6\text{ m/s}$ perpendicular to a uniform magnetic field. The path of the electron is a circle of radius $1.0 \times 10^{-3}\text{ m}$.

- What is the magnitude of the magnetic field?
- What is the magnitude of the electron's acceleration in the field?
- Sketch the magnetic field and the electron's path in the conductor.

- 44.** A beam of protons moves in a circle of radius 0.22 m at right angles to a 0.35-T magnetic field.
- What is the speed of each proton?
 - What is the magnitude of the centripetal force acting on each proton?

- 45.** A charged particle with a charge-to-mass ratio of $5.7 \times 10^8\text{ C/kg}$ travels in a magnetic field of strength 0.75 T in a circular path that's perpendicular to the magnetic field. What is the period of revolution for this particle?

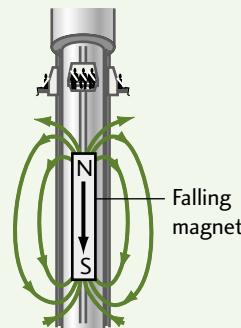
- 46.** A particle of mass $6.0 \times 10^{-8}\text{ kg}$ and charge $+7.2 \times 10^{-6}\text{ C}$ is travelling west. The particle enters a magnetic field of magnitude 3.0 T , where it completes one-half of a circle before exiting the field moving east. How much time does this charge spend inside the magnetic field?

9.7 Electromagnetic Induction — From Electricity to Magnetism and Back Again

- 47.** A bar magnet is dropped with its south end down through a horizontal wire loop. Looking down on the loop, what is the direction of the current in the loop? What is the direction of the current as the magnet falls out through the bottom of the coil?

- 48.** In the "Drop Zone" ride at Paramount Canada's Wonderland, riders are dropped from a great height and then decelerated safely to a stop before hitting the ground. One possible technological application of Faraday's principle and Lenz's law is the ride's braking mechanism. Figure 9.53 simulates the ride by using a magnet dropped into an open copper pipe.

Fig.9.53



- What is the direction of conventional current flow in the pipe?
- What is the shape and direction of the induced magnetic field?
- Does this situation result in decreased acceleration of the magnet/amusement park ride? Explain.
- Would this situation be any different if the north end of the magnet was dropped down? Explain.

- 49.** A long loop of copper wire is rotated in a magnetic field around an axis along its diameter. Why does the loop resist this type of motion? Would an aluminum loop make any difference to the resistance of rotation of one of these loops? Explain.



The Mass of an Electron

Purpose

To determine the mass of an electron

Background Information

The magnetic force applied to an electron, $F_B = qvB$, provides the centripetal force, $F_c = \frac{mv^2}{r}$, required to keep the electron moving in a circle, where q and m are the charge and mass on an electron, respectively, v is its velocity, B is the magnetic field strength, and r is the radius of curvature.

$$\text{If } F_B = F_c \\ \text{then } qvB = \frac{mv^2}{r} \\ v = \frac{qBr}{m}$$

An increase in kinetic energy is due to the applied voltage, V .

$$\frac{1}{2}mv^2 = qV \\ \frac{1}{2}m\left(\frac{qBr}{m}\right)^2 = qV$$

$$m = \frac{q(Br)^2}{2V}$$

where m is the required mass in kilograms (kg), q is 1.602×10^{-19} C, B is the magnetic field in tesla (T), r is the radius of curvature in metres (m), and V is the potential between electrodes in volts (V).

Safety Consideration

This lab uses high voltages. To avoid all shock hazards, be sure that the circuit is set up properly before turning the power on.

Equipment

- Vacuum tube, 6E5 (RCA)
- Socket base with leads (for vacuum tube)
- Air core solenoid
- Variable resistor (5 A)
- Power supply (100–250 VDC) for vacuum tube
- Power supply (6 VDC) for cathode heater (vacuum tube)
- Power supply (5 VDC, 5 A) for solenoid
- Ammeter (5 A)
- Connecting wires

Ruler

Wood dowels (various diameters) or circular templates

Procedure

Note: Most of these procedures are very specific for the type of solenoids and vacuum tubes that are being used in this lab. This lab is based on equipment from The Science Source (Waldoboro, Maine 04572).

- Set up the equipment as shown in Figures Lab.9.1, Lab.9.2a, and Lab.9.2b.

Fig. Lab.9.1

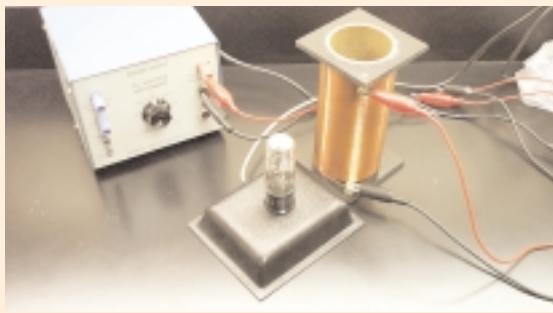


Fig. Lab.9.2 The proper wiring of the solenoid

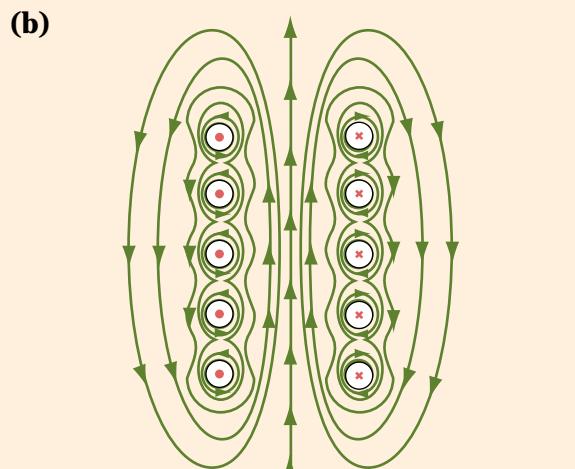
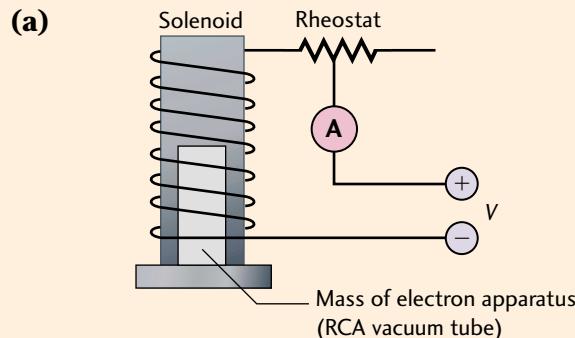
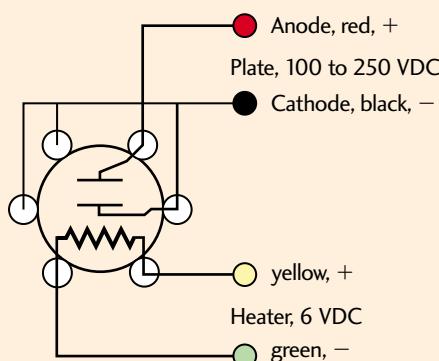


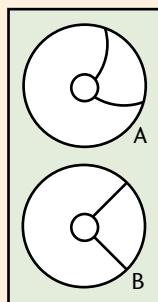
Fig. Lab.9.3 The proper wiring of the 6E5 RCA vacuum tube

(a)



(b)

Top view during lab



- Turn on the 6-V power supply to the air solenoid as well as the power supply to both the cathode heater and the parallel plates in the vacuum tube.
- Adjust the voltage across the plates and the rheostat to the solenoid in order to set the circular path of electrons (looking through the top of the tube) to an equivalent radius of curvature as the dowel or circular template.
- Measure the radius r of the beam path (by comparing it with the wooden dowels or templates), the solenoid current, and the potential across the plates in the vacuum tube.

- Measure the solenoid current from the ammeter and the plate potential from the voltmeter mounted on the power supply.

Uncertainty

Adjust the rheostat to higher and lower settings to measure an acceptable range for the solenoid current. The radius should have a precision of ± 0.50 mm. Assign acceptable uncertainty values for all measurements.

Analysis

- Find the magnetic field, B , from the written materials that came with your solenoid; for example, $B = 0.0036$ Tesla/amp(I).
- Calculate the magnetic field and the mass of an electron using the equation $m = \frac{q(Br)^2}{2V}$.
- Compare your answer for electron mass as calculated in this lab to the accepted value by finding the percent difference from 9.11×10^{-31} kg.

Discussion

- What happens to the radius of curvature of the electron beam when the voltage across the parallel plate is increased?
- What is the radius of curvature if the magnetic field strength is increased by increasing the solenoid current?

Conclusion

Write a concluding statement that summarizes your results. Include a sample calculation. Did you verify the accepted value for mass of the electron, within experimental uncertainty?

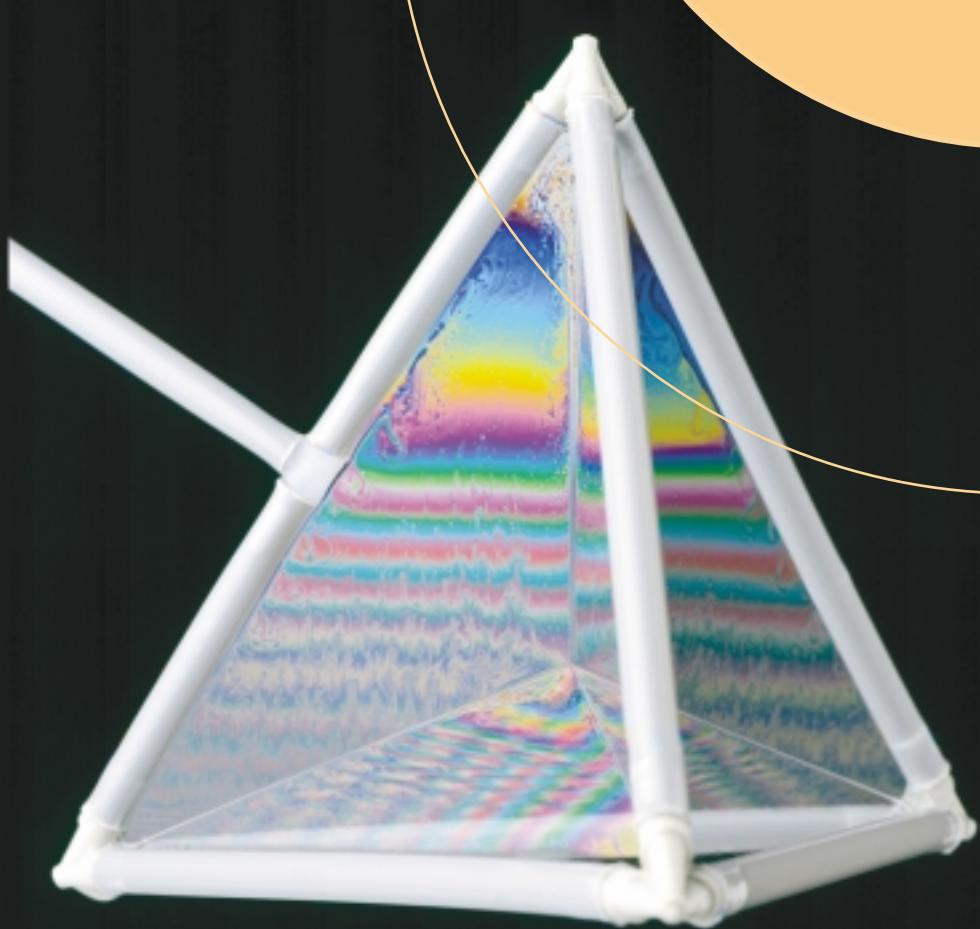
UNIT

D

10 The Wave Nature of Light

11 The Interaction of
Electromagnetic Waves

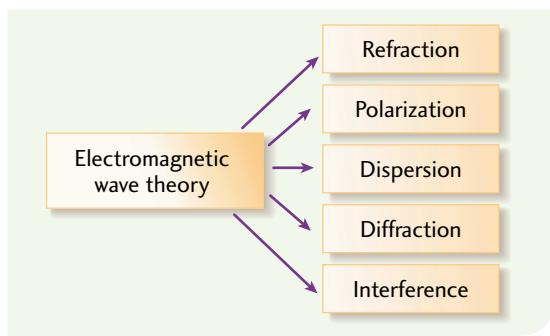
The Wave Nature of Light



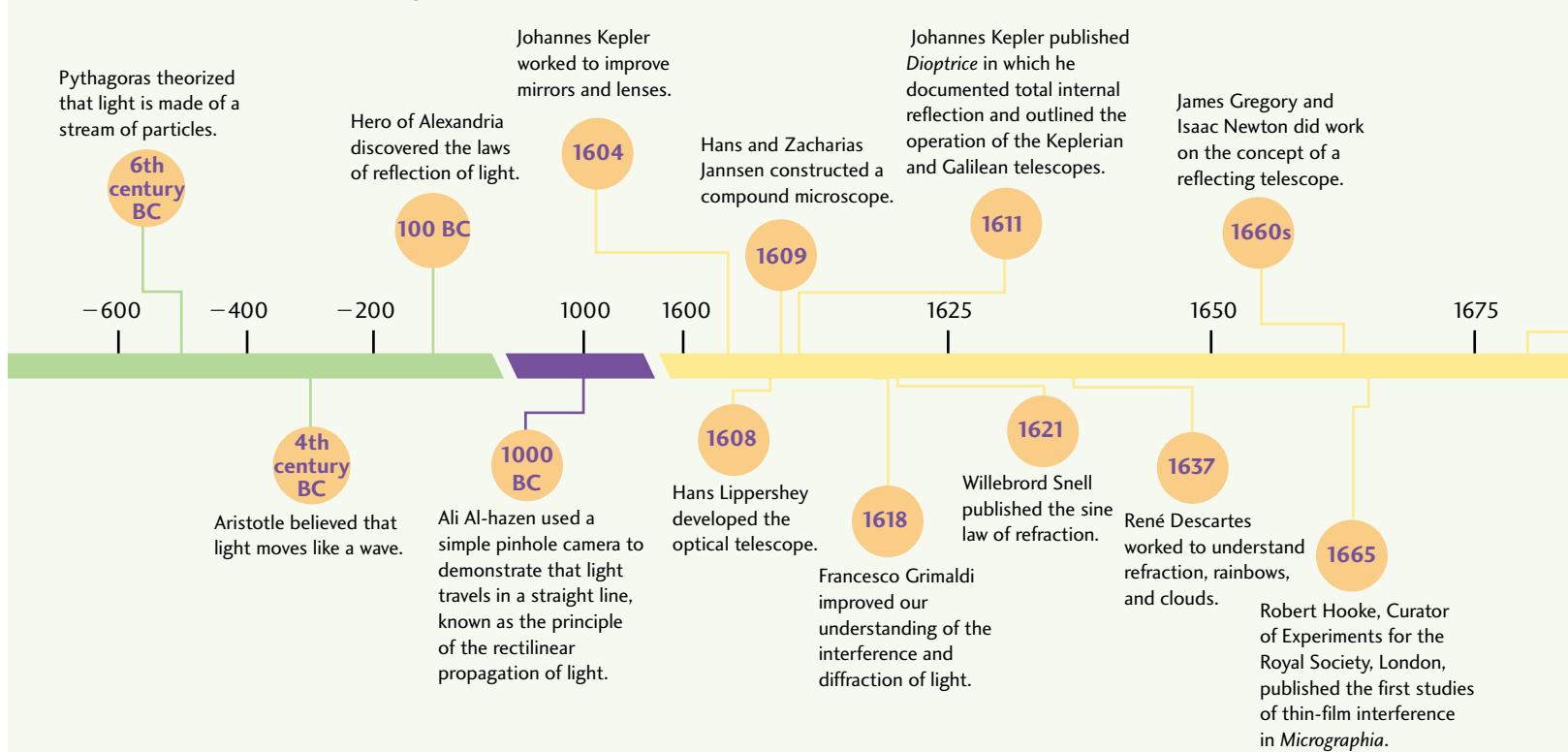
What is light? This question has been posed for centuries. Even though light has been studied extensively, its fundamental nature is still a mystery. In the sixth century BC, Pythagoras postulated that light was a particle. In the 1600s, the wave theory of light was being developed by Christian Huygens, while Isaac Newton was formulating corpuscular theories as to the particle nature of light. In the early 1900s, photon (particle) theory was being developed by scientists like Max Planck and Albert Einstein. Physicists like Thomas Young and Augustin Fresnel used the effects of refraction, interference, and diffraction to support their view that light was a wave.

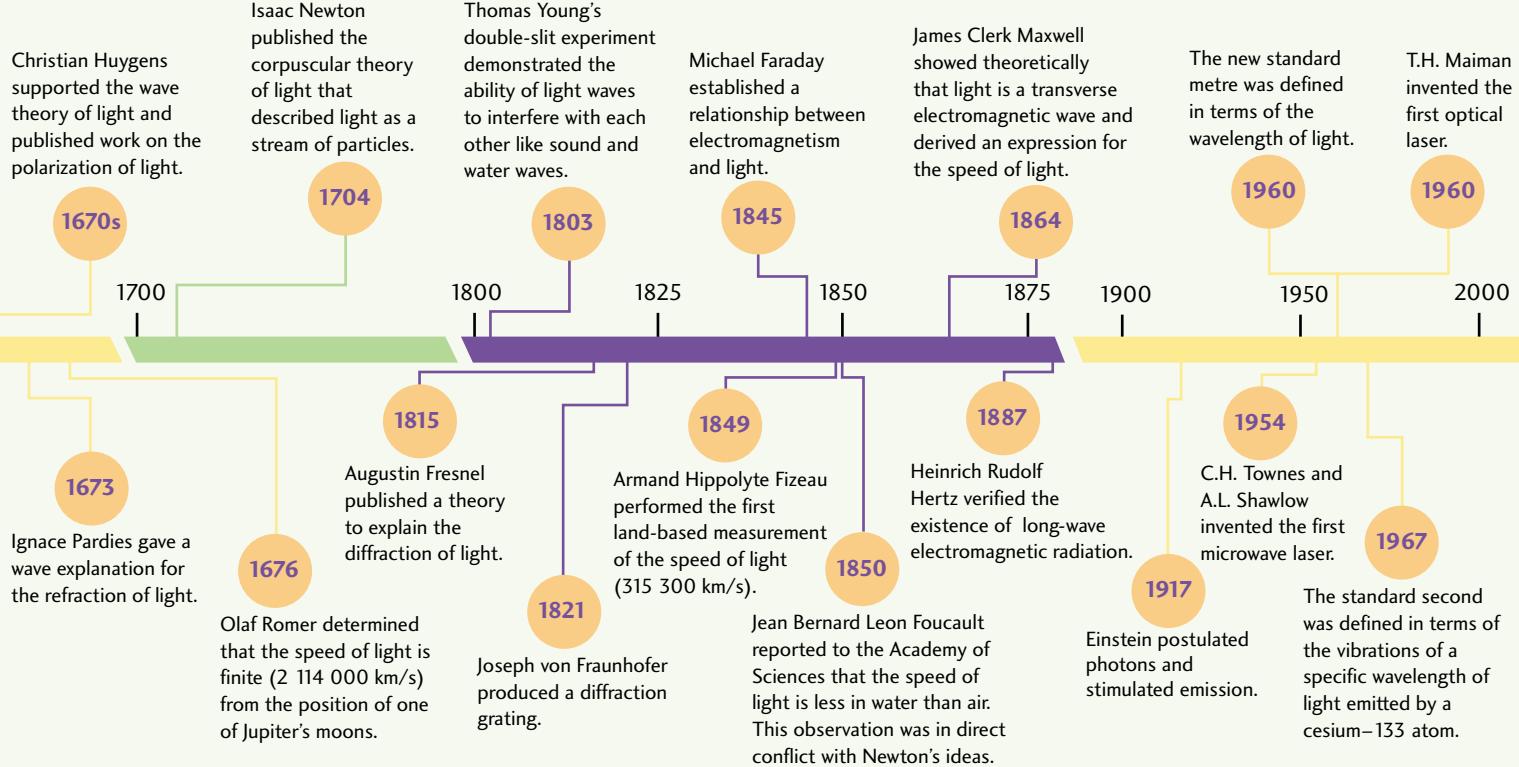
Light seemed to behave like both a particle and like a wave, depending on the experiment used to study it. Experiments involving the interference of light through single and double slits indicated that light was a wave. Einstein showed that light also behaved like a particle, or photon. In the photoelectric effect, photons knocked electrons out of metal in a manner that could only be explained by particle theory. In fact, it can be said that light is both a particle and a wave, and neither!

In this unit, we will investigate aspects of the wave nature of light and the aspects that corroborate the theory that light is a wave. We will also explain how various technological devices work in terms of the wave theory. From spectrometers to Polaroid sunglasses to CD technology, the wave nature of light is the basis of much of our technology today!



Timeline: The Wave Nature of Light





10

The Wave Nature of Light

Chapter Outline

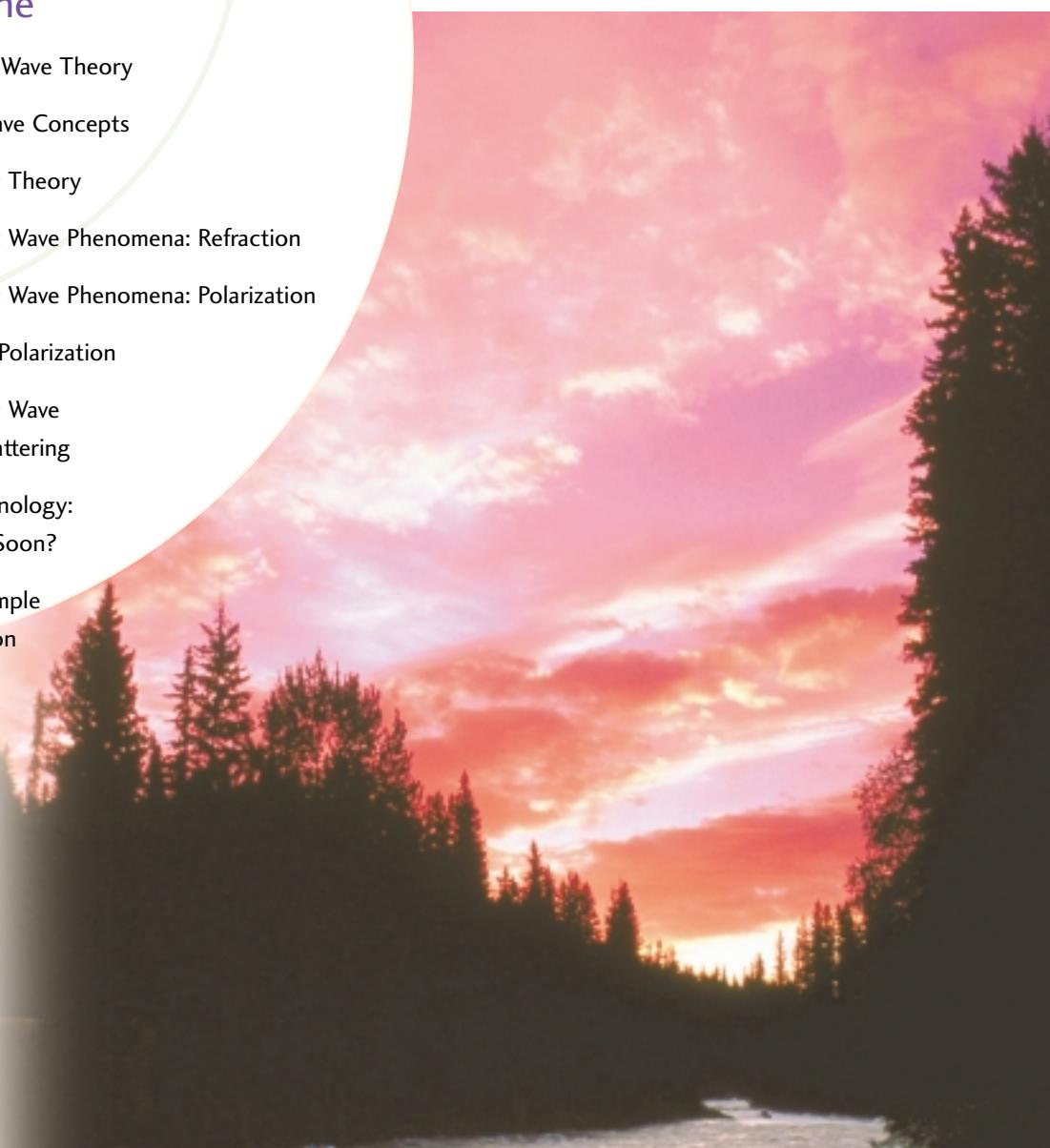
- 10.1 Introduction to Wave Theory
- 10.2 Fundamental Wave Concepts
- 10.3 Electromagnetic Theory
- 10.4 Electromagnetic Wave Phenomena: Refraction
- 10.5 Electromagnetic Wave Phenomena: Polarization
- 10.6 Applications of Polarization
- 10.7 Electromagnetic Wave Phenomena: Scattering

  Microwave Technology:
Too Much Too Soon?

LAB 10.1 Investigating Simple Harmonic Motion

LAB 10.2 Polarization

LAB 10.3 Malus' Law



By the end of this chapter, you will be able to

- use wave theory and refraction to explain how light behaves like a wave
- explain the various methods of polarizing light
- use the polarization of light to explain how light behaves like a wave
- explain where refraction and polarization are used in industry as well as where they occur in nature
- describe the various characteristics of different electromagnetic waves
- describe possible effects of using cell phones

10.1 Introduction to Wave Theory

Definitions

A stone is dropped into a still lake, a person shouts to someone across the room, a string is plucked and vibrates visibly, and oscillating electrons in an antenna send out radio waves to stereo receivers. These effects are all examples of wave motion. In general, **waves** are a *travelling* disturbance; that is, *they carry energy from one location to another*. We can break up waves into three broad categories: mechanical waves, electromagnetic waves (see Figure 10.1), and matter waves.

Fig.10.1a Water waves



Fig.10.1b A sound-wave collector



Fig.10.1c A radio-wave receiver



Mechanical waves are waves that are governed by Newton's laws. They require a physical medium in which to travel. Examples include water waves, sound waves, vibrating air columns in musical instruments, and waves travelling in springs.

Electromagnetic waves are waves that can travel through a vacuum, such as outer space. Electromagnetic waves all travel at 299 792 458 m/s, better known as the speed of light. Visible light is an example of this type of wave, along with infrared, ultraviolet, radio, and cosmic rays. Electromagnetic waves will be studied in depth in the next two chapters.

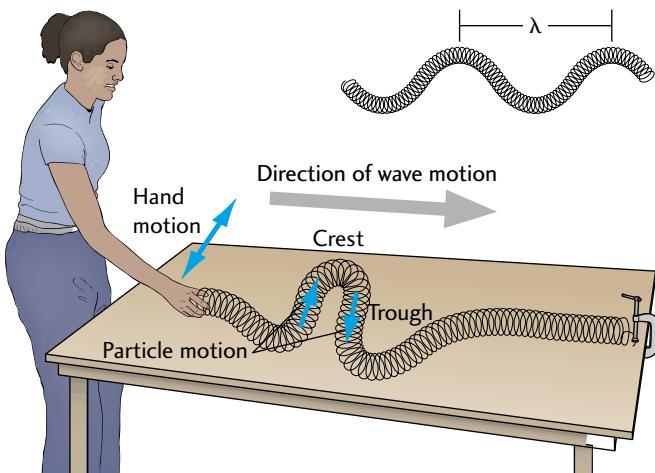
Matter waves are a model that amalgamates the particle and wave theories of energy and matter. Particles such as electrons, protons, neutrons, and other subatomic particles can all behave like waves. Many aspects of technology use these concepts. An example is electrons producing interference patterns similar to those of visible light or x-rays. The wave nature of matter will be covered in Chapter 12.

Types of Waves

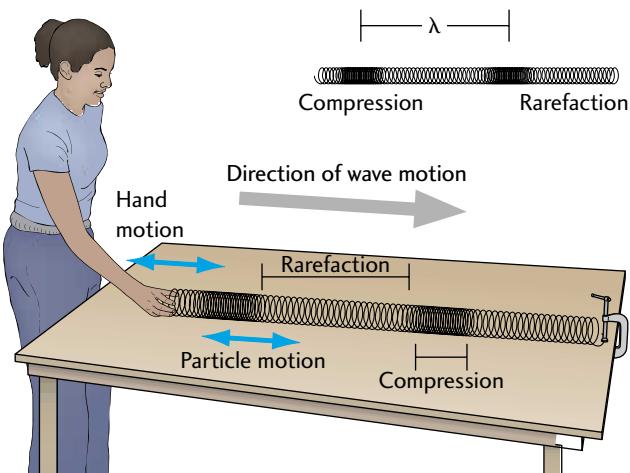
Fig.10.2a,b The action of sideways motion produces wave formation down the slinky. This motion is typical of transverse waves. The periodic pushing and pulling of the coils produces a compressed region that travels down the slinky, producing a longitudinal wave.

There are two general types of waves: **transverse** and **longitudinal**. Both types are generated from the action of an oscillating source. The repetitive motion of the source is called **simple harmonic motion (SHM)**.

Each type of wave can be illustrated using the mechanical motion of a spring, as shown in Figures 10.2a and 10.2b. These figures show the relationships between the direction of motion of the particle and the direction of wave travel for each type of wave.



Transverse wave production



Longitudinal wave production

Fig.10.3 Fans doing The Wave



Notice in both cases that it's not the particle itself that travels down the line. It's energy that is transmitted. The regular interaction between consecutive particles causes the wave to **propagate**. The relative position of two particles on the wave is called their **phase**.

In transverse waves, the particle motion is *perpendicular* to the direction of wave velocity. In longitudinal waves, particle motion is *parallel* to the direction of wave velocity. Both types of waves are called **travelling waves** because their energy travels from one point to another. They are also **periodic waves** because their cycles or patterns are repeated by the action at the wave source.

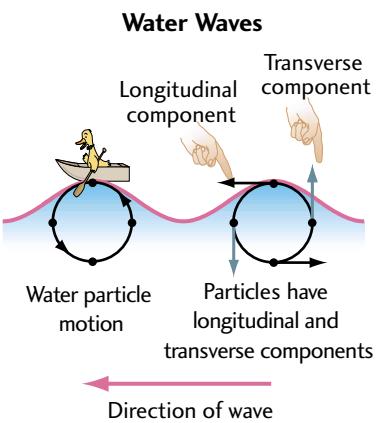
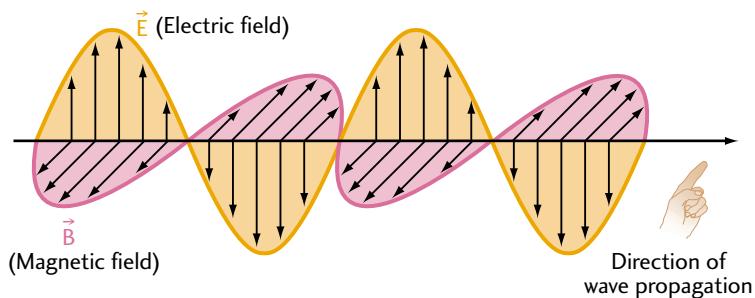


Fig.10.4 Water waves are a combination of the action of both kinds of waves, transverse and longitudinal. The particles of water move in circular paths, so sometimes they are parallel to the direction of wave motion, and at other times they are perpendicular to the direction of wave motion. People sitting in a boat find themselves moving in a circular clockwise path, in the direction of wave motion (see Figure 10.4).

An example of a *longitudinal wave* is sound. When a person speaks, the air is pushed out of the person's mouth (modulated by the mouth and vocal cords), causing compressions and rarefactions in the surrounding air. These pressure differences travel to the receiver of the sound, as illustrated in Figure 10.5.

An example of a *transverse wave* is light. Light is composed of oscillating electric (\vec{E}) and magnetic (\vec{B}) fields that are perpendicular to each other *and* to the direction of the wave's motion. Figure 10.6 illustrates this relationship. We will discuss the properties of light further in Section 10.3.

Fig.10.6 Light is a transverse wave



Note: In air, the only type of vibration possible is longitudinal vibration because air particles cannot sustain a transverse motion (they drift away).

1. Find examples of the three categories of waves.
2. Explain how a water wave is both a longitudinal and a transverse wave.
3. Tsunamis are waves generated by earthquakes in the sea. Sometimes they are referred to as tidal waves, even though tides have nothing to do with these mechanical waves. Research and find the wavelength and speed of these waves, along with historical examples of when they have occurred. Why are they so devastating?



Fig.10.7 The aftermath of a tsunami

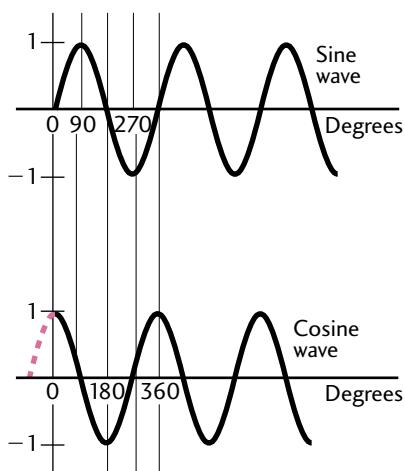


10.2 Fundamental Wave Concepts

Terminology

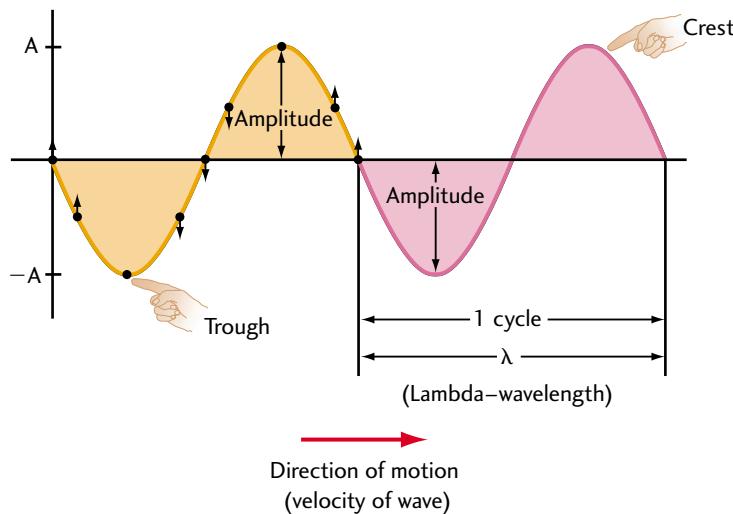
Note: The difference between the sine and cosine functions in Figure 10.8 lies in their starting points. At $t = 0$, the sine of zero is zero whereas the cosine of zero is one ($\sin 0^\circ = 0$ and $\cos 0^\circ = 1$). Thus, the wave starts at a different place in its cycle. When viewed together, the two waves are said to be phase-shifted.

Fig.10.8



In order to describe the periodic wave and all its properties, we use the sine (or cosine) function to represent this wave mathematically. Because both longitudinal and transverse waves are cyclic or periodic, we use the sine and cosine functions to represent both types of waves. These functions indicate the maximum and minimum displacements of the particle as well as the time periodicity of wave motion. Figure 10.9 illustrates the motion of a transverse wave. The particles transmitting the wave action are all moving perpendicular to the direction of motion, as indicated by the small arrows.

Fig.10.9 When the amplitude of the wave is a maximum (crest or trough), the particle is momentarily at rest because it is changing direction



The **period** of a wave (T) is the amount of time (t) it takes a wave to complete one cycle. The SI units for the period are seconds (s). Mathematically, $T = \frac{t}{N}$, where t is the total time and N is the total number of cycles.

The **frequency** of a wave (f) is the number of these cycles that can occur in a given time period, usually one second. The SI unit for frequency is hertz (Hz), which means “cycles per second” and is written as $\frac{1}{s}$ or s^{-1} . Mathematically, $f = \frac{N}{t}$

The **wavelength** (λ) is the length of one complete cycle. The SI unit for wavelength is the metre (m) and its symbol is the Greek letter *lambda*.

The **amplitude** of a wave is the maximum disturbance of the wave from its zero point. Waves have a positive and a negative amplitude.

By examining the units, we can see that period and frequency are reciprocals of each other. Thus, we can write

$$T = \frac{1}{f} \text{ and } f = \frac{1}{T}$$

EXAMPLE 1

Calculations involving T and f

Calculate the period and frequency of a propeller on a plane if it completes 250 cycles in 5.0 s.

Solution and Connection to Theory

Given

$t = 5.0 \text{ s}$, the total time of the event

$N = 250$, the total number of cycles

$$f = \frac{N}{t}$$

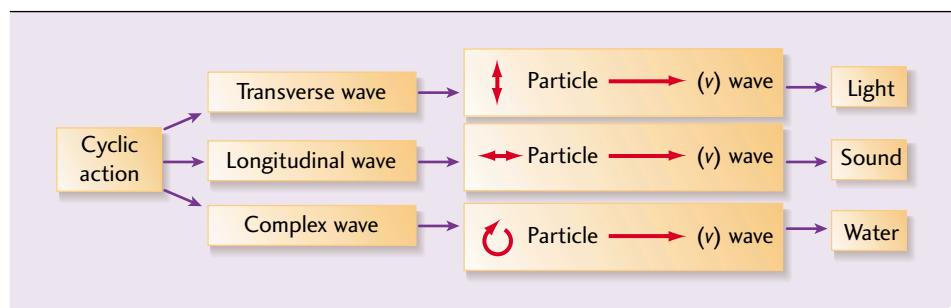
$$f = \frac{250 \text{ cycles}}{5.0 \text{ s}} = 50 \text{ s}^{-1} = 50 \text{ Hz}$$

To calculate the period,

$$T = \frac{1}{f} = \frac{1}{50 \text{ s}^{-1}} = 0.02 \text{ s}$$

Figure 10.10 summarizes the different types of cyclic-action waves.

Fig.10.10 Summary of Wave Types



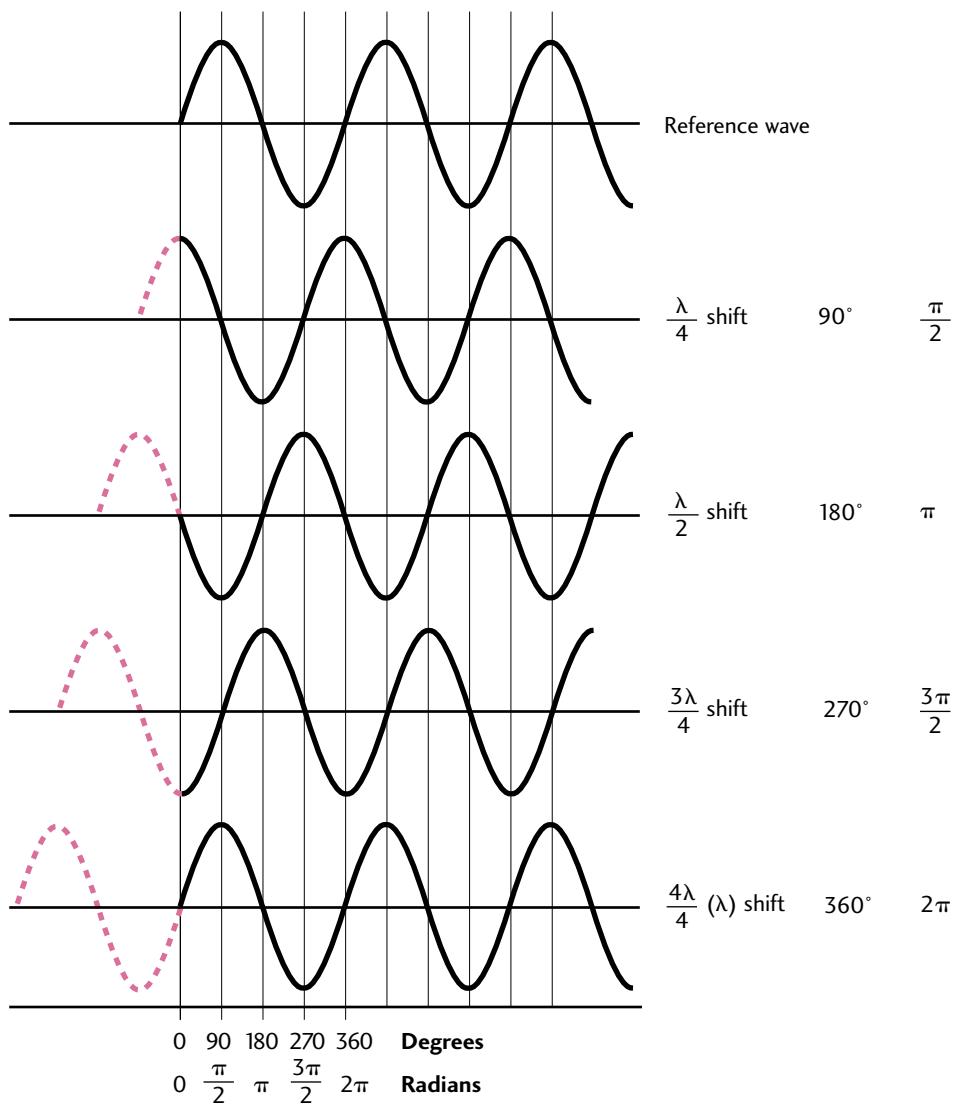
Phase Shift

Many kinds of periodic motion are also harmonic motion. The type of motion that can be represented by the sine or cosine wave is called *simple harmonic motion* and is represented by the equations

$$y = A \sin \theta \text{ or } x = A \cos \theta$$

where y and x are displacements in the vertical and horizontal directions, respectively, and A represents the wave's amplitude. A wave's displacement is the same as in kinematics; that is, it indicates how far in a given direction the wave has travelled. By changing the angle θ , the starting point of the wave also changes and the wave is said to be **phase-shifted**. A phase shift of 180° causes the crests and troughs to change position (i.e., a trough becomes a crest and vice versa). When the angle is shifted by a full 360° , the wave has completed one full cycle and once again looks like the original. The phase shift can also be expressed in terms of wavelength. A series of possible shifts is illustrated in Figure 10.11.

Fig. 10.11 Possible phase shifts. Notice that the shift can be expressed in terms of wavelength, degrees, and radians.



RADIAN MEASURE

In radian measure (see Chapter 7), a phase shift of 360° is equal to 2π radians. Thus, one radian is about 57.3° .

Simple Harmonic Motion: A Closer Look

To help us understand wave motion, and thus the properties of light, let's review the behaviour of mechanical waves. Consider Hooke's law, which we studied in Chapters 3 and 5. It states that the restoring force on a spring varies directly as the displacement of the spring ($F = kx$). In accordance with Newton's third law, the more you pull on a spring, the more the force trying to bring the spring back to equilibrium increases. This law holds true until the spring is deformed too much and thus cannot return to its original shape. In the ultimate stretch, the spring becomes a wire!

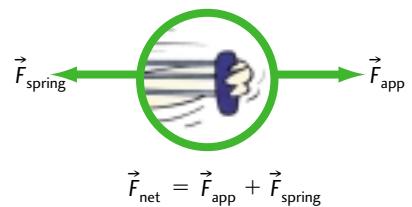
The spring chest expanders used by body builders are an example of this kind of force in action. In Figure 10.12a, the wannabe muscle-duck finds the pull easy at the beginning of the stretch. However, as the spring coils are pulled farther apart, the effort required becomes greater.

Fig.10.12a



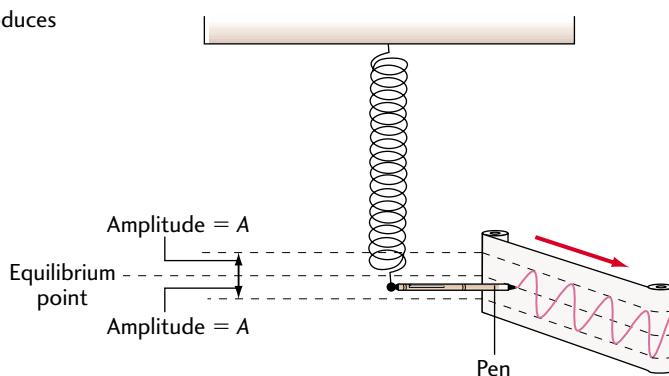
Fig.10.12b

The larger the stretch (x), the greater the applied force required



To illustrate how vibrating motion (up and down) can generate a sine wave, imagine a vertical spring with a pen attached to it horizontally. When you pull the spring down and then let go of it, the pen records the motion of the spring on a moving roll of paper. Figure 10.13 shows that the pattern produced by this imaginary device would be a sine wave. The wave action is caused by the inertia of the spring and pen. Inertia pulls the pen beyond the equilibrium position down the page, then the restoring force of the spring pushes the pen back up the page. The resulting repetitive movement illustrates simple harmonic motion.

Fig.10.13 Spring action produces sinusoidal motion



Simple Harmonic Motion in Two Dimensions

In the last section, we introduced the equations

$$y = A \sin \theta \text{ and } x = A \cos \theta$$

that represent the vertical and horizontal displacements, respectively, of simple harmonic motion. In two dimensions, x and y are perpendicular to each other. By combining the two displacements vectorially, we can calculate the resultant vector's magnitude using Pythagoras' theorem:

$$x^2 + y^2 = A^2 \cos^2 \theta + A^2 \sin^2 \theta$$

$$x^2 + y^2 = A^2 (\cos^2 \theta + \sin^2 \theta)$$

But $\cos^2 \theta + \sin^2 \theta = 1$; therefore,

$$x^2 + y^2 = A^2$$

$x^2 + y^2 = A^2$ is the equation of a circle, where A is the radius.

This equation shows that the magnitude of the wave's net displacement is constant, no matter what the angle is. For amplitudes that are equal in each direction, the result is circular motion. Therefore, the equations $y = A \sin \theta$ or $x = A \cos \theta$ represent the components of circular motion in the y and x directions, respectively, as shown in Figure 10.14.

Fig. 10.14 The components of circular motion

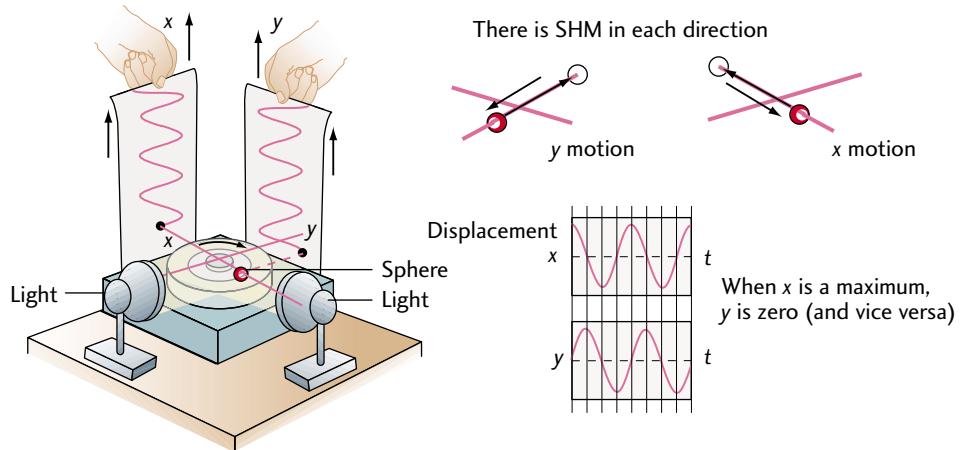


Figure 10.14 is a hypothetical setup that shows a small sphere rotating clockwise on a turntable. The lights shining on the turntable in the x and y directions cast shadows of the sphere onto two sheets of paper, labeled x and y , placed opposite each light on the other side of the turntable. As the sphere rotates, it undergoes simple harmonic motion in the x and y directions. Its two shadows trace a sine wave on each sheet of paper in both the x and y directions. When the wave on sheet x is a maximum, the wave on sheet y is a minimum and vice versa. Therefore, we can say that the shadow in the x direction traces a cosine wave, and the shadow in the y direction traces a sine wave.

EXAMPLE 2

Aspects of simple harmonic motion

What happens to the sine wave representation in Figure 10.15a if

- the amplitude is increased?
- the frequency is doubled?
- the phase is changed by 90° ($\frac{\pi}{2}$ radians)?

Solution and Connection to Theory

Fig.10.15b

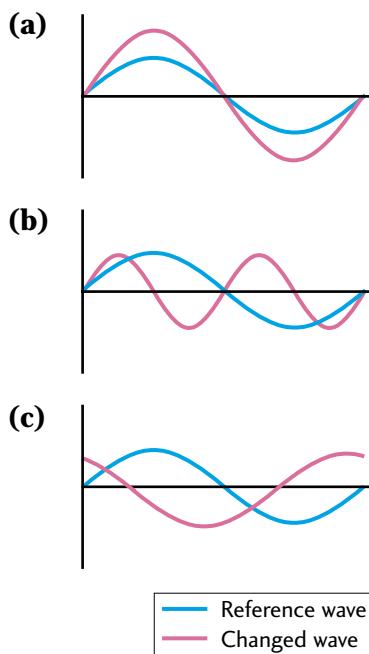
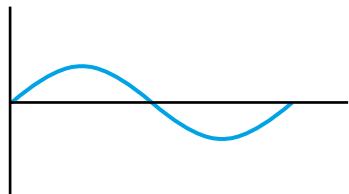


Fig.10.15a



Recall from Chapter 7 that $\theta = \omega t$, where ω is the angular velocity. Thus we can write the displacement as $x = A \cos \omega t$. Also, $r\omega = v$, where v is the linear speed and r is the radius of the circle. For circular motion, $v = \frac{2\pi r}{T}$. Then,

$$\omega = \frac{v}{r} = \frac{\left(\frac{2\pi r}{T}\right)}{r} = \frac{2\pi}{T}.$$

The equation $x = A \cos \frac{2\pi t}{T}$ is an expression for simple harmonic motion. If you know both the period, T , and the time, t , at which you look at the oscillating object, you can calculate its displacement from equilibrium.

- Calculate the period in seconds for the following cyclical events.
 - 5 classes every 375 minutes
 - 10 swings of a pendulum in 6.7 s
 - $33\frac{1}{3}$ turns of a turntable in 1 minute
 - 68 sit-ups in 57 s
- Calculate the frequency in Hz for the following cyclical events.
 - 120 oscillations in 2.0 s
 - 45 revolutions of a turntable in one minute
 - 40 pulses in 1.2 hours
 - 65 words keyed every 48 s
- Convert the period to a frequency for question 1, and the frequency to a period for question 2, above.
- Draw a wave of amplitude 2 cm and wavelength 4 cm. Redraw the wave for
 - a period change of $\frac{T}{4}$.
 - a phase change of 180° (π radians).



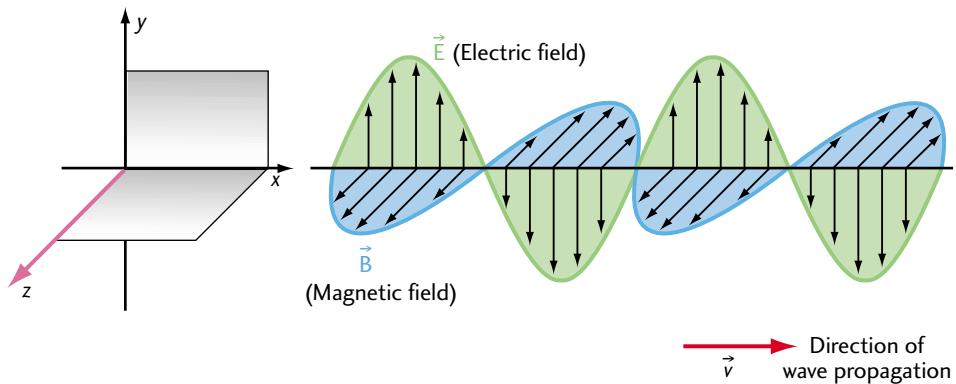
5. Find the distance of a spring, undergoing SHM from its rest position, if the defining equation is $x = 30 \cos \theta$, x is measured in centimetres, and the phase angle is
a) 30° . **b)** 180° . **c)** 270° . **d)** 360° . **e)** $\frac{\pi}{4}$ rad.
6. Explain why uniform motion in a circle is really simple harmonic motion.
7. Research the simple pendulum. Relate the pendulum's motion to simple harmonic motion. Find an expression for the period of the pendulum's swing.

10.3 Electromagnetic Theory

Properties of Electromagnetic Waves

Electromagnetic waves have the following properties, illustrated in Figure 10.16:

Fig.10.16 $(\vec{E} \perp \vec{B}) \perp \vec{v}$



FIELDS

Field theory was introduced over 100 years ago in order to explain forces between objects *not in contact* with each other. In 1935, Hideki Yukawa postulated that the fields themselves were created by the exchange of particles between the objects.

- 1) Electromagnetic waves are made up of alternating oscillating electric and magnetic fields.
- 2) The electric and magnetic fields are perpendicular to each other.
- 3) The vibration of the electric and magnetic fields is perpendicular to the direction of the wave's motion; therefore, electromagnetic waves are transverse waves.
- 4) The electric and magnetic fields vary **sinusoidally** in phase with each other; that is, they maintain the same sine-wave phase with respect to each other.
- 5) Electromagnetic waves travel at c (the speed of light) in a vacuum. In other media, they travel at different speeds, which causes refraction.

The Speed of Electromagnetic Waves

The speed of a wave is obtained from the equation $v = \frac{\Delta d}{\Delta t}$. Substituting wave variables for kinematics variables, we can replace Δd with λ (wavelength) and

Δt with T (period). When a wave travels a distance equivalent to λ , it takes a time T to do so. Substituting these new variables into the speed equation, we obtain

$$v = \frac{\lambda}{T}$$

We also know that $T = \frac{1}{f}$, where f is the frequency. Therefore,

$$v = \lambda f$$

This equation is known as the **universal wave equation**.

The Speed of Light

The term “electromagnetic wave” was born when James Clerk Maxwell proved in 1865 that light was a travelling wave of electric and magnetic fields. In Maxwell’s time, the only known electromagnetic waves were infrared, visible, and ultraviolet. Shortly after, Heinrich Hertz added radio waves to the list. Today, the list includes all the waves that comprise the electromagnetic spectrum, illustrated in Figure 10.17.

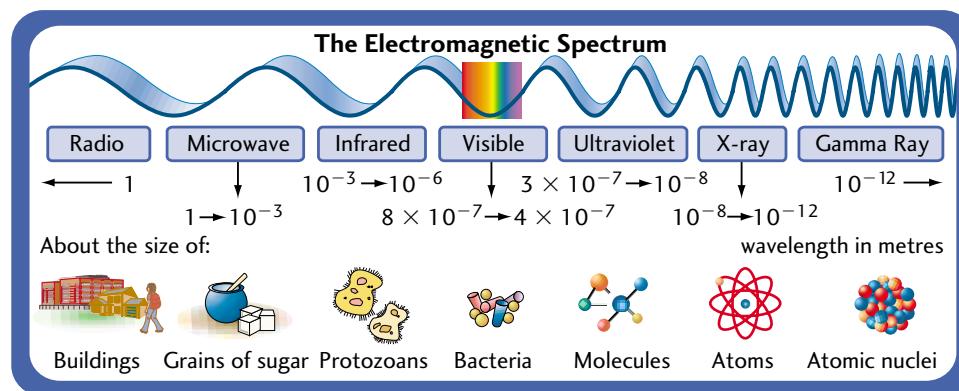
The equation for the speed of light derived by Maxwell was

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \text{ where}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{(N \cdot m^2)}$$

and is called the permittivity of free space (the electric field part of the equation), and $\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$ and is called the permeability of free space (the magnetic field part of the equation).

Fig.10.17



Maxwell proved theoretically that electromagnetic waves travel through a vacuum at the speed of light, c , or 3.0×10^8 m/s. His theory involved wave mechanics, and it corroborated the idea that light is a wave. It was found that electromagnetic waves could be encoded with information through modulation of their amplitudes (amplitude modulation or AM), their frequencies (frequency modulation or FM), or pulse code modulation (PCM) for digital transmission, then sent at the speed of light to antennae that intercept the waves. The electrons in the antennae are forced into oscillations corresponding to the radiation frequencies sent, which create changing magnetic and electric fields in the antennae. These field frequencies can then be decoded back to the original information so you can hear the top-40 hits on the radio. The wave theory of light has led to the development of our

The currently accepted value for the speed of light is 299 792 458 m/s.

whole communications field, from the wireless telegraph, to radio, to TV, to satellite communications.

EXAMPLE 3 Just how fast is c ?

Mach number is a relative measurement of speed in terms of a multiple of the speed of sound.

Calculate the time it would take for light to reach us from the Sun, which is 1.49×10^{11} m away. Compare it to the time it would take a supersonic plane to fly the same distance at Mach 3.

Solution and Connection to Theory

Given

Distance to the Sun = 1.49×10^{11} m (average orbital radius since Earth's orbit is elliptical), $c = 3.0 \times 10^8$ m/s, $v_{\text{plane}} = \text{Mach } 3 = 3$ times the speed of sound = $332 \text{ m/s} \times 3 = 996 \text{ m/s}$ (assuming 0°C)

For light:

Using the equation for speed, $v = \frac{\Delta d}{\Delta t}$, we substitute c for v :

$$c = \frac{\Delta d}{\Delta t}$$

$$\Delta t = \frac{\Delta d}{c}$$

$$\Delta t = \frac{1.49 \times 10^{11} \text{ m}}{3.0 \times 10^8 \text{ m/s}}$$

$$\Delta t = 497 \text{ s}$$

This time is equal to 8.3 minutes.

For the plane:

Using the same equation for speed, we obtain

$$\Delta t = \frac{\Delta d}{v}$$

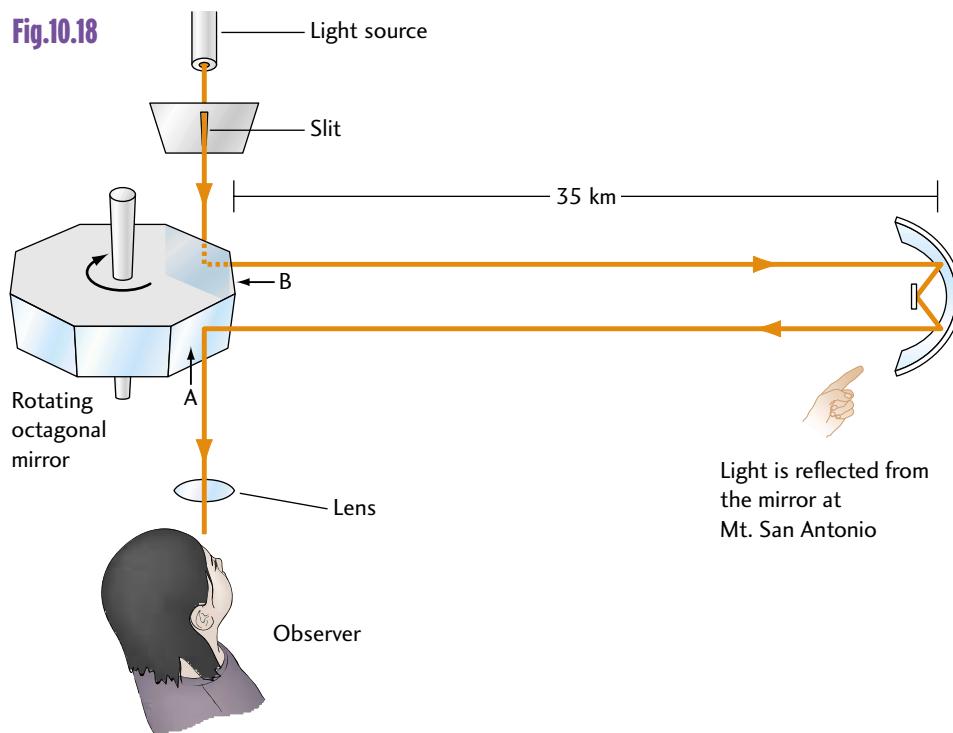
$$\Delta t = \frac{1.49 \times 10^{11} \text{ m}}{996 \text{ m/s}}$$

$$\Delta t = 1.50 \times 10^8 \text{ s}$$

This time is equal to 1730 days, or almost five years.

Compare this time to the time it takes to get to the Moon from Earth! The first lunar landing mission left Earth at 9:32 a.m. on July 16, 1969 and went into lunar orbit at 1:28 p.m. on July 19. This trip took just over three days. At the speed of light, this trip takes 1.3 s! (The mean radius of the Moon's orbit around Earth is 3.8×10^8 m.)

Fig.10.18



Measuring the Speed of Light

Albert A. Michelson measured the speed of light using an accurate distance measurement between Mt. Wilson and Mt. San Antonio, California. Without the rotating eight-sided mirror spinning, the position of the reflected light was measured at the observer position. The mirror was then set rotating and timed accurately. The rotational speed was adjusted in such a way as to have side B now reflecting the light to the observer rather than side A. Thus, the light now travelled to Mt. San Antonio and back in $\frac{1}{8}$ th of the period of rotation of the octagonal mirror. Knowing the distance and the time, a value of 2.997928×10^8 m/s was obtained for the speed of light.

The Production of Electromagnetic Radiation

The whole range of electromagnetic radiation is described as a spectrum, as illustrated in Figure 10.17. Another version of this diagram, shown in Figure 10.19, shows that the spectrum is continuous: the distinctions between the wave categories are blurred at the boundaries between different waves. Wave categories are named according to how waves are produced, as shown in Table 10.1. The “visible range” designation applies only to the range human beings can see naturally. Animals and insects “see” in different frequency ranges. They have their own defined visible light regions. Today, human beings can use infrared sensors to view objects in the dark, thus extending our range of vision. The next time you look up at the night sky, imagine the view if you could see all the frequencies of the electromagnetic spectrum!

Fig.10.19 The electromagnetic spectrum

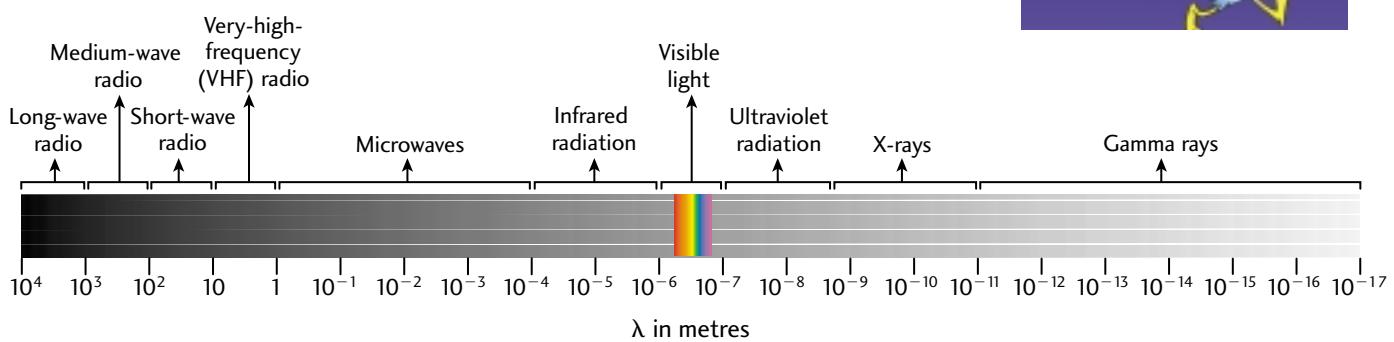


Fig.10.20



Table 10.1
Electromagnetic Waves

Formation method	Generator	Wave type	Typical uses	Detection
Electrons oscillate the length of an antenna, driven by electronic circuitry	Electric circuits	Radio waves	Carry AM signals in 1000 kHz range, FM in 100 MHz range, TV in 50 MHz and 500 MHz range	Antenna, crystal
High-frequency vibrations in small cavities	Klystron	Microwaves	Microwave ovens and weather radar	Electrical circuits
Electron transitions in outer orbits of atoms in outer orbits of atoms in inner orbits of atoms in innermost orbits of atoms	Hot bodies Lamps Sparks, lasers X-ray tubes	Infrared Visible Ultraviolet X-rays	Heat waves from the Sun Vision and laser communications Cause sunburn and skin cancer Penetrate soft tissue. Used in medical examinations and diagnostics	Thermopile (thermocouples) Eye Photoelectric photomultiplier Ionization chamber Photographic plates
Part of nuclear transformations and energy transitions in nucleus	Accelerators/reactors	Gamma rays	High penetrating power. Used to destroy malignant cells in cancer patients	Geiger and scintillation counters Bubble chambers

EXAMPLE 4

Calculating the frequency range of a band from the electromagnetic spectrum

Infrared light is invisible to the human eye except through special sensors. Given the range of wavelengths of infrared light, calculate their corresponding frequencies.

Solution and Connection to Theory

Given

The wavelength range for infrared light is from 1×10^{-3} m to about 7×10^{-7} m.

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{1 \times 10^{-3} \text{ m}} = 3 \times 10^{11} \text{ Hz.}$$

This frequency is for one end of the range. For the other end,

$$\frac{3.0 \times 10^8 \text{ m/s}}{7 \times 10^{-7} \text{ m}} = 4.3 \times 10^{14} \text{ Hz.}$$

The frequency of infrared light is 3×10^{11} Hz– 4.3×10^{14} Hz.

The delicate interactions between Earth, Earth's atmosphere, and electromagnetic waves determined how life evolved on this planet and the manner in which life continues to exist on it. The main source of electromagnetic waves on Earth is the Sun. (We also receive radiation from other celestial bodies, but it is much weaker than the radiation from the Sun.) The Sun generates the whole range of electromagnetic waves. The wavelengths in

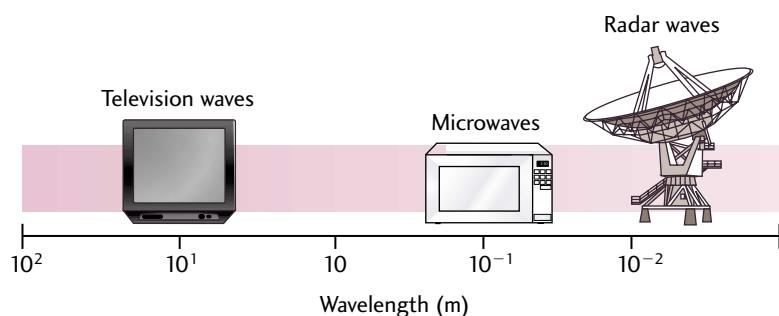
the ultraviolet region and longer penetrate to Earth's surface. Earth acts like a **black-body radiator**, absorbing the Sun's energy and re-emitting it mainly as heat in the infrared region of the spectrum. The air acts like an insulating blanket, capturing the Sun's energy for our use. Unfortunately, we are increasing this heating effect by adding gases, such as carbon dioxide, to the air. These gases increase the air's ability to reflect heat back to Earth's surface, which creates an overall warming effect termed "global warming."

The upper atmosphere, including the ozone layer, blocks harmful short-wave radiation, like cosmic rays, and reduces the intensity of ultraviolet rays. The upper atmosphere is also changing due to contamination. The holes that have formed in it cause it to transmit more harmful radiation than before.

Human beings generate a fair amount of electromagnetic waves through the use of modern telecommunications and electronic devices, especially in the radio end of the electromagnetic spectrum: telephones, radio, television, and satellites. Power lines and electronic equipment also create extra electric and magnetic fields in our environment. Use of these devices increases our total radiation exposure. It is a heavily debated subject as to how harmful this excess radiation is.

All the electromagnetic transmissions we send from Earth, including television shows, will travel to the far reaches of outer space. If aliens ever pick up our transmissions, they may get an interesting perspective of our civilization!

Microwaves and Microwave Ovens



Microwaves and water molecules are partners in one of the most common of all modern appliances — the microwave oven. A **magnetron** produces microwave radiation with a rapidly oscillating electric field (about 2.4×10^9 Hz). A metal fan distributes the waves by reflecting them (Figure 10.22b). Water molecules are polar (i.e., they have a positive and a negative pole) and are therefore attracted to and bonded to one another by weak intermolecular forces called **hydrogen bonds**. The rapidly oscillating electric field in the oven causes the polar water molecules in the food to change orientation billions of times a second. The net torque on the water molecules causes them to rotate and align their dipole moments with the electric field.

Fig.10.21 What would they think?



Fig.10.22a Wavelengths of microwaves and radio waves

Fig.10.22b A microwave oven

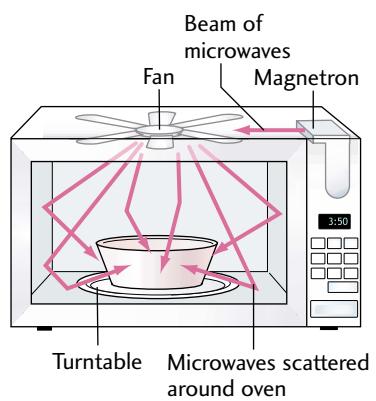
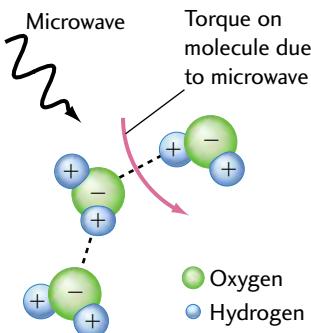


Fig.10.23 Intermolecular bonds form between water molecules because water molecules are polar. Torque exerted on a water molecule by a microwave causes a bond to break, releasing energy.



Hydrogen bonds are not connections between the hydrogen and oxygen atoms within the water molecule. Rather, they are a close-proximity attraction *between* water molecules that inhibits the molecules' translational and rotational motion.

When the water molecules absorb energy from the microwaves, the hydrogen bonds are broken (Figure 10.23) and the water molecules are free to find other partners. When new groups of water molecules are formed, the energy they gained from the microwaves is transferred into thermal energy or heat. The food cooks because the water in it is being heated.



1. Why does a ceramic dish with no food on it not heat up in a microwave oven, yet when food is prepared on it, it comes out hot?
2. Calculate the wavelength of microwaves.
3. What is the function of the metal grid on a microwave oven's door?
4. Calculate the frequency of
 - a) red light with wavelength 640 nm.
 - b) radio waves with wavelength 1.2 m.
 - c) x-rays with wavelength 2×10^{-9} m.
5. Calculate the wavelength of
 - a) infrared light of frequency 1.5×10^{13} Hz.
 - b) microwaves of frequency 2.0×10^9 Hz.
 - c) gamma rays of frequency 3.0×10^{22} Hz.

10.4 Electromagnetic Wave Phenomena: Refraction

The Refractive Index, n — A Quick Review

Refraction is a phenomenon of light that strongly supports the wave theory. Recall that light changes its speed when travelling through different optical mediums. If light enters the medium at an angle to the medium boundaries, it is bent either toward or away from the normal, depending on the refractive index of the material. (See Figure 10.24.)

Direction of Refraction

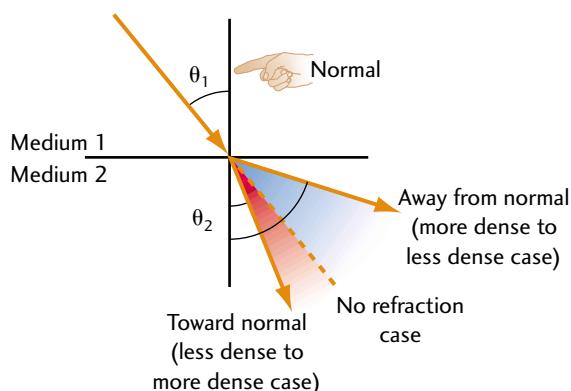


Fig.10.24 The normal is an imaginary line drawn perpendicular to the boundary between the media. All angles are measured from the normal.

The refractive index, n , is a measure of how much light slows down when it enters an optical medium. The greater the refractive index, the more the light slows down. Thus, n is defined as the ratio of the speed of light in a vacuum to the speed of light in a given medium. The equation for n is

$$n = \frac{c}{v}$$

where c is the speed of light in vacuum and v is the speed of light in a medium. We can see from this equation that the minimum value of n is 1. If n could be less than 1, then light could travel faster than the speed of light, which is impossible, according to Einstein.

Einstein postulated that the speed of light is constant, regardless of the reference frame in which it is measured. In fact, Einstein thought that it was the fastest speed at which an object could transmit information. This topic is further covered in Chapter 13.

EXAMPLE 5 The refractive index of diamond

Calculate the index of refraction of a diamond if the speed of light in a diamond is 1.24×10^8 m/s.

Solution and Connection to Theory

Given

$$c = 3.00 \times 10^8 \text{ m/s} \quad v = 1.24 \times 10^8 \text{ m/s} \quad n = ?$$

$$n = \frac{c}{v}$$

$$n = \frac{3.00 \times 10^8 \text{ m/s}}{1.24 \times 10^8 \text{ m/s}} = 2.42$$

The refractive index of a diamond is 2.42. Notice that the units have canceled out. Because n is a ratio, it is a unitless value.

The **phase velocity** of a wave can exceed the speed of light, c . However, no *information* carried by the wave can exceed c . For example, if an inch worm's linear motion along the ground corresponds to the speed c (also called **group velocity**), its up-and-down motion for propelling itself is faster than its linear speed. This speed is analogous to phase velocity.

EXAMPLE 6 Warp drive?

Calculate the speed of light in a hypothetical material you have discovered and named in honour of yourself. Its refractive index is 0.90.

Table 10.2 Index of Refraction	
Substance	n
Vacuum	1.000
Air	1.000 29
Water	1.33
Ethyl alcohol	1.36
Glycerin	1.47
Crown glass	1.50
Flint glass	1.91
Diamond	2.42

Solution and Connection to Theory

Given

$$c = 3.00 \times 10^8 \text{ m/s} \quad n = 0.90 \quad v = ?$$

$$n = \frac{c}{v}$$

$$v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{0.90}$$

$$v = 3.3 \times 10^8 \text{ m/s}$$

The speed of light in our hypothetical medium is greater than the speed of light in a vacuum!

Snell's Law: A More In-depth Look

The relationship between the angles of incidence and angles of refraction is given by **Snell's law**,

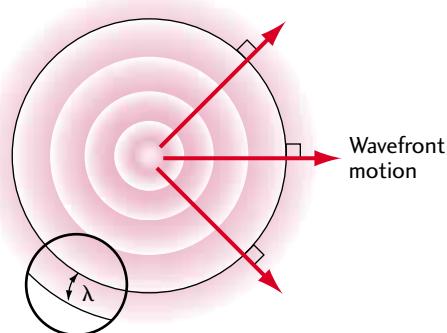
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where the subscripts 1 and 2 refer to the incident and refracted mediums, respectively. The derivation of this law assumes that light behaves like a wave. If we consider light to possess wavefronts much like the ripples produced by a disturbance in water (Figure 10.25), then light waves will bend as they enter a different medium *as long as they enter that medium at an angle*.

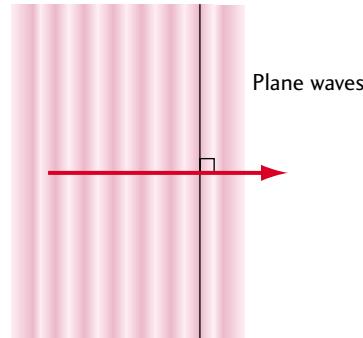
Fig.10.25

Wavefronts

(a) Circular wave



(b) Plane waves refracting



For a proof of Snell's law, see Figure 10.26. This proof involves wavefronts. To illustrate Snell's law, the ray diagram superposition with which we are familiar is shown in Figure 10.27.

Fig.10.26

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{\frac{\lambda_1}{AB}}{\frac{\lambda_2}{AB}} = \frac{\lambda_1}{\lambda_2}$$

f is constant and $v = \lambda f$

$$\text{so } \frac{\sin \theta_1}{\sin \theta_2} = \frac{\frac{v_1}{f}}{\frac{v_2}{f}} = \frac{v_1}{v_2}$$

$$n = \frac{c}{v}$$

$$\text{so } \frac{\sin \theta_1}{\sin \theta_2} = \frac{\frac{c}{n_1}}{\frac{c}{n_2}} = \frac{n_2}{n_1},$$

which produces

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Proof of Snell's Law

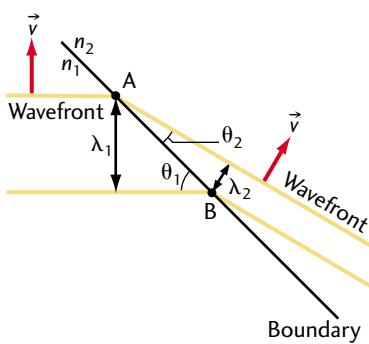
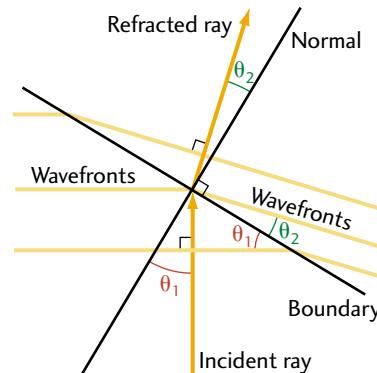


Fig.10.27



EXAMPLE 7 Using Snell's law

Find the angle of refraction for light travelling from air to diamond if the angle of incidence in air is 20° .

Solution and Connection to Theory

Given

$$n_1 = 1.00 \quad n_2 = 2.42 \text{ (from Table 10.2)} \quad \theta_1 = 20^\circ \quad \theta_2 = ?$$

The equation is

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

$$\sin \theta_2 = \sin \theta_1 \frac{n_1}{n_2}$$

$$\sin \theta_2 = \sin 20^\circ \frac{1.00}{2.42} = 0.342 \times 0.413 = 0.141$$

$$\theta_2 = \sin^{-1}(0.141)$$

$$\theta_2 = 8.1^\circ$$

The angle of refraction is 8.1° . The ray of light went from a less dense to a more dense medium. Therefore, the angle of refraction is less than the angle of incidence and the light is bent towards the normal.

EXAMPLE 8**Calculating the index of refraction using Snell's law**

Calculate the index of refraction for a substance where the angle of incidence is 30.0° , the angle of refraction is 50.0° , and the index of refraction of the second substance is 1.50.

Solution and Connection to Theory**Given**

$$n_2 = 1.50 \quad \theta_1 = 30.0^\circ \quad \theta_2 = 50.0^\circ \quad n_1 = ?$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_1 = \frac{n_2 \sin \theta_2}{\sin \theta_1}$$

$$n_1 = 1.50 \times \frac{\sin 50.0^\circ}{\sin 30.0^\circ}$$

$$n_1 = 1.50 \times 1.53 = 2.30$$

The index of refraction of the first substance is 2.30.

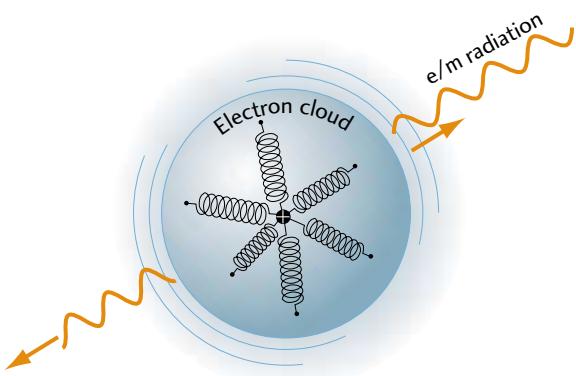
Refraction in an Optical Medium

Refraction in an optical medium is a complex effect. For the sake of simplicity, let's consider the optical medium to be made up of many simple electron oscillators. In this model, electron shells are held in place by springs attached to a positive stationary nucleus (Figure 10.28). When light enters the medium, its electric field interacts with the electron clouds, causing the electron oscillators to resonate in harmonic motion at the same frequency as the wave.

The oscillators then reradiate energy in the form of electromagnetic radiation, which has the same frequency as the incoming light. These created wavefronts travelling within the medium, called *secondary waves*, interfere with the incident primary wave to produce a net refracted wavefront. Figure 10.29 is a simplified representation of a wavefront entering a medium of regularly spaced atoms and creating scattered waves. The waves then add together to produce the net wavefront that moves through the medium at speed $v = \frac{c}{n}$. The phase relationship between the wavefronts determines how much light slows down. The greater the **phase lag** (retarding of one wavefront), the greater the speed reduction. In general, the index of

Fig.10.28 Electron Oscillators

When the electron oscillates at a certain frequency, it emits electromagnetic radiation of that frequency. The restoring force of the electron's motion is governed by Coulomb's law.



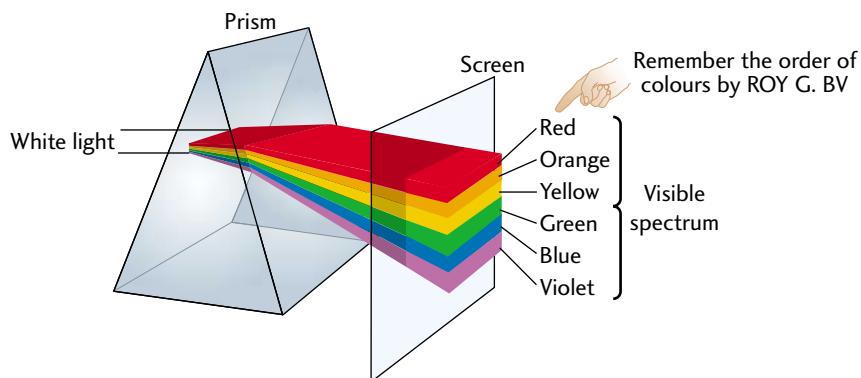
refraction is greater for shorter wavelengths due to the ability of the electron oscillators to absorb their energy more readily.

Any material that allows the electric and magnetic fields of a light wave to exist inside it also allows light to pass through it, hence making the medium transparent. According to the electron-oscillator model, light doesn't stimulate the oscillators because its frequency lies outside their natural frequency range. Substances like glass, quartz, diamond, and plexiglass all exhibit these properties.

Dispersion

When white light travels through a prism, a “rainbow” appears on the other side. This effect, shown in Figure 10.30a, is called **dispersion**. Dispersion is a method of demonstrating that white light is composed of many different wavelengths (colours) of light.

Fig.10.30a Dispersion by a prism separates white light into its component colours



Dispersion occurs because refractive indices are wavelength-dependent (see Figure 10.30a). Notice that the difference in the refractive index varies across the spectrum. In fact, the refractive index for crown glass ranges from 1.698 for violet light to 1.662 for red light. This 2% difference occurs each time the light refracts across the glass boundary. As you can see in Figure 10.30b, two refractions occur when light travels through a prism because light travels across two sets of boundary changes. The surfaces of the prism cause the light to bend in the same direction twice. This effect enhances the 2% difference in the indices of refraction and allows the different wavelengths to separate enough to be seen by the naked eye.

Another method of breaking white light into its component colours uses a **diffraction grating**. Diffraction gratings will be covered in Chapter 11.

Fig.10.29 A refracted wave in an ordered array of atoms

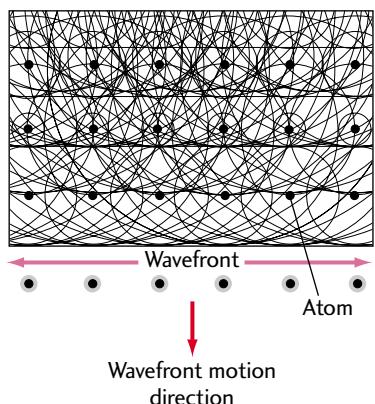


Fig.10.30b Because of the triangular shape of a prism, the refractions at 1 and 2 are both in the same direction, which enhances the separation of the different wavelengths of light

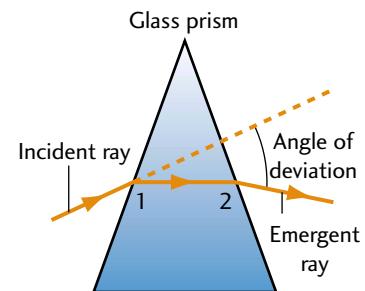
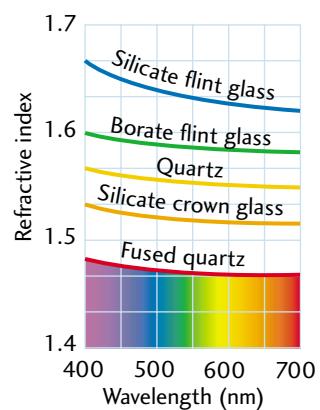


Fig.10.31 The index of refraction changes slightly with wavelength



The Spectroscope

Fig.10.32 A spectroscope

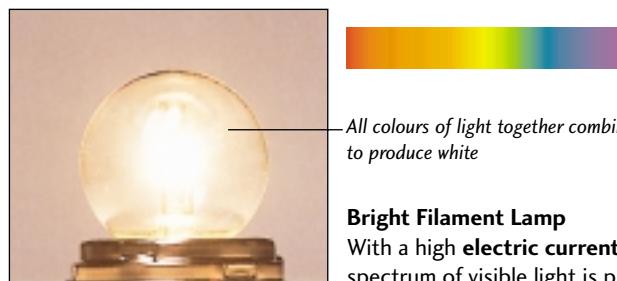


Spectra are distributions of energy emitted by radiant sources arranged in an order based on the wavelengths of the energy.

The **spectroscopic** is a device that produces **spectra** from a given source in terms of the wavelength of light it disperses. The spectroscope collects the light from a source, such as an incandescent lamp or a star, and then focuses it through a prism so we can study the resulting spectrum.

Incandescent sources, such as a bulb filament, produce **continuous spectra** that show all the colours in sequence flowing into each other (see Figure 10.33a). **Line spectra** (also referred to as **emission spectra**) consist of discrete vertical lines of different wavelengths, separated by dark bands (Figure 10.33b). Line spectra are formed by exciting atoms in gases, which we will study in Chapter 12. Each gas has its own “fingerprint” line spectrum. **Absorption spectra** are continuous spectra with gaps, or thin black lines, and are a characteristic of the Sun’s radiation. The black lines represent light from the hot interior of the Sun that has been absorbed by the Sun’s cooler exterior. Sometimes, we refer to these lines as **Fraunhofer lines** (see Figure 10.33c).

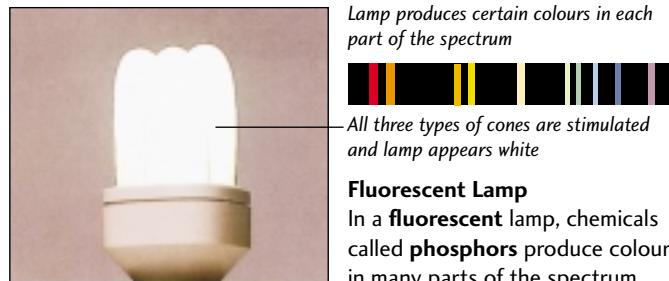
Fig.10.33a



All colours of light together combine to produce white

Bright Filament Lamp
With a high **electric current**, the whole spectrum of visible light is produced

Fig.10.33b

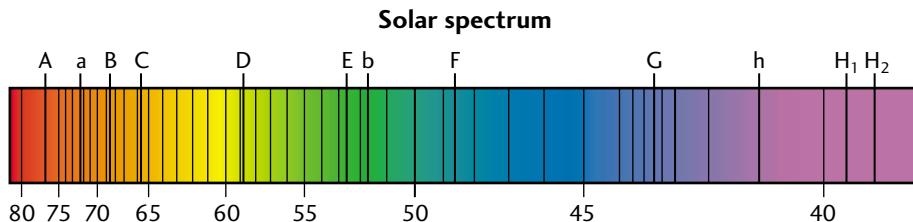


Lamp produces certain colours in each part of the spectrum

All three types of cones are stimulated and lamp appears white

Fluorescent Lamp
In a **fluorescent** lamp, chemicals called **phosphors** produce colours in many parts of the spectrum

Fig.10.33c The absorption spectrum of the Sun (Fraunhofer lines)





1. Make a list of everyday examples of refraction. (Hint: Think of examples of light passing through a transparent medium.)
2. Find examples of partial reflection and refraction. Look at situations when you view a medium at an angle.
3. What can you tell about a medium from the direction of the bend of the refracted ray relative to a normal?
4. Calculate the speed of light for the following mediums:
 - a) Water ($n = 1.33$)
 - b) Diamond ($n = 2.42$)
 - c) Plexiglass ($n = 1.51$)
5. Calculate the refractive index for a substance if the speed of light in the medium is
 - a) 2.1×10^8 m/s.
 - b) 1.5×10^8 m/s.
 - c) $0.79c$.
6. Calculate the angle of refraction for light as it passes from air into each of the mediums in problem 4 above at an angle of 25° ($n_{\text{water}} = 1.33$).

10.5 Electromagnetic Wave Phenomena: Polarization

In Section 10.3, we learned that electromagnetic radiation is a transverse wave made up of mutually orthogonal electric and magnetic fields that, under normal circumstances, are oriented randomly with respect to the propagation direction of the wave. In this section, we will only consider the electric field that occupies a two-dimensional plane (Figure 10.34).

From Chapter 2, we know that a vector can be broken into components (see Figure 10.35). If we remove one of the components of the electric field, we produce **polarized electromagnetic waves**. If both components are present, then the wave is said to be **unpolarized**. **Polarization** is the removal of one component of the *electric field*.

If one of the components of the electric field gets absorbed by a medium, then only one component remains. Now the electric field lines oscillate in one plane only, regardless of their original orientation. This type of electromagnetic wave is said to be **plane polarized** or **linearly polarized**. The effect is shown in Figure 10.36. It is analogous to a skipping rope vibrating up and down, side to side, and in all other possible directions. However, if the skipping rope is fed through a narrow slot in a wall (can be

Fig.10.34 The electric field of electromagnetic waves

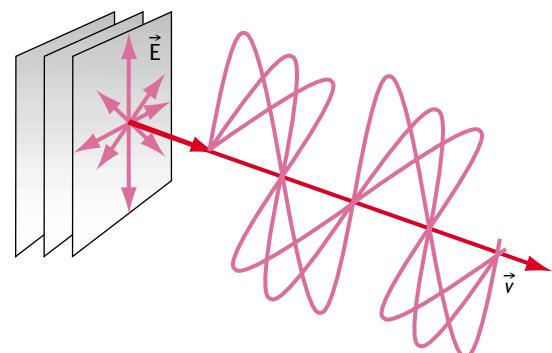


Fig.10.35 Any vector can be broken down to two components (x, y) that are perpendicular to each other. Two component vectors added together always produce the original vector.

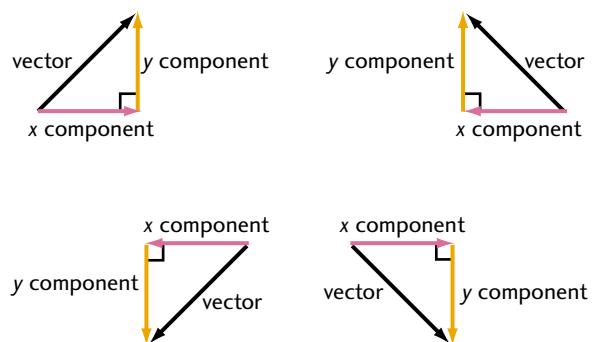
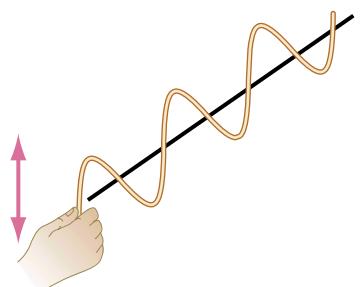
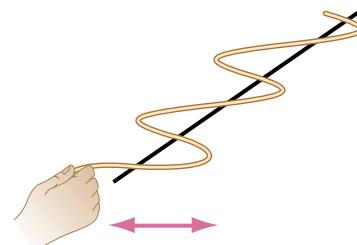


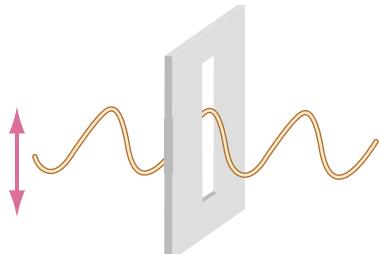
Fig.10.36



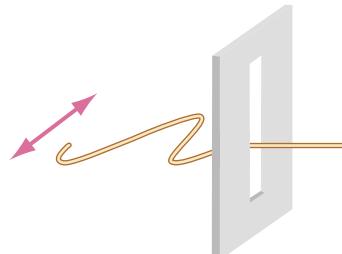
(a) Transverse waves on a rope polarized in a vertical plane



(b) Transverse waves on a rope polarized in a horizontal plane



(a) Vertically polarized wave passes through a vertical slit



(b) Horizontally polarized wave does not fit through the vertical slit

For radio waves, the direction of the antenna determines the direction of polarization. VHF television in North America uses horizontally polarized electromagnetic waves. The antennae are oriented horizontally so that the electrons can be driven in the same direction as the incoming wave (Figure 10.37). However, in Great Britain, the polarizing direction is vertical and the antennae are oriented vertically (Figure 10.38).

Fig.10.37



Fig.10.38



Dichroism is the name given to the method of polarizing light by absorbing one component of the electric field.

in any direction), the skipping rope can vibrate in the direction of the slot only. All the other directions hit the sides of the slot's walls and are damped. Thus, the electric field of a polarized electromagnetic wave vibrates in one plane only.

Polarization of Light using Polaroids (Polarizing Filters)

Because light is an electromagnetic wave, it can be polarized (normally, it is unpolarized). Light is produced by electrons oscillating in random directions in atoms (i.e., the oscillating electrons act like miniature antennae) and sending out light with random electric field orientations.

A **Polaroid** is a trade-marked name for an object created by Edwin Land in the early 1930s. In a sheet of clear plastic, he embedded tiny crystals of an iodine compound aligned in regular rows, much like a picket fence. As light passes through this polarizing material, one of its electric field components is absorbed. The other component moves through unhindered. Thus, the

Polaroid has a **preferential direction of transmission**. It is like the slot in a wall that allows a skipping rope to vibrate in one direction only (Figure 10.36). If you place two Polaroids with their transmission directions perpendicular to each other, virtually no light will pass through them, as shown in Figure 10.39a. Two Polaroids with their transmission directions oriented parallel to each other allow light to pass through both of them (Figure 10.39b).

METHOD OF PRODUCING POLAROIDS

A sheet of polyvinyl alcohol (plastic) is warmed and rapidly stretched in one direction, causing the long-chained molecules to align in the direction of the stretch. The sheet is then cemented to a rigid plastic sheet to prevent shrinkage and distortion. Later, the sheet, including the rigid plastic, is dipped into an iodine solution where the iodine atoms diffuse into the plastic, aligning themselves in long chains with the long-chained molecules of the polyvinyl. The alignment of the polyvinyl and iodine produces the polarizing effect.

Fig.10.39a A pair of “crossed” Polaroids with their transmission directions 90° to each other. No light is transmitted where they overlap.

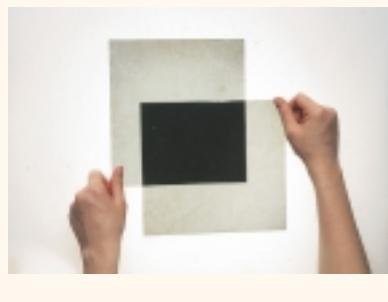
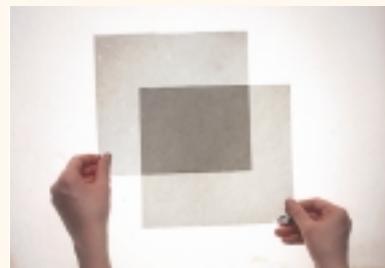


Fig.10.39b Two Polaroids with parallel transmission directions. Each Polaroid appears grey because it absorbs roughly half of the incident light. Light is transmitted where they overlap.



Malus’ Law: The Intensity of Transmitted Light

As unpolarized light passes through one Polaroid, it not only gets polarized in that direction, but it also loses some of its intensity. By removing one component of the electric field, we also decrease the intensity of the light by one-half. Thus, $I_1 = \frac{1}{2}I_0$, where I_0 is the intensity of the incident light and I_1 is the intensity of the ray exiting the polarizing filter. This effect is shown in Figure 10.40.

Now consider the situation in Figure 10.41. Here, the already polarized beam enters another Polaroid. This filter can be rotated in any direction. We will refer to the first Polaroid as the *polarizer*, and to the second Polaroid as the *analyzer*.

Fig.10.40 Light passing through one polarizing filter has one-half of its original intensity

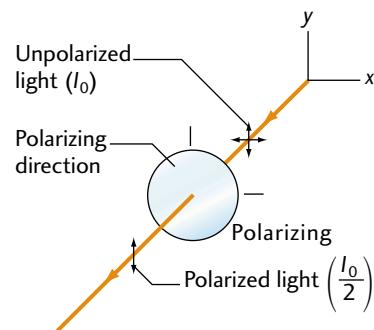
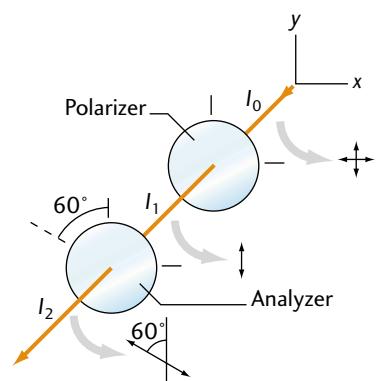


Fig.10.41 Light passing through two polarizing filters is angle-dependent



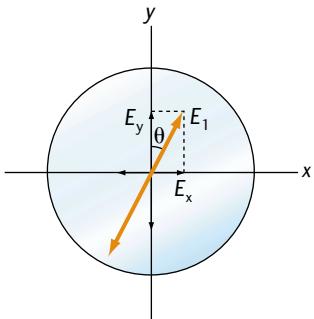
The **polarizer** produces polarized light. Rotating it doesn't affect the intensity of light. The emerging light's intensity is always $\frac{1}{2}I_0$.

The **analyzer** determines the plane of polarization. Rotating the analyzer causes the intensity of the emerging light to vary.

Fig.10.42 Polarizing direction

$$E_2 = E_1 \cos \theta$$

If E_1 is the entering amplitude, then $E_2 = E_y = E_1 \cos \theta$.



The emerging light is once again reduced in intensity. However, the intensity this time is angle-dependent. If the two Polaroids are aligned in the same direction, a maximum transmission of light occurs. If the Polaroids are 90° to each other, no light is transmitted. From Figure 10.42, we can see that the emerging electric field, E_2 , is equal to $E_1 \cos \theta$ because the cosine component of the electric field lies along the transmission axis. By definition, **intensity** is proportional to the *square of the amplitude* of the wave. In our case, the amplitude is $E_1 \cos \theta$, so we can write

$$E_2^2 = E_1^2 \cos^2 \theta \text{ which becomes}$$

$$I_2 = I_1 \cos^2 \theta$$

I_2 is the intensity of the ray of light emerging from the analyzer (the second Polaroid), and I_1 is the intensity of light emerging from the polarizer (the first Polaroid) and entering the analyzer. This equation is **Malus' law**.

EXAMPLE 9 Using Malus' law

If we wish to write the intensity in terms of I_0 , the incident light coming into the polarizer, then we use the equation $I_1 = \frac{1}{2}I_0$ to obtain $I_2 = \frac{1}{2}I_0 \cos^2 \theta$.

If two Polaroids are crossed with an angle of 60° between their polarizing directions, what percentage of light is transmitted through both Polaroids?

Solution and Connection To Theory

Given

$$\theta = 60^\circ \quad \frac{I_2}{I_1} = ?$$

$$I_2 = \frac{1}{2}I_0 \cos^2 \theta$$

$$I_2 = \frac{1}{2}I_0 \cos^2 60^\circ$$

$$I_2 = \frac{1}{2}(0.25) I_0$$

$$\text{Therefore, } \frac{I_2}{I_0} = 0.125$$

This value is 12.5% ($0.125 \times 100\%$). Thus, 12.5% of the light travels through both Polaroids.

If we wish to find the intensity of the light transmitted through the polarizer, we use the relationship $I_1 = \frac{1}{2}I_0$ or $I_1 = 0.5I_0$. Then, $I_2 = (0.5)I_0 \cos^2 60^\circ = (0.5)I_0(0.5)^2 = 0.125 I_0$.

Polarization by Reflection

Partial polarization also occurs when light reflects off a shiny surface. After reflection, the component of the electric field parallel to the surface is unchanged. The other component is partially absorbed, causing the light to become partially polarized. Figure 10.43 shows a ray diagram representation of this situation.

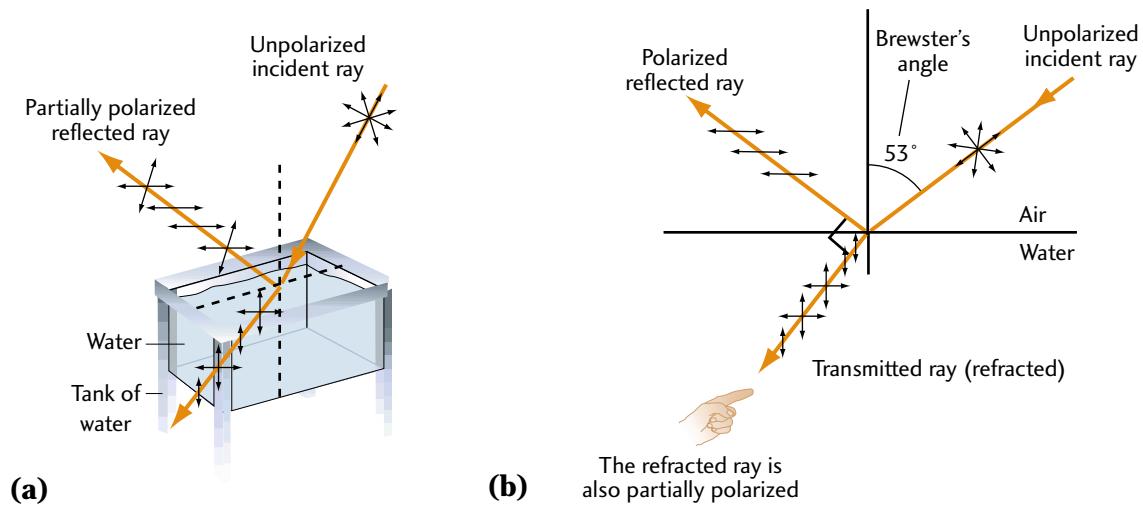
Most reflections create partially polarized light. However, 100% polarization can occur at a special angle of incidence. This angle is called **Brewster's angle**. At this angle, the reflected and refracted rays are 90° apart and the reflected ray is completely polarized. The conditions necessary for complete polarization by reflection are shown in Figure 10.43b. The main change from Figure 10.43a is that the refracted and reflected rays are separated by 90° and the reflected ray then has only one polarization component.

We can calculate Brewster's angle using the equation

$$\tan \theta_B = \frac{n_2}{n_1}$$

where n_1 is the refractive index of the incident medium, n_2 is the refractive index of the refraction medium, and θ_B is Brewster's angle.

Fig.10.43 An unpolarized wave reflects off the surface of the water. The reflected ray is partially polarized, predominately in one direction.



EXAMPLE 10

Brewster's angle for air–water boundary

Calculate the angle at which all of the reflected light is 100% polarized if light reflects from water.

Solution and Connection to Theory

Given

$$n_1 \text{ (air)} = 1.00 \quad n_2 \text{ (water)} = 1.33$$

$$\tan \theta_B = \frac{n_2}{n_1}$$

$$\theta_B = \tan^{-1} \frac{1.33}{1.00} = 53^\circ$$

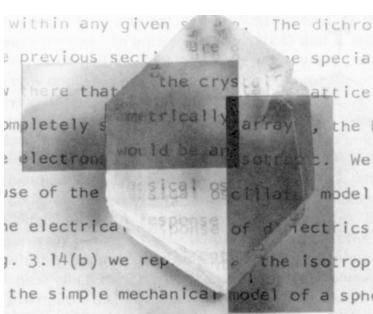
At an incident angle of 53° , the reflected light is totally polarized.

The transmitted (*refracted*) ray is partially polarized because it now contains all of one component and some of the other component of the electric field. Thus, you could produce almost 100% polarized refracted light by using a series of reflection plates, where each subsequent reflection removes a little more of one component of light.

Light scattered from the sky is also polarized. If you look at the sky while wearing Polaroid sunglasses, tilting your head from side to side will make the sky appear to change its tint (darker or brighter). The particles in the air preferentially scatter blue wavelengths over the other colours, so the sky appears blue. The scattering is like a reflection and thus produces some polarization.

Polarization by Anisotropic Crystals

Fig.10.44 The calcite crystal produces two images. Each is 100% polarized. When a Polaroid is placed on top of the crystal, one image disappears. A Polaroid rotated 90° to the first Polaroid causes the other image to disappear.

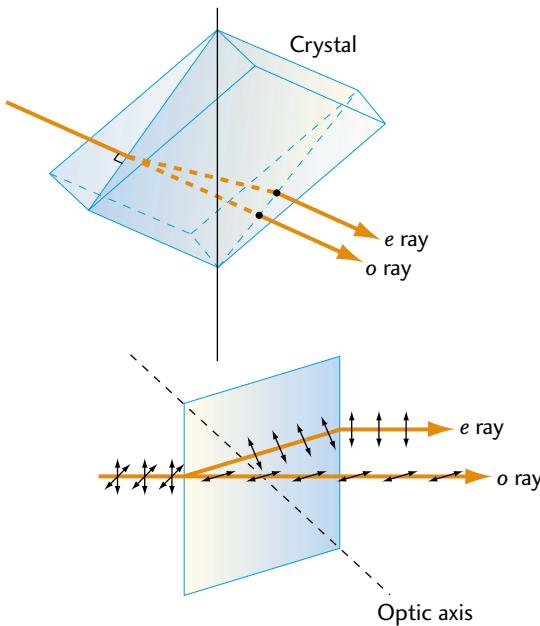


In 1669, Swedish physician Erasmus Bartholinus noticed that a piece of crystal, known as Icelandic spar (calcite), produced two images when light refracted through it. The cause of this phenomenon is the crystal's ability to separate the two components of the electric field. Each image is therefore 100% polarized. Figure 10.44 shows the two images of the text the crystal sits on. By rotating the crystal, one of the images rotates around the other. You can check the polarization of the images by placing a Polaroid filter on top of the crystal. One of the images will vanish. If the Polaroid is rotated, the other image will appear and the original one will vanish.

The two rays are named appropriately. The ***o* ray** is the **ordinary ray**, which means that it does nothing special. It obeys Snell's law and its speed is not changed as it travels through the crystal. The ***e* ray**, or **extraordinary ray**, on the other hand, obeys Snell's law in a more complicated way. Its

Fig.10.45

- e ray → Extraordinary ray:
Refractive index is angle-dependent
 o ray → Ordinary ray: Refractive index is constant



"Birefringence" means "refracting twice." The word "refringence" used to be used instead of "refraction." It stems from the Latin word *frangere*, meaning "to break." Many crystals are birefringent. Other examples of crystals are mica, sugar, and quartz. These crystals are important because they are used in various special optics instruments.

speed varies depending on the angle at which it enters the crystal. This angle is measured relative to an **optic axis**, which is an imaginary line through the crystal. The e ray has a refractive index that is dependent on an angle.

Materials exhibiting different refractive indices are said to be **birefringent**. What makes the optic axis special is that when unpolarized light enters along it, only one ray emerges. The representation of the different rays is given in Figure 10.45. Note that the optic axis is not a visible line in the crystal, but rather a measured direction.

To understand this effect better, let's study the term "**anisotropic**." This term indicates that certain properties of the crystal differ according to the direction of the measurement (such as the refractive index). The atoms that make up the crystal are arranged differently in different directions. When a ray of light enters the crystal, its electric fields line up differently relative to the electron positions in the atom for different incident angles. The relationship between the direction of the field and the electrons determines whether or not the electrons will vibrate (remember our electron oscillators from Section 10.4). In some cases, the electron will vibrate and absorb energy, and in other cases, it won't, depending on the alignment of the crystal's electrons and the light's electric fields.

In calcite, the o ray has a speed of 1.808×10^8 m/s, while the e ray has speeds varying from 1.808×10^8 m/s to 2.017×10^8 m/s.

1. a) Describe polarized light in terms of the rope analogy. Think of another analogy to explain polarization.
- b) Describe how polarization is produced by reflection, birefringent materials, and anisotropic crystals.



2. Describe what happens when a polarizing material is used to look at light coming from
 - a) a doubly refracting crystal.
 - b) a reflection from a glass window.
 - c) the blue sky.
 - d) a friend standing in a pool.
 - e) an LCD readout.
3. Research the following terms: wave plates, half and quarter, circular and elliptical polarization. Will Polaroids work on this type of light? Can the circular polarization of light be right-handed and left-handed?
4. The anisotropic crystals we studied are **uniaxial** (i.e., having only one optic axis). Research biaxial crystals and their effects.
5. a) Calculate the speeds of the *o* ray and *e* ray if $n_{\text{o ray}} = 1.658$ and $n_{\text{e ray}} = 1.486$.
b) What is the percent difference between the two speeds relative to the *o* ray?

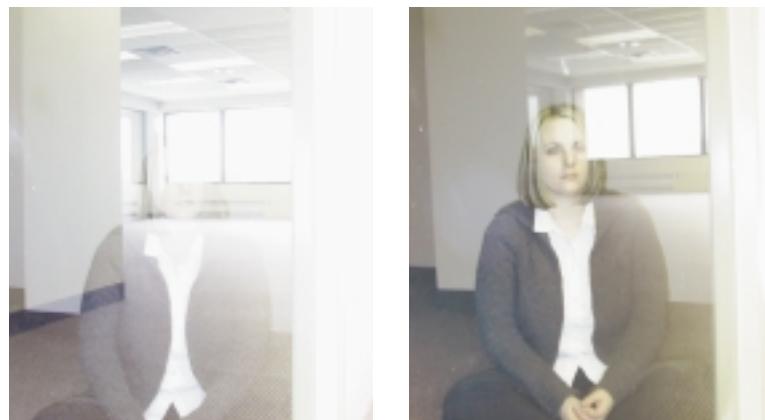
10.6 Applications of Polarization

Polarizing Filters in Photography

Like sunglasses, polarizing filters are used to remove the “visual noise” of glare from photos for cleaner and sharper-looking images. Many types of cameras have a coating on the lens that automatically polarizes light. Other polarizing filters on cameras can be rotated to change the polarizing axis relative to the view. The degree of rotation of the filter increases or reduces the amount of light that reaches the photographic film.

Polarizing filters improve images of the sky by deepening their hues. They also improve images taken through reflective surfaces like water and glass, such as photos of fish and animals at the zoo, or photos taken through windows.

Fig.10.46 Polarizing filters remove glare



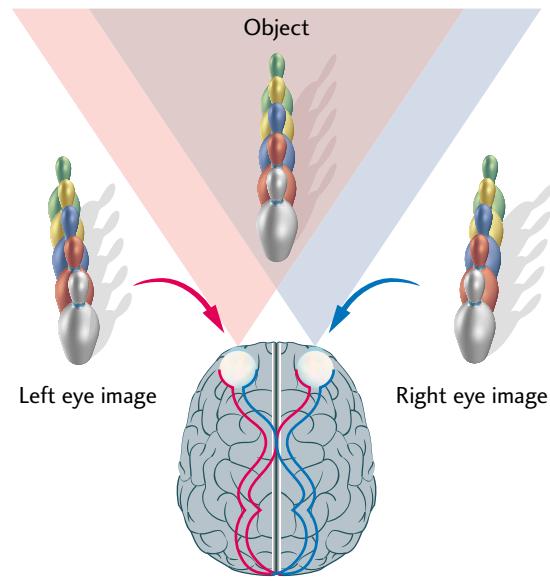


Fig.10.47 Each eye sees a different image. The brain combines the two perspectives to produce the final three-dimensional image.

3-D Movies

In the animal world, there are many different kinds of eyes. Their structure and location create different types of images and different ways of viewing the world. As illustrated in Figure 10.47, humans have eyes that are close together and side by side, which gives us binocular-type vision. Each eye picks up slightly different information from its surroundings. Our eyes produce the 3-D effect by blending two slightly different images together in the brain. To see this effect, try closing one eye, then the other eye while looking at a pencil held at arm's length. The image of the pencil shifts each time you look at it with the other eye. Our visual cortex produces the 3-D image by combining the similarities between the images of the two eyes, then adding in the differences.

In order to produce 3-D movies, two specially positioned cameras are used to film the scenes. The space between them duplicates the separation of our eyes. Two projectors are used to project the images on the screen, each using a polarizing filter. By placing the two polarizing filters in orthogonal orientations (90°) to each other, the images on the screen are also polarized in opposite directions. The scene on the screen appears doubled and blurry. When you put the special Polaroid sunglasses on, provided to you by the theatre, you see in 3D. The lenses of the 3-D glasses have their polarizing directions oriented at 90° to each other. The left eye receives images from the left projector only, and the right eye receives images from the right projector only. We thereby fool the brain into thinking that it's receiving two images of the same object, one from each eye. The brain puts the "two images" together to produce stereoscopic images. Although this technique produces superb results, the effect is diminished if the viewers tilt their heads, unless they are wearing achromatic, circularly polarized filters.

Fig.10.48 Viewing a 3-D movie



Achromatic refers to the ability of the lens to refract light without separating the colours, thus avoiding **chromatic aberration**.

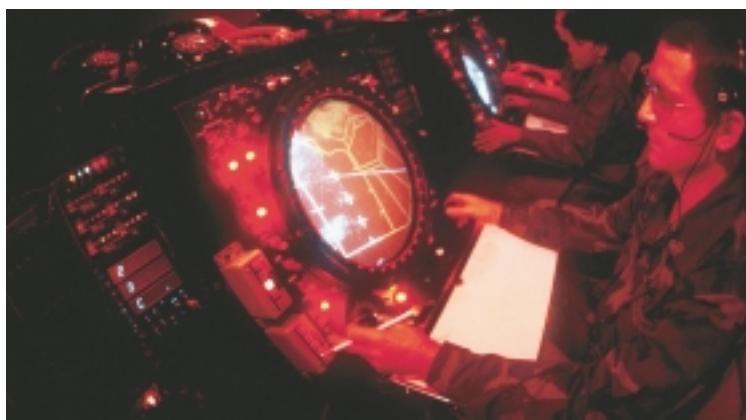
Radar

With radar, polarizing agents create **circularly polarized light**. This type of light is generated by adjusting the phases of the interfering waves to generate an electric field that rotates once around as the wave advances one wavelength. The direction of rotation can be either clockwise or counter-clockwise. When it is reflected, the light becomes polarized in the opposite circular direction and is almost 100% absorbed by the filter.

When viewing a radar screen, all possible information provided by the radar is important. The faint “blips” on a screen may be blocked out by light reflected off the surface of the oscilloscope screen. By placing a polarizing filter over the screen, any unwanted reflection is eliminated, so faint signals are able to come through and be seen on the screen. The light intensity of the screen is decreased by one-half as a result of polarization, but it is better than the reflected glare coming from a full light intensity.

The next time you are watching a movie showing a radar (or sonar) room, note the amount of light in the scene. Notice that it is usually dark in order to provide better contrast to view the radar screen and to remove any possible sources for reflection.

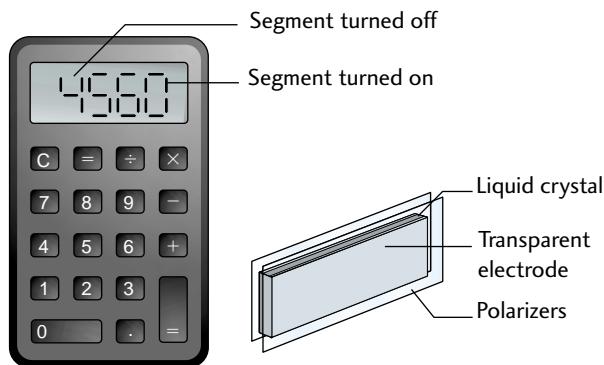
Fig.10.49 A radar screen



Liquid Crystal Displays (LCDs)

In calculators and other devices with numerical readouts, polarizing agents are used to create the various shapes on the readout screen. A series of liquid crystal grid-blocks is aligned across the screen. Each grid-block is sandwiched between transparent electrodes (see Figure 10.50). When a voltage is applied to it, the liquid crystal rotates the polarizing direction by 90° .

Fig.10.50 The LCD on a calculator



The liquid crystal grid-blocks are sandwiched between two polarizing filters. One filter acts as the polarizer while the other is the analyzer. Light passes from the polarizer through the liquid crystal to the analyzer. When there is no voltage, the direction of the polarized light is the same as that of the analyzer. The light passes through the analyzer, but its colour matches the background of the screen and nothing appears on the display. When the voltage is turned on, the direction of polarized light is 90° to the analyzer. No light passes through and we see a dark segment (Figure 10.51).

Fig.10.51 How an LCD works

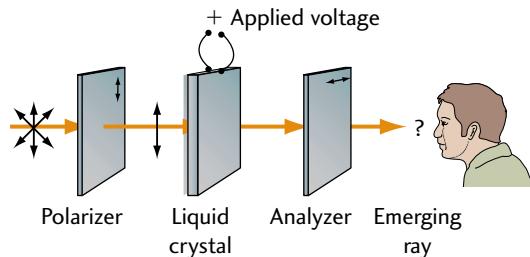


Table 10.4

Applied voltage	Liquid crystal	Emerging ray	What's seen
Yes	No rotation of light	None (absorbed by analyzer)	Dark band
No	Rotates light 90°	Not absorbed (passes through)	Nothing; light blends in with background

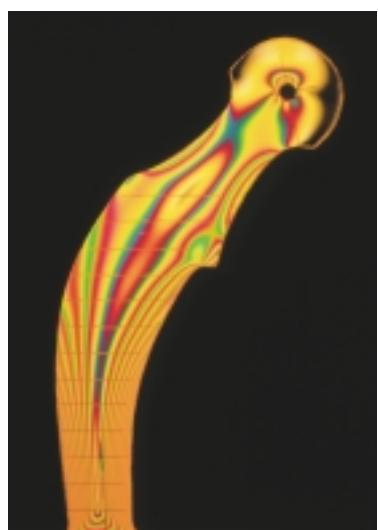
You can experiment with the polarizing filters in a calculator. Obtain a cheap calculator (one that does basic arithmetic only) with an LCD readout. View the readout through a Polaroid. Rotate the Polaroid and see the effect. Carefully dismantle the readout display and look for the various parts described in this subsection. Try removing the analyzer and rotating it 90°. Then try operating the calculator to see what happens to the readout.

Photoelastic Analysis

As we learned in Chapter 3, when building any structure consisting of parts that undergo great stresses, it's essential for engineers to know the limits and areas of weakness of the structure's components. The girders at the bottom of a 50-storey building, for example, support tremendous weight. If a hole was required in the girder to create a channel for wiring, would it cause excessive weakness in the building's structure? To study these types of stresses and strains, engineers use the **birefringence** properties of plastics to determine the areas of stress and strain of any object under a load.

To do so, a model of the object is made out of material such as lucite, which becomes birefringent when placed under stress. The amount of birefringence varies directly as the amount of stress on the object. When viewed between two crossed polarizing sheets, a series of coloured fringes appears. The closer the fringes, the higher the stress level on the object. In the lucite model of a prosthetic hip joint in Figure 10.52, the stress is distributed evenly over the whole structure and the weaknesses occur at the bases. Using birefringence, we can obtain a numeric value for the stress or strain that relates the number of fringes to the spacing created by the stress or strain acting on the object.

Fig.10.52 Lucite is used to study stress on a prosthetic hip joint



Stress and Strain

From Chapter 3, recall that stress is the *ratio* of the force required to cause a deformation and the area to which the force is applied. Strain is the *result* of the applied force.

Polarization in the Insect World

The eyes of certain insects, such as ants and bees, consist of **ommatidia** (Figure 10.54a), which are the repeated units that make up an insect's compound eye. The ommatidia allow insects to detect polarized light, which enables them to navigate by using the scattered sunlight from the sky.

To simulate what the insect sees, a series of eight triangular Polaroids is arranged in an octagon in Figure 10.54b. When the sky is viewed through this arrangement, a series of patterns is seen, which can be used to determine direction. Some airplanes are equipped with similar polarization indicators to help with navigation.

Fig.10.53 An insect's head



Fig.10.54a Ommatidium of an insect's eye (cross-section)

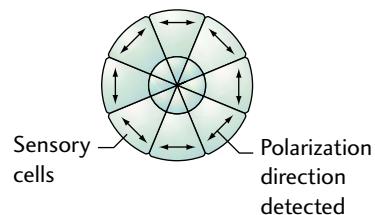
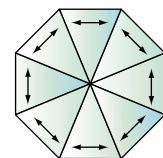


Fig.10.54b Eight polarizing sheets arranged in an octagonal shape simulate the ommatidium of an insect's eye

Each section detects a different polarization direction



Polarized Light Microscopy

Polarized light microscopy is mainly used for studying birefringent objects. In this type of light microscope, one filter is placed in the microscope head and another is placed over the lamp or condenser, usually at the base of the instrument. The slide, which rests on the mounting stage between the light source and the head, is between the two polarizing filters. By rotating either one of the filters, various aspects of the studied sample can be viewed. Birefringent material between crossed polarizing filters produces a coloured interference pattern. Different organelles or structures in the cells have different birefringent properties that become more apparent and in better contrast depending on the polarizing filter's angle of rotation. As the polarizing filter is rotated, one organelle or structure fades into the background while another comes into view.

Measuring Concentrations of Materials in Solution

Optical activity is the property of certain substances in solution, such as sugar, to rotate the plane of polarization without changing any other aspect of light. The amount of rotation varies with the concentration of the solution

and the distance the light must travel through it. By injecting a solution into a cell of known length and then shining polarized light through it, the angle of rotation of the polarizing plane can accurately determine the concentration of the solution. Industries involved in food chemistry and organic biochemical analysis use this technique to obtain high-precision concentration measurements.

1. Why does tilting your head while viewing a 3-D movie through linearly polarized glasses decrease the 3-D effect?
2. Research other methods used to produce the 3-D effect. Include the effect shown on TV during a Super Bowl intermission with special glasses using coloured filters and vectography.
3. Research how a circular polarizing filter works to block out unwanted glare. Relate reflection and the orientation of the *E* vector in your explanation.
4. Postulate how a bee can use the Sun's position in the sky to navigate.
5. Research the life and work of scientist Karl von Frisch, who investigated the ability of bees to navigate by using the Sun and polarization directions.
6. Describe a Frisch experiment that proved that bees use polarized light to navigate.
7. Research polarization directional equipment, and general methods and areas of use.
8. Research the types of materials studied using polarized light microscopy in the field of medicine and explain why birefringent materials are visible using this method.

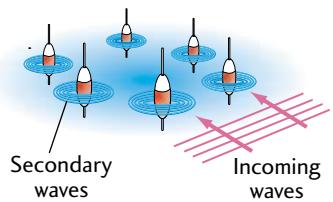


10.7 Electromagnetic Wave Phenomena: Scattering

In Section 10.5, we mentioned that light scattered by air particles is polarized. This section explains the scattering process of light. As the sunlight passes through the atmosphere, it gets randomly redirected by air. This redirection of light gives the sky its colour.

Scattering is similar to a bobber in the water. In Figure 10.55, as waves go by the bobber, they cause it to start moving up and down with the same frequency as that of the original wave. This motion causes more waves to emanate from the bobber. Now imagine thousands of bobbers all doing the same thing. The ordered wave that first came in is now a mass of ripples moving in all different directions.

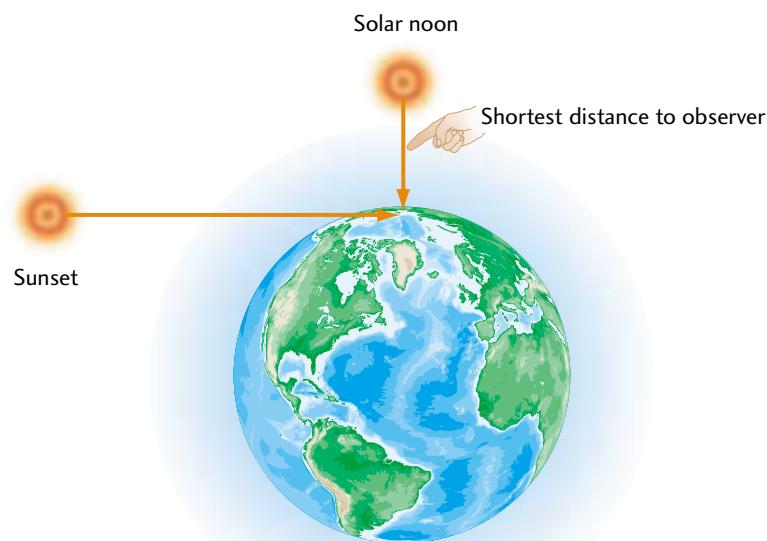
Fig.10.55 A single wave comes in and bends around each bobber, creating secondary sources of waves



Like the bobber, air molecules absorb and then re-emit light waves. The shortest wavelengths are scattered more easily than the longer ones because the electrons in molecules are able to absorb energies of the shorter wavelengths more easily. Objects tend to absorb energy readily if it causes them to vibrate at their **natural resonant frequency**. The natural resonance of electrons in air molecules is closest to the ultraviolet end of the spectrum. Therefore, the longer the wavelength of light, the less energy is absorbed by the electrons and the less scattering of light occurs.

In Figure 10.56, when the Sun is at solar noon (directly overhead), the distance the light travels through the atmosphere is a minimum. At sunset, this distance is a maximum.

Fig.10.56 Light travels a greater distance through the atmosphere at sunset than at noon



SCATTERING

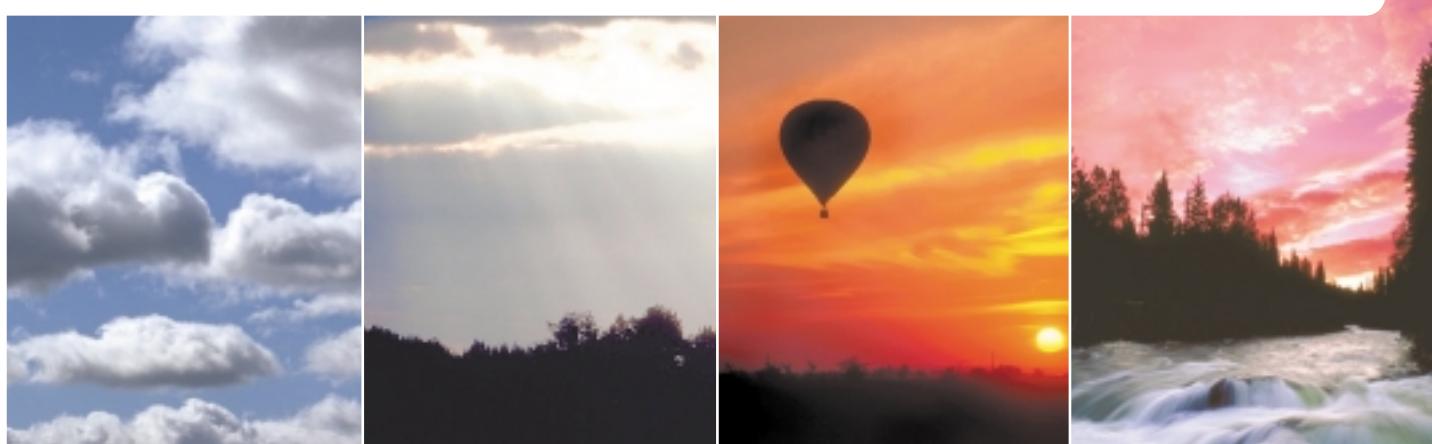
The extent of scattering of light by air molecules is proportional to $\frac{1}{\lambda^4}$. The wavelengths of visible light range from about $0.70 \mu\text{m}$ (red) to $0.40 \mu\text{m}$ (violet). Table 10.5 shows the relative amounts of various colours that are scattered.

Table 10.5
Extent of Scattering

Colour	red	orange	yellow	green	blue	violet
Wavelength (μm)	0.70	0.60	0.58	0.52	0.48	0.40
Relative number of scattered waves	1	2	3	4	5	10

At solar noon, the sky is blue because it is a mixture of the colours of light that scatter best: an unequal mixture of violet, blue, green, and yellow light. At sunset, the light has to travel the extra distance through the atmosphere. By the time it nears the surface of Earth, most of the short wavelengths have been scattered. The remaining longer wavelengths, which are in the red end of the spectrum, reach our eyes and we see a red Sun at sunset. The atmosphere near Earth's surface has more dust particles and, near cities, more pollutants, which are of the right size to scatter red wavelengths better. Thus, on nights when pollution is high, we see red sunsets (see Figure 10.57). However, in some areas of Earth, the pollution level is so high, that no Sun is seen at all. In theory, you could then create whatever colour of sky you wish by putting particles in the atmosphere that are most suited to scatter that particular wavelength of light.

Fig.10.57 The colours of the sky are caused by the scattering of light



1. Describe the scattering effect in terms of wavelength of light and colours as seen by a person looking at the sky.
2. How could you use the scattering effect of light to measure the pollution count in the air?





Microwave Technology: Too Much Too Soon?

Fig.STSE.10.1



With the proliferation of cellphone technology, studies are abounding on how microwaves affect human cells. The proximity of the phone to the head poses possible problems with electromagnetic radiation penetrating our skulls. Microwaves (with a frequency of 2450 MHz) break covalent bonds when molecules absorb their energy. Many studies dispute whether microwaves are capable of disrupting cellular activity through bond breaking because the microwave has an energy of 10^{-5} eV, whereas it takes 10 eV to break a covalent bond.

How could such low energies cause any harm to DNA? Microwaves have a heating effect. Polar molecules such as water (Figure STSE.10.2) rotate in the electromagnetic field of the wave due to a net torque produced on the molecule. The angular momentum of the atoms breaks the bonds and releases the energy in the form of heat, which is transferred to molecules in the form of kinetic energy, thus raising the temperature of the material. Microwaves cook food using this method. However, this effect is not the one that causes the breaking of DNA bonds.

A current theory suggests that the energy of the microwave accumulates in water molecules bound to the DNA (see Figure STSE.10.3). The disruption of the intermolecular hydrogen–oxygen bond creates oxygen radicals that can dissociate DNA bonds. Thus, a smaller amount of microwave energy can

Fig.STSE.10.2

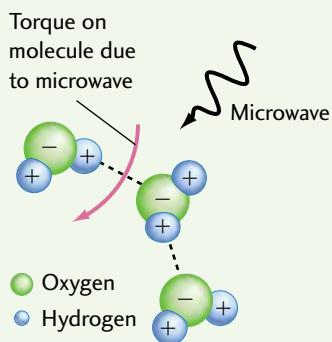
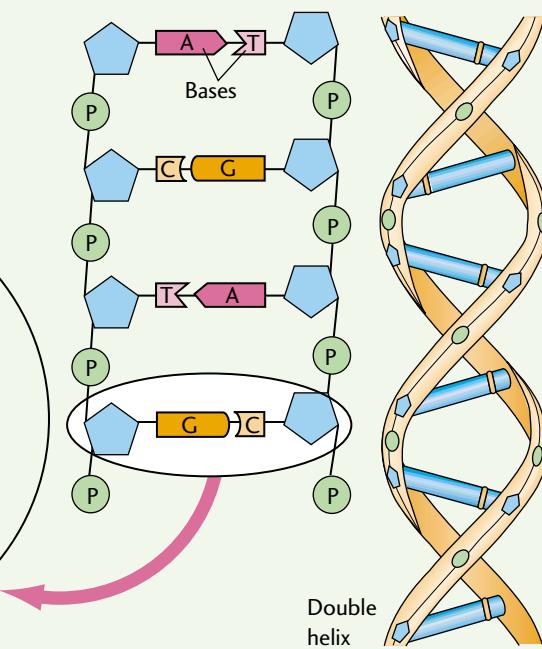
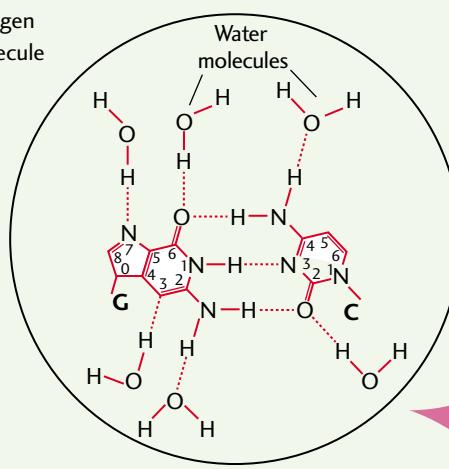


Fig.STSE.10.3a Nucleic acid hydration

The bases in opposite side chains of DNA bond together, adenine (A) with thymine (T) and cytosine (C) with guanine (G)

Fig.STSE.10.3b Guanine and cytosine form a hydrogen bond with a water molecule



damage living material than was earlier believed possible. If this theory is true, then using even low-energy cellphones may be harmful in the long term.

There is a historical similarity developing between microwave studies and studies of low-dose nuclear radiation on people. Where once only short-term large doses were thought to be harmful, further studies done as a result of data accumulated over the years have shown that the *cumulative effect* of low doses is also dangerous.

Design a Study of Societal Impact

Cellphones use microwave technology. Since the phone piece has to be positioned close to the ear, the energy of the microwave is inadvertently directed toward the brain. Research the current debate about the possible harmful effects of using cellphones on a regular basis.

Design an Activity To Evaluate

Intensity Drop of Radio Waves with Distance: Use a radar oscillator and a receiver–amplifier to study the effect of distance between a transmitter and a receiver on the strength of the signal. Use a log plot (see Appendix I) of intensity versus distance to obtain a relationship between the two. If radar equipment is not available, design a similar experiment using light and a light meter.

Research and compare the variations in intensity of microwaves emanating from cellphones. Design an experiment to determine whether the medium through which waves travel affects the rate at which their intensity decreases.

Build a Structure

Polarization of Electromagnetic Waves: Use a radar-transmitting dipole focused by a parabolic reflector to beam radio waves through a grid made from wires to a receiver. The wires must run in one direction only. Rotate the wire grid to different angles and measure the intensity of the wave. Relate your findings to the polarization of light using polarizing materials.

SUMMARY SPECIFIC EXPECTATIONS

You should be able to

Understand Basic Concepts:

- Define and explain the concepts and units related to the wave nature of light.
- Describe the different types of waves and their properties.
- Explain dispersion, polarization, and refraction in general wave terms.
- Describe what the electromagnetic spectrum is and provide specific examples of the different types of radiation.
- Describe simple harmonic motion and relate it to the generation of electromagnetic radiation.
- Provide examples of electromagnetic energy interacting with matter.
- Provide descriptions of how different types of radiation are created.
- Describe different methods of polarizing light.
- Define and explain the terms “anisotropic properties,” “birefringence,” “plane,” and “circularly polarized light.”

Develop Skills of Inquiry and Communication:

- Use the phenomena of dispersion, polarization, and refraction to develop the theoretical basis of light behaving like a wave.
- Make predictions based on the wave model of light about what to expect in experiments involving polarization, dispersion, and refraction.
- Predict what happens to light as it is transmitted through more than two polarizing filters.
- Perform experiments relating the wave model of light to refraction, dispersion, and polarization.
- Expand and develop new extensions to current labs in studying the aspects of the wave nature of light.

- Analyze and interpret experimental evidence indicating that light has similar characteristics and properties to those of mechanical waves and sound.
- Describe how the conceptual models and theories of light changed scientific thought.

Relating Science to Technology, Society and the Environment:

- Describe how the conceptual models and theories of light have led to the development of new technologies.
- Describe the contribution of physicists involved in the area of electromagnetic radiation to devices and instrumentation we use today.
- Describe how researchers working on electromagnetic wave theory have influenced the scientific processes and ideas of the era they lived in.
- Describe and explain the design and operation of the prism spectrometer.
- Describe the applications of polarized light in areas such as the military, photography, and leisure.

Equations

$$T = \frac{1}{f} \text{ and } f = \frac{1}{T}$$

$$v = \lambda f \text{ and } c = \lambda f$$

$$y = A \sin \theta \text{ or } x = A \cos \theta$$

$$n = \frac{c}{v}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$I_1 = \frac{1}{2} I_0$$

$$I_2 = I_1 \cos^2 \theta$$

$$\tan \theta_B = \frac{n_2}{n_1}$$

EXERCISES

Conceptual Questions

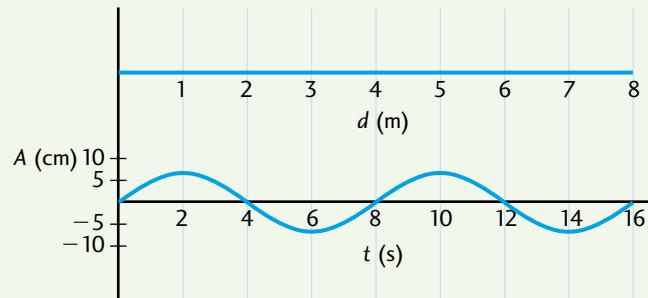
1. Relate the motion of a spring vibrating back and forth to the motion of a light wave.
2. Speculate as to what happens to the magnetic field when the electric field of an electromagnetic wave decreases.
3. Why is “visible light” a relative term?
4. Illustrate reflection of light using wavefronts.
5. Why can’t you put metallic objects in a microwave oven?
6. In an arcade shooting gallery, a row of ducks moves back and forth across the target area. In any direction, the speed is constant. Explain why this motion isn’t simple harmonic motion.
7. Galileo stated that simple harmonic motion is uniform circular motion viewed edge-on. Explain this statement (you may use diagrams to help in the explanation).
8. Explain refraction in terms of electron oscillators and speed changes.
9. Newton postulated that the refraction of light, as it passed from air to a more optically dense medium, was caused by gravity. In his opinion, light was a particle that was drawn toward the masses in the denser medium. Which aspect of his theory of refraction is correct and which aspect is incorrect for light entering the medium at an angle?
10. For an object to be invisible, what has to be true about its refractive index?
11. What can you tell about optical densities, using a laser?
12. Explain dispersion in terms of refraction.
13. Why is the prism shape optimal for creating dispersion?

14. Can sound waves be polarized? Explain.
15. What is the difference between a polarizer and an analyzer? What happens to light if the light path is reversed and it enters the analyzer first?
16. Your friend plays a trick on you by rotating the polarizing filters in your circular sunglasses 90° . What effects will you experience?
17. Does the effectiveness of Polaroid sunglasses vary throughout the day? Explain.
18. Are Polaroid sunglasses effective on circularly polarized light?
19. How could you use the scattering effect of light to measure the pollution count in the air?
20. Summarize the wave effects of polarization, scattering, and refraction.

Problems

10.2 Fundamental Wave Concepts

Fig.10.58



21. Copy the diagram of a wave into your notebook (Figure 10.58). From measurements and information taken directly from the diagram, find the
 - a) wavelength.
 - b) amplitude.
 - c) period.
 - d) frequency.
 - e) the speed of the wave.

- 22.** A plastic fish at the end of a spring is pulled down and released. If the fish moves up and down 10 times in 3.2 s, find the period and frequency of oscillation.
- 23.** What is the period and frequency of a person's heart if it beats 72 times in one minute?
- 24.** An electric shaver blade vibrates at 60 Hz. What is its period of vibration?
- 25.** A piston moves up and down in a car engine 150 times per minute (150 rpm). Find
a) the frequency in Hz (rps).
b) the period of vibration.
- 26.** In the olden days, there were three rotational "speeds" used in playing vinyl records, namely, 78 rpm, 45 rpm, and $33\frac{1}{3}$ rpm. Convert each of these values to Hz and then find the period of rotation.
- 27.** Find the displacement of a spring with a maximum amplitude of $A = 1$ from equilibrium for phase angles of
a) 10° . **b)** 95° . **c)** $\frac{3\pi}{4}$ rad. **d)** 2π rad.
- 28.** For the SHM displacement of a spring, $x = A \cos \theta$, the velocity of the wave varies directly as $(-\sin \theta)$. Sketch the velocity and displacement curves, drawing the velocity wave under the displacement wave. Discuss how the two curves are related in terms of the motion of the spring (compare maximum displacement and velocity).
- 29.** The acceleration of the spring varies directly as $(-\cos \theta)$. Draw this wave under the two waves you drew in problem 28. Discuss what the object is doing in terms of its acceleration and the motion of the spring (e.g., is it speeding up, changing direction, or slowing down?).
- b)** A bungee cord jumper of mass 100 kg, swinging from a cord 80 m long.
c) A pendant of mass 30 g, on a chain 15 cm in length.
- 31.** Repeat problem 30, only pretend that you are on
a) the Moon, where the gravitational field constant is 1.6 m/s^2 .
b) Jupiter, where the gravitational field constant is 24.6 m/s^2 .
- 32.** Calculate the period for the following:
a) A spring with constant $k = 23.4 \text{ N/m}$, with a 0.30-kg mass hanging from it.
b) A spring pulled down 20 cm from equilibrium, with a spring constant of 20 N/m , and a 0.40-kg mass hanging from it.
c) A spring on the Moon ($g = 1.6 \text{ m/s}^2$), with a spring constant of 2.0 N/cm , pulled down 1.0 m, with a 0.21-kg mass hanging from it.
- 33.** **a)** Calculate the spring constant for a spring with a hanging mass of 402 g and a frequency of 12 Hz.
b) How much force is required to pull the spring down 35 cm?

10.3 Electromagnetic Theory

- 34.** For the following wavelengths of light, calculate the corresponding frequency.
a) Red: 650 nm
b) Orange: 600 nm
c) Yellow: 580 nm
d) Green: 520 nm
e) Blue: 475 nm
f) Violet: 400 nm
- Note that these wavelengths are representative values: each colour has a range of frequencies associated with it.
- 35.** Calculate the time it would take light leaving Earth to reach
a) the Sun ($1.49 \times 10^{11} \text{ m}$ away).

Problems 30–33 pertain to Lab 10.1

- 30.** Calculate the period for the following objects:
a) A pendulum of length 2.1 m with a mass of 1.3 kg at the end of it.

- b)** the Moon (3.8×10^8 m away).
- c)** Pluto (5.8×10^{12} m away).
- d)** Mercury (9.1×10^{10} m away).

Convert the times to minutes and hours as well.

- 36.** Find the distance light travels in one year. This distance is referred to as a light year.
- 37.** If we see light coming from a galaxy 100 light years away, how long ago did the light leave the galaxy?
- 38.** A light bulb is turned on at one end of a football stadium. How much time elapses before the light reaches you? Assume a distance of 160 m.
- 39.** Calculate the time it would take light to travel around the world once ($r_{\text{Earth}} = 6.38 \times 10^6$ m).
- 40.** UV light is invisible to the human eye, unless we use special sensors. Given the range of wavelengths of UV light (4×10^{-7} m to about 8×10^{-8} m), calculate the corresponding frequencies.
- 41.** British Columbia is about a 50-h drive from Southern Ontario. Assume a distance of 4000 km. How much faster would it be to travel this distance at the speed of light?

10.4 Electromagnetic Wave Phenomena: Refraction

- 42.** For the following angles, find the sine of the angle.
 - a)** 30°
 - b)** 60°
 - c)** 45°
 - d)** 12.6°
 - e)** 74.4°
 - f)** 0°
 - g)** 90°
- 43.** For the following inverse sine values (\sin^{-1}), find the corresponding angle.
 - a)** 0.342
 - b)** 0.643

- c)** 0.700
- d)** 0.333
- e)** 1.00

- 44.** Calculate the speed of light in a material with a refractive index of 0.90. Comment.
- 45.** Find the angle of refraction for light travelling from air to a medium ($n = 1.98$), if the angle of incidence in air is 2.0 times the angle of refraction.
- 46.** Calculate the index of refraction for a substance where the angle of incidence in a material with $n = 1.5$ is 30° and the angle of refraction is 50° . Comment.
- 47.** Sketch a light ray passing through a rectangular piece of glass. The exiting ray should be parallel to the incident ray. Draw the wavefronts.
- 48.** Calculate the speed of light in
 - a)** diamond ($n = 2.42$).
 - b)** crown glass ($n = 1.52$).
 - c)** water ($n = 1.33$).
 - d)** ice ($n = 1.30$).
- 49.** Calculate the relative index of refraction for light travelling from the material to air for the substances listed in problem 48.
- 50.** Given that the refractive index of water is 1.33, how long does it take light to travel from one shore of a lake to the opposite shore if the lake is 12 km long?

10.5 Electromagnetic Wave Phenomena: Polarization

- 51.** A beam of light is reflected from a surface that has an index of refraction of 1.42. If the reflected beam is 100% polarized, what is the angle of
 - a)** incidence?
 - b)** refraction?
 - c)** reflection?
- 52.** What should be the Sun's angle of elevation over a lake in order for Polaroid sunglasses to be the most effective?

- 53.** What percentage of light intensity is transmitted through a polarizer–analyzer combination if the angle between their axes is
- a)** 30° ?
 - b)** 50° ?
 - c)** 70° ?
- 54.** Describe the image you would see through a doubly refracting crystal. What would you see if another crystal was placed on top of the first crystal and rotated?
- 55.** How can you determine if light is polarized, unpolarized, or partially polarized?
- 56.** Calculate the angle at which light reflected off water is 100% polarized.
- 57.** Calculate Brewster's angle for the following combination of mediums:
- a)** Air–water ($n_{\text{water}} = 1.33$)
 - b)** Air–glass ($n_{\text{glass}} = 1.50$)
 - c)** Glass–water
 - d)** Ice–water ($n_{\text{ice}} = 1.30$)
- 58.** What is the refractive index of a medium that has a Brewster's angle of 60° ?
- 59.** Two Polaroids are crossed such that no light is transmitted. Now a third Polaroid is placed *in between* and *at an angle* to the first two Polaroids. Why is light once again transmitted?
- 60.** Calculate the percentage of light travelling through two crossed polarizing filters if the angle between the polarizing directions is
- a)** 10° .
 - b)** 30° .
 - c)** 70° .
 - d)** 85° .
- 61.** At what angle should two polarizing filters be positioned to reduce the intensity of light by 60%?
- 62.** Three polarizing filters are placed on top of one another. If the angle between the first two filters is 60° and the angle between the first and third filter is 70° , find the percentage of light exiting the last polarizing filter.

Investigating Simple Harmonic Motion

Purpose

To investigate the factors affecting the period of a pendulum undergoing simple harmonic motion

Equipment

Various lengths of string

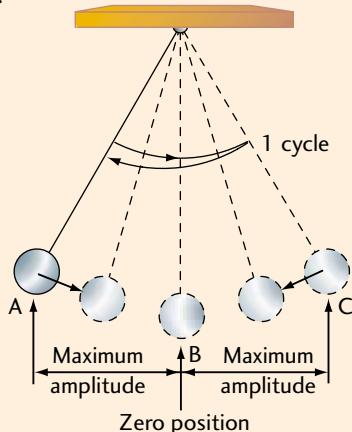
Various masses

Retort stand plus clamp

Timing device (photo gates or stopwatch)

Log paper

Fig. Lab.10.1



Procedure A: Length Dependence

- Set up the experiment as shown in Figure Lab.10.1.
- Draw back the pendulum from the zero position and measure the amplitude, as shown in Figure Lab.10.1.
- Release the pendulum. Record the time it takes to complete 10 cycles.
- Repeat steps 1–3 for at least 5 more lengths of string. Make sure that the pendulum is drawn back the same distance each time.

Procedure B: Amplitude Dependence

- Release the pendulum from a different measured zero position. Measure the time it takes to complete 10 cycles.
- Repeat step 1 using a different starting point. Perform this step at least 6 times.

Procedure C: Mass Dependence

- Release the pendulum with a known recorded mass from a standard position relative to the zero position.
- Record the time it takes to complete 10 cycles.
- Repeat for 5 more different masses from the same starting position.

Data

Record the data in chart form.

Analysis (see Appendix D for log analysis)

- Calculate the period of oscillation for each trial.
- Plot graphs of T versus length, T versus amplitude, and T versus mass.
- For any graph that is not a straight line, plot a $\log T$ versus \log length, amplitude, or mass graph. If using log paper, then there is no need to plot the logarithms of T or the other variables; use the values recorded in the chart.
- From the log graph, determine the equation of the line, hence the relationship between T and the x -axis variable.
- Assign tolerances (uncertainties) to your time and length measurements.

Discussion

- Which factors affect period?
- Derive as well as look up the derivation for $T = 2\pi\left(\frac{1}{g}\right)^{1/2}$. State all your assumptions.
- Does your experimental relationship match the theoretical one? If not, why not?
- Find the percent deviation between your constant and 2π . Are the two values in agreement? Compare them to your uncertainties in measurements.
- How does this experiment show simple harmonic motion?

Conclusion

Summarize your results and draw a conclusion from your observations.

Extension

Purpose

To study the harmonic motion of a mass oscillating on a spring

Procedure

- Design an experiment to study the factors affecting the period of oscillation of a mass on a spring pulled down from an equilibrium position.
- Experimentally determine the equation for the period of an oscillating mass.
- Derive or look up the theoretical equation for the period of a mass oscillating on a spring.
- Research the oscillator model for atoms. Compare the qualitative features of the oscillating spring and the oscillating atom (or electron).



Polarization

Purpose

To study various aspects of polarization

Equipment

3 Polaroids per group

Calcite crystal

Crumpled cellophane

Thin piece of mica

Unstained sample slides

Light microscope

Calculator

Lucite ruler with holes of various shapes (or a broken piece of lucite ruler)

Procedure

Record your observations for the following:

1. Take two Polaroids and cross them. Hold them up to the light and rotate one of the Polaroids around.
2. Position two Polaroids in a manner such that no light gets through. Put a third Polaroid in between them at an angle to the first two.
3. Place a calcite crystal on a page of written text. Rotate the crystal around.
4. Place one Polaroid on top of the crystal.
5. Rotate the Polaroid over top of the crystal.
6. Sandwich the mica between the Polaroids, hold them up to a light, and rotate the Polaroids.
7. Repeat step 6 for the ruler piece.
8. Place a Polaroid on top of a calculator LCD readout and rotate the Polaroid.
9. If it is a sunny day, look out the window through a Polaroid. Either tilt your head or rotate the Polaroid.
10. Stand to one side of a reflection in the window, such as that of the window in a class door. Observe the reflection in the window. Put a Polaroid in front of your eyes. Rotate the filter and adjust your position slightly until the image disappears.
11. Have a group member measure the angle relative to a normal to the glass. Use a protractor and metre stick.

12. Sandwich the sample slide between two Polaroids and view this combination under the microscope. Rotate one Polaroid and note any changes to the viewed object.

Analysis

Create a chart summarizing your results using the following headings: Method of Polarization, Expected Result, Viewed Result.

Discussion

1. Why does the intensity of the transmitted light change as you rotate the Polaroids around?
2. What law calculates the amount of light transmitted?
3. Why does light pass through three Polaroids positioned in the manner described in the procedure, but no light passes through with two Polaroids?
4. Why does the calcite crystal produce two images such that one image rotates around the other? Which image is produced by the α ray?
5. Why does a Polaroid cut only one image at a time from the calcite crystal?
6. Why are colours produced in the mica and the LCD readout when a polarizing filter is put on top of them?
7. How do you know where the stressed areas or points are when viewing the broken lucite ruler between polarizing filters?
8. Why does the blue sky change its tint with polarizing filters and not with ordinary sunglasses?
9. Find the refractive index of glass and calculate Brewster's angle. Compare it to the measured angle from the experiment.
10. Did you see any colour changes in the object you were looking at when one Polaroid was rotated?

Conclusion

Summarize the characteristics of polarized light and the phenomena that prove its existence.



Malus' Law

Purpose

To study the effect of angle on the intensity of transmitted light through a polarizer-analyzer setup

Equipment

Light meter

2 Polaroids mounted on stands. One Polaroid mount should allow the Polaroid to rotate.

Protractor

Incandescent light source

Procedure A: Setting Polaroid Transmission Directions

1. Place the two Polaroids together and rotate one of them until the maximum amount of light is transmitted.
2. Mark each Polaroid with an arrow indicating its relative axis.
3. Set the Polaroids into the mounts such that the polarizer direction is either vertical or horizontal.

Procedure B: Proving Malus' Law

1. Measure the light intensity of the source directly and record the measurement.
2. Measure the light intensity of the light exiting the polarizer and record the measurement.
3. Align the analyzer transmission direction parallel to the polarizer. Measure and record the light intensity exiting the analyzer.

4. Align the analyzer 90° to the polarizer. Measure and record the light intensity exiting the analyzer.
5. Align the analyzer parallel to the polarizer. Measure the light intensity for the following angles of the *analyzer* relative to the polarizer: $10^\circ, 30^\circ, 60^\circ, 80^\circ, 120^\circ, 160^\circ$.

Analysis

1. Calculate the expected transmitted intensity for one Polaroid.
2. Calculate the expected transmitted intensities for the angles in step 5 above.
3. Calculate percent deviations in all cases.

Discussion

1. By how much was the intensity of the light decreased through one Polaroid?
2. How much light was transmitted through the polarizer-analyzer combination?
3. Were your results consistent for the various angle measurements? If not, provide a reason for the discrepancy.
4. Within the deviations, did your results corroborate Malus' law?

Conclusion

Summarize your findings and draw a conclusion from your analysis.

11

The Interaction of Electromagnetic Waves

Chapter Outline

- 11.1 Introduction
- 11.2 Interference Theory
- 11.3 The Interference of Light
- 11.4 Young's Double-slit Equation
- 11.5 Interferometers
- 11.6 Thin-film Interference
- 11.7 Diffraction
- 11.8 Single-slit Diffraction
- 11.9 The Diffraction Grating
- 11.10 Applications of Diffraction
-  CD Technology
- LAB** 11.1 Analyzing Wave Characteristics using Ripple Tanks
- LAB** 11.2 Qualitative Observations of the Properties of Light
- LAB** 11.3 Comparison of Light, Sound, and Mechanical Waves
- LAB** 11.4 Finding the Wavelength of Light using Single Slits, Double Slits, and Diffraction Gratings



By the end of this chapter, you will be able to

- describe the wave theory of light using interference and diffraction phenomena
- compare single-slit, double-slit, and diffraction patterns of light
- use equations related to interference and diffraction of light
- describe how various technologies use the theories associated with interference and diffraction of light

11.1 Introduction

In Chapter 10, the wave nature of light was demonstrated through phenomena such as refraction, polarization, and Maxwell's electromagnetic theorems. Two more aspects of the wave nature of electromagnetic radiation will be covered in this chapter: interference and diffraction.

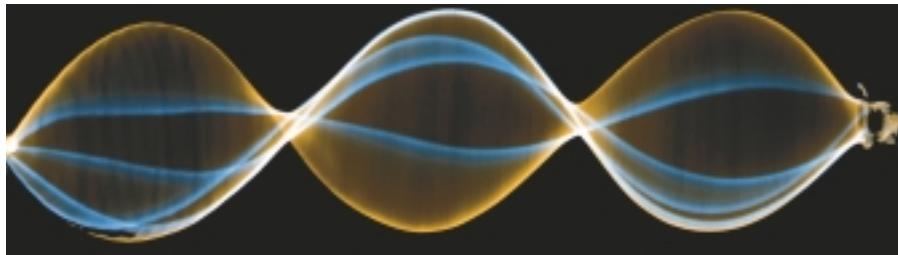


Fig.11.1a Two kinds of standing waves



From our studies of sound in Grade 11, we have already encountered these two aspects of waves. The interference of waves produces the characteristic standing wave patterns seen in strings (Figure 11.1a). It is also responsible for the variations in sound intensities you hear when walking around a room in which two speakers are sending out sound waves. In Figure 11.1b, the loud areas are spots where the sound waves interfere constructively, and in the quieter areas, the waves interfere destructively.

Diffraction is the bending of waves. We are surrounded by its effects. When we hear a person around a corner or from behind an obstacle, the effect is caused by waves bending around objects (Figure 11.2).

Fig.11.1b Interference of two speakers sounding the same frequency

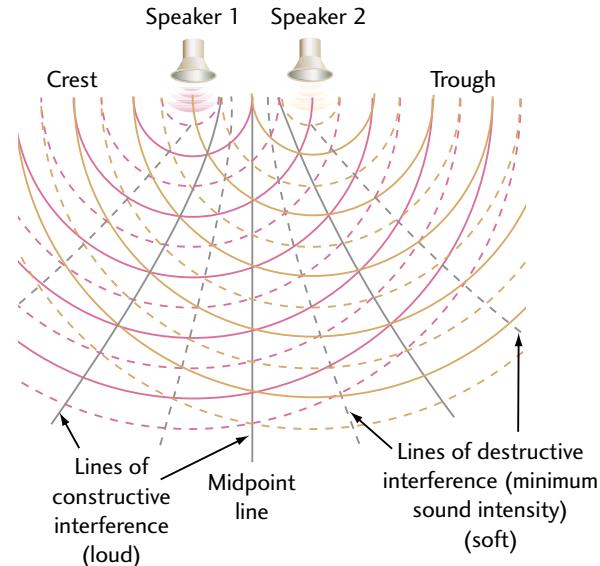
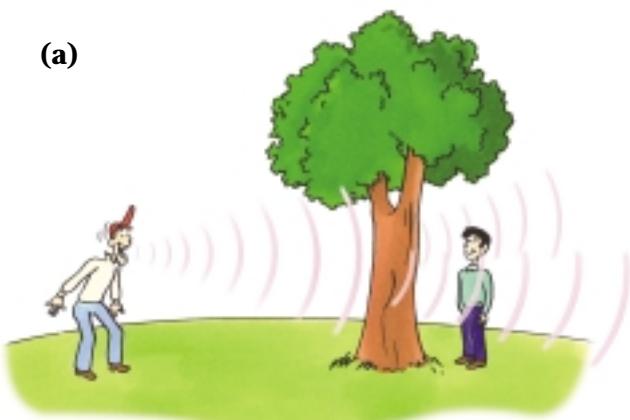


Fig.11.2 Waves diffract around obstacles



We will use mechanical waves such as water waves to show the effects of interference and diffraction. The next time you are in a pool or close to a body of water, look for interference and diffraction patterns in the water (see Figure 11.3a and Figure 11.3b).

Fig.11.3a

Diffraction

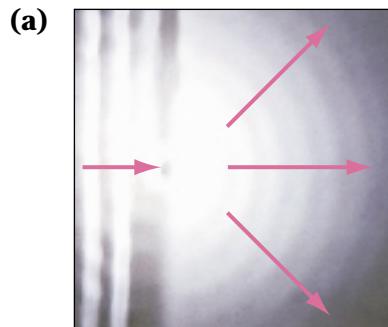
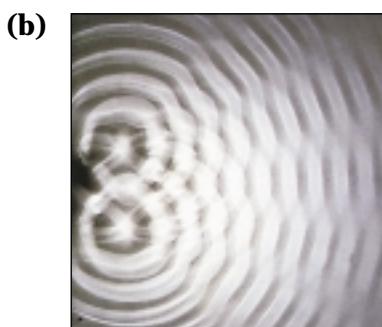


Fig.11.3b

Interference

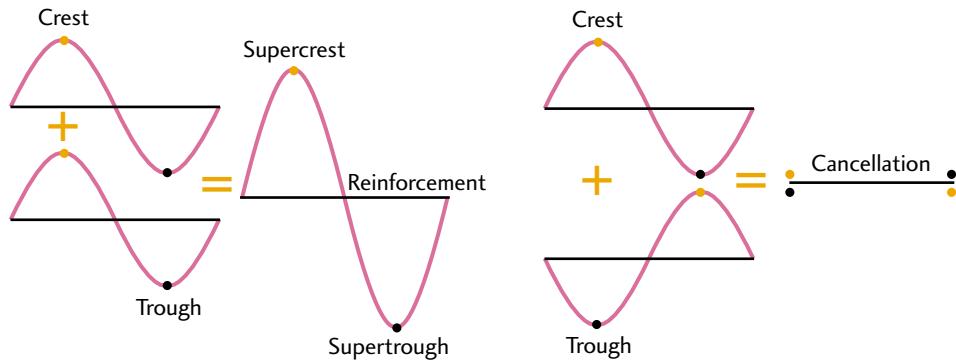


1. Review the concepts of interference and diffraction of sound.
2. Find specific examples of interference and diffraction effects. Include the production of different musical notes and the shapes of waves produced by various instruments.

11.2 Interference Theory

Combining two or more waves to produce a single wave is called the **principle of superposition**. As the waves meet, they occupy the same space at the same time. At this point, the *amplitudes* of the waves combine in one of two ways, as illustrated in Figure 11.4. When the amplitudes are both in the same direction, they are added together. This combination is called **constructive interference**. When the amplitudes are in opposite directions, they cancel out, or subtract. This combination is called **destructive interference**.

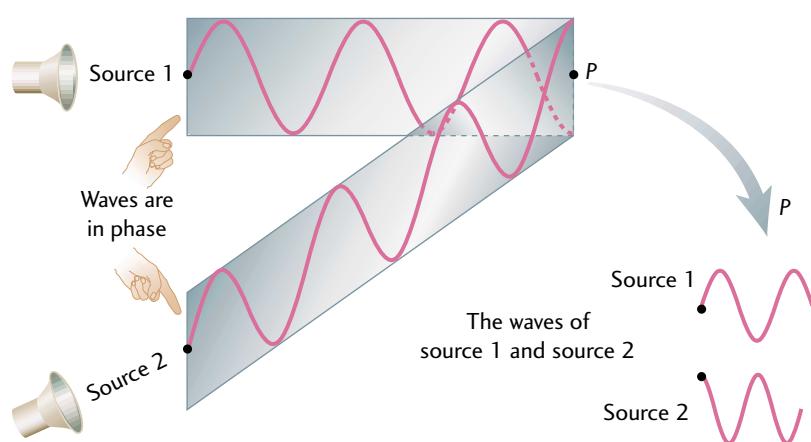
Fig.11.4 Constructive and destructive interference



Path Difference

In Figure 11.5, waves from two different sources travel a different distance to the observer. The waves may arrive at the same point *shifted* relative to one another. In Section 10.2, this effect was called a *phase shift*.

Fig.11.5 Phase shift



When the two waves add according to the principle of superposition, a net shifting effect occurs. Figure 11.7 shows a series of possible shifts. The possibilities for degrees of shifts are endless: the wave can move an extra distance of 1.0λ , 0.1λ , 0.1101λ , etc.

PHASE-SHIFTED WAVES

Fig.11.6 Stare at the two waves. It is very difficult to tell them apart.

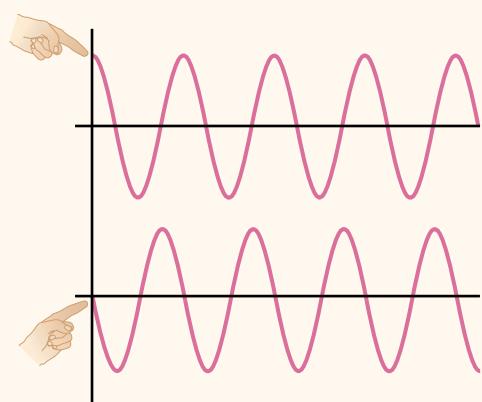
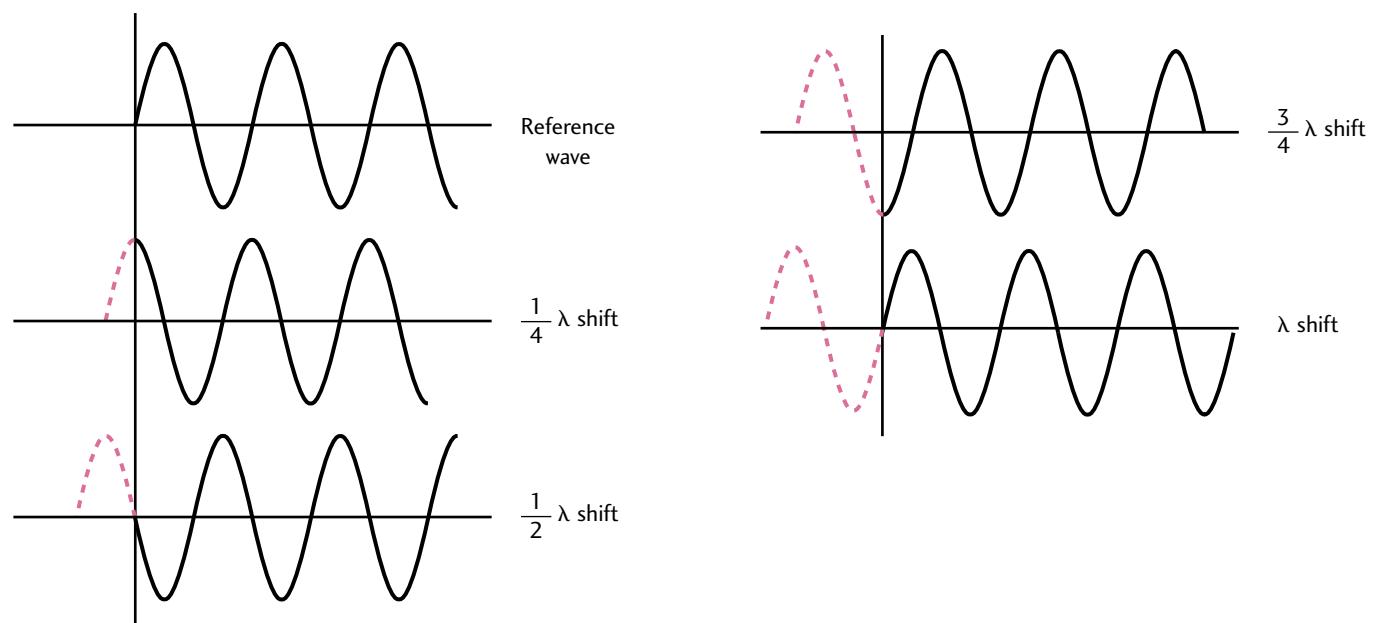


Fig.11.7 Possible phase shifts



The net shape of the resultant wave is complex in most cases. These complexities balance out when there are large numbers of paired waves and we see a net effect. In our study, we will focus on the two extremes of wave interaction, constructive and destructive interference.

When two waves arrive at one spot *in phase* (shifted by $m\lambda$, where m is an integer), the net effect is **constructive interference** and a **maximum** occurs (pl. maxima).

When two waves arrive *out of phase* (shifted by $(m + \frac{1}{2})\lambda$, where m is an integer), the net effect is **destructive interference** and a **minimum** occurs (pl. minima).

The term **node** is sometimes used to refer to a minimum.

Two-dimensional Cases

Interference is also visible in two dimensions. In Figures 11.8a and b, we use the wavefront representation of waves (see Section 10.4) to see the effect of two sources producing waves at the same time. Where two crests or two troughs overlap, maxima occur. Where a crest and a trough overlap, minima occur.

Fig.11.8a Wave interference pattern for two identical wave sources in water

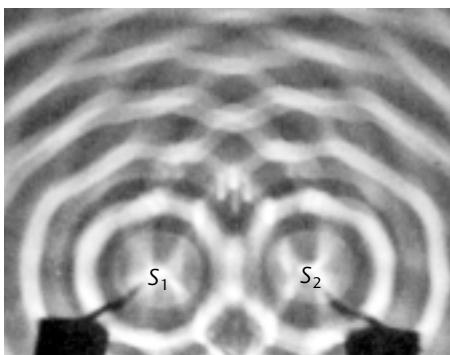
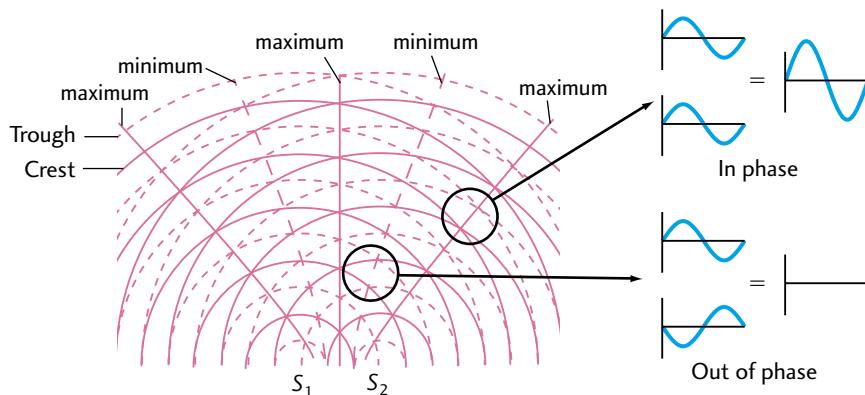


Fig.11.8b



1. Sketch the waves in Figure 11.7 into your notebook. Add the shifted wave to the reference wave, one at a time, and draw the resultant wave.
2. Sketch a series of concentric half-circles from a point (source 1), about 1 cm apart. From another point (source 2), about 1–2 cm away from source 1, sketch another series of concentric half-circles. Mark the maxima and minima. Find the central maximum and label it zero. Number the maxima on either side of the central maximum.
3. Describe the shapes of the maxima and minima in problem 2. Where do you see such patterns in everyday life?

11.3 The Interference of Light

In the early 1800s, Thomas Young, an English scientist, performed a series of experiments using light, which at the time could only be explained using wave theory. Using two opaque cards, Young punched a small hole in one of the cards, and two small pinholes placed close to each other in the other card. The single-holed card was placed in the direct path of a light source, with the double-holed card a certain distance behind it. A screen was then placed behind the double-holed card. When light was shone on the cards, an interference pattern was produced on the screen (see Figure 11.9). Using the wave equation ($v = \lambda f$), Young calculated a value for the wavelength of light. The pattern in Figure 11.9 shows the characteristic light and dark areas associated with wave interference.

Fig.11.10 A water-wave representation of Young's experiment

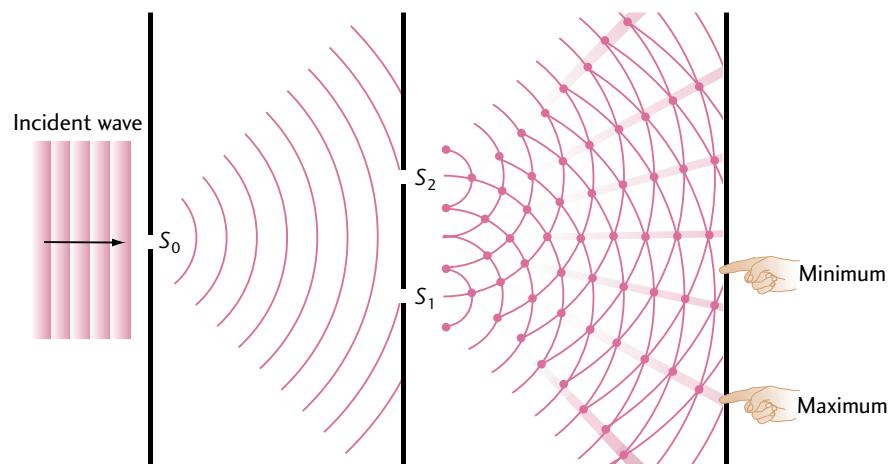


Figure 11.11 shows a fringe pattern for an experiment using slits instead of pinholes. Notice how the bands are numbered on each side from the central maximum, denoted as a zero. The integers are called **order numbers**. Two characteristics of this pattern are its regular spacing and the gradual drop-off in intensity of light as the order number increases.

COHERENCE

In order to see the effects of interference, the light sources must be **coherent**. The waves of both sources must maintain a constant phase relationship at all times. In the double-slit experiment, if two incandescent or fluorescent light bulbs were used, there would be no interference pattern because each light bulb emits light in random orientations. There is no stable relationship between the two waves arriving at any given point. Therefore, they cannot establish either a maximum or a minimum. Coherence can be achieved by placing one light bulb behind a barrier with two small openings. You could also use two lasers as long as one laser is tunable. The tunable laser can be set to the same phase as the other laser.

Fig.11.9 The two openings act as in-phase (coherent) light sources. The light from these sources travels to the screen and interferes in a manner similar to the water waves illustrated in Figure 11.10. The single opening collimates the original beam, creating a sharper image.

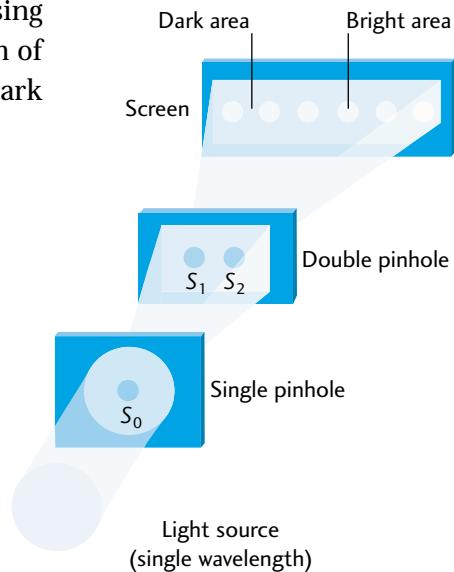
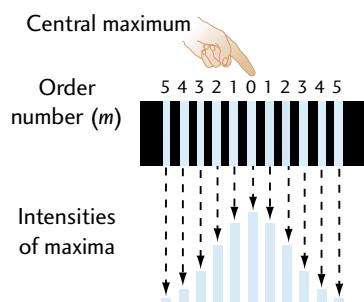


Fig.11.11 Intensities of the double-slit pattern



11.4 Young's Double-slit Equation

Fig.11.12a Young's double-slit experiment

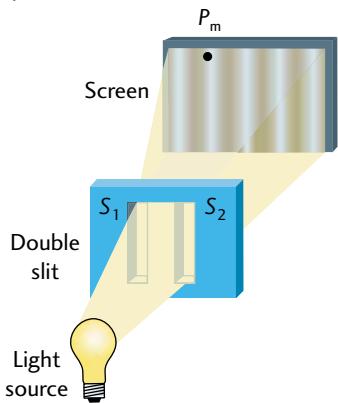


Fig.11.12b

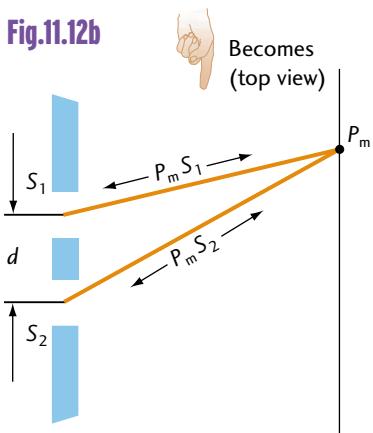


Figure 11.12b represents Young's experiment viewed from above. The distance between slits is labeled d , and P_m is a point on any maximum, where m is the order number. For example, a point on the second-order maximum would be labeled P_2 . S_1 and S_2 represent the positions of the two slits and hence the two light sources.

The distances from slit 1 and slit 2 to the point P_m are P_mS_1 and P_mS_2 , respectively. The **path difference** (the difference in length between the two distances) is $|P_mS_2 - P_mS_1|$. Recall from Section 11.2 that for a maximum (constructive interference), the path difference must be a whole number of wavelengths, $m\lambda$, where m is the order number and λ is the wavelength. Therefore, for *constructive interference*, the **first equation for Young's double-slit experiment** is

$$m\lambda = |P_mS_2 - P_mS_1|$$

EXAMPLE 1

Using the path-difference equation

Light from a red monochromatic source is shone through a pair of slits, creating an interference pattern. At the second-order maximum, light travels 0.800 000 1 m from slit 1 and 0.800 001 4 m from slit 2. Find the wavelength of light used.

Solution and Connection to Theory

Given

$$m = 2 \quad P_mS_1 = 0.800\ 000\ 1\text{ m} \quad P_mS_2 = 0.800\ 001\ 4\text{ m}$$

$$m\lambda = |P_mS_2 - P_mS_1|$$

$$\lambda = \frac{|P_mS_2 - P_mS_1|}{m}$$

$$\lambda = \frac{|0.800\ 001\ 4\text{ m} - 0.800\ 000\ 1\text{ m}|}{2}$$

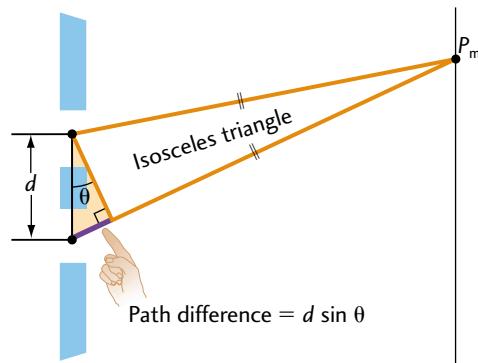
$$\lambda = \frac{1.3 \times 10^{-6}\text{ m}}{2}$$

$$\lambda = 6.5 \times 10^{-7}\text{ m or }650\text{ nm}$$

The wavelength of light used is 650 nm.

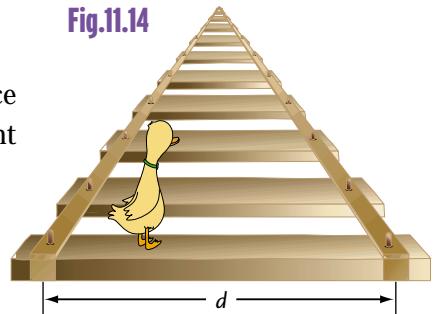
To derive the second and most common double-slit equation, we construct an isosceles triangle, as in Figure 11.13. The base of the isosceles triangle is the adjacent side of a right-angle triangle, where the hypotenuse is the distance between the midpoints of the two slits, d .

Fig.11.13 Young's double-slit equation



The approximation that the shaded triangle is a right-angle triangle is like looking at railroad tracks into the distance from d (Figure 11.14). The rails appear to converge, even though we know that they are parallel. In the case of light interference, the light rays actually converge, but extremely slowly, so they appear parallel close to the slits.

Fig.11.14



If the distance to the pattern at point P_m is much greater than the distance d between the slits, then d is approximately the same length as the adjacent side of the right-angle triangle in Figure 11.13. Thus,

$$\sin \theta = \frac{\text{path difference}}{d} \quad \text{or} \quad d \sin \theta = \text{path difference}$$

The angle θ depends on the order number (i.e., the maximum we select): the farther our chosen maximum is from the central maximum, the greater the angle θ . Therefore, the angle can be written as θ_m . For constructive interference, the path difference is $m\lambda$, as we noted above. The **second equation for Young's double-slit experiment** is

$$m\lambda = d \sin \theta_m$$

EXAMPLE 2

Young's double-slit experiment calculation

A monochromatic source of 450 nm illuminates two slits that are 3.0×10^{-6} m apart. Find the angle at which the first-order maximum occurs. For a screen that is 1.0 m away from the slit, how far will the first-order maximum be from the centre line?

Solution and Connection to Theory

Given

$$m = 1 \quad d = 3.0 \times 10^{-6} \text{ m} \quad \lambda = 450 \times 10^{-9} \text{ m} = 4.50 \times 10^{-7} \text{ m} \quad \theta_1 = ?$$

$$m\lambda = d \sin \theta_m$$

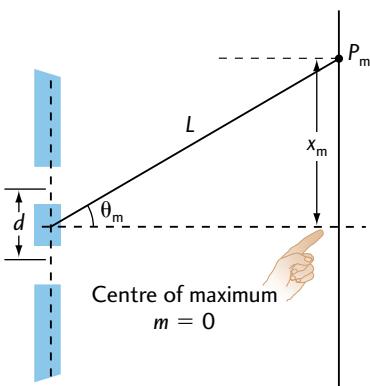
$$\theta_m = \sin^{-1} \frac{m\lambda}{d}$$

$$\theta_1 = \sin^{-1} \frac{1(4.50 \times 10^{-7} \text{ m})}{3.0 \times 10^{-6} \text{ m}}$$

$$\theta_1 = 8.6^\circ$$

For the screen 1.0 m away, the distance from the centre line for the first-order maximum is given by $\sin 8.6^\circ = \frac{x_1}{1.0 \text{ m}}$. Therefore, the maximum is 0.15 m or 15 cm from the centre line.

Fig.11.15 An alternative double-slit equation: x_m is the distance from the central maximum to the m th maximum.



The angle θ_m in Figure 11.15 is the same as θ in Figure 11.13. Using geometry, can you prove that they are equal?

The third equation for Young's double-slit experiment involves a linear measurement from the centre of the pattern to the bright band at point P_m (Figure 11.15). In this case, we have a triangle formed from the bright band to the halfway point between the slits. From this triangle, $\sin \theta_m = \frac{x}{L}$. We substitute this expression into $m\lambda = d \sin \theta_m$ to obtain the **third equation for Young's double-slit experiment**,

$$m\lambda = \frac{dx_m}{L}$$

We have added a subscript m to the distance x because x depends on which bright band is selected.

EXAMPLE 3 Using $m\lambda = \frac{dx_m}{L}$

A monochromatic light source of wavelength 450 nm illuminates two slits that are 6.0×10^{-6} m apart. Find the distance to the third-order maximum if the screen is 1.3 m away.

Solution and Connection to Theory

Given

$$m = 3 \quad d = 6.0 \times 10^{-6} \text{ m} \quad \lambda = 4.50 \times 10^{-7} \text{ m} \quad L = 1.3 \text{ m} \quad x_3 = ?$$

We can approximate L to be simply 1.3 m because the slit separation $d = 6.0 \times 10^{-6}$ m is insignificant compared to the perpendicular distance to the screen. Using Young's third equation

$$m\lambda = \frac{dx_m}{L}$$

and substituting for the third maximum, we obtain

$$3\lambda = \frac{dx_3}{L}$$

Isolating x_3 and substituting the given values,

$$x_3 = \frac{3\lambda L}{d}$$

$$x_3 = \frac{3(4.50 \times 10^{-7} \text{ m})(1.3 \text{ m})}{6.0 \times 10^{-6} \text{ m}}$$

$$x_3 = 0.29 \text{ m} = 29 \text{ cm}$$

This distance is equivalent to an angle of 13° , where $\theta = \sin^{-1}\left(\frac{0.29 \text{ m}}{1.3 \text{ m}}\right)$.

If we wish to solve the problem using nodal lines (minima) instead of maxima, then the path difference must be an integral number of half-wavelengths, or $(m + \frac{1}{2})\lambda$, instead of $m\lambda$.

EXAMPLE 4 Using minima instead of maxima

Find the wavelength of light used if the second-order minimum is located 21 cm from the central maximum on a screen 90 cm away. The separation between the double slits is $6.0 \times 10^{-6} \text{ m}$.

Solution and Connection to Theory

Given

$$m = 2 \quad x_2 = 21 \text{ cm} = 0.21 \text{ m} \quad d = 6.0 \times 10^{-6} \text{ m}$$

$$L = 90 \text{ cm} = 0.90 \text{ m} \quad \lambda = ?$$

Because we wish to find the minimum, we use the equation

$$(m + \frac{1}{2})\lambda = \frac{dx_m}{L}$$

Isolating λ and substituting the given values, we obtain

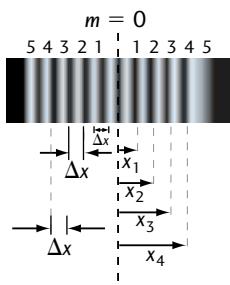
$$\lambda = \frac{dx_m}{L(m + \frac{1}{2})}$$

$$\lambda = \frac{(6.0 \times 10^{-6} \text{ m})(0.21 \text{ m})}{(0.90 \text{ m})(2 + \frac{1}{2})}$$

$$\lambda = 5.6 \times 10^{-7} \text{ m} \text{ or } 560 \text{ nm}$$

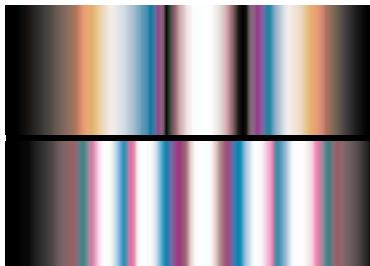
The wavelength of light is 560 nm.

Fig.11.16 The spacing in a double-slit pattern is constant



If we choose the third and fourth lines corresponding to x_3 and x_4 , then their difference, $x_4 - x_3$, is the distance between them, Δx .

Fig.11.17 A double slit separates white light into its component colours



One of the characteristics of double-slit patterns is the equal spacing between the bands. The distance between any two consecutive bands can be obtained by using the equation

$$m\lambda = \frac{dx_m}{L}$$

If we choose the m th and $(m + 1)$ st lines corresponding to the distances x_m and x_{m+1} , respectively, between two maxima (or two minima because the spacing in a pattern is regular) and subtract the distance between them, we obtain the distance between two consecutive lines, Δx .

Thus,

$$(m + 1)\lambda - m\lambda = \frac{dx_{m+1}}{L} - \frac{dx_m}{L}$$

$$m\lambda + \lambda - m\lambda = \left(\frac{d}{L}\right)(x_{m+1} - x_m)$$

which simplifies to

$$\lambda = \frac{d\Delta x}{L}$$

From this equation, we can see that the spacing, Δx , is proportional to the wavelength of light used. Thus, bands of red light will be spaced farther apart than bands of violet light. If we shine white light through a double slit, the slit separates the different colours of light and we observe the pattern shown in Figure 11.17.

EXAMPLE 5 Checking the spacing of different colours passing through a double slit

Compare the band-spacing patterns of red light (650 nm) to those of violet light (450 nm) when both colours of light are shone through slits separated by 6.0 μm from a distance of 1.0 m.

Solution and Connection to Theory

Given

$$\lambda_1 = 6.50 \times 10^{-7} \text{ m} \quad \lambda_2 = 4.50 \times 10^{-7} \text{ m} \quad d = 6.0 \times 10^{-6} \text{ m} \quad \Delta x = ?$$

For red light,

$$\lambda = \frac{d\Delta x}{L}$$

To calculate the band spacing, we isolate Δx and substitute the given values:

$$\Delta x = \frac{L\lambda}{d}$$

$$\Delta x = \frac{(1.0 \text{ m})(6.50 \times 10^{-7} \text{ m})}{6.0 \times 10^{-6} \text{ m}}$$

$$\Delta x = 0.11 \text{ m}$$

For violet light,

$$\lambda = \frac{d\Delta x}{L}$$

$$\Delta x = \frac{L\lambda}{d}$$

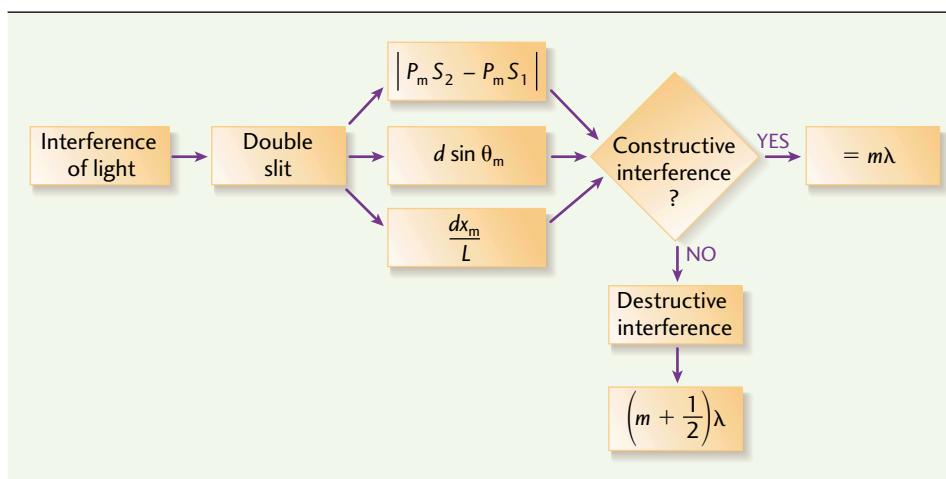
$$\Delta x = \frac{(1.0 \text{ m})(4.50 \times 10^{-7} \text{ m})}{6.0 \times 10^{-6} \text{ m}}$$

$$\Delta x = 0.08 \text{ m}$$

The spacing between red bands is 11 cm, whereas the violet bands are only 8 cm apart.

Figure 11.18 summarizes Young's double-slit equations.

Fig.11.18 Summary of Double-slit Equations



- When white light is shone through a double slit, which colour has the greatest spacing, Δx ? Which colour of light occurs first after the central maximum?
- Calculate the wavelength of light used in a double-slit experiment with a slit separation of $5.6 \mu\text{m}$ and a spacing of 28 cm between three light bands if the screen is 1.1 m away.
- In problem 2, what would be the spacing (Δx) for light of wavelength 510 nm?
- In problem 2, what is the distance from the centre to the third maximum?

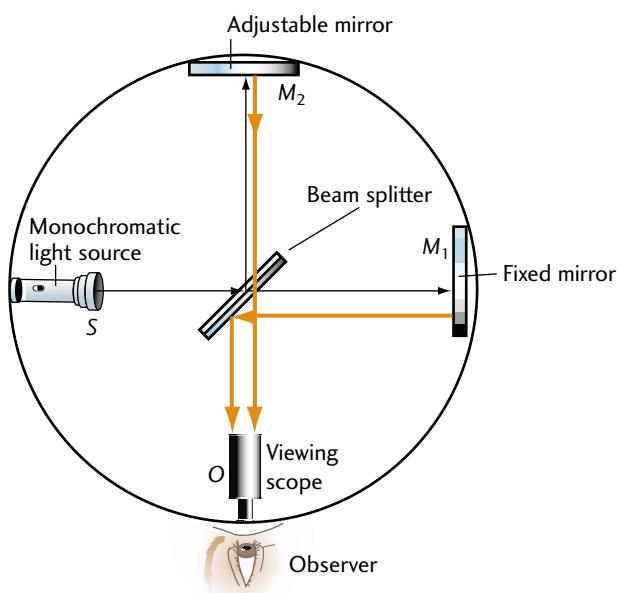


11.5 Interferometers

In Section 11.4, we learned that because of the wave nature of light, two light rays travelling different distances can interfere to produce characteristic dark and bright bands. The magnitudes of their path differences are approximately 10^{-7} m, or the range of visible light (400 nm–700 nm). Based on this principle, we should be able to measure the sizes of objects with lengths in this range.

In 1881, Albert A. Michelson used the interference properties of light to create such a measuring instrument, the **interferometer**. Figure 11.19 shows a simplified schematic representation of his instrument.

Fig.11.19 The optics of an interferometer



Light of unknown wavelength leaves the source, S , and travels to a fixed mirror, M_1 , through a **beam splitter**. The beam splitter is a piece of glass with a thin silver coating. The thin coating causes some of the light to be transmitted and the rest of it to be reflected; that is, it splits the original beam into two beams. The reflected beam then hits the adjustable mirror, M_2 , and reflects back through the beam splitter to the observer, O . The light transmitted through the beam splitter reflects off M_1 and the beam splitter to the observer. The two beams combine in a telescope, where the observer can then compare the path SM_1O to the path SM_2O . From Section 11.4, we found that if the paths differ by $m\lambda$, we see a bright band (a maximum) and if the paths differ by $(m + \frac{1}{2})\lambda$, we observe a dark band (a minimum). By shifting the adjustable mirror, M_2 , we can shift the interference pattern we observe. By counting the number of maxima we see, we can obtain an extremely precise measurement of the distance mirror M_2 has moved.

EXAMPLE 6 The interferometer

The adjustable mirror of an interferometer is moved back a distance of $\frac{\lambda}{4}$. What pattern is observed in the telescope?

Solution and Connection to Theory

From Figure 11.20, we can see that the light now travels an extra $\frac{\lambda}{4}$ twice between the original and second positions of M_2 . The extra distance produces a total shift of $\frac{\lambda}{2}$, or destructive interference and a minimum. The observer therefore sees a dark band and the fringe pattern moves by half a band.

Extension: Measuring Thickness using an Interferometer

If a piece of material with refractive index n_m and thickness t is placed into an interferometer, the number of wavelengths of light in the material is $\frac{2t}{\lambda_m}$. From Figure 11.21, we can see that light travels through the material twice. The number of wavelengths of light in a comparable amount of air is $\frac{2t}{\lambda_{air}}$. We can omit the subscript “air” because the wavelength in air is the standard wavelength. Therefore, we can write the expression as $\frac{2t}{\lambda}$. The path difference (in wavelengths) caused by inserting the material can be calculated using the expression $\frac{2t}{\lambda_m} - \frac{2t}{\lambda}$. From the sidebar (Adjusted Wavelength), we know that $\lambda_m = \frac{\lambda}{n_m}$. From this equation, we can calculate the path difference, ΔPD , in terms of wavelength, which allows us to accurately measure the thickness of a material:

$$\Delta PD = \left(\frac{2t}{\lambda} \right) (n_m - 1)$$

When measuring the thickness of a material using an interferometer, the observer counts the shift in the number of light or dark bands when the material is inserted, and compares it with the original pattern *before* the material was inserted. The shift corresponds to the path difference in terms of the number of wavelengths. If the refractive index of the material is known, its thickness can be calculated using the path difference equation. Interferometers are used to obtain extremely precise measurements of properties of materials related to their molecular and atomic structures.

Fig.11.20

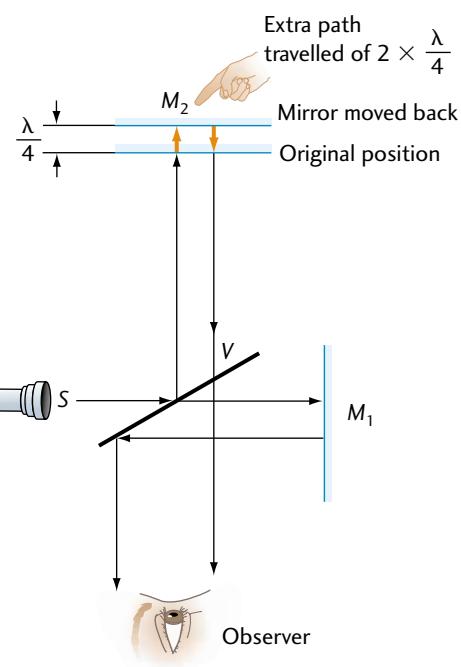
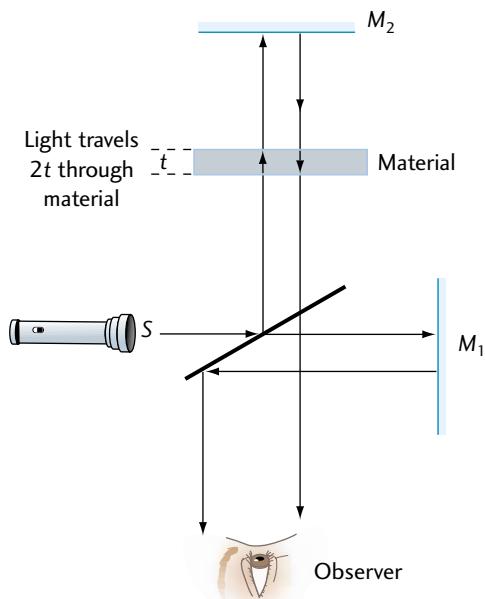


Fig.11.21 Measuring thickness using an interferometer

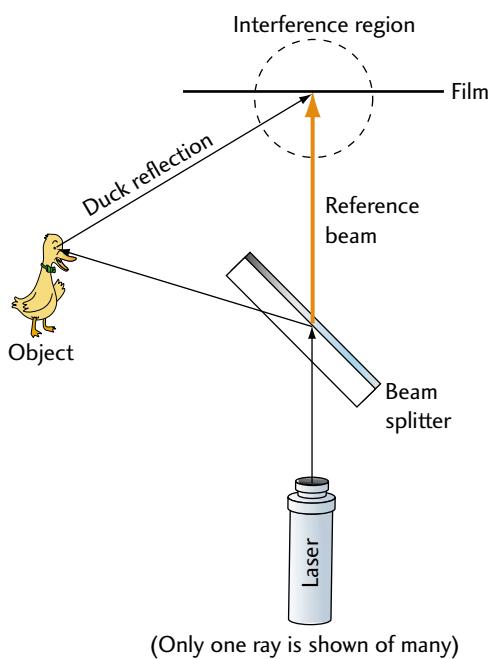


Adjusted Wavelength

$n = \frac{c}{v}$ (where c is the speed of light in vacuum and v is the speed of light in a medium) and $v = \lambda f$. The frequency of light doesn't change across a medium boundary, so $n = \frac{\lambda f}{\lambda_m f}$ or $\lambda_m = \frac{\lambda}{n}$.

Holography

Fig.11.22 The optics of a hologram



The term **LASER** stands for Light Amplification by Stimulated Emission of Radiation.

Fig.11.23 A ruby laser

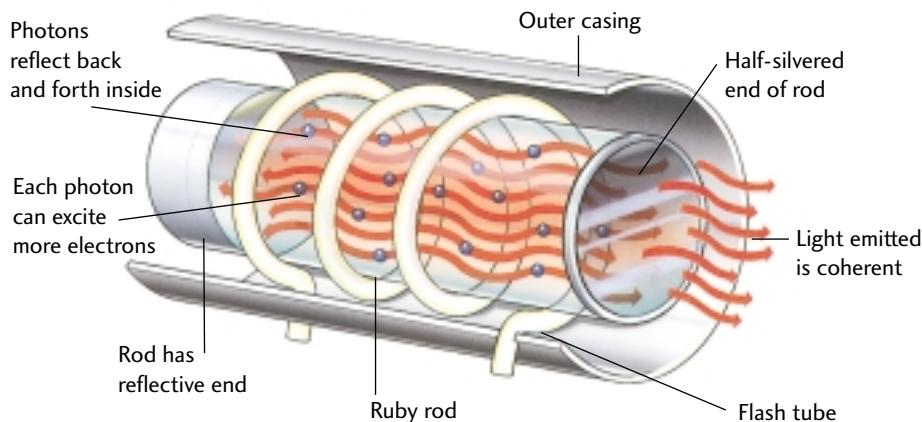
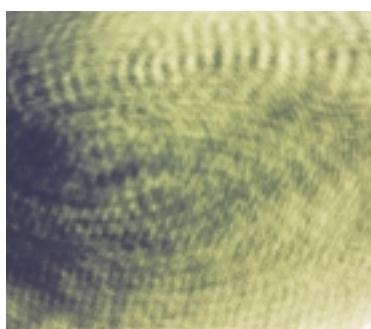


Fig.11.24 A holographic interference pattern on a film



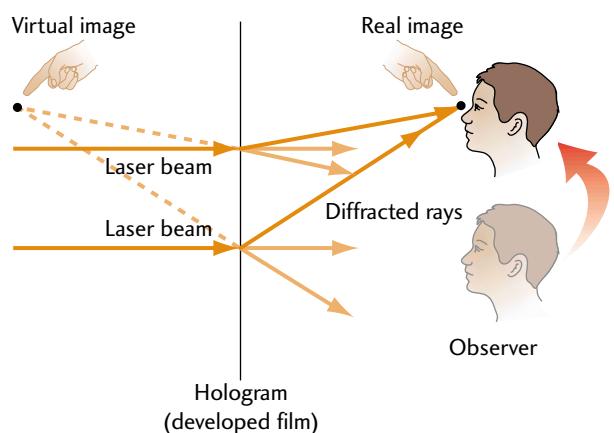
Notice in Figure 11.24 that, when viewing the holographic film directly, only a fancy interference pattern is observed. In order to see the image, we must use a laser beam to illuminate the plate. Laser light is shone through the interference pattern, which acts similarly to a diffraction grating (described in Section 11.9). The pattern on the film splits the light according to the spacing in the interference pattern. The image forms at the points of intersection of the emerging light rays (see Figure 11.25).

Because the interference pattern was created from rays of light travelling different distances to the plate, the information from both beams is contained in the hologram. Thus, depending on your perspective, you'll see a different image. Different angles produce different views (see Figure 11.26).

Fig.11.26 When viewing a hologram, what you see depends on where you look



Fig.11.25 A laser is required to view a hologram



- Derive the equation $\Delta PD = \left(\frac{2t}{\lambda}\right)(n_m - 1)$, where ΔPD is the path difference in wavelengths.
- If a shift of three bright bands is noticed when glass with refractive index 1.52 is inserted into an interferometer, find its thickness when viewed with light of wavelength 624 nm.

The Length of the Standard Metre

Michelson determined the length of the standard metre (the distance between two lines scratched on a platinum–iridium bar, kept at 0°C and stored at Sèvres, near Paris, France) to be 1 553 163.5 wavelengths of red cadmium light. In 1907, Michelson was awarded the Nobel Prize in physics for this measurement. Scientists saw the advantage of having a length standard not based on a solid object; there was no fear that the object, and therefore the standard, could be destroyed. A definition of length based on a wavelength was also portable, and thus available to everyone around the world. In 1961, the platinum–iridium bar was replaced by a multiple of the wavelength of the orange-red light of krypton-86 (1 650 763.73 wavelengths).

- Research the history of the metre from the platinum–iridium bar to its current standard definition in terms of the speed of light.
- Research how holograms, including those on credit cards, are produced. (They can be viewed with regular white light.) Why are they included on the credit card?

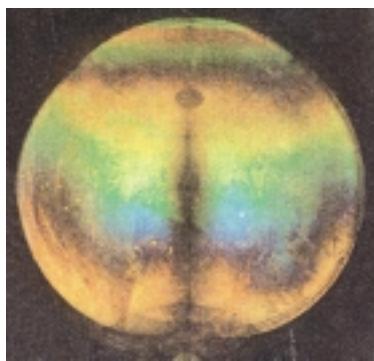


Fig.11.27 A laser is used to destroy a cataract



5. Research how laser scanners are used for product identification (such as checkout counters in stores).
6. Research how and why holograms are used in fighter planes (pilots view the controls on the panel beyond the windshield of the plane). Some car manufacturers have experimented with this technology. Why hasn't it been adopted?
7. Research other uses of the laser, such as in eye surgery (Figure 11.27).

Fig.11.28 Colours produced by light interference in thin films (soap bubbles)



11.6 Thin-film Interference

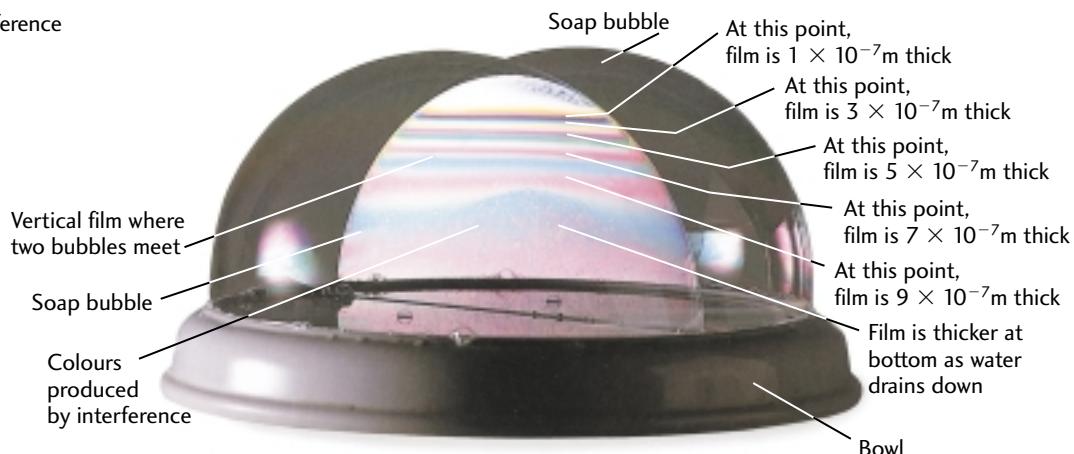
The colours we see on soap bubbles and films, as well as on gas and oil slicks on water, are caused by the interference of light (Figure 11.28). The film's thickness and the refractive index of the medium play important roles in causing this effect.

Path Difference Effect

Light hits the surface of the film and partially reflects. It also partially enters the film, reflects off the lower surface, comes out again, and combines with the light reflected from the upper surface to produce interference. The path difference in the film in terms of number of wavelengths determines the relative phase difference between the two waves. For example, consider the situation in Figure 11.29. If the thickness of the film is 2λ , then the total path the light travels in the film is 4λ . The light wave looks the same as the original wave and should interfere constructively. Similarly, if the total path difference is a multiple of $\frac{\lambda}{2}$, destructive interference is produced. Since each wavelength obeys these rules, the thickness of the film will cause constructive interference for some colours and destructive interference for others.

In the cases we have just described, we must remember to compensate for the refractive index when using the wavelength of light. The adjustment to the wavelength is given by $\lambda_{\text{medium}} = \frac{\lambda_{\text{air}}}{n_{\text{medium}}}$.

Fig.11.29 Reflection and interference in a thin film



The Refractive Index Effect

The other effect that plays a role in thin-film interference has to do with the incident medium's refractive index compared to the refractive index of the soap or oil slicks.

When light travels from a less optically dense to a more optically dense medium, the *reflected* ray undergoes a phase shift equivalent to $\frac{\lambda}{2}$. When light travels from a more optically dense to a less optically dense medium, no phase shift occurs in the reflected wave.

We have seen this effect when studying wave motion using springs in Grade 11. When a pulse on a spring reflects from a fixed end, it inverts (flips over). (See Figure 11.30a.) By analogy, a fixed end for a spring represents a more optically dense medium for light. When light passes from a less optically dense to a more optically dense medium, its phase shifts by 180° (it flips over). When the spring is attached to another spring or string that is free to move (i.e., a free end), the pulse doesn't flip over, as in Figure 11.30b. By analogy, a free end for light represents a less optically dense medium. When light passes from a more optically dense to a less optically dense medium, no phase shift occurs (i.e., the light wave doesn't flip over).

Fig.11.30a Reflection of a wave pulse at a fixed end has its amplitude inverted

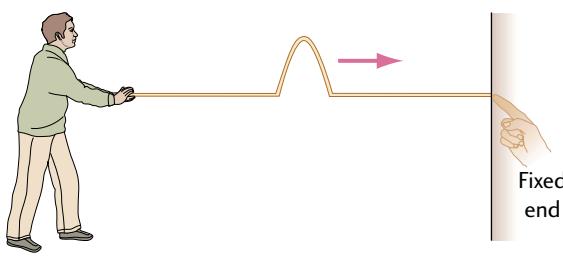
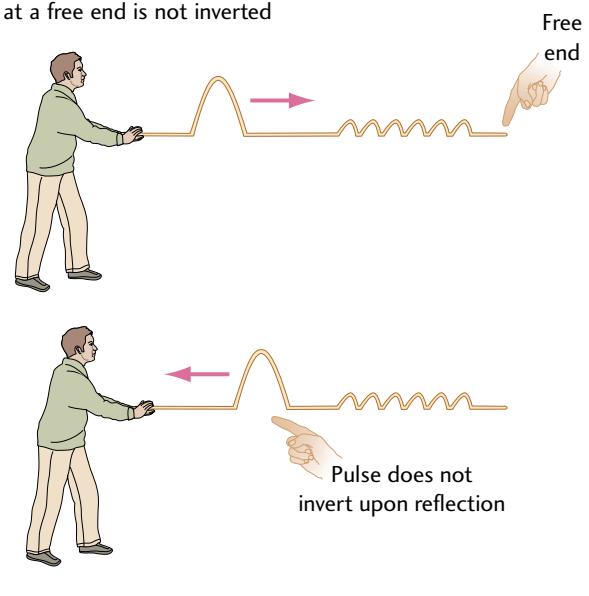


Fig.11.30b A wave pulse reflected at a free end is not inverted



Combining the Effects

When a phase-shifted light ray combines with a non-phase-shifted light ray, *net interference* occurs. If the net effect is constructive interference, a bright colour is seen. If the net effect is destructive interference, no colour is seen. We also need to remember that the wavelength of light changes when it enters a new medium.

EXAMPLE 7

Film thickness in wavelengths

A film of gasoline of thickness 510 nm formed on water is illuminated by light of wavelength 476 nm. The refractive index of gasoline is 1.40.

- How thick is the film, in wavelengths of light?
- How many wavelengths does the light travel in the film?
- Is a bright band or a dark band produced?

Solution and Connection to Theory

Given

$$\begin{aligned} n_1 &= 1.00 \text{ (air)} & n_2 &= 1.40 \text{ (gasoline)} & n_3 &= 1.33 \text{ (water)}, \\ \lambda_{\text{air}} &= 476 \text{ nm} = 4.76 \times 10^{-7} \text{ m} & \lambda_{\text{gas}} &= ? & t_{\text{wavelengths}} &= ? \end{aligned}$$

From the wave equation, $c = f\lambda$, the frequency of the light is constant and

$$\begin{aligned} v_1 &= c = f\lambda_1 \\ v_2 &= f\lambda_2 \end{aligned}$$

Dividing the first equation by the second equation,

$$\frac{c}{v_2} = \frac{f\lambda_1}{f\lambda_2}$$

$$\frac{c}{v_2} = \frac{\lambda_1}{\lambda_2}$$

$$\text{but } \frac{c}{v_2} = n_2$$

$$\text{so } n_2 = \frac{\lambda_1}{\lambda_2}$$

where v_2 is the speed of light in the new medium, λ_1 and λ_2 are the wavelengths of light in air and the second medium, respectively, and n_2 is the index of refraction of the material.

- First we calculate the wavelength of light in gasoline.

$$\lambda_{\text{gas}} = \frac{\lambda_{\text{air}}}{n_2}$$

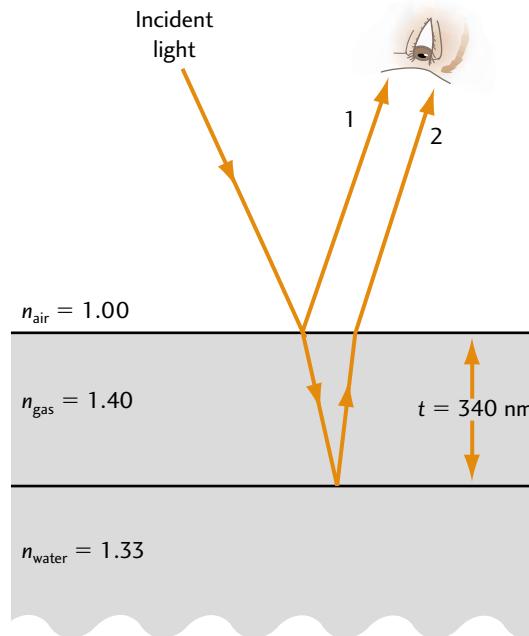
$$\lambda_{\text{gas}} = \frac{4.76 \times 10^{-7} \text{ m}}{1.40}$$

$$\lambda_{\text{gas}} = 3.40 \times 10^{-7} \text{ m} = 340 \text{ nm}$$

The thickness of the gasoline film is 510 nm. Therefore, the number of wavelengths of light in the gasoline medium $= \frac{510 \text{ nm}}{340 \text{ nm}} = 1.5$.

- Because the path of light in the film is twice the thickness of the film (i.e., light enters the film and is reflected at the bottom of the film), the total number of wavelengths travelled by the light is $2(1.5\lambda) = 3.0\lambda$.

Fig.11.31

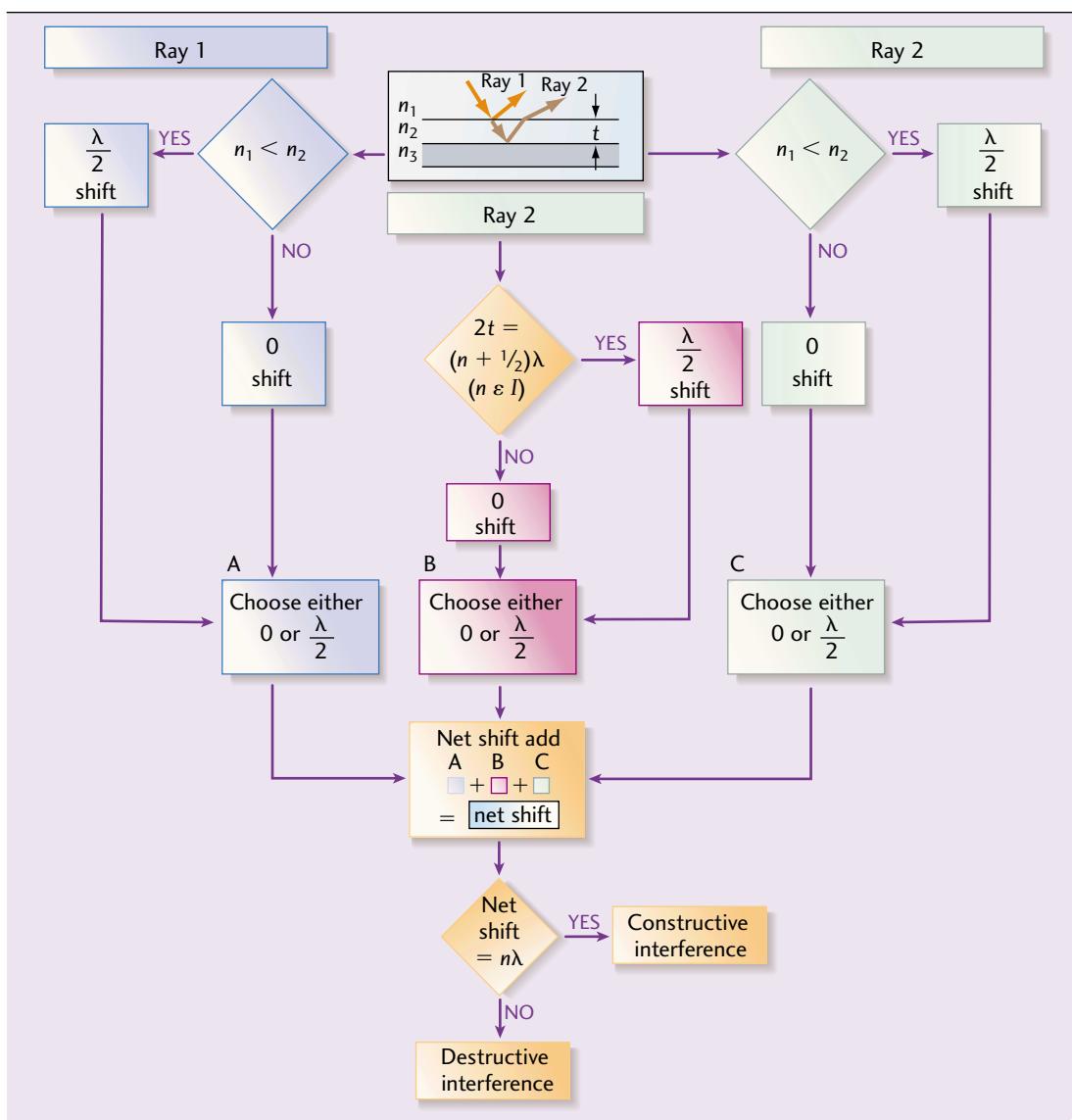


- c) The light wave reflects off a gasoline–water boundary at the bottom of the thin film. Since this reflection is between a more optically dense and a less optically dense medium, no phase shift occurs. However, the incident light that reflects off the surface of the gas film undergoes a $\frac{\lambda}{2}$ shift because it is going from a less optically dense medium (air) to a more optically dense medium (gas). The $\frac{\lambda}{2}$ shift from the top of the film combines with the 3λ shift from the bottom of the film to produce a dark band.

For constructive interference in Example 7, we would need a film thickness that, when doubled (because the light travels down and back up through the material), is an odd number of half-wavelengths of light. Thus, the minimum film thickness is $\frac{\lambda}{4}$ because when doubled, it becomes $\frac{\lambda}{2}$. In Example 7, this thickness is $\frac{1}{4} \times 5.10 \times 10^{-7} \text{ m} = 1.28 \times 10^{-8} \text{ m}$. The two waves are now *both shifted by $\frac{\lambda}{2}$* and will interfere constructively.

Figure 11.32 summarizes how to solve problems involving interference of light.

Fig.11.32 Method of Solving Thin-film Problems





Air Wedges

We can measure the thickness of a thin object (like a strand of hair) by using an air wedge.

Fig.11.33a Interference in an air wedge, top view. Notice the dark and bright bands.

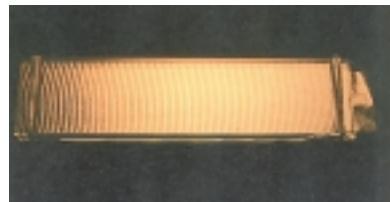
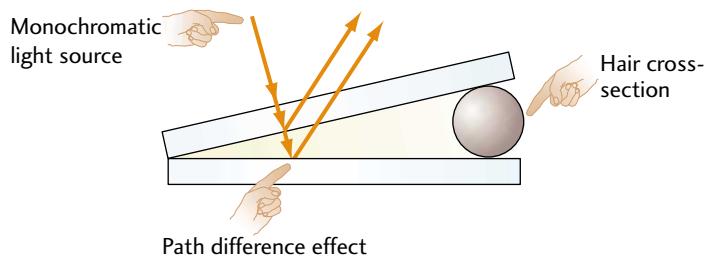


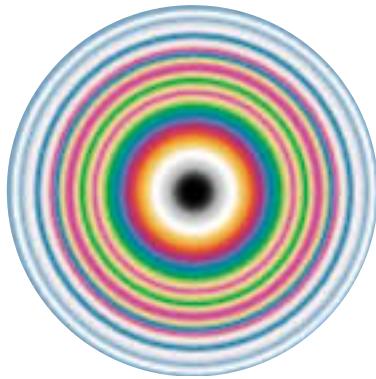
Fig.11.33b An air wedge



In Figure 11.33a, a hair is sandwiched between two glass plates, at one end. Consequently, the air between the plates forms a wedge shape (Figure 11.33b). When light enters the wedge, it travels different path-lengths, depending on the thickness of the air wedge. When the wedge is viewed from above using **monochromatic** light, a series of dark and light bands appears. The light bands occur at points of constructive interference where light travels a net multiple of half-wavelengths in relation to the thickness of the air wedge, such that $2t = (m + \frac{1}{2})\lambda$. The dark bands occur at points of destructive interference where light travels a net multiple of whole wavelengths in relation to the wedge's thickness such that $2t = m\lambda$, where t represents the thickness of the air wedge and m is the number of bands encountered at that point. If we know the number of bands and the wavelength of light, we can determine the thickness of the hair.

1. Explain how the two equations were obtained.
2. Find the thickness of a strand of hair if 22 dark bands were seen using light of wavelength 625 nm.
3. Find the total number of bright bands seen if a sheet of paper 1.75×10^{-5} m thick is wedged between two glass plates.
4. Research how Newton's rings are formed (shown in Figure 11.34). Explain their appearance.

Fig.11.34 Newton's rings



Non-reflective Coatings

Non-reflective plastic (many are scratch-resistant) coatings are evaporated onto the lenses of eyeglasses and camera lenses. The index of refraction of the non-reflective coating is less than that of the lens, so both medium surfaces reflect light with a phase change. The thickness of the coating is $\frac{\lambda}{4}$, so the path difference is $\frac{\lambda}{2}$.

5. How does the non-reflective coating eliminate reflection from a lens?
6. Research multiple lens coatings and their effectiveness. Which colours do these coatings affect the most?

Fig.11.35 Non-reflective coatings eliminate reflection



11.7 Diffraction

Shadows form because light travels in straight lines, the property called the **rectilinear propagation of light**. Yet, when a solid, thin object is illuminated by a monochromatic source of light, instead of producing the expected outline of the object, a shadow with a series of fringes appears (see Figure 11.36).

In 1666, Francesco Grimaldi at the University of Bologna postulated that the fringes were caused by light bending around corners, much as sound waves do. He called this effect **diffraction**. In the 1800s, French mathematician Augustin Fresnel suggested that light diffraction isn't easily seen because it depends on the ratio of the wavelength of the wave to the width of the opening it passes through. Simon Poisson, another famous French mathematician, objected to Fresnel's idea. Poisson argued that if the wave theory of light was correct, then a bright spot should be seen behind an opaque object in the centre of the shadow region. Fresnel set the experiment up, and to the disbelief of many, a bright spot was indeed observed (see Figure 11.37). This experiment confirmed Fresnel's theory and therefore the wave nature of light.

There is a somewhat arbitrary distinction between interference and diffraction. We usually define **interference** as a *superposition effect originating from two or more discrete sources of waves*. **Diffraction**, on the other hand, is *the interference effect from waves originating from a single source or wavefront*.

Wavelength Dependence

When water waves bend around solid obstacles, the amount of bending depends on the size of the object. In Figure 11.38, observe how the amount of bending increases as the size of the object gets closer to the wavelength of the wave. Similarly, sound waves can bend around corners and large objects such as trees because their wavelengths are comparable to the size of these objects.

Fig.11.36 The fuzzy edges and ripples in and around the razor blade are caused by the diffraction of light



Fig.11.37 The bright region in the centre of the shadow area caused by diffraction around an opaque object is referred to as Poisson's bright spot

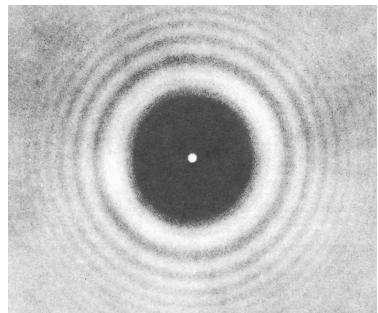
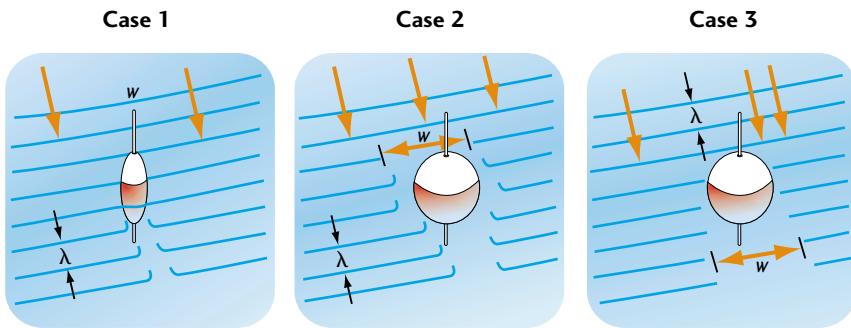


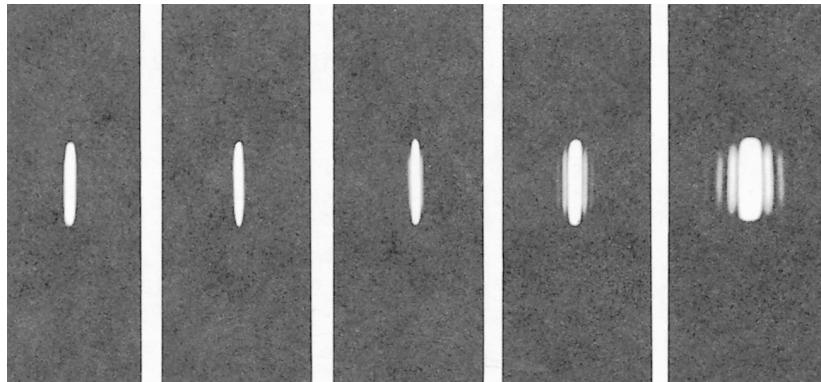
Fig.11.38 The amount of diffraction depends on the size of the object in relation to the wavelength of the water



From the equation $v = \lambda f$, a note of frequency 125 Hz travelling at 340 m/s has a wavelength of 2.72 m.

Because water waves and sound waves are **macroscopic**, we can easily observe their diffraction directly. Light waves, on the other hand, are **microscopic**; light has a wavelength in the range of 10^{-7} m. The only way we can observe the diffraction of light waves is through experimentation. Figure 11.39 shows a series of progressively narrower slits through which a parallel beam of monochromatic light is shone. As the slit narrows, it approaches the wavelength of the light, creating a diffraction pattern similar to that observed with long water waves and sound waves diffracting around obstacles. Diffraction is one of the most convincing arguments for the wave theory of light.

Fig.11.39 Slit widths are 1.5 mm, 0.7 mm, 0.4 mm, 0.2 mm, and 0.1 mm, from left to right

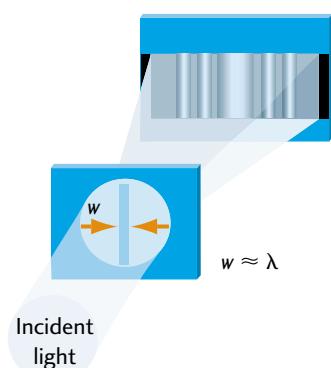


1. Find examples of the diffraction of light.
2. How does diffraction support the wave theory of light?

11.8 Single-slit Diffraction

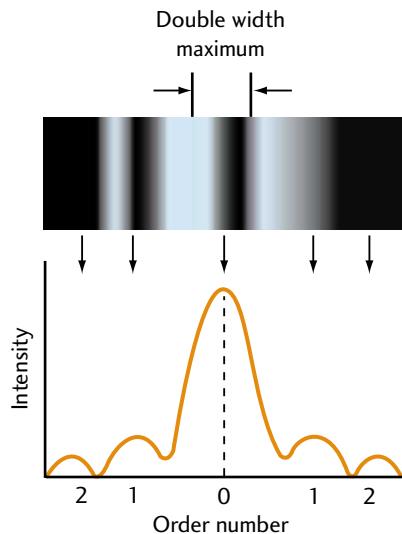
When shining light through a single slit (opening) comparable in size to the wavelength of light, the pattern illustrated in Figure 11.40 is seen. The main features of the diffraction pattern are shown in Figure 11.41.

Fig.11.40 Single-slit diffraction



Incident light

Fig.11.41 The characteristics of single-slit diffraction



They are summarized as follows:

- 1) The central maximum has a width double the size of a single maximum.
- 2) Away from the centre, the bright and dark bands are equally spaced.
- 3) The intensity of bright bands decreases rapidly the farther away they are from the slit.

The Single-slit Equation

In order to obtain a quantitative expression for the behaviour of light passing through a single slit, let's consider the circular wavefront shown in Figure 11.42a. According to Christian Huygens, we can consider a wavefront to be made up of a series of points, where each point acts like a new source of circular waves. These new sources produce a series of wavelets (new waves) that move forward. The whole wave then advances to a surface created by the overlap of all the Huygens wavefronts. Huygens' wavelets are illustrated in Figure 11.42b.

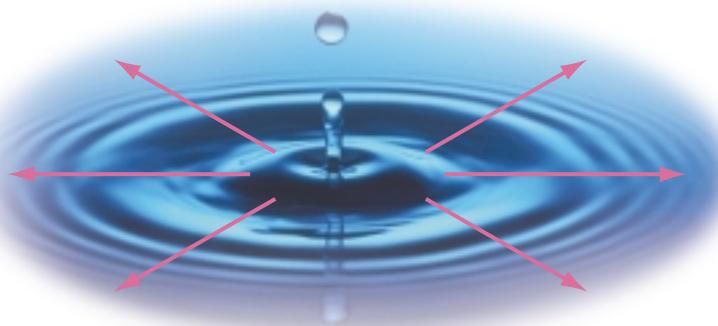
Similarly, when a light beam is shone through a slit, the waves that pass through the slit are tiny new sources of light that generate wavelets. Figure 11.43 illustrates the propagation directions (rays) of the wavelets.

Fraunhofer/Fresnel Diffraction

When the screen is far away from the slit, the rays hitting the screen are effectively parallel and the effect is referred to as a **Fraunhofer diffraction**. These waves appear as plane waves. If the screen is close to the slit, the curvature of the wavefronts becomes significant and the waves hit the screen obliquely. In this case, the term **Fresnel diffraction** is used. We can state that Fresnel diffraction is the general case. As the screen is moved farther away (lenses can be used to create the same effect), the effect changes to Fraunhofer diffraction.

Fig.11.42 Huygens' principle

(a) Wavefronts spreading out from a disturbance



(b)

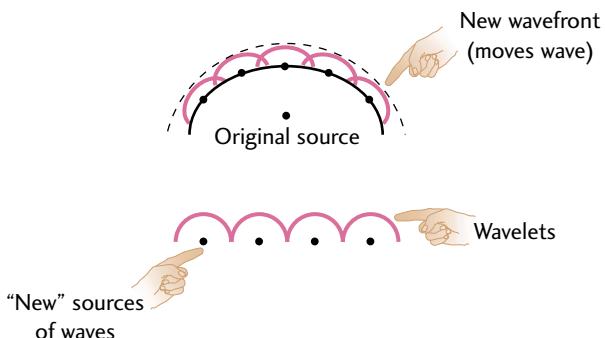
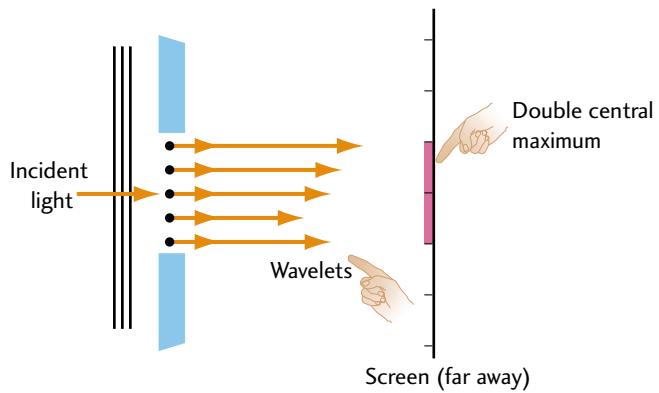


Fig.11.43 The single-slit opening is composed of secondary sources of light emitting Huygens' wavelets



In Figure 11.43, the light rays travel in phase in the same direction to the screen. Therefore, they interfere constructively, producing the bright double maximum on the screen.

In Figure 11.44, let's select an angle, θ_1 , such that the top ray travels a path difference of λ through the slit. Then, the path difference of the ray passing through the centre of the slit is $\frac{\lambda}{2}$. When the central ray and the bottom ray combine, they cancel each other out to produce a minimum. The same is true of the top and central rays. Similarly for any pair of rays equidistant from the central ray passing through the slit. If a ray passing through the bottom half of the slit combines with the corresponding ray passing through the top half of the slit, their path difference is $\frac{\lambda}{2}$ and they cancel each other out. The net effect is a minimum, or destructive interference at the point P_1 on the screen.

Fig.11.44 Condition for destructive interference

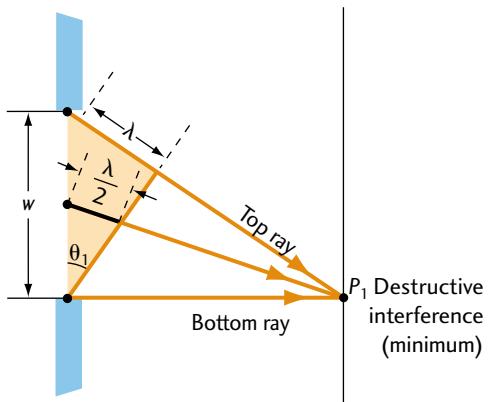
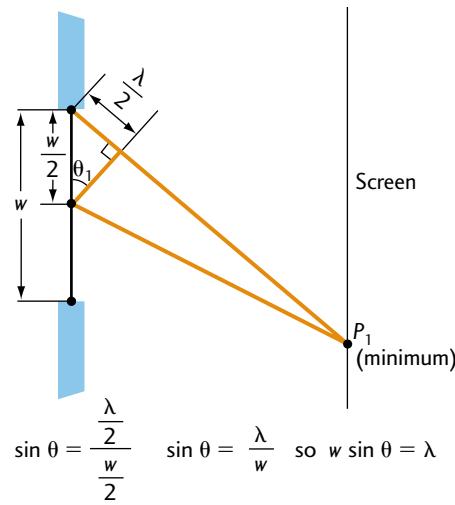


Fig.11.45 The single-slit equation



From Figure 11.45, we can derive the equation for the minimum at point P_1 :

$$\lambda = w \sin \theta_1$$

where w is the width of the slit, λ is the wavelength of the light, and θ_1 is the angle of the path difference.

In Figure 11.46, a wider angle, θ_2 , gives the top ray and the bottom ray a path difference of 2λ . Notice in this case that corresponding ray pairs on either side of the central ray have path differences of $\frac{\lambda}{2}$. Once again, even with a larger angle, all rays through the slit cancel to produce another minimum (destructive interference). From Figure 11.46, we generate the equation

$$\sin \theta_2 = \frac{2\lambda}{w}$$

or

$$2\lambda = w \sin \theta_2$$

In general, we can state that when the path difference is *an even-number multiple of $\frac{\lambda}{2}$* , the result is a minimum, and **the single-slit equation for destructive interference** is

$$m\lambda = w \sin \theta_m$$

where $m = 1, 2, 3, \dots$

Now, let's consider Figure 11.47, where the path difference between the top and bottom rays is $\frac{3\lambda}{2}$. If we pair rays from the bottom third of the slit with the rays from the middle third of the slit, each pair cancels out because their phases differ by a half-wavelength, as we saw in Figure 11.44. However, if the rays passing through the *bottom* two-thirds of the slit cancel, then the rays passing through the *top* one-third of the slit don't have any matching pairs left to cancel with. These rays reach the screen as a maximum. But since only a fraction of the light passing through the slit reaches the screen, its intensity is reduced. From the shaded triangle in Figure 11.47, we can derive the equation

$$\frac{3\lambda}{2} = w \sin \theta_1$$

for the first-order maximum (excluding the central maximum). If we increase the angle from the slit, we can create a path difference between the top and bottom rays of $\frac{5\lambda}{2}, \frac{7\lambda}{2}, \frac{9\lambda}{2} \dots$ to $(m + \frac{1}{2})\lambda$, where $m \in \mathbb{I}$. When the path difference is *an odd-number multiple of $\frac{\lambda}{2}$* , the rays passing through the slit don't all cancel out with corresponding rays that are shifted by $\frac{\lambda}{2}$, producing higher-order maxima. In general, the **single-slit equation for constructive interference** is

$$(m + \frac{1}{2})\lambda = w \sin \theta_m$$

Using our knowledge of constructive and destructive interference, we can now understand why there is a bright spot behind the solid object illuminated by a point source (Poisson's bright spot in Figure 11.37). The light diffracts

Fig.11.46 The second-order minimum. θ has increased until the path difference between the top and bottom rays is 2λ . P_2 is a minimum.

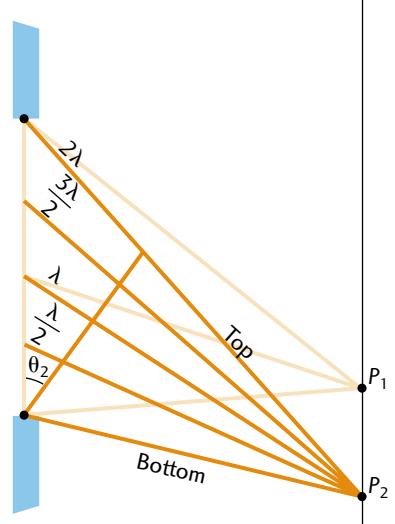
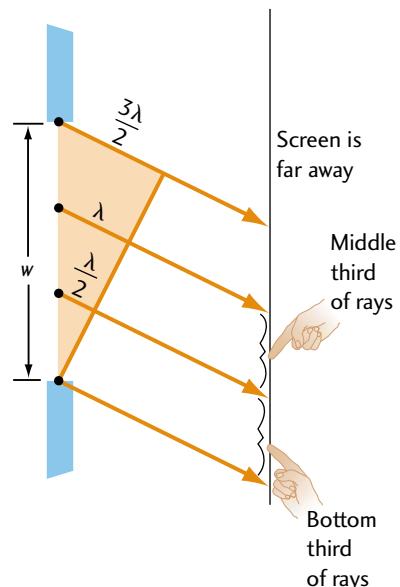


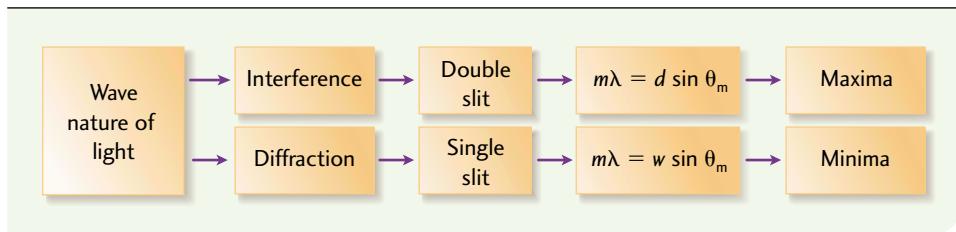
Fig.11.47 Condition for constructive interference



around the edges of the opaque object. At the bright spot in the centre, all the paths of light are the same length, causing constructive interference. This phenomenon is an example of Fresnel diffraction, and a strong argument for the wave theory of light.

Figure 11.48 summarizes the difference between interference and diffraction.

Fig.11.48 Interference versus Diffraction



EXAMPLE 8

Calculating the angle of the second nodal line for a single slit

A slit with a width of 2.0×10^{-5} m is illuminated by red light of wavelength 620 nm. At what angle does the third-order minimum occur?

Solution and Connection to Theory

Given

$$w = 2.0 \times 10^{-5} \text{ m} \quad m = 3 \quad \lambda = 620 \text{ nm} = 6.20 \times 10^{-7} \text{ m} \quad \theta_3 = ?$$

$$m\lambda = w \sin \theta_m$$

$$\sin \theta_m = \frac{m\lambda}{w}$$

$$\theta_3 = \sin^{-1} \frac{3(6.20 \times 10^{-7} \text{ m})}{2.0 \times 10^{-5} \text{ m}}$$

$$\theta_3 = 5.3^\circ$$

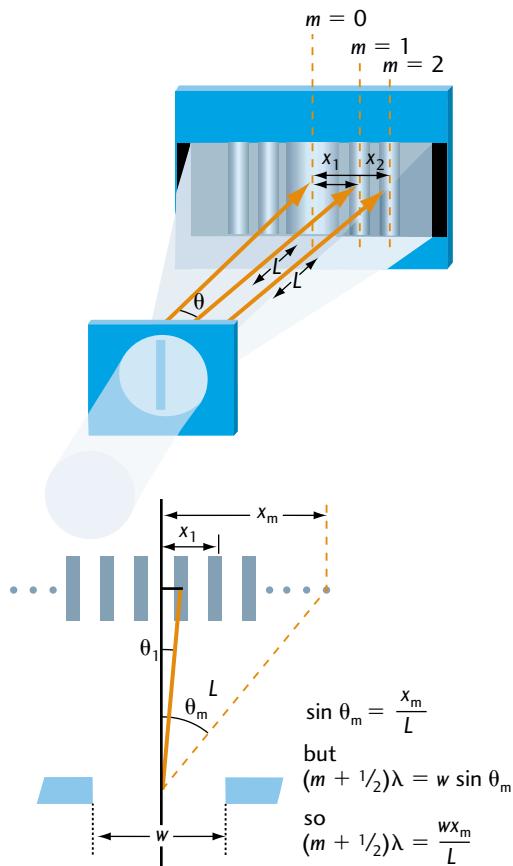
If we wish to solve for a maximum instead of a minimum, we need only change $m\lambda$ to $(m + \frac{1}{2})\lambda$.

The effects of light diffraction are not commonly seen by us because they are only present in situations where the size of the obstacle or aperture is comparable to the wavelength of light (too small for us to notice).

More Single-slit Equations (but they should look familiar)

If we wish to calculate distances from the centre of the pattern, we use the diagram in Figure 11.49. In Figure 11.49, consider the distance L (slit to screen) to be the same for any point on the pattern. This statement is true if $L \gg w$.

Fig.11.49 The $\frac{wx_m}{L}$ equation



From Figure 11.49, $\frac{x_m}{L} = \sin \theta$. We know that for destructive interference, $m\lambda = w \sin \theta_m$. When we combine these two equations, we obtain an alternative form of the **single-slit equation for destructive interference**:

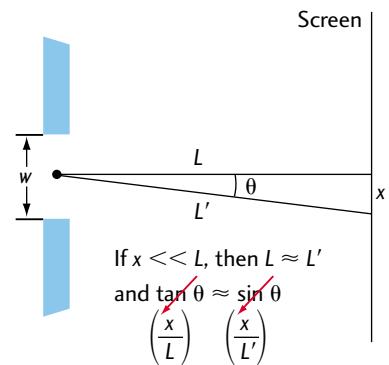
$$m\lambda = \frac{wx_m}{L}$$

Similarly,

$$\left(m + \frac{1}{2}\right)\lambda = \frac{wx_m}{L}$$

is an alternative form of the **single-slit equation for constructive interference**, where m is the order number of the dark and light bands, respectively, w is the width of the slit, x is the width of the band, and L is the distance from the slit to the screen.

Fig.11.50 The approximation used in the single-slit equation



Example:

If $x = 0.030$ m and $L = 1.50$ m,

then $\tan^{-1} \frac{0.030 \text{ m}}{1.50 \text{ m}} = 1.146^\circ$

Thus, $L' = \frac{x}{\sin 1.146^\circ}$

$L' = 1.49998$ m = 1.50 m

Therefore, $L = L'$.

If L is the perpendicular distance, then $\tan \theta \approx \sin \theta$ because the angle is small. (Try this approximation for an angle of 5° .) Therefore, a small shift in angle has no effect on L .

EXAMPLE 9**Using the alternative form
of the single-slit equation**

A single slit of width 9.5×10^{-6} m is illuminated by a monochromatic source of light of $\lambda = 640$ nm. If the screen is 1.3 m away, find the distance from the centre of the pattern to the first-order minimum.

Solution and Connection to Theory**Given**

$$m = 1 \quad L = 1.3 \text{ m} \quad w = 9.5 \times 10^{-6} \text{ m}$$

$$\lambda = 640 \text{ nm} = 6.40 \times 10^{-7} \text{ m} \quad w = ?$$

Because we need to find the distance to the minimum, we use the equation

$$m\lambda = \frac{wx_m}{L}$$

For a first-order minimum,

$$\lambda = \frac{wx_1}{L}$$

$$x_1 = \frac{L\lambda}{w}$$

$$x_1 = \frac{(1.3 \text{ m})(6.40 \times 10^{-7} \text{ m})}{9.5 \times 10^{-6} \text{ m}}$$

$$x_1 = 8.8 \times 10^{-2} \text{ m}$$

The first minimum is 8.8 cm from the centre of the central maximum.

EXAMPLE 10**The width of the central maximum**

From Example 9, what is the width of the central maximum?

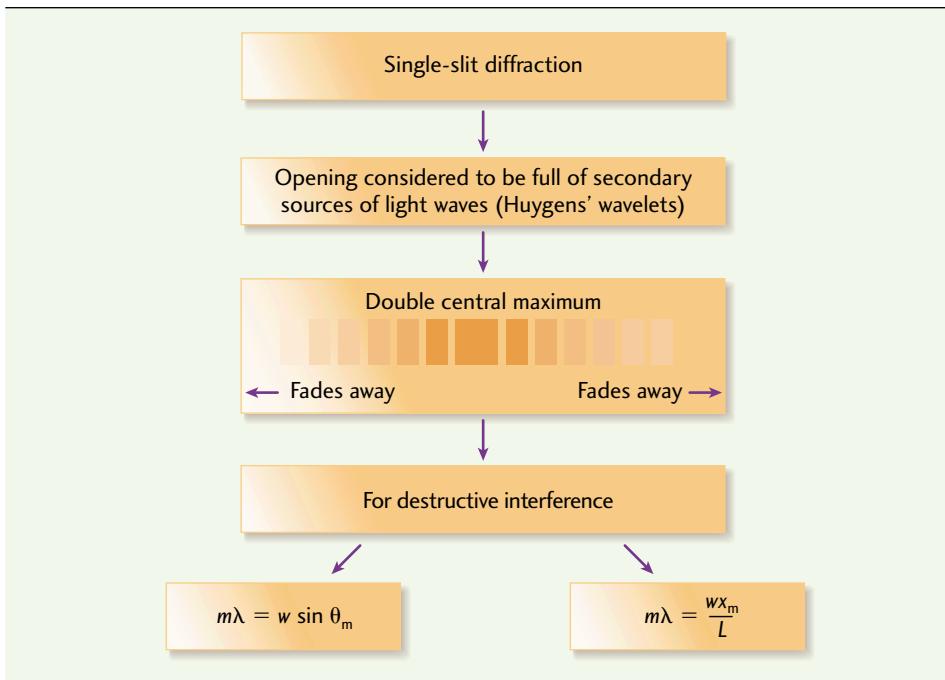
Solution and Connection to Theory

In Figure 11.49, we can see that the central maximum is framed by the first-order minima. From the last example, we calculated the first-order minimum to be 8.8 cm from the centre of the pattern, so the width of the central maximum must be two times 8.8 cm or 17.6 cm.

To calculate the angle subtended by the central maximum, we can use the equation $m\lambda = w \sin \theta_m$ and set $m = 1$. This equation gives us the angle subtended by half the central maximum. To find the whole angle, we multiply our answer by two. From Example 10, the total angle turns out to be 7.7° . Try to obtain this value yourself.

Figure 11.51 summarizes the main concepts of single-slit diffraction.

Fig.11.51 Summary of Single-slit Diffraction



Resolution

When we view two objects that are close together from far away (like two stars or two letters on a distant sign), sometimes they look like one object. As we move closer or as the objects move farther apart, it becomes easier to distinguish them (see Figure 11.52a). The apparent overlapping between object images is caused by diffraction patterns from each object overlapping and creating a smeared image. Satellite photos rely on computerized cleaning to remove such diffraction effects (Figure 11.52b).

Lord Rayleigh (1842–1919) suggested that two images are resolvable if the central maximum of one image lies on the first-order minimum of the other image. This concept is known as the **Rayleigh criterion**. The Rayleigh criterion can be calculated using the equation

$$\theta_R = \frac{1.22\lambda}{d}$$

where d is the diameter of a circular aperture and θ_R is the minimum angle, measured in radians, between the two objects that are at the Rayleigh criterion. Objects separated by this angle are resolvable. Although this definition is only an approximation, it is still useful in determining the limits of resolution in optical devices, including the eye.

Fig.11.52a Diffraction effects decrease as objects move closer or farther apart

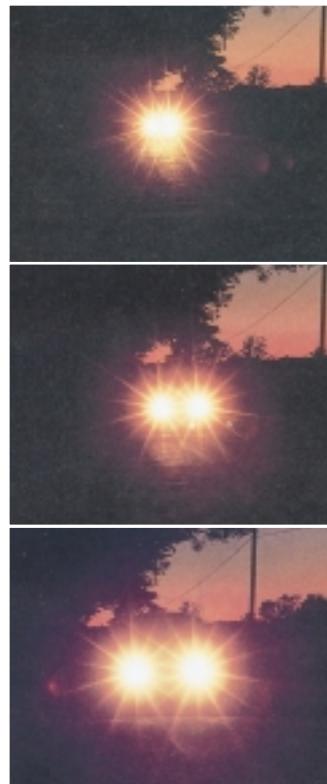
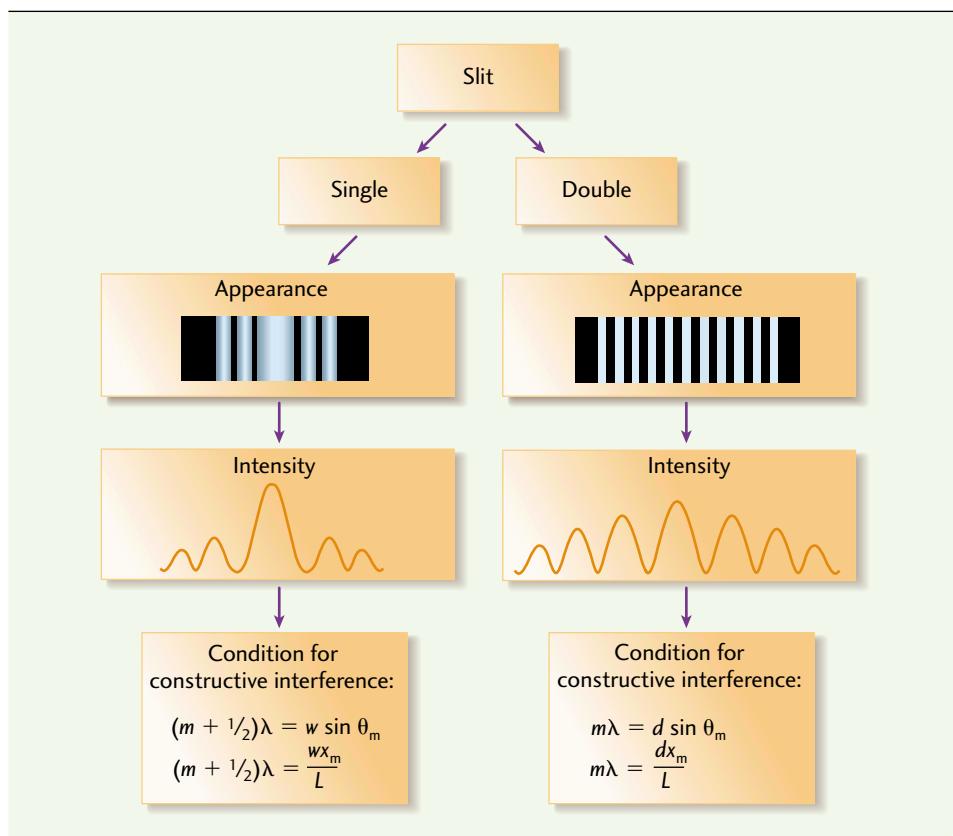
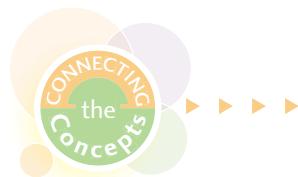


Fig.11.52b Image-processing software is used to remove diffraction effects from satellite images



Fig.11.53 summarizes the difference between single- and double-slit diffraction.

Fig.11.53 Comparison of Single- and Double-slit Patterns



1. A single slit of width 5.5×10^{-6} m is illuminated by light of wavelength 550 nm. If the screen is 1.10 m away, find
 - a) the angle of the second-order minimum.
 - b) the distance from the centre of the pattern to the second-order minimum.
2. For problem 1 above, what is the width of the central maximum in
 - a) centimetres?
 - b) degrees?
3. What is the spacing between consecutive maxima for the slit in problem 1?
4. What other factors can affect the resolvability between two objects?
5. In designing a telescope, use the words “largest,” “smallest,” “wavelength,” and “aperture” to describe the optimum conditions for resolvability.
6. a) If θ_R for the Hubble Telescope is 1×10^{-7} rad and its collector mirror is 2.4 m in diameter, what wavelength of light and what type of light does the telescope use?
 b) If two objects are 1.0 mm apart, how far away can you observe them using the value of the resolvability in part a) above?

11.9 The Diffraction Grating

By measuring spacings (distances between dark and light bands) in an interference or diffraction pattern, we have learned that we can calculate the wavelength of light. If we could sharpen the distinction between the light and dark areas in an interference pattern, we could measure the spacings between them more accurately. We do so by increasing the number of slits. The effect of shining light through 20 000 slits (in one centimetre!) produces sharper and more intense maxima. A comparison of double- and multiple-slit patterns is illustrated in Figure 11.54. The term used for an arrangement of multiple-spaced parallel slits is a **diffraction grating**.

Fig.11.54 A multiple-slit pattern is sharper than a double-slit pattern

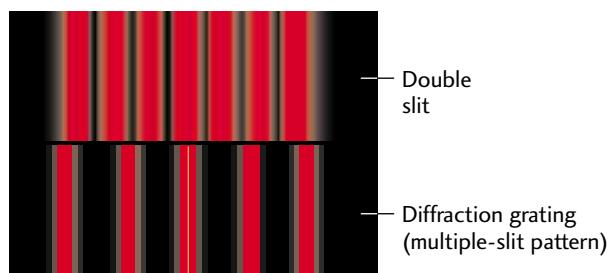


Fig.11.55 A CD acts like a reflection grating



Transmission gratings are the kinds of gratings referred to in this chapter. If patterns are formed by reflection from a series of ruled grooves (like off a CD), the grating is referred to as a **reflection grating**, shown in Figure 11.55.

The Diffraction-grating Equation

In Figure 11.54, the pattern produced by multiple slits is similar to that produced by a double slit. The diffraction grating is much like a composite of many double slits. Thus, it is no surprise that the diffraction-grating equation is the same as the double-slit equation,

$$m\lambda = d \sin \theta_m$$

where d represents the slit separation. To calculate the slit separation for a diffraction grating, we divide the grating width (w) by the total number of slits (N) to obtain

$$d = \frac{w}{N}$$

where d is the slit separation.

EXAMPLE 11

Finding the spacing between slits

For a given diffraction grating, there are 4500 slits in 3.6 cm. Find the slit separation.

Solution and Connection to Theory

Given

$$w = 3.6 \text{ cm} = 3.6 \times 10^{-2} \text{ m} \quad N = 4500 \quad d = ?$$

$$d = \frac{w}{N}$$

$$d = \frac{3.6 \times 10^{-2} \text{ m}}{4500}$$

$$d = 8.0 \times 10^{-6} \text{ m}$$

Note that the spacing approaches the wavelength of visible light.

The derivation of the equation $m\lambda = d \sin \theta_m$ for a diffraction grating is similar to that for double-slit interference. In Figure 11.56, the rays reaching a point P on a screen far away from the grating are approximately parallel. We pair up the slits and use the same logic as for the double-slit pattern. The pattern sharpens and brightens as more pairs of slits contribute to the maxima. The minima also become sharper because more pairs of rays cancel each other.

Fig.11.56 The diffraction-grating equation

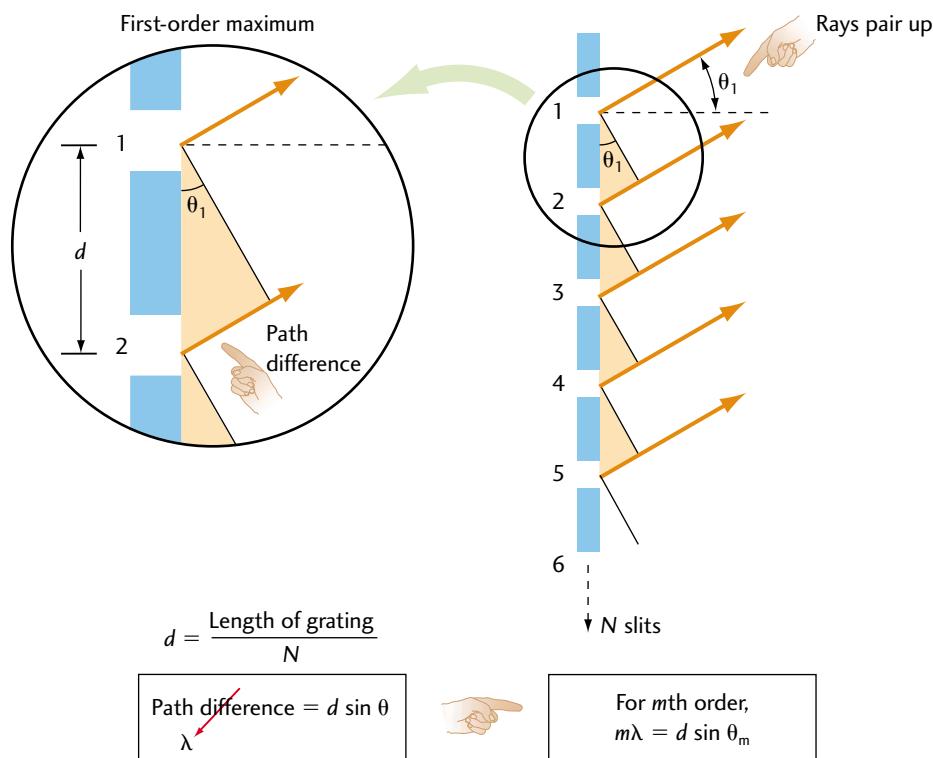
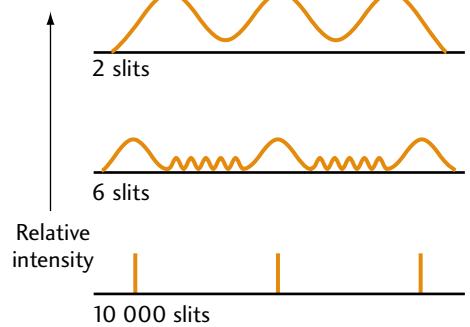


Fig.11.57 As we increase the number of slits from two, the intensity pattern becomes more complicated than the double-slit pattern until we get the sharp pattern produced by a diffraction grating



EXAMPLE 12 The diffraction grating

What are the angular positions of the first-order maxima for violet light (450 nm) and red light (650 nm) when using a diffraction grating with 5400 slits over 2.8 cm?

Solution and Connection to Theory

Given

$$\begin{aligned} \lambda_{\text{violet}} &= 4.50 \times 10^{-7} \text{ m} & \lambda_{\text{red}} &= 6.50 \times 10^{-7} \text{ m} & m &= 1 \\ w &= 2.8 \times 10^{-2} \text{ m} & N &= 5400 & d &=? & \theta_1 &=? \end{aligned}$$

First we find the slit separation, d :

$$d = \frac{w}{N}$$

$$d = \frac{2.8 \times 10^{-2} \text{ m}}{5400}$$

$$d = 5.2 \times 10^{-6} \text{ m}$$

For violet light,

$$m\lambda = d \sin \theta_m$$

For first-order maxima, $m = 1$; therefore,

$$\lambda = d \sin \theta_1$$

$$\sin \theta_1 = \frac{\lambda}{d}$$

$$\sin \theta_1 = \frac{4.50 \times 10^{-7} \text{ m}}{5.2 \times 10^{-6} \text{ m}}$$

$$\sin \theta_1 = 0.087$$

$$\text{Therefore, } \theta_1 = 5.0^\circ$$

For red light, we use the same equation as for violet light, so

$$\sin \theta_1 = \frac{6.50 \times 10^{-7} \text{ m}}{5.2 \times 10^{-6} \text{ m}}$$

$$\sin \theta_1 = 0.125$$

$$\text{Therefore, } \theta_1 = 7.2^\circ$$

EXAMPLE 13 More diffraction-grating calculations

Which maximum occurs closest to the central axis if the diffraction grating used has 12 678 lines in 2.40 cm: the second-order red (730 nm) maximum, the third-order violet (400 nm) maximum, or the second-order green (510 nm) maximum?

Solution and Connection to Theory

Given

$$N = 12\,678 \quad w = 2.40 \times 10^{-2} \text{ m} \quad \lambda_{\text{red}} = 7.30 \times 10^{-7} \text{ m}$$

$$\lambda_{\text{violet}} = 4.00 \times 10^{-7} \text{ m} \quad \lambda_{\text{green}} = 5.10 \times 10^{-7} \text{ m} \quad m_{\text{red}} = 2$$

$$m_{\text{violet}} = 3 \quad m_{\text{green}} = 2$$

First we find d , the spacing between the slits:

$$d = \frac{w}{N}$$

$$d = \frac{2.40 \times 10^{-2} \text{ m}}{12\,678}$$

$$d = 1.89 \times 10^{-6} \text{ m}$$

Now we can apply the equation $m\lambda = d \sin \theta_m$ to all three wavelengths of light and substitute the given values:

Red:

$$2\lambda = d \sin \theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{2\lambda}{d}\right)$$

$$\theta_2 = \sin^{-1} \frac{2(7.30 \times 10^{-7} \text{ m})}{1.89 \times 10^{-6} \text{ m}}$$

$$\theta_2 = \sin^{-1}(0.772)$$

$$\theta_2 = 50.6^\circ$$

Violet:

$$3\lambda = d \sin \theta_3$$

$$\theta_3 = \sin^{-1}\left(\frac{3\lambda}{d}\right)$$

$$\theta_3 = \sin^{-1}(0.635)$$

$$\theta_3 = 39.4^\circ$$

Green:

$$2\lambda = d \sin \theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{2\lambda}{d}\right)$$

$$\theta_2 = \sin^{-1}(0.540)$$

$$\theta_2 = 32.7^\circ$$

Since the angle of the green wavelength of light is the smallest, the green second-order maximum occurs closest to the centre.

Notice from the different angles in Example 13 that the spacing between bands is different for each wavelength. So, when white light is used to illuminate the grating, each spectral colour will appear in the pattern (see Figure 11.58a).

Fig.11.58a First- and second-order spectra of white light produced by a diffraction grating



If the composition of the light is a series of discrete frequencies, the resulting pattern is called a **line spectrum** (Figure 11.58b). If the range of frequencies is extensive, then the pattern is called a **continuous spectrum** (Figure 11.58c).

Fig.11.58b A line spectrum
(produced by a fluorescent lamp)

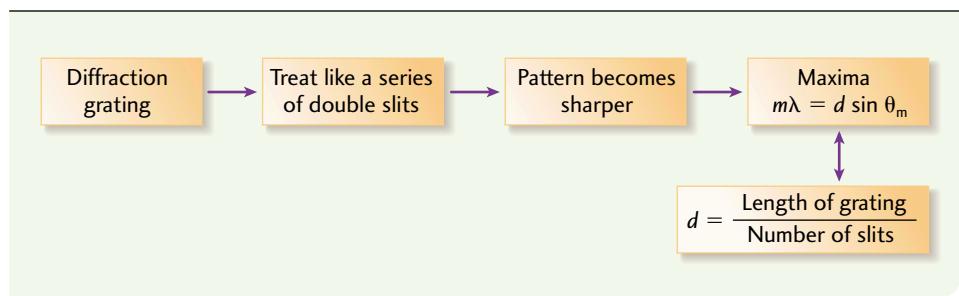


Fig.11.58c A continuous spectrum
(produced by a bright filament lamp)



Figure 11.59 summarizes the main concepts for a diffraction grating.

Fig.11.59 Diffraction Grating Summary



- Given a diffraction grating with 8500 slits in 2.2 cm, illuminated by monochromatic light of wavelength 530 nm, find the angles at which the first three maxima occur.
- For the diffraction grating in problem 1, find the maximum order number for the following wavelengths:
 - 650 nm
 - 550 nm
 - 450 nm
- If the second-order maximum occurs at 8.41° for red light of wavelength 614 nm,
 - what is the slit spacing?
 - how many slits are in the grating if it is 1.96 cm long?

11.10 Applications of Diffraction

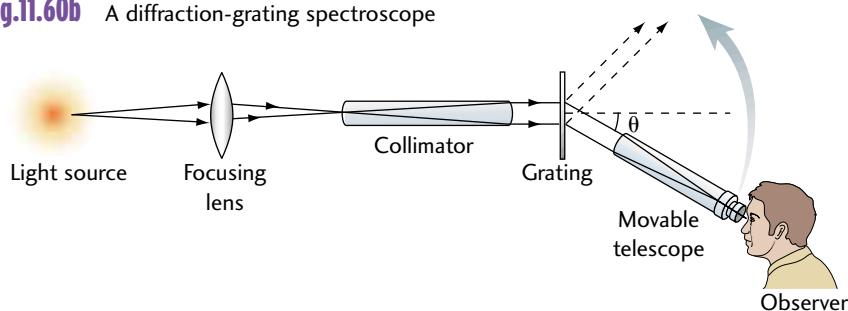
A Grating Spectroscope

Any source emitting light consists of a set of wavelengths making up that light. Different sources, such as incandescent bulbs, fluorescent lamps, fireflies, and the Sun, have different signature patterns or **spectra** (see Figure 11.58). Spectra can be viewed through a **grating spectroscope**, which separates the different wavelengths of emitted light. The patterns are characteristic of the specific processes and substances involved in producing the light from a particular source (Figures 11.60a and b).

Fig.11.60a A spectroscope is used to view a spectrum. If the spectrum from the source is recorded (on film), the device is called a spectrometer.



Fig.11.60b A diffraction-grating spectrograph



Extension: Resolution — What makes a good spectrometer?

The **resolution** of a diffraction grating is the minimum separation between adjacent spectra that the grating is able to distinguish. As the number of lines in a diffraction grating is increased, the maxima become sharper. The finer the maxima, the more accurate the measurement that can be obtained. If two adjacent lines in the spectrum are separated by a distance of $\Delta\lambda$ (i.e., λ and $\lambda + \Delta\lambda$), the resolution of the diffraction grating may be too low to separate the two lines.

The resolution of a grating can be calculated by using the following equation:

$$\frac{\Delta\lambda}{\lambda_{\text{avg}}} = \frac{1}{Nm}$$

where $\Delta\lambda$ is the difference in wavelength between the two lines, N is the number of slits in the diffraction grating, and m is the order number of the maxima. We now let $R = Nm$, where R is the **resolving power** of the diffraction grating. The equation then simplifies to

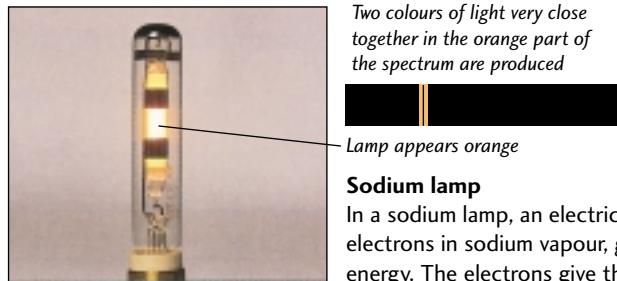
$$R = Nm = \frac{\lambda}{\Delta\lambda}$$

where λ represents λ_{avg} . Thus, the resolution of a grating is the inverse of its resolving power.

EXAMPLE 14 Sodium *d* lines

How many slits in a spectrometer's grating does it take to be able to resolve the sodium doublets, 589.00 nm and 589.59 nm, when viewing light from sodium in a flame?

Fig.11.61



Sodium lamp

In a sodium lamp, an electric current excites electrons in sodium vapour, giving them extra energy. The electrons give the energy out as light.

Solution and Connection to Theory

Given

$$\lambda_1 = 5.8900 \times 10^{-7} \text{ m} \quad \lambda_2 = 5.8959 \times 10^{-7} \text{ m}$$

$$\lambda_{\text{avg}} = \lambda = \frac{5.8900 \times 10^{-7} \text{ m} + 5.8959 \times 10^{-7} \text{ m}}{2} = 5.89295 \times 10^{-7} \text{ m}$$

We can substitute the given values into the equation for resolving power,

$$R = Nm = \frac{\lambda}{\Delta\lambda}$$

$$\frac{\lambda}{\Delta\lambda} = \frac{5.89295 \times 10^{-7} \text{ m}}{(5.8959 \times 10^{-7} \text{ m} - 5.8900 \times 10^{-7} \text{ m})}$$

$$\frac{\lambda}{\Delta\lambda} = 999 \text{ lines, or approximately } 10^3 \text{ lines.}$$

For the first-order maximum, $m = 1$; and since $Nm = R$, we obtain

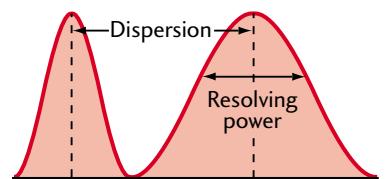
$$Nm = N(1) = 10^3 \text{ slits}$$

$$N = 1000 \text{ slits}$$

Therefore, we need a diffraction grating with 1000 slits.

The quality of a diffraction grating also depends on its ability to separate or spread out the spectral lines. The effect is called **dispersion**. It is different from the resolving power, which is a measure of the thicknesses of spectral lines. Dispersion depends on slit separation, while resolving power depends on the number of slits. Figure 11.62 illustrates the difference between dispersion and resolving power for two spectral lines. The ability of a spectrometer to create a clear spectrum depends on the number of slits in the grating and on the spacing between them.

Fig.11.62 Dispersion and resolving power



X-ray Diffraction

The patterns created by interference and diffraction of visible light may also be observed in other types of electromagnetic waves as long as the object or opening creating the pattern is of comparable size to the wavelength of the wave. For visible light, the openings must be in the range of 10^{-7} m . For x-rays, the openings must be in the range of 10^{-10} m . The spacing between layers of atoms in a regular crystalline structure (such as table salt) is in this range, so the crystal acts like a diffraction grating for x-rays.

Figure 11.64 shows a schematic diagram of two possible reflecting planes in a salt crystal (NaCl).

Fig.11.64 Two possible reflecting planes in the NaCl crystal

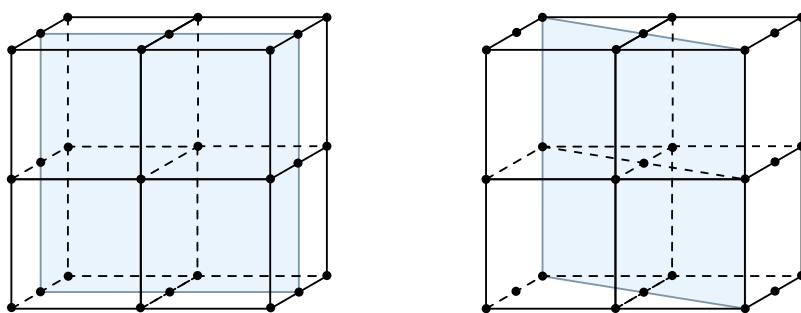


Fig.11.63 Production of x-rays

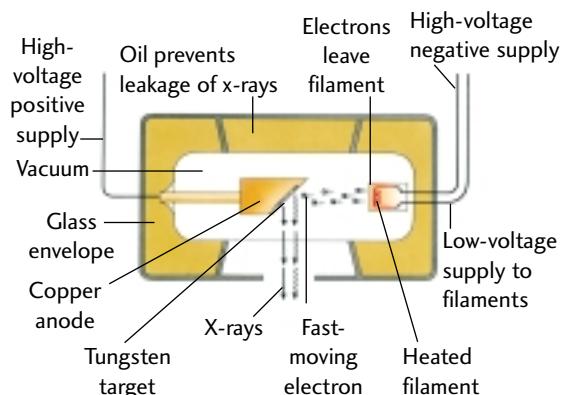
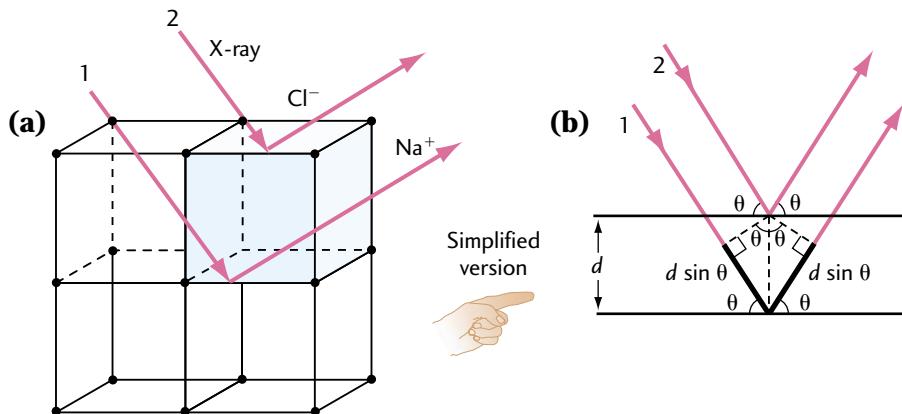


Figure 11.65a is a simplified diagram of how the salt crystal simulates a diffraction grating. The crystal planes don't actually reflect the x-rays, but the *net effect* produced on the x-rays through the crystal is similar to the diffraction grating and can be explained mathematically using this analogy.

Fig.11.65 X-ray diffraction through an NaCl crystal



In 1915, British physicist W.L. Bragg (along with his dad) received the Nobel Prize in physics for his work on the applications of x-rays in the study of crystalline structures.

Figure 11.65a illustrates one plane of the salt crystal. For the sake of simplicity, Figure 11.65b shows only two surfaces of the crystal. Ray 1 enters the crystal and reflects off the bottom surface. Ray 2 reflects off the upper surface. Notice that ray 1 has travelled an extra distance given by $2d \sin \theta$ (because the ray travels into and back out of the crystal). This effect is similar to thin-film interference (Section 11.6), where one ray of light reflects off the top surface of a thin film while the other ray travels an extra distance of $2t$, where t is the thickness of the film. For constructive interference, the two rays must be in phase (i.e., shifted by $m\lambda$, where m is a whole number) when they exit the film. Similarly for x-ray diffraction: for a maximum to occur, the path difference of the two x-rays travelling through the crystal equals $2d \sin \theta$ and their phase shift equals $m\lambda$. The expression

$$m\lambda = 2d \sin \theta$$

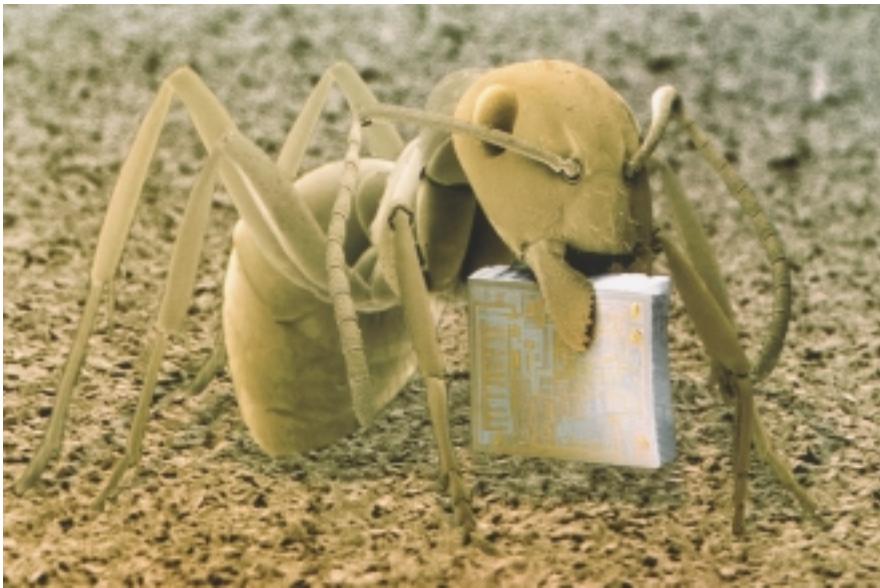
is known as **Bragg's law**.



1. Which spectrometer produces the best resolution: one with 3000 lines/cm or one with 20 000 lines in 20 cm?
2. a) Research the effect of spreading lines in a spectrum by a diffraction grating (dispersion). What factor(s) does it depend on?
b) Compare dispersion to the concept of resolving power.
c) Look up the equation for dispersion ($D = \frac{m}{d \cos \theta}$). Use this equation to find the dispersion for a grating with 10 000 slits and a slit separation of 2500 nm.
d) What does R equal for this grating?

3. For Example 13, find the sequence of colours up to the fourth order for each colour. What happens to the fourth-order green and red maxima?
4. Research the experimental setup for x-ray diffraction. Why is the crystal structure rotated during the experiment using a beam of continuous x-rays (more than one wavelength)? What are Laue spots?
5. X-rays are beamed at an NaCl crystal with a planar spacing of 2.5×10^{-10} m at an angle of 12° . What wavelength of x-rays will produce a pattern? Assume $m = 2$.
6. For problem 5, find two other possible angles at which diffraction can occur.
7. Research possible uses of x-ray diffraction in research and industry.
8. The electron microscope uses beams of electrons that behave like waves. (The wave-like behaviour of particles is covered in Chapter 12.) High-energy electrons have wavelengths 10^{-5} times the wavelength of light. Light microscopes, with a maximum magnification of about $\times 500$, are used to look at objects in the 250-nm range.

Fig.11.66 An ant seen through an electron microscope



Explain why using the electron microscope allows you to study smaller structures. Relate the reason to the size of the waves used by the instrument, the size of the object, and diffraction.



CD Technology

Fig.STSE.11.1a The cracked plastic surface of a CD reveals the musical layer beneath (magnified $\times 1000$)

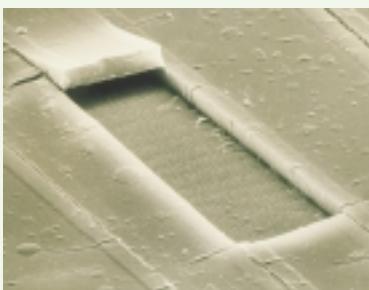


Fig.STSE.11.1b A pit on a CD

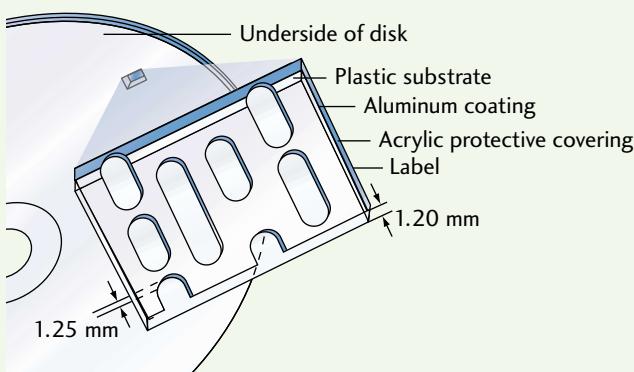
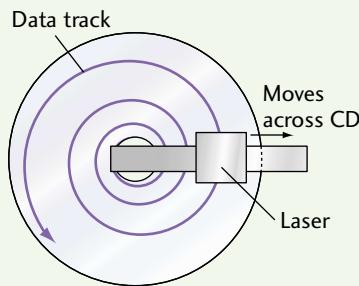


Fig.STSE.11.2 The spiral data track on a CD



We have come a long way in storing information: from analog recordings on vinyl records, to bits and bytes stored in computers the size of large rooms, to our current micro-technology. The CD (compact disc) has changed the way we store information.

A CD is made of a plastic substrate that has a series of pits and flats burned into it. The substrate is covered by a thin aluminum layer, which is then covered by a protective plastic coating. When viewed from the underside (the topside is the side with the label), the pits of the disc appear as bumps to the laser reading the disc (Figure STSE.11.1).

In general, the bump or pit is only about $0.5 \mu\text{m}$ wide and $0.83 \mu\text{m}$ long. The series of pits and flats represents zeros and ones (off and on) in binary code. They are grouped together to form bytes of information. A typical 12-cm CD holds 783 megabytes! The pits and flats are burned into the CD in a spiral arrangement (Figure STSE.11.2). The plane spiral of data on the CD winds from inside to outside. If we could stretch it out in a straight line, it would be about 5 km long!

As the disc spins, a laser scans the underside of the disc, running along the radius of the disc from centre to edge (Figure STSE.11.3). In Chapter 7, we learned that as an object's radius of rotation increases, its tangential velocity increases. Therefore, the pits and flats on the outside edge of the disc move by the laser faster than the pits and flats along the disc's inside edge. Typically, the disc spins at 200–500 revolutions per minute (rpm). In order to keep the data collection rate of the laser constant, the speed of the rotating disc is slowed as the laser scans across the disc, away from the centre.

Reading the Disc

The laser light reflects off the disc into a light-sensitive photodiode. When reflecting from a pit, the path difference is such that the light shifts $\frac{\lambda}{2}$ in the coating. As a result, destructive interference occurs between this ray and the ray reflecting from the flat, and the signal is weakened. When the laser reflects off a flat, the signal is stronger. The net effect is a series of fluctuations in intensity of laser light as it passes over the pits and flats. The photodiode converts the amplitude fluctuations to electrical signals, thus generating the on/off signals, which are sent on to amplifiers and into a computer processor.

Design a Study of Societal Impact

Is privacy important?

Because of today's amazing capabilities of collecting and storing data, various organizations, such as government departments, employers, and commercial companies, have access to more and more information about their constituents. Research the various methods of data collection: contest applications, government questionnaires, Internet use, phone technology, spy satellites, etc. How has access to personal information about us affected business, marketing, and international trade?

Information technology has opened up many new job possibilities. Research what they are.

Design an Activity to Evaluate

Tracking System: $1.6 \mu\text{m}$ separates the data tracks. Investigate how a three-beam tracking system works using the flats between the data tracks.

Optical System: In many CD systems, the laser light is polarized. Find out why. Draw a schematic arrangement of a playback system or build a model of it.

Storage Devices: Compare the old flexible floppy disc to the current CD-ROM and hard-drive methods of storing and retrieving data, as well as data storage capacities and speed of retrieval.

Build a Structure

Research the physics of a photodiode. Use a set of photodiodes and a laser to send messages in binary code. Use the voltage drop in the diode to represent ones (on) and zeros (off).

Investigate the wave nature of radio waves. Use a radio-wave generator and two dipole antennae hooked up in parallel to generate two waves. Use a radio receiver and antennae to investigate maxima and minima, depending on the position of the receiving antennae and the separation of the transmitting antennae. Extend the experiment to obtain single- and double-slit patterns by placing metal sheets with openings cut in them in front of the sources (transmitting antennae). You will need to amplify the signals from the receiving antennae.

Use a laser, a viewing scope, and beam splitters to construct a simple interferometer.

Fig.STSE.11.3 A laser scans the CD from centre to edge as the disc spins

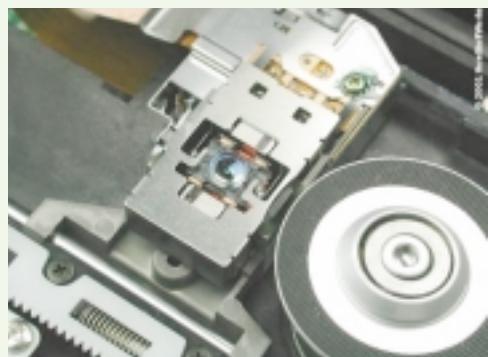
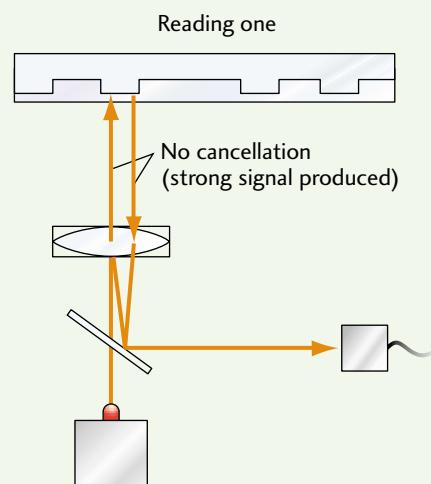
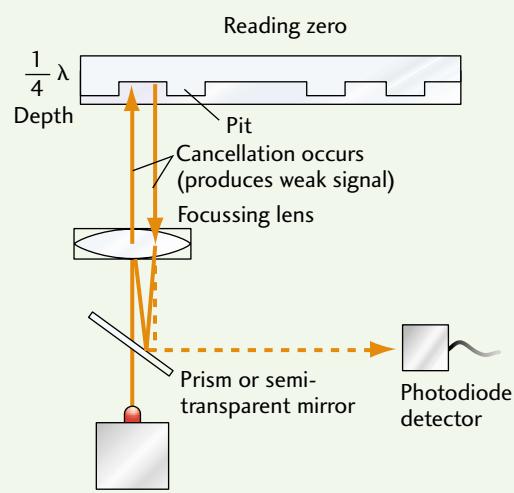


Fig.STSE.11.4 Reading binary code from a CD



SUMMARY | SPECIFIC EXPECTATIONS

You should be able to

Understand Basic Concepts:

- Define and explain the concepts and units related to the wave nature of light.
- Explain diffraction and interference in general wave terms.
- Use diffraction and interference to further develop the wave model of light.
- Explain how Young's experiments furthered the wave model of light.
- Describe wave interference of light in qualitative and quantitative terms using diagrams and equations.
- Explain the reasoning behind the interference equations.
- Describe and explain wave diffraction of light in quantitative terms using diagrams.
- Explain the reasoning behind the diffraction equations.
- Describe resolving power.

Develop Skills of Inquiry and Communication:

- Use interference and diffraction to develop the theoretical basis for light behaving like a wave.
- Make predictions based on the wave model of light about what to expect in experiments involving diffraction and interference.
- Predict diffraction and interference patterns produced in ripple tanks based on the wave model of light.
- Predict the effect of shining a laser onto a fine structure such as a human hair or razor edge.
- Identify and compare patterns produced by light passing through a single slit, a double slit, and a diffraction grating.
- Analyze quantitatively aspects of single-slit, double-slit, and diffraction-grating patterns.

- Describe the consequences of the Rayleigh criterion.
- Compare the dispersion of light by a grating and a prism.
- Explain how soap film colours are another example of the wave nature of light.
- Predict whether a colour or dark band will appear when viewing a thin film.
- Analyze and interpret experimental evidence indicating that light has characteristics and properties that are similar to those of mechanical waves and sound.
- Conduct experiments to test aspects of single-slit, double-slit, and diffraction-grating interference patterns.
- Develop extensions to Lab 11.1–Lab 11.4 and devise other labs to test the aspects of interference in thin films as well as single slits, double slits, and diffraction gratings.

Relating Science to Technology, Society, and the Environment:

- Analyze phenomena involving light and colour, and explain how the wave model of light provided a basis for the development of various technological devices.
- Describe how changes in scientific theories led to the development of devices such as the electron microscope, the x-ray spectrometer, and various types of interferometers.
- Use the principles of colour separation by way of diffraction gratings and thin films to explain instruments based on these principles (such as lens coatings and spectrometers).
- Describe the technology behind holograms.
- Describe how information is stored and retrieved using compact discs and laser beams.

Equations

Double Slit and Diffraction Grating

$$m\lambda = |P_m S_2 - P_m S_1| \text{ (constructive interference)}$$

$$m\lambda = d \sin \theta_m$$

$$m\lambda = \frac{dx_m}{L}$$

$$(m + \frac{1}{2}\lambda) \text{ (destructive interference)}$$

$$\lambda = \frac{d\Delta x}{L}$$

$$R = Nm = \frac{\lambda}{\Delta\lambda}$$

$$\lambda_m = \frac{\lambda}{n_m}$$

$$\Delta PD = \left(\frac{2t}{\lambda}\right)(n_m - 1)$$

Single Slit

$$m\lambda = w \sin \theta_m \text{ (destructive interference)}$$

$$m\lambda = \frac{wx_m}{L}$$

$$(m + \frac{1}{2}\lambda) \text{ (constructive interference)}$$

$$\lambda = \frac{w\Delta x}{L}$$

$$\theta_R = \frac{1.22\lambda}{d}$$

$$m\lambda = 2d \sin \theta$$

Conceptual Questions

1. Which aspects of light indicate that it is a wave?
2. Which aspects of the wave nature of light can and cannot be demonstrated using water waves?
3. Why do the colours of a soap bubble floating in the air or the soap film on a child's plastic hoop change continually?
4. A gasoline spill on water starts to evaporate. How does its evaporation affect which colours are seen?
5. Why do glass camera lenses appear a certain colour when viewed in sunlight, but the windshield of your car doesn't?
6. Young's wave model of light seemed to explain the nature of light better than Newton's particle model, yet Newton's model was still accepted. Henry Brougham, a British politician and amateur scientist, severely criticized Young and his results in the *Edinburgh Review* in 1803. He was quoted as saying: "We wish to raise our feeble voice against innovations that can have no other effect than to check the progress of science." Even though some of Newton's ideas were incorrect, the majority of scientists believed his theories.
 - a) Research Newton's ideas on the nature of light and compare them to Young's. Include Fresnel's (1818), Foucault's (1850), and Fizeau's (1850) contributions to the argument of whether light is a wave or a particle.
 - b) Discuss why innovation sometimes scares people.
 - c) How can old accepted ideas and laws hinder innovation and knowledge breakthroughs?
7. How can a dominating person affect scientific progress? Research other examples of this occurrence.
8. Do headlights from a car form interference patterns? Why?
9. Air wedges require glass plates that are optically flat. Why?
10. Why can you hear but not see around corners?
11. In the distance, you see a single headlight. As it approaches, you realize it's two headlights. Explain.
12. Can an object be resolved further by using a magnifying glass if it has reached the resolution limit set by diffraction?
13. Compare the grating spectroscope to the prism spectroscope.
14. What is the difference between continuous spectra and line spectra? Give an example of each.
15. A hologram can be cut into smaller pieces. Each piece produces a complete holographic image. Why can't you do the same with a normal photograph?
16. How is the diffraction grating similar to an interference grating?
17. What is the benefit to having a grating with close spacing?
18. If you have researched resolving power, explain why gratings have large numbers of slits.
19. Is there a limit to increasing the number of slits in a diffraction grating to produce a better spectrometer?
20. Describe the relationship between the relative intensities of fringes through a diffraction grating and a single-slit pattern.
21. You draw a duck on a page using dots instead of lines. As you move back to admire your work of art, the dots become indistinguishable and create a beautiful "continuous" normal sketch. Why? Assume the pupil of the eye is circular.

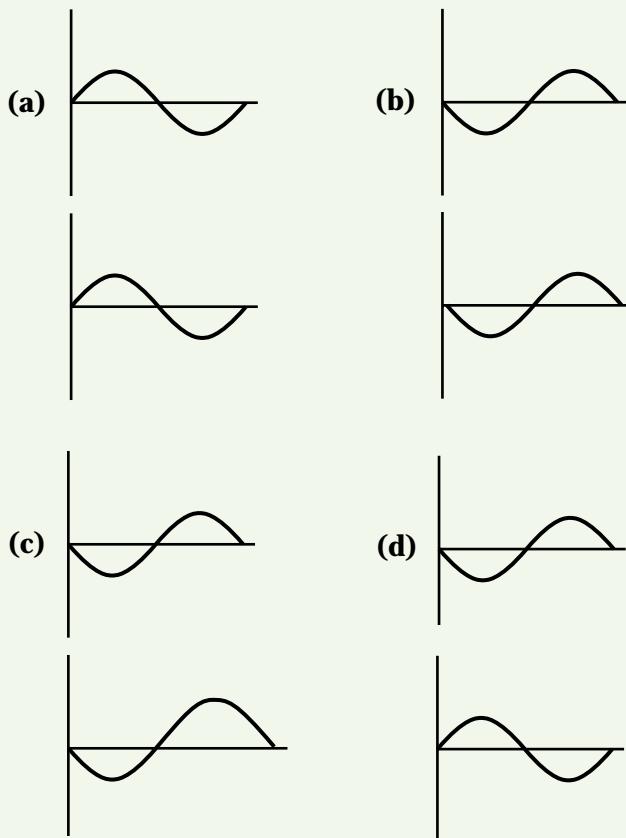
- 22.** Why is the electron microscope so much better at magnifying small objects than the conventional optical microscope?

Problems

11.2 Interference Theory

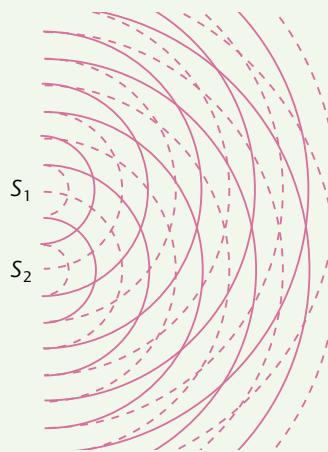
- 23.** For the wave pairs in Figure 11.67, determine if the interference is constructive, destructive, or partial.

Fig.11.67



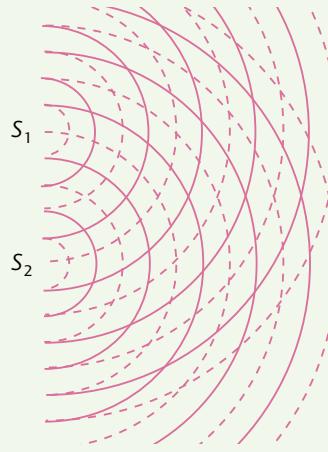
- 24.** Sketch the two-dimensional pattern in Figure 11.68 in your notebook. Draw in the maxima and label the order numbers.

Fig.11.68



- 25.** Sketch the two-dimensional pattern in Figure 11.69 in your notebook. Draw in a series of minima. Draw in the nodal lines and label the order numbers.

Fig.11.69

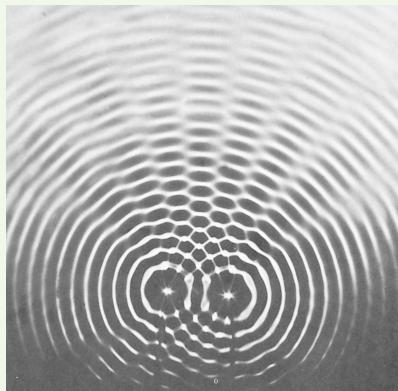


11.4 Young's Double-slit Equation

- 26.** Calculate the angle of the second-order maximum for monochromatic light of wavelength 550 nm if it illuminates
- a double slit with a slit separation of 2.0×10^{-6} m.
 - a diffraction grating with 10 500 slits in 1.0 m.
- 27.** For problem 26, given that the screen is 1.0 m away from the slits, find the distance of the second-order maximum from the centre.

- 28.** Sketch the wave interference pattern in Figure 11.70 into your notebook. Label the nodal lines and maxima with the appropriate order number. Use measurements from the figure to calculate the wavelength.

Fig.11.70



- 29.** In Young's double-slit experiment, a monochromatic source of wavelength 560 nm illuminates slits that are 4.5×10^{-6} m apart. Find
- the angle at which the first-order maximum occurs.
 - the angle at which the first-order minimum occurs.
 - the angle at which the third-order maximum occurs.
 - the angle at which the third-order minimum occurs.
- 30.** For light of wavelength 610 nm, hitting a double slit, the second-order maximum occurs at 23° . What is the slit separation?
- 31.** Two slits are 0.15 mm apart, the second-order maximum is 7.7 m away from the centre line, and the screen is 1.2 m away. What is the wavelength of light used?
- 32.** In an interference experiment, yellow light of wavelength 585 nm illuminates a double slit. If the screen is 1.25 m away and the distance between the centre line and the ninth-order dark spot is 3.0 cm, find the slit separation.

- 33.** What is the maximum order number possible for red light (630 nm) illuminating a double slit with separation 3.0×10^{-5} m?

11.5 Interferometers

- 34.** Refer to the interferometer diagram in Figure 11.20. Explain what happens to the observed pattern if M_2 is shifted back

- $\frac{\lambda}{4}$.
- $\frac{\lambda}{2}$.
- $\frac{3\lambda}{4}$.
- λ .

- 35.** A shift of four bright bands occurs when a material of refractive index 1.42 is inserted into an interferometer. Find the thickness of the material if the pattern is created by using light of wavelength 600 nm.

- 36.** What is the refractive index of a material that causes a shift of 12 bright bands if the thickness of the material is 3.60 microns and the wavelength of light used is 640 nm (1 micron = 10^{-6} m)?

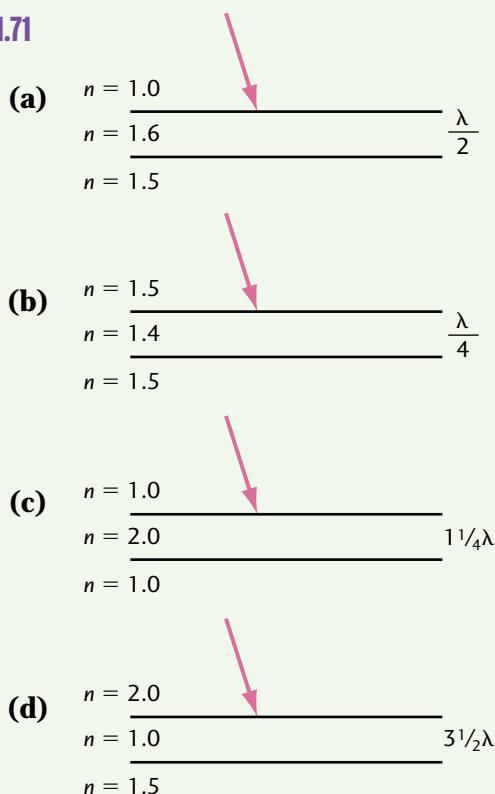
- 37.** Calculate the wavelength of light used in an interferometer if 10 bright bands shift for a material in which the speed of light is 1.54×10^8 m/s and the thickness is 2.80 microns.

11.6 Thin-film Interference

- 38.** A thin film of gasoline on water, with a thickness of 364 nm, is illuminated by light of wavelength 510 nm. If the refractive index of gas is 1.40 and that of water is 1.33, will constructive or destructive interference occur for light falling perpendicular to this surface?

- 39.** For the thin-film reflections in Figure 11.71, state whether the interference is constructive or destructive.

Fig. 11.71



- 40.** Light reflects off a thin film of gas ($n = 1.40$) on water ($n = 1.33$). If the wavelength of light is 560 nm and the thickness of the film is 4.80×10^{-6} m, will a bright or dark area result?

- 41.** For light of wavelength 500 nm, what is the minimum thickness of a film that will produce a maximum if the refractive index of the film is
a) 1.44?
b) 1.23?

Assume the film is on top of water ($n = 1.33$).

- 42.** Light of wavelength 580 nm strikes a soap film ($n = 1.33$), which is surrounded by air. What is the minimum thickness needed to produce
a) a dark spot?
b) a bright spot?

11.7 Diffraction

- 43.** Calculate the wavelengths for
a) sound travelling at 350 m/s with frequency 250 Hz.
b) light travelling at 2.50×10^8 m/s with a frequency of 4.81×10^{14} Hz.
c) radio waves travelling at c with a frequency of 1.20×10^8 Hz.
- 44.** Calculate the frequency of
a) gamma rays with a wavelength of 2.0×10^{-12} m.
b) water waves moving at 14 km/h with a crest-to-crest separation of 1.2 m.
- 45.** In your notebook, draw apertures of 0.5 cm, 1.0 cm, 2.0 cm, and 5.0 cm. For each aperture, draw approaching plane waves of wavelength 1.5 cm. Sketch the possible diffraction pattern through the aperture.
- ## 11.8 Single-slit Diffraction
- 46.** **a)** Calculate the angle of the second-order maximum for monochromatic light of wavelength 580 nm if it illuminates a single slit of width 2.2×10^{-5} m.
b) Calculate the angle of the second-order minimum for monochromatic light of wavelength 550 nm if it illuminates a single slit of width 2.2×10^{-5} m.
- 47.** Monochromatic light illuminates a single slit of width 1.2×10^{-2} mm. If the first-order minimum occurs at 4° , what is the wavelength of light used?
- 48.** For problem 47, given that the screen is 1.0 m away from the slit, find the distances of the second-order minimum and maximum from the centre of the pattern.
- 49.** For a single slit of width 1.1×10^{-5} m illuminated by red light of wavelength 620 nm, find the angle at which
a) the second-order minimum occurs.
b) the second-order maximum occurs.

- 50.** For a single-slit pattern, the width of the central maximum is 6.6° . Given that violet light of wavelength 400 nm was used, find the width of the slit.
- 51.** Light of angular wavelength 585 nm passes through a slit of width 1.23×10^{-3} cm. Given that the screen is 1.2 m away, calculate the position relative to the centre line of
a) the third-order minimum.
b) the second-order maximum.
- 52.** What is the angular width of the central maximum produced by a single slit of width 1.10×10^{-3} cm if illuminated by blue light of wavelength 470 nm?
- 53.** Light of wavelength 493 nm shines on a single opening 5.65×10^{-4} m wide. If the screen is 3.5 m away and the first nodal line is 3.1 mm from the centre of the pattern, find the width of the central maximum
a) in millimetres.
b) in degrees.
- 54.** What is the minimum slit width at which no interference pattern occurs for light of wavelength 450 nm?
- 11.9 The Diffraction Grating**
- 55.** Green light of wavelength 530 nm is beamed at a diffraction grating with 10 000 slits per centimetre. Find the angle at which the first-order maximum occurs.
- 56.** Red light of wavelength 650 nm is beamed at a diffraction grating with 2000 slits per cm. Find the order number of the nodal line occurring at 11.25° .
- 57.** What is the distance to the second-order maximum for a diffraction grating with 2.3×10^4 slits/mm if the screen is 0.95 m away and orange light of wavelength 610 nm is used?
- 58.** For a diffraction grating with 10 000 slits in 1.2 cm, calculate the maximum order number for
a) red light (600 nm).
b) violet light (440 nm).
- 59.** For a diffraction grating of 1000 slits/cm, how many orders of the entire spectrum are produced for wavelengths in the range of 400 nm–700 nm?
- 60.** For a diffraction grating with a slit separation of 1.0 microns, what is the maximum order number possible for
a) red light (610 nm)?
b) yellow light (575 nm)?
c) violet light (430 nm)?

11.10 Applications of Diffraction

- 61.** If sodium *d* lines are 589.00 nm and 589.59 nm, what is their angular separation for a grating spectrometer with 10^4 slits in 2.5 cm?
- 62.** How many slits are required to resolve the sodium doublet for the second-order number?
- 63.** How many orders of green light are visible in a spectrometer with 10^6 slits in 2.5 cm? ($\lambda_{\text{green}} = 520$ nm)
- 64.** When observing a gas mixture of hydrogen and deuterium through a grating with 4000 slits, will a red first-order doublet (656.30 nm and 656.48 nm) be resolved?
- 65.** X-rays of wavelength 0.55 nm illuminate a grating with 2.5×10^6 slits/m. At what angle will the first-order maximum occur? Is diffraction apparent?
- 66.** Given a crystal with spacing 0.40 nm, at what angle will a beam of 0.20-nm x-rays produce a third-order maximum?



Analyzing Wave Characteristics using Ripple Tanks

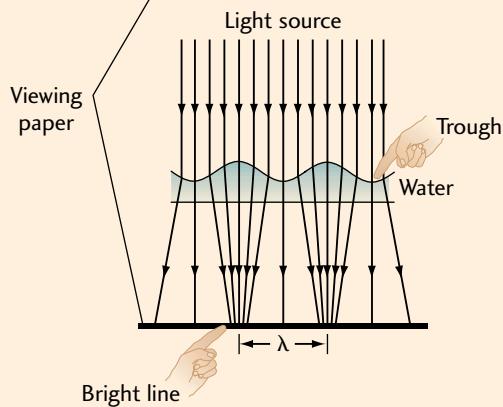
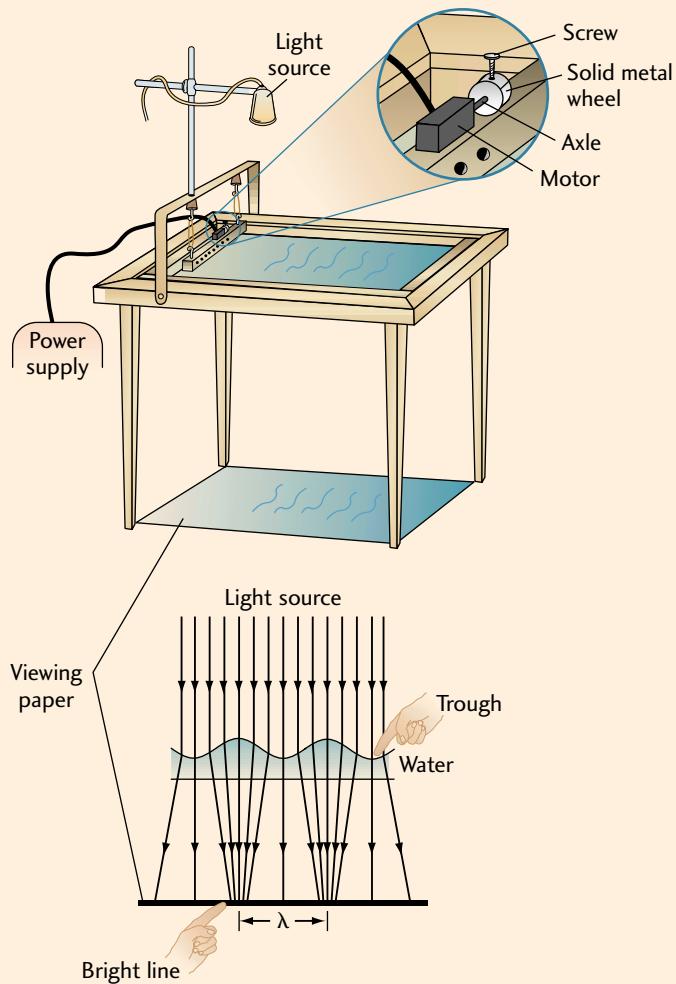
Purpose

To investigate characteristics of diffraction, refraction, and interference

Equipment

Ripple tank apparatus
Hand strobe
Wood or wax barriers
Glass or clear plastic plates

Fig. Lab.11.1



Procedure A:

Measuring Frequency

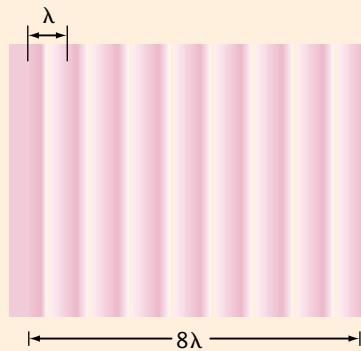
- Set up the ripple tank such that plane waves are generated. Use lower frequencies (about 5 Hz).
- Look at the waves through the hand strobe while rotating the hand strobe. Practise turning the hand strobe at a speed such that the waves in the tank appear to stop.
- When you are comfortable using the hand strobe, have a partner time 10 rotations of the hand strobe. The frequency of the waves is then (number of slots \times 10 turns)/time.

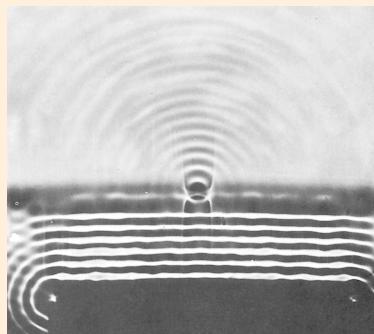
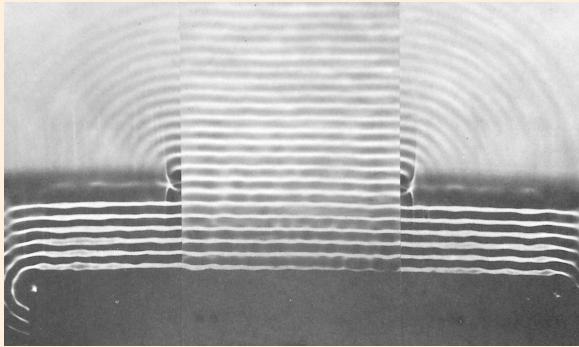
Measuring Wavelength

- Place two straight edges on the viewed pattern, roughly in line with the wavefronts.
- Look through the hand strobe and turn it at a rate such that the waves appear to stop.
- While viewing the "stopped" waves through the hand strobe, direct a group member to move the rulers such that they each line up with a bright wavefront. Count the number of wavelengths between the rulers. (If you see three bright bands, then you have two wavelengths. See Figure Lab.11.2.)
- Measure the distance between rulers. Use the number of wavelengths you counted to find the observed wavelength. Apply the equation

$$\frac{\text{distance between rulers}}{\text{number of wavelengths}} = \lambda.$$

Fig. Lab.11.2



Procedure B: Diffraction**Fig.Lab.11.3a****Fig.Lab.11.3b**

1. From your knowledge of the wave properties of light, predict what will happen to the interference pattern as the gap between the barriers is altered (see Figures Lab.11.3a and b).
2. Measure the observed wavelength as per Procedure A: Measuring Wavelength to get an idea of the wavelength of the water wave.
3. Adjust two barriers so that they are far apart; that is, much greater than the wavelength.
4. Move the barriers closer and observe the diffraction pattern. Sketch the amount and location of the bending of the waves. Record an estimate of the ratio of the size of the opening to the wavelength.
5. Remove one of the barriers and adjust the frequency of the waves from large to small wavelengths. Observe the amount of bending occurring at the corner of the barrier. Sketch the diffraction patterns. Record an estimate of the ratio of the wavelength to the barrier width.

Procedure C: Interference**Fig.Lab.11.4**

1. Set up the ripple tank so that two point sources are in the water (see Figure Lab.11.4). Practise generating interference patterns. Observe the relationship between frequency and number of maxima and minima.
2. The three equations you derived in Section 11.4 are $m\lambda = \text{path difference}$, $m\lambda = d \sin \theta_m$, and $m\lambda = \frac{dx_m}{L}$. In this lab, you will calculate wavelength using these equations by first choosing a point on a maximum near the wave sources and then a point on the maximum far from the wave sources. Predict and explain which point will produce better results.
3. For a pattern with at least three visible orders of maxima, sketch the pattern seen on the viewing paper. Make sure you locate the sources. You can draw a line down the centre of each maximum.
4. Measure the wavelength of the waves using Procedure A: Measuring Wavelength.

Procedure D: Refraction (optional)**Fig.Lab.11.5**

- In this experiment, the different depths of water represent different refractive indices. Thus, the water waves will adjust as they pass from one depth to another (see Figure Lab.11.5). Predict what happens to the wavelength, frequency, and speed of the wave as it passes from a deep end to a shallow end.
- Set up a depth boundary using a plastic plate. Place the plate at an angle to the plane wave generator. Make sure there is a minimum amount of water on the plate (so little that some dry spots may appear).
- Adjust the frequency until you see a bending occurring at the boundary.
- Using the hand strobe and Procedure A: Measuring Wavelength, sketch the boundary and the set of wavefronts on each side of the boundary.
- Measure the frequency of the waves on each side of the boundary using Procedure A: Measuring Frequency.

Data and Analysis B: Diffraction

- Organize your observations so that you can see the trend in the amount of diffraction compared to the ratio opening/wavelength.
- For the barrier observations, use the ratio width of barrier/wavelength. (Unless you have actual measurements, you can give estimates: much greater than 1, greater than 1, about 1, less than 1.)

Data and Analysis C: Interference

- Mark a point near the sources and a point far from the sources on one of the maxima.
- Calculate the wavelength from the near point using the three equations. Make appropriate measuring diagrams for each equation.
- Repeat step 2 for the far point.
- Calculate percent deviations of the wavelengths you measured.
- Average the results for each set of three calculations.

Data and Analysis D: Refraction (optional)

- Record the frequency, wavelength, and angle relative to the boundary for deep and shallow water.

- Calculate the ratios of the sines of the angles ($\frac{\sin \theta_{\text{shallow}}}{\sin \theta_{\text{deep}}}$), the speeds ($\frac{v_{\text{shallow}}}{v_{\text{deep}}}$), and the wavelengths ($\frac{\lambda_{\text{shallow}}}{\lambda_{\text{deep}}}$).

Discussion B: Diffraction

- In theory, what are the necessary conditions for producing maximum diffraction?
- Did your results agree with the theoretical expectations? If not, find possible reasons and repeat the procedure if necessary.
- Relate your results to the diffraction of light. Think of everyday examples when light behaves this way.

Discussion C: Interference

- In theory, which of the three equations used should be the best? Explain why.
- In theory, which point should produce more consistent results? Explain why.
- Do your percent deviations confirm the aspects discussed in questions 1 and 2 above?
- Relate your observations of water waves to patterns that light produces when shone through two slits.

Extension

Repeat the calculations using a minimum instead of a maximum.

Discussion D: Refraction (optional)

- What is the theoretical value of the ratios you have calculated?
- Do your results corroborate the theoretical ratios? If not, why not?
- By analogy to light, which depth of water represents the less optically dense medium? Relate the speeds in each depth to light passing from one medium to another.
- Superimpose a ray representation for refraction on your sketch.

Conclusion

Summarize your results in terms of the wave theory of light.

Qualitative Observations of the Properties of Light

Purpose

To study the qualitative aspects of the diffraction and interference of light

Equipment

Laser
Showcase lamp
Colour filters
Biconcave and biconvex lenses
Variable widths of single slits, double slits, diffraction gratings (if available; if not, use a painted (opaque) glass slide and 2 razor blades)
Wire loop
Soap solution

Procedure to Make Slits

If commercial slits are not available, use the razor blade to make a single stroke to cut the painted slide. To make a double slit, put the blades together and tape them together. In a single stroke, run the pair of blades down a painted slide. **Note: Use extreme caution when handling the razor blades.** You will be able to do the single- and double-slit parts of this lab.

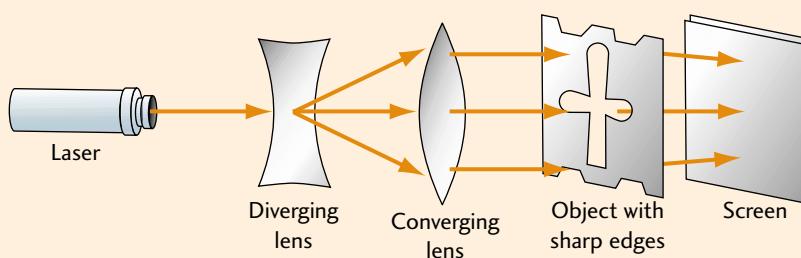
Procedure A: Single-slit Diffraction

- Shine a laser through the single slit. Observe and sketch the pattern. Note the number and intensity of the maxima.
- Use different widths of slits to observe what happens to the pattern. Correlate the width of the slit to the number and spacing of the maxima observed.
- Repeat steps 1 and 2 using a showcase lamp.
- Repeat steps 1 and 2 using various coloured filters in front of the slits.

Procedure B: Double-slit Diffraction

Repeat Procedure A using a double slit.

Fig. Lab.11.6



Procedure C: Diffraction Gratings

Repeat Procedure A using diffraction gratings.

Procedure D: Observing Diffraction along Edges of Obstacles

Arrange a convex lens, a concave lens, a razor blade or a strand of hair, and a screen in a row, as shown in Figure Lab.11.6. Shine a laser through this series of objects to the screen. Adjust the positions of all the elements to maximize the effect. Describe what you see.

Procedure E: Observing Thin Films

- Darken the room.
- Dip the wire loop into the soap solution. If the bubbles do not last long, add a touch of glycerine to the solution. Hold the loop in a vertical position.
- Use the laser or the showcase lamp plus a colour filter to illuminate the soap film. Observe light reflected from the soap film as it drains.
- Repeat step 3 using the showcase lamp without any filters.

Procedure F: Poisson's Bright Spot

- Paste the edge of a small opaque disk onto a drinking straw or any kind of holder (looks like a lollipop now).
- In a very dark room, shine a laser through the concave–convex lens arrangement (Figure Lab.11.6) onto the disk with a screen behind it.
- Observe the shadow region carefully, especially in the centre. If you have a photodiode, use it to scan the shadow region.

Observations

For each section, create a summary chart indicating what you studied, what you expected, and what you saw.

Discussion

1. How does the width of the single slit affect the number and spacing of the maxima in the interference pattern?
2. How does the separation of the two slits affect the number and spacing of the maxima in the interference pattern?
3. How does the diffraction grating spacing affect the number and spacing of the maxima in the interference pattern?
4. Compare the three patterns in terms of general shape, clarity, and intensity.
5. Why do colours appear in the pattern produced by the diffraction grating?
6. Which colour has the greatest spacing? Why?
7. What causes the lines to appear around the hair or other object illuminated by the laser? Why don't we normally see these lines?
8. What happens to the pattern in the soap over time? What is causing this pattern?
9. Describe what causes a maximum (light band) to become a minimum (dark band) in the pattern.
12. Why is there a bright spot on the screen behind the opaque disc?
13. How does each of these experiments show that light behaves like a wave?

Conclusions

Summarize your arguments supporting the wave theory of light.



Comparison of Light, Sound, and Mechanical Waves

Complete a chart with various aspects of wave theory described in general terms. For each aspect, find an experiment or observation that demonstrates this feature for each type of wave (light, sound, and mechanical). For example, in earlier labs, you demonstrated interference between two sources using light waves and mechanical waves. Now come up with a demonstration for the interference of sound waves.

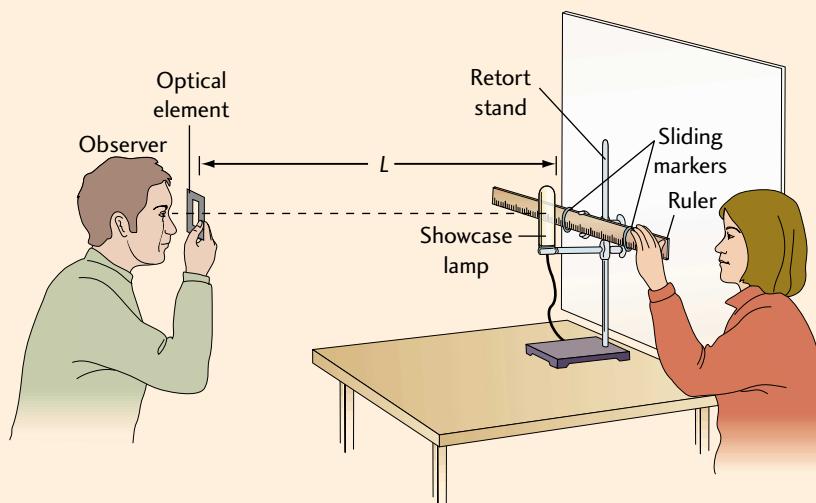
The lab write-up is essentially a summary chart. If a wave type doesn't exhibit certain characteristics or you can't demonstrate them using school resources, then state the reasons why.

Possible categories for your chart: Refraction, two-source interference, single-slit diffraction, diffraction grating, polarization, Doppler effect.



Finding the Wavelength of Light using Single Slits, Double Slits, and Diffraction Gratings

Fig. Lab.11.7



Purpose

To study the wavelength of light using single slits, double slits, and diffraction gratings

Equipment

Single slit, double slit, diffraction grating (with known values of w and d)

Showcase bulb plus red filter

Screen

String

Metre stick clamped horizontally onto a retort stand

2 position markers (paper or paperclips, etc.)

Procedure

- Working in groups of four, hold one of the optical instruments (slit or grating) while standing 1–2 m away from the bulb–ruler arrangement (Figure Lab.11.7).
- Look at the light source through the filter and optical instrument. Observe the fringe pattern.
- Have two group members position markers at the maxima. Count the number of fringes between the two markers.
- Record the distance between the two markers.
- Use the string to measure the distance of the slit(s) to the metre stick (L).
- Repeat this procedure with the other two optical instruments.

Analysis

- Calculate the separation distance between consecutive maxima by dividing the total distance between the markers and the total number of maxima seen between the markers minus 1. Call this value Δx .
- Use the equation $\Delta x = \frac{L\lambda}{d}$ to find the wavelength of red light used.

Discussion

- For which of the three elements used was it easiest to obtain values? Why?
- Look up the range of wavelengths of red light. Did your values fall within this range? If not, give possible reasons.
- Did the three values agree within a reasonable range? Which value do you think is the most reliable?
- If you know the value of the wavelength of light your filter transmits, use the equation $\Delta x = \frac{L\lambda}{d}$ to check the slit separation values.
- Red is the easiest colour of light to use in this experiment. Why?

Conclusion

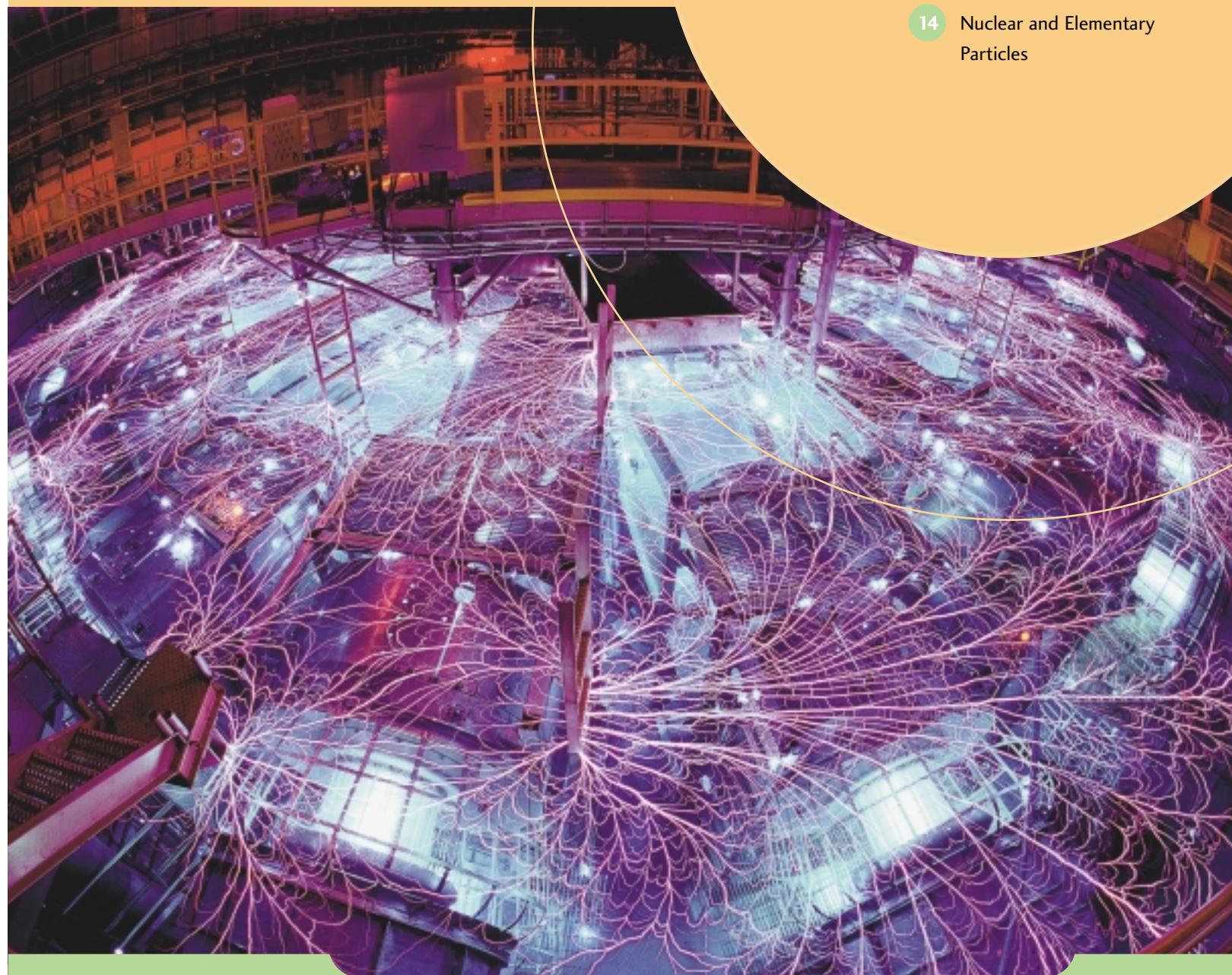
Summarize your findings. Is the equation derived from the wave theory of light valid?

UNIT

E

- 12 Quantum Mechanics
- 13 The World of Special Relativity
- 14 Nuclear and Elementary Particles

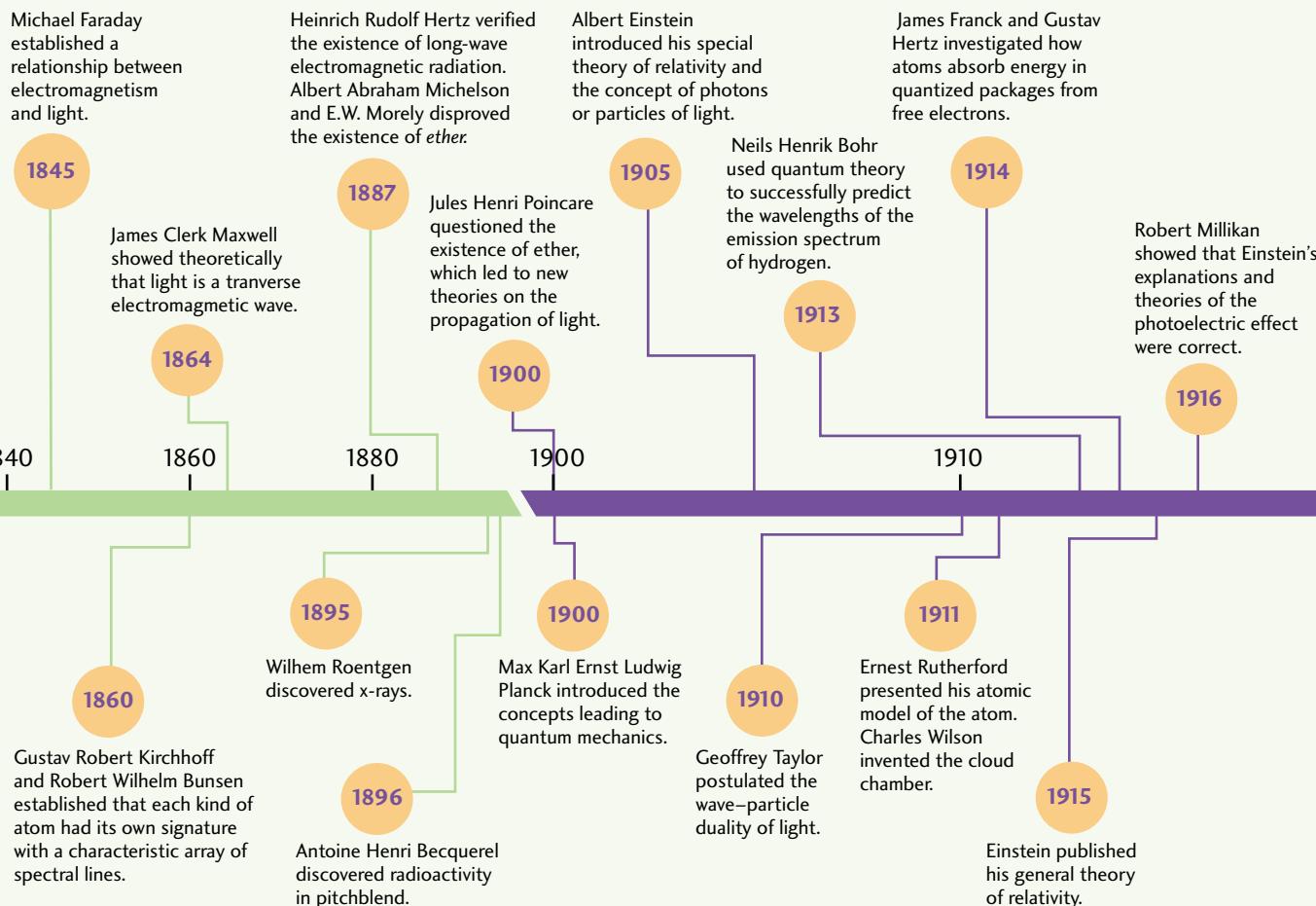
Matter–Energy Interface



As the 19th century drew to close, so did a chapter in physics. Newtonian physics explained the motion of objects on Earth as well as the motion of heavenly bodies. Christian Huygens, Thomas Young, Augustin Fresnel, and others explained the nature of light in terms of the wave theory. James Clerk Maxwell completed “classical” physics by amalgamating electricity and magnetism and, in the process, bringing light into the electromagnet family. It seemed as though the world around us was fully explained. Physics as a field of study seemed complete with only a few minor problems left to solve.

Then came the discoveries that brought the classical physics era to a close and ushered in the modern age. In December, 1895, two events occurred that changed the world forever. Louis Lumière invented the first motion-picture camera, and Wilhelm Roentgen discovered x-rays. These mysterious rays could not be explained by any laws of physics known at the time. A host of other discoveries followed: black-body radiation curves, light behaving like a particle in the photoelectric effect, x-ray photons with momentum, and particles exhibiting wave properties. In 1905, the most famous of all theories was born — special relativity. Where once we assumed such steadfast principles as a fixed reference point in the universe, absolute time, and relative speeds (speed of

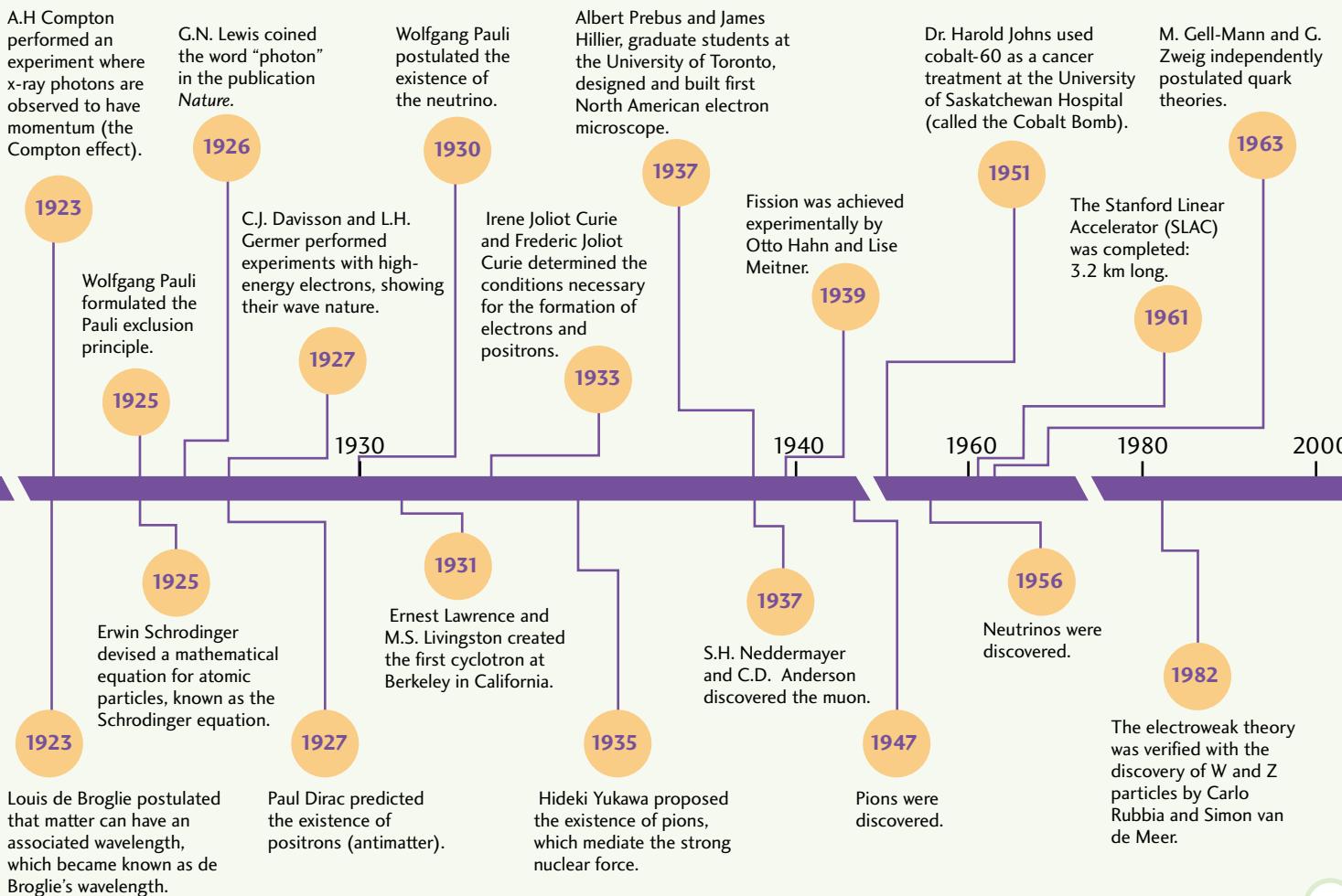
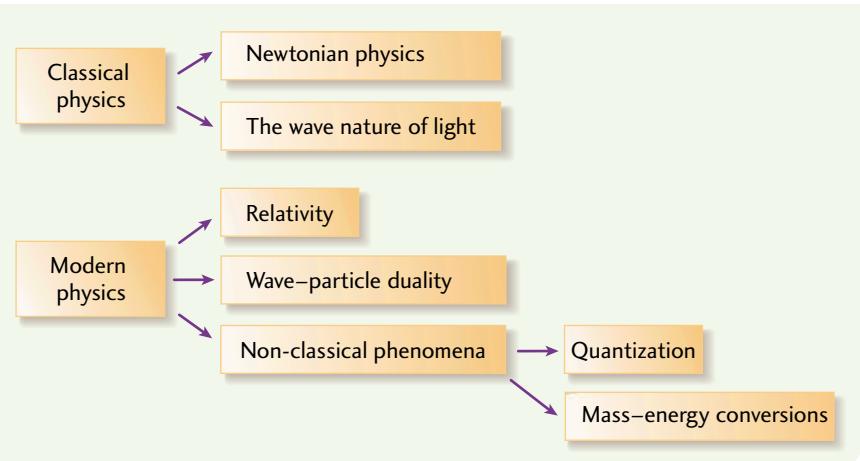
Timeline: The History of Matter–Energy Interface



medium adds to object speed), Einstein turned the known scientific world upside down with the new concepts of relative time, length contraction, a constant speed of light in all reference frames, and mass that increased with speed.

The theory of the atom evolved quickly. Ernest Rutherford's planetary model with an orbiting electron around a central nucleus replaced J.J. Thomson's plum-pudding model. Rutherford's model led to Bohr's quantization of electron orbits, which in turn led to a quantum-mechanical explanation of the atom based on wave mechanics.

In this unit, we will study the development of these theories and how they came together to form what is now called the *modern age of physics*. Technologies involving lasers and electron microscopes are a direct result of these theories. This unit discusses the transition from long-standing Newtonian physics to the age of Albert Einstein, Niels Bohr, and Stephen Hawking.



12

Quantum Mechanics

Chapter Outline

12.1 Introduction

12.2 The Quantum Idea

12.3 The Photoelectric Effect

12.4 Momentum and Photons

12.5 De Broglie and Matter Waves

12.6 The Bohr Atom

12.7 Probability Waves

12.8 Heisenberg's Uncertainty Principle

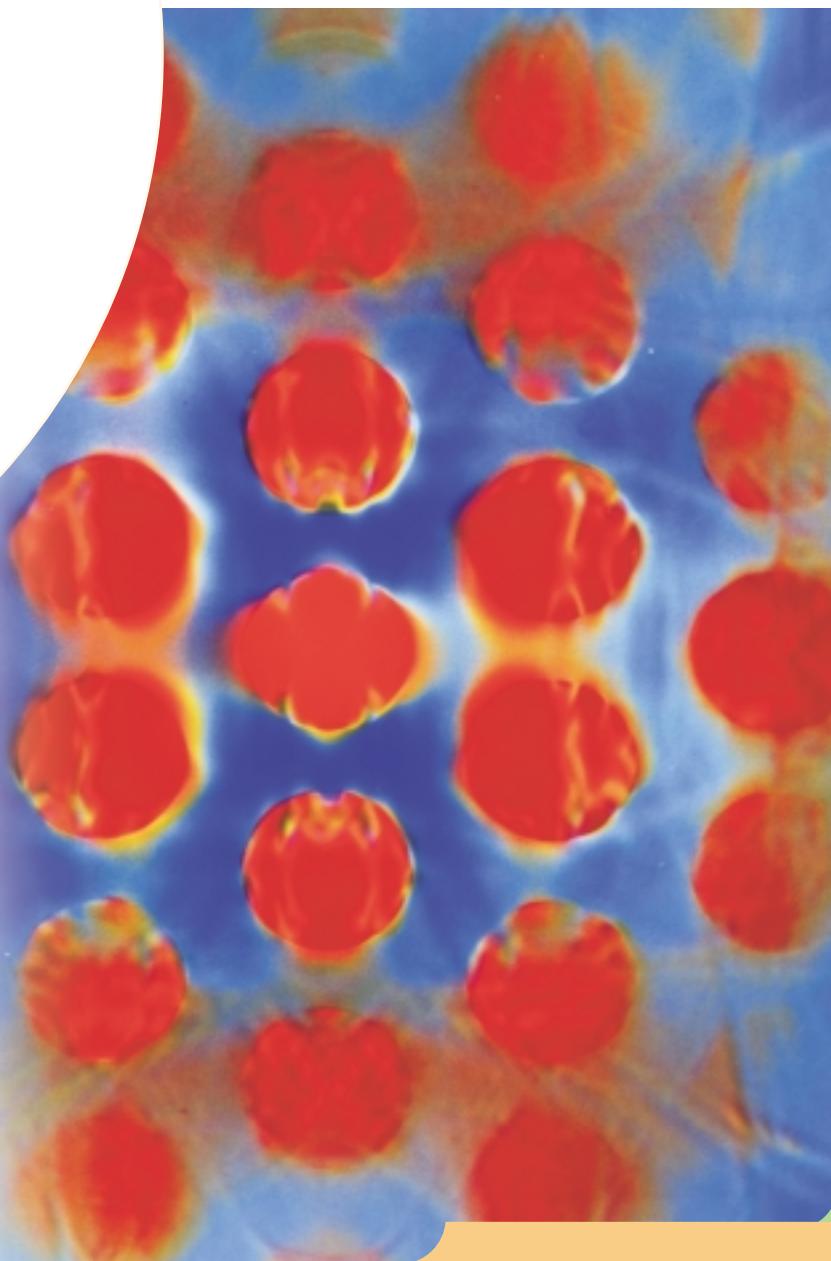
12.9 Extension: Quantum Tunnelling

 The Scanning Tunnelling Microscope

LAB 12.1 Hydrogen Spectra

LAB 12.2 The Photoelectric Effect I

LAB 12.3 The Photoelectric Effect II



By the end of this chapter, you will be able to

- outline the experimental evidence that supports the quantum energy idea and a particle model for light
- describe the Bohr model of the atom as a synthesis of classical and early quantum mechanics
- explain why probability is needed to describe photon diffraction
- understand that neither particles nor waves alone are an adequate model for light

12.1 Introduction

Thus far, our study of physics has kept the ideas of matter and energy separate. We have learned that neither matter nor energy can be created or destroyed. We have also learned that they are very different phenomena. However, we need to consider what happens when matter and energy interact and if they are ever indistinguishable. **Quantum theory** (also known as **quantum** or **wave mechanics**) is the theory of how atoms, matter, and energy are interrelated.

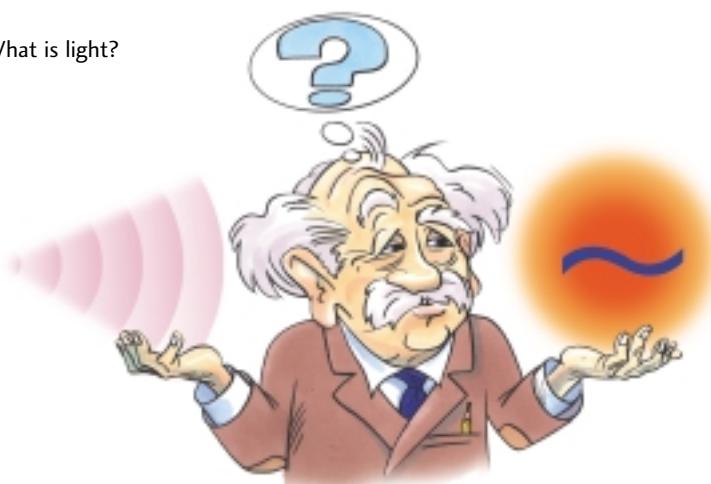
Problems with the Classical or Wave Theory of Light

At the end of the 19th century, there were a number of characteristics of light and properties of subatomic particles that the wave theory of light could not explain. Some of them were:

- 1) According to the wave theory of light, the energy of a system can be of any value, but this phenomenon is not observed; the spectra of atoms and electrons have very specific and consistent energy values (see Sections 12.2 and 12.6).
- 2) In some experiments, light exhibits particulate properties, such as momentum, a phenomenon that can't be explained in terms of the wave theory of light alone because a wave doesn't have mass (see Section 12.4).
- 3) Electrons, protons, and neutrons are particles and therefore should not exhibit wave characteristics. Yet, diffraction of all three types of particles was observed in laboratory experiments (see Sections 12.5 and 12.7).
- 4) If moving charged particles produce electromagnetic radiation, then electrons orbiting an atom should lose energy as they emit this radiation and fall into the nucleus, which doesn't occur (see Section 12.6).

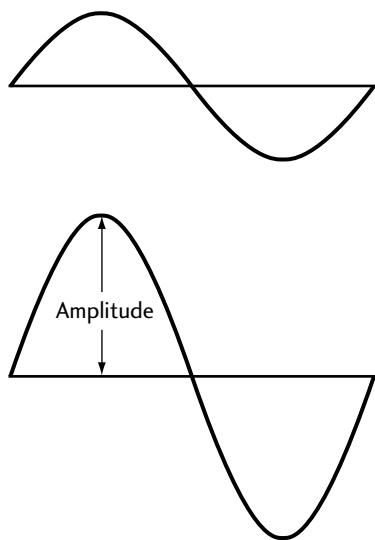
This chapter is an introduction to quantum theory, which addresses these problems with the wave theory of light and allows us to make accurate predictions about the behaviour of atoms and photons.

Fig.12.1 What is light?



12.2 The Quantum Idea

Fig.12.2 According to the wave model of light, waves with higher amplitude have more energy



According to the wave theory of light, the energy of a system can be of any value. For mechanical waves, increasing the amplitude of a wave increases the amount of energy transferred. Is the same true of light; that is, is the amplitude or brightness of a wavelength of light related to the amount of energy it transfers?

If we don't wear sunscreen or protective clothing when outside on a summer day, we get a sunburn (Figure 12.3).

Fig.12.3 Sunburn is cellular damage from ultraviolet (UV) radiation penetrating the dermal layer of the skin. Increased blood flow to capillaries causes redness.

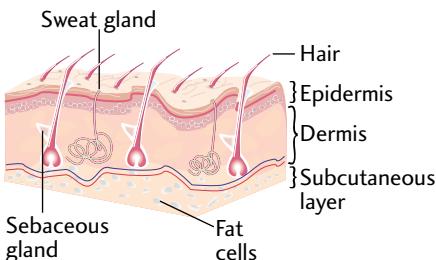


Fig.12.4 A welder must wear protective gear when using a welding torch to prevent painful burns caused by UV radiation



Fig.12.5 Bright stage lights don't cause sunburn



Similarly, welders must look away from the arc or wear protective goggles, and cover any exposed parts of skin (see Figure 12.4) to avoid receiving a painful burn that's very similar to sunburn. Why do sunlight and light from a welding torch affect our skin in a way that other bright lights don't?

Sunlight and welder light are harmful because they contain ultraviolet (UV) light that is more energetic than visible light, which is why it penetrates skin cells instead of reflecting off them. *The brightness of a light source is therefore not related to its penetrating power.*

In 1900, German physicist Max Planck suggested that light travels in packets called **quanta**. These packets define the amount of energy transferred by a given wavelength of light. Planck suggested that the smallest possible packet that can be associated with a given wavelength is given by the equation

$$E_\gamma = hf$$

where E is the energy of a given quantum in joules (J), h is Planck's constant ($6.626 \times 10^{-34} \text{ J}\cdot\text{s}$), and f is the frequency of the light in hertz (Hz or s^{-1}).

Substituting the wave equation for frequency,

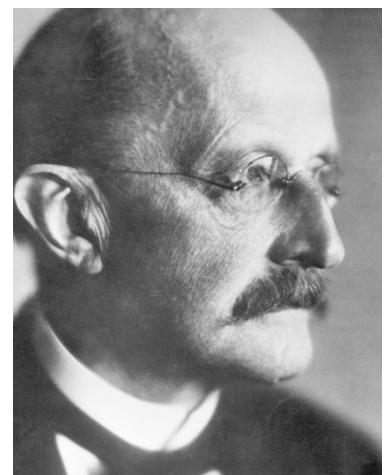
$$c = f\lambda \text{ where } f = \frac{c}{\lambda}$$

into Planck's equation, we obtain

$$E_\gamma = \frac{hc}{\lambda}$$

where E_γ is the smallest amount or *quantum* of energy (in joules) that can be transferred for a given wavelength of electromagnetic radiation. This idea is the main postulate of quantum mechanical theory.

Fig.12.6 Max Planck



Often, energy is expressed in terms of electron volts (eV), where
 $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Black-body Radiation

Any opaque object that has a temperature above absolute zero radiates photons. We can feel the warmth of a fireplace, a stove element, or the Sun without touching them. This effect is known as **black-body radiation**. The spectrum of any of these radiating objects is a continuous spectrum, like that of the rainbow (see Figure 12.7).

Fig.12.7 The spectrum of a radiating body

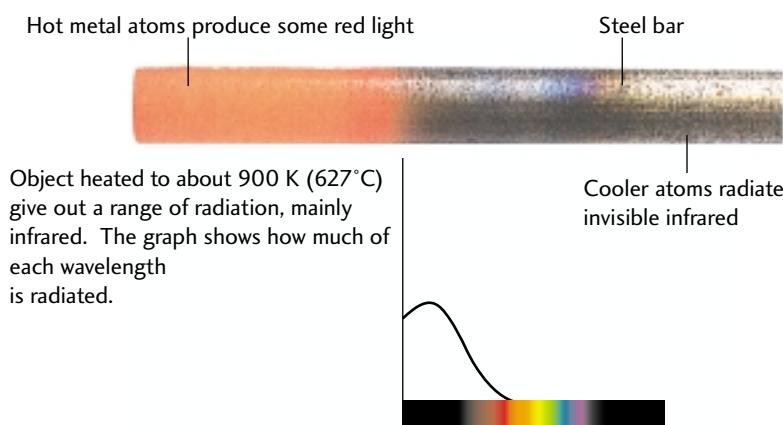
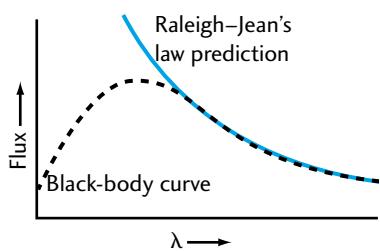


Fig.12.8a The Rayleigh-Jeans law used wave theory to describe the flux of photons of any wavelength as emitted from a black body. According to wave theory, the flux generated by shorter wavelengths should tend to infinity, which wasn't observed.

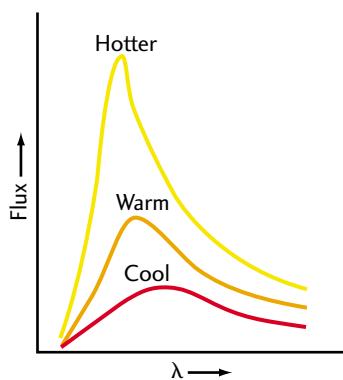


Wave theory predicted that radiation emitted by a hot object could be due to the oscillation of electric charges in the molecules of the material. Even though wave theory explained where the light came from, it didn't correctly predict the *spectrum* of the emitted light. A theoretical equation based on wave theory, developed by Lord Rayleigh and later modified by James Jeans, became known as the **Rayleigh-Jeans law**. It correctly predicted the intensity of visible light, but as the wavelength decreases (i.e., for UV light), it predicted that the energy of the wave approaches infinity, which was not observed (see Figure 12.8a). This problem was known as the **UV catastrophe**. Using his quantum hypothesis to modify the Rayleigh-Jeans equation, Planck was able to make it agree with all experimental observation, thereby solving the UV catastrophe. This modification of an earlier theory was very strong evidence for the correctness of Planck's idea.

FLUX

If we look at a 100-W light bulb, the filament (a small piece of tungsten wire heated to incandescence by the electricity flowing through it) is dazzling. If a larger piece of material radiates the same amount of energy (100 W), the light appears progressively dimmer as the surface area of the radiating object is increased. The smaller the object, the greater the energy flow per unit area, or **flux**. Flux allows scientists to compare energy flow rates from any object. If we consider more hazardous photons, such as x-rays, then experiments can be done to determine a safe level of flux. For a given emitter of x-rays, the inverse-square law determines how close we can safely come to the source and how much time we can spend at a certain distance from the source.

Fig.12.8b Using Planck's black-body equation, as an object heats up, the peak of its spectrum moves to shorter wavelengths. The object also emits more photons, which is detected by an increase in intensity.



The Black-body Equation

The graph of light emitted from a black body (Figure 12.8b) shows a definite peak in the most common wavelength of electromagnetic radiation (EMR) emitted. The equation that predicts the *maximum intensity* of the wavelength of EMR is called **Wien's law**,

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}}{T}$$

where λ_{\max} is the wavelength associated with the most common photons in a black-body curve, in metres, and T is the temperature in Kelvin. For visible light, the wavelength of this peak is seen as the dominant colour of the light.

E X A M P L E 1**Black bodies and their characteristics**

If a metal bar is heated in a shop so that the peak wavelength in its spectrum is $2.6 \text{ } \mu\text{m}$ long, determine

- its temperature.
- the energy in the photons at the peak wavelength, in joules and electron volts.
- In which part of the electromagnetic spectrum are the peak photons found?

Solution and Connection to Theory**Given**

$$\lambda = 2.6 \text{ } \mu\text{m} = 2.6 \times 10^{-6} \text{ m} \quad h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s} \quad c = 3.0 \times 10^8 \text{ m/s}$$

- Using Wien's law, we solve for the temperature:

$$T = \frac{2.898 \times 10^{-3}}{\lambda}$$

$$T = \frac{2.898 \times 10^{-3}}{2.6 \times 10^{-6} \text{ m}}$$

$$T = 1115 \text{ K}$$

The temperature of the metal bar is 1115 K or 842°C .

- Using Planck's equation,

$$E_\gamma = \frac{hc}{\lambda}$$

$$E_\gamma = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(3.0 \times 10^8 \text{ m/s})}{2.6 \times 10^{-6} \text{ m}} = 7.64 \times 10^{-20} \text{ J}$$

$$E_\gamma = \frac{7.64 \times 10^{-20} \text{ J}}{1.609 \times 10^{-19} \text{ J/eV}} = 0.48 \text{ eV}$$

The energy of the photons at the peak wavelength is $7.64 \times 10^{-20} \text{ J}$ or 0.48 eV.

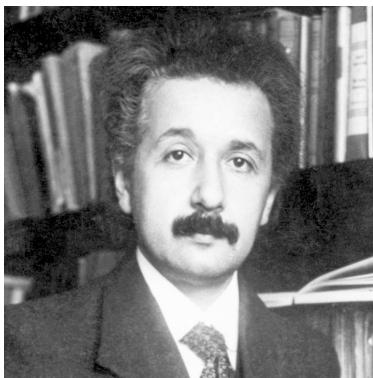
- $\lambda = 2.6 \text{ } \mu\text{m} = 2600 \text{ nm}$, which is found in the infrared (IR) region of the electromagnetic spectrum.



1. The star Rigel has a surface temperature of about 12 000 K.
 - a) What is its peak wavelength, and which part of the electromagnetic spectrum is it in?
 - b) What colour would the star appear to be if observed through a telescope? Why?
 - c) Could this star harbour planets with life? Would Earth have life if it orbited this star?
2. a) In a light bulb, if the tungsten filament has a temperature of 900 K, what is the peak wavelength?
 - b) In which part of the spectrum is this wavelength?
 - c) Why does a light bulb have to produce so much heat?
3. If light energy is quantized, why is a black-body curve continuous?

12.3 The Photoelectric Effect

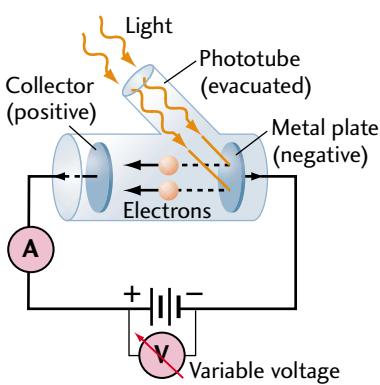
Fig.12.9 Albert Einstein (1879–1955), born in Ulm, Germany, was one of the most profound thinkers of his age



The **photoelectric effect** is a phenomenon that occurs when light shone on a metal surface causes electrons to be emitted from the surface. Experiments revealed that for a given frequency of light, the kinetic energy of the electrons ejected from a metal surface was the same. Also, even though increasing the brightness of the light caused more electrons to be ejected, their individual energies remained the same. However, if only a small amount of light of a higher frequency (different colour) was used, the kinetic energy of the electrons immediately increased as the wavelength of light decreased. This effect completely contradicted the wave theory of light; that is, that the energy in a wave is a function of its amplitude. Increasing the wave's amplitude, or brightness, should increase the energy of the ejected electrons.

Albert Einstein decided to undertake an explanation of the photoelectric effect from a theoretical point of view using Planck's quantum idea. For all his work, Einstein received the Nobel Prize in physics in 1921.

Fig.12.10a A diagram of the photoelectric effect apparatus



The Apparatus

The photoelectric effect apparatus consisted of a shiny metal surface enclosed in a vacuum tube to prevent oxidation. When light was shone on the metal surface, some electrons were ejected from it. The metal being struck by the light was negative (the cathode) and the terminal collecting the electrons was made positive (the anode), so the electrons zipped from one terminal to the other as soon as they were liberated by the photons of light. The anode and cathode were connected via a power supply providing a potential, and an ammeter that measured the amount of current. Then the potential was reversed so that the anode became negatively charged, causing electrons to be

repulsed from the anode. The kinetic energy of the electrons could then be measured by finding the minimum potential required to prevent electrons from being ejected from the metal surface. This potential energy is equal to the kinetic energy of the electrons and is called the **stopping potential**, V_{stop} . If the anode is positive, the electrons are attracted to the anode, causing current to flow. If the potential is reversed, the liberated electrons are repelled from the anode and no current flows (see Figure 12.10b).

Experiment 1: The Energy of Ejected Electrons Compared to the Intensity of Incident Light

If the quantum idea is valid, then the energy of the ejected photons shouldn't change when the intensity of light is increased. From the definition of electric potential,

$$V = \frac{E}{q}$$

so

$$E = Vq$$

Since we are considering electrons only, then

$$E = Ve$$

where e is the elementary charge ($1.6 \times 10^{-19} \text{ C}$).

If we change the potential so that the current on the ammeter in our circuit is zero, then the potential across the boundary is just enough to keep the electrons from passing to the anode. Since the kinetic and potential energies have to balance in order for the ejected electrons to stay away from the anode, then we can measure the kinetic energy of the ejected electrons very accurately.

By placing filters in front of the light source that have varying amounts of translucency (such as smoked glass), the level of light intensity can be varied. However, the energy of the electrons remains the same, regardless of the *intensity* of a given wavelength or colour of light. Einstein interpreted this result as evidence that *radiant energy was transmitted in bundles or quanta*, each with a specific energy. While more of these bundles impinging on the metal liberated more electrons, the energy imparted to each electron was the same, or at least within the distribution of incident photon energies.

Experiment 2: Changing the Colour of the Light

The second photoelectric effect experiment involved maintaining a constant *intensity level* of light while varying the *colour* of the incident light. Special filters designed to transmit only a small part of the EMR spectrum were used.

Fig.12.10b The effects of switching the polarity of the collector plate

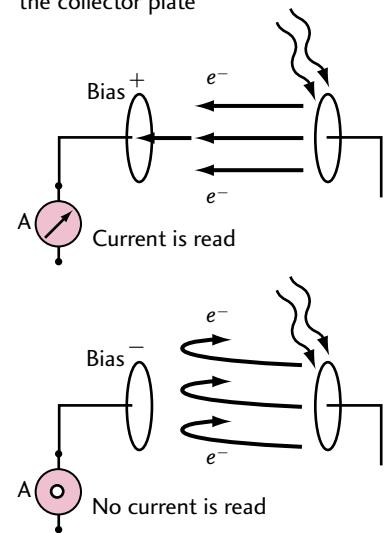


Fig.12.11a Lower-energy photons don't possess enough energy to liberate electrons from a metal surface

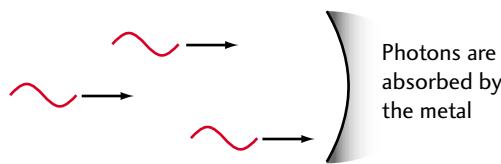


Fig.12.11b If a higher-energy photon hits the metal and is not reflected, it interacts with the electrons in the metal and transfers its energy to an electron. If the energy transferred by the photon is greater than the minimum energy required to evict the electron from the metal, then the electron will be emitted. The electron's kinetic energy is the energy of the photon minus the energy required to liberate the electron.

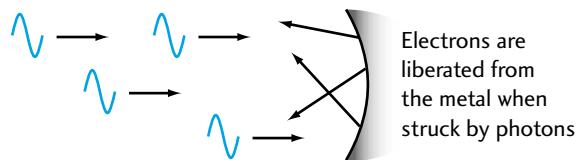


Table 12.1 Work Functions of Some Common Metals	
Metal	W_0 (eV)
Silver	4.65
Aluminum	4.28
Gold	5.37
Copper	4.65
Nickel	5.15
Platinum	5.65

If we choose a longer wavelength of light, such as red light, then no matter how bright the light source is, the photoelectric effect doesn't occur because photons of red light don't have enough energy to liberate any electrons from the metal surface (see Figure 12.11a). Einstein suggested that each metal used for the cathode has a specific minimum energy that permits electrons to be ejected from it. Energy is required to do the work of liberating electrons. Einstein therefore called this specific minimum energy the **work function**, W_0 .

If we apply Planck's equation for the energy of a photon,

$$E = hf \text{ or } E = \frac{hc}{\lambda}$$

to the energy of an electron, we obtain

$$\frac{hc}{\lambda} = hf = E_{k_{\max}} + W_0$$

Note: If energy is expressed in electron volts (eV), then Planck's constant becomes

$$\begin{aligned} & \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{1.60 \times 10^{-19} \text{ J/eV}} \\ &= 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}. \end{aligned}$$

where hf is the energy of the incident photon, $E_{k_{\max}}$ is the maximum amount of energy of the liberated electron, and W_0 is the work function (all in joules). Thus, the kinetic energy of the electron as measured by the photoelectric effect is *slightly less than* the incident photon's energy because some of the photon's energy becomes the work function that liberates the electron:

$$E_{k_{\max}} = E_{\text{photon}} - W_0$$

Therefore, in order for the photoelectric effect to occur, the energy of the incident photon must be greater than the work function: $E_{\text{photon}} > W_0$.

To properly analyze the energy required to liberate the electron, we need to measure the work function value for a given material. In the equation $E_{\text{photon}} = E_{k_{\text{max}}} + W_0$, the kinetic energy of the electrons equals the *stopping potential* times the electron charge:

$$E_{k_{\text{max}}} = eV_{\text{stop}}$$

From the filter we are using, we know the energy of the photons incident on the cathode, so we can plot the stopping potential versus the frequency for different incident photon energies:

If

$$hf = E_k + W_0$$

then

$$hf = eV_{\text{stop}} + W_0$$

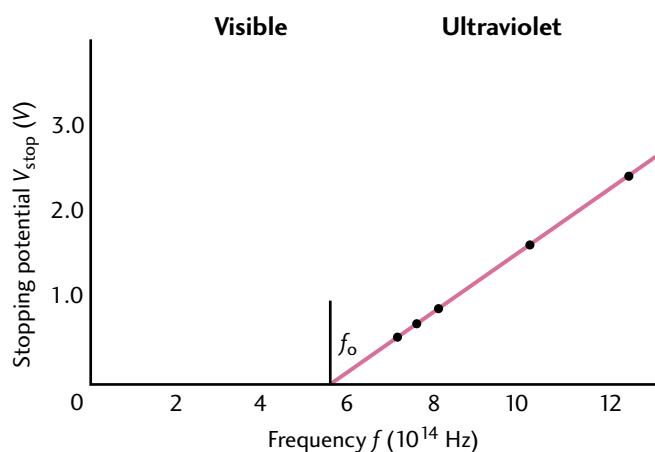
and

$$eV_{\text{stop}} = hf - W_0$$

$$V_{\text{stop}} = \left(\frac{h}{e}\right)f - \frac{W_0}{e}$$

This graph is a straight line with slope $\frac{h}{e}$ (see Figure 12.12a).

Fig.12.12a A graph of the photoelectric effect



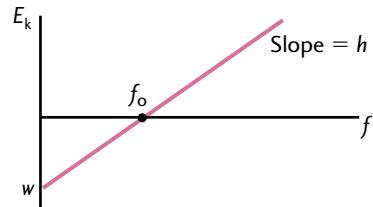
Since the only unknown variable for a new material is W_0 , we can calculate it by letting $V_{\text{stop}} = 0$. The frequency of light used, f_0 , can either be measured or is known from the type of filter being used. Then,

$$0 = hf_0 - W_0$$

$$W_0 = hf_0$$

$$\begin{array}{l} eV = hf - W_0 \\ y = mx + b \\ E_k \end{array}$$

Fig.12.12b



eV_{stop} is a measure of the kinetic energy of the electron. By plotting E_k versus f of the photoelectric effect equation ($hf = E_k + W_0$), we can obtain the work function (y intercept) and h , Planck's constant (slope).

EXAMPLE 2

The photoelectric effect

A material with a known work function of 2.3 eV is shone with incident light of wavelength 632 nm (typical HeNe laser).

- Will this light cause the metal to exhibit the photoelectric effect?
- If not, then what maximum wavelength will cause the photoelectric effect?

Solution and Connection to Theory

Given

$$W_0 = 2.3 \text{ eV} \quad \lambda = 632 \text{ nm} \quad E_{\text{photon}} = ?$$

- Using the photoelectric effect equation,

$$E_{\text{photon}} = eV_{\text{stop}} + W_0$$

If $E_{\text{photon}} > W_0$, then the material will exhibit the photoelectric effect.

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

$$E_{\text{photon}} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.0 \times 10^8 \text{ m/s})}{6.32 \times 10^{-7} \text{ m}}$$

$$E_{\text{photon}} = 3.143 \times 10^{-19} \text{ J} = 1.96 \text{ eV}$$

$$1.96 \text{ eV} < 2.3 \text{ eV}$$

Therefore, the electron cannot be liberated by the photoelectric effect.

- To determine the maximum wavelength of the light that will force an electron to escape, we set the photon energy equal to that of the work function:

$$W_0 = 2.3 \text{ eV} = 3.68 \times 10^{-19} \text{ J}$$

Then using $E_{\text{photon}} = \frac{hc}{\lambda}$, we obtain

$$\lambda = \frac{hc}{E_{\text{photon}}}$$

$$\lambda = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.0 \times 10^8 \text{ m/s})}{3.68 \times 10^{-19} \text{ J}}$$

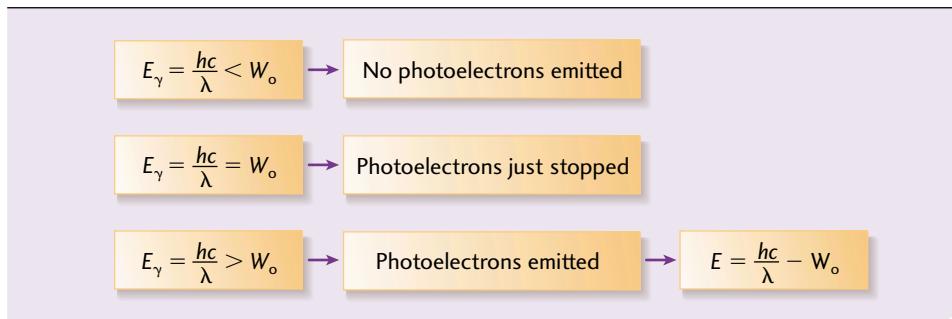
$$\lambda = 510 \text{ nm}$$

Thus, it would take a photon with a wavelength shorter than 510 nm to demonstrate the photoelectric effect on this material.

Satellites and spacecraft can be significantly affected by the photoelectric effect. The vacuum of space and the UV component of solar radiation ensure that all spacecraft will become positively charged. This positive charge can result in a static discharge when the spacecraft dock.

Figure 12.13 summarizes the conditions required for the photoelectric effect to occur.

Fig.12.13 Incident Photon Energy and the Work Function



- Using the equation $V = \left(\frac{h}{e}\right)f_0 + \frac{W_0}{e}$, calculate the values of h and W_0 given the data in Table 12.2. Assume that $e = 1.9 \times 10^{-19} \text{ C}$. Plot a graph of V_{stop} versus f_0 .
- If the work function of a material was 1.5 times the amount you calculated in 1.a),
 - how would this change affect the data from the table?
 - would the slope of the line in 1.a) change?
- From the equation you derived in 1.a), what is the maximum permissible kinetic energy of an electron liberated by a photon with a wavelength of 230 nm?
- Explain the following graphs.

Fig.12.14

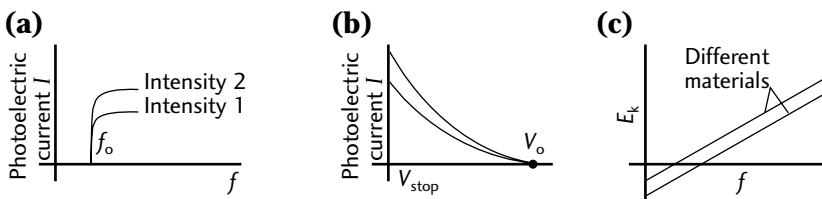


Table 12.2
Stopping Potential
and Frequency

V_{stop} (V)	f_0 (Hz)
0.7	7.2×10^{14}
0.95	7.7×10^{14}
1.1	8.05×10^{14}
2.0	10.4×10^{14}
2.85	12.5×10^{14}

12.4 Momentum and Photons

To determine how photons react with matter, their properties had to be analyzed both experimentally and theoretically. In a series of experiments performed by Arthur Holly Compton (Figure 12.15a) in St. Louis in 1923, a beam of x-rays was directed at a thin foil and a target made of carbon (see Figure 12.15b). By arranging detectors behind the target, he found that the x-rays were deflected in many directions as they hit the carbon. In atomic situations, this effect is called *scattering*. Compton analyzed the energies of these photons and found that they were different from the energy of the incident x-ray.

Recall from Section 10.4 that if an electromagnetic wave interacts with a particle such as an electron, the particle oscillates at the same frequency as the wave (see Figure 12.15c).

Fig.12.15a Arthur Compton

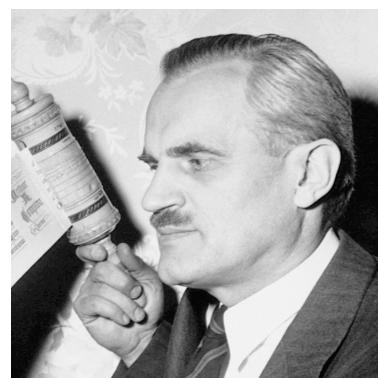


Fig.12.15b Compton's experiment

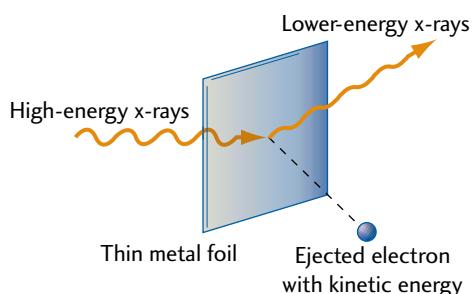
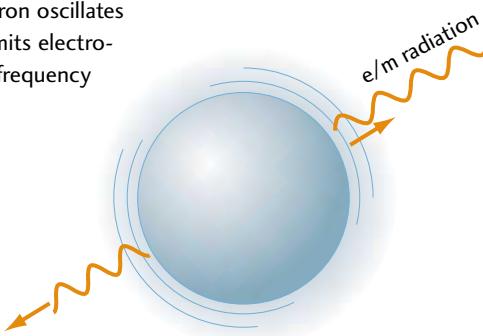


Fig.12.15c When an electron oscillates at a certain frequency, it emits electromagnetic radiation of that frequency



However, if the particle absorbs the wave, then the wave cannot be scattered. More importantly, if photons are entirely energy, then they should not experience changes to their energy during collisions because they have no mass.

Compton thought that if the photons could be treated as particles that were colliding with the electrons, he could analyze the observed scatter using the same techniques used to handle ordinary collisions, which obey the laws of conservation of energy and of linear momentum. But using this technique requires that photons possess momentum. How could a photon of energy with no mass possess momentum? Compton used Einstein's theory of special relativity to determine a mass-like property for a photon.

If the law of conservation of energy is valid, then the energy of the initial incident x-ray equals the kinetic energy given to the electron plus the energy of the emitted photon:

$$E_{\text{x-ray}} = hf_f + \frac{1}{2}mv^2$$

where $\frac{1}{2}mv^2$ represents the kinetic energy imparted to the electron that interacted with the x-ray photon (in joules) and the value of f_f reflects the new lower-energy frequency of the emitted x-ray photon.

Besides energy, momentum is also conserved during collisions (recall Chapter 4):

$$\vec{p}_{\text{x-ray}_i} = \vec{p}_{\text{electron}} + \vec{p}_{\text{x-ray}_f}$$

To calculate the momentum of photons, Compton returned to Einstein's famous equation relating mass and energy,

$$E = mc^2$$

Rearranging for mass,

$$m = \frac{E}{c^2}$$

The right side of this equation, $\frac{E}{c^2}$, is known as the **mass equivalence**. Recall from Chapter 4 that in classical physics, linear momentum (p) is given by the equation

$$p = mv \quad (\text{magnitude only})$$

where m is the object's mass in kilograms and v is its velocity in m/s. To calculate the momentum of the photon, Compton substituted the mass equivalence from Einstein's equation into the equation for linear momentum to obtain

$$p = \left(\frac{E}{c^2}\right)v$$

Substituting Planck's equation for the energy of a photon as a function of wavelength, we obtain

$$p = \left(\frac{hc}{\lambda c^2}\right)v$$

The velocity, v , can be replaced with c because all photons travel at the speed of light. Our final equation for the momentum of a photon is

$$p = \frac{h}{\lambda}$$

where h is Planck's constant and λ is the wavelength, in metres, associated with the photon. Note that momentum is a vector quantity.

Compton's work showed that photons collide and exchange energy with particles according to the law of conservation of energy, that they possess momentum, and that their momentum is conserved during collisions. His work lent support to the idea that light possessed both wave and particle properties at a fundamental level. It indicated that investigations of these phenomena were at the interface between what is considered matter and what is considered energy.

EXAMPLE 3 Photon momentum in one dimension

A 25-eV x-ray photon collides with an electron. What is the momentum of the original photon?

Solution and Connection to Theory

Given

$$E = 25 \text{ eV} \quad p = ?$$

For photon momentum,

$$p = \frac{h}{\lambda}$$

$$\frac{\text{J}\cdot\text{s}}{\text{m}} = \frac{(\text{N}\cdot\text{m})\text{s}}{\text{m}} = \text{N}\cdot\text{s}$$

Solving Planck's equation $E = \frac{hc}{\lambda}$ for λ ,

$$\lambda = \frac{hc}{E}$$

Substituting into the momentum equation, we obtain

$$p = \frac{Eh}{hc} = \frac{E}{c}$$

$$p = \frac{(25 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{3.0 \times 10^8 \text{ m/s}}$$

$$p = 1.3 \times 10^{-26} \text{ N}\cdot\text{s}$$

The momentum of the original photon is $1.3 \times 10^{-26} \text{ N}\cdot\text{s}$ in the same direction as the photon's motion.



1. Why did Compton use x-rays and carbon in his experiment?
2. An 85-eV x-ray photon collides with an electron. The resultant photon is deflected 60° from the original line of travel and has a wavelength of 214 nm.
 - a) What is the momentum of the original photon?
 - b) What is the momentum of the resultant photon?
 - c) How much energy was imparted to the electron?
 - d) How much has the energy calculated in c) above increased the electron's speed?
 - e) What implications does this speed have for the electron?

12.5 De Broglie and Matter Waves

Fig. 12.17 Louis de Broglie



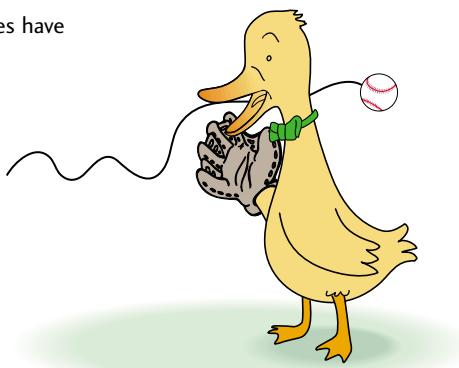
In 1924, Louis de Broglie (pronounced “de Broy”), a French graduate student, decided to expand on Compton’s idea of photon momentum. He suggested that since photons have detectable linear momentum, a property of matter, then matter might be explained in terms of waves. His argument was based on the supposition that so many concepts in physics are reversible. For example, changing electric fields produce changing magnetic fields and vice versa. So, if photons exhibited the property of momentum even though they are apparently massless, then perhaps objects with mass have wave properties.

De Broglie extended the momentum equation for photons,

$$\lambda = \frac{h}{p} \quad \text{or} \quad \lambda = \frac{h}{mv}$$

to matter.

Fig.12.18 Do particles have wave properties?



EXAMPLE 4 Using de Broglie's equation

Find the wavelength of a 1-g steel bearing, moving at 10 m/s.

Solution and Connection to Theory

Given

$$m = 1 \text{ g} = 0.001 \text{ kg} \quad v = 10 \text{ m/s} \quad h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s} \quad \lambda_{\text{bearing}} = ?$$

The ball's momentum is

$$p = mv$$

$$p = (0.001 \text{ kg})(10 \text{ m/s})$$

$$p = 0.01 \text{ N}\cdot\text{s}$$

Applying de Broglie's equation, we obtain

$$\lambda_{\text{bearing}} = \frac{h}{p}$$

$$\lambda_{\text{bearing}} = \frac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{0.01 \text{ N}\cdot\text{s}}$$

$$\lambda_{\text{bearing}} = 6.626 \times 10^{-32} \text{ m}$$

The wavelength of the bearing is $6.626 \times 10^{-32} \text{ m}$. Compare this wavelength to that of visible light, which has a wavelength of about 550 nm or $5.50 \times 10^{-7} \text{ m}$.

In Chapter 11, we learned that for interference and diffraction, the size of the slits must be close to the wavelength of incident light. Consider a cruise ship of length 300 m. Waves with wavelengths of 3 cm won't cause the ship to rock (Figure 12.19a). However, if these waves had wavelengths of around 30 m, the boat would begin to rock (Figure 12.19b). By analogy, if matter has wave properties, as de Broglie suggested, then the wavelengths calculated for common macroscopic objects are so tiny that we can't observe their effects. Experiments have corroborated this theory.

Fig.12.19a

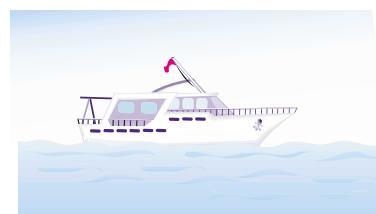
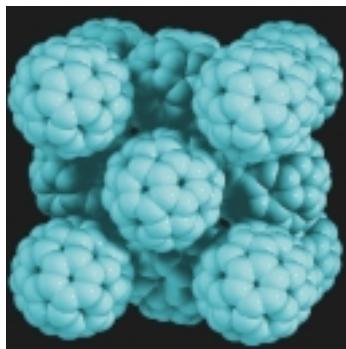


Fig.12.19b





Fig.12.20 The lattice structure of a Buckminsterfullerene (C_{60}) molecule (bucky ball)



Electron Diffraction

In 1927, Clinton J. Davisson and L.H. Germer at Bell Labs in New Jersey and George Thomson in Scotland independently tried to determine whether matter had wave properties by beaming electrons at a nickel crystal. By rotating the nickel crystal, they were able to measure the angles at which the electrons were scattered. They found that at certain angles, there was a peak intensity of electron scattering. From these results, they inferred that the electrons were diffracted by the regular atomic structure of the nickel crystal and that the pattern appeared to be a double-slit diffraction pattern. This experiment supported de Broglie's theory that matter has wave properties. More recently, objects the size of **Buckminsterfullerene (C_{60})**, also exhibited a diffraction pattern when beamed through a double slit.

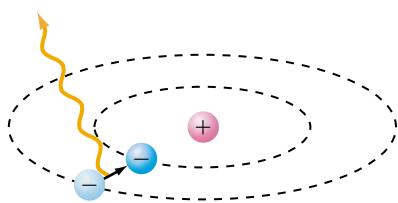
1. What is the de Broglie wavelength of an electron moving at 1 km/s?
2. Explain how beaming electrons at a nickel plate produces the same results as beaming them through a double slit.

12.6 The Bohr Atom

Fig.12.21a Niels Bohr



Fig.12.21b Bohr's model of the atom

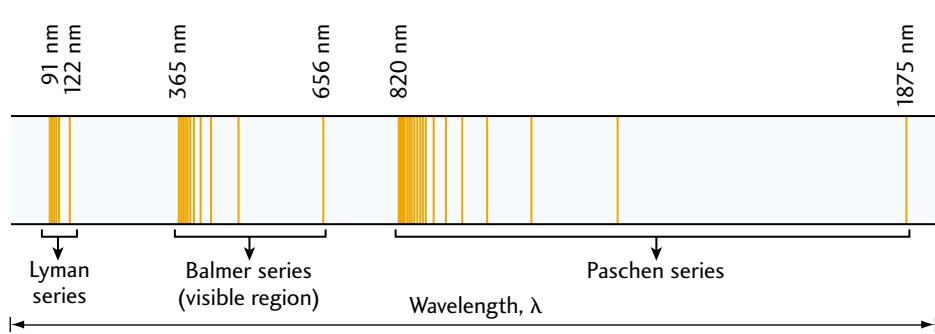


In Section 12.2, we learned that hot, opaque objects produce continuous spectra. To properly understand what is happening to an individual atom, scientists conducted experiments on only a few atoms of the same element (i.e., a rarefied gas) and at the same temperature using a non-ionized gas at a low enough temperature. When they examined the spectrum emitted by this gas, they found that instead of being continuous, the spectrum consisted of distinct lines that appeared grouped in various sets. In 1884, Johann Balmer, a Swiss high-school teacher and mathematician, invented an empirical relationship that predicted the energy levels of the set of spectral lines that could be seen in visible light. This set was named the Balmer series.

This observation was a mystery until 1913, when Danish scientist Niels Bohr introduced a new model of the hydrogen atom based on Rutherford's planetary model that explained the spectral lines produced by hydrogen gas at various temperatures (see Figure 12.21b). In Bohr's model of the hydrogen atom, an electron emits energy (a photon) when it drops from a higher energy level to a lower energy level. In Figure 12.22a, notice that the hydrogen atom emits photons at very specific energies. This effect suggests that energy at the atomic level is quantized. What causes these spectral lines?

In the Rutherford (planetary) model of the hydrogen atom, the electron orbits the nucleus at high speed. The electron is accelerating because its direction is constantly changing as it circles the atom's nucleus (see Figure 12.22b); therefore, it should lose energy and spiral into the nucleus. But this

Fig.12.22a The spectra produced by hydrogen gas



Balmer predicted that

$$\lambda = \frac{(364.5)n^2}{(n^2 - 4)}, \text{ where}$$

$n = 3, 4, 5, 6$, λ is the wavelength of a given Balmer line (in visible light only!) and n is an integer representing levels of transition in the Bohr atom.

effect isn't observed, otherwise all atoms would be neutrons! Bohr reasoned that electrons maintain their distance from the atomic nucleus because of the laws of conservation of energy and of angular momentum.

The Conservation of Energy

An electron orbiting a nucleus has two kinds of energy: kinetic and potential, such that

$$E_e = E_k + E_p$$

$$E_e = \frac{1}{2}m_e v_e^2 + \frac{k e^2}{r}$$

But the electron's potential energy is negative because the electron is in an **energy well** or an **electric field** created by the attraction of the nucleus to the electron. (Gravity acts on a mass in a similar way, as we learned in Chapter 6.) The total energy of the electron, E_e , is therefore

$$E_e = \frac{1}{2}m_e v_e^2 - \frac{k e^2}{r}$$

But the centripetal force causing the electron to "orbit" equals the electrostatic force if the "orbit" is circular; so,

$$F_c = \frac{m_e v_e^2}{r} = \frac{k e^2}{r^2}$$

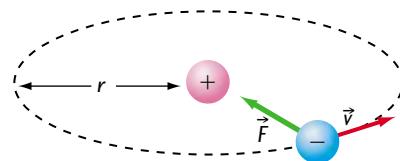
$$F_c = m_e v_e^2 = \frac{k e^2}{r}$$

Therefore, the electron's total energy is

$$E_e = \frac{k e^2}{2r} - \frac{k e^2}{r}$$

$$E_e = \frac{-k e^2}{2r}$$

Fig.12.22b The electron doesn't spiral into the atom's nucleus because of the laws of conservation of energy and momentum



The Conservation of Angular Momentum

Angular Momentum

When you spin a bicycle wheel, you know that the effort to slow it down depends on its speed, on the radius of the wheel, and on the mass. It is always easier to stop smaller-diameter wheels than larger ones of equal mass. The resistance of the wheel to change in its rotational motion is known as **angular momentum**.

I is the moment of inertia, the rotational equivalent to mass.

In 1923, de Broglie came up with an explanation for Bohr's assumption that $mv_n r_n = \frac{nh}{2\pi}$. If we consider the electron to be a standing wave (i.e., an integer number of wavelengths, $n\lambda$) around the atom's nucleus, then its total length is the circumference of its orbit:

$$2\pi r = n\lambda.$$

But $\lambda = \frac{h}{p}$ (de Broglie's equation), where $p = mv$. Therefore,

$$2\pi r = \frac{nh}{p} = \frac{nh}{mv}$$

$$mv r = n\left(\frac{h}{2\pi}\right) = n\hbar$$

The electron's standing-wave pattern results in resonance. No energy is lost, so the electron doesn't spiral into the nucleus.

The other fundamental quantity that needs to be conserved is angular momentum, L . In Chapter 7, we learned that

$$L = I\omega = mr^2\omega \text{ and } \omega = \frac{v}{r}$$
$$L = mvr$$

In creating his model of the hydrogen atom, Bohr assumed that angular momentum could be quantized because it is related to energy. (Recall from Chapter 5 that the momentum of a particle is $\sqrt{2mE_k}$.) But to quantize L , he had to quantize the speed (v) and the radius (r) because if even one of the terms in the equation could have any random value, then the result using the equation would also be a random value and thus would not be quantized. The equation for angular momentum was modified as follows:

$$L_n = mv_n r_n$$

where n is a positive integer that represents the energy level of the electron. Bohr then had to determine the smallest possible division or quantum of L . Planck had defined the smallest unit of energy to be h . Bohr suggested that the limit on L was $\frac{h}{2\pi}$. It has the short form \hbar , which is called "h bar." Thus we have

$$L_n = mv_n r_n = n\hbar$$

Solving for v_n , we obtain

$$v_n = \frac{n\hbar}{mr_n}$$

This expression is the quantization of v , the velocity of the electron. We can substitute this equation into the force equation, where centripetal force is caused by the electric force:

$$F_c = \frac{m_e v_e^2}{r} = \frac{ke^2}{r^2}$$

$$m_e v_e^2 = \frac{ke^2}{r} \text{ and } v = \frac{n\hbar}{mr_n}$$

$$\frac{m_e n^2 \hbar^2}{m_e^2 r_n^2} = \frac{ke^2}{r_n}$$

Simplifying and solving for r_n , we obtain

$$r_n = \frac{\hbar^2 n^2}{m_e k e^2}$$

Since the values of \hbar , m , k , and e are constant, this equation can be simplified by substituting their values. Therefore,

$$r_n = \frac{(1.0546 \times 10^{-34} \text{ J} \cdot \text{s})^2 n^2}{(9.11 \times 10^{-31} \text{ kg})(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})^2}$$

$$r_n = (5.29 \times 10^{-11} \text{ m})n^2$$

where r is the theoretical radius of the orbit of an electron in metres, and n is the quantum number. The electron's orbit has the smallest radius when $n = 1$. This radius is known as the **Bohr radius**. For hydrogen, $r_1 = 5.29 \times 10^{-11} \text{ m}$.

EXAMPLE 5 The radius of an energy level

What is the orbital radius of an electron in a hydrogen atom

- a) in the fourth energy level?
- b) in the fifth energy level
- c) What is the difference in radius between these two energy levels?

Solution and Connection to Theory

Given

$$r_n = (5.29 \times 10^{-11} \text{ m})n^2$$

- a) $n = 4$

$$r_4 = (5.29 \times 10^{-11} \text{ m})(4)^2$$

$$r_4 = 8.46 \times 10^{-10} \text{ m} = 0.846 \text{ nm}$$

The orbital radius at the fourth energy level is 0.846 nm.

- b) For $n = 5$,

$$r_5 = 1.323 \times 10^{-9} \text{ m} = 1.323 \text{ nm}$$

The orbital radius at the fifth energy level is 1.323 nm.

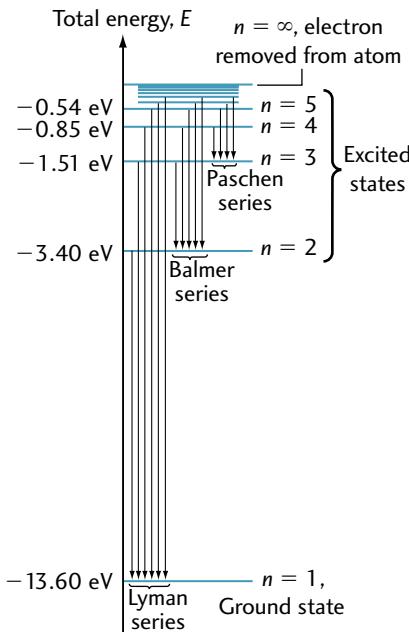
- c) $\Delta r = r_5 - r_4 = 1.323 \text{ nm} - 0.846 \text{ nm} = 0.477 \text{ nm}$

The difference between the two orbital radii is 0.477 nm.

Electron Energy

Since the nature of spectral lines is a function of photon energy, we can derive an expression for the energy of electrons and how this energy is quantized in Bohr's model of the atom.

Fig.12.23a Transitions in the hydrogen atom



Recall that the total energy of the electron is given by the equation

$$E = \frac{-ke^2}{2r} \quad \text{or} \quad E_n = \frac{-ke^2}{2r_n}$$

$$r_n = \frac{\hbar^2 n^2}{m k e^2}$$

Therefore,

$$E_n = \frac{-ke^2 m k e^2}{2(n^2 \hbar^2)}$$

$$E_n = \frac{-k^2 e^4 m}{2n^2 \hbar^2}$$

In this equation, \hbar , e , m , and k are constants; so,

$$E_n = \frac{-2.18 \times 10^{-18} \text{ J}}{n^2}$$

1 eV = $1.9 \times 10^{-19} \text{ J}$

If $n = 1$, then

$$E_1 = -2.18 \times 10^{-18} \text{ J}$$

$$= \frac{-2.18 \times 10^{-18} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}}$$

$$= -13.6 \text{ eV}$$

where E_n is the energy of an electron in joules (J).

In electron volts,

$$E_n = \frac{-13.6 \text{ eV}}{n^2}$$

EXAMPLE 6 Electron energies in the hydrogen atom

What is the energy of an electron in a hydrogen atom in the fourth and fifth energy levels?

Solution and Connection to Theory

Given

$$E_n = \frac{-2.18 \times 10^{-18}}{n^2}$$

$$E_4 = \frac{-2.18 \times 10^{-18} \text{ J}}{(4)^2}$$

$$E_4 = -1.36 \times 10^{-19} \text{ J} = -0.85 \text{ eV}$$

$$E_5 = \frac{-2.18 \times 10^{-18} \text{ J}}{(5)^2}$$

$$E_5 = -8.72 \times 10^{-20} \text{ J} = -0.545 \text{ eV}$$

The electron energies in the fourth and fifth energy level are -0.85 eV and -0.545 eV , respectively.

Bohr's work allowed scientists to predict the energies of the photons emitted from hydrogen gas perfectly! Bohr was awarded the 1922 Nobel Prize in physics for his work.

Photon Wavelength

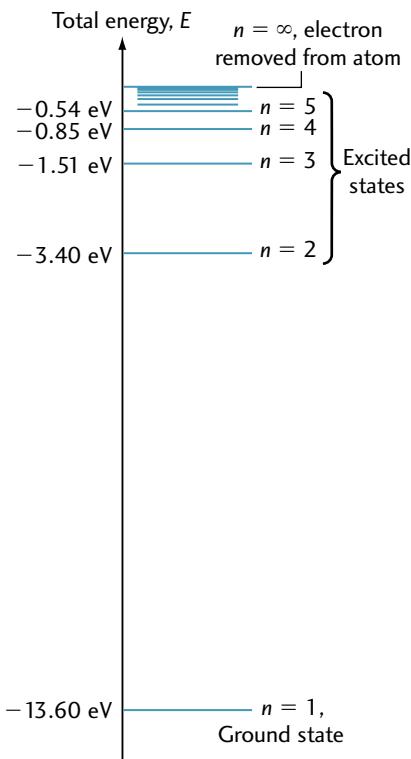
Now that we have defined the energies of electrons at various energy levels, we can study how the photons emitted by electrons appear as different wavelengths in a spectrum. If an electron moves from a higher energy level to a lower energy level (i.e., closer to the nucleus), it releases energy according to the equation $E_n = -2.18 \times 10^{-18} \text{ J}/n^2$, as we saw in the previous subsection.

Let us first consider photons in the visible range of the electromagnetic spectrum since they were the first to be analyzed by Balmer. The wavelengths in the visible spectrum range from 400 nm to 700 nm. This range of wavelengths corresponds to photon energies from 1.8 eV to 3.1 eV, calculated using Planck's equations. From Table 12.3, a photon jumping from $n = 2$ to $n = 1$ has an energy of 10.2 eV, which is in the UV range; so, electrons dropping from $n = 3$ or higher down to $n = 2$ radiate photons that have wavelengths in the visible range of the electromagnetic spectrum. The jump from energy level $n = 3$ to $n = 2$ releases 1.89 eV of energy, which corresponds to the wavelength of red light. An electron jumping from $n = 5$ to $n = 2$ emits a 2.86-eV photon, which is in the blue region of the spectrum.

Table 12.3 The Energy Levels of a Hydrogen Atom ($n = 1$ to $n = 5$)	
n	E_n (eV)
1	-13.6
2	-3.40
3	-1.51
4	-0.85
5	-0.54

Ionization Energy

Fig.12.23b The energy levels are closer together the farther they are from the atom's nucleus



If a hydrogen atom is ionized by having its electron removed, then *after removal*, the electron's energy level, n , is infinity and $E_n = 0$. By subtracting the energy of the lowest energy level ($n = 1$) from the new energy of the electron, we find that the energy of the ground state is also the **ionization energy** (see Figure 12.23b).

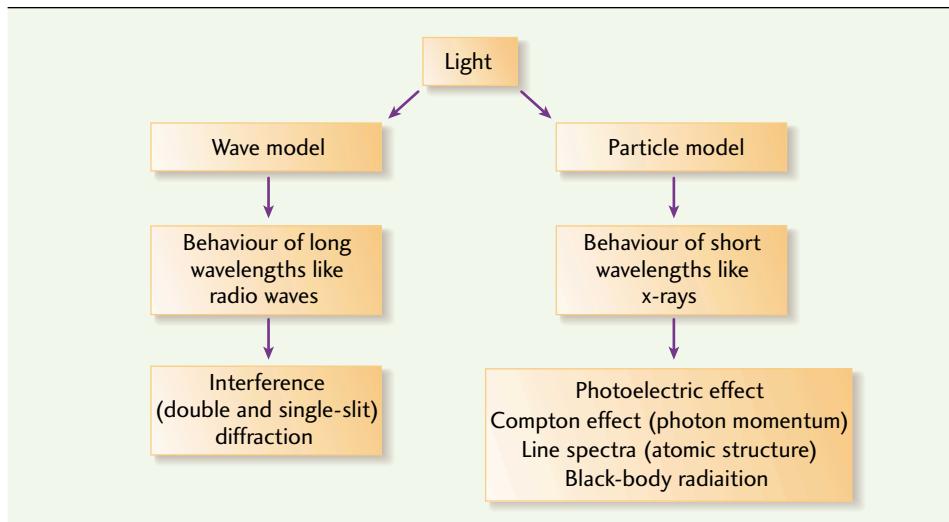
Bohr's Model applied to Heavier Atoms

For nuclei having more than one proton, Bohr's model requires only a few simple modifications to the above equations. However, the atoms can have only one electron because Bohr's model doesn't consider the effects of other electrons orbiting the nucleus. Therefore, the only other atoms (ions) to which Bohr's model can be applied are He^+ and Li^{2+} .

The Wave–Particle Duality of Light

Light is not a wave and it is not a particle; it is some kind of combination of the two that we cannot model or visualize. Physicists have come to the conclusion that this duality of light is a fact of life. It is referred to as the **wave–particle duality**. Niels Bohr suggested a **principle of complementarity**. It states that to understand a given experiment, we must use either the wave or the photon theory of light but not both, and we must be aware of both models of light if we are to fully understand light. Figure 12.24 lists some characteristics of light and the part of the model that best explains them.

Fig.12.24 Summary of the Wave–Particle Duality of Light





1. Choose three electron transfers between energy levels not discussed in this section and calculate the wavelengths of the resultant photons. Compare your answers with the values of the hydrogen spectra at the beginning of this section.
2. Astronomers spend a great deal of time studying spectra of stars and other objects. They call the accumulation of spectral lines at the end of a series a “forest.” Using the Lyman and Balmer series only, compute the wavelength separation of the last two spectral lines from $n = 8$ and $n = 7$ jumping down to $n = 1$ and $n = 2$, respectively.
3. Calculate the boundaries of the four spectral series (Balmer has been given to you).
 - a) Is there any overlap of series?
 - b) If so, are any spectral lines coincident?

12.7 Probability Waves

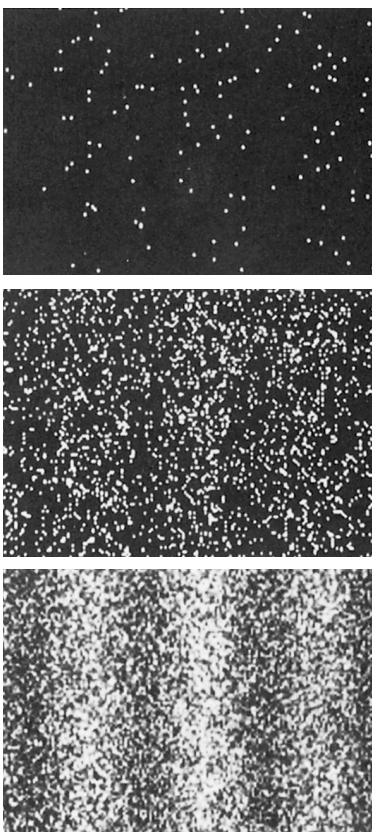
Probability is a very important concept in quantum mechanics because we are dealing with matter so tiny that we can't directly observe its behaviour. At best, we can know roughly where it is. To illustrate the role of probability and statistics in quantum mechanics, recall Young's double-slit experiment (Section 11.3) and the interference pattern produced by the two point sources of light (Figure 11.5). If we consider light as a waveform, like water, then the pattern observed is expected. However, in this chapter, we have learned that light is emitted in discrete packets called photons. We now need to reconcile the observed interference pattern with quantum theory.

If we had a source of single photons and a photographic detector, and emitted the photons slowly over time, would we see the pattern in Figure 12.25? Surprisingly, the answer is yes: a photographic plate would yield an image of an interference pattern even though the photons were emitted one by one and had no other photons to interfere with! Why? How can a single photon “know” which slit to go through and to hit the screen at a particular point?

The answer first has to consider the minuscule scale of subatomic particles. When we measure a tabletop, its length doesn't change in an unpredictable manner while we are measuring it. When we measure photons, we know that they are small packets of energy that can be observed discretely (a characteristic of a particle). If we wish to measure something with the photons, then the wavelength associated with them limits the precision of our measurement. Thus, observations of photons have to consider both photon aspects (energy and wavelength).

For example, if we were to scale the wavelength of a green-light photon (around 550 nm) to 1 m long, then proportionally, a millimetre would be about 1.8 km long! We cannot shine a light into the region around the slits to see what is going on because if a single photon from our light interacts with the light

Fig.12.25 Particles as well as waves exhibit diffraction patterns. In this experiment, when a sufficient number of electrons was beamed through a double slit, a diffraction pattern was observed



passing through the slits, then its path will be changed. If the photons from our light don't interact with the photons creating the diffraction pattern, we won't obtain any information about the diffraction pattern. *What we are measuring* is comparable in size to *what we are attempting to measure with (a photon)*. Since photons can interact, the measuring instrument affects the thing measured. This concept is known as **determinacy**.

To draw an analogy, suppose we wish to measure the amount of pitching (up-and-down motion) of a large cruise ship. If we use the wavelength of light as our measuring instrument, then we can arrive at a very accurate measurement because the wavelength of light is a factor of 10^9 less than the length of the ship. If we measure the ship's motion with mechanical waves having wavelengths of about a metre, the measurement of how much the boat moves relative to the water becomes less accurate. If we use water waves that are about the same length as the boat, these waves will make the boat move and thus interfere with our measurement.

Fig.12.26 The accuracy of a measurement depends on how much the measuring instrument interferes with the object being measured. The waves don't change the boat's length but by moving the boat, they change our ability to measure it accurately.

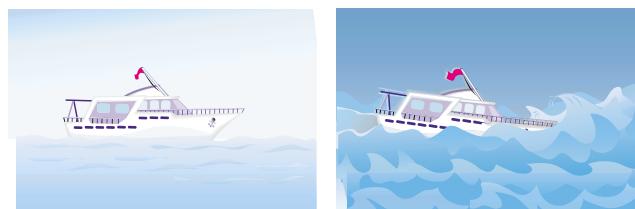


Fig.12.27 A photon probability distribution through a double slit

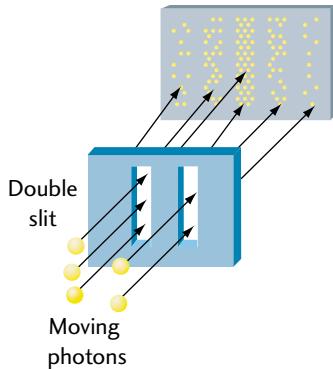
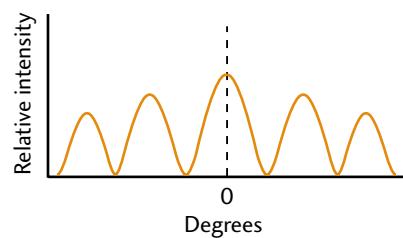


Fig.12.28 An intensity plot of a double-slit diffraction pattern, or the probability distribution of the photons after they pass through the slits. Regions of the graph where the plot is low are areas where photons tend not to occur; that is, regions of low photon probability. The more intense regions are where many more photons hit the target.

At the macroscopic level, we don't experience determinacy to an extent that will cause any precision problems. From de Broglie's work, we see that the wavelength of a baseball let alone a ship is incredibly small compared to light, so these waves don't interact and produce any measurable effects for us.

In Figure 12.27, because of determinacy, we can't know what is happening between the source of the photons and the detector screen. We can only look at the screen and state that it is *quite probable* that the photons will arrive at this or that location (i.e., the bright bands) and much less probable that they will arrive in another location (i.e., the dark bands). Thus, the location of a photon (or any subatomic particle) in an interference pattern can be determined by using wave equations, which predict the *probability* of a photon landing in a particular spot; that is, the photon's **probability distribution**.

The probabilistic approach to photon behaviour led to one of the most important theories of modern physics: the uncertainty principle.



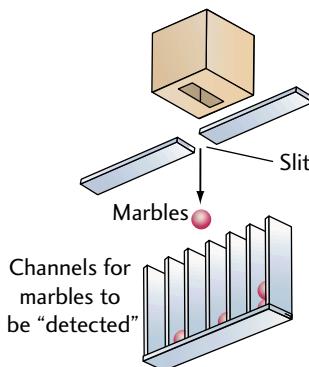
12.8 Heisenberg's Uncertainty Principle

In Section 12.7, we learned that the diffraction pattern produced by photons can be described as a probability distribution at the target. Let's consider two examples of probability distributions, one using mechanical forces and materials; and the other, a similar approach to the diffraction pattern.

A Hypothetical Mechanical Example of Diffraction

Suppose we set up a device such as the one shown in Figure 12.30.

Fig.12.30 A marble analogy for probability distribution



Let's imagine that the distance from the slit to the target is much greater than the width of the slit. In this experiment, the marbles are dropped from a stationary box one by one into channels on the target. If a typical marble has a diameter of 1 cm and the slit has a width of 1 m, then very few of the marbles will strike the edge of the slit and be deflected. As the width of the slit is reduced, a greater percentage of the marbles will be deflected by impacting on the edges of the slit, changing the distribution pattern (i.e., the number of marbles collected in each bin of the target). The narrower the slit, the greater the deflection and distribution of marbles.

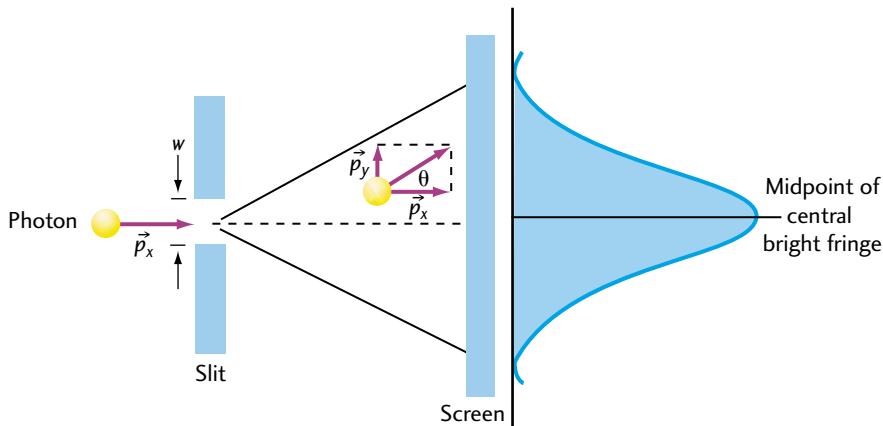
Returning to photons, we know from the Compton effect that a photon can excite an electron and be scattered. If we set up a slit of width close to the wavelength of light, then many photons will interact with the electrons in the material at the edge of the slit, causing the photons to scatter. Since most of the collisions will be glancing, the effects on the photons will be very slight and we would see a diffraction pattern similar to the marble experiment. Because we cannot observe the path of the photon from source to detector (due to determinacy), we can only know the *probability* of where most photons will land. This situation has important implications.

When the particles are sent from the source, they have momentum in the x direction only (i.e., straight ahead). If they are deflected, then they acquire momentum in the y direction as well (i.e., to either side). Considering the bright region in the centre of the diffraction pattern only, where about 85% of the photons land, we can determine a mathematical relationship that takes into account the uncertainty of where the photons will land (see Figure 12.31).

Fig.12.29 Werner Heisenberg



Fig.12.31 Most of the particles land in the bright region at the centre of a diffraction pattern



If the particles are scattered (deflected), then the particles that are scattered the most will have a larger y component of momentum p_y . Thus, any photon can have a p_y from zero to some maximum at the extreme end of the pattern. We know that the width of the slit coupled with the distance from the slit to the target determines the width of the diffraction pattern. After a photon is scattered, its momentum can be defined as

$$p = (p_x, p_y) = m(v_x, v_y)$$

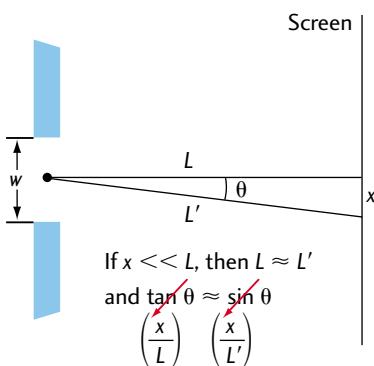
$$p = \sqrt{p_x^2 + p_y^2}$$

$$\theta = \tan^{-1}\left(\frac{p_y}{p_x}\right)$$

where θ is the angle measured from the original photon path central to the slit and the scatter path the photon took. These angles are typically quite small, and so we can approximate that

$$\theta = \frac{p_y}{p_x} = \frac{v_y}{v_x}$$

Fig.12.32 When θ is small, then $\tan \theta = \sin \theta = \theta$ (in rad)



where θ is measured in radians (see Figure 12.32).

In Section 11.8, we learned that the first-order minimum in a diffraction pattern can be located using the equation

$$\lambda = w \sin \theta_1$$

where λ is the wavelength of light passing through the slit, w is the width of the slit, and θ_1 is the angle of the path difference. Again, since θ is small, we can approximate that

$$\theta = \frac{\lambda}{w}$$

$$\frac{p_y}{p_x} \sim \frac{\lambda}{w}$$

Substituting de Broglie's equation, $p_x = \frac{h}{\lambda}$, we obtain

$$p_y \sim \frac{h}{w}$$

To incorporate the uncertainty aspects, we must consider that the photon can pass through any part of the slit, of width w . For diffraction to occur, the uncertainty in the y position at the slit is $\Delta y = w$. The uncertainty in y leads to uncertainty in the momentum, p_y , since h is constant; therefore,

$$\Delta p_y \Delta y \sim h$$

Heisenberg did a more rigorous analysis and obtained

$$\Delta p_y \Delta y \geq \hbar$$

For a photon to experience maximum diffraction, it must be near the edge of the slit to interact with the electrons in the particles of the edge (see Figure 12.33). Even though we know its location with some certainty, there is greater uncertainty in where it might go (its momentum). In the marble example, the marbles deflect if they hit the edge of the slit with any amount of their mass. Even the slightest change in their position changes the deflection angle greatly.

This idea can also be incorporated into energy considerations using the ideas of Planck and de Broglie. Recall from Planck that $E = \frac{hc}{\lambda}$.

$$\Delta p_y \Delta y \sim h$$

But $\Delta y = w$; therefore,

$$\Delta p_y \sim \frac{h}{w}$$

But $h = \left(\frac{E}{c}\right)\lambda$; therefore,

$$\Delta p_y \Delta y = \left(\frac{E}{c}\right) \left(\frac{\lambda}{w}\right) w$$

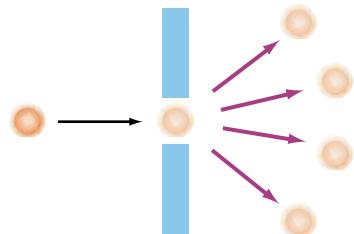
The ws cancel and $\frac{\lambda}{c}$ is time, so

$$\Delta E \Delta t \sim h$$

Heisenberg's actual expression was

$$\Delta E \Delta t \geq \hbar$$

Fig.12.33 Depending on the location of the ball when it hits the slit edge, its scattering angle can change a great deal. A slight change in the impact location results in a large change in the scattering angle.



The Uncertainty Principle: If we know the position of a particle, we cannot know its momentum and vice versa. Similarly, if we know the energy of the particle, we cannot know the length of time it has that energy and vice versa.

This phenomenon isn't observed with macroscopic objects since the speeds of all the particles that comprise them oscillate in all directions. Your position isn't uncertain because it is the average of the 10^{23} bonded atoms that constitute you on any given day.

EXAMPLE 7

Electron diffraction and uncertainty

In a diffraction pattern, an electron is deflected with a speed of 1000 m/s in the y direction. How precisely do we know its position in the slit?

Solution and Connection to Theory

Given

$$\hbar = 1.0546 \times 10^{-34} \text{ J}\cdot\text{s} \quad m = 9.11 \times 10^{-31} \text{ kg} \quad v = 1000 \text{ m/s}$$

From $\Delta p_y \Delta y \geq \hbar$, we solve for Δy :

$$\Delta p = m \Delta v$$

$$\Delta y \geq \frac{\hbar}{m \Delta v}$$

$$\Delta y \geq \frac{1.0546 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.11 \times 10^{-31} \text{ kg})(1000 \text{ m/s})}$$

$$\Delta y \geq 0.11 \mu\text{m}$$

The uncertainty in the electron's position is greater than or equal to 0.11 μm .

EXAMPLE 8

The uncertainty principle — position of an alpha particle

An alpha particle (ionized helium nucleus) is emitted from the decay of U^{238} . If this particle has an energy of 34 keV, what is the uncertainty in its position?

Solution and Connection to Theory

Given

$$E = 34 \text{ keV} \quad m_\alpha = 6.7 \times 10^{-27} \text{ kg}$$

First we need to calculate the momentum of this particle from its kinetic energy,

$$E_k = \frac{1}{2}mv^2$$

$$v^2 = \frac{2E}{m_\alpha}$$

$$v^2 = \frac{2(34\,000 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{6.7 \times 10^{-27} \text{ kg}}$$

$$v^2 = 1.63 \times 10^{12} \text{ m}^2/\text{s}^2$$

$$v = 1.27 \times 10^6 \text{ m/s}$$

For momentum,

$$\Delta p = mv$$

$$\Delta p = 2(6.7 \times 10^{-27} \text{ kg})(1.27 \times 10^6 \text{ m/s})$$

$$\Delta p = 8.5 \times 10^{-21} \text{ N}\cdot\text{s}$$

To find the uncertainty in position,

$$\Delta p_y \Delta y \geq \hbar$$

$$\Delta y \geq \frac{\hbar}{\Delta p}$$

$$\Delta y \geq \frac{1.055 \times 10^{-34} \text{ J}\cdot\text{s}}{8.5 \times 10^{-21} \text{ N}\cdot\text{s}}$$

$$\Delta y \geq 1.24 \times 10^{-14} \text{ m}$$

The uncertainty in the particle's position is $1.24 \times 10^{-14} \text{ m}$.

Heisenberg's Uncertainty Principle and Science Fiction

Heisenberg's uncertainty principle is the reason why some of the devices in science fiction movies are impossible in practice. Consider the transporter beams in the television series *Star Trek* that take a person apart molecule by molecule, keep all the molecules organized, beam them somewhere, and then put them all back together in the same manner. According to the uncertainty principle, it's impossible to identify individual particles and keep track of them.

Another aspect of science fiction that is currently restricted to us is time travel. While relativistic travel might be possible for going into the future, going back in time is theoretically impossible. To do so would require an undoing of the random events, including random atomic oscillations, which is impossible, because the information is lost and could not be obtained with sufficient precision due to the uncertainty principle. Thus, we have an "arrow of time," where time progresses in one direction only.



1. A particle's velocity is known to an uncertainty of $1 \mu\text{m/s}$. What is the uncertainty of the particle's position? Put this quantity into macroscopic perspective.
2. What are the units of the time-based version of the uncertainty principle? Do these units balance with the units of \hbar ? What quantity do these units represent? What is the significance of this quantity in interactions?
3. Discuss the marble analogy at the beginning of this section. Is it a reasonable approximation of what is observed with light? Why or why not?
4. Why did it take so long to discover the uncertainty principle?
5. Research and explain an observation from elementary chemistry suggesting the randomness of atomic behaviour.
6. What is the uncertainty in position of a proton with mass $1.673 \times 10^{-27} \text{ kg}$ and kinetic energy 1.2 keV ?
7. Current research suggests that time is indeterminate at subatomic levels. What effect, if any, does this indeterminacy have on macroscopic objects?

12.9 Extension: Quantum Tunnelling

According to the uncertainty principle, we cannot know for certain the exact speed, location, or energy of the particle, only average values. The consequence of this principle is that particles that should be unable to cross given energy boundaries, based on the amount of energy they possess, can cross these boundaries because the given energy of the particles isn't known exactly.

Suppose we have an electron contained by a potential barrier such as an electric or magnetic field. In classical physics, this electron would have to remain there until the potential barrier was reduced or the kinetic energy of the electron was increased so that it could bounce out. In quantum mechanics, an electron's location cannot be known with certainty, so it is described by a probability function (Figure 12.34).

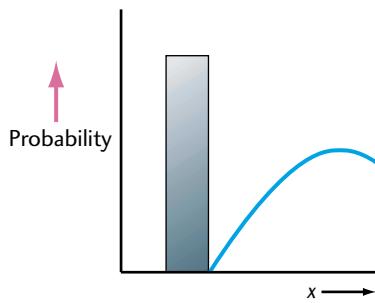


Fig.12.34 The probability of an electron's location between two potential barriers (classical physics). The particle is bound.

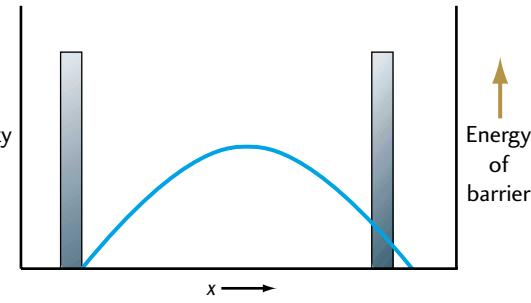
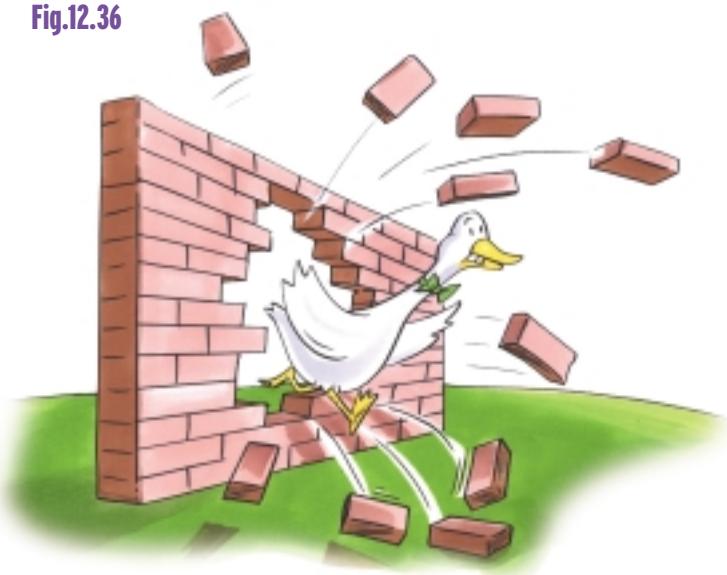


Fig.12.35 The probability of an electron's location between two potential barriers (quantum mechanics). The particle can extend beyond the barrier and escape.

In Figure 12.35, the curved line represents the probability of the particle's position. The two rectangles at the end of the particle's probability plot are potential barriers, such as an electric field. In this classical plot, the particle must remain between the potential barriers. The application of the uncertainty principle, however, requires that the positions of the particle be permitted into the potential barriers. If the barrier is sufficiently narrow, the particle can pass through it.

When we throw a baseball against a wall, we would be very surprised if it passed through the wall! Why don't larger objects tunnel through barriers? Large particles *can* tunnel; however, the probability is so small that for practical purposes, it's impossible. For a large object to tunnel, all the particles in the object would have to have exactly the same probability function at the same time, so when they arrive at the potential barrier, they *all* would have the same random chance to proceed into the barrier at the same instant. Even for small atoms consisting of only a few particles, the likelihood of all the particles having the same probability function is very small. If one of the particles tunnelled, the other particles would bounce off the potential barrier and the bonds connecting them would pull the tunnelling particle back. Considering that the average person consists of about 10^{23} atoms, you shouldn't be running into any brick walls with the hope of tunnelling through!

Fig.12.36



A Demonstration of Quantum Tunnelling

You can demonstrate the tunnelling effect by using a glass of water. Place your hands on either side of the glass, without touching it. When you view the glass from the top, you can't see your hands through the glass because the internal reflections of the light rays at the boundary between the water and the glass don't allow any light from your hands to reach your eyes. When you grip the glass firmly with your hand, you can now see your hand faintly through the glass. Although the majority of photons are reflected internally in the glass, some light waves are transmitted through the water-glass boundary and dissipate quickly. Some of them are reflected back into the glass and water by your hand to your eyes. You can see your hand because it reflects the photons that had tunnelled through the glass.





The Scanning Tunnelling Microscope

Fig.STSE.12.1 A scanning tunnelling microscope



Fig.STSE.12.2 DNA as seen through the STM; magnification X2 000 000

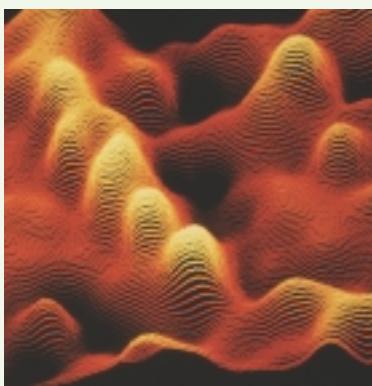
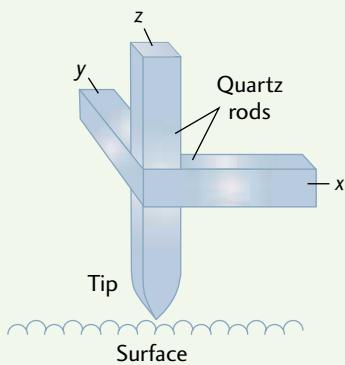


Fig.STSE.12.3 The tip and probe of the STM



Some of the most impressive images in our new technological era are super-high-magnification, 3-D images of surfaces that reveal the locations of individual atoms. These images are produced by a special device called a **scanning tunnelling microscope** or **STM**, which operates on the principle of quantum tunnelling. To increase the probability of quantum tunnelling, the object being studied and the probe of the STM must be placed very close together, a distance measurable in picometres ($1 \text{ pm} = 10^{-12} \text{ m}$). The probe is located near the object's surface and moved around to scan a small area. To avoid contamination from the atmosphere and other materials in the air, scanning tunnelling is usually done in a vacuum.

The problem of positioning an incredibly small probe with atomic precision was solved by using a unique property of crystals, called **piezoelectricity**. When a crystal, typically of quartz, has a small voltage applied across it, the dimensions of the crystal change slightly. These changes are linear and controllable by the electric field set up by the external potential. One polarity causes the crystal to increase its length, while the other polarity causes it to contract. If the voltage is applied to the piezoelectric arms carefully and in small increments, then the probe of the STM can be located quite reliably and made to follow the contour of any atomic-scale surface by this incredibly accurate positioning technique.

Three piezoelectric crystalline rods, each in one of the x , y , z directions, support the probe. Changing the potential difference across one crystal will move the probe in the desired direction. The z rod moves the probe up and down, making sure that the conducting tip is located at a proper distance from the material.

A potential difference of about 10 mV is applied to the tip of the STM. Electrons from the object being studied are able to quantum tunnel across the small distance to the tip. The voltage applied to the tip causes an electric field that reduces the potential barrier for electrons from the target to tunnel up through the probe, creating a probe current. The current is extremely sensitive with respect to distance (due to the sensitivity of tunnelling and distance), which allows us to map the surface of the object in remarkable resolution. The bumps in the image that look like round balls (Figure STSE.12.2) represent the locations where the probability functions of the atoms on the object and the tip overlap most strongly.

The STM has many commercial applications. Its ability to image crystalline structures and to look for impurities or variations in bond structure is of great use to semiconductor manufacturers. The STM is used to observe large molecules such as proteins and other genetic material. It is also used to scan the surfaces of metals in order to determine the conformation and size of aggregates and molecules, to characterize surface roughness, and to observe defects.

Fig.STSE.12.4 The tip of the STM



Design a Study of Societal Impact

Research applications of the STM not mentioned here. Evaluate its contribution to research and to our society. The STM is reasonably new, so innovative techniques for using its capabilities are still being developed.

Research the history of this device and when it was invented. Compare its performance to that of the electron or other powerful microscopes. Discuss the differing theories of its operation and its ability to image small items.

Design an Activity to Evaluate

Find out the approximate size of the atoms in an STM image and also the size of the smallest detail in the image.

Investigate piezoelectricity and determine what potential differences might be used to shunt the tip around very short distances. Which crystals would work best? Why?

Build a Structure

We can simulate an STM's operation using magnets. Construct a device using permanent magnets to represent atoms and an electromagnet to represent the STM probe. Support the vertical (z) axis of this probe by a spring with a scale on it so that you can observe the force. The magnetic probe needs to be held secure in the x and y directions until it is moved to map the object. You can calculate the spring's force by reading a properly calibrated spring scale or by using force-sensing technology like CBL-type equipment. This exercise can also be extended to map the size and shape of an object.

SUMMARY SPECIFIC EXPECTATIONS

You should be able to

Understand Basic Concepts:

- Explain why classical physics was inadequate to explain observed phenomena.
- List and describe three or four of the major developments in the birth of quantum mechanics.
- Describe the photoelectric effect experiment and its significance.
- Understand how photons have momentum and how it is computed.
- Explain why de Broglie made the suggestion of matter waves.
- Understand how single photon diffraction experiments cannot be physically explained and why we need a probability function.
- State the mathematical description of the uncertainty principle and state its significance.
- Explain how quantum tunnelling occurs, and the principle behind the STM.

Develop Skills of Inquiry and Communication:

- Carry out experiments to simulate quantum tunnelling in action.
- Describe and carry out experiments related to the photoelectric effect.
- Using computer technology, create models of wave packets and, if possible, animate them.

Relate Science to Technology, Society, and the Environment:

- Describe the benefits of the STM to the technological world.
- Describe how only certain types of light give us sunburn.
- Explain why sunburn occurs at a molecular level.
- Explain the significance of randomness at the atomic level.

Equations

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}}{T}$$

$$E = hf = \frac{hc}{\lambda}$$

$$E_{k_{\max}} = E_{\text{photon}} - W_0$$

$$p = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p}$$

$$E_{\text{x-ray}} = hf_f + \frac{1}{2}mv^2$$

$$\Delta p_y \Delta y \geq \frac{h}{2\pi}$$

$$\frac{h}{2\pi} = \hbar$$

$$\Delta E \Delta t \geq \frac{h}{2\pi}$$

$$L_n = mv_n r_n$$

$$r_n = \frac{\hbar^2 n^2}{m_e k e^2}$$

$$r_n = \frac{5.29 \times 10^{-11} \text{ m}}{n^2}$$

$$E_n = \frac{-2.18 \times 10^{-18} \text{ J}}{n^2}$$

EXERCISES

Conceptual Questions

1. How would you explain a photon of electromagnetic radiation to a person not versed in physics?
2. Explain why we find sunburn painful. What is the actual cause?
3. Using concepts learned in this chapter, explain why we don't get a sunburn from visual light.
4. How would the universe as we know it be different if $\hbar = 0$?
5. Investigate how the electron volt was created and why it was given this particular name.
6. How is Wien's law related to a black-body spectrum?
7. What is the meaning of the work function, W_0 ?
8. Why do you think mass equivalence is valid for photons when considering momentum conservation in the Compton effect?
9. What is an empirical relationship?
10. What is determinacy? Give an example of an everyday event that would be affected if determinacy existed at the macroscopic level.
11. Why doesn't Heisenberg's uncertainty principle affect how we observe objects' positions and speeds in the macroscopic world?
12. What other device besides the STM operates using the principle of quantum tunnelling?
13. Why do the spectral lines in the hydrogen atom become closer together farther away from the nucleus?
14. As a black body, a mercury lamp emits much of its light in the visible spectrum.
 - a) Why is this type of lamp hazardous to use?
 - b) What might be done to reduce this hazard?

15. What would be the implication of two particles of radically different mass having the same de Broglie wavelength?
16. Explain why it was necessary to quantize angular momentum in Bohr's model of the atom.
17. Suppose an x-ray photon strikes a carbon atom as described in Section 12.4 and the scatter angle is observed. Does knowing both the speed of light and the scatter angles violate the uncertainty principle? Explain your answer.
18. Research Brownian motion. How much does the uncertainty principle affect this phenomenon?

Problems

-  **12.2 The Quantum Idea**
19. You observe a hot piece of metal. Your spectroscope indicates that the brightest wavelength is 597 nm. What is the temperature of the metal in degrees Celsius?
 20. In 1965, Arno Penzias and Robert Wilson discovered cosmic background radiation (CBR). This radiation is thought to be an afterglow of the Big Bang when the universe was much denser than today. CBR indicates a temperature of 2.7 K. What is the peak wavelength associated with this temperature?
 21. Jupiter's cloud tops have been measured to have a temperature of about 125 K. What is the peak wavelength for this radiation and to which part of the electromagnetic spectrum does it belong?
 22. A 2-W laser emits a coherent light beam at a wavelength of 632.4 nm. Assuming that all the power is radiated, how many photons leave the laser tube every second?

12.3 The Photoelectric Effect

23. If you were performing the photoelectric effect experiment on a surface that is covered with gold, what stopping potential would you expect if the incident photons had an energy of 4.5 eV?
24. If light of wavelength 440 nm is shone on a nickel plate, will the nickel plate exhibit the photoelectric effect? Why or why not?
25. If the headlight of a car radiates at 30 W and the peak wavelength of the emitted light is 540 nm, how many photons per second does this light radiate?
26. If the work function of a particular metal is 3.0 eV and the incident radiation has a wavelength of 219 nm,
- what is the cut-off frequency for this material?
 - what is the maximum energy of any ejected photons?
27. Controllers of satellites have to be watchful of the photoelectric effect because satellites are covered with metal and are in a vacuum. If too many electrons are liberated, the bonding structure of the satellite skin can change or create unwanted electrical currents.
- How does the work function of a given metal influence your choice of the material to use to build a satellite?
 - What is the longest wavelength that could affect this satellite?
28. What is the longest wavelength of a photon that could generate the photoelectric effect on a piece of platinum?
29. What would be the significance of the photoelectric-effect graph if it
- passed through the origin?
 - had a positive y intercept?

12.4 Momentum and Photons

30. A photon has a wavelength of 400 pm.
- What is its frequency?
 - What is its momentum?
 - What is its mass equivalence?
31. What is the momentum of a photon with an energy equal to the rest energy of a proton?
32. An electric stove produces many infrared photons. If the peak wavelength of the radiation coming from a stove element is 10 μm , what is the momentum of the released photons?
33. X-rays of wavelength 1 nm are scattered from a carbon target at an angle of 43° to the original path of the x-ray beam. What is the difference between the original wavelength and the one observed from the scattered photons? (This effect is known as the Compton shift.)
34. An electron at rest is struck by an x-ray photon. If the scatter angle is 180° and the final speed of the electron is $7.12 \times 10^5 \text{ m/s}$, what was the wavelength of the incident photon?
35. If a photon with an incident wavelength of 18 pm loses 67% of its energy, what is the corresponding Compton shift as a percentage?
- ## 12.5 De Broglie and Matter Waves
36. A 45-g golf ball is struck and leaves the club at a speed of 50 m/s. What is the de Broglie wavelength associated with this ball?
37. In some scattering experiments, the speed of the particles is tuned so that their de Broglie wavelength has a specific value. If a wavelength of 0.117 nm is required, how fast must a neutron be travelling to achieve this wavelength?
38. How fast would a proton have to travel to possess the same de Broglie wavelength of the golf ball in problem 37? Is this speed possible?

- 39.** **a)** What is the de Broglie wavelength of an electron with a kinetic energy of 50 eV?
b) How does this wavelength compare with the size of a typical atom?

12.6 The Bohr Atom

- 40.** What wavelength is released if a photon drops from energy level $n = 5$ to energy level $n = 2$? In which part of the spectrum is this wavelength? If it is in the visible part of the spectrum, what is its colour?
- 41.** Using the equations given in this chapter, calculate the energy in eV required to cause an electron's transition from
a) $n = 1$ to $n = 4$.
b) $n = 2$ to $n = 4$.
- 42.** What is the difference in radius between the second and third energy levels of the Bohr atom?
- 43.** Calculate the centripetal force required to maintain an electron in the first energy level.
- 44.** How often does the electron in problem 43 "orbit" the nucleus?

- 45.** Compare the frequency you calculated in problem 44 with the frequency of a photon emitted by a drop to this energy level.
- 46.** We know that electrons "orbit" the atom in strange paths or zones called orbitals. Orbitals don't have the shape of simple planetary orbits as in the Bohr model. How can this discrepancy in the location of electrons be reconciled with Bohr's model given that Bohr's model predicts energy levels very accurately?

- 47.** Research the significance of the width of the emission or absorption lines generated by the Balmer series.

- ### 12.8 Heisenberg's Uncertainty Principle
- 48.** If an electron is travelling at 1 km/s, how uncertain is its position?
- 49.** An air bubble in a glass of water has a diameter of 1 mm. What is the maximum speed of an O_2 molecule that is in 0.1 mm of the bubble?

Purpose

To measure the emission lines of hydrogen and compare them to those predicted by the Bohr model of the atom

Equipment

Hydrogen vapour lamp

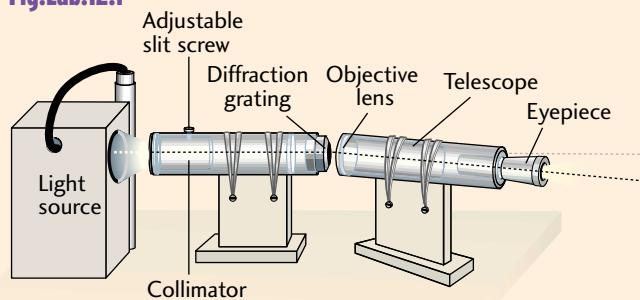
Spectroscope (calibrated)

Procedure

- Assemble all pieces of equipment as shown in Figure Lab.12.1.
- Switch on the hydrogen lamp and turn off all other sources of light. (Remember that incandescent lights produce a continuous spectrum and fluorescent lights produce large bands of emission. Both types of illumination will affect what you see through the spectroscope.)
- Observe the light from the hydrogen vapour lamp through the spectroscope.
- Record the wavelengths of all the emission lines visible in the spectroscope.

Optional: Using a Web cam or other digital equipment, attempt to image the lines through the spectroscope. (The spectral lines may be too dim for most devices, but using digital equipment would permit the creation of an actual spectrum during image processing.) Imaging should be done in black and white, if possible.

- Switch your spectroscope with two other groups and repeat step 4 each time.
- Determine your best estimate of the value for each emission line and the distance between each line.
- Calculate the uncertainty of these observations.
- If available, attempt this experiment on singularly ionized helium (He II or He^+).

Fig. Lab.12.1**Discussion**

- How did the measured wavelengths compare to the expected values?
- Were the values within the uncertainty of the measurement? Why or why not?
- How wide were the spectral lines? What is the significance of this width? What properties of the vapour might be inferred from this information?
- How many Balmer lines were present? If some of the lines were missing, give a reason why.
- Does the width of the lines affect the ability to discern the Balmer lines from $n = 5, 6, 7$, etc.?
- Do the lines vary in brightness?
- Optional:** If you used a digital camera, do *not* manipulate the image digitally because doing so will change the relative intensities of the light seen. Using suitable image processing software, take a profile of the image through the spectrum. Use the intensity values of your profile as your data. Combine your data from five different images to obtain a representative spectrum.

Conclusion

Explain how your observations of emission spectra confirm Balmer's and Bohr's predictions of the spectra for the hydrogen atom. Discuss your ability to make these observations with precision and the significance of the line width.



The Photoelectric Effect I

Purpose

To observe the photoelectric effect

Equipment

Arc lamp or shielded UV light

Electroscope

Polished piece of zinc

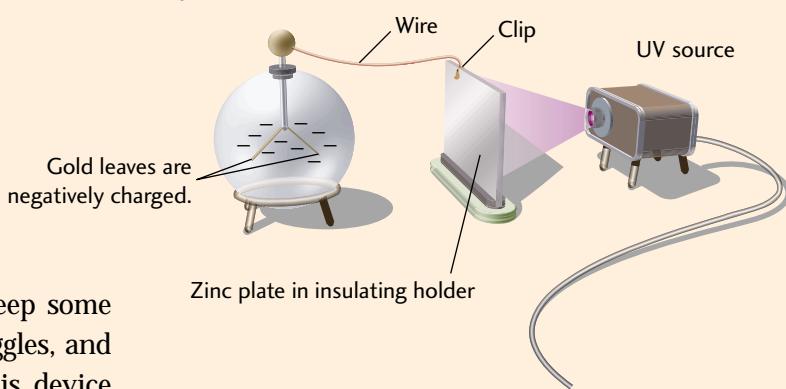
Safety Consideration

Arc lamps are sources of UV light. Keep some distance from them, wear protective goggles, and do not stare at the arc. The use of this device should be kept to a minimum.

Procedure

1. The photoelectric effect releases electrons from the surface of a metal when it is struck by sufficiently energetic photons. This release of electrons tends to make the piece of metal increasingly positive. If the electroscope is positively charged, then we need only observe what happens to it. If the electrons are leaving the plate, then the positive charge will be dissipated. If the net charge on the zinc plate is not changing, then the electroscope will not move. Make sure that the electroscope is positively charged.
2. A standard piece of glass prevents the transmission of UV light. Hold a piece of glass between the arc lamp and the zinc plate. Start the arc lamp and observe the electroscope.
3. Now remove the piece of glass and observe the effect on the electroscope.

Fig. Lab.12.2



Discussion

1. What other explanation could be given for the photoelectric effect phenomenon?
2. Why was zinc chosen as the metal?
3. Why does it have to be polished?
4. What type of light emanates from a carbon lamp?
5. What methods might you use to measure the current flow from the metal? These methods would have to measure the amount of charge on the electroscope without discharging it.

Conclusion

Explain the photoelectric effect in your own words as based on the collection of data from this experiment. Discuss how this experiment is strong evidence for the quantum model compared to the classical model of light propagation.



The Photoelectric Effect II

Purpose

To measure the photoelectric effect and analyze the data to determine the value of Planck's constant

Safety Consideration

This experiment uses UV light. To reduce the amount of exposure to the skin and eyes, stand a short distance away from the light source and wear protective goggles.

Fig. Lab.12.3a A photoelectric effect apparatus

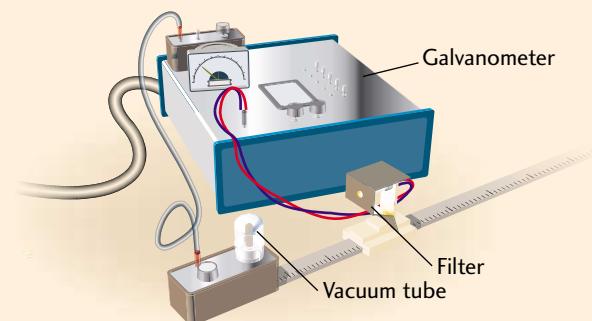
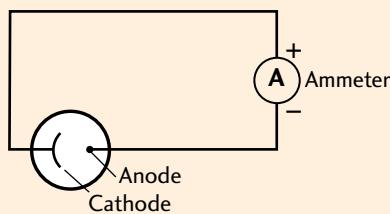


Fig. Lab.12.3b In the vacuum tube, the cathode emits electrons when struck by light. The anode collects the electrons.



Equipment

A photoelectric effect apparatus

Procedure

- Set up the photoelectric effect apparatus as shown in Figure Lab.12.3a.
- Choosing the reddest line filter, expose the vacuum tube to the various possible intensities of incident light. Record the values registered on the galvanometer for each

intensity of light. Adjust the stopping potential to obtain the maximum current.

Optional: If you have voltage-sensing equipment and know how to amplify voltages, the readings of the current as the stopping potential is adjusted can be stored in the computer or calculator for an even more careful analysis.

- Repeat step 2 with each available line filter.
- If you have another vacuum tube containing a different material, repeat steps 2 and 3 using this vacuum tube.
- Determine the uncertainty in all your measurements and calculations.

Discussion

- Using the data from the experiment, plot a graph of the energy of the photons versus the stopping potential. Extend the line back to the y axis and determine the significance of all the coefficients. What type of regression is appropriate here?
- Determine the work function, W_0 , of the material inside the tube. Does this value agree with the accepted value for this substance?
- Repeat step 2 for any other tubes you used.
- Derive the value of Planck's constant (h) from your measurements, given the work function of the material you calculated.
- Interpret the consequences if your graph in step 1 had
 - no y intercept.
 - a positive y intercept.

Conclusion

Explain the photoelectric effect in your own words based on what you have learned in this experiment. Discuss how this experiment is strong evidence for the quantum model as compared to the classical model of light propagation.

The World of Special Relativity



Chapter Outline

- 13.1 Inertial Frames of Reference and Einstein's First Postulate of Special Relativity
- 13.2 Einstein's Second Postulate of Special Relativity
- 13.3 Time Dilation and Length Contraction
- 13.4 Simultaneity and Spacetime Paradoxes
- 13.5 Mass Dilation
- 13.6 Velocity Addition at Speeds Close to c
- 13.7 Mass–Energy Equivalence
- 13.8 Particle Acceleration
- The High Cost of High Speed

LAB 13.1 A Relativity Thought Experiment

By the end of this chapter, you will be able to

- explore abstract ideas through thought experiments
- describe the influence of special relativity on science and technology
- solve relativistic mass, time, and length problems
- apply $E = mc^2$ to the laws of conservation of mass and energy

13.1 Inertial Frames of Reference and Einstein's First Postulate of Special Relativity

Fig.13.1 Albert Einstein

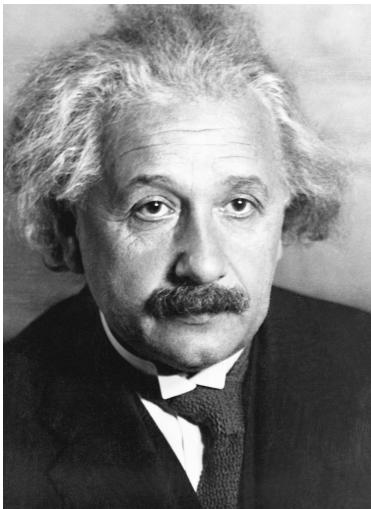
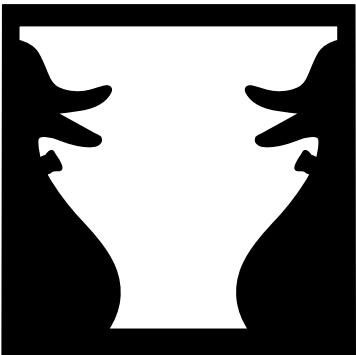


Fig.13.2 Vase or ducks?



For several centuries, Newton's laws of motion seemed to describe nature perfectly. Then, in 1905, a 26-year-old scientist named Albert Einstein (Figure 13.1) showed that Newton's second law was invalid at speeds close to that of light, c . Prior to Einstein, when net force was equated to mass times acceleration ($\vec{F}_{\text{net}} = m\vec{a}$), it was assumed that mass didn't change. However, using two basic postulates that we will study in this chapter, Einstein was able to show that mass depends on the speed of the object. When travelling at highway speeds ($v = 30 \text{ m/s}$), the increase in mass is too small to measure. It works out to be only one part in a trillion because this speed is only a tiny fraction of the speed of light. This correction is so minute for speeds less than c , that the Newtonian laws of mechanics accurately describe a wide range of basic phenomena, from planetary to atomic physics.

In Figure 13.2, do you see a vase or two silhouettes of Albert the duck's profile? Is only one viewpoint correct? Each of us has our own point of view or frame of reference based on our perception and past experience. In physics, a **frame of reference** is the point of view from which we observe motion. For example, eating a dinner at home is about the same as eating dinner in an airplane as long as the plane flies at a steady velocity and doesn't hit an air pocket, which jolts everything. If food happens to slip off your fork, it will fall straight to the floor because your velocity relative to your dinner is the same at home or on the plane; that is, your frame of reference with respect to your dinner is the same in both situations.

Consider the perspective of two objects moving side by side, in opposite directions to one another. For example, you are sitting by the window in a Via Rail train waiting to depart from the station and right beside you is a commuter train that appears to start moving backward. Or perhaps your train is starting to move forward? Sometimes, it's hard to tell. In physics, we say that your velocity with respect to, or relative to, the commuter train is the opposite of the train's velocity relative to you. Algebraically,

$$_y V_t = -_t V_y$$

Relative Motion

You moving away from me at 4 m/s [W] is the same as me moving away from you at 4 m/s [E].

$$_y V_m = -_m V_y, \text{ or}$$

$$4 \text{ m/s} [W] = -4 \text{ m/s} [E]$$

Now consider watching an object fall while you are moving sideways at a constant velocity. A conductor standing in the moving commuter train drops his watch and it falls to the floor. From his perspective, he correctly states that the vertical velocity of the watch increased as the watch fell, while its horizontal velocity was zero. From the stationary Via Rail train, you observe that the watch's motion is parabolic, accelerating downward

while travelling at a uniform horizontal velocity away from you. The laws of physics for you and the conductor are the same, but the path of the watch described by each of you is different (see Figure 13.3).

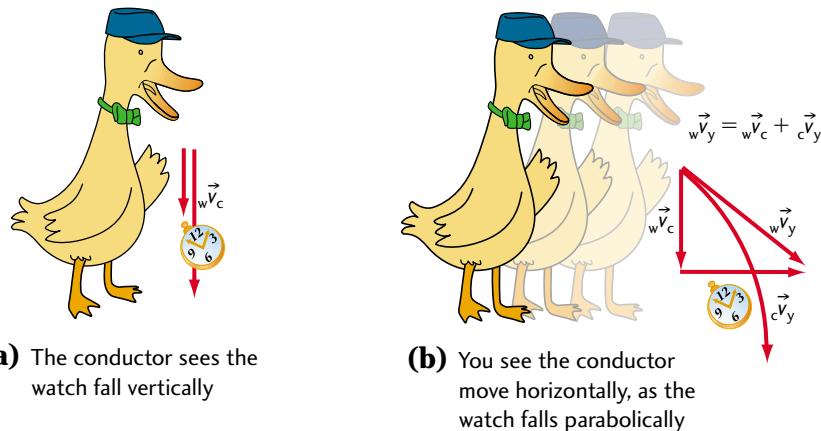


Fig.13.3 The fall of a watch from two inertial frames of reference

Since both you and the conductor have uniform velocities, there is no unbalanced force acting on either of you. Both your frames of reference or viewpoints satisfy Newton's first law of motion. Such non-accelerating viewpoints are called **inertial frames of reference**.

Einstein's First Postulate of Special Relativity: The laws of physics are the same for observers in all inertial frames of reference.

Let's examine how the laws of physics are the same for a stationary and a moving observer, both witnessing the same event.

EXAMPLE 1 Turkey trouble

Nadia is sitting at the dinner table when her overstuffed 6.0-kg turkey suddenly explodes into two equal pieces. One piece moves 2 m/s [L] and the other travels 2 m/s [R]. At that very moment, Jerry walks by the table at 2 m/s [L]. Find the change in kinetic energy, ΔE_k , of the turkey from both Nadia's and Jerry's points of view. Refer to Figure 13.4.

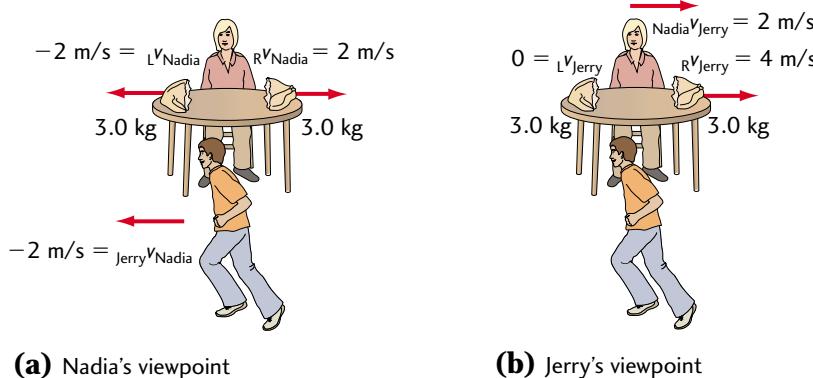


Fig.13.4 The change in kinetic energy in two inertial reference frames

Solution and Connection to Theory

Given

$$m_0 = m_L + m_R = 3.0 \text{ kg} + 3.0 \text{ kg} = 6.0 \text{ kg}$$

$$v_{Nadia}^0 = 0 \text{ m/s}$$

$$v_{Nadia}^R = 2 \text{ m/s [R]}$$

$$v_{Nadia}^L = 2 \text{ m/s [L]} = v_{Jerry}^Nadia$$

$$\Delta E_k = ?$$

Nadia's stationary reference frame:

Using the equation $E_k = \frac{1}{2}mv^2$, we find the change in kinetic energy of the turkey by subtracting its original kinetic energy, $E_k = 0$, from the kinetic energy after it explodes. After the explosion, the kinetic energy of the turkey, E'_k , is

$$E'_k = \frac{1}{2}(3.0 \text{ kg})(2 \text{ m/s})^2 + \frac{1}{2}(3.0 \text{ kg})(2 \text{ m/s})^2 = 12 \text{ J}$$

$$\Delta E_k = E'_k - E_k$$

$$\Delta E_k = 12 \text{ J} - 0 \text{ J}$$

$$\Delta E_k = 12 \text{ J}$$

Jerry's moving reference frame:

Since Jerry is moving 2 m/s [L], then relative to Jerry, the turkey is moving 2 m/s [R]. From his reference frame, the initial kinetic energy of the turkey is

$$E_k = \frac{1}{2}(6.0 \text{ kg})(2 \text{ m/s})^2 = 12 \text{ J}$$

After the explosion, the 3-kg half going left has a speed of 0 m/s relative to Jerry, while the other half has a speed of 4 m/s away from him. Thus,

$$E'_k = \frac{1}{2}(3.0 \text{ kg})(0 \text{ m/s})^2 + \frac{1}{2}(3.0 \text{ kg})(4 \text{ m/s})^2 = 24 \text{ J}$$

$$\Delta E_k = E'_k - E_k$$

$$\Delta E_k = 24 \text{ J} - 12 \text{ J}$$

$$\Delta E_k = 12 \text{ J}$$

Since the laws of physics are the same in all inertial reference frames, Nadia and Jerry agree that 12 J of energy have been transferred, even though they disagree on the initial and final values of E_k .



1. In Example 1, show that momentum is conserved in both Nadia's and Jerry's frames of reference.
2. In society, conflicting points of view often lead to complex legal trials. Think of a situation, real or imagined, involving differing points of view.

3. Find or create an image of an optical illusion (similar to Figures 13.2 or 13.5), or an image expressing contrary points of view to share with the class.

Fig.13.5 A spatial illusion by M.C. Escher (1898–1972)



13.2 Einstein's Second Postulate of Special Relativity

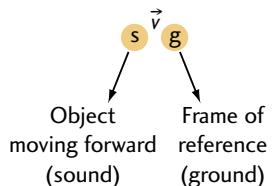
When a friend calls out to you on a windy day, does she seem to be farther away than she really is when she shouts into the wind? Recall from Chapter 2 that in Newtonian physics, we use the relative additions of wind and sound velocities to show the velocity of sound with respect to the ground:

$$_s\vec{V}_g = _s\vec{V}_w + _w\vec{V}_g$$

where $_s\vec{V}_g$ is the velocity of sound with respect to the ground, $_s\vec{V}_w$ is the velocity of sound with respect to the wind, and $_w\vec{V}_g$ is the velocity of the wind with respect to the ground (see Figure 13.6).

As we learned in Unit D, light also has a wave nature. Like sound, should it not therefore have a speed relative to the ground when it travels through a moving medium? In the late 1900s, most scientists thought that all waves required a medium in which to travel. They postulated that the medium through which light travelled was a universal, incompressible, viscous, transparent medium that they named **ether**. They reasoned that if the Sun was at rest relative to the ether, then the velocity of sunlight relative to Earth, $_{LE}\vec{V}_E$, would depend on both the velocity of light from the Sun, $_{LS}\vec{V}_S$, and the velocity of Earth through the ether about the Sun, $_{SE}\vec{V}_E$. The following example illustrates their calculations of the relative velocity of light through ether.

Fig.13.6



EXAMPLE 2

The velocity of light in the ether wind

Find the speed of light relative to Earth if Earth is moving at right angles to the Sun at a speed equal to its orbital velocity about the Sun.

Solution and Connection to Theory

Given

$$v_s = 3.0 \times 10^8 \text{ m/s} \text{ (speed of light relative to the Sun)}$$

$$r_{E-S} = 1.50 \times 10^{11} \text{ m} \text{ (orbital radius of Earth about the Sun)}$$

$$T_{E-S} = 3.16 \times 10^7 \text{ s} \text{ (orbital period of Earth about the Sun)}$$

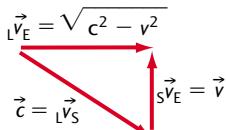
To find the velocity of Earth relative to the Sun, we need to divide Earth's orbital circumference by one year, the period of Earth's motion. Then the velocity of the Sun relative to Earth is

$$v_E = \frac{2\pi r_{E-S}}{T_{E-S}}$$

$$v_E = \frac{2(3.14)(1.50 \times 10^{11} \text{ m})}{3.16 \times 10^7 \text{ s}}$$

$$v_E = 2.98 \times 10^4 \text{ m/s}$$

Fig.13.7



From Figure 13.7, we can calculate the speed of light relative to Earth, v_E , using velocity addition in two dimensions:

$$lV_E = lV_S + sV_E = 3.0 \times 10^8 \text{ m/s [outward]} + 2.98 \times 10^4 \text{ m/s [tangentially]}$$

$$lV_E = \sqrt{(3.0 \times 10^8 \text{ m/s})^2 - (2.98 \times 10^4 \text{ m/s})^2}$$

$$lV_E = 299\,999\,998 \text{ m/s}$$

The scientists calculated that the difference between the speed of light relative to the Sun and the speed of light from the Sun through the ether was

$$v_s - v_E = 3.0 \times 10^8 \text{ m/s} - 299\,999\,998 \text{ m/s} = 2 \text{ m/s}$$

If scientists could measure this small difference of 2 m/s, they could prove that ether existed. When Einstein was eight years old, the great experimentalists A.A. Michelson and E.W. Morley tried to determine if there was a solar ether through which Earth moved (see Figure 10.18). Using an interferometer (see Figures 13.8a and b), a beam of light was split into two separate beams, one of which travelled perpendicular to Earth's motion and the other of which travelled parallel to Earth's motion. Both beams travelled the same distance and were reflected by mirrors back to the place where they separated. Depending on the travel time difference (if any), the waves meet crest to crest (constructive interference) or crest to trough (destructive interference). (See Section 11.5 to review how an interferometer works.)

Fig.13.8a The Michelson interferometer

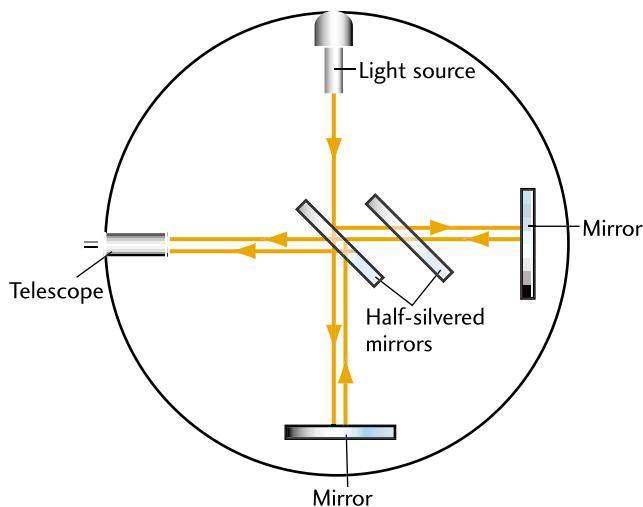
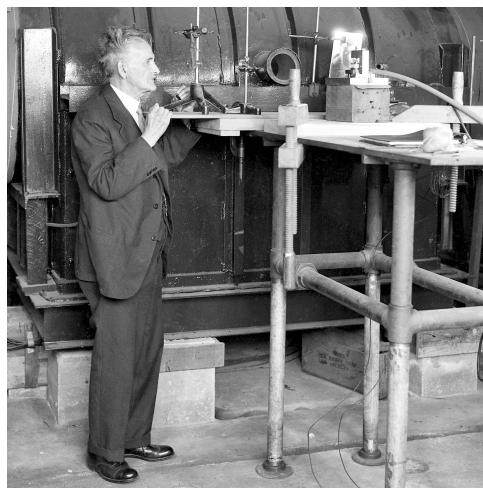


Fig.13.8b A.A. Michelson at his interferometer



Observations were made during the day and at night (as Earth spins on its axis) and through all seasons of the year (as Earth orbited the Sun). Even though their interferometer was about 40 times more sensitive than necessary, Michelson and Morley couldn't detect any difference in travel times between the perpendicular and parallel cases. They concluded that there was no ether at all; therefore, light can travel through a vacuum! In 1907, Michelson was awarded the Nobel Prize in physics for his experimental work.

The **null result** of the Michelson-Morley experiment was a blow to those who believed light needed a medium through which to travel. Based on Michelson's null result and his conviction that the laws of physics are the same in all inertial systems, Einstein came up with his second postulate of special relativity.

Einstein's Second Postulate of Special Relativity: The speed of light in a vacuum has the same value, c (3.0×10^8 m/s), in all inertial systems; that is, the speed of light is absolute!

A passenger in the space shuttle and a person sitting on Earth both measure the same value for the speed of light. We know from Chapter 10 that light can be slowed down when it enters a refractive medium such as water, but c is the ultimate speed of our universe. Nothing can travel faster than c !

Einstein's second postulate has been experimentally corroborated. For example, electrons accelerated through a potential difference of one million volts (1 MV) have a speed of $0.9411c$ (see Figure 13.10). When the potential is increased to 4 MV, their speed doesn't double to $1.8822c$, as we might expect from Newtonian physics, but increases to $0.9936c$ — a change of only 5.58%! The addition of energy doesn't cause the expected increase in speed of the electron. Some of this energy must be converted to another form. We will discuss what happens to this energy in Section 13.5.

Fig.13.9 A universal traffic sign



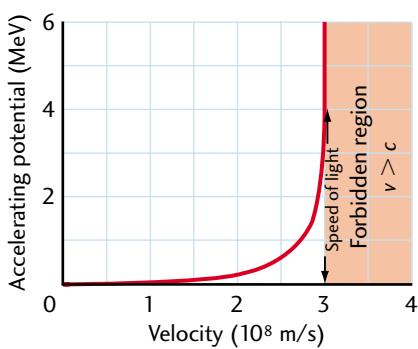
Classical Electron Acceleration

Electrical work (qV) increases kinetic energy ($\frac{1}{2}mv^2$), so

$$qV = \frac{1}{2}mv^2, \text{ or}$$

$$v = \sqrt{\frac{2qV}{m}}$$

Fig.13.10 The speed of electrons accelerated through an electric potential difference





- From the kinetic energy-versus-speed graph of electrons in Figure 13.10, at what speed do the effects of special relativity begin to appear?
- Captain Picard is travelling through the universe in his starship at a velocity of $0.6c$ [L]. A Klingon warship is approaching head-on with a velocity equal to $0.5c$ [R]. Use classical kinematics to find the speed of the Klingons relative to Picard. Which postulate of special relativity do these moving masses contradict?
- Find the speed of an electron accelerated from rest through a potential difference of one million volts (using Newtonian physics). How does this value compare to c ?

13.3 Time Dilation and Length Contraction

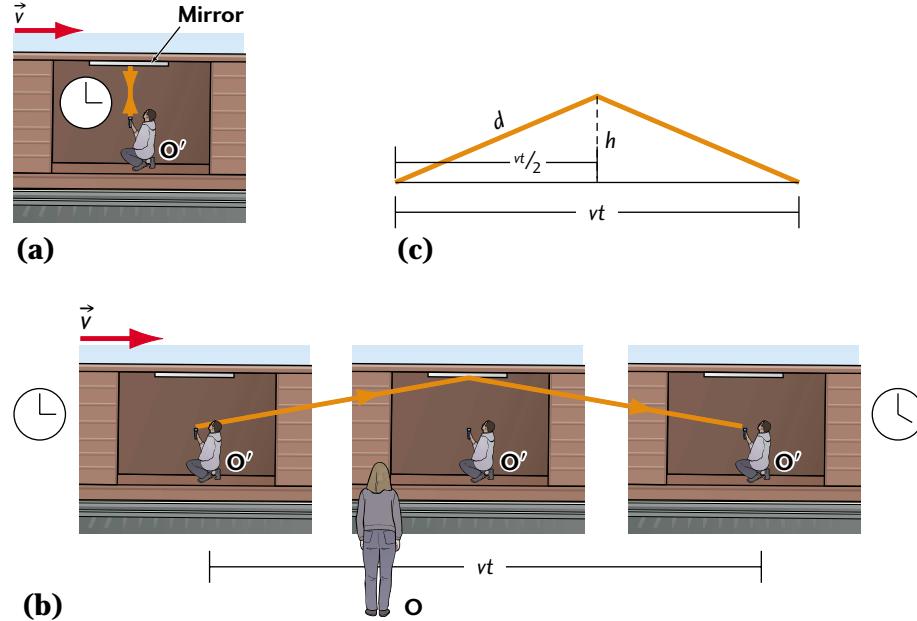
Moving Clocks Run Slow

One of the effects on objects travelling at speeds approaching the speed of light is **time dilation**. Using Einstein's two postulates of special relativity and Pythagoras' theorem, we can show that the measurement of time depends on how fast you are going!

Thought Experiment 1: The Relationship between Time and Speed

Phillip, a physicist, is travelling at a speed v in his personal boxcar while performing a physics experiment. He transmits a pulse of light from the floor up to the ceiling, where the pulse reflects off a mirror and travels back to the floor. Knowing that the height of the ceiling is h and the speed of light is c , from $v = \frac{d}{t}$, he is able to calculate the time, t_0 , for the experiment: $t_0 = \frac{2h}{c}$ (see Figure 13.11a).

Fig.13.11



Meanwhile, his friend Barb, a bystander, observes Phillip's experiment as he rides by. Barb's view of the experiment is a little different than Phillip's (see Figure 13.11b). From Barb's point of view, while the light travels up to the ceiling, the vehicle moves sideways. Therefore, Barb observes the light to travel farther than Phillip does, and she measures a time t greater than t_0 . Because the laws of physics must be the same for both Phillip's and Barb's inertial frames of reference (Einstein's first postulate of special relativity) and light travels at a speed c (Einstein's second postulate) in each frame, Barb and Phillip each measure a different time for the experiment.

From Figure 13.11c,

$$t = \frac{2\frac{d}{c}}{2\sqrt{\left(\frac{vt}{2}\right)^2 + h^2}}$$

But

$$h = \frac{ct_0}{2}$$

Therefore,

$$t = \frac{2\sqrt{\left(\frac{vt}{2}\right)^2 + \left(\frac{ct_0}{2}\right)^2}}{c}$$

or

$$(c^2 - v^2)t^2 = c^2t_0^2$$

Thus, the time measured by Barb is given by the equation

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where **t is relativistic time**, measured in a frame of reference where the beginning and end of the experiment occur *at two different points in space*; and **t_0 is proper time**, which is the time interval measured in a reference frame where the beginning and the end of the experiment occur *at the same point in space*. Notice in Figure 13.11a that Phillip's vehicle needed only one clock at one place to measure the time, t_0 . In Figure 13.11b, Barb needed two synchronised clocks for accuracy — at the points where she saw the experiment begin and end. Her time is therefore labeled t .

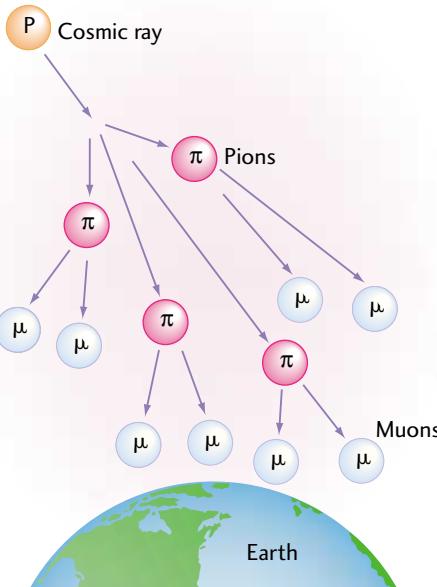
For Newtonian speeds, the values of t and t_0 are much the same, but at speeds comparable to c , they are quite different. In 1937, while studying cosmic radiation entering our atmosphere, scientists discovered the muon. Although it has the same charge as an electron, it is 207 times more massive. The muon is unstable, having an average lifetime at rest of 2.2 μs . However, travelling at high speed in the upper atmosphere, the measured lifetime of the muon is found to be somewhat longer, as we will see in the following example.

EXAMPLE 3

The extended lifetime of cosmic muons

What is the mean lifetime of a muon, measured by scientists on Earth, if it is moving at a speed of $v = 0.70c$ through the atmosphere? Assume that its lifetime at rest is $2.2 \mu\text{s}$

Fig.13.12 Cosmic rays striking the upper atmosphere lead to showers of secondary cosmic rays consisting of muons



Solution and Connection to Theory

Given

$$t_0 = 2.2 \times 10^{-6} \text{ s} \quad v = 0.70c \quad c = 3.0 \times 10^8 \text{ m/s} \quad t = ?$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{2.2 \times 10^{-6} \text{ s}}{\sqrt{1 - \frac{(0.70c)^2}{c^2}}}$$

$$t = \frac{2.2 \times 10^{-6} \text{ s}}{0.71}$$

$$t = 3.1 \times 10^{-6} \text{ s} = 3.1 \mu\text{s}$$

The lifetime of the muon is therefore extended by $0.9 \mu\text{s}$. This time difference means that the muon travels farther than would have been predicted by Newtonian mechanics. In fact, it would decay long before reaching Earth if it were not for the time dilation effect.

The time interval measured by an observer in relative motion to an event is longer than proper time. Time is relative. Absolute time does not exist!

Our experience is governed by a time that is proper to our own frame of reference and we cannot rely blindly on clocks that are moving at high speeds relative to us!

Moving Objects Appear Shorter

From the relationship between time and speed, we can see that at speeds close to c , time is relative. Using this relationship, we can easily show that length measurements are also relative. Like time dilation, this effect, known as length contraction in the direction of travel, appears insignificant at everyday speeds, but becomes significant at speeds approaching c .

Thought Experiment 2: The Relationship between Length and Speed
 Katrina the cosmonaut is travelling via spacecraft at a speed v from Earth to Mars, which we will assume are at rest relative to each other and located a distance L_0 apart.

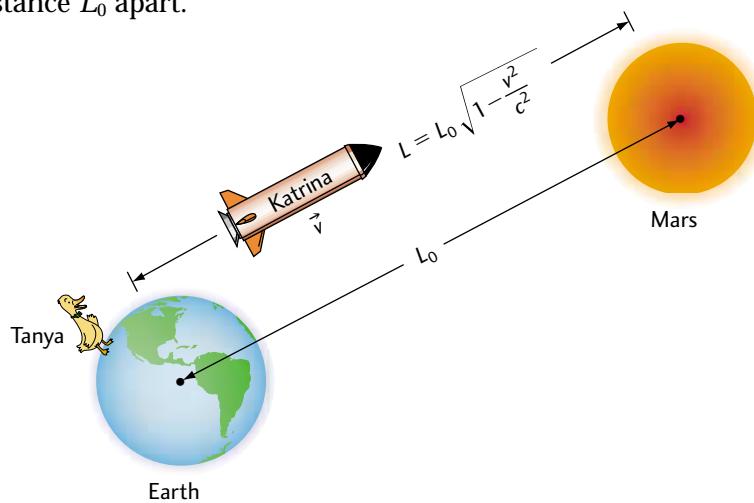


Fig.13.13 Katrina observes the Earth–Mars distance to contract such that $L < L_0$

For Tanya, a technician on Earth, the time taken for this trip is $t = \frac{L_0}{v}$ (where $L_0 = d = vt$). **Proper length**, L_0 , is the rest length or the length measured in a reference frame in which *the observed object* (in this case, the distance between the two planets) *is at rest*. For Katrina, proper time t_0 (measured with the one clock in her spaceship) is less because time dilates according to the equation $t_0 = t \sqrt{1 - \frac{v^2}{c^2}}$. From Katrina's point of view, she sees herself at rest with Mars approaching at speed v . Thus, Katrina calculates the **relativistic length**, L , of her trip to be $vt_0 = vt \sqrt{1 - \frac{v^2}{c^2}}$.

Using $vt = L_0$, we find the distance in Katrina's direction of motion is contracted or decreased according to the **equation for relativistic length**

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Just as the cosmonaut measures shorter interplanetary distances, a stationary observer measures the length of a moving object to be shorter in the direction of travel than when it is at rest. Again, we must be careful to distinguish between L and L_0 . In Tanya's case, the ends of the observed length (Earth and Mars) were at rest relative to her. Her distance is therefore the proper length, L_0 .

The length of a moving object in the direction of travel is shortened. Length is a relative, not an absolute, concept. Absolute length does not exist!

EXAMPLE 4 Our shrunken sky

A muon, created 12 km above Earth, travels downward at a speed of $0.98c$. Determine the contracted relativistic length the muon experiences as it travels to Earth.

Solution and Connection to Theory

Given

$$L_0 = 12\,000 \text{ m} \quad v = 0.98c \quad L = ?$$

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$L = (12\,000 \text{ m}) \sqrt{1 - (0.98)^2}$$

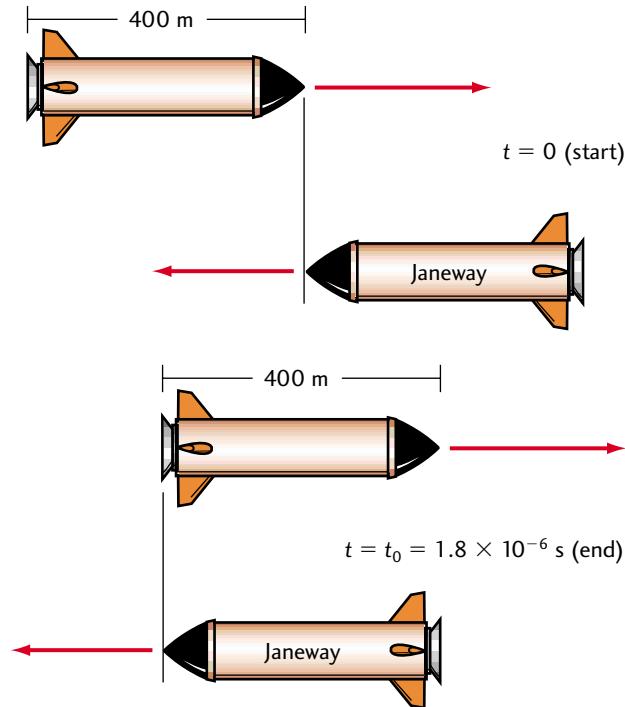
$$L = 2.4 \times 10^3 \text{ m}$$

To the muon, 12 km of our atmosphere seems like only 2.4 km!

EXAMPLE 5 Like ships passing in the night

Two identical spacecraft, each 400 m long when measured at rest, pass each other while heading in opposite directions (Figure 13.14). Captain Janeway, piloting one of the vehicles, measures a proper time interval of 1.80×10^{-6} s for the second ship to pass her. Find the relative speed of the two ships.

Fig.13.14 Using length contraction, the captain calculates relative velocity



Solution and Connection to Theory

Given

$$L_0 = 400 \text{ m} \quad t_0 = 1.8 \times 10^{-6} \text{ s} \quad c = 3.0 \times 10^8 \text{ m/s} \quad v = ?$$

$$v = \frac{L}{t_0}$$

But $L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$; therefore,

$$v = \frac{L_0 \sqrt{1 - \frac{v^2}{c^2}}}{t_0}$$

$$v = (400 \text{ m}) \frac{\sqrt{1 - \frac{v^2}{9.0 \times 10^{16} \text{ m}^2/\text{s}^2}}}{1.8 \times 10^{-6} \text{ s}}$$

$$v^2 = \left(4.94 \times 10^{16} \text{ m}^2/\text{s}^2\right) \left(1 - \frac{v^2}{9.0 \times 10^{16} \text{ m}^2/\text{s}^2}\right)$$

$$v^2 = 3.189 \times 10^{16} \text{ m}^2/\text{s}^2$$

$$v = 1.8 \times 10^8 \text{ m/s}$$

The relative speed of the two ships is therefore $1.8 \times 10^8 \text{ m/s}$.



1. In Example 3, calculate how much farther the muon travelled than Newtonian physics predicts.
2. In Thought Experiment 1, find the time, t_0 , that Phillip measured for the light pulse to travel from the floor to the ceiling and back if the height is 3.0 m. Find the time t that Barb measured if Phillip's vehicle is travelling at a speed $v = 0.6c$.
3. Marc Garneau is orbiting Earth in an ultra-fast space shuttle. His heart is pulsing at a rate of 52 beats per minute. If the shuttle is travelling at a speed of $0.28c$, how many beats per minute will a sensitive detector on Earth measure?
4. On another trip from Mars to Saturn, Katrina measures the distance, L , to be exactly one-half of its proper length. What is her speed, v ?
5. In Example 5, why is Janeway's time the proper time?
6. If you wanted to travel from Earth to Pluto in one hour (by your watch), what would be your speed? The Earth–Pluto distance is 5.75×10^{12} m.

Women in Physics

Einstein met his first wife, Mileva, in Zurich, where they were both studying physics. Some historians of science feel that at a time when few women studied physics, Mileva may have made significant contributions to the theory of special relativity through her relationship with Albert.

7. Investigate women's contributions to physics in the early 1900s, and in particular, the evidence related to Mileva's possible contributions to special relativity.

Fig. 13.15 Mileva Maric (1875–1948)



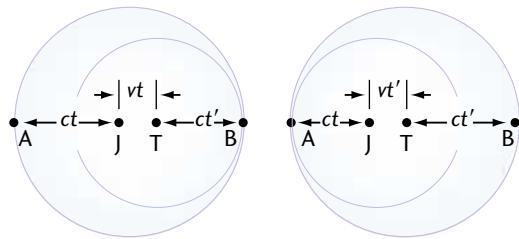
13.4 Simultaneity and Spacetime Paradoxes

Simultaneity

In Section 13.3, we learned that a stationary observer will have a different experience of space and time than an observer travelling at relativistic speeds, even though the laws of physics are the same in both their (inertial) reference frames. In other words, events occurring at the same time or place from your viewpoint may not be doing so for someone else in relative motion. Let's look at a situation involving two people in relative motion.

Thought Experiment 3: Relativity Can Make You Cross-eyed

Ted and Jane are moving toward each other at a relative speed, v . For a moment, their paths cross, and at that instant, they both send off a flash of light. A short time later, they both have quite a different view of what took place (see Figures 13.16a and b).



(a) Jane's viewpoint: $t > t'$ **(b)** Ted's viewpoint: $t' > t$

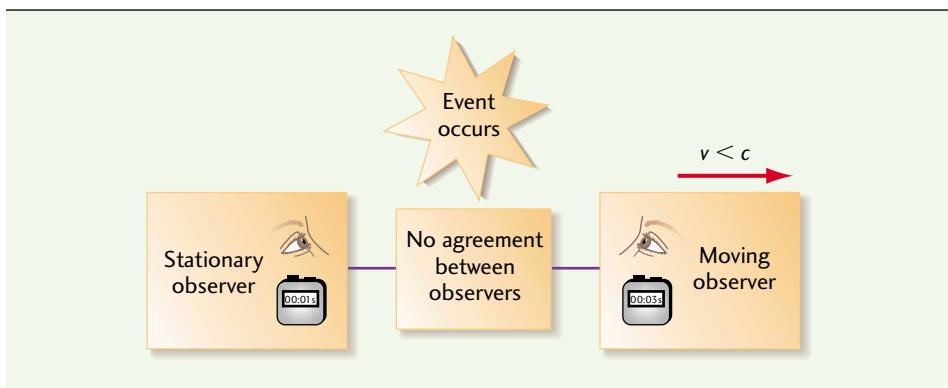
Fig.13.16 Observers in relative motion disagree

From Jane's point of view (Figure 13.16a), Ted has moved a distance vt to the right. The spherical shell of light expanding outward from Jane has a radius $AJ = ct$. Since Ted emitted a flash of light centred about himself, Jane knows that the radius of Ted's sphere $TB = ct'$. It's smaller than the radius of Jane's shell, ct , because both spheres of light were emitted at the same place and time and thus reach point B at the same time. According to Figure 13.16b, Ted has a similar view, but in reverse. He knows the radius of the shell of light expanding from Jane has a radius ct , the distance from Jane to point A . From his viewpoint, the distance from him to A is greater and equals ct' . He thinks that $t' > t$, but Jane sees $t > t'$. Is $t > t'$ or is $t' > t$? We could go cross-eyed trying to decide which viewpoint is correct. For an observer at rest, the light from both Jane and Ted would reach points A and B simultaneously!

An event that is simultaneous for one observer isn't necessarily simultaneous for another observer. Simultaneity is a relative, not an absolute, concept.

The concept of simultaneity is summarized in Figure 13.17.

Fig.13.17 Simultaneity



Paradoxes

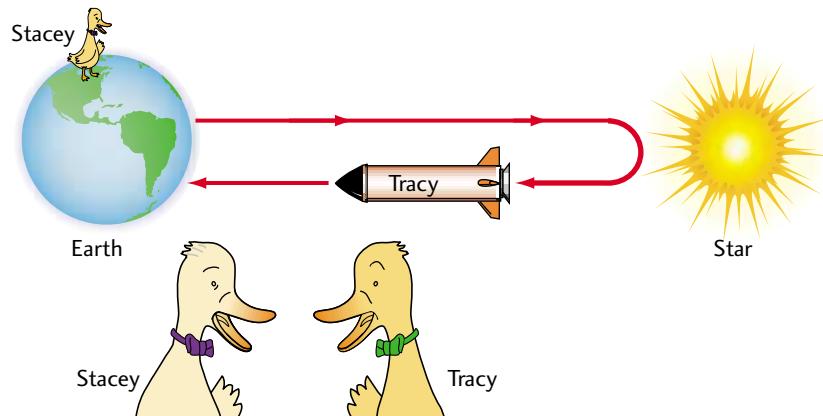
A **paradox** is a situation in which people reach contradictory conclusions using valid deductions from premises that are acceptable to everyone. In special relativity, even though thought experiments can have the same

beginning, sometimes our analysis can lead each of us on different paths of thought to contradictory outcomes that seem irresolvable. The **twin paradox** is one of the most famous paradoxes in relativity.

EXAMPLE 6 Stacey and her travelling twin, Tracy

Tracy the astronaut travels at a speed of $0.95c$ to a star that is 8.00 light years away, then immediately turns around and returns to Earth. She greets her twin sister Stacey, who had stayed on Earth, and now appears much older than Tracy! Find the age difference of the sisters when they reunite.

Fig.13.18 Tracy the traveller meets her “older” twin, stay-at-home Stacey



One light year (ca) is the distance light travels in one year or 3.16×10^7 s. It is a more practical unit for measuring distance in astronomy. Thus, the time light takes to travel 2 ca is $t = \frac{d}{v} = 2 \frac{ca}{c} = 2$ a.

Solution and Connection to Theory

Given

$$L_0 = 8.00 \text{ ca} \quad v = 0.95c \quad t_0 = ? \quad t = ?$$

For Stacey, the total time for the trip is

$$t = \frac{2L_0}{v} = \frac{2(8.00 \text{ ca})}{0.95c} = 16.8 \text{ a}$$

For Tracy, the total time is

$$t_0 = \frac{2L}{v}, \text{ where } L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = (8.00 \text{ ca}) \sqrt{1 - (0.95)^2} = 2.50 \text{ ca}$$

$$t_0 = \frac{2(2.50 \text{ ca})}{0.95c} = 5.26 \text{ a}$$

The twins’ age difference when Tracy returns is $16.8 \text{ a} - 5.26 \text{ a} = 11.6 \text{ a}!$

Perhaps you think it is unfair that Stacey ages more than Tracy. Many students feel that way. Others argue that, from Tracy’s point of view, Stacey’s clock should run slow because she is moving relative to Tracy. We would have an unresolved paradox unless we note that Tracy is not always in an

inertial reference frame. She must accelerate to leave Earth, turn around when she reaches the star, and brake upon her return to Earth. For this reason, it is Tracy's and not Stacey's clock that runs slow. She isn't getting more out of life than her sister; she is just experiencing 5.26 years of living while Stacey experiences 16.8 years.

When scientists questioned Einstein about this paradox, he argued that even though both observers need only one clock, the proper time, t_0 , was the time measured by the traveller. Experiments measuring the dilated average lifetimes of unstable particles, such as muons travelling at relativistic speeds in particle accelerators, have since confirmed his conclusion. The effects of time dilation were also experimentally corroborated in 1977, when atomic clocks were carried around the world on commercial airline flights, once eastward and once westward. When the travelling clocks (like Tracy's) were compared to those that stayed home (like Stacey's), the ones that were flown were slower!

Spacetime Invariance

If space and time are both related to speed, then space and time must be related to each other. From geometry, we know that Euclidian space has three dimensions: x , y , and z . In relativity, on the other hand, **spacetime** is a framework that has four dimensions: x , y , z , and t (time).

To see how length and time are joined in relativity, consider Orson, the international gourmet, travelling on the high-speed *Occident Express*. His train is travelling at a speed of $0.69c$, while he eats a meal in 21 minutes from a 29-cm plate (Figure 13.19). Yet for Tory the timer, at rest relative to Earth, the plate is contracted to a diameter of

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$L = (29 \text{ cm}) \sqrt{1 - (0.69)^2}$$

$$L = 21 \text{ cm}$$

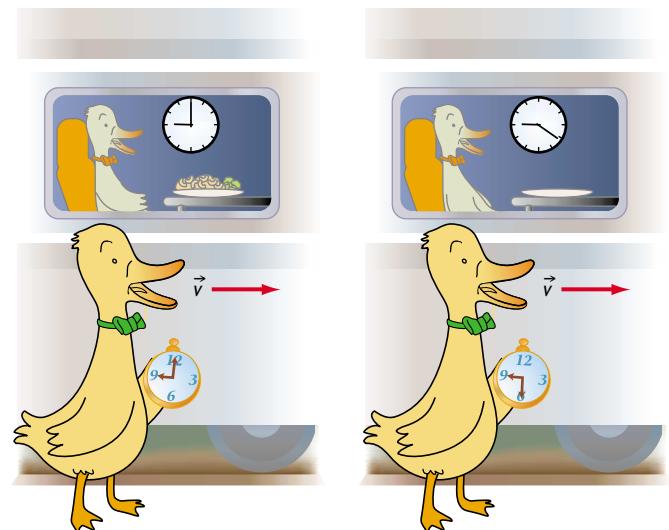
and the time is dilated to

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{(21 \text{ min})}{\sqrt{1 - (0.69)^2}}$$

$$t = 29 \text{ min}$$

Fig.13.19 Tory measures a longer time but a shorter plate than Orson



	Speed	Time (min)	Length (cm)
Orson	$0.69c$	21	29
Tory	0	29	21

Thus to Tory, what the food loses in size it gains in time! Even though length contracts and time dilates, length and time together are invariant.

The **spacetime interval** is the interval between two events in space and time, *here and now* and *there and then*. We can show that the spacetime interval is the same for observers in all inertial reference frames. If two flashes of lightning occur a distance Δx and a time Δt apart, then Δx and Δt are linked in an unusual way. The equation

$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$$

is called the **square of the spacetime interval** for one dimension. It is constant or absolute in all inertial reference frames. An observer travelling at high speed may measure different Δx and Δt values than an observer at rest, but both observers will get the same value for $(\Delta s)^2$! Let's look at this invariant quantity by revisiting Orson and Tory.

EXAMPLE 7

Spacetime invariance

Show that the spacetime interval for Orson is the same as for Tory.

Solution and Connection to Theory

Given

$$\begin{aligned}\Delta t \text{ (Orson)} &= 21 \text{ min} & \Delta t' \text{ (Tory)} &= 29 \text{ min} & v &= 0.69c \\ c &= 3.0 \times 10^8 \text{ m/s} & (\Delta s)^2 &=?\end{aligned}$$

For Orson,

$$\Delta t = (21 \text{ min})(60 \text{ s/min}) = 1.26 \times 10^3 \text{ s}$$

$\Delta x = 0$ (He sat at the table.)

$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$$

$$(\Delta s)^2 = (3.0 \times 10^8 \text{ m/s})^2(1.26 \times 10^3 \text{ s})^2 - 0$$

$$(\Delta s)^2 = 1.4 \times 10^{23} \text{ m}^2$$

For Tory,

$$\Delta t' = (29 \text{ min})(60 \text{ s/min}) = 1.74 \times 10^3 \text{ s}$$

$$\Delta x' = v\Delta t'$$

$$\Delta x' = (0.69)(3.0 \times 10^8 \text{ m/s})(1.74 \times 10^3 \text{ s})$$

$$\Delta x' = 3.6 \times 10^{11} \text{ m}$$

$$(\Delta s)^2 = (3.0 \times 10^8 \text{ m/s})^2(1.74 \times 10^3 \text{ s})^2 - (3.6 \times 10^{11} \text{ m})^2$$

$$(\Delta s)^2 = 1.4 \times 10^{23} \text{ m}^2$$

Therefore, the squares of the spacetime intervals for Orson and Tory are both equal to $1.4 \times 10^{23} \text{ m}^2$.

When we wish to calculate the distance between two points in Euclidean space, we use the equation

$$\Delta d = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

Notice that this equation is quite similar to the spacetime interval (for three spatial dimensions), except for the negative signs:

$$\Delta s = \sqrt{c^2(\Delta t)^2 - [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]} \text{ or}$$

$$\Delta s = \sqrt{c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2}$$

To calculate the distance between two points in spacetime, scientists use ct instead of t to represent the time axis so that it will have the same units (m) as the position coordinates.

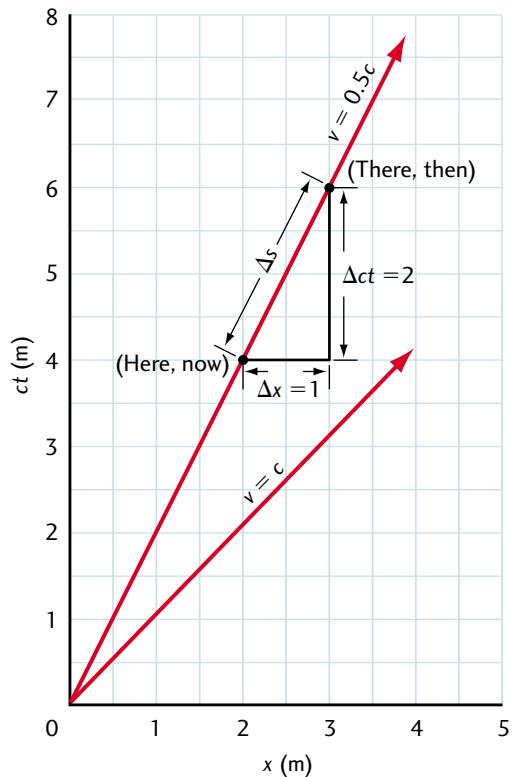
From the spacetime graph in Figure 13.20, we can see that if the span along the ct axis is squared, then it will have the same units as Δx^2 , namely m². From the vertical ($\Delta ct = 2$ m) and horizontal ($\Delta x = 1$ m) displacements, we can find the spacetime interval, in metres:

$$\Delta s = \sqrt{c^2(\Delta t)^2 - (\Delta x)^2}$$

$$\Delta s = \sqrt{2^2 - 1^2}$$

$$\Delta s = 1.73 \text{ m}$$

Fig.13.20 A spacetime graph shows $(\Delta s)^2$ for an object with speed $v = 0.5c$



- Find the speed of a rocket that takes three years longer than light (according to the rocket's clock) to travel a distance of 7.0 ca.
- Find Tracy's speed if the twins were 20 years old when Tracy left, and were 5.0 years apart in age when Tracy returned from a round trip to a star that is 5.0 ca away.
- If you vacation to a star 200 ca away by travelling at a speed of 0.9986c, will you get there before you are 60 years old? How long will it take?
- Research the average speed of commercial aircraft. Using time dilation, determine how much shorter a trip around the globe is for the passenger in the plane than for a stationary observer. Find information on the type of clocks that have been developed to measure extremely small intervals of time.
- Today, people are travelling more than ever. In terms of relativity, do travellers age differently than people who stay home?
- While travelling in a high-speed boxcar, Rashad hits a ping-pong ball against a wall. The ball bounces back to him in 1.5 s. For



Kareem, who is standing beside the tracks when Rashad zooms by, the ball takes 2.0 s to return to Rashad. What space interval did Kareem measure for this event?

7. During fission in nuclear reactor cores, emitted high-speed beta (β) particles travel into the surrounding water, causing a glowing blue light to be emitted. Investigate the phenomena of Cerenkov radiation to explain why the blue glow occurs and to possibly explain whether or not particles can travel faster than light.

13.5 Mass Dilation

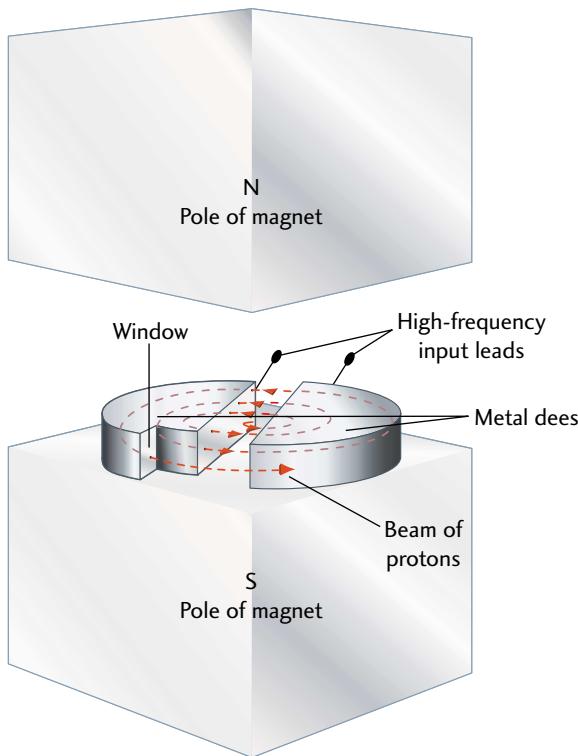
Until the turn of the 19th century, the fastest observed object was considered to be the planet Mercury, with an average speed that is 0.016% the speed of light. At this speed of 4.8×10^4 m/s, the difference between the relativistic and the classical (Newtonian) mass was only one part in 79 million. However, technology was becoming more precise and exacting. If a theoretical physicist hadn't discovered special relativity in 1905, it is likely that within the decade, an experimentalist would have.

In a 1909 experiment, H. Bucherer showed that electrons emitted from the beta-decay of radioactive particles and travelling at a speed of $0.69c$ had a significantly smaller charge-to-mass (e/m) ratio than expected. Bucherer claimed that the best fit to his e/m data was given by Einstein's (then recent) equation for mass dilation. Another possible explanation for this observation was that the electron charge decreases as the velocity increases, but this idea was discarded and mass dilation theory was universally accepted by 1916.

The Cyclotron

Advances in electrical engineering led to particle accelerators such as E.O. Lawrence's first cyclotron in 1932 (see Figure 13.21), used for nuclear studies. In the cyclotron, a beam of particles is bent into a circular path by a magnetic field. The particles orbit inside two semicircular metal chambers called "dees" (because they are shaped like the letter D). Inside the dees, the particles experience no electric force, but in the gap between the dees, they are given an accelerating voltage, thus gaining a little energy with each cycle. By the end of the 1930s, the speed to which particles could be accelerated in a cyclotron reached its limit, beyond which there was no possible way to compensate

Fig. 13.21 The first cyclotron (1932) had a radius of 12.5 cm. Protons spiral outward within the hollow dees. An electric field accelerates the protons each time they cross the gap.



for the effect of mass dilation. In 1947, scientists at Berkeley overcame this problem by building a frequency-modulated cyclotron (synchrocyclotron) that was about ten times more energetic.

Thought Experiment 4: The Dilated Bohr Electron

We can combine the ideas of circular motion and electrical force (studied in Chapter 8) with that of relativistic length contraction to support the idea that mass is relative, and that at high speeds, it appears dilated. Recall from Chapter 12 that when an electron travels in a circular orbit around a proton, as in early models of the hydrogen atom, we say that the electron's centripetal force,

$$F_c = \frac{mv^2}{r}$$

is provided by the electron's electrostatic attraction towards the proton,

$$F_e = \frac{ke^2}{r^2} \text{ (Coulomb's law)}$$

When we equate these two forces

$$\frac{mv^2}{r} = \frac{ke^2}{r^2}$$

and isolate the mass, we obtain the equation

$$m = \frac{ke^2}{v^2 r}$$

At low speeds, the mass of a moving object is negligibly different from its stationary or **rest mass**, m_0 . This equation suggests that as the radius, r , becomes contracted at high speeds, the mass of the electron becomes dilated. For relativistic mass, we substitute the equation for length contraction for r such that

$$m = \frac{ke^2}{v^2 r_0 \sqrt{1 - \frac{v^2}{c^2}}}$$

Substituting the rest mass, $m_0 = \frac{ke^2}{r_0 v^2}$, into this equation, we obtain the equation for mass dilation:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Fig.13.22 Circular motion of an electron about a proton

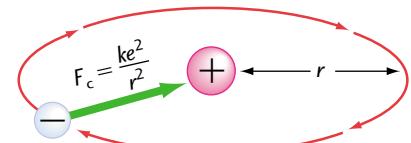


Figure 13.23 summarizes the derivation of the equation for relativistic mass.

Fig.13.23 Understanding Mass Dilation

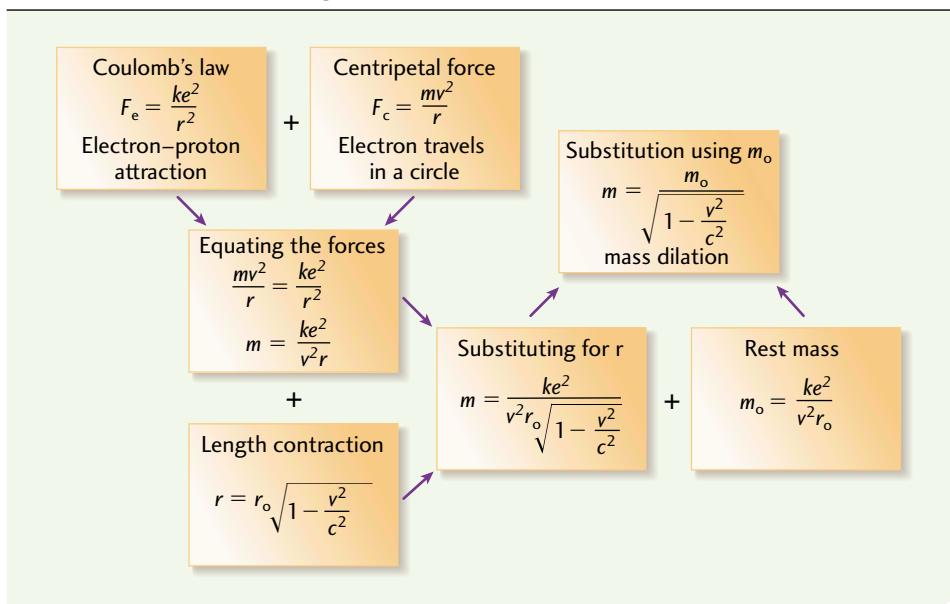
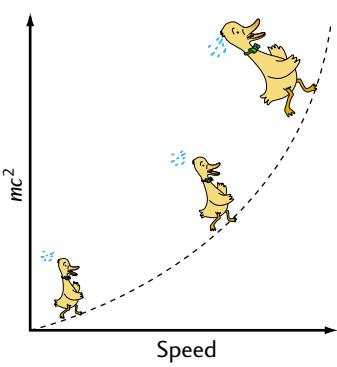


Fig.13.24 As speed increases, mass increases as well



Einstein's second postulate of special relativity can be linked to key aspects of Newton's second law of motion. As an object accelerates, its inertia, or resistance to change, increases. In other words, we observe the mass of an object in motion to be greater than when it is at rest. In classical mechanics, if we apply a constant unbalanced force to an object, the object accelerates; that is, its speed increases by the same amount per second; for example, from 500 m/s to 600 m/s at one stage. If a sufficient force is applied, this increase continues until the object accelerates from 300 000 000 m/s to 300 000 100 m/s in the same time interval. At this speed, the object exceeds the speed of light, c , which violates Einstein's second postulate of special relativity! (Recall our discussion of the electron in Figure 13.10.) As an object approaches the speed of light, its inertia or mass increases as described by mass dilation so that the object never travels faster than the speed of light. The unbalanced force is no longer sufficient to cause the same acceleration because the mass is increasing (see Figure 13.24). As higher speeds are reached, the mass increases, making it harder to accelerate further.

The mass of a moving object is dilated. Mass is relative. Absolute mass does not exist!

In 1947, cosmic rays colliding with atomic nuclei in Earth's upper atmosphere were observed to create short-lived high-speed particles called pions, π , that have an average rest life of 2.6×10^{-8} s. The pion decays into a muon, μ , and a particle called a neutrino, ν . Let's use the pion in an example to show mass dilation.

EXAMPLE 8 The mass of the cosmic pion

Find the dilated mass of a pion of rest mass $m_0 = 2.5 \times 10^{-28}$ kg if it is travelling at a speed of $0.99c$.

Solution and Connection to Theory

Given

$$m_0 = 2.5 \times 10^{-28} \text{ kg} \quad v = 0.99c \quad m = ?$$

Applying the mass dilation equation and substituting,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m = \frac{2.5 \times 10^{-28} \text{ kg}}{\sqrt{1 - (0.99)^2}}$$

$$m = \frac{2.5 \times 10^{-28} \text{ kg}}{0.14}$$

$$m = 1.8 \times 10^{-27} \text{ kg}$$

The dilated mass of the pion is 1.8×10^{-27} kg, or over seven times its rest mass.

Infinitesimal mass increases are happening all around us. They occur whenever macroscopic objects move. A simple application of the binomial theorem allows a very close approximation to the exact answer when $v \ll c$ (relatively slow speeds).

EXAMPLE 9 Low-speed mass

Determine the increase in mass of a car travelling at 25 m/s if its rest mass is 2000 kg.

Solution and Connection to Theory

Given

$$v = 25 \text{ m/s} \quad c = 3.0 \times 10^8 \text{ m/s} \quad m_0 = 2000 \text{ kg} \quad \Delta m = m - m_0 = ?$$

$$\Delta m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0$$

Binomial Theorem Derivation

In mathematics, the binomial expansion of

$$\sqrt{1 - \frac{v^2}{c^2}} = 1 - \frac{v^2}{2c^2} + \frac{v^4}{8c^4} - \dots$$

When $v \ll c$ (slow speeds),

$$\sqrt{1 - \frac{v^2}{c^2}} \approx 1 - \frac{v^2}{2c^2}$$

is a good estimate for length contraction because the higher-order terms like $\frac{v^4}{c^4}$ are comparatively insignificant.

For mass and time dilation, we use

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

The binomial expansion of this expression is approximately

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 1 + \frac{v^2}{2c^2}$$

for the same reason.

Note the minus sign for length contraction and the plus sign for mass and time dilation.

$$\Delta m = m_0 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

$$\Delta m = m_0 \left(1 + \frac{v^2}{2c^2} - 1 \right)$$

$$\Delta m = \frac{1}{2} \frac{m_0 v^2}{c^2}$$

$$\Delta m = \frac{1}{2} (2000 \text{ kg}) \left(\frac{25 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} \right)^2$$

$$\Delta m = 6.9 \times 10^{-12} \text{ kg}$$

Therefore, the mass of the car increases 6.9×10^{-12} kg. Because this increase is so small, Newtonian mechanics is more than adequate for most everyday situations.

Electrons Moving in Magnetic Fields

The electron has been the easiest atomic particle for scientists to accelerate to high speeds because it has a mass that is about 1800 times less than the mass of the proton. Since mass dilates at relativistic speeds, we should re-examine the movement of electrons in magnetic or electric fields. From Chapter 9, we know that the centripetal force,

$$F_c = \frac{mv^2}{r}$$

of an electron moving in a circular path is provided by the magnetic force,

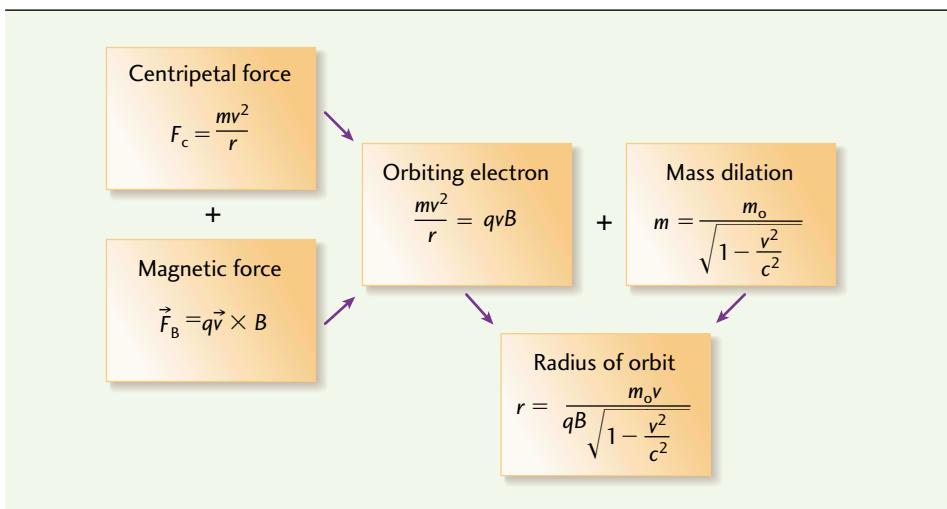
$$F_B = qvB$$

where q is the charge, v is the speed, and B is the magnetic field strength. Now we must also include mass dilation,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Combining these three equations, we can derive the equation for the orbital radius of an electron, as shown in Figure 13.25.

Fig.13.25 The Orbital Radius of a High-speed Charge in a Magnetic Field



EXAMPLE 10 The curved path of electrons

An electron travels at $0.69c$ in a circle at right angles to a uniform magnetic field of strength 2.0 T . Find the radius of the circle and compare it to the radius calculated without considering mass dilation.

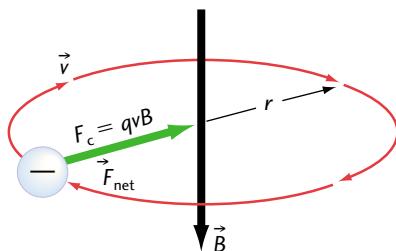
Solution and Connection to Theory

Given

$$\begin{aligned} m_0 &= 9.11 \times 10^{-31}\text{ kg} & v &= 0.69c & B &= 2.0\text{ T} \\ q &= 1.602 \times 10^{-19}\text{ C} & c &= 3.0 \times 10^8\text{ m/s} & r &=? \end{aligned}$$

Without considering mass dilation:

Fig.13.26 Circular motion of an electron in a uniform magnetic field



When electron motion is perpendicular to the magnetic field, the centripetal force is

$$F_c = \frac{mv^2}{r} = qvB, \text{ where } m = m_0; \text{ thus,}$$

$$r = \frac{m_0 v^2}{qvB}$$

$$r = \frac{m_0 v}{qB}$$

$$r = \frac{(9.11 \times 10^{-31} \text{ kg})(0.69)(3.0 \times 10^8 \text{ m/s})}{(1.602 \times 10^{-19} \text{ C})(2.0 \text{ T})}$$

$$r = 5.9 \times 10^{-4} \text{ m}$$

Considering mass dilation:

When the mass is dilated, $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, so

$$r = \frac{m_0 v}{qB \sqrt{1 - \frac{v^2}{c^2}}}$$

$$r = \frac{(9.11 \times 10^{-31} \text{ kg})(0.69)(3.0 \times 10^8 \text{ m/s})}{(1.602 \times 10^{-19} \text{ C})(2.0 \text{ T}) \sqrt{1 - (0.69)^2}}$$

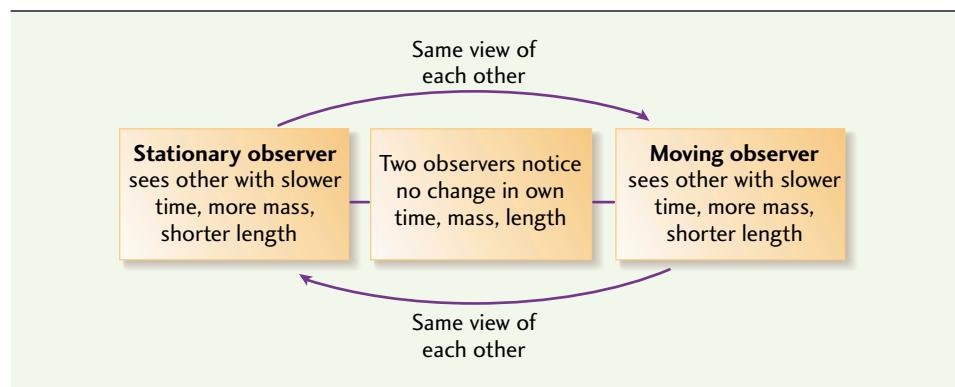
$$r = 8.1 \times 10^{-4} \text{ m}$$

With mass dilation, the electron's radius of orbit is larger.

In Bucherer's 1909 experiment, the observed radius of the electrons in the magnetic field was 37% larger than that predicted by Newtonian mechanics.

Figure 13.27 summarizes the three relativistic effects predicted by Einstein.

Fig.13.27 Relativistic Effects



- Find the dilated mass of Earth if its rest mass is $m_0 = 5.98 \times 10^{24} \text{ kg}$ and its orbital speed is $2.96 \times 10^4 \text{ m/s}$.
- Which increase in speed represents the greater gain in mass for a proton: accelerating from $0.90c$ to $0.99c$, or accelerating from $0.99c$ to $0.999c$? Explain.
- Find the increase in mass of Tomiya, a sprinter running at 10 m/s, if his rest mass is 60 kg. (Hint: Use the low-speed binomial approximation.)

- The cost of building larger cyclotrons scales roughly as the size of the magnet used or as the cube of the energy. In 1980, a 500-MeV cyclotron cost about \$100 million. Show why building a cyclotron of 5 GeV (1 GeV = 1000 MeV) is unrealistic. Investigate how the scientists at CERN are able to build a 7-TeV (1 TeV = 1000 GeV) accelerator, due for completion in 2006.
- Moving at right angles to a magnetic field, how would the radius of curvature of a high-speed electron's path compare to that of a low-speed electron?
- If a proton and an electron were each travelling at $0.69c$ perpendicular to a uniform magnetic field, which particle would have a greater radius of curvature? Explain.
- A cosmic-ray proton travelling at $0.996c$ enters the upper atmosphere in the plane of the equator at right angles to Earth's magnetic field of 5.0×10^{-5} T. Use relativistic considerations to calculate the radius of its curved path in this region.

13.6 Velocity Addition at Speeds Close to c

Einstein's second postulate of special relativity states that the speed of light in a vacuum is c , regardless of the speed of the light's source or of the observer. If the classical addition of velocities were valid at high speeds, then a serious contradiction would occur. For example, when a radioactive nucleus of tellurium-128 decays by a process of double-beta emission to xenon-128, it emits two electrons with equal speeds of $0.55c$ but in opposite directions. What is the speed of one electron relative to the other?

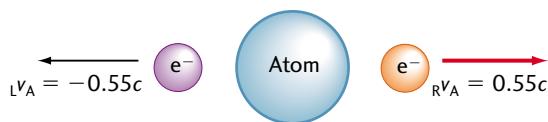


Fig.13.28 The Newtonian addition of velocities is incorrect at high speeds

When we add these two velocities using the method of classical mechanics, we obtain

$$\begin{aligned}\vec{v}_R &= \vec{v}_A + \vec{v}_R \\ \vec{v}_R &= \vec{v}_A - \vec{v}_R \\ v_R &= -0.55c - 0.55c \\ \vec{v}_R &= -1.1c = 1.1c [L]\end{aligned}$$

In Newtonian physics, the electron on the left in Figure 13.28 sees the one on the right travelling away at $1.1c$! The Newtonian relative velocity exceeds the speed of light — a violation of Einstein's second postulate! We need a new equation for velocity addition that gives an answer similar to

the answer obtained using classical vector addition at low speeds, but that also yields an answer that is less than c at high speeds. This **equation for relativistic velocity addition in one dimension** is

$${}_{\text{A}}\vec{v}_{\text{C}} = \frac{{}_{\text{A}}\vec{v}_{\text{B}} + {}_{\text{B}}\vec{v}_{\text{C}}}{1 + \frac{{}_{\text{A}}\vec{v}_{\text{B}} \cdot {}_{\text{B}}\vec{v}_{\text{C}}}{c^2}}$$

where the numerator, ${}_{\text{A}}\vec{v}_{\text{B}} + {}_{\text{B}}\vec{v}_{\text{C}}$, is the Newtonian solution to the velocity of A relative to C at low speeds. The relativistic correction is found in the denominator,

$$1 + \frac{{}_{\text{A}}\vec{v}_{\text{B}} \cdot {}_{\text{B}}\vec{v}_{\text{C}}}{c^2}$$

which keeps the resultant velocity, ${}_{\text{A}}\vec{v}_{\text{C}}$, always less than c .

We will practise using this equation in the following example.

EXAMPLE 11

Relative velocities in particle decay

A ${}^{128}\text{Te}$ radioactive nucleus at rest emitted two electrons with equal speeds of $0.55c$ but in opposite directions. What was the speed of one electron relative to the other?

Solution and Connection to Theory

Given

The velocity of the left electron relative to the Te nucleus = ${}_{\text{L}}v_{\text{T}} = -0.55c$

The velocity of the right electron relative to the Te nucleus = ${}_{\text{R}}v_{\text{T}} = 0.55c$

The velocity of the right electron relative to the left electron = ${}_{\text{R}}v_{\text{L}} = ?$

$${}_{\text{R}}\vec{v}_{\text{L}} = \frac{{}_{\text{R}}\vec{v}_{\text{A}} + {}_{\text{A}}\vec{v}_{\text{L}}}{1 + \frac{{}_{\text{R}}\vec{v}_{\text{A}} \cdot {}_{\text{A}}\vec{v}_{\text{R}}}{c^2}}$$

$${}_{\text{R}}v_{\text{L}} = \frac{0.55c + 0.55c}{1 + \frac{(0.55c)(0.55c)}{c^2}}$$

$${}_{\text{R}}v_{\text{L}} = \frac{1.1c}{1 + \frac{0.3025c^2}{c^2}}$$

$${}_{\text{R}}v_{\text{L}} = \frac{1.1c}{1.3025}$$

$${}_{\text{R}}v_{\text{L}} = 0.84c$$

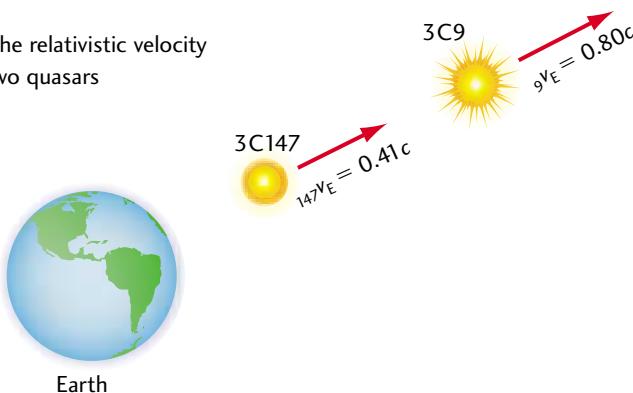
Therefore, the speed of the two electrons relative to each other is $0.84c$. In relativistic addition, like in vector addition, $1 + 1$ may no longer equal 2!

When studying the velocity of remote galaxies, astronomers measure the cosmological red shift caused by the Doppler effect. The amount of shift toward the redder wavelengths indicates the speed at which the galaxy is receding from us. Let's look at such a case.

EXAMPLE 12 Receding galaxies

From its red shift, quasar 3C9 is determined to be receding from us at a speed of $0.80c$. In line with 3C9 but closer to us is quasar 3C147. It is receding from us at a speed of $0.41c$. How fast is 3C9 receding from 3C147? (See Figure 13.29.)

Fig.13.29 The relativistic velocity addition of two quasars



Solution and Connection to Theory

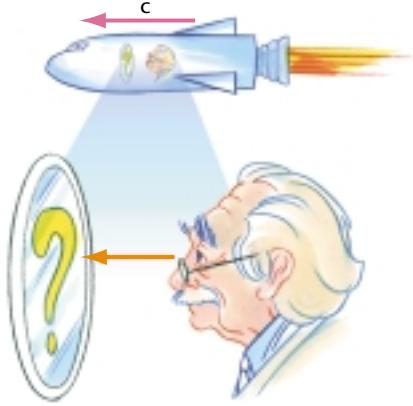
Given

$$9v_E = 0.80c \quad _E v_{147} = -0.41c \quad 9v_{147} = ?$$

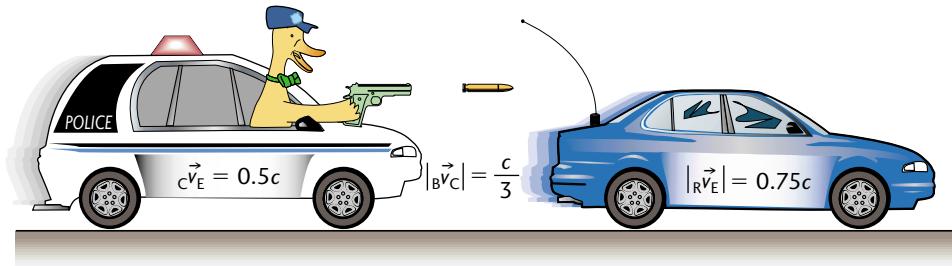
In Newtonian physics, we would calculate the speed of 3C9 relative to 3C147 to be $0.80c - 0.41c = 0.39c$, but we have seen that this approach is inadequate at high speeds. Instead, we use the equation

$$\begin{aligned} {}_A \vec{V}_C &= \frac{{}^A \vec{V}_B + {}_B \vec{V}_C}{1 + \frac{{}^A \vec{V}_B \cdot {}_B \vec{V}_C}{c^2}} \\ {}_9 \vec{V}_{147} &= \frac{{}^9 \vec{V}_E + {}_E \vec{V}_{147}}{1 + \frac{{}^9 \vec{V}_E \cdot {}_E \vec{V}_{147}}{c^2}} \\ {}_9 v_{147} &= \frac{0.80c - 0.41c}{1 + \frac{(0.80c)(-0.41c)}{c^2}} \\ {}_9 v_{147} &= \frac{0.39c}{0.672} \\ {}_9 v_{147} &= 0.58c \end{aligned}$$

The two quasars are receding from each other at a speed of $0.58c$.

**Fig.13.30** Albert's dilemma

- If Albert holds a mirror in front of his face while travelling at the speed of light, will he be able to see himself in the mirror, or will the light from his face never reach the mirror? (See Figure 13.30.)
- A high-speed nuclear particle travelling at a velocity of $|_{N}\vec{v}_L| = 0.999c$ away from a lab observer emits a gamma ray with a velocity of $|_{\gamma}\vec{v}_N| = c$ toward the observer. Find the speed of the gamma ray relative to the lab.
- As seen from Earth, cosmic police are travelling at $0.5c$ [N] while pursuing bandits travelling at $0.75c$ [N] (see Figure 13.31). In order to stop the criminal, the police fire a bullet that travels at $\frac{c}{3}$ [N] relative to the police. Will the bullet ever reach the bad guys?

Fig.13.31 Cosmic cops and robbers

- The expansion of our universe is described by Hubble's law, $v = Hr$, where v is the velocity of receding galaxies, $H = 1.7 \times 10^{-2}$ (m/s/ca), and r is the distance to the galaxy in light years (ca). Using $v = c$, use Hubble's law to find a limit to the radius of our universe. Investigate the larger speeds that astronomers have discovered for certain celestial bodies and discuss the dilemma or contradictions that may arise in extreme cases.

13.7 Mass–Energy Equivalence

In switching from Newtonian physics to relativistic physics, we needed to alter our concepts of space, time, and mass. Now we must also change the way we think about momentum (since it involves mass) and energy (since work involves force \times distance).

Relativistic Momentum

In the case of momentum, our definition $\vec{p} = m\vec{v}$ is only valid for classical physics. Once again, the Newtonian concepts and equations that relate to momentum need to be generalized for all velocities to include relativistic ones. If we replace the mass m with the equation for dilated mass, then the **relativistic momentum equation** becomes

$$\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We use this equation when the law of conservation of momentum is applied to relativistic situations. To see how the relativistic conservation of momentum applies to our lives, we need look no further than the television set.

EXAMPLE 13 TV tube electrons

In many ways, a television is like a particle accelerator. Both devices need a source of charged particles, an electric field to accelerate them, focusing devices to keep the beam sharp, deflectors to aim the beam, a target for the beam to strike, and a high vacuum chamber to house all the components. TV electrons in a beam reach speeds of about 9.2×10^7 m/s. Determine the momentum of TV electrons

- a) using classical mechanics (valid for low speeds only).
- b) using relativity (valid for all speeds less than c).

Solution and Connection to Theory

Given

$$m_e = 9.11 \times 10^{-31} \text{ kg} \quad v = 9.2 \times 10^7 \text{ m/s}$$

a) Classical case:

Here, we simply substitute the values for mass and velocity into the momentum equation:

$$\begin{aligned} p &= mv \\ p &= (9.11 \times 10^{-31} \text{ kg})(9.2 \times 10^7 \text{ m/s}) \\ p &= 8.37 \times 10^{-23} \text{ kg}\cdot\text{m/s} \end{aligned}$$

The momentum of each electron is 8.37×10^{-23} kg·m/s.

b) Relativistic case:

To save time, we can simply divide the classical result by $\sqrt{1 - \frac{v^2}{c^2}}$

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \frac{8.37 \times 10^{-23} \text{ kg} \cdot \text{m/s}}{\sqrt{1 - \frac{(9.2 \times 10^7 \text{ m/s})^2}{(3.0 \times 10^8 \text{ m/s})^2}}}$$

$$p = \frac{8.37 \times 10^{-23} \text{ kg} \cdot \text{m/s}}{\sqrt{1 - (0.307)^2}}$$

$$p = 8.79 \times 10^{-23} \text{ kg} \cdot \text{m/s}$$

The momentum of each electron is now $8.79 \times 10^{-23} \text{ kg} \cdot \text{m/s}$.

If we use the classical equation, $\vec{p} = m\vec{v}$, our answer has an error of only about 5%. This error would be significantly greater at higher velocities.

Relativistic Energy

Fig.13.32 The rise and fall of humanity



Recall from Chapter 5 that a change in the mechanical energy of a particle, ΔE , is work. We know that the work done on an object by the net force (or rate of change in momentum) can result in a change in the object's kinetic energy. Since the momentum equation was modified for relativistic speeds, we also need to modify our equation for energy. Knowing that mass is dilated at high speed, it is easy to assume that we can modify the classical kinetic energy equation, $E_k = \frac{1}{2}mv^2$, by simply substituting the equation for mass dilation, $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$. Instead, however, we use Einstein's famous equation for mass–energy equivalence, $E = mc^2$.

From the mass dilation equation,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{If } E = mc^2, \text{ then } mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{But } \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \dots \text{ (binomial expansion)}$$

$$\text{Therefore, } mc^2 = m_0 c^2 \left(1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \dots \right)$$

$$mc^2 = m_0 c^2 + \frac{1}{2} m_0 v^2 + \dots$$

$$\text{But relativistically, } E_k = \frac{1}{2} m_0 v^2$$

Therefore,

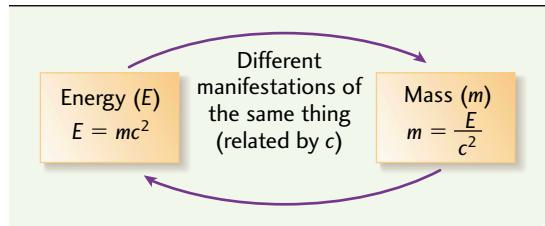
$$mc^2 = m_0 c^2 + E_k \quad \text{or} \quad E = m_0 c^2 + E_k$$

Einstein called the term $m_0 c^2$ the energy that an object has when it is at rest, or **rest energy**. It is the energy that makes up the internal structure of that object. The term mc^2 is called the **total energy**, E , and is the sum of the rest energy and the kinetic energy:

$$\text{total energy} = \text{rest energy} + \text{kinetic energy}$$

In Section 13.4, we learned that in special relativity, the concepts of space and time can no longer be separated; they must be considered together as *spacetime*. Similarly, energy and mass must also be considered together. An increase in energy is accompanied by an increase of mass (or inertia). If mass is equivalent to energy, then we should be able to transform it into energy and vice versa (see Figure 13.33).

Fig.13.33



The transformation of mass to energy has been observed in radioactive decay, such as the decay of muons in the upper atmosphere. New particles of smaller mass are created and pure electromagnetic energy is emitted. The power provided by CANDU nuclear reactors comes from the energy released by the fission of uranium-235. The Sun radiates about 1.23×10^{34} J of energy each year, causing its mass to continually decrease. In all these cases, if a system changes its energy by an amount ΔE , the mass of the system will also change by an amount Δm given by

$$\Delta E = (\Delta m) c^2$$

E X A M P L E 14 Our Sun's life

The mass of our Sun is continually decreasing due to the energy it radiates outward through the process of fusion. Its current mass is about 2.0×10^{30} kg, and it transfers energy to the solar system at a rate of 3.9×10^{26} W. If 35% of the Sun's core (by mass) is hydrogen, the fuel responsible for this energy output, how long will it take for all the hydrogen to be converted into radiant energy?

Solution and Connection to Theory**Given**

$$m_S = 2.0 \times 10^{30} \text{ kg} \quad P = 3.9 \times 10^{26} \text{ J/s} \quad 1 \text{ a} = 3.16 \times 10^7 \text{ s}$$
$$m_H = 0.35m_S \quad \Delta t = ?$$

First, we need to convert the Sun's mass of hydrogen to its equivalent energy using the equation $E = m_H c^2$:

$$E = (2.0 \times 10^{30} \text{ kg})(0.35)(3.0 \times 10^8 \text{ m/s})^2 = 6.3 \times 10^{46} \text{ J}$$

Next, we use the equation for power, $P = \frac{E}{t}$, to find the time t :

$$t = \frac{6.3 \times 10^{46} \text{ J}}{3.9 \times 10^{26} \text{ J/s}}$$

$$t = 1.6 \times 10^{20} \text{ s}$$

Finally, to convert this time to years, we divide by 3.16×10^7 s/a:

$$t = \frac{1.6 \times 10^{20} \text{ s}}{3.16 \times 10^7 \text{ s/a}}$$

$$t = 5.1 \times 10^{12} \text{ a}$$

Therefore, it will take about 5.1×10^{12} years to burn all the Sun's hydrogen.

This span of time is much longer than we need to worry about. Many other changes in solar processes will occur long before then. Of course, only a small part of the hydrogen is converted to energy. The “embers” of this fire are helium.

Often, the change in mass corresponding to a change in energy is too small to measure, such as in chemical reactions, where heat is lost or gained. The difference between the reactant and the product masses is so small that we may state that mass is conserved during most chemical reactions.

EXAMPLE 15 Is tea fattening?

Have you ever considered that boiling water for a cup of tea actually causes an equivalent increase in the mass of tea because energy has been added? Binder, desiring to brew some tea, heats 250 g of water at 10°C up to the boiling point at 100°C. Find the increase in mass, Δm , due to this increase in energy, ΔE .

Solution and Connection to Theory

Given

$$m = 250 \text{ g} \quad c = 3.0 \times 10^8 \text{ m/s} \quad T_1 = 10^\circ\text{C}, T_2 = 100^\circ\text{C}$$
$$c_{\text{water}} = 4.2 \times 10^3 \text{ J/kg}^\circ\text{C}$$

First, we need to find the thermal energy increase in the 250 g of water by using the specific heat of water, the temperature change, and the average mass of water:

$$\begin{aligned}\Delta E_H &= mc_{\text{water}}\Delta T \\ \Delta E_H &= mc_{\text{water}}(T_2 - T_1) \\ \Delta E_H &= (0.250 \text{ kg})(4.2 \times 10^3 \text{ J/kg}^\circ\text{C})(100^\circ\text{C} - 10^\circ\text{C}) \\ \Delta E_H &= (0.250 \text{ kg})(4.2 \times 10^3 \text{ J/kg}^\circ\text{C})(90^\circ\text{C}) \\ \Delta E_H &= 9.5 \times 10^4 \text{ J}\end{aligned}$$

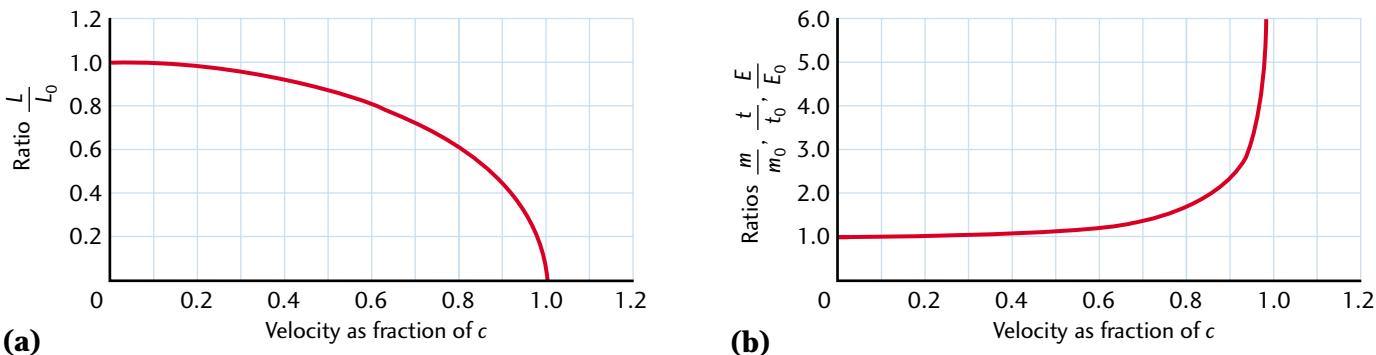
Next, we equate the energy change, ΔE , to the corresponding mass change, Δm . Using the equation $\Delta E = (\Delta m)c^2$, we obtain

$$\begin{aligned}\Delta m &= \frac{\Delta E}{c^2} \\ \Delta m &= \frac{9.5 \times 10^4 \text{ J}}{(3 \times 10^8 \text{ m/s})^2} \\ \Delta m &= 1.1 \times 10^{-12} \text{ kg}\end{aligned}$$

The mass increase in our cup of tea is therefore 1.1×10^{-12} kg, so eating our food hot or cold doesn't affect our mass intake noticeably! At the same time, with this amount of energy, we could lift a 250-g cup of tea to a height of 39 km!

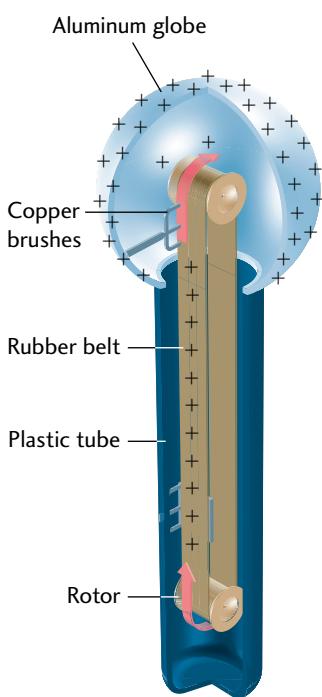
The two graphs in Figure 13.34 illustrate what we have learned in Sections 13.3, 13.5, and 13.7: as objects approach the speed of light, their length decreases in the direction of motion, while their mass and energy increase and time dilates.

Fig.13.34 Effects on length, time, mass, and energy as an object approaches the speed of light



- Explain why a proton gains more momentum when accelerating from $0.5c$ to $0.8c$ than from $0.2c$ to $0.5c$.
- Which particle has the greater rest mass: particle A with $E = 125$ J and $E_k = 87$ J, or particle B with $E = 54$ J and $E_k = 15$ J?
- How many grams of matter are equivalent to the energy needed to power an 80-W light bulb for one year?
- Abdullah of mass 65 kg is beamed up as pure electromagnetic energy while onboard the starship *Enterprise*. Calculate the energy equivalent to his mass.

Fig.13.35 A 250-kV Van de Graaff generator



13.8 Particle Acceleration

The Van de Graaff generator was invented in 1931. It allowed positive charges to build up on a metal sphere to very high voltages. When two generators are used in tandem, particles are accelerated through 30 MV. By 1960, the Van de Graaff generator had become the workhorse of low-energy nuclear physics.

At such high voltages, the classical equation for change in kinetic energy, $\Delta E_k = \frac{1}{2}m_0(\Delta v)^2 = qV$, is no longer valid because the electrons travel at very high speeds. Using this equation, the electrons' speed works out to be 3.2×10^9 m/s, which exceeds the speed of light. If we use the equation for relativistic energy, $E = m_0c^2 + E_k$, and substitute $E_k = qV$ when the electrons are accelerated from rest, we obtain the equation

$$E = m_0c^2 + qV$$

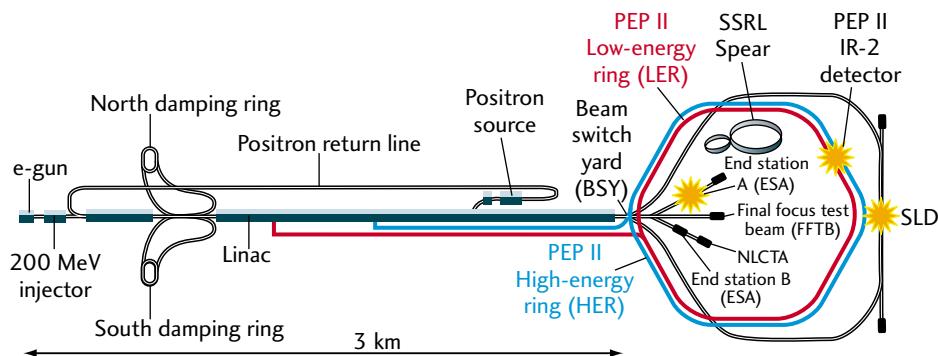
Scientists use this equation to determine the high-speed velocities of charges in particle accelerators such as the Stanford Linear Accelerator Center (SLAC) (see Figure 13.36), which accelerates electrons numerous times through a series of hollow tubular electrodes. The purpose of linacs (linear accelerators) is to move ions in a straight path at energies high enough to penetrate deeply into a target nucleus in order to produce elementary particles, to learn about nuclear structure, and to study particle collisions.

Fig.13.36 The Stanford Linear Accelerator Center (SLAC)



EXAMPLE 16 The three-kilometre accelerator

Fig.13.37 A schematic diagram of the inside of the Stanford Linear Accelerator



The SLAC accelerates electrons from rest through a potential of 50 GV (5.0×10^{10} V) over a distance of 3 km. Determine the speed of these electrons.

Solution and Connection to Theory

Given

$$m_0 = 9.11 \times 10^{-31} \text{ kg} \quad q = 1.602 \times 10^{-19} \text{ C} \quad V = 5.0 \times 10^{10} \text{ V}$$

$$c = 3.0 \times 10^8 \text{ m/s} \quad v = ?$$

To calculate the total energy, E , we substitute our given values for m_0 and c into the equation

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0 c^2 + qV$$

$$E = \frac{(9.11 \times 10^{-31} \text{ kg})(3.0 \times 10^8 \text{ m/s})^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= (9.11 \times 10^{-31} \text{ kg})(3.0 \times 10^8 \text{ m/s})^2 + (1.602 \times 10^{-19} \text{ C})(5.0 \times 10^{10} \text{ V})$$

$$\frac{8.19 \times 10^{-14} \text{ J}}{\sqrt{1 - \frac{v^2}{c^2}}} = (8.19 \times 10^{-14} \text{ J}) + (8.00 \times 10^{-9} \text{ J})$$

Bringing the $\sqrt{1 - \frac{v^2}{c^2}}$ up to the right side of the equation and squaring both sides, we obtain

$$\left(\frac{8.19 \times 10^{-14} \text{ J}}{8.00 \times 10^{-9} \text{ J}} \right)^2 = 1 - \frac{v^2}{c^2}, \text{ or}$$

$$1.048 \times 10^{-10} = 1 - \frac{v^2}{c^2}$$

We can simplify the last part of the solution by using a simple mathematical trick to obtain a very accurate answer for the particle velocity as it approaches the speed of light.

Since the electrons have a speed very close to the speed of light, we can approximate $(1 - \frac{v^2}{c^2}) = 2(1 - \frac{v}{c})$. Our equation becomes

$$1.048 \times 10^{-10} = 2 \left(1 - \frac{v}{c} \right) \text{ or}$$

$$5.24 \times 10^{-11} = 1 - \frac{v}{c}$$

Multiplying both sides of the equation by c (using $c = 3.0 \times 10^8 \text{ m/s}$), we obtain

$$3.0 \times 10^8 \text{ m/s} - v = (3.0 \times 10^8 \text{ m/s})(5.24 \times 10^{-11})$$

$$v = 3.0 \times 10^8 \text{ m/s} - 0.016 \text{ m/s}$$

The electrons are travelling only 1.6 cm/s slower than light!

High-speed Approximation

Since $1 - \frac{v^2}{c^2} = (1 - \frac{v}{c})(1 + \frac{v}{c})$, when $v \approx c$, then $(1 + \frac{v}{c}) \approx 2$.

Thus, $1 - \frac{v^2}{c^2} \approx 2(1 - \frac{v}{c})$

Converting kg to MeV/c²

$$E = mc^2$$

$$m = \frac{E}{c^2}$$

$E = qV$, where q is the elementary charge, e ; therefore,

$$m = \frac{eV}{c^2}$$

$$m = \left(\frac{eV}{c^2} \right) \left(\frac{\text{MeV}}{1 \times 10^6 \text{ eV}} \right)$$

$$m = \frac{\text{MeV}}{c^2}$$

For particle physicists, there is a more convenient unit of mass to use than the kilogram. Since mass and energy are equivalent, they are given the unit of MeV/c² (from Einstein's equation $m = \frac{E}{c^2}$), where **eV** is the **electron volt**, or the work done in accelerating an electron from rest through a potential of one volt.

When working on relativistic energy problems, using MeV/c^2 units instead of kilograms is usually more convenient, especially if we make use of the $E = mc^2$ triangle (Figure 13.38), as we will do in Example 17.

Fig.13.38 The energy triangle of special relativity

Deriving the Energy Triangle

$$mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

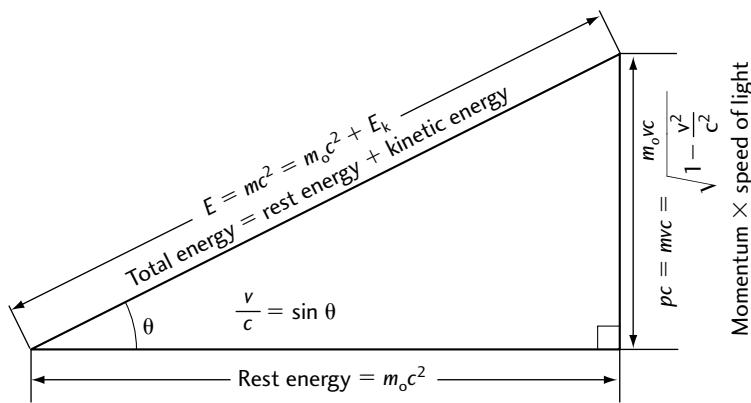
$$(mc^2)^2 = \frac{(m_0 c^2)^2}{1 - \frac{v^2}{c^2}}$$

$$(mc^2)^2 - (mv)^2 = (m_0 c^2)^2$$

$$(m_0 c^2)^2 + (mv)^2 = (mc^2)^2$$

Compare this equation with Pythagoras' theorem,

$$a^2 + b^2 = c^2$$



EXAMPLE 17 The 10-MeV electron linac

Hospitals use 10-MeV electron linacs to treat tumours (see Figure 13.39). Determine the final speed of these electrons.



Fig.13.39 A 10-MeV electron linac

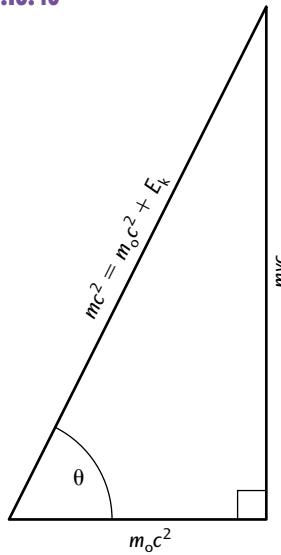
Solution and Connection to Theory

Given

$$m_0 = 0.511 \text{ MeV}/c^2 \quad qV = 10 \text{ MeV}$$

Using the $E = mc^2$ triangle (Figure 13.40), we substitute the given value for m_0 into a trigonometric function, $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, and use the equation $E_k = qV$ to obtain the angle θ .

Fig.13.40



$$\cos \theta = \frac{m_0 c^2}{m_0 c^2 + qV}$$

$$\cos \theta = \frac{0.511 \text{ MeV}}{0.511 \text{ MeV} + 10 \text{ MeV}}$$

$$\cos \theta = \frac{0.511 \text{ MeV}}{10.511 \text{ MeV}}$$

$$\cos \theta = 0.0486$$

$$\theta = \cos^{-1}(0.0486) = 87.21^\circ$$

Once we know the angle, we can calculate the electrons' speed:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{mv}{mc^2}$$

$$\frac{v}{c} = \sin 87.21^\circ$$

$$\frac{v}{c} = 0.9988$$

$$v = 0.9988c$$

$$v = 0.9988(3.000 \times 10^8 \text{ m/s})$$

$$v = 2.996 \times 10^8 \text{ m/s}$$

The speed of the electrons is $2.996 \times 10^8 \text{ m/s}$. These high-speed electrons penetrate deeper than alpha particles. They ionize cellular water molecules, creating free radicals that attack proteins, enzymes, and nucleic acids, thereby killing cells, including cancerous ones.

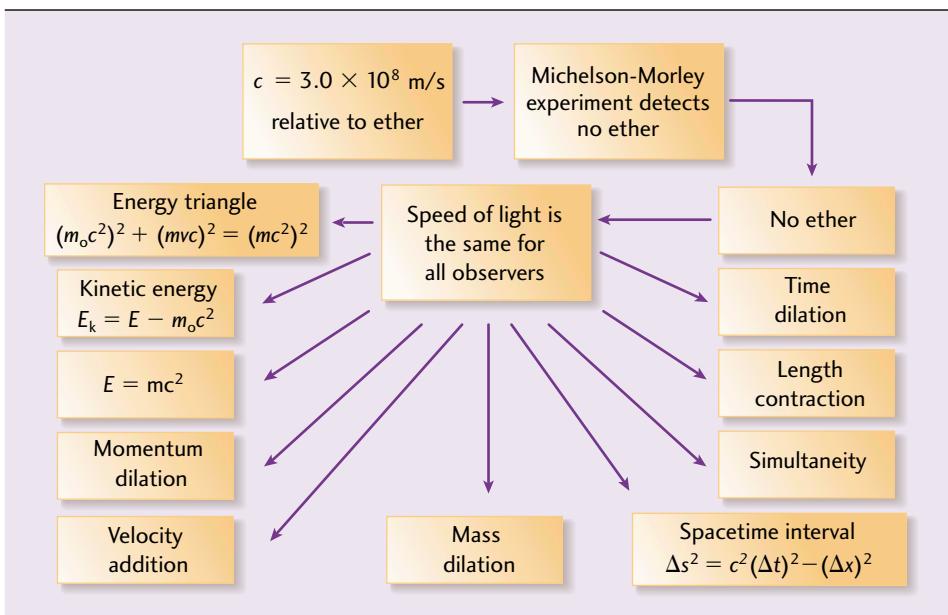
Pion Poetry

A pion that sped close to c
On a path that seemed longer to me
Had a half-life inflated,
And a mass quite dilated.
Its E_k was mega eVs.

In this chapter, we learned that in order for all the laws of physics, the speed of light, and the spacetime interval to be *absolute* in all inertial frames of reference, then space, time, mass, and energy must be *relative*; that is, we find their magnitude by comparing them to something similar, such as “how big is it (compared to what)?” or “what time is it (compared to when)?”

Figure 13.41 summarizes all the concepts we have studied in this chapter and how they are related.

Fig.13.41 Summary of Special Relativity



1. Convert the rest mass of a muon ($m_0 = 106 \text{ MeV}/c^2$) to kilograms.
2. Express the rest mass of a proton ($m_p = 1.67 \times 10^{-27} \text{ kg}$) in MeV/c^2 .
3. If the kinetic energy of a proton is five times its rest energy, find the proton's speed. (Hint: Use the energy triangle.)
4. A future linac accelerates protons such that their mass becomes 4×10^6 times their rest mass. How many metres per second less than c are these protons travelling?





The High Cost of High Speed

Fig.STSE.13.1 “People slowly accustomed themselves to the idea that the physical states of space itself were the final physical reality.”

—Professor Albert Einstein



The publication of Einstein’s theory of special relativity in 1905 not only dramatically altered society’s traditional perception of spacetime (see Figure STSE.13.1), but also the direction of technological development in the century that followed. By 1916, the theory was firmly established and today, it is the foundation of pure and applied research in nuclear and particle physics.

According to Einstein’s equation $E = mc^2$, a few eV/ c^2 of atomic mass are converted to energy during ionization. During nuclear decay, a few MeV/ c^2 s of nuclear mass are converted to energy. Scientists trying to understand the nature of the most elementary constituents of matter are looking to the next level of energy, where particle interactions in the TeV (10^{12} eV) range will reveal the rules governing their behaviour. Energizing ions to these immense energy levels requires innovative ideas and careful planning because the cost of construction can be astronomical.

In California, at the SLAC lab where high-energy electron beams have provided useful information about nuclear structure, scientists are now busy building a \$177-million upgrade. By colliding beams of 9.1-GeV electrons with beams of 3-GeV anti-electrons, they hope to explain why we have matter in the universe rather than antimatter. The present-generation 500-MeV cyclotrons, such as TRIUMF in Vancouver, British Columbia (see Figure STSE.13.2), cost about \$100 million. The TRIUMF cyclotron is used for nuclear reaction experiments involving pi (π) mesons. The cost to extend this design to a high-energy 5-GeV cyclotron would approximate the U.S. gross national product! At CERN, near Geneva, Switzerland, a new system called the Large Hadron Collider (LHC) is scheduled for completion in 2005 at a cost of over \$1.8 billion (see Figure STSE.13.3). One of LHC’s magnets is as massive as the Eiffel Tower in Paris, France! The LHC will be used to study collisions among ions with energies in the TeV range in an effort to detect the Higgs boson, the particle that is believed to give mass to subatomic particles. It seems that as the speed of the accelerated ions approaches ever closer to c , the cost of the particle accelerator approaches societies’ financial limits!

Fig. STSE.13.2 TRIUMF, Canada’s national laboratory for particle and nuclear physics



Fig. STSE.13.3 The new LHC at CERN will share the 27-km LEP tunnel in order to cut costs



Design a Study of Societal Impact

Research one of the modern particle accelerator labs, such as the linac facility in Darmstadt, Germany, or the Tevatron at Fermilab in Batavia, Illinois. Describe the purpose of the research the lab is undertaking. Determine the financial costs of constructing and maintaining the lab. Comment on any innovative techniques that were used to economize on construction or maintenance costs. Find some of the useful or beneficial spin-offs of the lab's research. Argue for or against whether such huge expenditures for research are justified in view of other pressing societal concerns, such as adequate funding for education, healthcare, or environmental protection.

Design an Activity to Evaluate

The intensity of naturally occurring radiation from the interaction of cosmic rays with Earth's upper atmosphere increases with altitude. In Canada, the average amount of exposure to cosmic radiation that a person receives almost doubles for every 2000-m increase in elevation. Use a Geiger detector/counter to perform a correlation study on the amount of background radiation at ground level to that obtained at higher elevations such as nearby mountains. Investigate ways in which airline pilots protect themselves against the harmful effects of high-altitude radiation exposure.

Build a Structure

Demonstration of high-speed particles need not be restricted to multi-billion-dollar colliding-beam accelerators. By researching electrostatic generators, you can construct a low-cost, effective, manually powered, reliable device similar to a Van de Graaff generator or a Whimshurst machine (see Figure STSE.13.5). Conduct a variety of experiments, such as estimating the efficiency of your electrostatic generator by measuring and comparing the energy input and the resultant electrical potential energy, or investigating the conductivity of air through measurements of the maximum discharge distance. During construction, remember that smooth, round metal components are better than sharp ones.

Fig.STSE.13.4 A radiation detector



Fig.STSE.13.5 A homemade electrostatic generator



SUMMARY SPECIFIC EXPECTATIONS

You should be able to

Understand Basic Concepts:

- Define and describe inertial and non-inertial frames of reference relative to a person at rest or moving at low or high speeds.
- Describe Einstein's first and second postulates of special relativity and how they revolutionized physics in the early 1900s.
- Use basic kinematic equations and relativity principles to derive the formulas of high-speed physics.
- Describe qualitatively and calculate quantitatively the mass, time, and length effects of special relativity.
- Apply relativistic velocity addition to astronomy and particle physics.
- Recognize the equivalence of mass and energy and quantitatively apply $E = mc^2$.

Develop Skills of Inquiry and Communication:

- Use analogies from other areas of physics to illustrate the concepts of special relativity.
- Illustrate, through examples, the relative concept of simultaneity.
- Carry out thought experiments based on your understanding of special relativity.

Relate Science to Technology, Society, and the Environment:

- Identify benefits arising from the development of expensive particle accelerators.

Equations

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0 c^2 + E_k$$

$$E = m_0 c^2 + qV \text{ (linacs)}$$

$$\text{Energy triangle: } (mv)^2 + (m_0 c^2)^2 = (mc^2)^2$$

$$\text{Relative velocity: } {}_A \vec{V}_C = \frac{{}_A \vec{V}_B + {}_B \vec{V}_C}{1 + \frac{{}_A \vec{V}_B \cdot {}_B \vec{V}_C}{c^2}}$$

$$\text{Spacetime interval: } (\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$$

$$\text{Cyclotron: } r = \frac{m_0 v}{Bq \sqrt{1 - \frac{v^2}{c^2}}}$$

EXERCISES

Conceptual Questions

- When is your car in an inertial frame of reference and when is it in a non-inertial frame? Give examples of each.
- At the Edmonton World Games, Donovan is running the 100-m dash while Leah is competing in the 1500-m race.

Fig.13.42



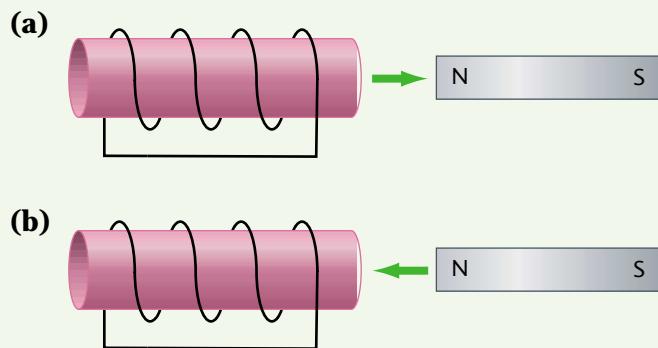
If they both run at a constant speed, will they both be in inertial reference frames during their respective events? Explain your answer.

- If you were riding inside the closed cabin of a steadily moving luxury cruise ship (no windows), could you devise a simple physics experiment to see if you were truly moving or not? Explain your answer.
- Which would take the least time: to swim upstream (parallel to the current) and back down, or to swim the same distance across the stream (perpendicular to the current) and back?
- Explain why the failure of the Michelson-Morley experiment was a benefit for science.
- If you were driving toward an intersection at a speed close to the speed of light and the traffic light suddenly turned amber, would it appear

redder or more yellowish than normal? (Hint: Think of the Doppler effect for sound.)

- Changing the magnetic field inside a coil of wire by inserting a magnet induces electrons to flow in the loops. Use the first postulate of special relativity to argue that moving electrons across a magnetic field should also force the electrons to flow in the loops. See Figure 13.43. (Inserting the magnet induces current according to Faraday's law, while moving the coil forces current according to the motor principle.)

Fig.13.43



- For a high-speed atmospheric muon, why is the proper time, t_0 , for its average lifetime measured in a reference frame moving with the muon, and not in the reference frame of the observing scientist on Earth?
- In terms of real numbers, use the relativity equation for length or time to explain why the speed, v , of an object must always be less than c .
- If you were moving north, parallel to a stationary copper wire in which the electrons were moving south (and the protons are at rest), from your point of view, would the wire seem positively charged (have a greater concentration of protons) or negatively charged (have a greater concentration of electrons)? Explain your answer.

- 11.** If you travelled to a star that was 5 ca away in a time of one year according to your watch, have you travelled faster than the speed of light?
- 12.** Based on what you have learned in this chapter, is it possible to go back in time?
- 13.** Does it matter if an observer approaches a stationary source of sound, or if the source approaches a stationary observer with an equal speed? Will the Doppler shift to a higher frequency be the same in each case? Does your answer seem to contradict the first relativity postulate? (The perceived frequencies are given by the equations $f_2 = \frac{f_1 v_s}{v_s - v_0}$ and $f_2 = \frac{f_1(v_s + v_0)}{v_s}$, respectively.)
- 14.** In Thought Experiment 1, both Barb and Phillip would be correct in saying that the other person's clocks ran slow. Explain.
- 15.** How do we know that the charge of an electron is constant and not changed by its motion in the same way as its mass is?
- 16.** What happens to the radius of the orbit of a proton travelling at right angles to a uniform magnetic field if the magnetic field is increased?
- 17.** As an object speeds up, does its density dilate at the same rate as its mass? Explain.
- 18.** If you are in a rocket moving with a speed $0.7c$ toward a star, at what speed will the starlight pass you?
- 19.** As stationary observers, we would see the relativistic effects of length contraction, mass dilation, and time dilation when observing a spacecraft go by at $0.90c$. What would the occupants of the spacecraft say that they observed about us?
- 20.** Light from your camera flash reflects off the mirror of a car moving with a speed v away from you. Is the speed of the returning light $c - v$? Explain.
- 21.** What if a particle could travel faster than c ? Describe the unusual properties of these hypothetical particles (known as tachyons) in terms of our understanding of physics.
- 22.** Which particle would have the greater speed: particle A whose kinetic energy is twice its rest energy, or particle B whose total energy equals twice its rest energy? Why?
- 23.** Does one kilogram of ice have the same rest energy as one kilogram of water? Explain your answer.
- 24.** Is it possible for light (electromagnetic radiation) to carry inertia (or mass) between emitting and absorbing bodies? Explain.
- 25.** Explain why a 100-eV electron is described as a classical particle but a 100-MeV electron is called a relativistic particle.
- 26.** What does it mean when we say that the rest mass of a muon is $106 \text{ MeV}/c^2$?
- 27.** Is it more accurate to state that particle accelerators speed electrons up to high speeds or that they increase the mass of electrons?

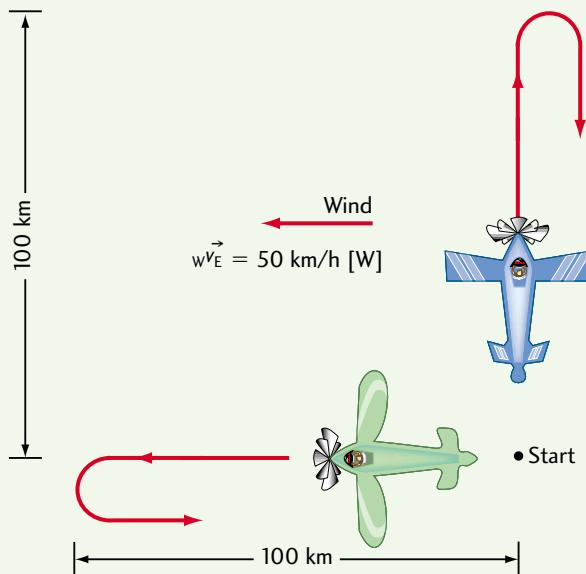
Problems

13.1–13.2 Einstein's Postulates of Special Relativity

- 28.** What fraction of the speed of light does each of the following represent?
- The rate of continental drift (3 cm/year)
 - The drift speed of electrons in a current-carrying wire (0.1 mm/s)
 - The speed of a human sprinter (10.8 m/s)
 - The speed of a fast aircraft (Mach 6.54)
 - The orbit speed of the electron in the Bohr model of the hydrogen atom ($2.2 \times 10^6 \text{ m/s}$)
- 29.** Two airplanes hold a Michelson-Morley race. With respect to the ground, Snoopy flies from north to south and back, while the Red Baron flies from east to west and back, each one

covering a total of 200 km. Both planes fly at 130 km/h in still air. However, during the race, the wind blows 50 km/h [W].

Fig.13.44 A Michelson-Morley race



Determine

- Snoopy's speed with respect to the ground while going north as well as south.
- the Red Baron's speed with respect to the ground while travelling east as well as west.
- Who wins the race and by how many seconds?
- Show that $\frac{\text{Snoopy's time}}{\text{Red Baron's time}} = \sqrt{1 - \frac{w^2}{v^2}}$, where w = wind speed and v = plane speed.

13.3 Time Dilation and Length Contraction

- A rod lying parallel to and moving parallel to the x axis with a speed of $0.80c$ has a proper length of 1.0 m. What is its length in the rest frame?
- A spaceship appears to be shortened to one-third of its proper length. What is its relative speed?

32. A moving stopwatch reads zero as it passes the origin of a co-ordinate frame at rest. What time does it read when it passes the 180-m mark of the rest frame if it travels at $0.7c$?

33. Muons (μ) are produced from the decay of pions (π) that have an average rest frame half-life of 2.6×10^{-8} s and travel at $0.998c$ in the upper atmosphere. Calculate how far they travel into our atmosphere before decaying.

34. In Thought Experiment 2, find the length L Katrina would measure, if the distance from Earth to Mars is $L_0 = 7.83 \times 10^{10}$ m and she is travelling at a speed of $v = 0.25c$.

35. Henry is driving his car at 35 m/s to take his girlfriend to see a movie. He thinks he arrived on time, but she thinks he is late. Find the difference in their two times if the distance from Henry's house to his girlfriend's house is 35 km.

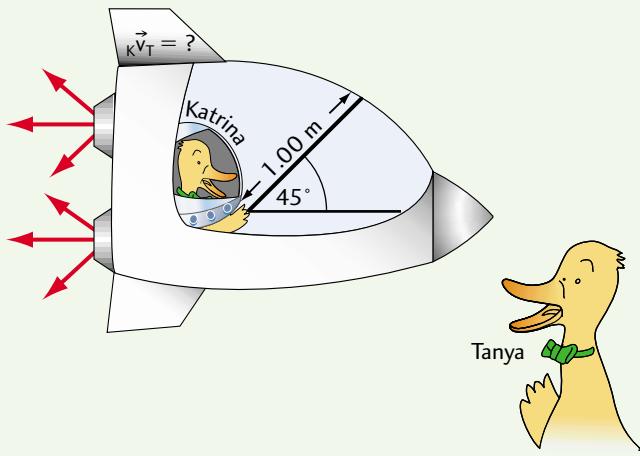
36. A high-speed muon in the CERN storage ring makes one complete orbit during its lifetime, which scientists in the lab measure to be 2.8×10^{-6} s. Find the radius of its orbit. The muon's average lifetime at rest is 2.2×10^{-6} s.

Fig.13.45 Inside the ring at CERN



- 37.** While travelling to Mars, Katrina the cosmonaut holds a 1.00-m stick at an angle of 30° from the direction of motion. A stationary observer, Tanya, measures the angle to be 45° as Katrina passes by (see Figure 13.46). How fast is Katrina moving? (Hint: Length is contracted in the direction of travel only.)

Fig.13.46 Tanya's viewpoint



- 38.** If an airplane travelled once around Earth ($r_E = 6.38 \times 10^6$ m) at 300 m/s, how far behind would its clock be, compared to a clock left behind at the airport? (Hint: Use the binomial approximation.)

13.4 Simultaneity and Spacetime Paradoxes

- 39.** Calculate the length of one light year (ca) in metres.
- 40.** In Jane's inertial reference frame, event Y occurs $1.0 \mu\text{s}$ after event X, 600 m away. In Ted's reference frame, the events occur simultaneously ($\Delta t' = 0$). Find the distance between events X and Y from Ted's viewpoint. (Hint: Use the equation for $(\Delta s)^2$.)
- 41.** In problem 40, find Ted's speed relative to Jane.

- 42.** A soccer ball is kicked the length of a 12-m boxcar and bounces off the far wall, returning to the player after a time of 4.0 s. A stationary observer on the outside, watching the train fly by at supersonic speed, records a time of 5.0 s for the soccer ball event. According to the stationary observer, how far did the train travel in 5.0 s?

- 43.** In problem 42, determine the speed of the train relative to the stationary observer.
- 44.** Trevor travels at a speed of $0.95c$ to a distant planet and immediately returns to Earth. On landing, he finds that his twin sister is one year older! How far is it from Earth to the planet?

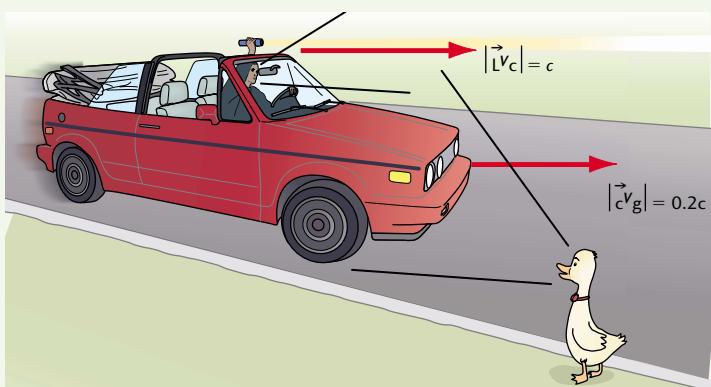
13.5 Mass Dilation

- 45.** Find the radius, r , of the circular path of an electron with speed $v = 0.8c$, travelling at right angles to a uniform magnetic field strength of 1.5 T. (Use the relativistic mass equation.)
- 46.** In Thought Experiment 4, if the electron orbits the proton with a speed of $0.6c$, what is the orbital radius, r ? (Use the relativistic mass equation.) How does this radius compare to the size of a proton? ($r_{\text{proton}} \approx 1.2 \times 10^{-15}$ m)
- 47.** Determine the mass increase of a 60-kg student when travelling at the speed of Earth in its orbit, $v = 3.0 \times 10^4$ m/s.
- 48.** What is the mass of an electron at SLAC that has a speed of $0.999\ 999\ 999\ 67c$? (Use the high-speed approximation.)
- 49.** Find the magnetic field strength required to keep an anti-electron with a speed of $0.999\ 999\ 986c$ orbiting in a circle of radius 450 m. The anti-electron rest mass is 9.1×10^{-31} kg and its charge is 1.6×10^{-19} C. (Use the relativistic mass equation.)
- 50.** At what speed is the density of an object dilated twice as much as its density at rest?

13.6 Velocity Addition at Speeds Close to c

51. A duck, standing by the side of the road at night, sees a car approaching at a speed of $0.2c$. If the driver is shining a flashlight forward as the car advances, find the speed of the light from the flashlight relative to the duck. (See Figure 13.47.)

Fig.13.47 Relative velocity of light



52. An astronomer observing stellar red shifts finds star *A* receding at a speed of $0.2c$ and star *B* receding at $0.3c$ in the exact opposite direction. Find the speed of star *A* relative to star *B*.
53. If rocket *A* moves with a velocity of $\vec{u} = 0.8c$ [N] relative to rocket *B*, and rocket *B* moves with a velocity of $\vec{v} = 0.7c$ [N] relative to Earth, find the velocity of rocket *A* relative to Earth.
54. A positron with a speed of $0.95c$ collides head-on with an electron going $0.85c$ in the opposite direction. Find the speed of the positron relative to the electron.

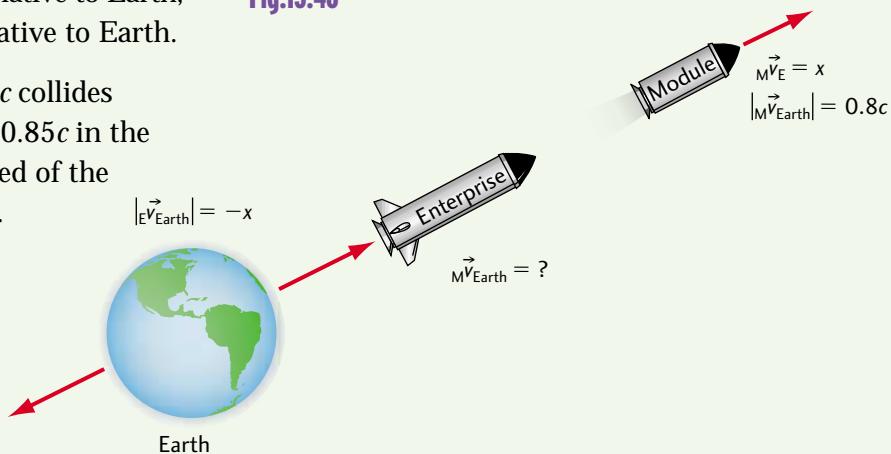
55. Officer Bob travels at a speed of $0.3c$ while pursuing Nicole Felon, who is escaping at a speed of $0.9c$. Bob fires a phaser bullet at Nicole with a speed of X relative to himself, and it just manages to reach Nicole. Find the value of X .

56. Captain J. Kirk of the SS *Enterprise* measures Earth receding from him at the same speed at which he fires an explorer module in the forward direction. If the speed of the explorer module relative to Earth is $0.80c$, then find the speed of the *Enterprise* relative to Earth. (See Figure 13.48.)

13.7 Mass–Energy Equivalence

57. In a chemical reaction, 3.2×10^4 J of heat energy are released when 1.0 g of coal is burned. Find the mass equivalence of this energy.
58. In a nuclear reaction, 9.2×10^{10} J of energy are released when 1.0 g of deuterium is fused. Find the mass equivalence of this energy.
59. Bananas cost \$1.29/kg. If we could convert 1.0 kg of bananas into energy, how many kilowatt-hours would we get? How does this banana rate of energy compare to a typical hydro consumer rate of \$0.08/kWh?

Fig.13.48



- 60.** Does it take more work to increase an electron's speed from $0.5c$ to $0.9c$, or from $0.9c$ to $0.95c$?
- 61.** If 8.19×10^{-14} J of energy were transformed into an object, would it have the mass of an electron or a proton?
- 62.** Find the difference between the classical momentum of a 125-kg meteorite travelling 75 km/s and its relativistic momentum.
- 63.** Mercury of mass 3.28×10^{23} kg moves along its solar orbit at an average speed of 4.78×10^4 m/s. How much mass converted to energy could accelerate Mercury from rest to this speed?
- 65.** Through what potential difference must an electron be accelerated from rest so that its mass equals that of a proton (938.3 MeV/ c^2)?
- 66.** Which particle has the greater speed: particle A with $m_0c^2 = 21$ J and $E_k = 8$ J, or particle B with $m_0c^2 = 22$ J and $E_k = 7$ J? (Use the energy triangle.)
- 67.** A cosmic-ray proton has a speed of $0.996c$. Express its total energy in units of MeV.

- 68.** What is the speed of the 3.1-GeV positrons used in the PEP II ring at Stanford? ($E_k = 3.1$ GeV, where 1 GeV = 10^9 eV)

- 69.** Which particle has the greater speed: particle A with a momentum of 4×10^{-8} N·s and a rest energy of 20 J, or particle B with momentum 5×10^{-8} N·s and total energy = 30 J?

13.8 Particle Acceleration

- 64.** The meson, meaning "in the middle," was first discovered in cosmic rays and has a rest mass of 135 MeV/ c^2 . Convert this mass to kilograms and show that it is between the mass of an electron and a proton.



A Relativity Thought Experiment

Purpose

To explore the daily observable effects of special relativity in a hypothetical world where the speed of light is 30 m/s

Equipment

Since this lab is a thought experiment (or Gedanken experiment, as Einstein would call it), you will need to be equipped with the equations and concepts of special relativity.

Fig. Lab.13.1 Albert Einstein



Procedure

Imagine a world where the speed of light was only 30 m/s. Newtonian mechanics would no longer be valid over a wide range of speeds and the effects of relativity would be very pronounced.

- Based on the classical definition of density, perform a thought experiment to determine the relativistic equation for density.
- Using your equation, complete Table Lab.13.1 below for various values of speed by calculating the density. Remember: $c = 30 \text{ m/s}$.
- From the values calculated in the table, plot a graph of density versus speed for $0 < v < c$. Use $\rho_0 = 1.0 \text{ g/cm}^3$.

- Assuming your rest density is 1.0 g/cm^3 , determine from your graph, the speed at which you would need to move in order to appear as dense as gold ($\rho_{\text{gold}} = 19.3 \text{ g/cm}^3$).

Discussion

Remember: $c = 30 \text{ m/s}$.

- If you were standing upright, facing forward but moving at a relativistic velocity to your right, which of the following quantities would change: height, width, pulse rate, number of atoms in your body, mass, temperature, rate of ageing, girth (waistline), or net electrical charge?
- If you spent 10% of your time travelling at a speed of 15 m/s and the other 90% of your time at rest, how would your life expectancy be affected? Calculate how long you would live from the viewpoint of stationary people who have an average lifetime of 75 years.
- As the head traffic engineer for the department of highways, you are in charge of setting a safe but fuel-efficient speed limit on cars. Calculate a speed limit using relativistic equations and explain your reason for your choice of speed limit. Keep in mind that collisions cause changes in kinetic energy.
 - If you wished to store your valuable 5.0-m-long Rolls Royce in your tiny 4.0-m-long garage, at what speed would you need to drive it into the garage? What paradox can arise from this situation involving relative motion?

Table Lab.13.1
Density versus Speed

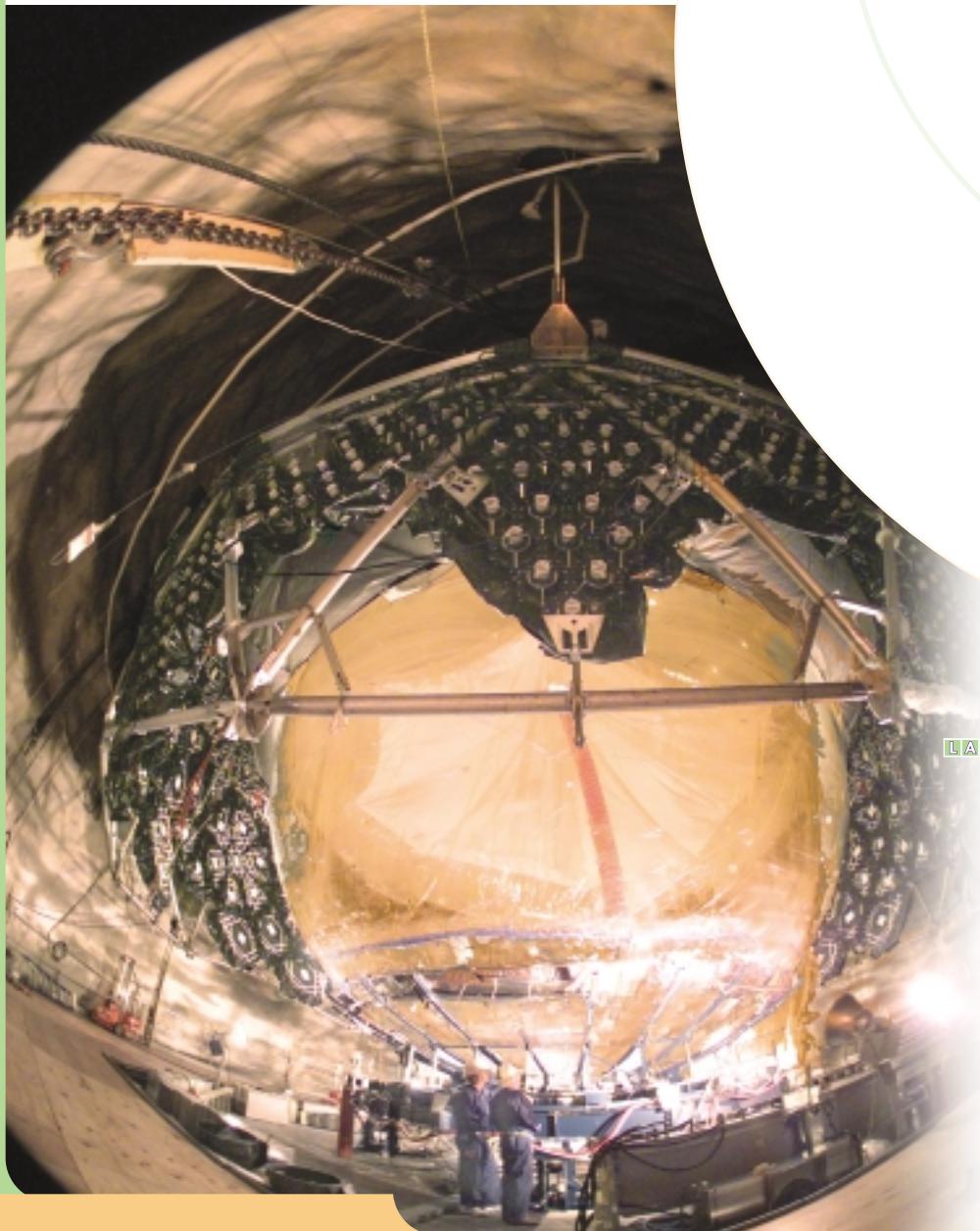
Speed (m/s)	0	0.50c	0.70c	0.85c	0.93c	0.96c	0.98c	0.99c
Density (g/cm ³)	1.0							

- c) While driving through an intersection at the posted speed limit, the traffic light was blue and you proceeded through. A patrol officer pulled you over and gave you a ticket for going through a red light! Explain what happened.
- d) You resume driving and decide to pass the lady in the car ahead by speeding up to 20 m/s. Her speedometer reads 15 m/s, but you are closing in on her at 7.5 m/s! How is that possible?
4. a) While wearing an electronic heart monitor, you sprint around a circular track at a speed of 10 m/s. You observe a pulse rate of 100 beats/min but your coach, resting at the centre of the stadium, records a different rate. What is the difference?
- b) Hungry after jogging, you decide to buy some “fast” food that costs \$5.00 per “quarter pounder,” but the bill comes to \$10.00! Calculate just how fast that quarter pounder was!
5. a) Aliens visiting your planet from another quadrant of the galaxy, where the speed of light is 3×10^8 m/s, say your food seems relatively cold. Explain.
- b) According to these aliens (called *Homo sapiens*), lots of other unusual effects are occurring on your planet — everyone is constantly resetting their watches! Can you explain to them why it’s necessary to do so on your planet?

Conclusion

Describe other everyday phenomena that would seem different to these alien visitors, and explain why they occur.

Nuclear and Elementary Particles



Chapter Outline

- 14.1 Nuclear Structure and Properties
- 14.2 Natural Transmutations
- 14.3 Half-life and Radioactive Dating
- 14.4 Radioactivity
- 14.5 Fission and Fusion
- 14.6 Probing the Nucleus
- 14.7 Elementary Particles
- 14.8 Fundamental Forces and Interactions — What holds these particles together?
 Positron Emission Tomography (PET)

LAB

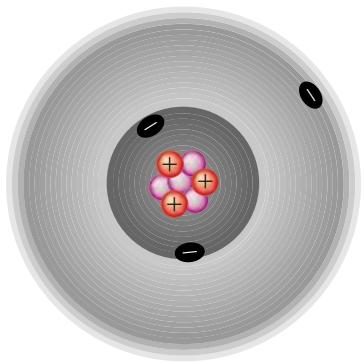
- 14.1 The Half-life of a Short-lived Radioactive Nuclide

By the end of this chapter you will be able to

- describe the concepts of radioactivity, quantum electrodynamics, and the Standard Model
- compare alpha, beta, and gamma radiation and their applications
- apply $E = mc^2$ to elementary-particle interactions
- describe how quantum theory has led to technological advances benefiting our society

14.1 Nuclear Structure and Properties

Fig. 14.1 Model of an atom of lithium (${}^7_3\text{Li}$). The nucleus is greatly enlarged to show its three protons (${}^1_1\text{p}$) and four neutrons (${}^1_0\text{n}$). The neutral atom has three electrons (${}^{-1}_0\text{e}$) in two shells to balance the charge on the nucleus.



All matter is composed of atoms that are in turn composed of a heavier, central, positively charged core surrounded by a less massive negatively charged cloud of electrons. The **nucleus**, or positively charged core of the atom, is composed of **neutrons** that have no charge, and positively charged **protons**. Protons and neutrons have about the same mass and are known as **nucleons**. An element or atom, and in particular the composition of its nucleus, is described using the notation



where **Z** is the **atomic number** (number of electrons or protons), **A** is the **atomic mass number** (number of protons + number of neutrons = number of nucleons), and **X** is the generic symbol for the atom or element. The **number of neutrons**, $N = A - Z$.

EXAMPLE 1

Particles in a nucleus

An atom has a mass number of 109 and an atomic number of 47. Find the name of the element, its symbol, the number of protons (or electrons), and the number of neutrons using a periodic table.

Solution and Connection to Theory

Given

$$A = 109 \quad Z = 47 \quad \text{element name} = ? \quad X = ? \quad N = ?$$

First, we can look in the periodic table (see Appendix I) and find the element with an atomic number of 47. It is silver, with symbol Ag.

The number of protons, Z , is 47 and the number of electrons is also 47.

For the number of neutrons,

$$N = A - Z$$

$$N = 109 - 47 = 62$$

Silver has 47 protons, 47 electrons, and 62 neutrons. It can be written as ${}^{109}_{47}\text{Ag}$ or just ${}^{109}\text{Ag}$.

Isotopes

You may have noticed in the periodic table that the atomic mass, A , of silver was 107.9 instead of 109. Silver is composed mainly of two types of atoms, called **isotopes**. In nature, 48% of all silver atoms have 62 neutrons while the remaining 52% have only 60 neutrons. Finding the weighted average of the two types of silver yields a mean value of

$$0.48 \times 109 + 0.52 \times 107 = 107.9$$

These two silver isotopes have similar chemical properties. Many elements have two or more isotopes. For example, hydrogen has three isotopes: hydrogen (^1H), **deuterium** (^2H), and **tritium** (^3H).

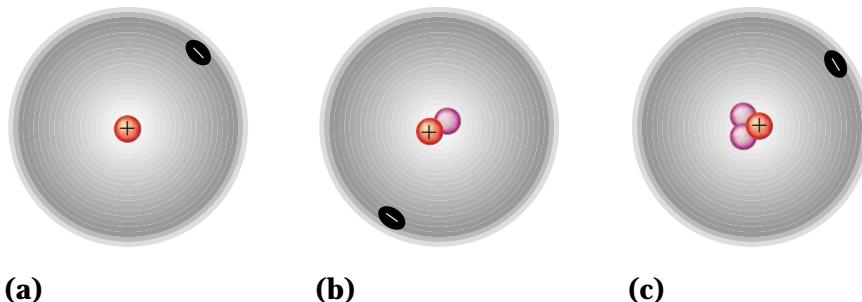


Fig.14.2

Three isotopes of hydrogen:

- (a) ^1H is normal hydrogen
- (b) ^2H is deuterium
- (c) ^3H is tritium

Unified Atomic Mass Units

Nuclear masses are specified in **unified atomic mass units**, **u**. Unified atomic mass units are based on a mass scale that defines the mass of the neutral carbon isotope, ^{12}C , to be exactly 12 u. The masses of other basic particles are given in Table 14.1.

Table 14.1
Rest Masses in Kilograms, Unified Atomic Mass Units, and MeV/c^2

Particle	kg	u	MeV/c^2
Electron	9.1164×10^{-31}	0.000 549	0.511
Proton	$1.672\ 62 \times 10^{-27}$	1.007 276	938.27
Neutron	$1.674\ 93 \times 10^{-27}$	1.008 665	939.57
Hydrogen	$1.673\ 53 \times 10^{-27}$	1.007 825	938.78
Deuterium	$3.344\ 49 \times 10^{-27}$	2.014 102	1876.12
Tritium	$5.008\ 27 \times 10^{-27}$	3.016 049	2809.43
Helium	$6.646\ 48 \times 10^{-27}$	4.002 603	3728.40

Isotopes of elements have been discovered using mass spectrometers. The atomic masses of nuclei can be determined by measuring the radius of the path of high-speed nuclei moving at right angles to a magnetic field. In Chapter 9, we learned that the motor principle equates the magnetic force, F_B , to the centripetal force, F_c . Since heavier nuclei travel a path of greater radius, the spectrometer can separate the different isotopes.

As we saw in Chapter 13, mass can also be expressed in MeV/c^2 , a unit used by particle physicists.

Mass Defect and Mass Difference

When we consider the mass of a stable nucleus, it is always less than the sum of the masses of the individual protons and neutrons that compose it. The difference between the actual atomic mass (in u) and the atomic number (A) is known as the **mass defect**. In nuclear processes, a **mass difference** corresponds to an energy difference or transformation.

EXAMPLE 2

The mass difference between fluorine and its nucleons

Compare the mass of the fluorine nucleus to that of its constituent protons and neutrons.

Solution and Connection to Theory

Given

Atomic mass (including electrons) = 18.9984 u $A = 19$ $Z = 9$

$$m_n = 1.008\ 665 \text{ u} \quad m(^1\text{H}) = 1.007\ 825 \text{ u}$$

$$N = A - Z = 19 - 9 = 10$$

The mass of 10 neutrons and 9 protons (including 9 electrons) is

$$10m_n = 10 \times 1.008\ 665 \text{ u} = 10.086\ 65 \text{ u}$$

$$9m(^1\text{H}) = 9 \times 1.007\ 825 \text{ u} = 9.070\ 425 \text{ u}$$

The total mass of all the neutrons, protons, and electrons is

$$10.086\ 65 \text{ u} + 9.070\ 425 \text{ u} = 19.157\ 0751 \text{ u}$$

Therefore, the mass difference between ^{19}F and its components is

$$19.1571 \text{ u} - 18.9984 \text{ u} = 0.1587 \text{ u}$$

When using the masses of neutral atoms, we need to keep track of the electron masses, which is why the mass of ^1H is used instead of the proton mass alone.

EQUIVALENT MASS UNITS

$$\begin{aligned}1 \text{ u} \\ 931.5 \text{ MeV}/c^2 \\ 1.661 \times 10^{-27} \text{ kg}\end{aligned}$$

Nuclear Binding Energy and Average Binding Energy per Nucleon

In Chapter 13, we learned from Einstein's energy equation, $E = mc^2$, that the mass difference between a nucleus and the sum of its constituent particles (i.e., the mass defect) is equivalent to a difference in energy, or $\Delta E = (\Delta m)c^2$. The lost mass appears as another form of energy, such as radiation.

In the converse process of breaking a nucleus into protons and neutrons, energy must be supplied from the outside. The amount of energy needed, equivalent to the mass difference, is called the **total binding energy** of the nucleus. It is this energy *deficiency* of the nucleus that keeps it together. The total binding energy is the work required to "unglue" the components of the nucleus. The **average binding energy per nucleon** (proton or neutron) is a measure of how tightly *each nucleon* is bound in the nucleus. We calculate it by dividing the total binding energy of the nucleus by the total number of nucleons it comprises.

EXAMPLE 3 The average binding energy of fluorine

Find the average binding energy per nucleon in the fluorine nucleus.

Solution and Connection to Theory

Given

From Example 2, we know that the mass difference of the fluorine nucleus is 0.1587 u.

The energy associated with this mass difference (expressed in MeV) is
 $E = (0.1587 \text{ u})(931.5 \text{ MeV/u}) = 147.8 \text{ MeV}$

In order to break up a fluorine atom's nucleus into its constituent nucleons, 147.8 MeV of energy are needed. In the case of fluorine, the average binding energy per nucleon is $\frac{147.8 \text{ MeV}}{19 \text{ nucleons}} = 7.78 \text{ MeV}$ per nucleon.

The atomic binding energy of the orbiting electron in the hydrogen atom is 13.4 eV due to the electrical force of attraction between the proton and the electron. Compared to this energy, the average binding energy for the nucleons in a fluorine nucleus is 7.78 MeV. In this case, the ratio of atomic energy to nuclear energy is about 1:10⁶, indicating that the nuclear force of attraction between nucleons is much greater than the force of attraction between the nucleus and the surrounding electrons. While it takes only a few electron volts to remove an electron from an atom, it takes a few *million* electron volts to remove a nucleon from a nucleus. We should therefore be able to obtain about a million times more energy from a nuclear process of energy extraction such as fission (see Section 14.5) than from a chemical process, such as burning coal.

1. What do the different isotopes of a given element such as hydrogen have in common? In what ways do they differ?
2. In the periodic table, the atomic masses of most elements are not whole numbers. Why not?
3. a) What is the total binding energy of the deuterium nucleus (in MeV)?
b) What is the average binding energy per nucleon of the deuterium nucleus (in MeV)?
4. If the isotope ^{35}Cl occurs naturally 75.8% of the time and ^{37}Cl occurs 24.2% of the time, determine the average atomic mass of chlorine. Compare your answer with the value found in the periodic table.
5. In 1992, the newly discovered 109th element, meitnerium, was named in honour of Lise Meitner, who in 1939, as a refugee from Nazi Germany, was the first to explain the process of nuclear fission. Research why Meitner declined to have any part in the building of nuclear weapons at the Manhattan Project and why she was denied the Nobel Prize in physics.



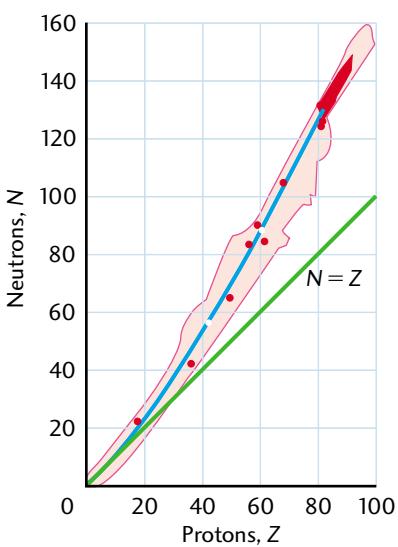
14.2 Natural Transmutations

Nuclear Stability

The **strong nuclear force** (i.e., the attractive force between nucleons: proton–proton, proton–neutron, and neutron–neutron) is very short-range (1.5×10^{-15} m). In a sense, nucleons are “glued” to their neighbours only. On the other hand, the electrical force of proton–proton repulsion, although weaker at shorter nucleon distances, is unlimited in range. It can overcome the strong nuclear force at larger nuclear distances (in nuclei where $Z > 82$) because all the nuclear protons repel one another. Heavy nuclei can become unstable if the localized nucleon attractive forces are unable to overcome the overall electrical proton–proton repulsion.

When the number of neutrons, N , is plotted against the number of protons, Z , for the various isotopes, we observe a pattern of nuclear stability. Stable nuclei tend to have the same number of protons and neutrons ($N = Z$) for the first 20 elements. Larger stable elements contain more neutrons than protons to counteract the increasing overall electrical proton–proton repulsion. Above $Z = 82$, no number of neutrons can produce the force required to form a stable nucleus. In Figure 14.3, the blue line represents the stable nuclei. The red areas represent naturally unstable nuclei. The pink region represents artificial unstable isotopes.

Fig. 14.3 A chart of isotope nucleons and their stability



Radioactivity is the spontaneous disintegration of atomic nuclei through the emission of radiation or particles. At the end of the 19th century, Henri Becquerel, a French physicist, discovered that uranium salts are spontaneously radioactive. By 1898, Marie and Pierre Curie discovered and isolated two unknown and highly radioactive elements called radium and polonium. In an attempt to find out more about the process of radioactivity, scientists tried reacting radium with other chemicals, and also heating it to high temperatures. They concluded that the process of radiation originated from the nucleus.

Ernest Rutherford and other scientists revealed that the radiation from radium consisted of three types of emissions: **alpha (α) particles**, **beta (β) particles**, and **gamma (γ) rays**. When a particle is emitted from a nucleus, the ratio of neutrons to protons (the $\frac{N}{Z}$ ratio) changes and the nucleus of the new element tends to be more stable. The changing of one element into another is called a **transmutation**.

Table 14.2 summarizes the characteristics of alpha, beta, and gamma emissions.

Table 14.2
Radioactive Emissions

Emission	Description	Rest Mass (in u)	Charge	Penetrating power	Relative Ionizing ability
Alpha	Helium nuclei (two protons and two neutrons)	4.002 603	+2	Stopped by a sheet of aluminum foil	Highest
Beta	Electrons	0.000 549	-1	Several millimetres of aluminum	Medium
Gamma	Short-wave electromagnetic radiation	0	0	30 cm of lead or 2 km of air	Lowest

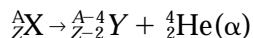
When a magnetic field is applied to a beam or stream of emitted alpha particles, they are deflected in one direction, and a beam of beta particles is deflected in the opposite direction, whereas gamma rays are not deflected at all. The direction of deflection indicates that alpha and beta particles have opposite charges and gamma rays are uncharged (see Figure 14.4). Alpha particles have the highest relative **ionizing ability** (ability to strip electrons from atoms) because they have the greatest mass and charge.

In a cloud chamber, alpha particles leave short, fat tracks, beta particles leave longer, skinny tracks, and gamma rays leave very long tracks with so few ions produced that they are difficult to detect.

Alpha Decay

During alpha decay, a nucleus emits an alpha (α) particle, which consists of two protons and two neutrons, which is equivalent to a helium nucleus. The new element formed is called the **daughter nucleus**. Many of the heavy nuclei (where $Z > 82$) decay through alpha emission (see Figure 14.5).

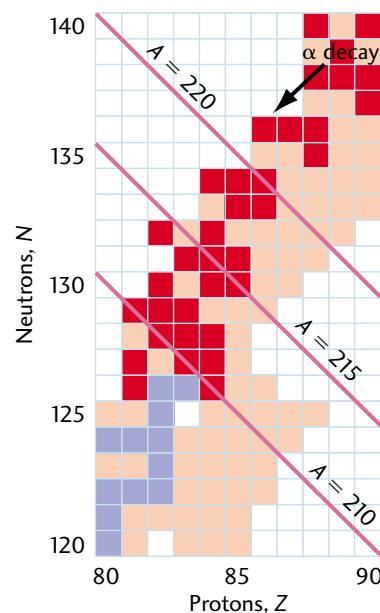
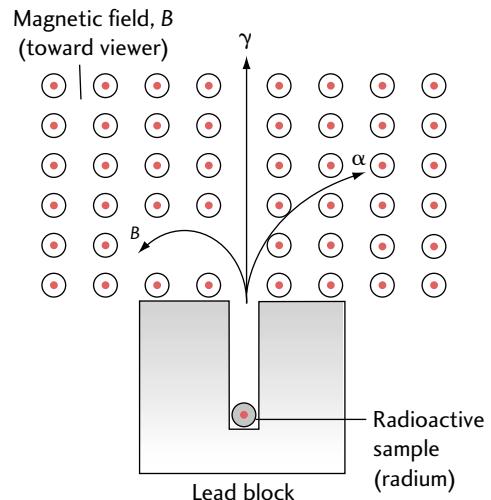
The general equation for alpha decay can be written as



In Figure 14.5, alpha decay is equivalent to moving down two squares and left two squares, as two protons and two neutrons are emitted by the nucleus. Alpha emission *increases* the $\frac{N}{Z}$ ratio of a nucleus, as illustrated in the following example.

Fig.14.5 Chart of the nucleons (nuclear particles) of the isotopes of elements from mercury to thorium. Blue nuclides are stable, red nuclides are naturally radioactive, pink nuclides are artificial and radioactive.

Fig.14.4 Alpha and beta particles are bent in opposite directions by a magnetic field while gamma rays are not deflected



EXAMPLE 4**The $\frac{N}{Z}$ ratio in decay processes**

Find the ratio of neutrons to protons in the radon nucleus and compare it with the neutron/proton ratio in the daughter nucleus (after the alpha particle has been emitted).

Solution and Connection to Theory**Given**

$$Z_{\text{Rn}} = 86 \quad A_{\text{Rn}} = 222 \quad Z_{\text{He}} = 2 \quad A_{\text{He}} = 4 \quad \frac{N}{Z} = ?$$

The neutron number for radon is

$$N = A - Z$$

$$N = 222 - 86$$

$$N = 136$$

The $\frac{N}{Z}$ ratio for radon is

$$\frac{N}{Z} = \frac{136}{86} = 1.581$$

The neutron number of helium is

$$N = 4 - 2$$

$$N = 2$$

Radon emits an alpha particle, which carries away two protons and two neutrons from the nucleus. In the remaining nucleus,

$$N = 136 - 2 = 134 \text{ and } Z = 86 - 2 = 84$$

The new $\frac{N}{Z}$ ratio is

$$\frac{N}{Z} = \frac{134}{84} = 1.595$$

From the periodic table and from $Z = 84$, we know that the new element formed is polonium-218. This isotope of polonium is also unstable and spontaneously decays by emitting an alpha particle.

Note that the mass of the daughter polonium nucleus (218.008 966 u) and the alpha particle (4.002 603 u) have a sum (222.011 569 u) that is slightly less than the mass of the original radon nucleus (222.015 353 u). From Section 14.1, we know that the missing mass represents a transformation of mass into energy, given by the equation $\Delta m = \frac{\Delta E}{c^2}$. Much of this energy becomes the kinetic energy of the emitted alpha particle.

EXAMPLE 5 The kinetic energy of an alpha particle

From 1899 to 1906 at McGill University in Montreal, QC, Ernest Rutherford and Frederick Soddy studied the radioactive alpha decay ($m = 4.002\ 602\ \text{u}$) of radium ($m = 226.025\ 402\ \text{u}$) to radon ($m = 222.017\ 571\ \text{u}$). Find the energy released, available as kinetic energy, for the alpha particle and the daughter nucleus.

Solution and Connection to Theory

Given

$$m_{\alpha} = 4.002\ 602\ \text{u} \quad m_{\text{radon}} = 222.017\ 571\ \text{u} \quad m_{\text{radium}} = 226.025\ 402\ \text{u}$$

The total mass of the final alpha particle and radon is
 $4.002\ 602\ \text{u} + 222.017\ 571\ \text{u} = 226.020\ 172\ \text{u}$

The mass available as kinetic energy when radium decays to radon is
 $226.025\ 402\ \text{u} - 226.020\ 172\ \text{u} = 0.005\ 23\ \text{u}$

Since $1\ \text{u} = 931.5\ \text{MeV}$, ΔE , available as kinetic energy, E_k , is
 $\Delta E = (0.005\ 23\ \text{u})(931.5\ \text{MeV/u}) = 4.87\ \text{MeV}$

This energy is given to the alpha particle and to the daughter nucleus. Since momentum is conserved during the decay, the momenta of the alpha particle and the daughter nuclei are equal and opposite ($p_{\alpha} = p_d$). However, their kinetic energies ($E_k = \frac{p^2}{2m}$) are different. The alpha particle receives most of the kinetic energy because its mass is much smaller. In this case, the energy of the alpha particle is about 4.86 MeV.

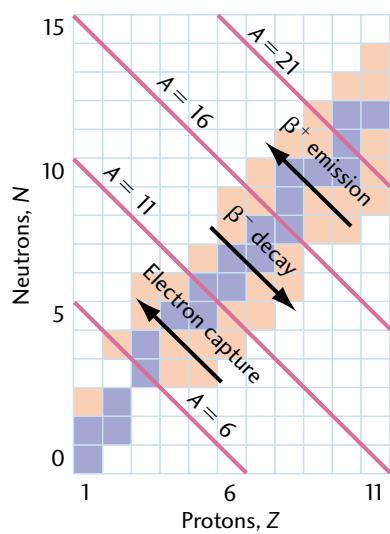
Beta Decay

There are two basic kinds of beta decay: β^- decay in which electrons are emitted, and β^+ decay in which positrons are emitted. Since the mass of the beta particle ($0.000\ 548\ \text{u}$) is extremely small compared to a proton's mass ($1.007\ 276\ \text{u}$), the daughter nucleus has the same atomic mass as the parent nucleus.

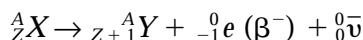
β^- Decay (Electron Emission)

The unstable isotopes to the left of the blue line in Figure 14.6 have *too many neutrons* in the nucleus. One way for such isotopes to achieve greater stability is for a neutron to become a proton. When a neutron decays to a proton, it emits an electron (e^-). When β^- emission takes place, a continuous spectrum of kinetic energy of the emitted electrons is observed. In other words, the electron's kinetic energy is often less than we would expect from the laws of conservation of mass and energy. In 1930, Wolfgang Pauli

Fig.14.6 Chart of the nucleons of the isotopes of elements from hydrogen to sodium



suggested that the remaining energy is given to a second particle that became known as the **neutrino**, a neutral particle that has a very small, or zero, rest mass. Its **antiparticle** is the **antineutrino**. When a neutron decays to a proton, it emits an electron and an *antineutrino* ($\bar{\nu}$). (Neutrinos and antineutrinos will be discussed further in Section 14.7.) This process of spontaneous electron emission is called **β^- decay** and is described by the equation

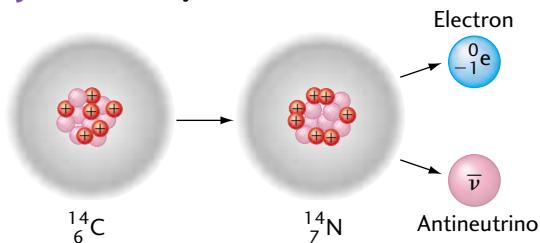


Notice that in β^- decay, the atomic number of daughter nucleus ($Z + 1$) is one greater than that of the parent nucleus. In both alpha and beta decay, one element becomes changed into another; that is, it undergoes a transmutation.

EXAMPLE 6 The missing mass

During the lifetime of a plant or animal on Earth's surface, a certain amount of the isotope carbon-14 is absorbed by the organism through the process of respiration. The unstable carbon-14 nucleus decays by β^- emission to nitrogen-14. Find the mass difference for the β^- decay of carbon-14 to nitrogen-14, and its energy equivalent.

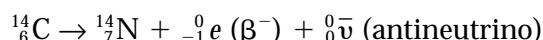
Fig.14.7 The decay of carbon-14



Solution and Connection to Theory

Given

$$m_C = 14.003\ 242\text{ u} \quad m_N = 14.003\ 074\text{ u}$$



To find the mass difference,

$$\Delta m = m_C - m_N$$

$$\Delta m = 14.003\ 242\text{ u} - 14.003\ 074\text{ u}$$

$$\Delta m = 0.000\ 168\text{ u}$$

To find the energy equivalent of the mass difference,

$$\Delta E = (\Delta m)c^2$$

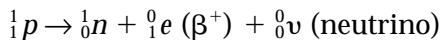
$$\Delta E = (0.000\ 168\text{ u})(931.5\text{ MeV}/\text{uc}^2)c^2$$

$$\Delta E = 0.156\text{ MeV} = 156\text{ keV}$$

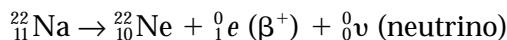
The mass difference of the β^- decay of carbon-14 to nitrogen-14 is 0.000 168 u, which is equivalent to 156 keV of energy.

β^+ Decay (Positron Emission)

Positron decay is like the mirror image of electron decay. The unstable isotopes to the *right* of the blue line in Figure 14.6 *do not have enough neutrons* to glue the protons together. To achieve greater stability, a proton decays to a neutron by emitting a positron (e^+) and a neutrino (ν) according to the statement



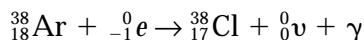
In **positron** or **β^+ emission**, the atomic number of the nucleus becomes one less, as in the following equation for the β^+ decay of sodium:



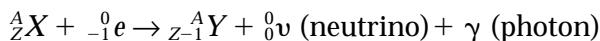
Positron emission was first observed in 1934 by Irene Curie and Pierre Joliet during the β^+ decay of phosphorus-30.

Electron Capture and Gamma Decay

Although related to β^+ decay, electron capture is different in that a particle is *taken into the nucleus* rather than emitted from it. In electron capture, an atomic electron strays too close to the nucleus and is absorbed, causing a proton to change into a neutron. Again, as in β^+ decay, a neutrino is emitted. A typical electron capture equation for the isotope argon-38 is



As the innermost electron shell of the atom becomes empty, a higher-energy electron drops down to fill this lower energy level, and an x-ray or a gamma ray is emitted:



As we learned in Chapter 12, excited electrons in the atom emit photons of light when they drop from a higher to a lower energy level. In a similar way, radioactive gamma emissions occur when the nucleus decays to a lower energy state. The quantized energy levels of nuclei are much farther apart than the energy levels of electrons in atoms. Gamma emission is observed in all nuclei of atomic mass greater than 5. It usually occurs during alpha and beta decay as the daughter nucleus is led into an excited state.

Table 14.3 summarizes the different types of spontaneous radioactive decays (natural transmutations).

Table 14.3
Summary of Spontaneous Radioactive Decays

Emission	Unstable parent nucleus	Daughter nucleus
Fig.14.8a		
	Alpha decay	
	${}^A_Z X$	
	Alpha particle	${}^{A-4}_{Z-2} \gamma$
Fig.14.8b		
	β^- -decay	
	${}^A_Z X$	
	Electron	${}^A_{Z+1} \gamma$
	${}^0_{-1} e$	
	Antineutrino	${}^0_0 \bar{\nu}$
Fig.14.8c		
	β^+ decay	
	${}^A_Z X$	
	Positron	${}^A_{Z-1} \gamma$
	${}^0_{+1} e$	
	Neutrino	${}^0_0 \nu$
Fig.14.8d		
	Electron capture	
	${}^A_Z X$	
	Electron	${}^A_{Z-1} \gamma$
	${}^0_{-1} e$	
	Neutrino	${}^0_0 \nu$
	Gamma ray	
Fig.14.8e		
	Gamma decay	
	${}^A_Z X$	
	Gamma ray	${}^A_Z X$

- Assume that nucleons can only bond with neighbouring nucleons. In this model, determine the maximum number of attractive bonds a nucleon can have. See how many tennis balls you can attach to a central ball.
- For the following parent nuclei undergoing alpha decay, determine the daughter nucleus.
 - $^{238}_{92}\text{U}$
 - $^{248}_{96}\text{Cm}$
 - $^{223}_{86}\text{Rn}$
 - $^{244}_{94}\text{Pu}$
 - $^{64}_{29}\text{Cu}$
- For the following parent nuclei undergoing β^- decay (electron emission), determine the daughter nucleus.
 - $^{32}_{15}\text{P}$
 - $^{23}_{10}\text{Ne}$
 - $^{35}_{16}\text{S}$
 - $^{45}_{20}\text{Ca}$
 - $^{64}_{29}\text{Cu}$
- For the following parent nuclei undergoing β^+ decay (positron emission), determine the daughter nucleus.
 - $^{19}_{10}\text{Ne}$
 - $^{22}_{11}\text{Na}$
 - $^{46}_{24}\text{Cr}$
 - $^{239}_{93}\text{Np}$
 - $^{64}_{29}\text{Cu}$
- Research and report on how cloud chambers show tracks from radioactive emissions.
- Scientific discoveries can arise from an unexpected result during experimentation. Becquerel expected the emission of x-rays to decrease in uranium salts that had not been exposed to sunlight, but this effect was not observed. Find other examples of important findings in science that arose unexpectedly or by chance.

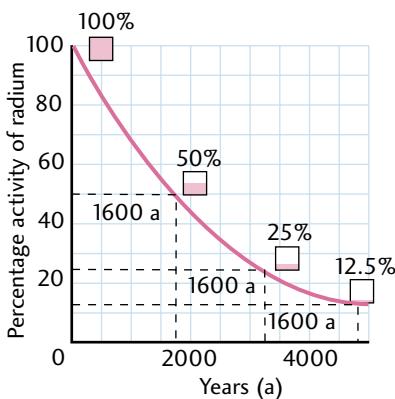
14.3 Half-life and Radioactive Dating

Half-life

The time it takes for one-half of the nuclei of a sample of a radioactive isotope to decay by spontaneous emission is called the **half-life** of that isotope. For carbon-14, one-half of the radioactive nuclei will decay in 5730 a. At the end of two half-life intervals or 11 460 a, one half of the remaining nuclei will decay, leaving $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ of the original sample. After three half-life intervals, $\frac{1}{8}$ of the original will be left, and so on. It's as if you had a new car-rental business where you started out with 128 cars, but after three years, half the cars were scrapped, leaving you with 64 cars. After six years, again half of the remaining 64 cars were scrapped and now you have only 32 cars to rent. We can therefore state that the half-life of a car rental is three years. In Figure 14.9, we see a plot of this pattern on a graph for the decay of radium-226.

Table 14.4 Half-lives of Common Radioactive Isotopes			
Radioisotope	Symbol	Decay	Half-life
beryllium-8	^8_4Be	α	2×10^{-16} s
polonium-214	$^{214}_{84}\text{Po}$	α	1.64×10^{-4} s
oxygen-19	$^{19}_8\text{O}$	β^-	29 s
magnesium-29	$^{29}_{12}\text{Mg}$	β^-	9.5 min
lead-212	$^{212}_{82}\text{Pb}$	β^-	10.6 h
iodine-131	$^{131}_{53}\text{I}$	β^-	8.04 d
argon-39	$^{39}_{18}\text{Ar}$	β^-	5.26 a
cobalt-60	$^{60}_{27}\text{Co}$	β^-	5.3 a
strontium-90	$^{90}_{38}\text{Sr}$	β^-	28.8 a
radium-226	$^{226}_{88}\text{Ra}$	α	1.62×10^3 a
carbon-14	$^{14}_6\text{C}$	β^-	5.73×10^3 a
americium-243	$^{243}_{95}\text{Am}$	α	7.37×10^3 a
plutonium-239	$^{239}_{94}\text{Pu}$	α	2.44×10^4 a
uranium-235	$^{235}_{92}\text{U}$	α	7.04×10^8 a
uranium-238	$^{238}_{92}\text{U}$	α	4.45×10^9 a

Fig.14.9 The activity of radium over three half-lives



Mathematically, the **radioactive decay** curve can be drawn from the function

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{\frac{1}{2}}}}$$

where N_0 is the initial number of nuclei (or initial concentration) and N is the number remaining (or final concentration) after a time interval t . $T_{\frac{1}{2}}$ is the half-life of the particular isotope. The number of nuclei decaying per second is called the **activity** and is measured in becquerels (Bq). Activity is proportional to the number of nuclei present and is described by the equation

$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{\frac{1}{2}}}}$$

where A_0 represents the initial activity and A is the activity after a time t .

EXAMPLE 7 Radioactive gas

Radon-222 is a colourless, odourless, inert gas that is a daughter product of uranium-238. If the half-life of radon-222 is 3.8 days, how long does it take for a sample to decay to 10% of its original concentration?

Solution and Connection to Theory

Given

$$T_{\frac{1}{2}} = 3.8 \text{ d} \quad N_0 = 100\% \quad N = 10\% \quad t = ?$$

Substituting into our decay equation,

$$10\% = 100\% \left(\frac{1}{2}\right)^{\frac{t}{3.8 \text{ d}}}$$

Dividing by 100%, we get

$$0.1 = \left(\frac{1}{2}\right)^{\frac{t}{3.8 \text{ d}}}$$

Taking the log of each side,

$$\log(0.1) = \frac{t}{3.8 \text{ d}} \log(0.5)$$

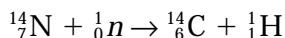
Solving for t ,

$$t = \left(\frac{\log(0.1)}{\log(0.5)}\right)(3.8 \text{ d}) = 12.6 \text{ d}$$

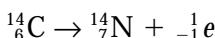
A sample of radon-222 takes 12.6 days to decay to 10% of its original concentration.

Radioactive Dating

Carbon-14 is a radioactive isotope that is used by archaeologists to determine the age of certain artifacts. Carbon-14 is created in the upper atmosphere when a neutron from cosmic radiation interacts with nitrogen, creating a daughter nucleus of carbon-14 and a hydrogen atom:



Eventually, the unstable carbon β^- decays back to nitrogen according to the equation



These two competing reactions create an atmospheric equilibrium concentration of carbon-14, which can react with oxygen to form carbon dioxide, ${}^{14}\text{CO}_2$. Since the half-life of ${}^{14}\text{C}$ is 5730 a, it is long-lasting and pervasive. During photosynthesis, carbon-14 is incorporated into plant sugars along with the stable carbon isotope ${}^{12}\text{C}$. Thus, all biological organisms contain a specific ratio of ${}^{14}\text{C}$ to ${}^{12}\text{C}$. However, when an organism dies, it no longer takes in new ${}^{14}\text{C}$; so the ratio of ${}^{14}\text{C}$ to ${}^{12}\text{C}$ in the dead organism begins to decrease as its supply of ${}^{14}\text{C}$ decays. By determining this ratio in an organic relic, archaeologists are able to determine its age.

EXAMPLE 8 Viking relics

The Norse settlement of L'Anse aux Meadows of Northwest Newfoundland and Labrador is the oldest known European colony in Canada. Two thousand four hundred Norse objects have been excavated at this site. If these early Norse colonies were built in 1006, find the percentage ratio of ${}^{14}\text{C}$ to ${}^{12}\text{C}$ found in these relics compared to the ratio in organic matter presently alive.

Solution and Connection to Theory

Given

N_0 = original number of nuclei in the relic

$$t = 2000 - 1006 = 994 \text{ a} \quad T_{\frac{1}{2}} = 5730 \text{ a} \quad N = ?$$

Using our decay equation and substituting the given values, we obtain

$$N = N_0 \left(\frac{1}{2} \right)^{\frac{994 \text{ a}}{5730 \text{ a}}} = N_0 (0.5)^{0.1735} = N_0 (0.8867)$$

The concentration is about 88.7% of the present-day ratio of ${}^{14}\text{C}:{}^{12}\text{C}$ in living organisms.

Fig.14.10 Norse relics found at L'Anse aux Meadows in Newfoundland and Labrador





One problem with carbon-14 dating is the assumption that the condition of the upper atmosphere hasn't changed over the centuries. Also, it's possible that relics become contaminated with other organisms during the decay of the original ^{14}C . However, carbon dating is still considered an adequate and fairly accurate means for approximating the age of organic relics.

Similar to the way the ratio of $^{14}\text{C}:\text{ }^{12}\text{C}$ is used for dating organic relics, the ratio of $^{235}\text{U}:\text{ }^{238}\text{U}$ can be used to find the age of geological formations.

1. You bake a round cake, cut it in half, and eat one side. Still hungry, you then cut the remaining piece in half and eat one of the quarters. If you ate 8 pieces in all, what fraction of the cake is left?
2. Traces of potassium-40 are found in some salts sold at your grocery store. If the half-life of ^{40}K is 1.28×10^9 a, determine the time it would take for a 5-mg sample to decay to 1 mg of ^{40}K .

An Ancient Natural Nuclear Reactor

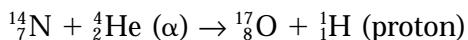
Normally, 0.72% of all uranium is ^{235}U , whether it is found on Earth, in Moon rocks, or in meteorites. However, the uranium extracted from a West African mine in Oklo, Republic of Gabon, contains only 0.44% ^{235}U , while the rest is ^{238}U . The reason for this low concentration of ^{235}U is believed to be due to a natural radioactive process that occurred in the distant past, which consumed a lot of the ^{235}U . For this process to occur, scientists estimate that a concentration of about 3.0% ^{235}U is needed.

3. Find how long ago the natural concentration of ^{235}U was 3.0% if it is 0.44% today.
4. What environmental changes could have been caused by the natural fission reactor in West Africa 1.7 billion years ago?
5. Data collection from the rings of long-lived trees (more than 1000 years old) indicate the carbon-14–carbon-12 ratio has changed with time. Describe some human activities that could be altering this ratio.

14.4 Radioactivity

Artificial Transmutations

A transformation of one element into another can also occur when a nucleus is struck by another particle. In 1919, Rutherford bombarded nitrogen with alpha particles (emitted from radium) and found that oxygen was created. As the alpha particles from radioactive decay travelled through nitrogen gas, some of them were absorbed and protons were emitted according to the reaction statement



In Section 14.3, we learned that ^{14}C becomes ^{14}N . This natural transmutation occurs in the upper atmosphere when high-speed neutrons interact with ^{14}N . Prior to 1932, **artificial** (or human-made) **transmutations** of elements were restricted to the available energies of alpha particles emitted during alpha decay; that is, 4 MeV to 8 MeV of energy. In 1932, John Cockcroft and Ernest Walton were able to produce the nuclear transformation of lithium-7 into beryllium-8 by means of a 400 000-V accelerator that accelerated protons up to speeds of 8.8×10^6 m/s. The unstable ^8Be then decayed into two alpha particles (see Figure 14.11).

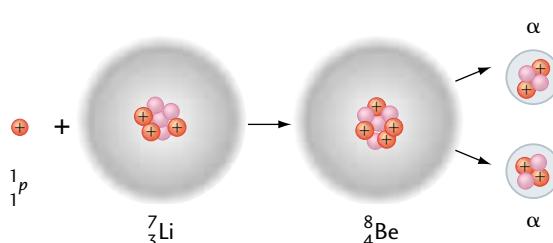


Fig. 14.11 In the Cockcroft–Walton experiment, a 400-keV proton interacts with lithium to create the unstable ^8Be , which quickly decays into two alpha particles

In that same year, James Chadwick discovered the neutron. When he bombarded beryllium with alpha particles from polonium decay, he found that the high-energy emission had no detectable charge. When he aimed these emissions at a block of paraffin, they collided with the hydrogen protons in the wax. By analyzing the ejected protons, he discovered that the emissions were about the same mass as a proton, only neutral in charge. Thus, the neutron became a tool for physicists to probe further into the positively charged nucleus. Unlike alpha particles, which are positive and repelled by the nucleus, the neutron could approach the nucleus without repulsion.

None of the elements with more than 83 protons in their nucleus are stable. When a heavy, naturally occurring element such as uranium is bombarded with neutrons, a transient element called neptunium ($_{93}\text{Np}$), with a half-life of 2.4 days, is created. In general, when heavy elements are bombarded by high-energy ions in a **cyclotron**, it is possible for even more massive elements to form that may not occur naturally.

EXAMPLE 9 The search for element 118

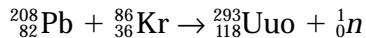
In 1999, the Lawrence Berkeley Lab announced that three atoms of the element 118 (ununoctium, Uuo) had been synthesized by accelerating a beam of krypton-86 ions to an energy of 449 MeV and directing it onto a target of lead-208. Write the nuclear reaction statement for this interaction if one neutron is also produced.

Solution and Connection to Theory

Given

From the periodic table, krypton is $^{86}_{36}\text{Kr}$ and lead is $^{208}_{82}\text{Pb}$.

Since Pb and Kr provide $208 + 86 = 294$ nucleons (on the left side), there must be the same number, $293 + 1 = 294$, for the Uuo and neutron (on the right side). In this manner, the nucleon number is conserved. Also, the amount of charge, $Z = 82 + 36 = 118$, on the right must be the same ($118 + 0$) on the left. In this way, charge is also conserved. From these considerations, we obtain



Using an 88-inch (224-cm) cyclotron, the team of scientists took 11 days to find the heaviest **transuranic** ($Z > 92$), or human-made, element of the time. Two years later, they retracted their announcement after several confirmation experiments failed to reproduce the results! However, these manufactured elements have extended the periodic table to at least 112 elements. For example, in 1996, lead and zinc atoms were fused to create ununbium, which quickly decayed to ununnilium. Ununnilium decays to hassium, which in turn decays to seaborgium. The nuclear reactions are as follows:

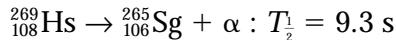
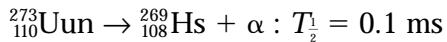
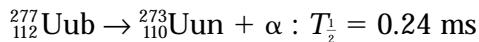
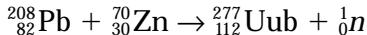
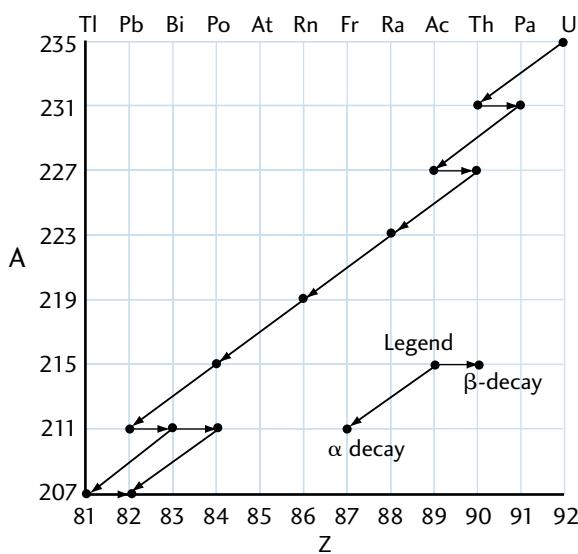


Fig. 14.12 A decay series beginning with $^{235}_{92}\text{U}$. Isotopes in the series are shown by a dot representing their A and Z numbers. Horizontal arrows show beta decay while diagonal arrows show alpha decay.



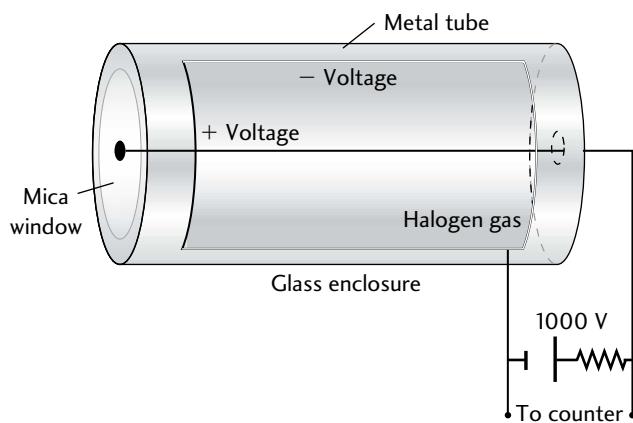
A succession of such decays is called a **decay series**. In the above series, ^{277}Uub and ^{273}Uun are very short-lived and would not normally exist in nature because they would not be around long enough to be detected. On Earth, ^{235}U decays naturally to the stable lead isotope ^{207}Pb through a series of seven alpha decays and four beta decays, as shown in Figure 14.12.

Since Earth is believed to be over 4.5 billion years old, many radioactive isotopes having short half-lives should have disappeared long ago. But certain other isotopes, such as ^{235}U , ^{238}U , and ^{323}Th , have long half-lives. These isotopes continually replenish the isotopes in their decay series that have short half-lives. For example, ^{235}U alpha-decays to ^{231}Th . The half-life of ^{231}Th is only 25.5 hours. ^{231}Th would not exist on Earth today if it were not replenished by the decay of ^{235}U .

Detecting Radiation

Regardless of our daily activities, we are exposed to radiation whether we realize it or not. Low-level radiation can be found in rocks, water, air, and in the food we eat. By far the largest contributor to our daily radiation exposure is the natural world.

To detect radiation, a variety of instruments is used, including a Geiger counter (see Figures 14.13a and b). This instrument consists of a metal tube filled with halogen gas such as argon at low pressure. A thin insulated wire is mounted in the centre of the tube. The potential difference between the wire and the outside of the tube is almost large enough to ionize the argon. When radiation enters the mica end window, it ionizes the gas, creating an ion pair. The negative ion (electron) accelerates toward the wire, reaching a high velocity and producing a large number of additional ion pairs due to repeated collisions. The new electrons also accelerate, thus creating an avalanche of negative charges upon the wire. The resulting current through the circuitry produces a voltage pulse that can be amplified and counted electronically. The voltage pulse can also drive a speaker, so we may hear the clicking noise associated with the detection of radioactivity.



The Measure of Nuclear Activity

Recall from Section 14.3 that the amount of *activity* of a radioactive source equals the number of nucleon disintegrations per second, measured in becquerels (Bq).

Fig.14.13a The Geiger counter

The **absorbed dosage** through exposure to radiation is measured in units called **grays (Gy)**. One gray is the amount of radiation that deposits energy at a rate of one joule per kilogram in an absorbing material; that is,

$$1 \text{ Gy} = 1 \text{ J/kg}$$

Alpha particles lose energy more rapidly, depositing all their energy over a much shorter path compared to beta particles and gamma rays. For this reason, the biological effect from a dose of 1 Gy of alpha radiation can be up to 20 times more damaging than from the same dose of beta or gamma radiation. Since the biological effect depends both on the type of radiation and on the **dose (D)**, a **dose equivalent (DE)**, measured in **sieverts (Sv)**, is found by using a **quality factor (QF)** from 1 to 20. For example, a 2-Gy dose of low-energy kilo-electron-volt neutron radiation with a quality factor of 4 would have an equivalent dose of

$$DE = D \times QF = 2 \times 4 = 8 \text{ Sv}$$

Fig.14.13b Detecting radiation using a Geiger counter



Table 14.5
Measuring Radiation

Quantity	Measure of	Unit
Activity (A)	Decay rate	becquerel (Bq)
Absorbed dose (D)	Energy absorption	gray (Gy)
Dose equivalent (DE)	Biological effectiveness	sievert (Sv)

EXAMPLE 10

Energy of an equivalent dose

A short-term dose equivalent (DE) of 4 Sv of gamma radiation will cause death to about 40% of the people exposed to it. If the quality factor (QF) of gamma radiation is 1, determine the amount of energy that a 70-kg person would absorb from such a dose.

Solution and Connection to Theory

Given

$$m = 70 \text{ kg} \quad QF = 1 \quad 1 \text{ Gy} = 1 \text{ J/kg} \quad DE = 4 \text{ Sv}$$

$$D = \frac{DE}{QF}$$

$$D = \frac{4 \text{ Sv}}{1} = 4 \text{ Gy} = 4 \text{ J/kg}$$

Thus, 1 kg of body mass absorbs 4 J of energy. A 70-kg person will absorb an energy, E , given by

$$E = \text{mass} \times \text{dose}$$

$$E = (70 \text{ kg})(4 \text{ J/kg})$$

$$E = 280 \text{ J} = 300 \text{ J}$$

The person would absorb 300 J of energy.

This amount of energy would only raise the temperature of a person by 0.001°C . Thus, it is not the heat, but rather the absorption of the radiation by water molecules in the cells of the body that causes damage. The gamma rays cause water molecules to dissociate into very reactive ions that attack the organic molecules, the basic building blocks of the cell. Fortunately, the average annual radiation dose per person is about 2 mSv, which is below the 360-mSv “no observable effect” level for mammals.



1. Which element has the most massive stable isotope?
2. If the average whole-mouth dental x-ray is a dose of 0.20 mSv, then how many dental x-rays could we have each year to still be under the 0.36-Sv “no observable effect” level?
3. In general, radioactive elements are bound to minerals in rocks deep within Earth and present no hazard to our health. However, all the radioactive series emit the radioactive gas radon. When rocks fracture or are used in construction materials, the gas may escape from the surface and enter the water we drink and the air we breathe. Table 14.6 shows the average levels of radon gas emissions in schools of British Columbia. Some places are close to exceeding the Canadian guideline recommendation to take action if the activity per cubic metre reaches 800 Bq/m³.
 - a) Comment on the irony of the present trend to conserve energy through the construction of well-insulated and tightly sealed homes and buildings.
 - b) Research the potential for earthquake prediction that radon gas detection may have in areas close to fault or fracture lines within Earth.

Table 14.6
Comparison of Radon Levels in Homes and in Schools

School district	Mean radon in schools (Bq/m ³)	Mean radon in homes (Bq/m ³)	% of schools above 150 Bq/m ³	% of homes above 150 Bq/m ³	% of schools above 750 Bq/m ³	% of homes above 750 Bq/m ³
Kelowna	26	85	4	7.8	0	0
South Okanagan	81	107	14	16.4	0	1.4
Penticton	38	107	5.6	16.4	0	1.4
Castlegar	100	240	38	41	15	6
Prince George	30	89	4.5	29	0	0
North Thompson	137	159	70	53	0	11
Vernon	57	74	5	9.2	0	0
Nelson	164	122	45	19.7	5	1.4
Trail	57	111	13	16.4	0	0

Decay Series and the Food Chain

Scientists are concerned about the radiation levels found in the wildlife of Northern Canada. Animals receive higher doses from radioactive isotopes in their food than do humans because they ingest more soil with their meals. Also, by living outside, they are exposed to more external cosmic and gamma radiation.

^{238}U , ^{235}U , ^{232}Th , and ^{40}K (found in 0.01% of all potassium) are the main sources of naturally occurring radioisotopes that animals are exposed to through the decay series. Polonium-218, though short-lived

Fig. 14.14 The caribou in Northern Canada receive higher doses of radiation than humans



(3.8 days), delivers the greatest radiation doses to caribou that breathe radon-222 gas oozing out of the soil. Caribou ingest high levels of polonium-210 because of their diet of moss-like lichen. As radon gas decays in the atmosphere, rainfall washes lead-210 (further down the decay series) to the ground, where it is absorbed by plants. People who eat caribou show elevated concentrations of ^{210}Po in the liver and kidneys. Small burrowing creatures, such as voles living near the uranium tailing facilities in Northern Saskatchewan, have shown particularly high concentrations of radium-226.

Artificial radioisotopes can get into the environment from processes like nuclear weapons testing and nuclear reactor accidents. Iodine-131, with a half-life of eight days, presents a hazard right after an accident because it can accumulate in the thyroid gland, causing cancer. Many of the beta-emitting isotopes decay after five years, but strontium-90 and cesium-137 have longer half-lives (28 to 30 years) and have found their way into all of the world's populations. This effect is due, in part, to the 1000 megatons of nuclear explosives released into the atmosphere between 1945 and 1963 as a result of nuclear testing. Cesium-137 is particularly hazardous in the tundra. Levels reached as high as 1100 Bq/kg in caribou of Northern Quebec after the 1986 Chernobyl nuclear power plant accident in the Ukraine. People eating this meat ran a significant health risk. Monitoring the amount of radiation in the environment and in the food chain helps scientists affect international policy.

4. If the quality factor (*QF*) of a 1.3-MeV gamma-ray emission is 1 and the activity per kilogram in Scandinavian reindeer due to this radiation is 29 000 Bq/kg, determine the annual dose these reindeer could be receiving in sieverts.
5. Research the history of nuclear weapons testing from 1945 until the 1963 Atmospheric Test Ban Treaty. Determine the societal and political factors that led countries to ban such testing and briefly discuss the main terms of the treaty.

14.5 Fission and Fusion

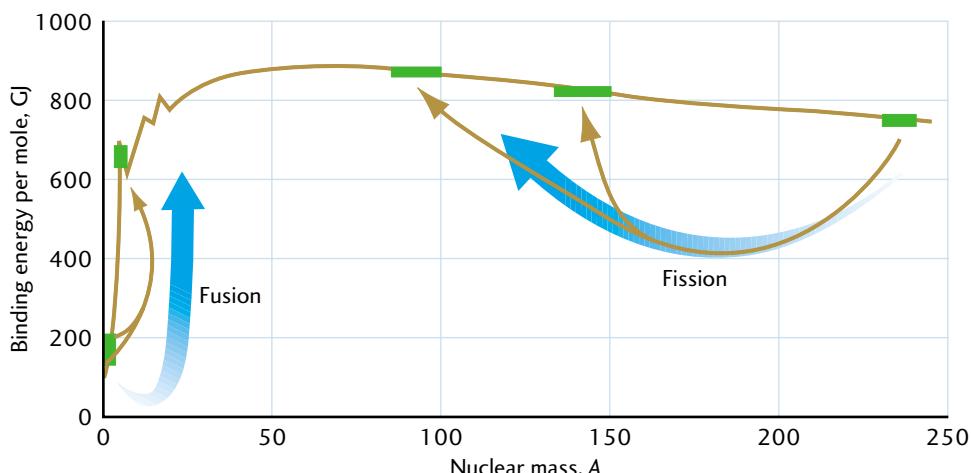
Avogadro's number

One mole represents 6.02×10^{23} of anything

When two nucleons are within 1 fm or 2 fm (10^{-15} m) from each other, they experience a binding nucleon force of attraction. At such small distances, this attractive force, called the *strong nuclear force*, is more powerful than the electrostatic force of repulsion between protons. Strong forces are associated with large energies. Compared to **exothermic** chemical reactions (involving electric forces), the energy released in nuclear reactions can be 4×10^7 times greater per mole. The attainment (or hope of attainment) of such large energy releases has made the processes of *fission* and *fusion* important to nuclear engineers. As we will see, each of these processes takes place with nuclei that reside at opposite ends of the periodic table.

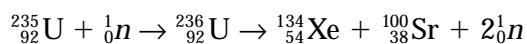
Fission

Fission is a nuclear process whereby heavier nuclei split into two smaller nuclei. This spontaneous decay occurs because the binding energy of massive nuclei is less than that of stable isotopes having atomic mass numbers between 36 and 56. The binding energy graph of Figure 14.16 shows the energy of nuclear fission. An isotope with a mass number of 240 and a binding energy of 750 GJ/mol breaks into two isotopes, each having a binding energy of about 850 GJ/mol for each nucleon in the nucleus.



The difference in energy between the parent and daughter nuclei means that, potentially, 100 GJ/mol of energy is released! To break radioactive isotopes such as ^{235}U into two, scientists bombard them with slow or **thermal neutrons**. If a neutron is moving slowly enough, then the uranium nucleus can absorb it. If the neutron is too fast, then it skips on by, like a rock skipping across a pond's surface. The deformed uranium-235 nucleus stretches to an elliptical shape, becoming unstable and splitting apart as it elongates. In Figure 14.17, it looks almost like biological fission or cell division as the electrical repulsion overcomes the strong nuclear force holding it together.

An example of a fission reaction is one that occurred in the first atomic bomb:



The fission fragments, ^{134}Xe and ^{100}Sr , are roughly half the mass of ^{236}U . They are just one combination of the many possible product nuclei. The number of neutrons produced can also vary. Regardless of the combination of product nuclei, the sum of the A and Z values balances on each side of the reaction statement. The excited ^{236}U nucleus lasts only 10^{-12} s, so the process is very quick (see Figure 14.18).

Fig.14.15

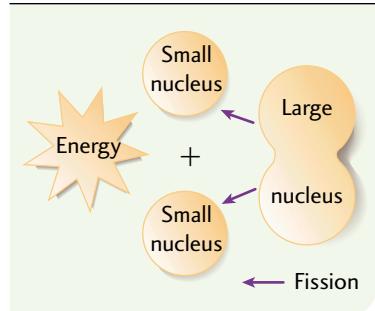


Fig.14.16 The binding energy of nuclear particles (nucleons) for the elements expressed in gigajoules of energy, plotted against the number of particles in atomic nuclei

Fig.14.17 An impacting neutron causes the uranium nucleus to vibrate, leading to fission

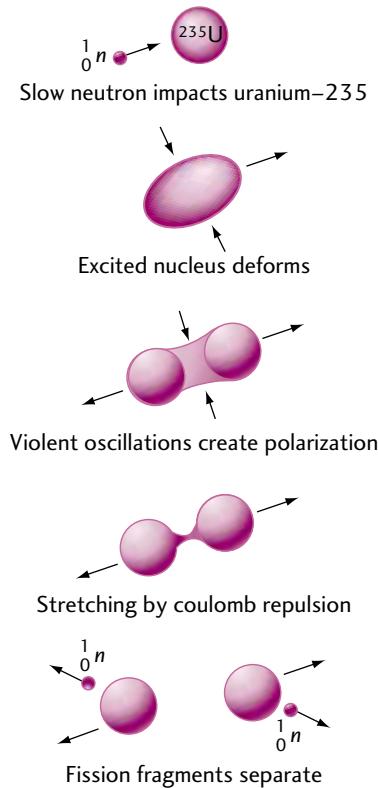
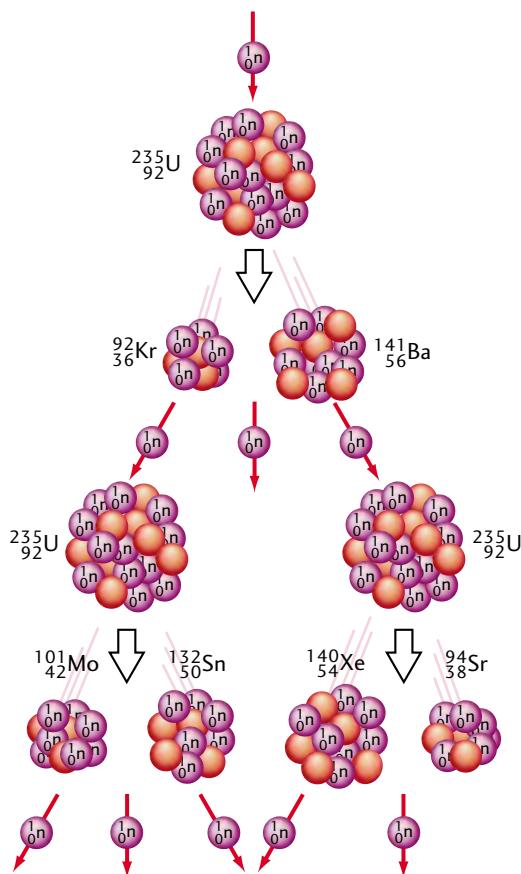


Fig.14.18 A fission of uranium-235



The Sum of a Geometric Series

For $1 + 2 + \dots + 2^n$,

the sum is

$$S_n = 2^{n+1} - 1$$

In the case where $n = 5$,

$$S_5 = 2^{5+1} - 1 = 63$$

If there is enough uranium available, then the two product neutrons could bombard two more nuclei, releasing energy. Then, the four new neutrons could interact with four more ^{235}U nuclei, releasing more energy and eight neutrons, and so on. This process is called a **chain reaction**. Each step in the process takes only about 10 ns, and the number of disintegrations quickly adds up as the sum of a geometric series: $1 + 2 + 4 + 8 + \dots$. For example, in a chain reaction involving 80 steps, the number of uranium nuclei disintegrating is enormous! Since $S_{80} = 2^{80+1} - 1 = 2.4 \times 10^{24}$, the number of moles of uranium undergoing fission would be

$$\frac{2.4 \times 10^{24} \text{ nuclei}}{6.02 \times 10^{23} \text{ nuclei/mol}} = 4.01 \text{ mol}$$

Expressed in mass units, this value represents

$$(4.01 \text{ mol})(235 \text{ g/mol}) = 0.942 \text{ kg of } ^{235}\text{U}$$

In atomic bombs, some neutrons escape through the surface of the material before they can cause further fission reactions.

Fig.14.19 Atomic bombs at the Russian Nuclear Weapons Museum at Arzamas-16, now known as Sarov



The chain reaction only becomes self-sustaining when there is enough mass to slow down or **moderate** enough of the neutrons, causing them to interact with, rather than escape, the radioactive material. This amount of material is called the **critical mass**. In Figure 14.20, the smaller purple circles represent uranium nuclei, and the red arrows show the paths that the neutrons could take after fission.

The lengths of the arrows represent the distances the neutrons travel before capture. If the amount of material is too small, as in Figure 14.20a, then many of the neutrons exit through the surface of the material. A smaller amount of material therefore has less chance of capturing a neutron than a larger amount of material (Figure 14.20b). Depending on the material being used, critical mass is typically a few kilograms.

EXAMPLE 11 The power of the atomic bomb

If 0.942 kg of ^{235}U undergoes fission in a chain reaction, find the approximate power of the explosion. Assume from the previous geometric example that the number of fission steps for 0.942 kg of ^{235}U is 80 and that each step takes about 10 ns.

Solution and Connection to Theory

Given

$$\Delta E/\text{mol} \approx 100 \text{ GJ/mol}$$

$$0.942 \text{ kg of } ^{235}\text{U} = \frac{942 \text{ g}}{235 \text{ g/mol}} = 4.01 \text{ mol}$$

$$n \text{ (number of geometric steps)} = 80 \quad \Delta t \approx 10 \text{ ns/step} \quad P = ?$$

To find the power, P , we need ΔE as well as the total time, $t = n\Delta t$, for the 4.01 mol to undergo fission through a chain reaction.

First, the energy released is

$$\Delta E = (100 \text{ GJ/mol/nucleon})(4.01 \text{ mol})(235 + 1 \text{ nucleons})$$

$$\Delta E = 95 \text{ TJ}$$

Power is energy per unit time (J/s), so

$$P = \frac{\Delta E}{n\Delta t}$$

$$P = \frac{95 \text{ TJ}}{(80 \text{ steps} \times 10 \text{ ns/step})}$$

$$P = \frac{9.5 \times 10^{13} \text{ J}}{8 \times 10^{-7} \text{ s}}$$

$$P = 1.2 \times 10^{20} \text{ W}$$

This example illustrates the tremendous power that was unleashed, first as a test over a desert of New Mexico in July, 1945, then later during the Second World War over the Japanese cities of Hiroshima and Nagasaki. Compared to a lightning flash where 1 GJ of energy is delivered in a time of 0.2 s, the atomic bomb is 19 000 times more powerful!

Fig.14.20 In a sub-critical mass at (a), too many neutrons escape before colliding; in the critical mass of (b), enough neutrons encounter other nuclei to maintain a chain reaction.

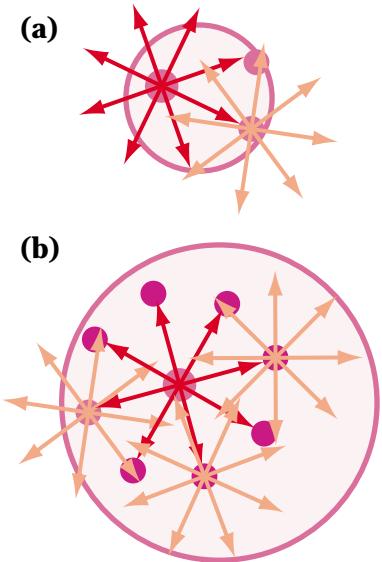


Fig.14.21 A 61-kiloton fusion device, detonated on June 4, 1953, called Climax at the Nevada Test Site

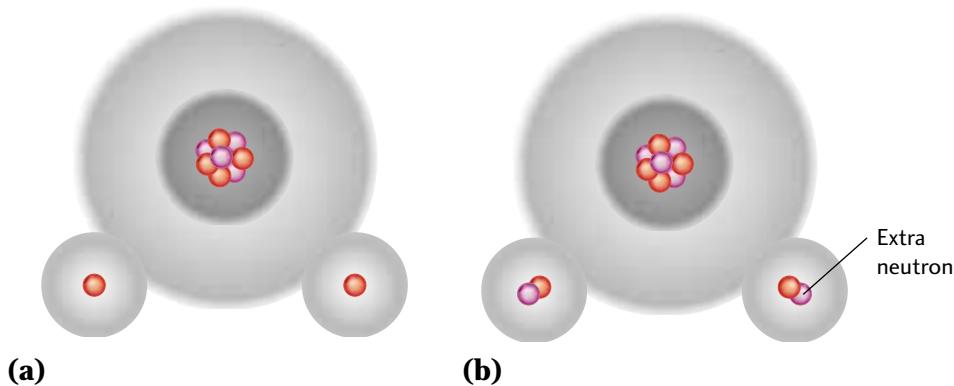


Fission Reactors

In a nuclear reactor, the fission chain reaction takes place under controlled conditions where the number of neutrons is kept constant instead of increasing geometrically. To maintain this chain reaction, the (fast) fission neutrons must be slowed down by using an effective **moderator** such as heavy water. Slow neutrons are less likely to be absorbed by ^{238}U and they trigger the fission of ^{235}U more readily than fast neutrons. A good moderator must absorb the *kinetic energy* of the neutrons, but not the neutrons themselves. Heavy water (D_2O) is a very efficient moderator because it contains nuclei that have about the same mass as neutrons. In collisions between particles of equal mass, more of the kinetic energy can be transferred to the stationary mass.

Fig. 14.22 The difference between ordinary and heavy water

- (a) Ordinary water, H_2O
- (b) Heavy water, D_2O



EXAMPLE 12 Comparing moderators

In a fission reactor, compare the effectiveness of two different moderators in slowing down fast neutrons to thermal speeds. Assume the fast neutron's speed is $2 \times 10^7 \text{ m/s}$ and that it collides head on with either a deuterium nucleus or a carbon nucleus at rest.

Solution and Connection to Theory

Given

$$v_{1_0} = 2 \times 10^7 \text{ m/s} \quad m_n = 1.0 \text{ u} \quad m_D = 2.0 \text{ u} \quad m_C = 12.0 \text{ u} \quad v_{1_f} = ?$$

Recall from Chapter 5 (see sidebar) that the final speed of the neutron is given by the equation

$$v_{1_f} = v_{1_0} \frac{(m_n - m_x)}{(m_n + m_x)}$$

For the deuterium nucleus,

$$v_{1_f} = (2 \times 10^7 \text{ m/s}) \frac{(1.0 \text{ u} - 2.0 \text{ u})}{(1.0 \text{ u} + 2.0 \text{ u})} = -6.7 \times 10^6 \text{ m/s}$$

For the carbon nucleus,

$$v_{1f} = (2 \times 10^7 \text{ m/s}) \frac{(1.0 \text{ u} - 12.0 \text{ u})}{(1.0 \text{ u} + 12.0 \text{ u})} = -1.7 \times 10^7 \text{ m/s}$$

Therefore, deuterium is much more effective in slowing down the neutrons. It reduces the speed of the neutron by $(1 - \frac{6.7}{20}) = 67\%$, whereas carbon reduces the speed by only $(1 - \frac{17}{20}) = 15\%$.

The CANDU Reactor

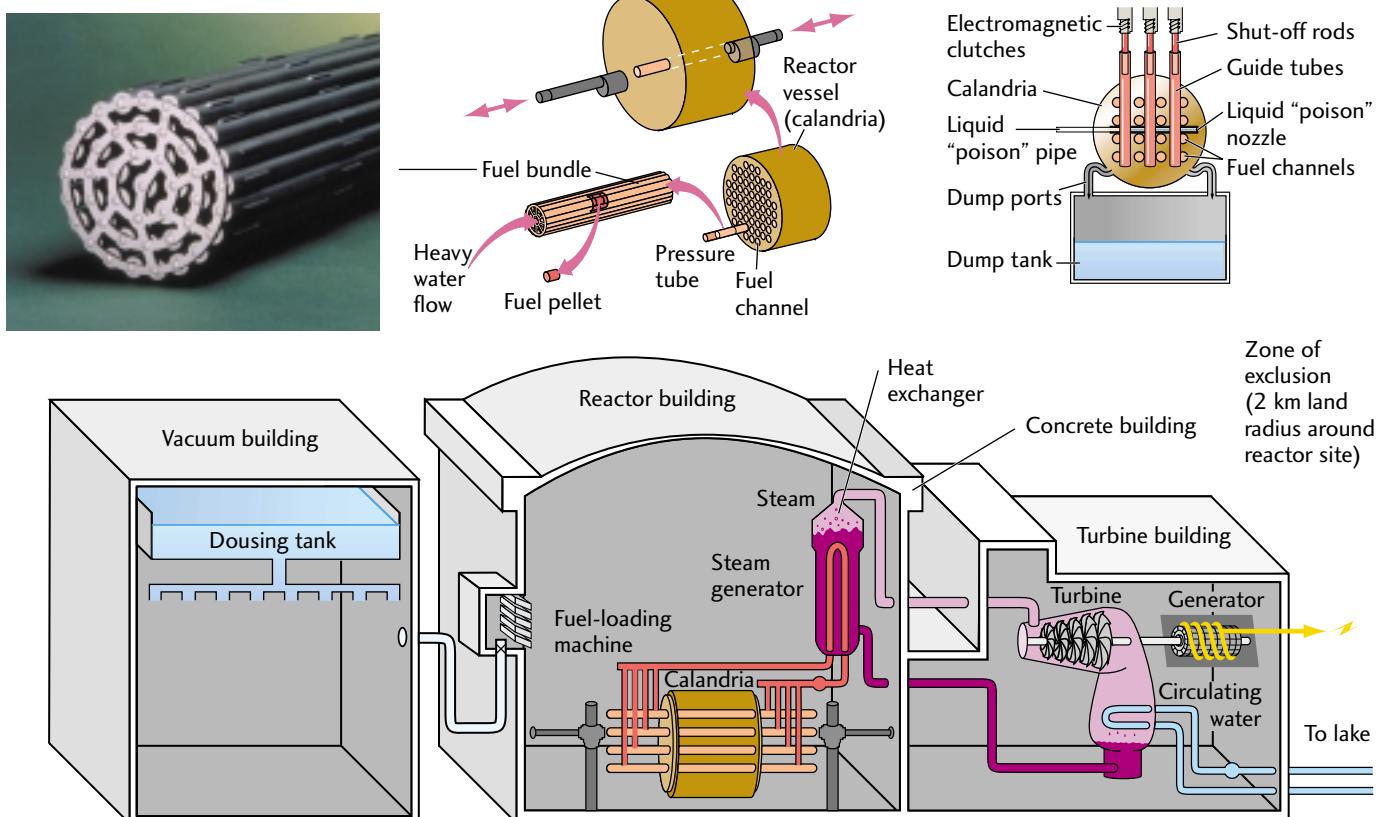
Deuterium or heavy water, ${}^2\text{H}_2\text{O}$, is the moderator used in the CANDU reactor, designed and built by Atomic Energy of Canada. The word CANDU is an acronym that stands for CANada Deuterium Uranium. It is unique in that it can be refuelled while on full-power operation.

In the CANDU reactor, the reactor core, or **calandria**, is composed of horizontal pressure tubes containing the fuel bundles. The heavy-water moderator flowing through the fuel bundles, shown in Figure 14.24, absorbs the energy released during the fission process. Heavy water

Fig.14.23 Aerial view of CANDU reactors at the Pickering nuclear plant



Fig.14.24 Parts of a CANDU nuclear reactor

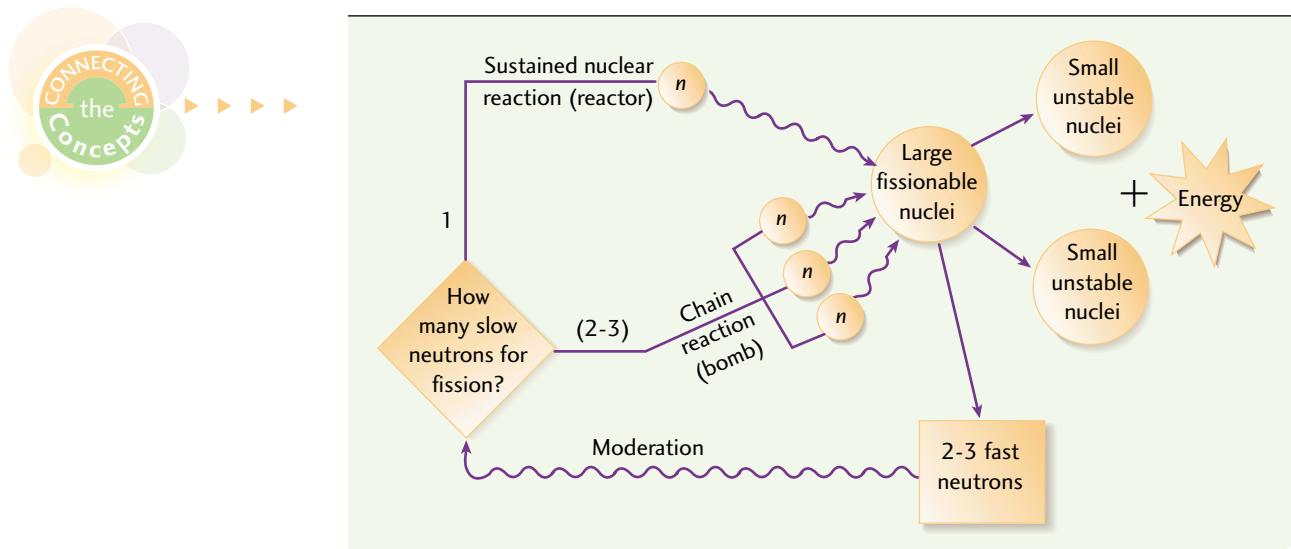


transfers heat via a heat exchanger, in which ordinary water is heated to steam. The steam drives a turbine connected to a generator. The steam is condensed by cooling it with nearby lake water, and is returned to the system in liquid form.

The heavy water in the calandria also acts as a moderator to slow down the (fast) fission neutrons. In the event of problems, a moderator dump system can be employed. Draining the moderator stops the reaction. There is also a neutron-absorbing boron solution that can be injected into the core through a “poison pipe” that, besides stopping the reaction, can also serve to cool the core. Other safety features include insertable cadmium rods to absorb slow core neutrons, and a low-pressure vacuum building to draw out radioactive steam and condense it. For safety, the entire structure housing the reactor core is encased in a second larger concrete containment enclosure with one-metre-thick walls.

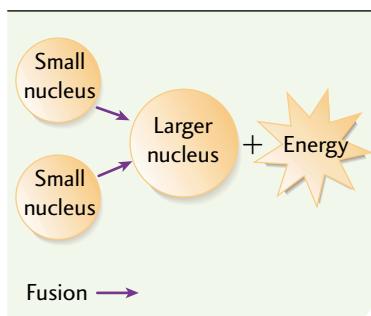
Figure 14.25 summarizes the neutron cycle in nuclear fission.

Fig.14.25 The Neutron Cycle in Nuclear Fission



Fusion

Fig.14.26



Fusion is a nuclear process of building larger nuclei by bringing together smaller nuclei. From Figure 14.16, we see that the binding energy per nucleon per mole increases up to a maximum at $A = 60$, where the nuclei are most tightly bound. From this graph, we can infer that we can gain energy by fusing smaller nuclei into larger, more stable nuclei. Fusion reactions are important in thermonuclear weapons, in powering the Sun, and in possible nuclear reactors.

EXAMPLE 13**Fusion reactor energy**

Find the energy released in creating one mole of helium in a reaction between deuterium (${}^2_1\text{H}$) and tritium (${}^3_1\text{H}$) according to the following reaction statement:

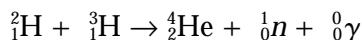


Fig.14.27 The fusion of deuterium (${}^2_1\text{H}$) and tritium (${}^3_1\text{H}$)

Proton Neutron

Solution and Connection to Theory**Given**

From Table 14.1,

$$\begin{aligned} m_{\text{D}} &= 2.014\ 102\ \text{u} & m_{\text{T}} &= 3.016\ 049\ \text{u} & m_{\text{n}} &= 1.008\ 665\ \text{u} \\ m_{\text{He}} &= 4.002\ 602\ \text{u} & 1\ \text{u} &= 1.661 \times 10^{-27}\ \text{kg} & \Delta m &=? \end{aligned}$$

To find the mass difference, Δm , we subtract the mass before the reaction from the mass after the reaction:

$$\Delta m = (m_{\text{D}} + m_{\text{T}}) - (m_{\text{He}} + m_{\text{n}})$$

$$\Delta m = (2.014\ 102\ \text{u} + 3.016\ 049\ \text{u}) - (4.002\ 602\ \text{u} + 1.008\ 665\ \text{u})$$

$$\Delta m = 0.018\ 884\ \text{u}$$

Then we convert the mass to kilograms and use the mass–energy equivalence equation to find the energy released for a helium nucleus:

$$\Delta E = (\Delta m)c^2$$

$$\Delta E = (0.018\ 884\ \text{u})(1.661 \times 10^{-27}\ \text{kg/u})(3.0 \times 10^8\ \text{m/s})^2$$

$$\Delta E = 2.82 \times 10^{-12}\ \text{J}$$

Last, we convert this energy to energy per mole using Avogadro's number:

$$\text{Energy/mol} = (2.82 \times 10^{-12}\ \text{J/nucleus})(6.02 \times 10^{23}\ \text{nuclei/mol})$$

$$\text{Energy/mol} = 1699\ \text{GJ/mol}$$

Therefore, the creation of one mole of helium releases 1700 GJ of energy.

In theory, fusion can provide 340 GJ per gram of reactants compared to about 86 GJ per gram for fission reactions.

The successful development of fusion reactors could benefit society greatly. To power a city of about one million people for 12 hours, the proponents of fusion power claim that it would take 3.5 million kg of coal, 2.5 million kg of oil, 250 kg of uranium (through fission), or 1 kg of fusion material!

In 2001, Clarington on Lake Ontario was proposed as the site for the International Thermonuclear Experimental Reactor (ITER), an international research facility to develop fusion energy (see Figure 14.28). At a cost of about \$12 billion, it may be the largest international research and development investment next to the International Space Station (see Chapter 6 STSE). It is to be the crucial last step before the world builds its first demonstration fusion power plant.

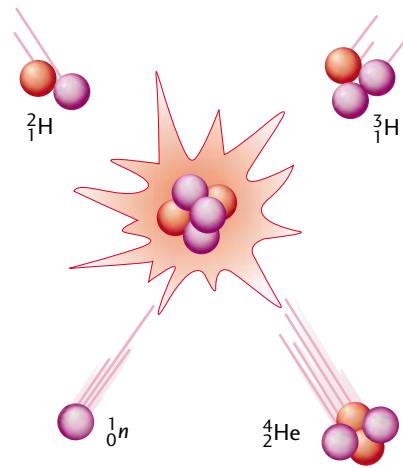
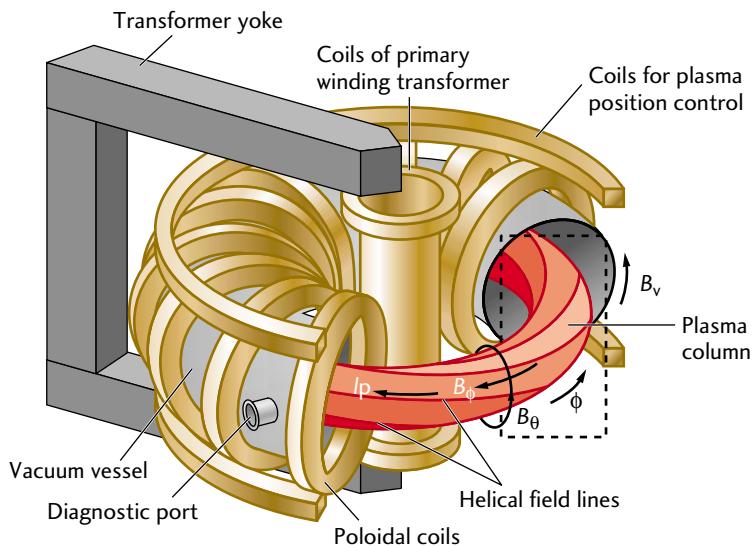


Fig.14.28 The proposed 180-hectare ITER fusion site on the north shore of Lake Ontario

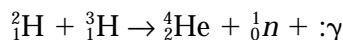


Fig.14.29 A fusion reactor that uses a magnetic field to confine hot plasma

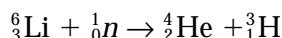


In the fusion process, a few grams of deuterium and radioactive tritium are fed into the machine's core. This fuel is heated to a temperature of at least 2×10^8 °C. The positively charged nuclei will fuse only if they collide violently, and the high temperature provides enough kinetic energy for them to overcome electrostatic repulsion. At this temperature, atoms are ionized into a **plasma** gas, a fluid composed of high-energy ions as we see in the Sun or in a flame. Superconducting magnets around a large toroidal or tire-shaped vessel confine the reacting plasma to keep it from touching the walls (see Figure 14.29). They also induce a current in the plasma that heats it to ignition. Upon ignition, the deuterium and tritium fuse to produce fusion energy. The resulting heat is removed using a water cooling system.

In the deuterium–tritium fusion reaction,



the alpha particle (^4_2He) deposits its energy within the fuel, thereby contributing to its heating. The neutron, unconfined because of its charge neutrality, carries away 80% of the energy and is captured by a surrounding blanket-like structure of cool liquid lithium through the reaction



The energetic helium and tritium heat up the lithium, which drives a steam generator that in turn runs a turbine for electrical power generation. The tritium is extracted from the lithium to produce (or **breed**) new tritium fuel for the reactor.

Fusion reactions have been produced for about the last 60 years. At first, there were small-scale studies in which only a few fusion reactions actually occurred. Presently, the world's largest fusion experiment is the Joint European Torus machine (JET) in England. It has shown that fusion power can be generated safely and effectively on a small scale. The larger ITER machine is needed to simulate the operation of a real power station.

The major safety feature of the fusion process is that it cannot get out of control. The process requires a precise, tightly controlled environment. If conditions are not ideal, the whole process simply stops. It cannot escalate out of control and cause a “core melt,” as in nuclear fission reactors.

Outside of experimental fusion reactors, fusion for the smaller nuclei ($A < 60$) doesn't occur under natural conditions on Earth because of the large amount of energy needed to bring two positively charged nuclei close together. The electrostatic force of repulsion between them becomes enormous as they come together. It's only when they are squeezed very close to one another that they feel the short-range strong nuclear force. At that point, the electrostatic repulsion is overcome and they fuse.

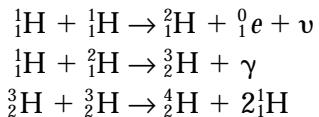
Fig.14.30 The Tokamak Fusion Test Reactor



Creating the Heavy Elements

Stellar fusion reactions have been going on for billions of years. The process of fusion in stars originally created many of the elements. In a way, stars are huge factories converting hydrogen into helium, then helium into heavier elements, and so on, while emitting a huge amount of energetic radiation. When we feel the Sun's warmth and see by its light, we are sensing a product of fusion. We can say that fusion is the basis for all life on Earth.

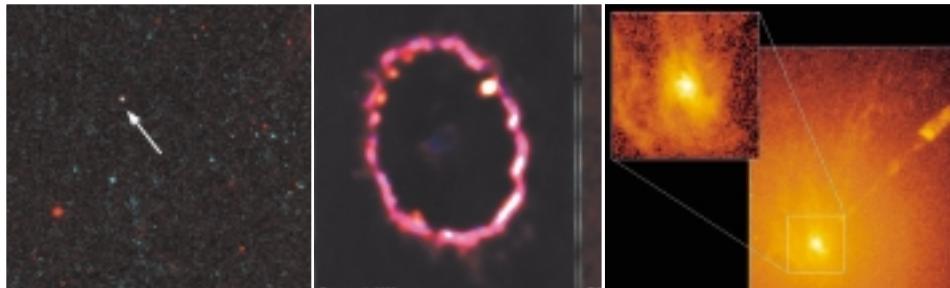
When a star is formed, it initially consists of hydrogen and helium. Under high temperatures and great gravitational pressure, hydrogen isotopes collide in a star and fuse. Thus begins a sequence of fusion reactions called the **proton–proton cycle**, which leads to the creation of helium and the output of energy:



From the third reaction, helium nuclei could be involved in forming heavier elements. In general, lighter elements fuse and form heavier elements. The mass of the elements that form inside a star depends on the star's temperature; the hotter the star, the heavier the elements it can create. These reactions continue within the star until the nuclei reach the mass numbers where the binding energy is at a maximum.

In Figure 14.16, we see that the maximum binding energy occurs around $A = 60$, or the element iron. At that stage, the fusion process slows down because the higher elements have less binding energy; therefore, the mass difference in the reaction would require inputting energy instead of releasing it (an endothermic reaction). Once a star has converted a large fraction of its core's mass to iron, it has almost reached the end of its life. Since the chain of fusion reactions begins to ebb, the energy output of the star begins to wither and it shrinks until it becomes a relatively cool iron sphere. As the temperature drops, the force of gravity can collapse the star (if it has enough mass). A tremendous, brilliant explosion can occur. The star will suddenly expand and produce, in a very short time, more energy than our Sun will produce in its lifetime! When a star explodes, we say that it has become a **supernova**. The supernova remnant may become a rapidly rotating **neutron star** or a **pulsar**. A **black hole** may form if the collapse of the burned-out star is so dense that it traps anything near it, including light (see Figure 14.31).

Fig. 14.31 Phases of stars in the universe: a neutron star (arrow), a supernova, and a black hole as seen through the Hubble Telescope



While a star is in the supernova phase, many important reactions occur. As the star collapses, the nuclei are accelerated to much higher velocities than the nuclei in fusing stars. With the added energy caused by their speed, nuclei can fuse and produce elements that are higher in mass than iron. The

extra energy in the explosion is necessary to overcome the energy barrier of a higher-mass element. Elements such as lead, gold, and silver found on Earth are the remnants of supernova explosions.

Comparing Energy Sources — A Debate

Long-range projections of energy use show that the total world energy demand by the year 2050 will be two or three times the 1990 level. As fossil fuels become depleted, proponents of fusion power argue that harnessing fusion power is one of the best options for a long-term, safe, and sustainable energy supply for future generations. On the other hand, proponents of fission power argue that fission is a relatively safe and inexpensive way to generate electricity. Both systems have risks and costs associated with them. Table 14.7 outlines some of the advantages of fission and fusion in a variety of areas.

Table 14.7
Fusion Development versus Fission Investment

	Fission	Fusion
Fuel availability	Uranium is indigenous to Canada, which makes us less dependent on importing expensive oil and natural gas. By using more nuclear energy, Canada becomes more self-reliant and free from world market price fluctuations. The diminishing supply of oil can then be reserved for transportation fuels and chemical feedstocks, not the generation of electricity.	Tritium is a waste product of the CANDU nuclear reactor. Deuterium is extracted from common water. One water molecule in 6000 has one ^2H atom. Initially, only one or two kilograms of fuel will be needed per year. The supply is inexhaustible.
Safety	History has shown that nuclear power isn't risk free. CANDU reactors have three safety systems: the moderator dump, the cadmium control rods, and the boron-filled moderator "poison" pipe. The safety of the CANDU has been proven. Compared to other daily activities, like driving a car, the risks of nuclear power to society are extremely small given the power that is generated for everyone.	The risk is in making it work. This new technology is based on the successful JET project that now needs the technology developed on a larger scale to be economically viable. The human safety risk is minimal; there are no runaway reaction possibilities, such as occurred at the fission plants at Three Mile Island and Chernobyl.
Environment	Fossil fuels (mainly coal) increase global warming through CO_2 emissions and contribute to the acid rain problem. Compared to using fossil fuels, CANDU reactors are much more environmentally friendly. The operation of a reactor has a negligible impact on background radiation levels. The highly radioactive waste produced doesn't take up much volume. Therefore, it can be isolated and contained.	Fusion creates no greenhouse gases. The very small amount of tritium that may be found in the air or in the cooling water of the power plant will be recycled as fuel. The amount of radioactive waste is similar to that of a hospital, and would be disposed of in the same way. Fusion has the least environmental impact of any current method of energy production.
Cost	High capital costs at the outset are offset by a plenitude of safe and inexpensive power for years to come. Maintenance costs and extensive safety regulations are other factors affecting profitability.	Initial research costs are high, but the inexhaustible power is immense. One kilogram of fusion fuel is comparable to 250 kg of fission fuel. The ITER machine will supply about 450 MW of power to the power grid. The development of Canada's expertise as a world leader in robotics and fuel processing is an added spin-off of fusion technology.



1. In a collision with (fast) fission neutrons, why would heavy-water molecules of tritium (T_2O) be less likely to absorb neutrons than ordinary water?
2. Calculate the effectiveness of a tritium nucleus in slowing down a (fast) fission neutron that is traveling at 2×10^7 m/s. (Hint: See Example 12.)
3. What are some good reasons for locating a fusion research facility in Clarington on Lake Ontario?
4. If 400 g of deuterium and 600 g of tritium are needed to fuel a city like Edmonton for 12 hours, calculate the power output of an ITER-type fusion plant. (Hint: Use the results of Example 13.)
5. In the solar proton–proton cycle, how many neutrinos are produced for every helium-4 nucleus?
6. As the leader of an interest group in the year 2025, you must choose and research arguments in favour of the type of new power generator you want to advocate for your small city. Viable fusion power technology has just become available. Hold a class debate on the method of power generation that you prefer. Begin by defining the demographical and geographical characteristics of your chosen city.
7. Research how lake water that is used to cool equipment in nuclear power plants and returned to the lake at a higher temperature affects the environment.

14.6 Probing the Nucleus

Fig. 14.32 The Pep II facility at SLAC, where energetic electrons and positrons circulate in opposite directions in two separate storage rings and collide with one another at a single crossing point. This experiment will produce mesons, or particles involved with the strong nuclear force.



In the early 1950s, scientists discovered that if the incoming particle in a nuclear reaction had sufficient speed, new types of particles could be produced from the interaction energy, and new interaction rules could be observed. Various particle accelerators have been constructed, such as synchrotrons and linear accelerators. Sometimes, they are used to probe the nucleus. Often electrons are used as probing particles because they are much smaller than nucleons, are easier to accelerate, and are not affected by the strong nuclear force. The electron is accelerated to a great enough velocity such that its *de Broglie wavelength* (recall Chapter 12),

$$\lambda = \frac{h}{mv}$$

is smaller than the size of a nucleus. The probe (electron) can then interact with objects inside the nucleus. If its matter (de Broglie) wavelength is too long, it will not be sensitive to the finer structures in the nucleus. For example, a 10-GeV electron has a de Broglie wavelength of about 0.1 fm, or small enough to interact with objects inside a nucleon. The faster the probing particle travels, the finer the details it can reveal about the nucleus.

EXAMPLE 14 The wavelength of a gigavolt electron

An electron with a rest mass of 0.511 MeV is accelerated to a kinetic energy of 22 GeV. Find its de Broglie wavelength.

Solution and Connection to Theory

Given

$$E_k = 22 \text{ GeV} = 2.2 \times 10^4 \text{ MeV} \quad m_0 = 0.511 \text{ MeV} \quad \lambda = ?$$

Using the mass–energy triangle from Chapter 13 (Figure 14.33),

$$(mvc)^2 = (m_0c^2 + E_k)^2 - (m_0c^2)^2$$

we substitute for E_k and m_0c^2 :

$$(mvc)^2 = (0.511 \text{ MeV} + 22\,000 \text{ MeV})^2 - (0.511 \text{ MeV})^2$$

$$(mvc)^2 = 4.84 \times 10^8 \text{ MeV}^2$$

$$mv = 22\,001 \text{ MeV}$$

$$mv = (22\,001 \text{ MeV})(1.6 \times 10^{-13} \text{ J/MeV}) = 3.52 \times 10^{-9} \text{ J}$$

Dividing both sides by c yields the term for momentum:

$$mv = \frac{3.52 \times 10^{-9} \text{ J}}{3.0 \times 10^8 \text{ m/s}}$$

$$mv = 1.17 \times 10^{-17} \text{ N} \cdot \text{s}$$

The de Broglie wavelength of the electron is

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{1.17 \times 10^{-17} \text{ N} \cdot \text{s}}$$

$$\lambda = 5.65 \times 10^{-17} \text{ m}$$

The result suggests that electrons of this energy could be used to probe target nucleons for objects that are 10 to 100 times smaller than a proton ($1.2 \times 10^{-15} \text{ m}$). Indeed, linear accelerators have been used to scatter electrons from nuclei in order to determine nuclear shapes and sizes. Presently, the 3-km-long accelerator at Stanford, California, is being used to collide a 9.0-GeV beam of electrons head-on with a 3.1-GeV beam of positrons.

The more massive protons take longer than electrons to accelerate to enormous speeds. As we learned in Section 13.5, cyclotrons are a good way to accelerate ions up to non-relativistic speeds (or energies of 10 MeV). As proton energies rise above 10 MeV to 400 MeV, the particle's mass increases (due to relativistic mass dilation). Correspondingly, the cyclotron frequency drops (from 32 MHz to 20 MHz). One solution to counteract the

Fig.14.33 The mass–energy triangle

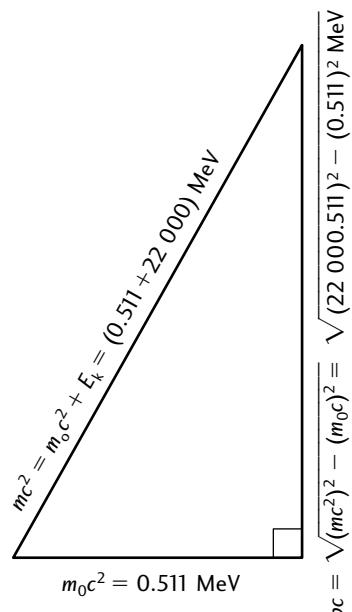


Fig.14.34 The BaBar particle detector at the Stanford Linear Accelerator Center. Here colliding beams are focused to a width comparable to a strand of human hair in order to improve the chance of a collision.



Matter versus Antimatter

At SLAC, scientists search through the collision debris of electrons with their antimatter opposite (positrons) for evidence of short-lived subatomic particles known as B mesons. From the collision studies, they are beginning to understand one of Nature's great secrets — why the universe has a preponderance of matter over antimatter.



frequency drop, used in the cyclotron of the TRIUMF project in Vancouver, is to increase its magnetic field as the protons approach speeds up to 75% of the speed of light. Another solution, used at the European Centre for Nuclear Research (CERN), is to increase the accelerator's size. The radius of the new Large Hadron Collider (LHC) is 4.3 km. In this 27-km-long superconducting ring, protons will be accelerated to energies of 7 TeV (1000 GeV = 1 TeV). Two beams will collide head-on in an attempt to discover the Higgs boson, a subatomic particle that explains the mechanism by which particles (such as electrons) are endowed with mass.

1. Find the de Broglie wavelength of a SLAC positron with a kinetic energy of 3.1 GeV. (Use the mass–energy triangle.)
2. Find the cyclotron frequency for the LHC (of radius 4.3 km) at CERN. (Assume $v \approx c$ and use $f = \frac{v}{2\pi r}$.)
3. a) What is the velocity of a proton with 10 MeV of kinetic energy?
b) What is the cyclotron radius of a 10-MeV proton orbiting with a frequency of 32 MHz? (Use the cyclotron frequency equation.)

14.7 Elementary Particles

We have learned in this chapter that there is more to matter than simply atoms arranged into molecules. Atoms consist of positive nuclei surrounded by negative electron clouds. Some nuclei appear to be stable, while other nuclei are unstable and decay to other forms of matter. We also learned that neutrons are a kind of glue that prevents proton–proton repulsion within the nucleus. In this section, we will learn about current theories of matter and the forces that hold it together. These theories explain or predict much of the behaviour of matter in high-energy physics experiments.

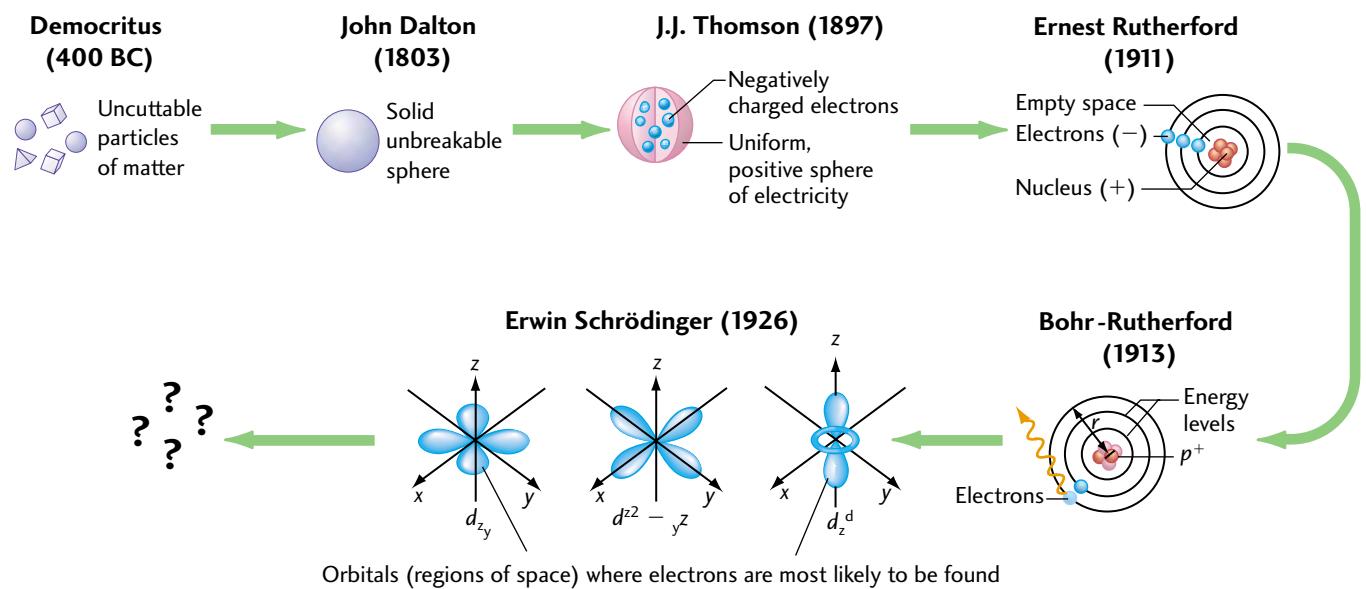
What is matter?

Since ancient times, humans have wanted to know what matter is and what it is made of. In the 5th century BC, the Greek philosopher Empedocles proposed that there were four primary elements: earth, air, fire, and water. In the 19th century, our list of elements expanded to eventually become the periodic table (see Appendix I), where each element is the smallest particle of its kind that still retains the properties of that substance. Dmitri Mendeleev (1834–1907), a Russian chemist, arranged the 63 known elements in ascending order of atomic mass.

What is matter composed of?

We now know that the smallest particle of each element is the atom. But what is the atom composed of? Figure 14.35 summarizes the evolution of the model of the atom, from ancient times to the present day.

Fig.14.35 The evolution of the model of the atom



What are the most fundamental particles of matter and how are they held together? Our studies now turn to the latest model of matter and forces, the Standard Model of fundamental particles and interactions.

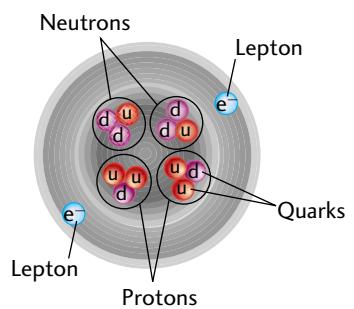
The Standard Model

The **Standard Model** of fundamental particles and interactions attempts to explain and predict particle interactions within the atom's nucleus and during nuclear decay processes. This model describes the atom as being made up of combinations of smaller, more fundamental particles called *leptons* and *quarks*. According to the Standard Model, the atom looks like Figure 14.36.

Leptons

The electron is a member of a family of fundamental particles called **leptons**. There are six leptons, all of which are so fundamental that they are believed to not possess any internal structure. Table 14.8 summarizes the characteristics of leptons.

Fig.14.36 According to the latest model, the atom is made up of smaller, more fundamental, particles called quarks and leptons

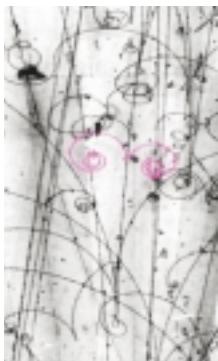


The **spin quantum number** describes the particles' spin direction. When many of these particles were discovered and identified, the divisions and categories of particles were based on their masses. However, as the number and complexity of the particles increased, spin became an important part of the classification system for subatomic particles.

Table 14.8
Leptons

Particle	Symbol	Antiparticle	Mass (MeV/c ²)	Charge	Spin
Electron	e^-	e^+	0.511	-1	$\frac{1}{2}$
Electron neutrino	ν_e	$\bar{\nu}_e$	≈ 0	0	$\frac{1}{2}$
Muon	μ^-	μ^+	105.7	-1	$\frac{1}{2}$
Muon neutrino	ν_μ	$\bar{\nu}_\mu$	≈ 0	0	$\frac{1}{2}$
Tauon	τ^-	τ^+	1784	-1	$\frac{1}{2}$
Tauon neutrino	ν_τ	$\bar{\nu}_\tau$	≈ 0	0	$\frac{1}{2}$

Fig.14.37a A bubble chamber photograph shows the tracks (coloured in magenta) of a particle and its antiparticle. Their tracks show that they spin off in opposite directions, indicating that they have opposite charges.

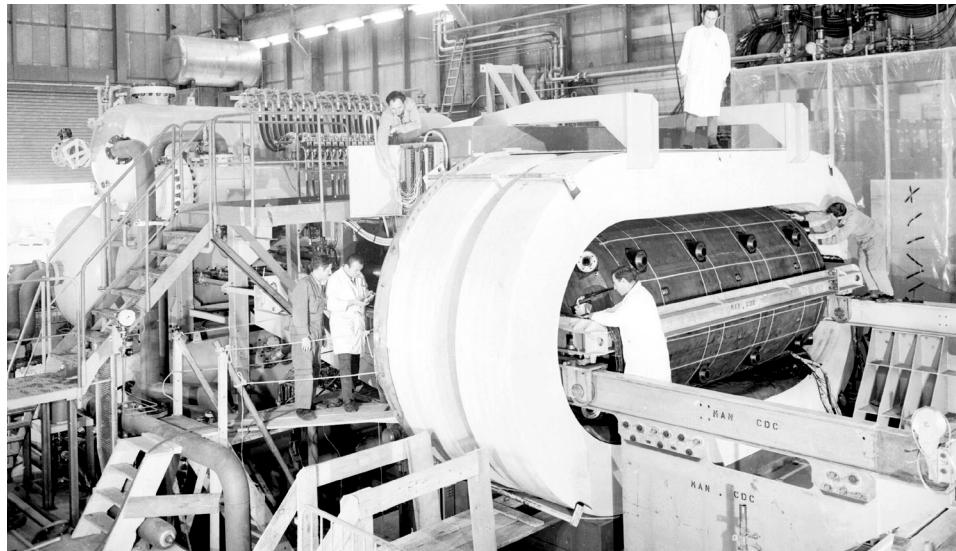


Leptons are particles that are totally unaffected by the strong nuclear force. Rather, they interact by way of the *weak nuclear force*. Leptons are grouped into three pairs, each containing one charged (positively or negatively) particle and one neutrino, which has no charge. The pairs are: the electron and the electron neutrino (e^- and ν_e), the muon and the muon neutrino (μ^- and ν_μ), and the tauon and the tauon neutrino (τ^- and ν_τ). Each particle has an associated mass and charge as well as a quantum-mechanical spin.

Neutrinos, one for each of the three main leptons, have an extremely small rest mass (possibly zero) and no charge. They travel at speeds very close to the speed of light. Even though billions of them pass through our body each second, they are extremely elusive and hard to detect. Experimental detection of neutrinos from nuclear reactor studies was first achieved in 1956. Presently, the Sudbury Neutrino Observatory detects about 8 to 10 neutrinos daily.

There was also evidence of particles similar to each lepton in every way but with the opposite charge. These particles, called *antiparticles*, are detected by the tracks they leave in a magnetic field (see Figure 14.37a).

Fig.14.37b Bubble chamber detectors were used extensively from 1953 until the 1970s. Charged particles injected into a superheated liquid leave tracks of photo-ready bubbles (see Figure 14.37a). The curved path of measurable radius is due to the action of an external magnetic field on the charges. From the radius, known strength of the magnetic field, and energy of the particles, we can calculate their charge and mass.



The positron (the antiparticle of the electron) was discovered in 1932. After that, scientists predicted that other particles also have antiparticles. Antiprotons were first observed in 1955 using the large Bevatron accelerator at the University of California in Berkeley. *All the subatomic particles have an associated antiparticle of the same mass but opposite charge.* Neutral particles, such as photons, neutral pions (π^0), and eta particles (η^0), are considered to be their own antiparticles (see Appendix J).

Besides the electron, the two other leptons, the *muon* and the *tauon*, are found in weak and electromagnetic interactions. They have the same charge as the electron but have larger masses.

Leptons make up only a small portion of the matter that effects our daily life. All other matter is made up of other subatomic particles called **quarks**.

Quarks

Particle physicists observed that leptons seemed to be elementary particles because they didn't break down into smaller particles. But other particles, called *hadrons*, did. In 1964, M. Gell-Mann and G. Zweig proposed that all remaining matter not classified as leptons is made up of a group of six smaller, more fundamental particles called **quarks**. Gell-Mann chose the name "quarks," (pronounced "kwords") after a nonsense word used by James Joyce in his novel *Finnegan's Wake*. Physicists have given the quarks fanciful and somewhat arbitrary names that help remember their interaction properties. Table 14.9 summarizes the characteristics of quarks.

Name	Mass (GeV/c^2)	Symbol	Charge
<i>Up</i>	0.004	<i>u</i>	$\frac{2}{3}$
<i>Down</i>	0.008	<i>d</i>	$-\frac{1}{3}$
<i>Strange</i>	0.15	<i>s</i>	$-\frac{1}{3}$
<i>Charm</i>	1.5	<i>c</i>	$\frac{2}{3}$
<i>Bottom</i>	4.7	<i>b</i>	$-\frac{1}{3}$
<i>Top</i>	176	<i>t</i>	$\frac{2}{3}$

Hadrons (Baryons and Mesons)

Hadrons are subatomic particles that are made up of quarks. There are two kinds of hadrons: *baryons* and *mesons*. All **baryons** are made up of three quarks. There are about 120 different types of baryons, all made up of different quark combinations, some of which are outlined in Table 14.10.

When a particle meets its antiparticle, they **annihilate** or destroy each other. The lost mass is converted to energy in the form of gamma radiation and/or a number of smaller particles according to the equation $E = mc^2$. For example, in pair annihilation, an electron and a positron combine to form a 1.022-MeV gamma ray. To find the energy of the gamma ray, substitute the mass of the electron into the equation $E = mc^2$ and convert your answer to electron volts.

For every quark, there is also an **antiquark** (such as *antiup*, *antidown*, etc.) that has the same mass but the opposite charge.

Strange was named after the "strangely" long lifetime of the *K* particle, the first composite particle found to contain this quark.

Charm was named on a whim and was discovered in 1974 almost simultaneously at both the Stanford Linear Accelerator Center (SLAC) and at Brookhaven National Laboratory.

Bottom was discovered at Fermi National Laboratory (Fermilab) in 1977, in a composite particle called *upsilon*.

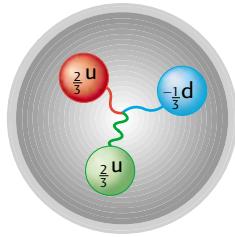
Top was discovered in 1995 at Fermilab. *Top*, the most massive quark, had been predicted for a long time before it was finally observed.

Top and *bottom* were initially called *truth* and *beauty*. The names *up*, *down*, *strange*, *charm*, *top*, and *bottom* have no real connection with our ordinary understanding of these words.

Table 14.10**Baryons and Antibaryons**

Symbol	Name	Quark content	Electric charge	Mass (GeV/c ²)	Spin
<i>p</i>	Proton	<i>uud</i>	1	0.938	$\frac{1}{2}$
\bar{p}	Antiproton	$\bar{u}\bar{u}\bar{d}$	-1	0.938	$\frac{1}{2}$
<i>n</i>	Neutron	<i>udd</i>	0	0.940	$\frac{1}{2}$
Λ	Lambda	<i>uds</i>	0	1.116	$\frac{1}{2}$
Ω^-	Omega	<i>sss</i>	-1	1.672	$\frac{3}{2}$

Fig. 14.38 Baryons are made up of three quarks. The three quarks of a proton are two *up* and one *down*.



Examples of baryons are protons and neutrons. A proton is made up of two *up* (*u*) quarks and one *down* (*d*) quark, or $p^+ = uud$. Since the charge of a proton is $1e$, we assign a fractional electrical charge of $+\frac{2}{3}e$ to *up* and $-\frac{1}{3}e$ to *down*. The proton's total charge is then

$$\left(\frac{2}{3}e\right) + \left(\frac{2}{3}e\right) + \left(-\frac{1}{3}e\right) = e$$

All **mesons** are made up of one quark and one antiquark. There are about 140 of them, some of which are outlined in Table 14.11. The quark combination for the positive pion (π^+) is one *up* quark and one *down* antiquark, or

$$\pi^+ = u\bar{d}$$

In 1935, Hideki Yukawa, a Japanese scientist, hypothesized the existence of the **meson** (meaning “middle”) because it had a mass that was midway between the mass of a proton and that of an electron. Twelve years later, the short-lived π or *pi meson* (or pion) was finally discovered in upper atmospheric cosmic-ray interactions.

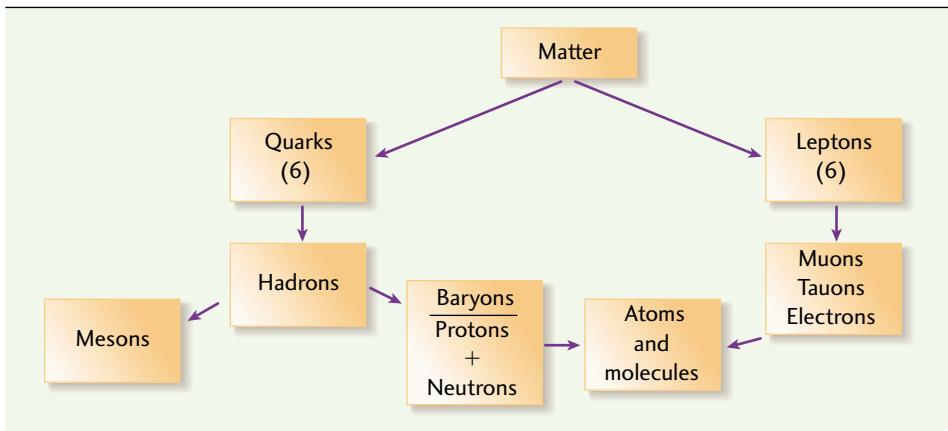
Table 14.11
Mesons

Symbol	Name	Quark content	Electric charge	Mass (GeV/c ²)	Spin
π^+	Pion	<i>u</i> \bar{d}	+1	0.140	0
K^-	Kaon	<i>s</i> \bar{u}	-1	0.494	0
ρ^+	Rho	<i>u</i> \bar{d}	+1	0.770	1
B^0	B-zero	<i>d</i> \bar{b}	0	5.279	0
η_c	Eta-c	<i>c</i> \bar{c}	0	2.980	0

Since the meson's discovery in 1947, many more subatomic particles have been detected in high-energy nuclear collision experiments carried out using particle accelerators.

Figure 14.39 summarizes the connections between elementary particles and atoms and molecules.

Fig.14.39 Elementary Particles



The binding energy of quarks in nucleons is so strong that physicists can only speculate on their masses by experimentation. Comparing information from Tables 14.9 and 14.10, we can see that the mass of the proton ($938.3 \text{ MeV}/c^2$) is much greater than the sum of the masses of the three quarks ($4 \text{ MeV}/c^2 + 4 \text{ MeV}/c^2 + 8 \text{ MeV}/c^2 = 16 \text{ MeV}/c^2$). This difference suggests that the proton has a mass “defect” of

$$938.3 \text{ MeV}/c^2 - 16 \text{ MeV}/c^2 = 922.3 \text{ MeV}/c^2.$$

EXAMPLE 15 The charge on a pion

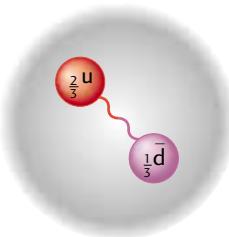
Calculate the total charge on a pion that has a quark combination of $u\bar{d}$, as shown in Figure 14.40.

Solution and Connection to Theory

From Figure 14.40, the total charge of the pion is $q_\pi = \left(\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) = e$.

Therefore, this particle is a positive pion, π^+ .

Fig.14.40 The quark combination of $u\bar{d}$ in a pion



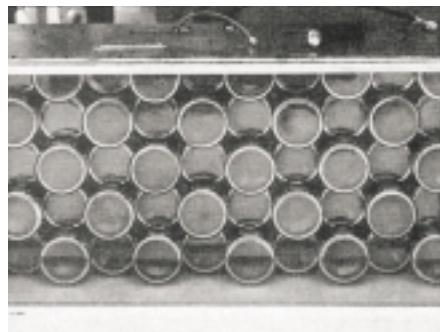
The Enigmatic Neutrino

In the early days of nuclear physics, scientists calculated that some energy seemed to be missing during beta decay. It was suggested that the missing energy was carried off by an unseen particle called the neutrino. Neutrinos were not detected until 1956, mainly because they only interact with particles through the weak force, are neutral, and have a very small mass, if any.

Two kilometres below the ground, in a nickel mine near Sudbury, Ontario, a neutrino observatory (SNO) has been constructed at a cost of \$73 million. About 100 scientists from Canada, the U.S., and the

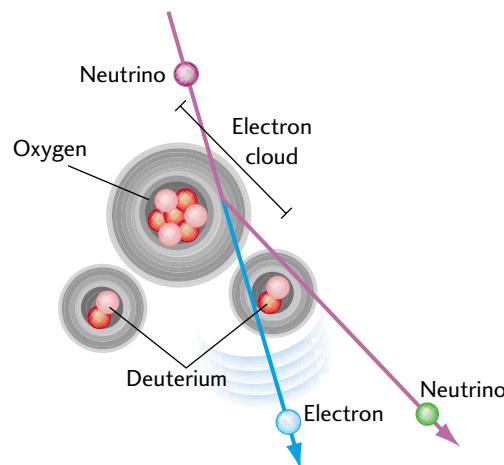


Fig.14.41 The photomultiplier bank used to discover the neutrino in 1956



U.K. measure the **flux** of electron neutrinos (ν_e) and the total flux of neutrinos, including the tau (ν_τ) and muon (ν_μ) neutrinos. From the nine or ten events recorded each day, researchers have found that the observed percentage of neutrinos flowing from the Sun is less than they predicted. Now they suspect that the electron neutrinos can change type during their travels, something that is not quite in agreement with the Standard Model. The scientists at Sudbury will be adding salt to \$300 million worth of heavy water on loan from Atomic Energy of Canada in order to make their experiment more sensitive. It is hoped that new research will lead to further extension or adaptation of the Standard Model to take into account these new-found characteristics of neutrinos.

Fig.14.42 A neutrino knocks an electron loose from an oxygen atom. A “sonic boom” of light is created by the energetic electron as it slows down to the local speed of light in the fluid.



1. It seems as though the boundaries of contemporary experimentation into the world of particle physics will only be pushed back if major expensive projects such as the Sudbury Neutrino Observatory (SNO), the Vancouver TRIUMF sector-focused cyclotron, or the proposed Clarington ITER project are undertaken. Write a short reflection paper to discuss how the magnitude and financial cost of primary scientific research has been of benefit or detriment to the human condition.
2. Calculate the overall charge of the following hadrons:
 - a) uud
 - b) $\bar{u}\bar{u}\bar{d}$
 - c) $u\bar{d}$
 - d) udd
 - e) $s\bar{u}$
3. For each hadron in problem 2 above, give the name of the hadron and state whether it is a baryon or a meson.
4. Show that the three-quark combination for the neutron gives an overall charge of 0.
5. If the rest mass of a B^0 meson is $527.9 \text{ MeV}/c^2$, calculate its mass “defect.”

14.8 Fundamental Forces and Interactions

— What holds these particles together?

The Standard Model postulates that matter is made up of complex combinations of six quarks and six leptons. But how are these particles held together?

Forces or interactions?

The four fundamental forces of nature are the gravitational force, the electromagnetic force, the strong nuclear force, and the weak nuclear force. All other forces, such as friction and magnetism, are caused by one of these four fundamental forces. In Chapters 8 and 9, we explained these forces at a macroscopic level in terms of *field theory*. Masses attract one another by way of the gravitational force, whereas electrons and protons, along with the atoms and molecules they comprise, interact by way of the electromagnetic force.

Recall from Chapter 9 that in terms of field theory, a mass or charge creates the field that causes a force to be exerted on any other mass or charge placed in the field (action at a distance). In terms of quantum mechanics and the Standard Model, forces are considered to be the process of interactions between fundamental particles exchanging force-carrying particles. *Masses and charges are continuously emitting and absorbing force-carrying particles that carry momentum and energy between the masses and charges.* These *force-carrying particles* are called **bosons**. Each of the four fundamental forces has its own specific boson that mediates the force. Table 14.12 lists the four fundamental forces and their respective bosons, along with their characteristics.

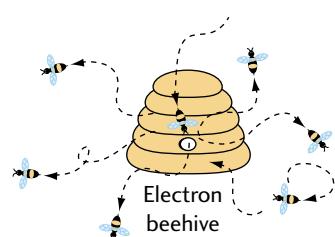
Force-carrier interactions follow the laws of conservation of energy and momentum. They cannot violate the law of conservation of energy (ΔE) for a time (Δt) longer than is permitted by Heisenberg's uncertainty principle,

$$\Delta E \Delta t \geq \frac{\hbar}{2\pi}$$

Bosons are named after Satyendra Nath Bose, an Indian physicist who, along with Albert Einstein, derived a statistical theory for them in the 1920s.

As yet, there is no evidence to support the existence of the graviton, but there has been a great deal of success through high-energy particle physics experimentation in describing the characteristics of the other three force bosons.

Fig.14.43 Virtual “photon” bees buzzing around an “electron” beehive



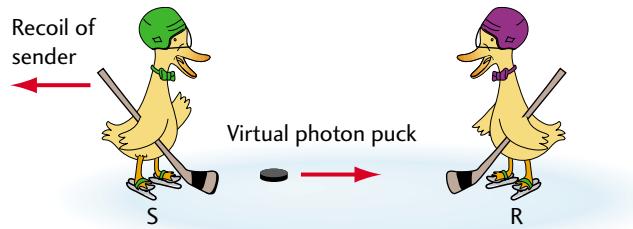
In general, each subatomic particle is surrounded by a cloud of swarming bosons that determine its interactions with other subatomic particles. For example, electrons are surrounded by virtual photons that mediate the electromagnetic force (see Figure 14.43).

Boson Exchange

Bosons are **virtual particles** if they are emitted and absorbed within such a short time interval that they are undetectable. Virtual particles may not appear in real space.

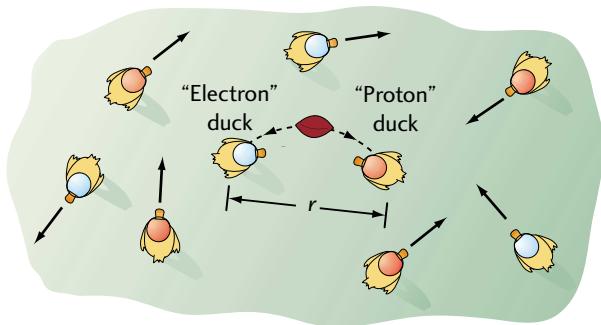
How can we explain effects of the fundamental forces in terms of the exchange of bosons? For electrostatic repulsion between two like-charged particles such as electrons, we know from Table 14.12 that photons are the bosons or force-carrying particles that mediate the electromagnetic force. When two electrons interact, photons carry momentum and energy between the two electrons. When a photon from one electron travels to another electron, it is absorbed by the second electron. Imagine two hockey players on an icy pond passing a hockey puck back and forth, as illustrated in Figure 14.44. Every time the puck is sent, conservation of momentum makes the sender recoil. Similarly, every time the puck is received, conservation of momentum pushes the receiver farther from the sender. Since a photon is absorbed by the second charge very shortly after it is emitted by the first charge, it is not detectable. For this reason, it is called a *virtual photon* (compared to a photon that is free to be detected, like a photon of light).

Fig.14.44 Two “electron” ducks recoil while exchanging a virtual “photon” puck



It is more difficult to envision the attraction of opposite charges by exchanging photons. Imagine watching a football game from high above the stadium. Any players not involved in throwing the ball back and forth appear to move about the stadium in a random fashion and may even leave the field of view. The two players throwing the football back and forth appear to remain within a fixed range of interaction relative to each other and therefore appear to be attracted to each other (see Figure 14.45).

Fig.14.45 A “electron” and “proton” duck remain within a fixed range while exchanging a virtual “photon” football



If the players decided to throw a more massive object like a bowling ball, they would have to decrease their radius of interaction (move closer to each other) in order to throw and catch the ball, meaning that there is a stronger force of attraction between them.

The ideas of quantum mechanics combined with electromagnetism create the more encompassing theory of **quantum electrodynamics**.

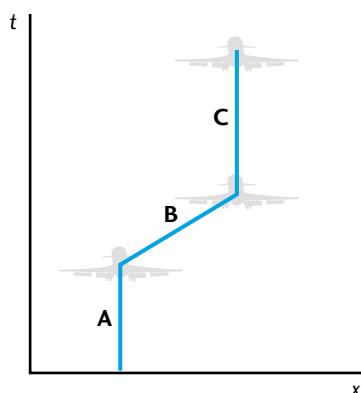
Feynman Diagrams

Richard Feynman, an American physicist, developed a visual way to show the exchange of virtual particles in spacetime graphs. In a **Feynman diagram**, time is the vertical axis and space is the horizontal axis. Table 14.13 summarizes the common elements of Feynman diagrams.

Table 14.13 Elements of Feynman Diagrams	
Particle	Symbol
Real particle	—
Virtual particle	- - - - -
Photon	~~~~~
Gluon	0000000

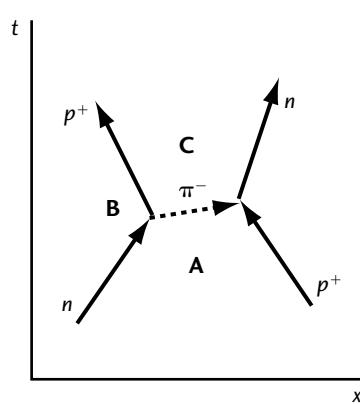
Figures 14.46a and b illustrate how to read Feynman diagrams. Notice in Figure 14.46b that the meson is not a horizontal line, which means that the meson exchange is not instantaneous but takes place over space and time.

Fig.14.46a A Feynman diagram is read horizontally and vertically



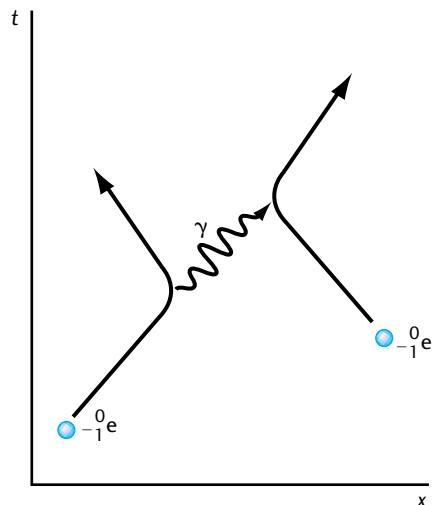
- A The plane sits at Toronto airport.
- B The plane flies to Ottawa.
- C The plane sits at Ottawa airport.

Fig.14.46b A Feynman diagram of the strong-force interaction via the exchange of a virtual meson



- A A neutron and proton approach.
- B At a close distance, they exchange a virtual meson.
- C A new proton and neutron recoil.

Fig.14.47 A Feynman diagram of the interaction of two electrons via the exchange of a virtual photon

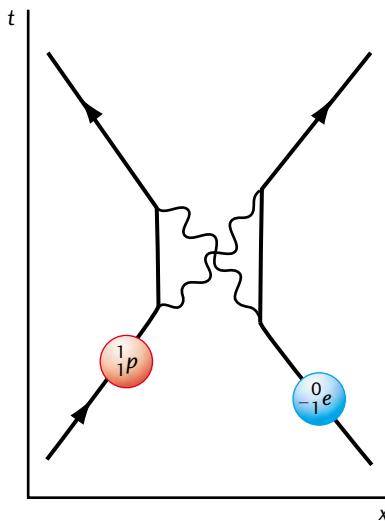


EXAMPLE 16

Reading Feynman diagrams

Describe the interactions that are taking place in Figure 14.48.

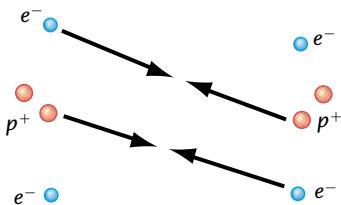
Fig.14.48



Solution and Connection to Theory

Figure 14.48 is a Feynman diagram showing an electron–proton electromagnetic interaction (solid lines) via the exchange of virtual photons (wavy lines).

Fig.14.49 The residual electromagnetic force: the atoms are electrically neutral, but the electrons in one atom are attracted to the protons in the other atom, and vice versa



The electromagnetic force explains how the attraction between electrons and protons in an atom neutralizes charge and how the **residual force**, illustrated in Figure 14.49, forms a mutual attraction between opposite charges in nearby atoms in a bound molecule. But how does the short-range strong nuclear force and gluon exchange work to overcome the seemingly overwhelming tendency of the positive protons (baryons) to repel? (Recall that gluons are the bosons that mediate the strong nuclear force.) The answer lies in another theory, quantum chromodynamics (QCD) and colour charge.

Quantum Chromodynamics (QCD): Colour Charge and the Strong Nuclear Force

The Pauli Exclusion Principle

No two particles can be in the same state or configuration at the same time.

We know that baryons are composed of three quarks. It is possible to have baryons with the quark combinations of *uuu*, *ddd*, and *sss* (the Ω^- baryon in Table 14.10). But three quarks in the same configuration violates the **Pauli exclusion principle**. Also, the charges of oppositely charged particles, like the proton and the electron, add up to zero. The photon, which mediates the interaction between them and itself has no charge, doesn't

affect their respective charges during the interaction. However, when two quarks, such as *up* ($\frac{2}{3}$) and *down* ($-\frac{1}{3}$), interact, the gluon, which mediates their interaction, would have to have a charge of $-\frac{1}{3}$ in order for the overall charge of this interaction to be conserved. But the gluon is a neutral particle. The theory of electrostatic charges is therefore inadequate for explaining how gluons mediate the interaction of quarks. We need a theory that has three different “charges” instead of two so that charge can be conserved during the interaction.

In 1965, Moo-Young Han and Yoichiro Nambu at Duke University suggested that quarks could be described as possessing **colour charge**. All quarks come in the three primary colours: red (R), green (G), and blue (B). Three secondary colours, cyan (antired, \bar{R}), yellow (antiblue, \bar{B}), and magenta (antigreen, \bar{G}), are assigned to all antiquarks. *All baryons have one quark of each primary colour; a combination that makes them colour-neutral* in a manner analogous to **additive colour theory**. For example, Ω^- can be written as $s_R s_B s_G$. Quark colour assignment is summarized in Table 14.14. Note that these colour assignments are conceptual only, for the purposes of explaining particle interactions: subatomic particles don’t have colours as we understand them.

Table 14.14 Quark and Antiquark Colours	
Quark colours	Antiquark anticolours
Red (R)	Cyan (\bar{R})
Green (G)	Magenta (\bar{G})
Blue (B)	Yellow (\bar{B})

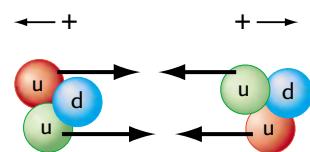
These colour assignments, along with a few simple rules, allow us to use the colour-charge theory to predict possible quark combinations of hadrons and what goes on during gluon exchange (see Table 14.15).

Recall that molecules are clusters of atoms that are bound together by the residual electromagnetic forces between adjacent neutral atoms (see Figure 14.49). Similarly, baryons (neutrons and protons) in a nucleus are bound together by the residual strong nuclear force between quarks in adjacent colour-neutral baryons (see Figure 14.50), despite the repulsive electromagnetic force.

The Weak Nuclear Force — Decay and Annihilations

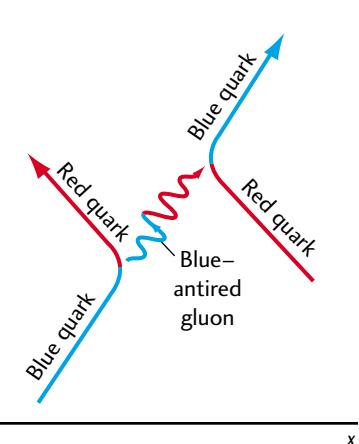
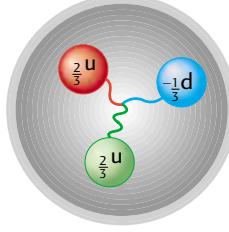
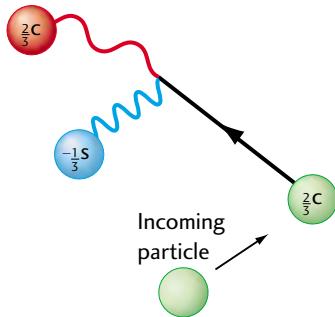
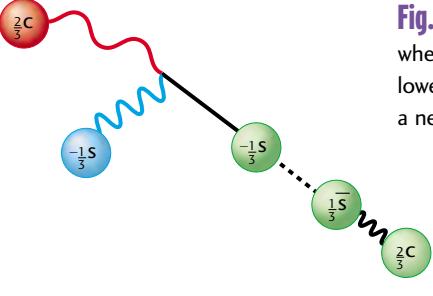
In Section 14.2, we learned that heavy nuclei decay into lighter nuclei by emitting alpha particles, beta particles, and gamma rays. Similarly, massive leptons and quarks decay to lighter leptons and quarks. This decay is caused by weak nuclear interactions. When a subatomic particle decays,

Fig.14.50 The positively charged protons in a nucleus are held together by the residual strong nuclear force between quarks in adjacent colour-neutral baryons



At short (nuclear) distances, nucleon attraction can overcome the Coulomb (electrostatic) repulsion of protons in the nucleus. At long atomic distances, the strong force is negligibly small — some particles (electrons) don’t feel the nuclear force at all.

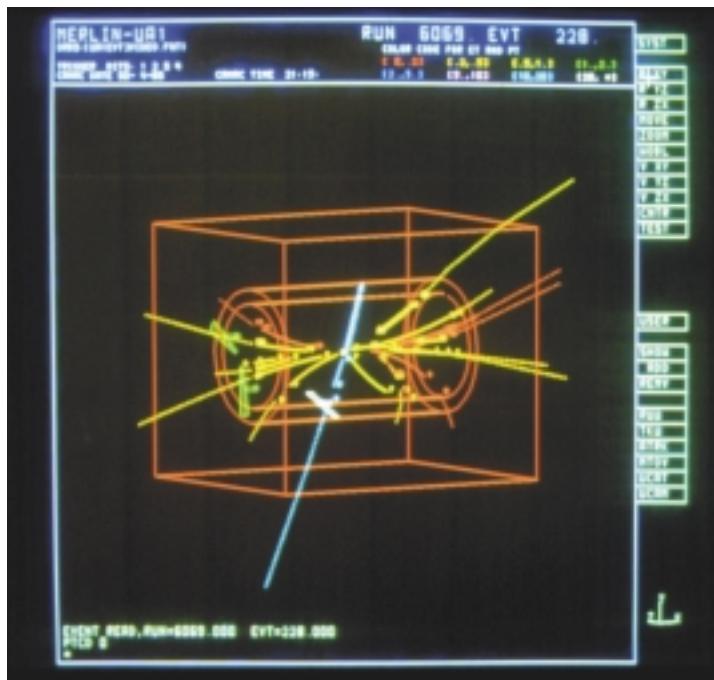
Table 14.15
The Implications of Colour Theory

Theory	Implication
<p>Quarks in a hadron exchange gluons (the strong-force bosons).</p> 	<p>Fig.14.51a The gluon leaving the quark at left has the colour forces of blue and antired. During the exchange of the virtual gluon, the emitting quark loses its blue-antired property and becomes a red quark, while the red quark gains a blue-antired property and becomes blue.</p> <p>A colour force field keeps the quarks together in the hadron. <i>Gluons carry a colour and an anticolour.</i> During a quark-quark interaction, a gluon takes colour away from the emitting quark and carries it to the absorbing quark. Thus, the quark colours (but not the quark type) are exchanged. Gluon colours are the difference between the interacting quarks' colours.</p>
<p>Colour charge is always conserved.</p> 	<p>14.51b In a proton, the continual exchange of colours by gluons holds the quarks together</p> <p>Hadrons consist of either three quarks (baryons) or two quarks (mesons) that are colour neutral. In hadrons, gluons are continuously exchanging the <i>colours</i> of the quarks. Therefore, the quarks exchanging gluons always remain bound in a colour-neutral state so that the overall colour charge of the hadron is conserved. A proton (<i>uud</i>) that is a quark composite of <i>ud</i> or <i>uddd</i> cannot exist because it isn't colour neutral.</p>
<p>Colour-charged particles cannot be found individually.</p> 	<p>Fig.14.51c The string of the lower charm quark stretches to restrain it, while the two upper quarks are unrestrained</p> <p>The colour force can be described as elastic bands holding the quarks together. When the quarks are close together, they are free to move, but when stretched during an interaction, the elastic bands quickly pull them back. In a violent collision, the band may break. Energy transferred by breaking a quark-quark bond can be converted into the mass of the new quark-antiquark pair ($E = mc^2$). For this reason, it's impossible to have an isolated quark.</p>
<p>New quark pairs can be created.</p> 	<p>Fig.14.51d An $s-\bar{s}$ quark pair form where a violent interaction breaks the lower charm string. A new baryon and a new meson are created.</p>

it is replaced by two or more new particles. During the decay process, the total mass and energy of the system is conserved, but some of the original particle's mass is converted into kinetic energy. As a result, the product particles always have less mass than the original particle that has decayed. *The stable matter that we observe is made up of the smallest leptons and quarks, which cannot decay any further;* that is, electrons, and *up* and *down* quarks (for protons and neutrons). (To compare the masses of these particles with those of more massive leptons and quarks, refer to Tables 14.8 and 14.9.)

When a massive quark or lepton decays to a less massive quark or lepton (e.g., when a muon decays to an electron), it is said to change **flavour**. Changes in flavour are due to the *weak force*. From Table 14.12, the force-carrying particles (bosons) for the weak interaction are the W^+ , W^- , and Z^0 particles. W particles are electrically charged (+ and -) and Z^0 particles are neutral.

W^- and Z^0 particles are not detected directly, but rather through their decays:
 $W^- \rightarrow e^- + \nu$ (see Figure 14.52)
 $Z^0 \rightarrow e^+ + e^-$



In 1979, the Nobel Prize was awarded to S. Glashow, A. Salam, and S. Weinberg, who were able to show that the weak and electromagnetic forces were two aspects of the same force. In the Standard Model, the electromagnetic and weak interactions are combined into one unified interaction called the **electroweak force**. Many examples of decays involve the weak or the electroweak interactions. Figure 14.54 shows the decay of a cosmic-ray pion. Another decay that has been observed is that of the kaon meson (see Figure 14.55).

Fig.14.52 An electronic display from a detector at CERN showing the decay of a W boson into an electron (blue track going toward the bottom) and a neutrino (reconstructed blue track upward), with the other particle tracks in red and yellow

Fig.14.53 The beta decay of a neutron via the weak-force W boson

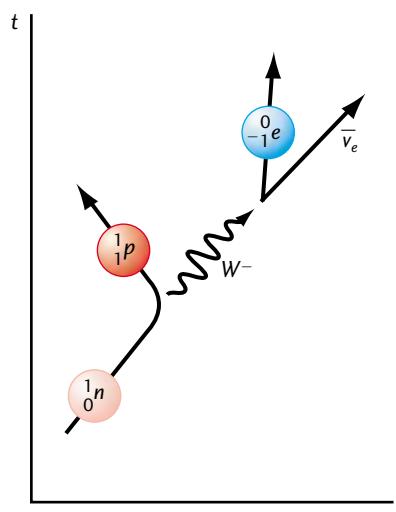
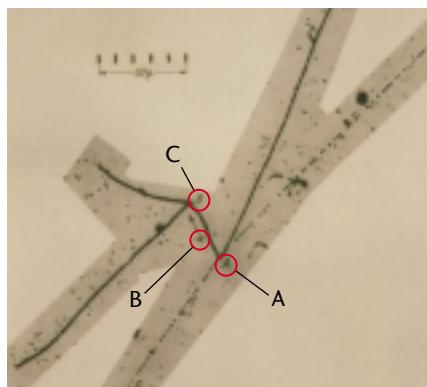


Fig.14.54 The decay of a cosmic-ray pion (entering at top) into a muon (travelling down) and a neutrino (invisible). At the bottom of the picture, the muon decays into an electron (going right) and another neutrino. This pion–muon–electron decay chain was photographed in 1948 using an emulsion photograph.



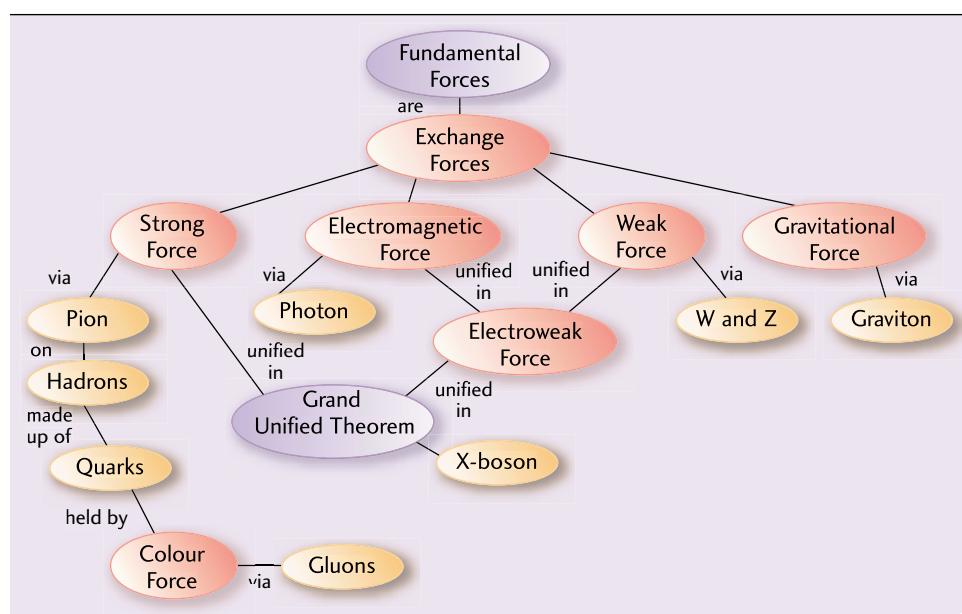
Fig.14.55 The first observed decay of a kaon (meson), coming in from top right, into three pions (mesons) at A. One pion moves slowly, leaving a dense track until it interacts at B, 25 μm farther up and left. The other two pions are faster and leave faint tracks (to top right and bottom left). The kaon contains *strange* quarks.



There is still much work being done to further our understanding of matter and the fundamental particles that make it up. After the unification of the weak and electromagnetic interactions, scientists have new hope of completing the **grand unified theory** linking the electroweak and quantum chromodynamic (QCD) theories. To see experimental evidence of this unified theory, scientists believe that energies of 10^{15} GeV will be required. This amount of energy is far beyond any accelerator presently contemplated. Two essential elements of the Standard Model that have yet to be discovered are the Higgs boson, which interacts with particles to give them mass, and the graviton.

Other questions still unanswered include: Why are there three types of quarks and leptons of each charge? Do they have a substructure or are they truly fundamental? Do neutrinos have a mass that can account for the missing **dark matter** of the universe? These questions and a host of others await answers from the curious physicists of the future.

Fig.14.56 A Summary of the Fundamental Forces and the Elementary Particles



1. For each of the following Feynman diagrams, describe the interactions that are taking place.

Fig.14.57a

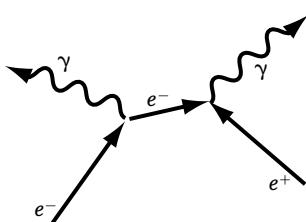


Fig.14.57b

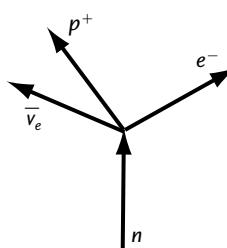
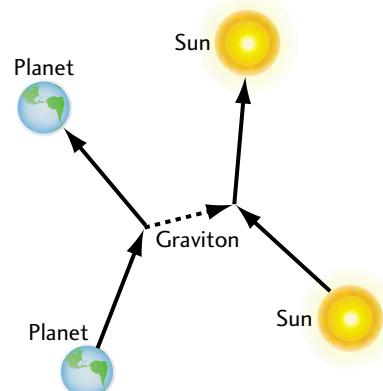


Fig.14.57c



Canadian Contributions to Physics

Canadians have made a significant contribution to many aspects of modern physics. The following are some of the major players in this area.



Bertram Brockhouse conducted experiments at Chalk River, Ontario in the field of solid-state physics. Using the neutron spectrometer, he was able to look inside the crystalline structure of solids to find out how solids like rocks and gems are held together. For his work, he was awarded a Nobel Prize in physics in 1994.

Richard Taylor studied mathematics and physics at the University of Alberta in Edmonton. While at Stanford University in California, he joined the High Energy Physics Laboratory. Together with Jerome Friedman and Henry Kendall of MIT, he used the new linear accelerator (SLAC) to smash protons and neutrons to pieces and discovered that they are made up of quarks. Taylor, Friedman, and Kendall were awarded the Nobel Prize in physics in 1990.

Werner Israel used mathematical techniques to show that black holes are the simplest big objects in the universe. "The surface of a black hole is as smooth as a soap bubble." Israel is currently working on several projects involving the internal geometric structure of black holes using superstring theory.

Harriet Brooks was the first female Canadian nuclear physicist. She graduated from McGill University, Montreal, in 1898, with a BA in Mathematics and Natural Philosophy. In 1899, she began research with Ernest Rutherford. In 1901, she became the first woman to study at the Cavendish Laboratory at Cambridge University, England. She also worked at Marie Curie's lab in France. She was the first person to realize that one element can change into another, and was one of the discoverers of radon.

William Unruh applied quantum mechanics to study gravity and the forces that existed at the moment of creation according to the Big Bang theory. His other areas of study include black hole evaporation and quantum computation: using quantum laws to design computers that can solve certain problems billions of times faster than traditional equipment.

Ian Affleck's early work was in elementary particle theory. Affleck's theories bridge the behaviour of elementary particles, such as the proton, neutron, and quark, with theories concerned with condensed matter such as semiconductors.

2. Choose a Canadian physicist who you think made the most interesting contribution to modern physics. Write a short report on his or her area of interest and the latest developments in that field.



Positron Emission Tomography (PET)

Positron emission tomography (PET) is a form of medical diagnostic imaging based on the detection of the concentrations of positron-emitting radioisotopes within the tissue of living subjects. In this form of tomography, the subject ingests a radioactive nuclide that decays by positron emission. A particular radioactive substance is selected to “label” a specific element or compound that takes part in the body’s own natural processes, such as oxygen in cellular respiration. The concentration of the labeled substance is detected by a series of gamma detectors, as illustrated in Figure STSE.14.1.

Fig. STSE.14.1 The doughnut-shaped scanner is composed of many gamma detectors to register any annihilation events from the subject. Figure STSE.14.1a shows a PET scanner manufactured in Knoxville Tennessee by CTI Inc.



Particle Love

A positron, while zipping through air,
Met an electron that oh was so fair.
It was love at first sight.
They embraced with delight
And presto! a γ -ray was there.

The gamma detectors don’t detect the positron-emitting radioisotope directly, but rather the gamma rays that are released 180° to each other in the annihilation reaction of an emitted positron (e^+) with an existing electron (e^-), as shown in Figure STSE.14.2, according to the reaction

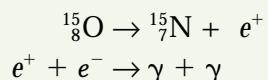
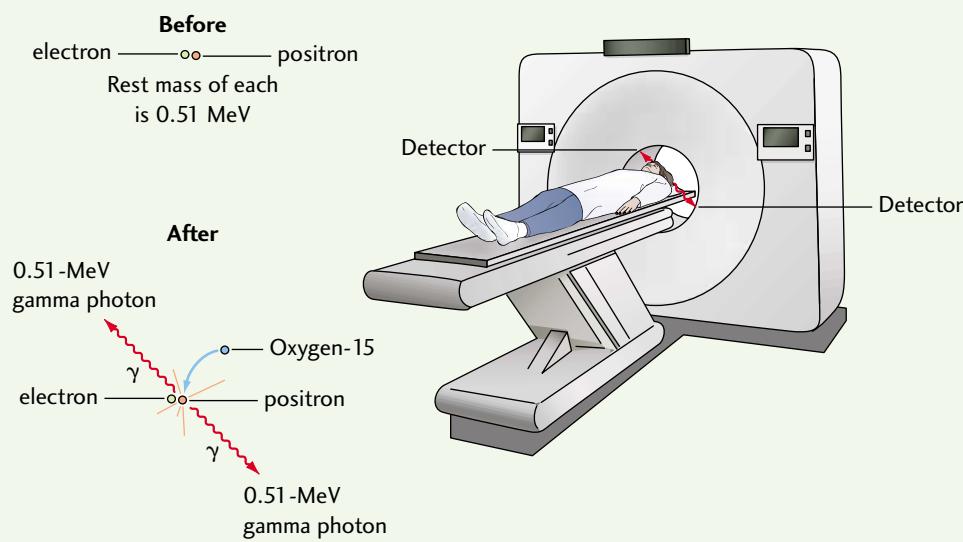


Fig. STSE.14.2 Simultaneous detection of two of these photons by detectors on opposite sides of an object places the site of the annihilation on or about a line connecting the centres of the two detectors. The computer maps the distribution of positron annihilations for later display.

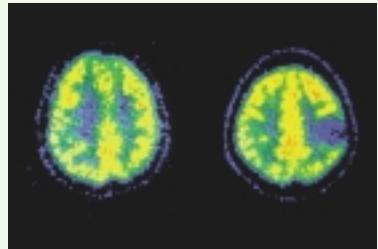


The tracers can label oxygen for respiration or simple sugars like glucose when they decay. High glucose metabolism detected by a PET scan can be due to the presence of a Hodgkin's lymphoma (malignant cancer tumour), as shown in Figure STSE.14.3.

The risks of PET scans are similar to those of other forms of computed tomography, such as CAT scans or MRI and nuclear medicine. Patients ingest a short-lived radioactive isotope, but the dose is so small, it is unlikely to cause cancer. The small risk is generally accepted by patients when they consider the benefits of the information that this procedure can provide.

All short-lived positron-emitting radioisotopes, or tracers, used in PET, such as ^{15}O (traces oxygen in cell respiration) must be generated by a cyclotron or other particle accelerator. Because the radioisotopes are short lived, this facility must be nearby. Such facilities are extremely expensive and, as a result, may only be funded by joint medical research and university facilities, such as McMaster University Medical Centre in Hamilton, Ontario. It has a cyclotron than generates radioisotopes for medical purposes.

Fig.STSE.14.3 Places of high glucose metabolism can be identified as areas of tumour growth, such as this brain tumour (image on right)



Design a Study of Societal Impact

Most people don't realize how the invisible particles released during radioactive decay affect their quality of life. Research an application of nuclear physics that affects the average Canadian, such as:

- What are natural sources of radiation, like radon in the basement of homes?
- How does upper-atmospheric cosmic radiation affect airline pilots?
- How does a smoke detector work? What is the proper way to discard it?
- How are isotopes produced? How are these isotopes used to treat illness in nuclear medicine?
- How useful is irradiation in the food industry to treat food contamination?
- How has nuclear weapons/isotope use affected the environment?
- How do biologists use nuclear chemistry in their studies?

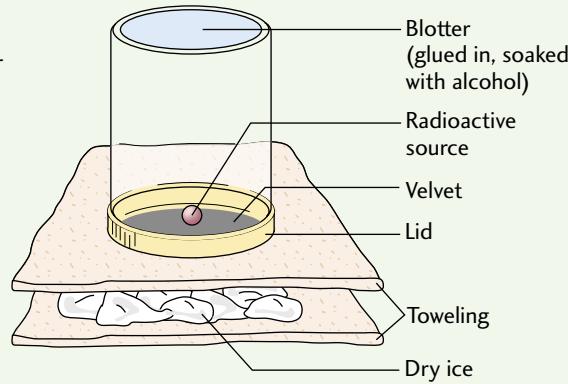
Design an Activity to Evaluate

Use a Geiger counter or scalar timer to perform a correlation study on the amount of background radiation in your home or school. Evaluate different house or building conditions for background radiation levels. For a more long-term study, see if there is any seasonal fluctuation in the amount of background radiation.

Build a Structure

Construct a cloud chamber that makes visible the paths of particles emitted as a result of radioactive decay. You will need to find a relatively safe radioactive sample, such as luminous watch or clock hands or alpha particle source, or some luminous paints. (Ask at a scientific supply store or your physics teacher.) Other materials you will need are black felt, dry ice, and rubbing alcohol. Construct your cloud chamber as shown in Figure STSE.14.4. To view the paths of particles, turn off the lights and use a flashlight to illuminate the area in front of the source above the velvet. Check the Internet for lots of different ideas on this project.

Fig. STSE.14.4 A cloud chamber



You should be able to*Understand Basic Concepts:*

- Describe nuclear components and how they relate to mass number, atomic number, and isotopes.
- Understand nuclear stability in terms of binding energy and calculations using mass defect with $E = mc^2$.
- Describe the basic types of radioactive decay and the characteristics of the particles emitted.
- Explain decay processes relative to the region of stability on a neutron–proton isotope graph.
- Understand the process behind the decay of carbon-14 and apply age dating to Canadian historical and geological events.
- Appreciate how the creation of a series of transuranic isotopes depends on nuclear interactions.
- Understand how human-made and natural radiation affects us personally and as world citizens.
- Identify the basic elements of nuclear fusion and fission in power generation and nuclear warfare.
- Determine energy available in nuclear fusion and fission reactions, and also estimate the underlying chain reaction times and the velocity moderation of fast neutrons.
- Describe how quantum theory explains both radioactive decay and virtual particle exchange.
- Apply quantitatively the uncertainty principle and de Broglie’s wavelength to estimate the size and range of the elementary particles and their associated forces.
- Describe the Standard Model of elementary particles in terms of the characteristic properties of quarks, leptons, and bosons, and identify the quarks that form familiar particles such as the proton and the neutron.

Develop Skills of Inquiry and Communication:

- Collect experimental data to determine the half-life of a short-lived radioactive nuclide.
- Analyze images of the trajectories of elementary particles in terms of their mass and charge.
- Illustrate elementary particle concepts and interactions using Feynman diagrams.

Relate Science to Technology, Society, and the Environment:

- Debate the pros and cons of nuclear fusion and fission power plants compared to the more traditional forms of power generation.
- Identify the radiation, both natural and human-made, to which we are exposed, and its affect on us and on the environment.
- Outline the historical development of nuclear and elementary particle experiments and theory, and how it led to the present Standard Model of matter and energy.

Equations

$$\Delta m = \frac{\Delta E}{c^2} \quad (\text{mass defect})$$



$$r = \frac{kq_1 q_2}{E_k} \quad (\text{stopping distance})$$



$$N = N_0 \left(\frac{1}{2} \right)^{\frac{t}{T_{\frac{1}{2}}}} \quad (\text{radioactive half-life})$$

$$r = 1.2 \sqrt[3]{A} \text{ fm} \quad (\text{radius of nucleus})$$

$$\text{Activity} = \frac{0.693N}{T_{\frac{1}{2}}} \quad (\text{disintegrations/s})$$

$$E^2 = p^2 c^2 + m_0^2 c^4 = m^2 c^4 = (m_0 c^2 + E_k)^2 \quad (\text{Mass-energy})$$

$$v_{1f} = v_{1o} \frac{(m_a - m_b)}{(m_a + m_b)} \quad (\text{moderated velocity})$$

$$1 + 2 + \dots 2^n = 2^{n+1} - 1 \quad (\text{geometric series})$$

$$\lambda = \frac{h}{mv} \quad (\text{de Broglie wavelength})$$

$$\Delta E \Delta t \geq \frac{h}{2\pi} \quad (\text{uncertainty principle})$$

$$r = \frac{mv}{Bq} \text{ and } f = \frac{qB}{2\pi m} \quad (\text{cyclotron radius and frequency})$$

EXERCISES

Conceptual Questions

1. Why can two atoms of the same element be chemically identical but physically different?
2. If the atomic mass number, A , equals the number of neutrons plus protons, then why do most of the elements in the periodic table have a non-integer value?
3. If carbon-12 has a mass of exactly 12 u, why does boron-10 have a mass of 10.012 936 u?
4. The sum of the masses of a proton and a neutron is more than the mass of a deuterium atom. Where did the missing mass go?
5. Discuss whether your body has more protons or neutrons.
6. When balancing a nuclear reaction, why is the atomic mass number, A , conserved but not the nuclear mass?
7. Is the average binding energy per nucleon greater in the more stable nuclear isotopes or in the less stable isotopes? Why?
8. During alpha decay of larger isotopes, such as uranium, the $\frac{N}{Z}$ ratio of the daughter nucleus becomes greater, but during beta decay, it becomes smaller. Why?
9. During alpha decay, which particle gets most of the available kinetic energy: the alpha particle or the daughter nucleus? Explain your answer.
10. If an alpha particle contacted the nucleus of a gold atom, would it be deflected?
11. Write the symbols used to describe the following:
 - a) proton
 - b) alpha particle
 - c) beta particle
 - d) neutrino
 - e) gamma ray
12. In what ways does the strong nuclear force differ from the Coulomb force of static electricity?
13. What early experimental evidence suggested that radioactivity was a nuclear process rather than a chemical one?
14. Why do larger stable isotopes require a greater ratio of neutrons to protons?
15. Is an alpha particle an ion or an atom of helium?
16. If 50% of the population lives past 76 years, then what age would you expect 25% of the population to reach? Why is human life expectancy not a good analogy for radioactive decay?
17. What isotope is the daughter of the beta decay of carbon-14?
18. Describe possible changes that have occurred to our atmosphere in the last century that could affect the ratio of ^{14}C to ^{12}C in the air we breathe.
19. Potassium-42 has a half-life of 12.4 h and is used in medicine to locate brain tumours. Think of two reasons why this isotope is suitable for diagnostic tests.
20. Why can't the remains of aquatic creatures be carbon dated?
21. Why can't relics over 60 000 years old be carbon dated?
22. Why can alpha and beta rays penetrate about 10 times farther into water than into lead?
23. Gamma decay does not involve transmutation of the elements, but alpha and beta decays do. Why?
24. Why are alpha particles 20 times more dangerous to mammals than beta or gamma particles?
25. Would an alpha particle with 4.2 MeV of kinetic energy go fast enough to cause a transmutation of nitrogen-14 to oxygen-14? (See problem 72.)

- 26.** Match the four states of matter: gases, liquids, plasmas, and solids, with the four ancient Greek elements of earth, wind, fire, and water.
- 27.** Why is it that nuclear fission does not take place in naturally occurring deposits of uranium?
- 28.** What force confines the plasma in that huge fusion reactor known as the Sun, and why can we not use this method of plasma confinement on Earth?
- 29.** Why do we not have to worry about disastrous meltdowns or runaway reactions in a fusion reactor?
- 30.** Why are high temperatures needed to start a fusion reaction but not a fission reaction?
- 31.** Why is a critical mass needed for fission but not for fusion?
- 32.** It has been said that using natural uranium to start a chain reaction is like using waterlogged wood to start a fire. Comment on this analogy.
- 33.** Why do neutrons not make a track in a bubble chamber like alpha and beta particles do?
- 34.** Why are high-energy accelerators needed for the creation of massive elementary particles?
- 35.** The cyclotron at TRIUMF is called a meson factory because it is so efficient at creating pions (also called pi-mesons). With which fundamental force is a pion associated?
- 36.** Why is beta decay described as a weak interaction?
- 37.** Why do you think the graviton or messenger of the gravitational force is so elusive or difficult to detect?
- 38.** In what ways do the weak and electromagnetic forces differ?
- 39.** Which process takes less time: a nuclear, a weak, or an electromagnetic interaction?
- 40.** Why does a high-energy particle that lasts only 10^{-23} s not make a track in a bubble chamber?
- 41.** Are all baryons more massive than all leptons?
- 42.** If each gluon has a colour and an anticolour, then how many possible kinds of gluons do you think there are?

Problems

14.1 Nuclear Structure and Properties

- 43.** Which elements are represented by
a) $^{35}_{17}X$? **b)** $^{222}_{86}X$? **c)** 9_4X ?
d) $^{238}_{92}X$? **e)** $^{256}_{101}X$?
- 44.** Determine the number of protons and neutrons in each of the isotopes listed in problem 43.
- 45.** Find the rest mass of a fluorine atom, in MeV/c^2 .
- 46.** A muon has a mass of $106 \text{ MeV}/c^2$. What is this mass in atomic mass units (u)?
- 47.** An element has two naturally occurring isotopes. The first isotope, with an atomic mass of 62.9296 u, occurs 69% of the time, and the second isotope has an atomic mass of 64.9278 u and occurs 31% of the time. What is the element and what is its mean atomic mass?
- 48.** Determine the average binding energy per nucleon of ^{14}C in MeV. (The mass of ^{14}C is 14.003 242 u.)

14.2 Natural Transmutations

- 49.** Calculate the change in the nuclear $\frac{N}{Z}$ ratio during the beta decay of carbon-14 into nitrogen-14.
- 50.** Calculate the binding energy of the last neutron in a He-4 nucleus (in MeV). (Hint: Compare the ^4He mass with that of ^3He [3.0160 u] + neutron.)

- 51.** Calculate the kinetic energy created during the alpha decay of uranium-232 ($m = 232.037\ 131\ \text{u}$) into thorium-228 ($m = 228.028\ 716\ \text{u}$). Express your answer in MeV.
- 52.** In a head-on collision of a 5.3-MeV alpha particle with a uranium-232 nucleus, determine the closest distance of the alpha particle before it is deflected back.
- 53.** Thorium-231 is radioactive. It emits a beta particle and is also the product of alpha decay. Write the reaction showing the product of ^{231}Th decay as well as the reaction statement that shows the mother isotope of ^{231}Th .
- 54.** Find the mass difference for the beta decay of a neutron into a proton. (Use Table 14.1.)
- 55.** If the kinetic energy of the electron emitted during the beta decay of a neutron is equivalent to about $\frac{1}{3}$ the mass difference, find the energy of the neutrino that is also emitted.
- 56.** Boron-12 decays by beta emission. The electron and neutrino are emitted at right angles to one another. The electron's momentum is $2.64 \times 10^{-21}\ \text{N} \cdot \text{s}$ and the neutrino's momentum is $4.76 \times 10^{-21}\ \text{N} \cdot \text{s}$. Find the momentum of the recoiling carbon-12 nucleus.
- 57.** What is the kinetic energy of the recoiling carbon-12 nucleus in problem 56? (Use $E_k = \frac{p^2}{2m}$.)
- 58.** If the alpha particles in Example 5 were accelerated to a kinetic energy of 449 MeV in the 88-inch (2.24-m) cyclotron at Berkeley, how close would they now get to the gold nucleus?
- 14.3 Half-life and Radioactive Dating**
- 59.** An injured athlete is given an injection of technetium-99m, which has a half-life of six hours. It collects in areas where bones have a high growth rate, like a stress fracture, and will show up under bone imaging. Plot a graph of percentage of the original dose that is still radioactive versus time for a total of 20 h. From the graph, find the percentage of technetium-99m remaining after 8 h.
- 60.** The Shroud of Turin has been carbon dated to the year 1350. What is the ratio of ^{14}C to ^{12}C in a relic that is 2000 years old compared to a relic from 1350?
- 61.** Polonium-210, present in tobacco, has a half-life of 138 days, while polonium-218 that clings to tobacco smoke has a half-life of 3.1 minutes. If a smoker had 1 μg of each isotope in his lungs to start with, how much radioactive Po would there be in total, after 7.0 minutes?
- 62.** A phosphorus-32 solution is injected into the root system of a plant. A Geiger counter is used to detect the movement of the phosphorus throughout the plant. After 30 days, the radioactivity level is down to 23% of its original level. Determine the half-life of ^{32}P .
- 63.** Assume that a rock originally had no lead-207, only ^{235}U . If uranium-235 decays through a series into ^{207}Pb in a half-life of $7.1 \times 10^8\ \text{a}$, find the age of the rock if it presently contains 5.12 mg of ^{235}U and 3.42 mg of ^{207}Pb .
- 64.** A 1.00-g sample of carbon from a living tree has a carbon-14 activity of 900 disintegrations/s compared to a 1.00-g sample of an ancient Viking axe handle that has 750 disintegrations/s. How old is the axe handle?

14.4 Radioactivity

65. Lead-208 is a doubly stable nucleus because both its neutron number ($N = 126$) and its proton number ($Z = 82$) are “magic” nuclear shell numbers: 2, 8, 20, 28, 50, 82, and 126 (analogous to electron shell numbers). Find the other three doubly stable isotopes.
66. The fallout from nuclear weapons testing includes ^{137}Cs , which has a half-life of 30.2 a before it beta decays into ^{137}Ba . Find the energy released during this decay. (Use $m_{\text{Cs}} = 136.9071$ u and $m_{\text{Ba}} = 136.9058$ u.)
67. In the average person of mass 70 kg, there is an activity of about 3700 Bq due to potassium-40 in the food we eat. If 5% of these beta emissions are absorbed by the body and have energy of 1.0 MeV each, determine the amount of grays absorbed per year.
68. The exposure to cosmic radiation doubles with every 2000-m increase in elevation. An airline pilot, while flying 20 h/week at an altitude of 10 000 m, receives a cosmic radiation dose of about 7.0×10^{-6} Sv/h. How many times greater is this amount than the average annual overall dose of 2 mSv per person?
69. A uranium-238 nucleus, in a series of decays, becomes lead-206. How many alpha and beta particles are emitted in this series?
70. If lead-208 is the end of a series of six alpha decays and four beta decays, find the isotope of the original nucleus.
71. The radius of a nucleus (in fm or 10^{-15} m) is given by $r = 1.2\sqrt[3]{A}$, where A is the atomic mass number. Find the distance separating their centres if an alpha particle and a nitrogen-14 nucleus are just touching.

72. Find the E_k needed, in MeV, for an alpha particle to just touch a nitrogen-14 nucleus in a head-on collision before being deflected by Coulomb repulsion. (Use the separation distance from the previous problem.)

73. During radioactive decay, the activity in Bq is given by the equation $\text{Activity} = \frac{0.693N}{T_{\frac{1}{2}}}$, where N is the initial number of atoms. Use this equation to calculate the number of disintegrations/s occurring in a 1.0-mg sample of pure hassium-269 with a half-life of 9.3 s. How close is your answer to one you would get by using the radioactive decay equation?

14.5 Fission and Fusion

74. In the solar proton–proton cycle, use Table 14.1 to determine the energy released during the second stage where hydrogen and deuterium fuse to make helium-3. (The mass of ^3He is 3.016 029 u.)
75. During a period of two years, a CANDU reactor produced 700 MW of power from the fission of ^{235}U and consumed one-half of its fuel. How much ^{235}U did it start with?
76. Find the percent of fission energy that creates 700 MW of electrical power if 2.5 kg of pure uranium-235 is fissioned daily in a CANDU nuclear reactor
77. If the earlier atomic bombs required a critical mass of about 50 kg of ^{235}U , and if all of this mass underwent fission during the explosion, how much energy would have been released?
78. If a fast 5.0-MeV neutron emitted in a fission reaction loses 90% of its kinetic energy in each collision with the moderating deuterium nuclei, how many collisions must it undergo before becoming a thermal or slow neutron of energy 0.050 eV?

- 79.** Find the speed of a fast 3.5-MeV fissioned neutron after it undergoes an elastic head-on collision with a stationary hydrogen nucleus. (Use $m_H = 1.007276$ u, $m_n = 1.008665$ u.)
- 80.** Find the missing daughter isotope in the following fission reaction:
- $$^{235}_{92}\text{U} + {}^1_0\text{n} \rightarrow {}^{141}_{56}\text{Ba} + \underline{\quad} + {}^3_0\text{n}$$

14.6 Probing the Nucleus

- 81.** How fast does a lead-207 ion travel when it is accelerated to a kinetic energy of 7.0 TeV in the LHC at CERN? (Use the $E = mc^2$ triangle.)
- 82.** Find the de Broglie wavelength of the lead ions in the previous problem.
- 83.** The TRIUMF cyclotron accelerates negative hydrogen ions to speeds of $0.75c$. Find the de Broglie wavelength of one of these ions.
- 84.** What is the strength of the magnetic field used in the TRIUMF cyclotron of problem 83 if the negative hydrogen ions make 23×10^6 rev/s? (Hint: Use $f = \frac{qB}{2\pi m}$.)
- 85.** A 9.0-GeV electron has the same de Broglie wavelength as a high-energy proton. What is the kinetic energy of the proton?

- 86.** In a cyclotron, a proton of kinetic energy 400 MeV is orbiting at a cyclotron frequency of 20 MHz. Find the strength of the cyclotron's magnetic field, B . (Hint: First calculate the dilated mass.)

14.7 Elementary Particles

- 87.** Calculate the overall charge of the following hadrons:
- a) uds
 b) $u\bar{d}$
 c) $d\bar{b}$
 d) $c\bar{c}$

- 88.** For each hadron in problem 87, give the name of the hadron and state whether it is a baryon or a meson.

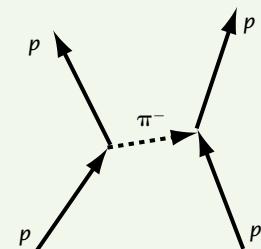
- 89.** Find the antiquark combination representing an antineutron.
- 90.** Using $1 \text{ u} = 931.5 \text{ MeV}/c^2$, determine the element on the periodic table whose atomic mass number is closest to the mass of the *top* quark.

- 91.** Using only *up* or *down* quarks (or antiquarks), find the combination for a π^- meson (its charge is $-e$).

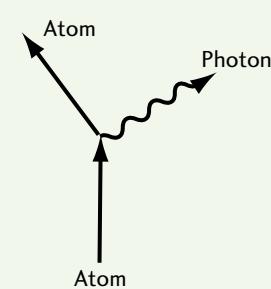
14.8 Fundamental Forces and Interactions — What holds these particles together?

- 92.** If the diameter of a nucleon is 2.4×10^{-15} m, use a velocity of 3×10^8 m/s to determine the typical time for a nuclear interaction involving the strong force.
- 93.** Describe the interaction taking place in each of the following Feynman diagrams.

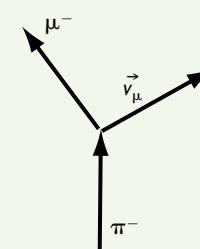
a) Fig.14.58a



b) Fig.14.58b

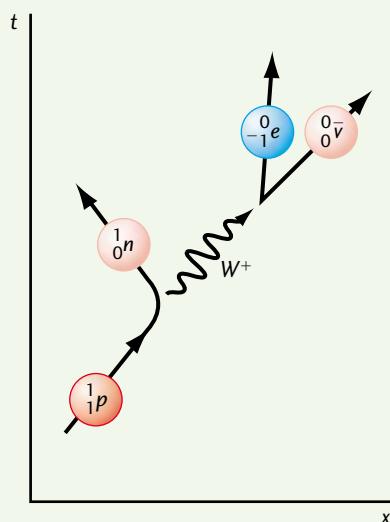


c) Fig.14.58c



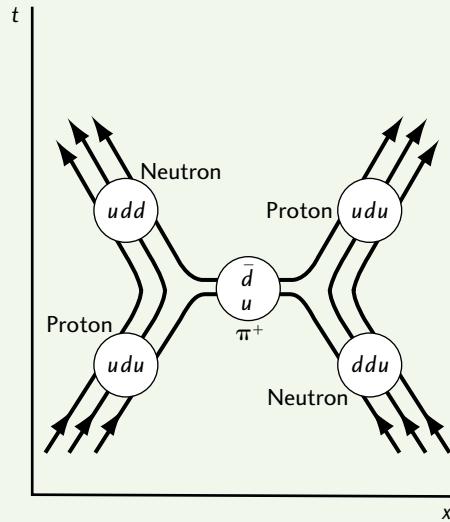
- 94.** Figure 14.59 shows the Feynman diagram for the decay of the proton via the W^+ weak boson. The end products are a neutron, an electron, and an antineutrino. Draw the Feynman diagram for the decay of an antiproton.

Fig.14.59 Proton decay via the W^+ weak boson



- 95.** Figure 14.60 shows the Feynman diagram of a proton–neutron interaction. The proton, p , recoils as a neutron, n , after giving up a virtual π^+ meson. The incoming neutron absorbs the π^+ meson to become a proton. Draw the Feynman diagram for an interaction of two nucleons where a virtual π^0 meson is exchanged.

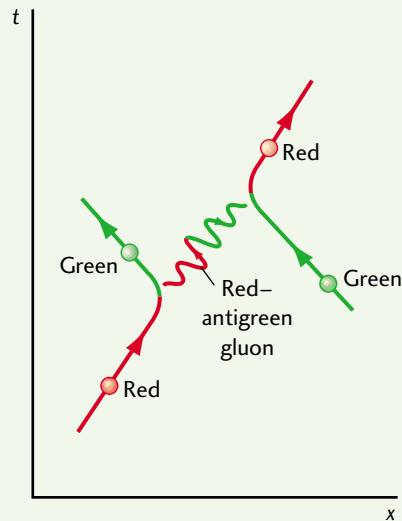
Fig.14.60 Proton–neutron interaction via the π^+ meson



- 96.** In the decay ${}^1_1 p \rightarrow {}^1_0 n + \pi^+$, the energy, E , for the π^+ meson comes from the mass difference between the proton and the neutron. It is given by $E^2 = p^2 c^2 + m_0^2 c^4$, according to Einstein's energy equation. Use $m_p = 938.3$, $m_n = 939.6$, and $m_\pi = 139.6$ (all in MeV/c^2) to show that the momentum, p , is imaginary, and thus that the meson is a virtual particle.

- 97.** If a meson consisted of a *strange* quark and an *antistrange* quark connected by a string, what kinds of particles (consisting of two quarks) can be created if the string breaks?
- 98.** A meson is made up of a *strange* quark and an *anticharm* quark. What is its charge?
- 99.** A baryon is made up of two *top* quarks and a *bottom* quark. What is its charge?

Fig.14.61 A red quark–green quark interaction



- 100.** The Feynman diagram in Figure 14.61 shows the exchange of a gluon between two quarks. The force holding them together involves a change of colour via a red-antigreen gluon. Show the Feynman diagram for a blue-to-green quark colour-force exchange.



The Half-life of a Short-lived Radioactive Nuclide

Purpose

To calculate the half-life of a radioactive isotope

Safety Consideration

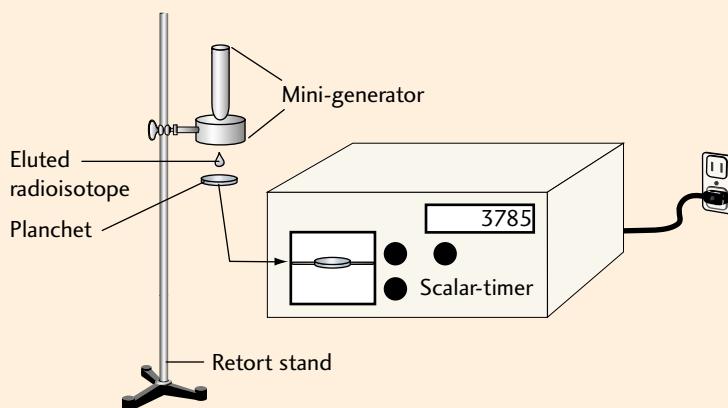
1. Wear latex rubber gloves when working with any radioactive substance.
2. All students must wash their hands at the end of this lab.
3. Any spillage of the radioactive eluant must be reported immediately to the teacher.
4. All eluant must be collected in a central container for proper handling and disposal.

Equipment

Cesium-137/barium-137 minigenerator or another source that produces a short-lived radioisotope ($T_{\frac{1}{2}} < \frac{1}{4}$ of lab time)

Geiger counter (scalar timer), 10 mL beaker

Fig. Lab.14.1



Procedure

1. Prepare a lab data table similar to the one provided.
2. Set up the lab equipment as illustrated in Figure Lab.14.1.
3. With the detector area free of any radioactive source, set the voltage of the scalar timer to zero and turn the detector on.

4. Start the detector and slowly increase the voltage until you see it begin to detect some of the background radiation. You have it set correctly if it registers about 5 counts in every 10 seconds. Note: If the voltage is too high, the detector will “avalanche.” This occurs when one event causes a burst of counts that are obviously increasing the visible counts very quickly. Too low a voltage will mean very few events are registered and the lab will take too long to perform.
5. Determine the background radiation level by counting the events that occur in 300 s (5 minutes). Record these values in your data table, noting that a second background count will be taken at the end of the lab.
6. Elute the minigenerator by collecting about 3 mL of the radioactive eluant into a 10-mL beaker.
7. Place the 10-mL beaker under the counter and record the count rate for one-minute intervals. Record the information for one-minute periods in the data chart as shown, being sure that you only measure for the odd-numbered time periods. Use the even-numbered time periods to record your previous data and reset the counter.
8. Repeat this procedure for at least four readings.
9. Return all the eluant to the container provided by your teacher, including any rinse water that you used to clean the beaker.

Uncertainty

The absolute uncertainty in statistical measurements of this nature is found by taking the square root of the count. For example, a count of 4000 would have an absolute uncertainty of ± 63 counts ($\sqrt{4000} \approx 63$). If this count was registered in 60 s, the count rate would be 67 ± 1 counts/s ($\frac{4000}{60} \approx 67$ and $\frac{63}{60} \approx 1$).

Analysis

- Calculate all of the raw count rates and record their values in counts per second. Depending on the strength of your source and the absorbers you used, you may want to record the count rates in counts per minute.
- Subtract the count rate for the background radiation and record those values, including the uncertainty.
- Plot a graph of the corrected activity versus time. Be sure that you plot your activity at a time value that is halfway through the first, third, and fifth minute. Draw the best fit curve for all the data points.

Discussion

- Why did you have to plot your count rate at the half-time of any given interval?
- Look up the half-life for the parent radioactive isotope used in this lab and compare it to the one that you calculated in this lab ($T_{\frac{1}{2}} = 2.6$ min for ^{137}Ba).

Conclusion

Write a concluding statement that summarizes how your experimental half-life value compared with the accepted value for your isotope, considering experimental uncertainty.

Background Radiation		
Background counts	Time(s)	Count rate (c/s)
Average count rate		

Half-life Data					
Time (h)	Counts	Counting period (s)	Count rate (c/s)	Background rate (c/s)	Count rate (c/s) (corrected for background)
		60			
		60			
		60			
		60			
		60			
		60			
		60			

Appendices



- APPENDIX A: Experimental Fundamentals
- APPENDIX B: Lab Report
- APPENDIX C: Uncertainty Analysis
- APPENDIX D: Proportionality Techniques
- APPENDIX E: Helpful Mathematical Equations and Techniques
- APPENDIX F: Geometry and Trigonometry
- APPENDIX G: SI Units
- APPENDIX H: Some Physical Properties
- APPENDIX I: The Periodic Table
- APPENDIX J: Some Elementary Particles and Their Properties

APPENDIX A: Experimental Fundamentals

Introduction

The reason for performing experiments lies in the need to test theories. In the research world, the experiment tests the ideas put forth by theoreticians. It can also lead to new ideas and subsequent laws as a result of the data obtained. In order to perform experiments safely, the proper use of equipment must be adhered to. The following sections outline safety concerns and the formal method of writing a scientific lab report.

Safety

In any situation involving the use of chemicals, electrical apparatuses, burners, radioactive materials, and sensitive measuring devices, the role of safety and proper use of instrumentation is of primary importance when performing labs. There is a system, developed Canada wide, which tries to ensure workplace safety standards. **WHMIS** stands for *Workplace Hazardous Materials Information System*. This system has formulated a set of rules and symbols that recognize potential hazards and appropriate precautions when using chemicals, hazardous materials, and equipment. The following symbols, illustrated and described in Table A.1, are the standard set of warning labels set out by WHMIS.

As well, there are a set of safety warning labels associated with household products. The symbols are referred to by the abbreviation **HHPS**, or *hazardous household product symbols*.

Table A.1
WHMIS Symbols

Symbol	Risks	Precautions
Compressed gas		Materials that are normally gaseous. Kept in a pressurized container Ensure container is always secured.
Flammable and combustible		Materials that will continue to burn after being exposed to a flame or other ignition source Store in properly designated areas. Work in a well-ventilated area.
Oxidizing		Materials that can cause other materials to burn or support combustion Store in areas away from combustibles. Wear body, hand, face, and eye protection.
Toxic, immediate, and severe		Poisons/potentially fatal materials that cause immediate and severe harm Avoid breathing dust or vapours. Avoid contact with skin or eyes.
Toxic, long term, concealed		Materials that have harmful effects after repeated exposures or over long periods of time Wear appropriate personal protection. Work in a well-ventilated area.
Biohazardous infectious		Infectious agents or biological toxins causing a serious disease or death Special training required. Work in designated biological areas with appropriate engineering controls.
Corrosive		Materials that react with metals and living tissue Wear body, face, and eye protection. Use a breathing apparatus.
Dangerously reactive		Materials that may have unexpected reactions Handle with care, avoiding vibration, shocks, and sudden temperature changes.

In physics at the high-school level, the use of chemicals is minimal. However, the use of high- and low-voltage supplies as well as measuring and timing devices is common. Pertinent excerpts from the safety manual include:

- Fused and grounded 110–120 V outlets should be used.
- Outlets should be away from sources of flammable gases.
- A master cutoff switch should be available and accessible.
- An appropriate fire extinguisher and fire blanket should be in the room.
- High-voltage sources should be clearly marked.
- Electrical cords should be free of cuts.
- Radioactive sources should be stored in a locked cupboard.

When performing experiments, the following safety practices should be adhered to:

Experiments involving an open flame:

- Long hair should be tied back.
- Loose clothing should not be worn on experimental days. Sleeves should be rolled up.
- Do not leave the candle or burner unattended.
- Always have something under the candle to catch the wax.
- Have a beaker of water nearby in case of emergency when using candles.

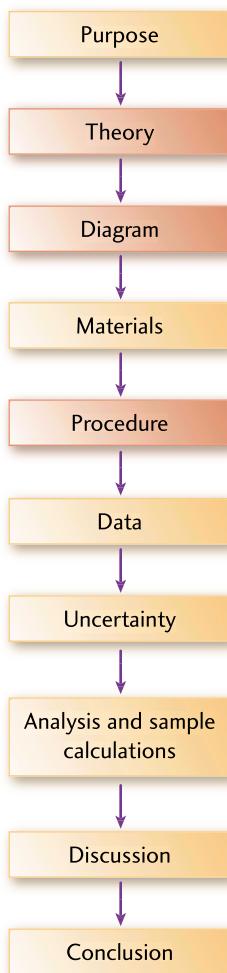
Experiments involving power supplies:

- Never short out the supply.
- Keep water and wet hands away from electrical equipment, especially when using ripple tanks.
- Be aware of wires connected to high-voltage supplies. Make sure they are securely attached and not touching grounded objects.
- Always have the supply turned off when connecting it to the experimental components.
- If you're not sure, ASK!

APPENDIX B: Lab Report

Lab Report

Fig.A.1 Outline for a lab report



The following outline is for a general lab report. Some sections may be omitted or modified by the teacher, depending on the type of experiment and the level of experimental expertise you have developed.

Purpose: A statement(s) that encompasses the aim or goal of the experiment.

Theory: (optional) This section briefly describes the theoretical background to the experiment. It can develop or state equations to be used in the analysis as well as possible logic outcomes that are being tested. It may predict certain ideal, theoretical outcomes that will be used as comparative values against the ones obtained from the experiment.

Diagram: (optional) A sketch or schematic of what the experiment looks like. It can include electrical representations of set-ups, circuit drawings, and labelled diagrams of the actual physical set-up.

Materials (Equipment): A list of equipment and support material needed to run the experiment.

Procedure: (optional) In many instances, the procedure is already provided and thus need not be recopied. In cases where you have designed the experiment, the procedure becomes an important part of the lab report because it clarifies the method and reasoning behind what the experiment is set up to accomplish.

Data: The data section should be organized and clear. Multiple data results should be organized in a chart.

Charts: The standard chart is a tool for recording and reading results. The chart proper should contain data values only, without the units. Units are indicated with the headings. No calculations should be done in the chart. The start of a sample chart is shown.

Trial #	m_1 (kg)	m_2 (kg)	m_3 (kg)	m_{total} (kg)	$F_{applied}$ (N)	d (m)	t (s)
1	1.2	2.0	4.1	7.3	71.5	0.6	0.34
2	1.2	2.0	5.0	8.2	80.4	0.6	0.44

A rough chart should be created before doing the experiment. This step helps organize your results as the experiment proceeds and allows you to easily refer back to them at a later date. A neat copy is then made for the actual report.

Uncertainty: This section defines the limits of the precision of the data you are collecting. It is an indicator of how accurately you can measure any event. The value is specified by the “ \pm ” sign. In some books, it is referred to as the *experimental error*. This term is somewhat misleading because it implies that you are making some kind of mistake while doing the lab. The uncertainty in a data measurement is only an indicator of the accuracy of the measuring device and the manner in which it is used.

Normally, uncertainty has two components. The **instrumental** part indicates the precision of the instrument. In the case of an instrument with divisions marked on it, the estimation uncertainty is $\pm .5$ of a division if the divisions are very close together and hard to see in between. If there is ample space between divisions, allowing you to estimate between the lines, then $\pm .1$ of a division is used. In some cases, you may use $\pm .2$ of a division if the spacing between markings does not allow you to easily estimate between them. For digital readouts, the uncertainty is provided by the manufacturer.

The **procedural** part of the uncertainty lies in the manner you use the instrument. A 30 cm long ruler marked in mm divisions has an instrumental uncertainty of $\pm .5$ mm. If you were to measure the length of the football field with it, the uncertainty would be far greater than 0.5 mm. The reason is because you must lift the ruler and place it back down. For each lift of the ruler, a repositioning or alignment uncertainty occurs, which must be taken into account. A value of 0.5 mm up to 1 mm may be assigned per lift. Timing and reflexes fall into the same category. Even though a stop watch can indicate hundredths of a second, timing an object dropped from a height of 10 cm will not have a value accurate to 0.01 s. Your reflexes are good to only 0.2 s–0.3 s because it takes a finite amount of time to press the start and stop buttons on the watch.

The total uncertainty is the addition of the two component uncertainties.

Statistical Deviation of the Mean

This calculation is used to obtain a scatter indicator of values that should, in theory, be the same, but are not because of factors that cannot be controlled in the experiment. For instance, if you were to roll a ball from a ramp off a table many times and measure its range from the edge of the table, you would find that the ball will land at slightly different places, even though you have made every attempt to keep the action of releasing the ball the same each time. The slight imperfections in the ball, ramp, and table all contribute to this effect. The standard deviation of the mean indicates how reproducible the event is. The value is given the Greek letter sigma (σ). The formula for standard deviation of the mean is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n \Delta_i^2}{n}}$$

where $\Delta_i = |\text{data value} - \text{average value}|$

n is the number of data values

When the σ (uncertainty) is quoted with the average for a set of data, the uncertainty combined with the mean will encompass 68% of the data values. If the \pm is quoted as 2σ , then the range encompasses 95% of the data values. If the \pm is quoted as 3σ , then the range encompasses 99.7% of the data values.

Example: Given the following five distances, find the average value and quote the scatter of the data in terms of one standard deviation (σ).

$$d \text{ (m)} \quad 2.003 \quad 2.008 \quad 2.000 \quad 2.005 \quad 2.005$$

$$\text{Average value} = \frac{(2.003 \text{ m} + 2.008 \text{ m} + 1.999 \text{ m} + 2.005 \text{ m} + 2.005 \text{ m})}{5}$$

$$= 2.004 \text{ m}$$

Δ	Δ^2	$\sum_{i=1}^5 \Delta_i^2$	$\frac{\sum_{i=1}^5 \Delta_i^2}{5}$	$\sqrt{\frac{\sum_{i=1}^5 \Delta_i^2}{5}}$
$ 2.003 - 2.004 = .001$	1×10^{-6}	4.4×10^{-5}	8.8×10^{-6}	.003
$ 2.008 - 2.004 = .004$	1.6×10^{-5}			
$ 1.999 - 2.004 = .005$	2.5×10^{-5}			
$ 2.005 - 2.004 = .001$	1×10^{-6}			
$ 2.005 - 2.004 = .001$	1×10^{-6}			

$\sigma = 0.003 \text{ m}$ and we quote our average value as $2.004 \pm .003 \text{ m}$

Analysis and Sample Calculations: In many experiments, a set of calculations is repetitive. In these cases, the values obtained from the calculations are summarized in table form. A sample calculation is then shown in full.

Discussion: In early lab reports, this section answers lab questions posed by the teacher. These questions are used to lead to an analysis and assessment of the validity of the experimental results. In cases where a comparison is made between two values or an accepted value and the experimental value, the **percent deviation** is used. This equation is

$$\frac{| \text{accepted value} - \text{experimental value} |}{|\text{accepted value}|} \times 100$$

or, for two values that should be the same,

$$\frac{| \text{value 1} - \text{value 2} |}{|\text{larger value}|} \times 100$$

If the percent deviation lies inside the accepted range of values based on your uncertainty assignments, then the values can be stated as being the same.

Not all experiments work successfully each time. If your results do not match the theoretical expectations, then in this section, you would discuss possible reasons for this discrepancy.

Conclusions: This last section is a summarizing statement of the results arrived at in the experiment. It is the bookend for the opening introductory section, the **Purpose**. This section brings together the whole intent of the experiment in terms of its degree of success.

APPENDIX C: Uncertainty Analysis

Accuracy versus Precision

When a value is measured, there are two parameters that affect the quality of the measurement: accuracy and precision. **Accuracy** is the amount that a measurement is removed from the true value. However, we rarely know what the true value is; it can only be predicted theoretically. Most values in nature have accepted values based on the results of repeated experiments. For example, the local acceleration due to gravity is 9.80 m/s^2 .

Precision is how closely subsequent measurements can be repeated. If you measure the length of a table to be 2.3 m the first time and 1.4 m the second time, your measurements would show a definite lack of precision! Your goal as an experimenter is to obtain the greatest possible precision from your equipment. All equipment has limited precision. Clever design of your experiment will permit better and more reliable data collection.

Note: A measured variable has a stated uncertainty, not an error.

Working with Uncertainties

To assess how precisely a parameter can be stated, we must consider two aspects:

- 1) How precise are the measurements? Have they been done using a procedure that other scientists will accept?
- 2) What is the procedure for combining variables that have stated uncertainties?

Making Measurements with Stated Uncertainties

No scientist is ever satisfied with one observation. A single observation is referred to as anecdotal. Determining the average of many measurements gives the most reliable value because scientists assume that the uncertainty in the measurements is due to random interpretations of the most precise reading possible of an instrument. For example, if you measure the length of a table using a ruler calibrated in millimetres, to obtain the most precise value, you will have to estimate fractions of a millimetre by eye. A number of such estimates will introduce variations in your values.

After averaging many numbers together, an additional observation does very little to change the average. The theory of large numbers suggests that 30 observations is a maximum and 5 is a minimum. The best experimenters calibrate their equipment very carefully in order to ensure that any anomalies observed are those of the phenomenon being investigated.

Manipulation of Data with Uncertainties

We first define two ways of stating uncertainty.

Absolute uncertainty is the actual value of the relative and instrumental uncertainties added together. The uncertainty associated with a measurement carries the same units as the measurement.

Relative uncertainty is the absolute uncertainty expressed as a percent of the data value.

Your answer should always include the number and its uncertainty: $z \pm \sigma_z$

Addition and Subtraction of Data

When adding or subtracting data,
ALWAYS ADD the *absolute uncertainties* of the measurements.

Example: $4.5 \pm .5 \text{ m} + 1.5 \pm .5 \text{ m} = 6.0 \pm 1.0 \text{ m}$

$$4.5 \pm .5 \text{ m} - 1.5 \pm .5 \text{ m} = 3.0 \pm 1.0 \text{ m}$$

The answer always carries the largest possible uncertainty associated with it.

Alternative Method:

Let $z = x_1 + x_2 + \dots + x_n$

$$z = \sum_{k=1}^n x_k$$

$$\text{Then } \sigma_z = \sqrt{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_n^2}$$

$$\sigma_z = \sqrt{\sum_{k=1}^n \sigma_k^2}$$

Multiplication and Division of Data

When multiplying and dividing data,
ALWAYS ADD the *relative or percentage uncertainties* of the measurements.

However, before we state our final answer for a calculation, the relative or percentage uncertainty is converted back into an absolute uncertainty by ***multiplying it by the answer itself***.

Example: $(5.0 \pm .5 \text{ m}) \times (2.5 \pm .5 \text{ m}) = (5.0 \pm 10\% \text{ m}) \times (2.5 \pm 20\% \text{ m})$
 $= 12.5 \pm 30\% \text{ m}^2 = 12.5 \pm 3.8 \text{ m}^2$

Example: Combining absolute and relative uncertainties; calculating the uncertainty of a slope.

Note: When the two types of calculations are combined, such as in a slope calculation, follow the order of operations.

Given: $v_1 = 10.0 \pm 1 \text{ m/s}$ $v_2 = 30 \pm 3 \text{ m/s}$ $t_1 = 5.0 \pm .4 \text{ s}$
 $t_2 = 10.0 \pm .1 \text{ s}$, find acceleration.

$$\begin{aligned} a &= \frac{v_2 - v_1}{\Delta t} = \frac{(30 \pm 3 \text{ m/s}) - (10 \pm 1 \text{ m/s})}{(10.0 \pm .1 \text{ s}) - (5.0 \pm .5 \text{ s})} \\ &= \frac{20 \pm 4 \text{ m/s}}{5.0 \pm .15 \text{ s}} = \frac{(20 \pm 20\%) \text{ m/s}}{(5.0 \pm 3\%) \text{ s}} = 4.0 \pm 23\% \text{ m/s}^2 \\ &= 4.0 \pm .9 \text{ m/s}^2 \end{aligned}$$

Alternative Method:

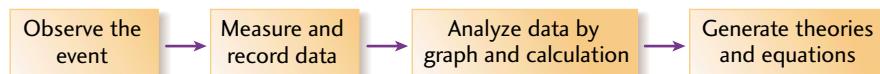
Let $z = (x_1)(x_2) \dots (x_n)$

Then $\sigma_z = z \sqrt{\frac{\sigma_1^2}{x_1^2} + \frac{\sigma_2^2}{x_2^2} \dots \frac{\sigma_n^2}{x_n^2}}$

APPENDIX D: Proportionality Techniques

In physics, like in any science, we observe nature in order to seek regularities or patterns. The patterns in our observed data form the basis for theories and laws that explain observed events. If the regularities that we observe follow mathematical functions, then we can derive equations that accurately model the behaviour of natural events.

Fig.A.2



Proportionality techniques involve data analysis from controlled experiments to examine the way in which two experimental quantities, the independent and the dependent variables, may be correlated. A proportionality statement is a simple expression that describes how one variable varies in relation to another, which allows us to predict an object's behaviour without our needing to understand why the object behaves that way. For example, scientists don't completely understand how the gravitational force works, but their predictions based on experimental data of the action of gravity have enabled them to send astronauts to the Moon and back!

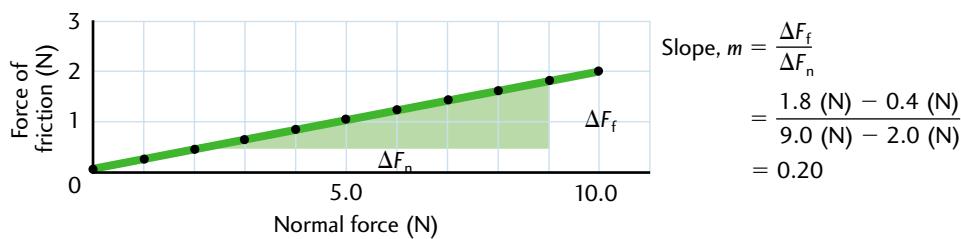
The simplest proportion is a direct proportion in which a change in one quantity by some multiple is met by a similar change in the other quantity.

Proportionality description	Proportionality Statement
The variable y is directly proportional to the variable x	$y \propto x$

Table A.2	
Normal force F_n (N)	Frictional force F_f (N)
0.0	0.0
1.0	0.2
2.0	0.4
3.0	0.6
4.0	0.8
5.0	1.0
6.0	1.2
7.0	1.4
8.0	1.6
9.0	1.8
10.0	2.0

All direct proportionalities are straight-line graphs when their quantities are plotted. (See Table A.2 and Figure A.3.)

Fig.A.3 Friction versus the normal force



Creating an Equation from a Proportionality

The generic equation for a straight line is $y = mx + b$, where m represents the slope of the straight line and b represents the y intercept. From the constant slope of the line in Figure A.3 and the intercept of the graph at 0 N,

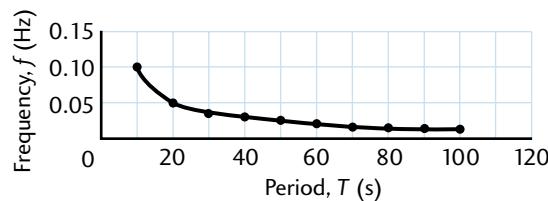
the equation of the relationship is $F_f = 0.20F_n$. This equation can be used to predict what force of friction will result from any particular normal force. For example, the equation allows us to predict that a normal force of 7.0 N will result in a force of friction of $F_f = (0.20)(7.0 \text{ N}) = 1.4 \text{ N}$.

To form an equation from this direct proportionality (straight-line graph), we replace the proportionality sign, \propto , with an equal sign and the slope of the straight-line graph, which is the **constant of proportionality, k** .

$$y \propto x \rightarrow y = kx$$

A proportionality statement describes a direct proportion between any two entities, including algebraic manipulations of data. For example, the data from a simple inverse relationship like frequency and period (see Table A.3), plotted in Figure A.4, are not directly proportional.

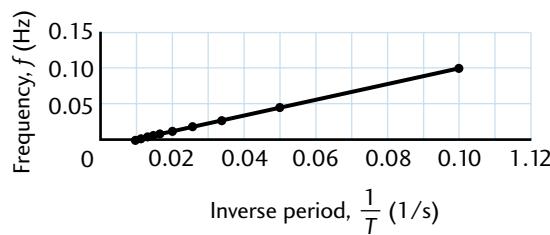
Fig.A.4 Frequency versus period



If the quantities of frequency and $\frac{1}{T}$ (inverse period) are plotted in Figure A.5, a straight-line graph results.

Period T (s)	Frequency f (Hz)	Reciprocal of Period T^{-1} (s^{-1})
10	0.100	0.100
20	0.050	0.050
30	0.033	0.033
40	0.025	0.025
50	0.020	0.020
60	0.017	0.017
70	0.014	0.014
80	0.012	0.012
90	0.011	0.011
100	0.010	0.010

Fig.A.5 Frequency versus inverse period



The *inverse* of the period is directly proportional to the frequency: $f \propto \frac{1}{T}$ or $f = \frac{k}{T}$. In this case, the slope is 1, so the true equation is $f = \frac{1}{T}$.

Finding the Correct Proportionality Statement

The Multiplier Method

A proportionality is the tracking of how quantities vary with respect to each other. In a direct proportionality, both quantities are related by the same **multiplier**. We find **multipliers** by dividing each data point by the first data point and recording the relationship, as shown in Table A.4.

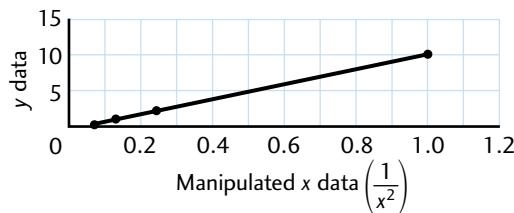
Table A.4

x	y
1.0	10
2.0	2.5
3.0	1.1
4.0	0.6

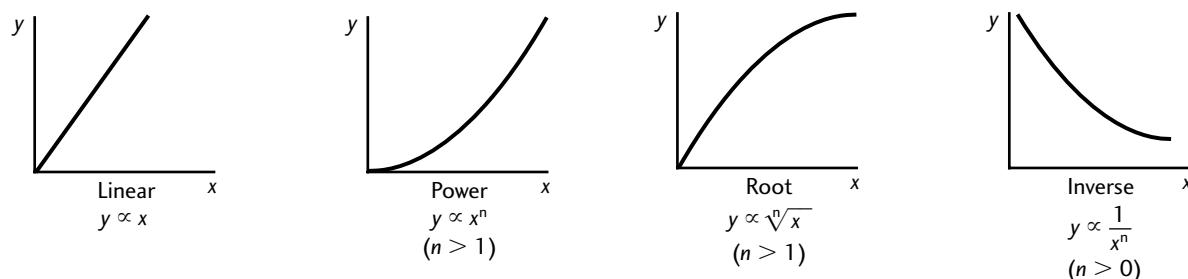
2x
 3x
 4x 0.250x
 0.111x
 0.062x

In Table A.4, the multipliers from the y data do not equal the x multipliers; therefore, the two quantities, x and y , are not directly proportional. However, we can manipulate the x multiplier by trial and error. If we find a relationship where the x and y multipliers are equal, then there is a direct proportionality for the data in Table A.4.

If we take the inverse of the x multiplier, then $\frac{1}{x} = \frac{1}{2} = 0.5$, which does not equal 0.25 (the y multiplier), so x and y are not related inversely. If we square the inverse of the x multiplier, then $(\frac{1}{2})^2 = 0.25$. In this case, the y multiplier equals the x multiplier; therefore, we have found the correct proportionality: y is directly proportional to $\frac{1}{x^2}$ or $y \propto \frac{1}{x^2}$. The resulting graph is a straight line (see Table A.5 and Figure A.6).

Fig.A.6 Y versus manipulated x data

Most of the time, we will be trying to find relationships between data gathered during an experiment that will have an associated level of uncertainty. Uncertainty will make it more difficult to find multipliers. For example, the multiplier 0.248 may actually indicate an inverse square relationship $(\frac{1}{x^2})$ from a y multiplier of 2 $(\frac{1}{2^2} = \frac{1}{4})$. In this case, compare the shape of the graph of your data with one of the four graphs in Figure A.7 to give you an idea of what the proportionality might be.

Fig.A.7 The shape of the graph can suggest an appropriate proportionality for the data

Finding the Constant of Proportionality in a Proportionality Statement

Once the correct proportionality statement is known, then the constant of proportionality and the final equation can be easily determined. The slope of the straight-line graph from the manipulated data is the constant of proportionality.

To determine the constant of proportionality without graphing any data, rearrange the generic equation for the constant of proportionality and selectively substitute the data points. Using our friction example, the proportionality statement is $F_f \propto F_n$ and the generic equation is $F_f = kF_n$. So $k = \frac{F_f}{F_n}$. We can calculate the constant of proportionality, k , by substituting any data points, including units:

$$k = \frac{F_f}{F_n}$$

$$k = \frac{0.8 \text{ N}}{4 \text{ N}}$$

$$k = 0.2$$

For data that has an associated uncertainty, calculate k for each set of data points, then calculate the average of all your values of k to determine the representative value for all the data. This method is called the **multiple k method**. The final equation can be determined by substituting the average k value into the equation.

Other Methods of Finding Equations from Data

The Log–Log Method

Typical data proportionalities can be described by the exponential proportionality, $y = kx^n$, where k is the constant of proportionality and n is the specific proportionality between the independent and the dependent variables. Table A.6 summarizes the relationship between n and the type of proportionality. Taking the logarithm of both sides of the generic proportionality $y = kx^n$, we obtain

$$y = kx^n$$

$$\log(y) = \log(kx^n)$$

$$\log(y) = \log(x^n) + \log(k)$$

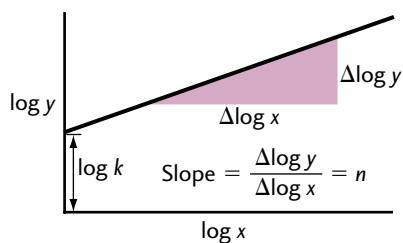
$$\log(y) = n\log(x) + \log(k)$$

Note the similarities between this equation and the generic linear equation, $y = mx + b$.

Graphing calculators are invaluable for plotting quick graphs to test proportionalities and determine slopes.

Table A.6		
n	Type of proportionality	Example
$n = 1$	Linear	$y = 3x$
$n > 1$	Power	$y = 2x^2$ (parabolic)
$0 < n < 1$	Root	$y = 2x^{1/2} = 2\sqrt{x}$ (square root)
$n = -1$	Inverse	$y = 2x^{-1} = \frac{2}{x}$
$-1 < n < 0$	Inverse root	$y = 2x^{-1/2} = \frac{2}{\sqrt{x}}$ (inverse square root)
$n < -1$	Inverse power	$y = 2x^{-2}$ (inverse square)

Fig.A.8 The plot of $\log y$ and $\log x$ is a straight line of slope n and intercept $\log k$.



Check your calculator manual to see if it can perform two-dimensional statistical calculations, such as linear or power regression.

Regression can also be done with the spreadsheet program Microsoft® Excel by highlighting the two columns of data and then accessing the Regression Tool from the Data Analysis option in the Tools menu.

Any data can be made linear by taking the logarithm of both quantities. Taking the logarithms of the experimental data first will allow linear regression to be used in all circumstances.

Lost your calculator's instruction manual? Web links to various calculator manufacturers are available at <www.irwinpublishing.com/students>. Find the manufacturer's Web site and download a printable set of instructions for your calculator.

Plotting the logs of the variables x and y always produces a straight-line graph with a slope n and a y intercept of $\log k$, as illustrated in Figure A.8.

Calculating the slope and the y intercept of this graph will yield both n and $\log k$ (k can be determined by finding the inverse log). Both n and k can then be directly substituted into the general proportionality equation $y = kx^n$.

Computed Regression

Most scientific calculators and computer spreadsheets have built-in algorithms that perform statistical regression on data. These functions on a calculator allow you to input two columns of data and find the equation that describes the relationship between the two quantities. Once the data has been entered, the calculator provides the user with two key pieces of information:

- 1) The **correlation coefficient, r** , is a number that varies from -1 to $+1$ and is a measure of how well the two quantities correlate. In **linear regression** mode, r measures how well the data resembles a linear proportion. In **exponential regression** mode or **logarithmic regression** mode, r measures whether the data fits an exponential or logarithmic proportionality. The better the fit, the closer r is to 1 .
- 2) A and B determine the equation that best describes the relationship between the two quantities, x and y . Table A.7 summarizes the meaning of the parameters A and B for each regression mode.

Table A.7

Mode	General equation	A	B
Linear regression	$y = A + Bx$	y intercept of straight line	Slope of a straight line
Exponential regression	$y = Ax^B$	Constant of proportionality, k	Power of exponent, n

Linear regression works when you know that the data you have input is linear, whereas exponential regression works for every set of data.

The General Method for Using Regression

- 1) Set your calculator to linear regression mode (two-dimensional statistics), as described in your calculator's instruction manual.
- 2) Input all the experimental data as ordered pairs $(x_1, y_1 \dots x_2, y_2 \dots x_3, y_3 \dots x_n, y_n, \text{etc.})$ according to the calculator manual's instructions.
- 3) Display the correlation coefficient (r). If r is very close to 1 ($0.997 > r \geq 1$), then the data are linearly proportional.
- 4) Display the A and B values for the regression. A represents the y intercept and B represents the slope of the line.

The same type of regression analysis can be done with a computer spreadsheet such as Corel Quattro® Pro or Microsoft® Excel. The software produces a statistics table that provides the correlation coefficient, slope, and y intercept of the graph, as illustrated in Table A.8.

Table A.8 Regression Output from Microsoft® Excel

SUMMARY OUTPUT						
Regression Statistics						
Multiple R	0.999995					
R Square	0.999989					
Adjusted R Square	0.999984					
Standard Error	0.012649					
Observations	4					
ANOVA						
	Df	SS	MS	F	Significance F	
Regression	1	30.40578	30.40578	190036.1	5.26E-06	
Residual	2	0.00032	0.00016			
Total	3	30.40610				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	2.006	0.010583	189.5492	2.78E-05	1.960465	2.051535
X Variable 1	2.466	0.005657	435.9313	5.26E-06	2.441661	2.490339

Slope (k value)

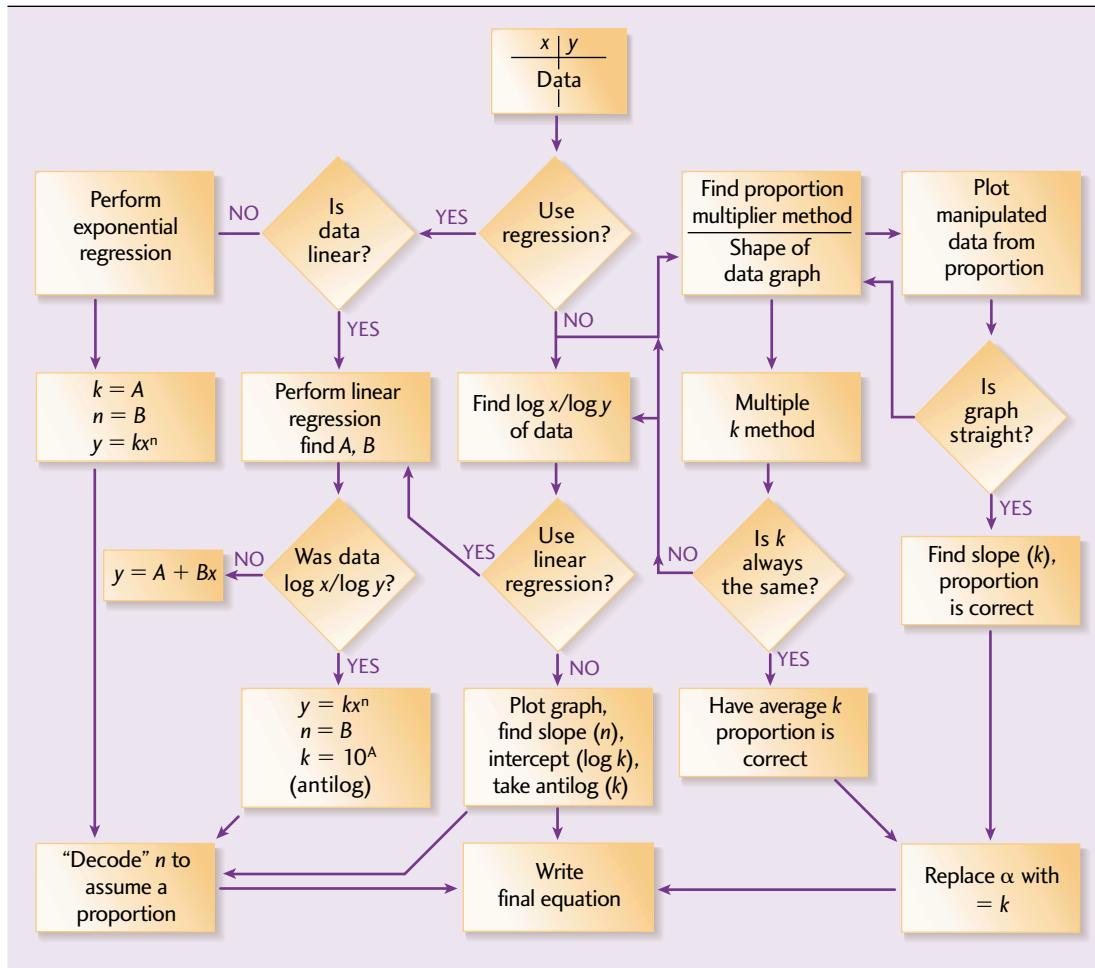
y intercept
(Linear regression)

The slope, y intercept, and correlation coefficient from Table A.8 yield the equation

$$y = 2.5x + 2$$

Figure A.9 summarizes the different methods for deriving an equation that relates a set of experimental data.

Fig.A.9 Data Analysis Methods



APPENDIX E: Helpful Mathematical Equations and Techniques

Mathematical Signs and Symbols

- = equals
- \approx equals approximately
- \sim is the order of magnitude of
- \neq is not equal to
- \equiv is identical to, is defined as
- $>$ is greater than (\gg is much greater than)
- $<$ is less than (\ll is much less than)
- \geq is greater than or equal to (or, is no less than)
- \leq is less than or equal to (or, is no more than)
- \pm plus or minus
- \propto is proportional to
- Σ the sum of
- \bar{x} the average value of x

Significant Figures

The number of significant digits in a value is the number of digits that are known with certainty.

These digits include

- 1) all non zero digits. Example: 1234 m (4 significant digits)
- 2) all embedded zeroes. Example: 1204 m (4 significant digits)
- 3) all trailing zeroes after a decimal. Example: 1.23400 m (6 significant digits)
- 4) any trailing zeroes without a decimal, if known to be measured.
Example: 12 000 m (5 significant digits if specified, otherwise 2 significant digits)

In scientific notation, all the significant digits are included. Thus, point 4 above becomes more obvious. Example: 1.2000×10^4 m if all the zeroes are significant and 1.2×10^4 m if the zeroes are not significant.

When **adding and subtracting** numbers, the answer carries the least number of decimal places used in the addition or subtraction. Example: $1.2 \text{ m} + 1.22 \text{ m} + 1.222 \text{ m} = 3.642 \text{ m}$, but is correctly stated as 3.6 m.

When **multiplying and dividing** numbers, the answer carries the least number of significant digits used in the multiplication or division. Example: $1.2 \text{ m} \times 1.333 \text{ m} = 1.5996 \text{ m}^2$, but is correctly stated as 1.6 m^2 .

The Quadratic Formula

Given: $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If you are solving for Δt , always select the positive square root.

Substitution Method of Solving Equations

In this method, there are two equations and two unknowns. Each equation on its own cannot provide the answer. However, by combining them through a common variable, we can obtain one equation in one unknown.

Given m and d_0 , find f (d_i is also unknown).

The two equations to be used are

a) $m = \frac{-d_i}{d_0}$ and **b)** $\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$

- 1) Rearrange equation **a**): $d_i = -md_0$
- 2) Substitute the expression for d_i into equation **b**) to produce the following equation: $\frac{1}{f} = \frac{1}{d_0} + \frac{1}{(-md_0)}$
- 3) You now have only one unknown, so you can solve for f .

Rearranging Equations

Many times, you find the appropriate equation, but the term to the left of the equal sign is not the one you are looking for. In this case, rearrange the equation and solve for the unknown. A guide to rearranging equations follows.

- 1) Move terms separated by the $+$ and $-$ first. Continue to do so until the term you are solving for is left alone on one side of the equal sign. When a term or group of terms in brackets moves across the equal sign, the sign of the term changes.
- 2) Separate the desired variable from other variables that are attached to it by multiplication and division. To do so, you use the opposite operation to the one that is attaching the desired variable to another one. Then, do the same thing to all the other terms on the other side of the equal sign.

Example: $\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$ Assume you need to solve for the acceleration.

- 1) Move terms first. $\Delta\vec{d} - \vec{v}_1\Delta t = \frac{1}{2}\vec{a}\Delta t^2$
- 2) Separate \vec{a} from $\frac{1}{2}$ and Δt^2 by dividing them out. $\frac{(\Delta\vec{d} - \vec{v}_1\Delta t)}{(\frac{1}{2}\Delta t^2)} = \vec{a}$

Thus, the equation for a reads $\vec{a} = \frac{(\Delta\vec{d} - \vec{v}_1\Delta t)}{(\frac{1}{2}\Delta t^2)}$ or

$$\vec{a} = \frac{2(\Delta\vec{d} - \vec{v}_1\Delta t)}{\Delta t^2}$$

Exponents

Exponents simplify multiplications of numbers.

Example: $10 \times 10 \times 10 \times 10 = 10^4$, which equals 10 000. This number can be written in scientific notation as 1.0×10^4 .

Fractions or decimals are treated the same way.

Example: $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.1 \times 0.1 \times 0.1 \times 0.1 = 10^{-4}$, which equals 0.0001. This number can be written in scientific notation as 1.0×10^{-4} .

When an unknown is being multiplied, the same rules apply.

$A \times A \times A \times A = A^4$ and $\frac{1}{A} \times \frac{1}{A} \times \frac{1}{A} \times \frac{1}{A} = \left(\frac{1}{A}\right)^4$, which can be written as A^{-4}

Multiplication and division rules show that the exponents add and subtract respectively.

$$A^n \times A^p = A^{n+p} \quad \text{Example: } 10^3 \times 10^5 = 10^8$$

$$\frac{A^n}{A^p} = A^{n-p} \quad \text{Example: } \frac{10^3}{10^5} = 10^{-2}. \text{ This equation could also have been written as } 10^3 \times 10^{-5} = 10^{-2}$$

The square root sign $\sqrt{}$ can be written as the exponent $\frac{1}{2}$. Thus, $\sqrt{4}$ can be written as $(4)^{1/2} = 2$.

Analyzing a Graph

The graph in Figure A.10 is based on pairs of measurements of a quantity y (measured in arbitrary units q) with x (in units p). The equation of the line is

$$y = mx + b$$

slope y intercept

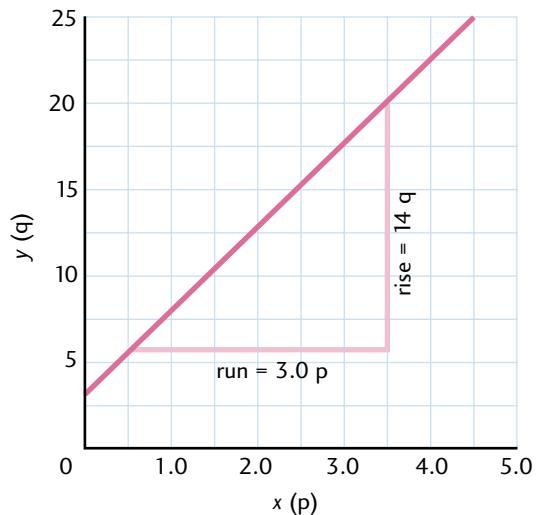
Fig.A.10

The slope of the line is

$$\begin{aligned} \text{slope} &= \frac{\text{rise}}{\text{run}} = \frac{20 \text{ q} - 6 \text{ q}}{3.5 \text{ p} - 0.5 \text{ p}} = \frac{14 \text{ q}}{3.0 \text{ p}} \\ &= 4.7 \text{ q/p} \end{aligned}$$

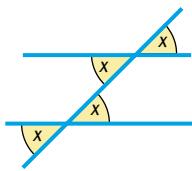
The y intercept of the line is 3 q. The equation of the line is

$$y = \left(4.7 \frac{\text{q}}{\text{p}}\right)x + 3 \text{ q}$$

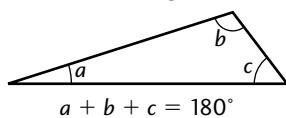


APPENDIX F: Geometry and Trigonometry

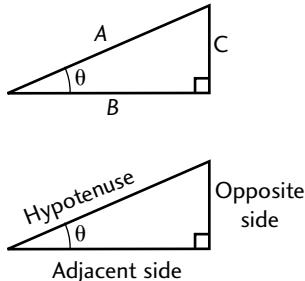
Equal angles



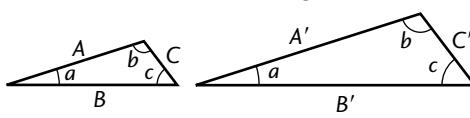
Triangles



90° Triangles



Similar Triangles



1. Angles are equal.
2. Ratios of sides are also equal. Example: $\frac{A}{A'} = \frac{B}{B'} = \frac{C}{C'}$

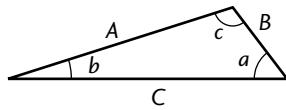
$$A^2 = B^2 + C^2$$

$$\frac{B}{A} = \left(\frac{\text{ADJ}}{\text{HYP}} \right) = \cos \theta$$

$$\frac{C}{A} = \left(\frac{\text{OPP}}{\text{HYP}} \right) = \sin \theta$$

$$\frac{C}{B} = \left(\frac{\text{OPP}}{\text{ADJ}} \right) = \tan \theta$$

Non 90° Triangles



$$\begin{aligned} & \text{Cosine law} \\ & A^2 = B^2 + C^2 - 2BC \cos a \\ & B^2 = C^2 + A^2 - 2CA \cos b \\ & C^2 = A^2 + B^2 - 2AB \cos c \end{aligned}$$

$$\begin{aligned} & \text{Sine law} \\ & \frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C} \end{aligned}$$

Trigonometric Identities

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2}(\alpha \pm \beta) \cos \frac{1}{2}(\alpha \mp \beta)$$

$$\csc^2 \theta - \cot^2 \theta = 1$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

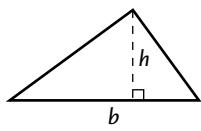
$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$

Lengths, Areas, and Volumes

Area of triangle with base b and altitude h

$$= \frac{hb}{2}$$



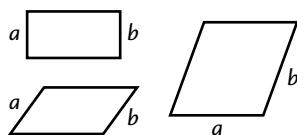
Perimeter of square with side a

$$= 4a$$



Perimeter of any other parallelogram with sides a and b

$$= 2(a + b)$$



Area of rectangle with sides a and b of unequal length

$$= ab$$



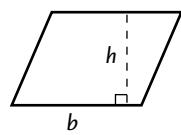
Area of square with side a

$$= a^2$$



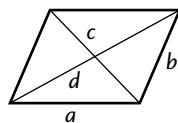
Area of any parallelogram with side b and with h as perpendicular distance from b to side parallel to b

$$= bh$$



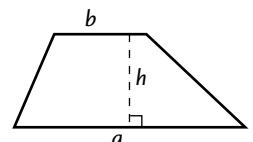
Area of rhombus with diagonals c and d

$$= \frac{cd}{2}$$



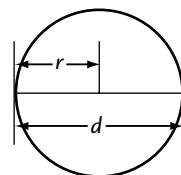
Area of trapezoid with parallel sides a and b and altitude h

$$= \frac{h(a + b)}{2}$$



Circumference of circle with radius r

$$= 2\pi r$$

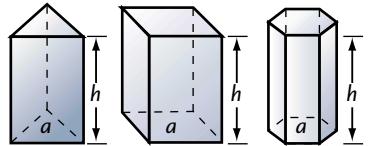


Area of circle with radius r and diameter d ($2r$)

$$= \pi r^2 = \frac{1}{4}\pi d^2$$

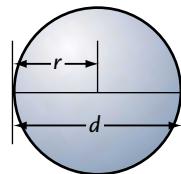
Volume of regular prism with a as area of base and h as altitude

$$= ah$$



Surface of sphere with radius r and diameter d ($2r$)

$$= 4\pi r^2 = \pi d^2$$

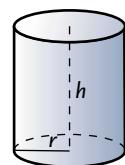


Volume of sphere with radius r and diameter d ($2r$)

$$= \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$$

Volume of right cylinder with r as radius of base and with h as altitude

$$= \pi r^2 h$$



APPENDIX G: SI Units

SI Base Units			
Quantity	Name	Symbol	Definition of Unit
Length	metre	m	Length of 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton 86 atom.
Mass	kilogram	kg	The mass of the international prototype of the kilogram kept at the International Bureau of Weights and Measures.
Time	second	s	Duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.
Electric current	ampere	A	Current in two straight parallel conductors of infinite length and negligible circular cross-section placed 1 m apart in a vacuum that would produce between those conductors a force equal to 0.2 $\mu\text{N}/\text{m}$ of length.
Thermodynamic temperature	kelvin	K	1/273.16 of the thermodynamic temperature of the triple point of water — the equilibrium temperature between pure ice, airfree water, and water vapour ($0.01^\circ\text{C} = 273.16 \text{ K}$).
Amount of substance	mole	mol	Amount of substance of a system containing as many elementary entities as there are atoms in 0.012 kg of carbon 12.
Luminous intensity	candela	cd	Luminous intensity perpendicular from a surface of $1/600\ 000 \text{ m}^2$ of a black body (full radiation) at the temperature of solidifying platinum at a pressure of 101.325 kPa.

SI Derived Units				
Quantity	Name	Symbol	Description	
<i>(a) Units with special names</i>				
Frequency	hertz	Hz	cycle per second	s^{-1}
Force	newton	N	kilogram metre per second squared	$kg \cdot m/s^2$
Pressure and stress	pascal	Pa	newton per square metre	N/m^2
Energy, work, quantity of heat	joule	J	newton metre	$N \cdot m$
Power	watt	W	joule per second	J/s
Electric charge, quantity of electricity	coulomb	C	ampere second	$A \cdot s$
Electric potential difference	volt	V	joule per coulomb	J/C
Electric resistance	ohm	Ω	volt per ampere	V/A
Electric conductance	seimens	S	reciprocal ohm	Ω^{-1}
Flux of magnetic induction, magnetic flux	weber	Wb	volt second	$V \cdot s$
Inductance	henry	H	volt second per amp	$V \cdot s/A$
Activity of radionuclides	becquerel	Bq	emission per second	s^{-1}
Dose equivalence	sievert	Sv	joule per kilogram	J/kg
Absorbed dose of radiation	gray	Gy	joule per kilogram	J/kg
<i>(b) Without special names</i>				
Area			square metre	m^2
Volume			cubic metre	m^3
Speed			metre per second	m/s
Acceleration			metre per second squared	m/s^2
Density			kilogram per cubic metre	kg/m^3
Torque, moment of force			newton metre	$N \cdot m$
Angular velocity			radian per second	rad/s
Angular acceleration			radian per second per second	rad/s^2
Electric field strength			volt per metre	V/m
			newton per coulomb	N/C
Entropy			joule per kelvin	J/K
Specific heat			joule per kilogram kelvin	$J/kg \cdot K$
			joule per kilogram celsius degree	$(J/kg \cdot ^\circ C)$

SI Prefixes			Some Units Permitted for Use with SI			
Multiplying Factor	Name of Prefix	Symbol for Prefix	Quantity	Name	Symbol	Definition
10^{18}	exa	E	Time	minute	min	$1 \text{ min} = 60 \text{ s}$
10^{15}	peta	P		hour	h	$1 \text{ h} = 3600 \text{ s}$
10^{12}	tera	T		day	d	$1 \text{ d} = 86\,400 \text{ s}$
10^9	giga	G		year	a	$1 \text{ a} = 365.24 \text{ d} \text{ (approx.)}$
10^6	mega	M	Volume	litre	L	$1 \text{ L} = 1 \text{ dm}^3 (10^{-3} \text{ m}^3)$
10^3	kilo	k	Temperature	degree Celsius	°C	$0^\circ\text{C} = 273.15 \text{ K}$
10^2	hecto	h				(However, for intervals, $1^\circ\text{C} = 1 \text{ K}$)
10^1	deca	da	Mass	tonne	t	$1 \text{ t} = 1000 \text{ kg}$
10^{-1}	deci	d	Energy	electron volt	eV	$1 \text{ eV} = 0.160\,219 \text{ aJ} \text{ (approx.)}$
10^{-2}	centi	c	Mass of an atom	unified atomic mass unit	u	$1 \text{ u} = 1.660\,565\,5 \times 10^{-27} \text{ kg} \text{ (approx.)}$
10^{-3}	milli	m				
10^{-6}	micro	μ				
10^{-9}	nano	n				
10^{-12}	pico	p				
10^{-15}	femto	f				
10^{-18}	atto	a				

The Greek Alphabet								
Alpha	A	α	Iota	I	ι	Rho	P	ρ
Beta	B	β	Kappa	K	κ	Sigma	Σ	σ
Gamma	Γ	γ	Lambda	Λ	λ	Tau	T	τ
Delta	Δ	δ	Mu	M	μ	Upsilon	Υ	ν
Epsilon	E	ϵ	Nu	N	ν	Phi	Φ	ϕ, φ
Zeta	Z	ζ	Xi	Ξ	ξ	Chi	X	χ
Eta	H	η	Omnicrom	O	\o	Psi	Ψ	ψ
Theta	θ	θ	Pi	Π	π	Omega	Ω	ω

APPENDIX H: Some Physical Properties

Air (dry, at 20°C and 1 atm)

Density	1.21 kg/m^3
Specific heat at constant pressure	$1010 \text{ J/kg}\cdot\text{K}$
Ratio of specific heats	1.40
Speed of sound	343 m/s
Electrical breakdown strength	$3 \times 10^6 \text{ V/m}$
Effective molar mass	0.0289 kg/mol

Water

Density	1000 kg/m^3
Speed of sound	1460 m/s
Specific heat at constant pressure	$4190 \text{ J/kg}\cdot\text{K}$
Heat of fusion (0°C)	333 kJ/kg
Heat of vaporization (100°C)	2260 kJ/kg
Index of refraction ($\lambda = 589 \text{ nm}$)	1.33
Molar mass	0.0180 kg/mol

Earth

Mass	$5.98 \times 10^{24} \text{ kg}$
Mean radius	$6.37 \times 10^6 \text{ m}$
Free-fall acceleration at the Earth's surface	9.8 m/s^2
Standard atmosphere	$1.01 \times 10^5 \text{ Pa}$
Period of satellite at 100-km altitude	86.3 min
Radius of the geosynchronous orbit	42 200 km
Escape speed	11.2 km/s
Magnetic dipole moment	$8.0 \times 10^{22} \text{ A}\cdot\text{m}^2$
Mean electric field at surface	150 V/m, down

Distance to:

Moon	$3.82 \times 10^8 \text{ m}$
Sun	$1.50 \times 10^{11} \text{ m}$
Nearest star	$4.04 \times 10^{16} \text{ m}$
Galactic centre	$2.2 \times 10^{20} \text{ m}$
Andromeda galaxy	$2.1 \times 10^{22} \text{ m}$
Edge of the observable universe	$\sim 10^{26} \text{ m}$

APPENDIX I: The Periodic Table

Periodic Table
(Based on Carbon 12 = 12.0000)

		Transition Elements																		Rare Earth Elements																																									
		I A						II A						III A						IV A						V A						VI A						VII A																							
		6 C		7 N		8 O		9 F		10 Ne		11 Ar		12 Cl		13 Br		14 Kr		15 Xe		16 Rn		17 He		18 Neon																																			
		Atomic number		Symbol		Carbon		Name		Hydrogen		Helium		Boron		Nitrogen		Oxygen		Fluorine		Neon		Boron		Helium		Neon		Hydrogen		Helium		Neon																											
		C		N		O		F		Ne		Ar		Cl		Br		Kr		Xe		Rn		He		He		He		He		He		He		He																									
		Carbon		Nitrogen		Oxygen		Fluorine		Neon		Helium		Boron		Nitrogen		Oxygen		Fluorine		Neon		Helium		Neon		Hydrogen		Helium		Neon		Hydrogen		Helium		Neon																							
		12.0000		14.0071		16.0000		19.0000		20.0000		22.0000		24.0000		26.0000		28.0000		30.0000		32.0000		34.0000		36.0000		38.0000		40.0000		42.0000		44.0000		46.0000		48.0000		50.0000		52.0000		54.0000		56.0000		58.0000		60.0000		62.0000		64.0000		66.0000		68.0000		70.0000	
		12.0000		14.0071		16.0000		19.0000		20.0000		22.0000		24.0000		26.0000		28.0000		30.0000		32.0000		34.0000		36.0000		38.0000		40.0000		42.0000		44.0000		46.0000		48.0000		50.0000		52.0000		54.0000		56.0000		58.0000		60.0000		62.0000		64.0000		66.0000		68.0000		70.0000	
		12.0000		14.0071		16.0000		19.0000		20.0000		22.0000		24.0000		26.0000		28.0000		30.0000		32.0000		34.0000		36.0000		38.0000		40.0000		42.0000		44.0000		46.0000		48.0000		50.0000		52.0000		54.0000		56.0000		58.0000		60.0000		62.0000		64.0000		66.0000		68.0000		70.0000	
		12.0000		14.0071		16.0000		19.0000		20.0000		22.0000		24.0000		26.0000		28.0000		30.0000		32.0000		34.0000		36.0000		38.0000		40.0000		42.0000		44.0000		46.0000		48.0000		50.0000		52.0000		54.0000		56.0000		58.0000		60.0000		62.0000		64.0000		66.0000		68.0000		70.0000	
		12.0000		14.0071		16.0000		19.0000		20.0000		22.0000		24.0000		26.0000		28.0000		30.0000		32.0000		34.0000		36.0000		38.0000		40.0000		42.0000		44.0000		46.0000		48.0000		50.0000		52.0000		54.0000		56.0000		58.0000		60.0000		62.0000		64.0000		66.0000		68.0000		70.0000	
		12.0000		14.0071		16.0000		19.0000		20.0000		22.0000		24.0000		26.0000		28.0000		30.0000		32.0000		34.0000		36.0000		38.0000		40.0000		42.0000		44.0000		46.0000		48.0000		50.0000		52.0000		54.0000		56.0000		58.0000		60.0000		62.0000		64.0000		66.0000		68.0000		70.0000	
		12.0000		14.0071		16.0000		19.0000		20.0000		22.0000		24.0000		26.0000		28.0000		30.0000		32.0000		34.0000		36.0000		38.0000		40.0000		42.0000		44.0000		46.0000		48.0000		50.0000		52.0000		54.0000		56.0000		58.0000		60.0000		62.0000		64.0000		66.0000		68.0000		70.0000	
		12.0000		14.0071		16.0000		19.0000		20.0000		22.0000		24.0000		26.0000		28.0000		30.0000		32.0000		34.0000		36.0000		38.0000		40.0000		42.0000		44.0000		46.0000		48.0000		50.0000		52.0000		54.0000		56.0000		58.0000		60.0000		62.0000		64.0000		66.0000		68.0000		70.0000	
		12.0000		14.0071		16.0000		19.0000		20.0000		22.0000		24.0000		26.0000		28.0000		30.0000		32.0000		34.0000		36.0000		38.0000		40.0000		42.0000		44.0000		46.0000		48.0000		50.0000		52.0000		54.0000		56.0000		58.0000		60.0000		62.0000		64.0000							

APPENDIX J: Some Elementary Particles and Their Properties

Family	Particle	Particle Symbol	Antiparticle Symbol	Rest Energy (MeV)	Lifetime (s)
Photon	Photon	γ	Self*	0	Stable
Lepton	Electron	e^- or β^-	e^+ or β^+	0.511	Stable
	Muon	μ^-	μ^+	105.7	2.2×10^{-6}
	Tau	τ^-	τ^+	1784	10^{-13}
	Electron neutrino	ν_e	$\bar{\nu}_e$	≈ 0	Stable
	Muon neutrino	ν_μ	$\bar{\nu}_\mu$	≈ 0	Stable
	Tau neutrino	ν_τ	$\bar{\nu}_\tau$	≈ 0	Stable
Hadron					
<i>Mesons</i>					
	Pion	π^+	π^-	139.6	2.6×10^{-8}
		π^0	Self*	135.0	0.8×10^{-16}
	Kaon	K^+	K^-	493.7	1.2×10^{-8}
		K_S^0	\bar{K}_S^0	497.7	0.9×10^{-10}
		K_L^0	\bar{K}_L^0	497.7	5.2×10^{-8}
<i>Baryons</i>	Eta	η^0	Self*	548.8	$< 10^{-18}$
	Proton	p	\bar{p}	938.3	Stable
	Neutron	n	\bar{n}	939.6	900
	Lambda	Λ^0	$\bar{\Lambda}^0$	1116	2.6×10^{-10}
	Sigma	Σ^+	$\bar{\Sigma}^-$	1189	0.8×10^{-10}
		Σ^0	$\bar{\Sigma}^0$	1192	6×10^{-20}
		Σ^-	$\bar{\Sigma}^+$	1197	1.5×10^{-10}
<i>*The particle is its own antiparticle.</i>					

Numerical Answers to Applying the Concepts

1.3

1. 2.6×10^6 s
2. 1.4 km
3. 5.5×10^2 ml

1.4

1. 14 m/s [N]
2. a) 1.6 km/h
b) 0.40 km/h [E]
3. a) 1.1 m/s [E]
b) 0

1.6

1. 9.4×10^3 m
2. 1.7×10^{-2} m/s
3. a) 1.8 s
b) 4.4 m
4. -7.7×10^5 m/s²
5. 14 m/s
6. 4.9 s
7. a) 2.7 s
b) 43 m/s

1.7

1. a) 330 m
b) 8.16 s
c) 16.3 s
2. a) 2.1 s
b) 2.9 s
3. 7.2 m/s [up]

1.8

1. a) 2.0 m/s^2 , 0 m/s^2 , 12 m/s^2
b) 455 m
2. a) S.D. Sr.: 6 s, S.D. Jr.: 5 s
b) S.D. Junior by 1 s
c) S.D. Senior by 1.5 s
3. a) 2.5 m/s , 0 m/s , -1.25 m/s , 1.5 m/s
b) 0.65 m/s

1.12

1. a) 5.0 m/s^2
b) 2.5 m/s^2
c) 2.5 m/s^2
2. 270 N
3. -2000 N
4. $F_{\text{engine}} = 4.66 \times 10^3 \text{ N}$,
 $F_f = -1.94 \times 10^3 \text{ N}$
5. -280 N

1.13

3. a) $2.25 \times 10^4 \text{ N}$
b) $1.35 \times 10^4 \text{ N}$, $6.0 \times 10^3 \text{ N}$
4. a) 0.83 m/s^2
b) 350 N

1.14

1. a) $-6.7 \times 10^3 \text{ N}$
b) 0.17 m/s^2
2. b) 1.5 m/s^2
3. 42 m

1.15

1. $5.5 \times 10^{-67} \text{ N}$
2. $2.1 \times 10^{20} \text{ N}$
3. a) $\frac{1}{8}F$
b) $\frac{2}{9}F$
c) F
4. $2.6 \times 10^6 \text{ m}$
5. 24 m/s^2

2.1

2. a) 49 m [S] + 12 m [E]
b) 100 m/s [S] + 173 m/s [W]
c) 12 m/s^2 [N] + 8.4 m/s^2 [E]
3. $v_x = 4.5 \text{ m/s}$, $v_y = -2.1 \text{ m/s}$
4. 5.0 m/s [up 53° forward]
5. a) 26 m/s [78° E]
b) 49 m [18° N]
c) 30.1 N [53° W]
6. 26 m/s [87° W]

2.2

1. a) [76° E]
b) 19 km/h [E]
c) 5.2 h
2. a) [9.6° E]
b) $t_{\text{girl}} = 169 \text{ s}$, $t_{\text{boy}} = 167 \text{ s}$
c) 83 m
d) girl
3. 12 km/h [59° E]
4. a) [76° E]
b) 2.6 s

2.3

1. a) 1.7 s
b) 44 m
2. a) 2.3 s
b) 120 m

3. a)

- 89 m
163 m/s 23° to vertical

4. 66 m/s 17° above horizontal

2.4

1. 1.8 m/s^2 [56° W]
2. a) 15.8 N [80° W]
b) 0.20 m/s^2 [80° W]
3. 104 N [3.3° W]
4. 1.38 m/s^2

2.5

1. 1.6 s
2. 1.2 s
3. 9.8 m
4. (3.36)(mass) N

2.6

1. a) 5.1 m/s^2 [right], 71 N
b) 3.5 m/s^2 [right], 32 N
c) 1.1 m/s^2 [left], $1.8 \times 10^2 \text{ N}$
d) 0.82 m/s^2 [left], $T_1 = 122 \text{ N}$,
 $T_2 = 106 \text{ N}$

2.7

1. 21 m/s^2
2. 8.9 m/s^2
3. a) increases by 4
b) halved
c) doubled
4. a) $2.7 \times 10^{-3} \text{ m/s}^2$
b) toward Earth
5. 0.31 m/s
6. 2724 rotations per day

2.8

1. a) 3.5 m/s
b) 24 N
2. 972 N
3. 3.4 m/s
4. b) 19 m/s
5. 22.8 days
6. $7.57 \times 10^3 \text{ m/s}$
7. $7.4 \times 10^4 \text{ s}$

3.3

1. $F_h = 5.0 \times 10^3 \text{ N}$, $F_v = 8.7 \times 10^3 \text{ N}$
2. 68.4 N
3. b) 0.39 m
c) 0.45 kg

- 4.** **a)** 20.7 N
b) 6.71 N
c) 19.6 N [down]
6. 1.11×10^3 N

3.4

- 1.** **b)** 425 N·m
2. **a)** 1.3×10^3 N
3. **a)** 98.0 N
b) B
c) 1.7×10^2 N·m

3.5

- 1.** 0.332 m
2. **a)** 147 N·m
b) 2.63 m
c) 2.6%
3. **a)** 24.5 N
b) 24.5 N [left], 49 N [up]
4. left: 919 N [up], right: 306 N [up]

3.6

- 1.** 41.6 N·m [clockwise]
2. 5.3×10^2 N
3. **a)** $T = 5.57 \times 10^3$ N,
 $F_h = 5.55 \times 10^3$ N [right],
 $F_v = 1.05 \times 10^3$ N [up]

3.7

- 1.** **a)** 49.8 cm
b) 36.5 cm
2. **a)** 3-wheel: 13.3° , 4-wheel: 31.0°

3.8

- 1.** **a)** 3.0×10^{-2} N
b) 1.11×10^1 m/s²
2. 8.0×10^{-2} m
3. 36.0 N

3.9

- 1.** 1.83×10^{-3} m
2. **a)** 9.8×10^4 N/m²
b) 2.0×10^{-5}
c) 3.0×10^{-4} m
3. **a)** 4.4×10^4 kg

4.2

- 1.** 1.3×10^2 kg·m/s [W 20° N]
2. 4.5×10^3 kg
3. **c)** 38.5 kg·m/s [N]

4.3

- 1.** **a)** 4.2×10^3 N·s [forward]
b) 6.0 N·s
c) 15 N·s [down]

- 2.** 2.4×10^3 N·s [up]
3. **a)** 1.3×10^4 N
b) 3.3 m
4. **a)** 62.5 N·s [S]
b) 1875 N·s [W]
c) 45 N·s [E]

4.4

- 1.** 2.5 m/s [forward]
2. 11.8 m/s [back]
3. 2.0 m/s [forward]
4. 6.9×10^{-23} kg
5. $\frac{5}{9} V$

4.5

- 1.** 4.1 m/s [S 37° W]
2. 8.3 m/s [N 16° E]
3. 1.7 m/s [R 47° D]
4. 10 m/s [S 5° W]

4.6

- 1.** **a)** 1.5 m
b) 17 cm (from the 5 kg ball)
c) 6.7 km (from the larger satellite)
2. **a)** $p_{1_0} = 0.22$ kg·m/s [S 20° E],
 $p_{2_0} = 0.17$ kg·m/s [S 10° W],
 $p_{1_f} = 0.26$ kg·m/s [S 5° W],
 $p_{2_f} = 0.15$ kg·m/s [S 30° E],
 $p_{cm} = 0.39$ kg·m/s [S 8° E]

5.2

- 1.** **a)** 6.0 J
b) 9.6×10^2 J
c) 4.4×10^2 J
2. 1.6×10^5 J
3. 4.5×10^2 J
4. 1.1×10^7 J
5. **a)** 9625 J
b) 0.80 J
6. 16 m

5.3

- 1.** **a)** 5.6×10^{11} J
b) 5.6 m/s, 15.4 J
c) 2.4×10^3 J
2. 5.6 m/s
3. 6.5 kg
4. 4.2×10^{-23} N·s
5. -5.1×10^3 J
6. **a)** 1.1×10^2 J
b) 1.1×10^2 J
c) 4.6×10^3 N

5.4

- 1.** **a)** 4.1×10^1 J

- b)** 0 J
c) 3.7×10^4 J
2. **a)** 23 m/s
b) 23 m/s
3. **a)** 60 m/s
b) 1.8×10^2 m
c) $E_k = 2.4 \times 10^3$ J,
 $E_p = 3.0 \times 10^3$ J
4. 3.0×10^5 N/m

5.5

- 1.** **a)** 2.0×10^2 N/m
b) 1.0 J
c) 7.0×10^{-2} J
2. 2.45 N/m
3. **a)** 4.4×10^{-2} J
b) 2.7×10^{-1} J
4. 8.0 m/s
5. 9 cm
6. 0.49 m

5.6

- 1.** 6.9×10^4 J
2. **a)** 590 W
b) 10 600 J
3. 2×10^7 W
4. 4.6×10^5 J
5. 5.7
6. **a)** 30 m/s [W]
b) 4.5×10^5 J
7. **a)** $p = 16.5$ kg·m/s, $E_k = 270$ J;
 $p = 0$, $E_k = 0$
b) -12 m/s
c) 36 J, 3.4 J
8. **a)** $v_{1_f} = -3.3$ m/s, $v_{2_f} = 1.7$ m/s
b) $v_{1_f} = -68.8$ cm/s, $v_{2_f} = 15.2$ cm/s
9. **a)** 1.0 J
b) 0.425 J
c) ≈ 28 J
d) ≈ 10 J
e) 64%

6.1

- 1.** **a)** 2.64×10^{33} J
b) -5.26×10^{33} J
c) -2.63×10^{33} J
2. 7.323 m/s²
3. **b)** 4.7×10^6 m

6.2

- 1.** **a)** 2.7×10^{11} m
b) 0.97
c) 55 000 m/s
2. 56 000 m/s

- 3.** **a)** 423 m/s
b) 3.84×10^{28} J
5. 297.2 days

6.3

- 1.** **a)** 0.872 85 J
b) 0
c) 1.9 m/s
2. **a)** 51.83 s
b) 1.32 m/s
c) 0.4224 J

7.2

- 1.** **a)** 0.17 rad
b) 1.0 rad
c) 1.6 rad
d) 3.07 rad
e) 4.47 rad
2. **a)** 180°
b) 45°
c) 675°
d) 639°
e) 2.3×10^{30} °
3. **a)** 1.57 rad
b) 4.56 rad
c) 2.62 rad
d) 161 rad

7.3

- 2.** **a)** 1.1×10^2 m/s
b) 5.0×10^{-5} rad/s²,
 0.090 rad/s
3. **a)** 0.13 rad/s
b) 24 m/s²
c) 0
d) 2.4

7.4

- 1.** **a)** 3.58×10^3 rad
b) 44 rad/s²
2. **a)** 8.3 s
b) 7.3 rad
c) 1.2 cycles
d) 5.9 s
3. **a)** 6.03 s
b) -0.266 rad/s²

7.5

- 2.** **a)** -0.086 N·m
b) -2.8 turns, -0.93 turns,
 -0.51 turns
3. 0.693 kg·m²
4. **a)** 4.13 kg·m²
b) 18.4 N·m
c) 4.46 rad/s²

7.6

- 1.** **a)** 29 J
b) 6.9 J
c) 7.7 J
2. **a)** 5.3 J
b) 0

7.7

- 1.** 0.23 J
2. 2.0×10^3 J

7.8

- 1.** **a)** 1.1×10^4 J
b) 4.1×10^5 J
c) 4.2×10^5 J
2. **a)** 2.6×10^2 J
b) 10.8 m/s
c) 1.9×10^2 rad/s

7.9

- 1.** 1.94×10^{31} kg·m²/s
2. 7.1×10^2 kg·m²/s
3. 1.003×10^{42} kg·m²/s,
 1.003×10^{42} kg·m²/s

7.10

- 2.** 4.69×10^4 rad/s, 1.34×10^{-4} s
3. 29.3 km/s

7.11

- 3.** 0.56 m/s²

8.4

- 1.** 49 N
2. 3.5×10^{-2} m
3. **c)** 56°

8.6

- 1.** **a)** -1.7 N [right]
b) 3.4 N [right]
2. -1.7 N [left]
3. **b)** 6.8×10^7 N/C, 1.7×10^7 N/C,
 7.5×10^6 N/C
c) decreases $\frac{1}{4}$, decreases $\frac{1}{9}$
e) 4.2 N [right]
4. **a)** 3.7×10^6 N/C [left],
 $0, 3.2 \times 10^6$ N/C [left]

8.7

- 1.** **a)** -6.8×10^{-1} J
b) -4.5×10^5 V
c) -4.5×10^5 V
2. **a)** $q_1: 2.0 \times 10^{-8}$ J, $q_2: 5.0 \times 10^{-9}$ J
b) 2

8.8

- 1.** 3.0×10^{-14} m
3. 6.0 m/s [left]
4. **a)** 3.8×10^5 m/s
b) 2.7×10^5 m/s
5. **a)** 3.2×10^{-15} J
b) 8.4×10^7 m/s

8.9

- 1.** 3.7×10^2 V
2. 4.7×10^4 N/C
3. 4.8×10^{-19} C

9.5

- 1.** 0.90 N
2. 18 A
3. **a)** 7.1×10^{-5} T
4. 2.4×10^{-2} A
5. **a)** 0.66 m
b) 4.7×10^{-1} m [S], 4.7×10^{-1} m
 below wire
6. **a)** 1.4×10^{-2} N/m
7. 0.36 N
8. 1.0×10^{-14} N [into page]

10.2

- 1.** **a)** 75 min
b) 0.67 s
c) 1.80 s
d) 0.838 s
2. **a)** 60 Hz
b) 0.75 Hz
c) 0.009 23 Hz
d) 1.35 Hz
3. **a)** **i)** 2.22×10^{-4} Hz
ii) 1.49 Hz
iii) 0.556 Hz
iv) 1.19 Hz
b) **i)** 0.0167 s
ii) 1.33 s
iii) 108 s
iv) 0.74 s
5. **a)** 26 cm
b) -30 cm
c) 0 cm
d) 30 cm
e) 21 cm

10.3

- 4.** **a)** 4.7×10^{14} Hz
b) 2.5×10^8 Hz
c) 1.5×10^{17} Hz
5. **a)** 2.0×10^{-5} m
b) 0.15 m
c) 1.0×10^{-14} m

10.4

4. a) 2.3×10^8 m/s
b) 1.24×10^8 m/s
c) 2.0×10^8 m/s
5. a) 1.43
b) 2.0
c) 1.27
6. a) 18°
b) 10°
c) 16°

10.5

5. a) 1.81×10^8 m/s,
 2.02×10^8 m/s
b) 11.6%

11.4

2. 713 nm
3. 20 cm
4. 42 cm

11.5

2. $1.8 \mu\text{m}$

11.6

2. $6.9 \mu\text{m}$
3. 55

11.8

1. a) 11.5°
b) 22 cm
2. a) 22 cm
b) 11.5°
3. 11 cm
6. a) 197 nm
b) 5 km

11.9

1. 12° , 24° , 38°
2. a) 4
b) 4
c) 5
3. a) $8.40 \mu\text{m}$
b) 2334 slits

11.10

1. 3000 lines/cm
3. $\theta_{\text{red}} = 22.7^\circ$, $\theta_{\text{violet}} = 12.2^\circ$,
 $\theta_{\text{green}} = 15.6^\circ$
5. 52 pm
6. 168° , 192°

12.2

1. a) 2.4×10^{-7} m
2. a) 3.2×10^{-6} m

12.3

1. a) 8×10^{-34} J·s, 2.9 eV
3. 5.79×10^{-19} J

12.4

2. a) 4.53×10^{-26} N·s
b) 3.1×10^{-27} N·s
c) 1.27×10^{-17} J
d) 5.27×10^6 m/s

12.5

1. 7.27×10^{-7} m

12.6

2. 3.05×10^{-7} m
3. Lyman: 10.2 eV, 13.6 eV;
Balmer: 1.89 eV, 3.4 eV;
Paschen: 0.66 eV, 1.51 eV

12.8

1. 6.3×10^{-2} m
6. 1.32×10^{-13} m

13.2

1. 1.5×10^8 m/s
2. $1.1c$ [R]
3. 5.93×10^8 m/s

13.3

1. 189 m
2. 2×10^{-8} s (Phillip),
 2.5×10^{-8} s (Barb)
3. 49.9 bpm
4. 2.60×10^8 m/s
6. 2.95×10^8 m/s

13.4

1. $0.7c$
2. $0.8c$
3. 10.59 a
6. 3.97×10^8 m

13.5

1. $5.980\ 000\ 03 \times 10^{24}$ kg
3. 3.33×10^{-14} kg
7. 6.98×10^5 m

13.6

2. c
4. 1.76×10^{10} ca

13.7

2. B
3. 2.8×10^{-5} g
4. 5.85×10^{18} J

13.8

1. 1.88×10^{-28} kg
2. $939.4 \text{ MeV}/c^2$
3. 2.96×10^8 m/s
4. 9.38×10^{-6} m/s

14.1

3. a) 2.23 MeV
b) 1.12 MeV/nucleon
4. 35.48 u

14.2

2. a) $^{234}_{90}\text{Th}$
b) $^{244}_{94}\text{Pu}$
c) $^{219}_{84}\text{Po}$
d) $^{240}_{92}\text{U}$
e) $^{60}_{27}\text{Co}$
3. a) $^{32}_{16}\text{S}$
b) $^{23}_{11}\text{Na}$
c) $^{35}_{17}\text{Cl}$
d) $^{45}_{21}\text{Sc}$
e) $^{64}_{30}\text{Zn}$
4. a) $^{19}_{9}\text{F}$
b) $^{22}_{10}\text{Na}$
c) $^{46}_{23}\text{V}$
d) $^{239}_{92}\text{U}$
e) $^{64}_{28}\text{Ni}$

14.3

1. $\frac{1}{256}$
2. 2.97×10^9 a
3. 1.7×10^9 a

14.4

1. Bi
2. 1800 doses
4. 191 mSv

14.5

2. 50% effective
4. 7.87 GW
5. 2

14.6

1. 4.0×10^{-16} m
2. 11 kHz
3. a) 4.35×10^7 m/s
b) 0.216 m

14.7

2. a) 1
b) -1
c) 1
d) 0
e) -1
5. $57.1 \text{ MeV}/c^2$

Numerical Answers to End-of-chapter Problems

Chapter 1

- 16.** **a)** 200 m
b) 0 m
- 17.** **a)** 23 m
b) 11 m [E]
- 18.** 32 ft/s²
- 19.** **a)** 18.5 km/h
b) 5.14 m/s
- 20.** 9.5×10^{17} cm
- 21.** 6.5 m/s, 7.1 m/s
- 22.** **a)** 2.1×10^{-3} m/s
b) 2.1×10^{-3} m/s [left]
- 23.** **a)** 5.3 s
b) 17 s
- 24.** 5.0 m/s
- 25.** 6.6 s
- 26.** 400 m/s² [E]
- 27.** 9.5 s
- 28.** **a)** 6.0 s
b) 50 m
c) 12.0 s
- 29.** **a)** 107
- 30.** **a)** 3.7 m
- 31.** 3.7 m
- 32.** **a)** 9.8 m/s² [down]
c) 2.3 m
- 33.** 1.4 s
- 34.** $\frac{h}{v_i}$
- 35.** **a)** B, C, D
b) A
c) 5 m/s, 0 m/s, -10 m/s
d) 9.1 m/s
e) 0 m/s
g) 30 m
- 36.** **a)** 1 m/s², 2 m/s², -2.0 m/s²
c) 73 m
- 37.** **a)** 0–5 s
b) 5–10 s
d) 5 s
e) -10 m/s²
- 38.** **a)** Curly: 0 m/s², Larry: 2.5 m/s², Moe: 5.0 m/s²
b) Curly: 100 m, Larry: 20 m, Moe: 40 m
c) Moe
- 44.** 16.4g
- 45.** 6.2×10^4 N
- 40.** 0.25 s
- 47.** 3.4 m/s²
- 48.** 4.7 N, -4.7 N
- 49.** 9800 N

- 50.** **a)** 39.2 m/s²
b) 6.1×10^3 N, 2.9×10^3 N
- 51.** -3.1 N
- 52.** 68 cm
- 53.** 4.2×10^3 N
- 54.** 6.0×10^{-6} N, 2.0×10^{-11} m/s²
- 55.** -19.6 m/s²
- 56.** 6.16×10^{17} N
- 57.** 894 N

Chapter 2

- 14.** **a)** 8.6 km [N] + 23 km [E]
b) 8.7 N [S] + 5 N [E]
c) 21 m/s² [S] + 21 m/s² [W]
d) 42 kg·m/s [N] + 2.2 kg·m/s [W]
- 15.** **a)** 7.7 m
b) 6.4 m
- 16.** $a_x = 3.3 \text{ m/s}^2$, $a_y = -2.3 \text{ m/s}^2$
- 17.** 4.9 km [W12°N]
- 18.** 22 m/s, 63° to horizontal
- 19.** 83 cm [S49°W]
- 20.** 56 m/s [N15°W]
- 21.** 33 m/s² [N2°W]
- 22.** **a)** 0.44 h
b) 0.22 km
c) 1.9 km/h [N16°E]
- 23.** **a)** [N16°W]
b) 1.7 km/h [N]
c) 0.46 h
- 24.** 83 m
- 25.** [E7.7°N]
- 26.** [N38°E]
- 27.** **a)** toward stern: $v = 0.5 \text{ m/s}$ [S]; toward port: $v = 0.5 \text{ m/s}$ [W]
b) toward stern: $v = 2.3 \text{ m/s}$ [N]; toward port: $v = 2.8 \text{ m/s}$ [N10°W]
- 28.** **a)** 4.8 m
b) 1.2 s
c) 6.4 m/s [E51°N]
- 29.** **a)** [W37°N]
b) 3.3 s
c) 3.0 m/s [N]
- 30.** 4.2 m
- 31.** **a)** 19.6 m
b) 28 m/s, 44.4° below horizontal
- 32.** 95 m
- 33.** **a)** 0.52 s
b) At tourist's feet
c) 26 m

- 34.** 59 m
- 36.** 36 m/s, 45° above horizontal
- 37.** **a)** 32 N [N72°E]
b) 51 N [S49°W]
c) 22 N [S42°E]
- 38.** **a)** 106 N [S8.5°E]
b) 0
- 39.** 1.4 m/s²
- 40.** 229 m/s [N26°E]
- 41.** **a)** 4.9×10^2 N
b) 6.4 m/s
- 42.** 0.68 m
- 43.** 9.6 kg
- 44.** 19°
- 45.** **a)** 4.9 m/s²
- 46.** **a)** 0.14 m/s²
b) 7.6 m/s
c) 53 s
- 47.** 57 m
- 48.** 17 s
- 49.** **a)** 4.9 m/s^2 , 98 N
b) 3.9 m/s² [right]; 137 N, 176 N
c) 4.2 m/s² [right], 84 N

- 50.** 3.8 m/s²
- 51.** 0.80
- 52.** 2.4 s
- 54.** **a)** 78 m/s²
- 55.** 6.0×10^{-3} m/s²
- 56.** 21 m/s
- 57.** 19 m/s
- 58.** 9.9 m/s
- 59.** **a)** 4.9 N
b) 9.7 N
- 60.** 49 N, 9.4 N
- 61.** **a)** 5.9×10^3 N
b) 95 m
- 62.** **a)** 2.0×10^{30} kg
b) $\rho_S = \frac{1}{4}\rho_E$

Chapter 3

- 21.** 20 N
- 22.** 17 N
- 23.** 566 N
- 24.** 128 kg
- 25.** 5.01×10^3 N, 1.04×10^3 N
- 26.** **a)** 617.4 N
b) 2.4 m
- 27.** 3.56×10^3 N
- 28.** 1.1 kg
- 29.** 75 N [left]

- 31.** **a)** 0.5 m from m_1 , 1.5 m from m_2
b) 39.2 N
- 32.** $F_1 = 1.1 \times 10^3$ N [down],
 $F_2 = 1.6 \times 10^3$ N [up]
- 33.** 0.75 m [right], 1.25 m [up]
- 34.** 1.25 m
- 35.** 3.3×10^2 kg
- 36.** 0.95 m from centre on
 17.0-kg side
- 37.** 29.4 N, 39.2 N
- 38.** Front legs: 1.05×10^2 N each,
 back legs: 4.4×10^1 N each
- 39.** **a)** 196 N [up]
b) 34.2 N [out horizontally]
- 40.** 2.7×10^2 N
- 41.** **a)** 3.1×10^2 N
b) 1.2 m
- 42.** 7.8×10^2 N [up]
- 43.** 0.29 m
- 44.** 1.9×10^3 N [up],
 2.5×10^3 N [down]
- 45.** 0.75 N, 0.25 N
- 46.** 9.5×10^2 N
- 47.** 26°
- 48.** 1.73 m
- 49.** 5.2 cm (21.8°)
- 50.** 26.6°
- 51.** 1.6×10^3 N/m
- 52.** 1.88×10^4 N/m
- 53.** 25.4 kg
- 54.** **a)** 7.5×10^2 N
b) 1.7×10^{-2} m
- 55.** **a)** 9.8×10^{-8}
b) 2.0×10^{-7} m
c) 1.7×10^6 kg
- 56.** 8.32×10^4 N
- 57.** 3.95×10^7 N/m
- 58.** 7.1×10^8 N·m
- 59.** **a)** 2.5×10^{-2} m
b) 3.01×10^{-4}
- 60.** **a)** Stress: 6.67×10^5 N/m²,
 strain: 6.67×10^{-5}
b) 2.0×10^{-4} m
- 61.** 22.000 0775 m

Chapter 4

- 16.** 9.0×10^5 kg·m/s
- 17.** 7.5×10^{-2} kg·m/s
- 18.** 6.3×10^{-1} kg·m/s
- 19.** 165.6 kg (glider)
- 20.** 6.0×10^{26} m/s
- 23.** 15 m/s
- 24.** **a)** 1.2×10^3 kg·m/s
b) 1.2×10^3 kg·m/s

- 25.** **a)** 1.86 s
b) 14.7 N
c) $27.3 \text{ kg}\cdot\text{m/s}$
- 26.** **a)** $66.5 \text{ kg}\cdot\text{m/s}$
b) $66.5 \text{ kg}\cdot\text{m/s}$
- 27.** 9 kg·m/s
- 30.** **a)** 7.5×10^2 kg·m/s
b) 3.7×10^{-2} s
- 31.** **a)** -5.3×10^5 N
b) -2.7×10^4 N
- 32.** **a)** 1.1×10^1 kg·m/s
b) -1.3×10^7 m/s²
c) -3.9×10^5 N
d) 2.8×10^{-5} s
e) -1.1×10^1 kg·m/s
- 33.** **b)** 6.0×10^7 N·s
- 34.** 24.75 N·s [forward]
- 35.** 1.4×10^3 N·s
- 36.** 5.6×10^3 m/s
- 37.** 2.5 m/s [S]
- 38.** 4.8 m/s
- 39.** 1.5 m/s
- 40.** 0.33 m/s
- 41.** 0 m/s
- 42.** 4.8×10^4 kg
- 44.** 4.4×10^6 m/s
- 45.** $\frac{3}{7}V$
- 46.** 2×10^3 s
- 47.** **b)** 763 kg·m/s [E24.7°N]
- 48.** 17 m/s [N1.4°W]
- 49.** 35 m/s [E]
- 50.** 6.7×10^{-25} kg, 1.7×10^7 m/s
 [S32°W]
- 51.** 3.3°
- 52.** 1.058×10^3 kg
- 53.** 5.63 m/s [U40°R]
- 54.** 7.7 m/s [R20°U]
- 55.** **a)** $v_{1_0} = 23$ mm/s, $v_{2_0} = 0$,
 $v_{1_f} = v_{2_f} = 23$ mm/s
b) $v_{1_0} = 23$ mm/s [E], $v_{2_0} = 0$,
 $v_{1_f} = 23$ mm/s [E45°S],
 $v_{2_f} = 23$ mm/s [E45°N]
c) $p_{T_0} = 0.0069$ N·s [E],
 $p_{T_f} = 0.0098$ N·s [E]
d) $p_{1_{oh}} = +0.0069$ N·s,
 $p_{1_{ov}} = 0$, $p_{2_{oh}} = 0$, $p_{2_{ov}} = 0$,
 $p_{1_{fh}} = p_{2_{fh}} = p_{2_{fv}} = +0.0049$ N·s,
 $p_{1_{fv}} = -0.0049$ N·s
e) 0.0098 N·s [E]
- 56.** 24.1 m/s [S26.6°W]
- 57.** **a)** 15 000 kg
b) 133 m away from the
 larger mass
- 59.** 0.0069 N·s [E], 0.0098 N·s [E]

Chapter 5

- 11.** **a)** 2.0×10^4 J
b) 46 J
c) 2.7×10^{-18} J
- 12.** **a)** 2.7×10^3 J
b) 2.5×10^3 J
c) 9.1×10^2 J
- 13.** 18 m, 36°
- 14.** 1.4×10^8 J
- 15.** 2100 J
- 16.** 0 J
- 18.** **a)** 3.4×10^2 N
b) 5.8×10^2 N
c) 1.2×10^2 N
- 19.** 5.4×10^4 J
- 20.** **a)** 8.5×10^2 J
c) 3.8 m/s
- 21.** **a)** 2.3×10^3 J
b) 3.9×10^{-4} J
c) 5.8×10^6 J
- 22.** 1.4 $\times 10^2$ kg
- 23.** 3.0×10^4 m/s
- 24.** 2.9×10^4 J
- 26.** 14%
- 27.** **a)** -2.8×10^5 N
b) 2.8×10^5 N
- 28.** 1 m, 50 J, 8 m/s; 2 m, 225 J,
 17 m/s; 3 m, 425 J, 24 m/s
- 29.** 55 N·s
- 30.** **a)** 5 m/s
b) 12.5 J
d) 4.2 N
- 31.** **a)** 38 J
b) 1.5 J
c) 9.2×10^5 J
d) 0 J
- 32.** **a)** 4.5×10^2 kg
b) 1.5×10^4 J
- 33.** 7.6 m/s
- 34.** 20 cm
- 35.** 4
- 36.** **a)** A, F
b) 38 m/s
c) 19 m/s
d) 1.4×10^5 N
- 37.** 17 m
- 38.** 5.8 cm
- 39.** 1.7 m
- 40.** 1.1×10^3 m
- 41.** 5.3×10^2 N/m
- 42.** **a)** 50 J
b) 1.4×10^2 J
- 43.** 2.7 m/s
- 44.** 1.8×10^4 N/m

- 45.** 34 m/s
46. a) 0.77 m/s
b) 30 cm
47. 1.1 m/s
48. 6.0×10^2 N
49. 1.4×10^3 N/m
50. 2.3×10^2 m
51. 1.7×10^7 J, 4.8 kWh
52. a) 4.3×10^4 W
b) 58 hp
54. a) 1.6×10^5 W
55. 511 W
57. a) 2 m/s, 5 m/s
b) 38 J
58. 5.8 m/s, 26 m/s
59. a) 7.0 kg·m/s, 7.7 J
b) 1.1 m/s
c) 3.8 J
60. 0.45 m/s
61. 5 m/s [W], 3 m/s [E]
62. a) 2.5 m/s
b) 7.5 m/s
63. a) 52 m/s
64. a) 1.7 m/s
b) 70 m/s

Chapter 6

- 13.** 7.9968×10^{11} J
14. a) 776.4 km
b) 5.75×10^{10} J
c) 1.11×10^4 m/s
15. a) 1.66×10^{10} J
b) 1.66×10^{10} J
16. 1.1×10^4 m/s
17. 8.92×10^{-3} m
18. 1.91×10^8 m from Earth's centre
19. 5.87×10^7 J/kg
20. 7671 m/s, 5552 s (92.5 min)
21. 35 872 km
22. 1.48×10^{10} J
23. b) $T \propto r^{\frac{3}{2}}$
24. -3.84×10^{28} J
25. 2.5×10^4 m/s
26. 2.31×10^3 m/s
27. 7086 s or 1 h 58 min
28. a) 24 000 m/s
b) 3500 m/s
29. 2370 m/s
30. 0.25 Hz
31. 0.87 s
32. a) 2.93 J
b) 1.71 m/s
c) 1.27 m/s
33. a) 9.75 m/s^2
b) 6.5 m/s^2

- 34. a)** 6.53 kW
35. 5.7×10^8 N/m
36. 0.011 J
37. 3.3×10^{-4} m
38. a) 8.39 cm
b) 6.96 cm
c) 1.08 cm
d) 3.41×10^{-10} cm
e) 0 cm
39. 5.2 s
40. a) 5.2 s
b) i) 0.357 J
ii) 0.0186 J
iii) 7.828×10^{-10} J
iv) 0

Chapter 7

- 17. a)** 0.0175 rad
b) $\frac{\pi}{2}$ rad
c) 3.84 rad
d) 8.01 rad
e) 20.9 rad
18. a) 96.1 rad
b) $\frac{3\pi}{2}$ rad
c) 2.3 rad
d) 7.46 rad
19. a) 0°
b) 120°
c) 3600°
d) $2.67 \times 10^{4^\circ}$
20. a) 0.56 cycles
b) $\frac{1}{2}$ cycle
c) 0.14 cycles
d) 1.25 cycles
21. a) 80π m
b) 268π m
c) 86 m
d) 3.9×10^2 m
22. a) 30π rad
b) 27 rad/s
23. 0.97 rad/s
24. a) 178.0 rad/s
b) 1.0×10^2 rad
25. a) 0.0222 rad/s²
b) 0.406 Hz
26. -0.21 rad/s^2
27. a) -0.818 rad/s^2
b) 198 rad
c) 31.5 cycles
d) 11 rad/s
28. a) -0.92 rad/s
b) $-3.0 \times 10^{-3} \text{ rad/s}$
c) 12 rad/s
29. 4.3 rad/s
30. 2.4×10^2 m/s²
- 31. a)** 99.9 m/s
b) 0
c) 1.10×10^3 rev
d) 2.70×10^5 m
32. a) 2.6×10^2 rad/s
b) 2.1×10^2 m/s
33. 1.2 s
34. 0.93 s
35. a) 9.2 rad/s
b) 19 rad
36. a) -42 rad/s^2
b) 3.5×10^2 rad
c) $2.0 \times 10^{4^\circ}$
d) 4.5 s
37. a) 25 rad/s
b) 38 rad/s
38. a) 20π rad
b) 63 rad/s
c) 17 rad/s^2
39. a) 1.4 s
b) 1.5×10^4 rad/s²
40. 2.3 s
41. a) 38 rad
b) 7.2 rad/s
42. 5.63×10^6 s
43. 3.56 s
44. c, a, b
45. a, b and c, e, d
46. 189 kg·m²
47. a) $0.15 \text{ kg}\cdot\text{m}^2$
b) $0.077 \text{ kg}\cdot\text{m}^2$
c) $0.088 \text{ kg}\cdot\text{m}^2$
d) $0.44 \text{ kg}\cdot\text{m}^2$
48. a) $0.010 \text{ kg}\cdot\text{m}^2$
b) 377 rad/s
49. 1.08 kg·m²
50. a) 3.0 kg·m²
b) 20.9 rad/s
c) 1.8 kg·m²
51. -4.8×10^4 J
52. a) 330 kg·m²
b) 3.24×10^2 J
c) 0.945 m/s
d) 1.4
53. a) 1.92×10^{24} J
b) 1.27 m/s
54. a) 1.0×10^{-23} kg·m²
b) 6.3×10^3 rad/s
c) 2.0×10^{-16} J
55. a) 2.3×10^{-51} kg·m²
b) 4.6×10^{16} rad/s
c) 2.4×10^{-18} J
56. a) 5.7 m/s
b) 29 rad/s

- 57.** **a)** 4.9 m/s
b) 25 rad/s
- 59.** 6.4 m/s
- 60.** **a)** $0.0264 \text{ kg} \cdot \text{m}^2$
b) $4.0 \text{ kg} \cdot \text{m}^2/\text{s}$
- 61.** **a)** $0.108 \text{ kg} \cdot \text{m}^2$
b) 250 rad/s
c) $27 \text{ kg} \cdot \text{m}^2/\text{s}$
d) 71.4 rad/s^2
e) 7.7 N·m
- 62.** **a)** 0.26 s
b) 71 rad/s
c) $0.11 \text{ kg} \cdot \text{m}^2/\text{s}$
- 63.** **a)** $1.6 \times 10^{-4} \text{ kg} \cdot \text{m}^2$
b) $2.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2/\text{s}$
- 64.** **a)** $1.7 \text{ kg} \cdot \text{m}^2$
b) $22 \text{ kg} \cdot \text{m}^2/\text{s}$
- 65.** 3.5 kg·m²
- 66.** 1.56
- 67.** increases by 4
- 68.** **a)** $5.7 \times 10^4 \text{ rad/s}$
b) 10 rad
c) $5.7 \times 10^4 \text{ rad}$
d) $7.1 \times 10^2 \text{ rotations}$
- 69.** **a)** 5.7 rad/s
b) 6.0 rad/s
c) -0.96 rad/s
- 70.** **a)** -9.2 rad/s
b) -12 rad/s
c) -6.5 rad/s
d) 40 rad/s
- 71.** 2.6 rad/s
- 73.** **a)** 0.138 m/s^2
b) 3.99 s
c) 0.551 m/s
d) 184 rad/s
e) 0.0205 J
f) 1.43 J
g) 1.46 J
- 74.** **a)** 0.138 m/s^2
b) 1.03 s
c) 1.14 m/s
d) 380 rad/s
e) 0.0880 J
f) 6.16 J
g) 6.24 J
- 36.** **a)** –
b) e^-
- 37.** **a)** glass +, silk –
- 39.** +
- 40.** **a)** +
- 41.** 9.38×10^{19}
- 42.** 6.9×10^{12}
- 43.** $6.4 \times 10^{-8} \text{ C}$
- 44.** $-4.3 \times 10^{-11} \text{ C}$
- 45.** $1.5 \times 10^7 \text{ electrons}$
- 46.** **a)** $\frac{1}{16}$
b) 4 times
c) $\frac{1}{4}$
- 47.** $\frac{1}{2}r$
- 48.** $2.3 \times 10^{-8} \text{ N}$
- 49.** **a)** $3.00 \times 10^{-8} \text{ C}$
b) $4.5 \times 10^{-8} \text{ C}$
- 50.** $\frac{-1}{3}$
- 51.** **a)** 5.1 m
- 52.** **a)** 3.3 N [right]
b) 7.4 N [right]
c) 12 N [left]
d) 0.29 or 0.15 m [left of left charge]
- 54.** $8.9 \times 10^2 \text{ N}$ [90° away from line connecting other charges]
- 55.** **a)** 43.1 N [out from centre of square]
b) 0 N
- 59.** $1.8 \times 10^5 \text{ N/C}$
- 60.** $2.2 \times 10^{-2} \text{ C}$
- 61.** $3.6 \times 10^4 \text{ N/C}$ toward smaller charge
- 62.** **a)** $3.8 \times 10^6 \text{ N/C}$ [left]
b) $-1.86 \times 10^2 \text{ N}$
- 63.** $3.6 \times 10^8 \text{ N/C}$ [left]
- 64.** $3.25 \times 10^5 \text{ N/C}$ [right]
- 65.** $5.1 \times 10^{11} \text{ N/C}$
- 66.** $1.2 \times 10^{-1} \text{ m}$
[from larger charge]
- 67.** 0 N/C
- 68.** $1.1 \times 10^6 \text{ N/C}$ [90° from line connecting other charges]
- 69.** 6.0 J
- 70.** $1.2 \times 10^2 \text{ C}$
- 71.** $2.3 \times 10^4 \text{ V}$
- 72.** 2.3 J
- 73.** $1.9 \times 10^5 \text{ V}$
- 74.** **a)** 0.18 J
b) 0.14 J
- 75.** $2.5 \times 10^2 \text{ V}$
- 76.** **a)** $5.0 \times 10^{-4} \text{ N}$
b) $5.0 \times 10^4 \text{ J}$
c) $1.6 \times 10^{-4} \text{ kg}$
- 77.** $4.78 \times 10^5 \text{ m/s}$
- 78.** 1.41 times faster
- 79.** **a)** 1.1×10^{16}
b) $7.3 \times 10^7 \text{ m/s}$
- 80.** **a)** $3.0 \times 10^{10} \text{ m/s}^2$
b) $1.202 \times 10^{-15} \text{ J}$
- 81.** $1.9 \times 10^{-14} \text{ m}$
- 82.** **a)** 2.5 cm
b) $6.0 \times 10^5 \text{ m/s}$
- 83.** $7.80 \times 10^2 \text{ N/C}$
- 84.** $1.81 \times 10^3 \text{ V}$
- 85.** **a)** $2.04 \times 10^{-7} \text{ N/C}$
b) $6.1 \times 10^{-9} \text{ V}$
- 86.** $7.7 \times 10^2 \text{ N/C}$
- 87.** $3 \times 10^3 \text{ V}$
- 88.** $5.0 \times 10^{-3} \text{ m}$
- 89.** $2.67 \times 10^{-1} \text{ m}$
- 90.** **a)** $4.2 \times 10^{-19} \text{ C}$
b) $\approx 3e^-$
- 91.** **a)** $1.26 \times 10^7 \text{ m/s}$
b) $7.26 \times 10^6 \text{ m/s}$
- 92.** **a)** $4.5 \times 10^3 \text{ N/C}$
b) $1.2 \times 10^{-4} \text{ N}$
c) $1.2 \times 10^{-4} \text{ N}$
d) $2.7 \times 10^{-8} \text{ C}$

Chapter 9

- 22.** $8.1 \times 10^{-2} \text{ m}$
- 23.** $7.5 \times 10^{-5} \text{ T}$
- 24.** $4.2 \times 10^{-3} \text{ m}$
- 25.** $1.6 \times 10^{-4} \text{ T}$
- 26.** $1.8 \times 10^{-2} \text{ T}$
- 27.** **a)** 0
b) $4.0 \times 10^{-4} \text{ T}$
- 28.** $2.5 \times 10^{-4} \text{ T}$
- 29.** 24 A
- 30.** **a)** 0.57 N [up]
b) 0.57 N [down]
- 31.** **a)** 4900 A
- 32.** **a)** $6.8 \times 10^{-2} \text{ N}$
b) 2.7 N/kg
- 33.** $4.3 \times 10^{-3} \text{ m}$
- 34.** $1.4 \times 10^3 \text{ m}$
- 36.** $1.3 \times 10^{-9} \text{ N}$
- 37.** **a)** $1.12 \times 10^{-15} \text{ N}$
[toward wire]
b) away from wire
- 38.** **a)** 0
b) $2.36 \times 10^5 \text{ T}$
- 39.** 4750 m/s
- 40.** 2500 V
- 41.** $2.44 \times 10^3 \text{ N}$

Chapter 8

- 34.** **a)** 0
b) –
c) +
d) 0
e) +
- 35.** **a)** –
b) +
- 71.** $2.3 \times 10^4 \text{ V}$
- 72.** 2.3 J
- 73.** $1.9 \times 10^5 \text{ V}$
- 74.** **a)** 0.18 J
b) 0.14 J
- 75.** $2.5 \times 10^2 \text{ V}$
- 76.** **a)** $5.0 \times 10^{-4} \text{ N}$

- 42.** 1.56 N [perpendicular to wire],
0.78 N [at 30°]
- 43. a)** 2.8×10^{-2} T
b) 2.5×10^{16} m/s²
- 44. a)** 7.4×10^6 m/s
b) 4.2×10^{-13} N
- 45.** 1.5×10^{-8} s
- 46.** 8.7×10^{-3} s
- 47. a)** clockwise
b) counterclockwise
- 48. a)** clockwise (from top)
b) linear (at south end)
- Chapter 10**
- 21. a)** 4 m
b) 5 cm
c) 8 s
d) 0.1 s^{-1}
e) 0.4 m/s
- 22.** 3.125 cycles/s, 0.32 s/cycle
- 23.** 1.2 cycles/s, 0.83 s/cycle
- 24.** 0.017 s/cycle
- 25. a)** 2.5 Hz
b) 0.4 s/cycle
- 26. i)** 1.3 Hz, 0.77 s/cycle
ii) 0.75 Hz, 1.33 s/cycle
iii) 5/9 Hz, 1.8 s/cycle
- 27. a)** 0.98 m
b) -0.087 m
c) -0.71 m
d) 1 m
- 30. a)** 2.9 s /cycle
b) 18 s/cycle
c) 0.78 s/cycle
- 31. a) i)** 7.2 s/cycle
ii) 44 s/cycle
iii) 1.9 s/cycle
b) i) 1.8 s/cycle
ii) 11 s/cycle
iii) 0.49 s/cycle
- 32. a)** 0.711 s/cycle
b) 0.889 s/cycle
c) 0.204 s/cycle
- 33. a)** 2.3×10^3 N/m
b) 8.0×10^2 N
- 34. a)** 4.62×10^{14} Hz
b) 5.00×10^{14} Hz
c) 5.17×10^{14} Hz
d) 5.77×10^{14} Hz
e) 6.32×10^{14} Hz
f) 7.50×10^{14} Hz
- 35. a)** 8.28 min (0.138 h)
b) 2.1×10^{-2} min (3.5×10^{-4} h)
c) 3.2×10^2 min (5.4 h)
d) 5.1 min (8.4×10^{-2} h)
- 36.** 9.46×10^{15} m
- 37.** 100 a
- 38.** 5.33×10^{-7} s
- 39.** 0.314 s
- 40.** 8×10^{14} Hz– 4×10^{15} Hz
- 41.** 1.8×10^7 times
- 42. a)** 0.5
b) 0.866
c) 0.707
d) 0.218
e) 0.963
f) 0
g) 1
- 43. a)** 20°
b) 40°
c) 44.4°
d) 19.5°
e) 90°
- 44.** 3.3×10^8 m/s
- 45.** 8.1°
- 46.** 0.98
- 48. a)** 1.24×10^8 m/s
b) 1.97×10^8 m/s
c) 2.26×10^8 m/s
d) 2.31×10^8 m/s
- 49. a)** 0.413
b) 0.658
c) 0.752
d) 0.769
- 50.** 5.31×10^{-5} s
- 51. a)** 54.9°
b) 35.2°
c) 54.9°
- 52.** 36.9°
- 53. a)** 37.5%
b) 20.7%
c) 5.85%
- 56.** 53°
- 57. a)** 53.1°
b) 56.3°
c) 40.9°
d) 45.7°
- 58.** 1.73
- 60. a)** 48.5%
b) 37.5%
c) 5.85%
d) 0.380%
- 61.** 26.6°
62. 6.06%
- 29. a)** 7.14°
b) 10.8°
c) 21.9°
d) 25.8°
- 30.** $3.12 \mu\text{m}$
- 31.** $481 \mu\text{m}$
- 32.** 0.23 mm
- 33.** 4.76×10^{11}
- 35.** $2.86 \mu\text{m}$
- 36.** 2.07
- 37.** 531 nm
- 41. a)** 86.8 nm
b) 203 nm
- 42. a)** 218 nm
b) 109 nm
- 43. a)** 1.40 m
b) 520 nm
c) 2.5 m
- 44. a)** 1.5×10^{20} Hz
b) 3.2 Hz
- 46. a)** 3.78°
b) 2.87°
- 47.** 837 nm
- 48.** 140 mm, 174 mm
- 49. a)** 6.47°
b) 8.10°
- 50.** $6.95 \mu\text{m}$
- 51. a)** 171 mm
b) 143 mm
- 52.** 4.90°
- 53. a)** 6.1 mm
b) 0.10°
- 54.** 450 nm
- 55.** 0.304°
- 56.** 1
- 57.** 27 m
- 58. a)** 2
b) 2
- 59.** 11
- 60. a)** 1
b) 1
c) 2
- 61.** 1.39×10^{-2} °
- 62.** 500
- 63.** 0
- 65.** 7.9×10^{-2} °
- 66.** 49°
- Chapter 11**
- 26. a)** 33.4°
b) 0.662°
- 27.** 0.55 m
- Chapter 12**
- 19.** 4581.27°C
- 20.** 1.07×10^{-3} m
- 21.** 2.32×10^{-5} m, infrared
- 21.** 6.36×10^{18} photons/s
- 23.** 0
- 25.** 8.15×10^{19} photons/s

- 26.** **a)** 2.83×10^{52} Hz
b) 4.28×10^{-19} J
- 28.** 2.2×10^{-7} m
- 30.** **a)** 7.5×10^{17} Hz
b) 1.66×10^{-24} N·s
c) 5.53×10^{-33} kg
- 31.** 5.02×10^{-19} N·s
- 32.** 6.63×10^{-29} N·s
- 33.** 4.48×10^{-12} m
- 34.** 2.04×10^{-9} m
- 35.** increases by 202%
- 36.** 2.9×10^{-34} m
- 37.** 3371 m/s
- 38.** 1.37×10^{27} m/s
- 39.** **a)** 1.73×10^{-10} m
- 40.** 4.34×10^{-7} m, violet
- 41.** **a)** -12.75 eV
b) -2.55 eV
- 42.** 2.64×10^{-10} m
- 43.** 8.22×10^{-8} N
- 44.** 6.56×10^{15} Hz
- 48.** 7.27×10^{-7} m
- 49.** 1.98×10^{-5} m/s
- 43.** 1.8×10^8 m/s
- 44.** 0.691 ca
- 45.** 1.52×10^{-3} m
- 46.** 7.81×10^{-15} m
- 47.** 3.00×10^{-7} kg
- 48.** 3.67×10^{-26} kg
- 49.** 2.26×10^{-2} T
- 50.** 2.6×10^8 m/s
- 51.** 3.00×10^8 m/s
- 52.** 1.42×10^8 m/s
- 53.** 2.88×10^8 m/s
- 54.** 2.99×10^8 m/s
- 55.** 2.47×10^8 m/s
- 56.** 1.50×10^8 m/s
- 57.** 3.56×10^{-13} kg
- 58.** 1.02×10^{-6} kg
- 59.** $\$2 \times 10^9$
- 60.** $0.5c - 0.9c$
- 61.** electron
- 62.** 2.9×10^{-1} N·s
- 63.** 4.16×10^{15} kg
- 64.** 2.4×10^{-28} kg
- 65.** 937.8 MV
- 66.** A
- 67.** 10 501 MeV
- 68.** $2.999\ 999\ 96 \times 10^8$ m/s
- 69.** A
- 52.** 5.0×10^{-14} m
- 54.** 0.789 MeV
- 55.** 0.546 MeV
- 56.** 5.44×10^{-21} N·s
- 57.** 7.42×10^{-16} J
- 58.** 5.07×10^{-16} m
- 59.** 39.7%
- 60.** 0.85:1
- 61.** 0.79×10^{-6} g
- 62.** 14 d
- 63.** 5.78×10^8 a
- 64.** 1507 a
- 65.** ${}_2^4\text{He}, {}_{8}^{16}\text{O}, {}_{20}^{48}\text{Ca}, {}_{20}^{40}\text{Ca}, {}_{28}^{78}\text{Ni}, {}_{50}^{132}\text{Sn}$
- 66.** 0.6996 MeV
- 67.** 0.013 mG
- 68.** 3.64
- 69.** 8 alpha, 6 beta
- 70.** ${}_{90}^{232}\text{Th}$
- 71.** 4.8 fm
- 72.** 4.2 MeV
- 73.** 1.67×10^{17} Bq, 1.61×10^{17} Bq
- 74.** 5.49 MeV
- 75.** 883 kg
- 76.** 24.2%
- 77.** 5×10^{15} J
- 78.** 8
- 79.** 1.782×10^4 m/s
- 80.** ${}_{36}^{92}\text{Kr}$
- 81.** 0.999639 c
- 82.** 1.73×10^{-19} m
- 83.** 1.16 fm
- 84.** 2.28 T
- 85.** 8.0 GeV
- 86.** 1.87 T
- 87.** **a)** 0
b) 1
c) 0
d) 0
- 89.** $\overline{udd}, \overline{uud}$
- 90.** Os
- 91.** \overline{ud}
- 92.** 8×10^{-24} s
- 96.** $p^2 = -2.165 \times 10^{-13} \text{ N}^2 \cdot \text{s}^2$
- 98.** -1
- 99.** 1

Chapter 13

- 28.** **a)** 3.16×10^{-18}
b) 3.3×10^{-16}
c) 3.6×10^{-8}
d) 7.28×10^{-6}
e) 7.33×10^{-3}
- 29.** **a)** 120 km/h
b) 180 km/h [W], 80 km/h [E]
c) Snoopy by 0.139 h
- 30.** 0.60 m
- 31.** 2.83×10^8 m/s
- 32.** 6.12×10^{-7} s
- 33.** 1.04×10^2 m
- 34.** 7.58×10^{10} m
- 35.** 6.81×10^{-12} s
- 36.** 82.7 m
- 37.** 2.45×10^8 m/s
- 38.** 6.68×10^{-8} s
- 39.** 9.47×10^{15} m
- 40.** 500 m
- 41.** 1.66×10^8 m/s
- 42.** 9×10^8 m

Chapter 14

- 43.** **a)** Cl
b) Rn
c) Be
d) U
e) Md
- 44.** **a)** $17\ p^+, 18\ n$
b) $86\ p^+, 136\ n$
c) $4\ p^+, 5\ n$
d) $92\ p^+, 146\ n$
e) $101\ p^+, 155\ n$
- 45.** $17\ 697\ \text{MeV}/c^2$
- 46.** 0.114 u
- 47.** Cu, 63.55 u
- 48.** 7.5 MeV/nucleon
- 49.** $4/3 : 1/1$
- 50.** 20.55 MeV
- 51.** 5.41 MeV
- 52.** 5.0×10^{-14} m
- 54.** 0.789 MeV
- 55.** 0.546 MeV
- 56.** 5.44×10^{-21} N·s
- 57.** 7.42×10^{-16} J
- 58.** 5.07×10^{-16} m
- 59.** 39.7%
- 60.** 0.85:1
- 61.** 0.79×10^{-6} g
- 62.** 14 d
- 63.** 5.78×10^8 a
- 64.** 1507 a
- 65.** ${}_2^4\text{He}, {}_{8}^{16}\text{O}, {}_{20}^{48}\text{Ca}, {}_{20}^{40}\text{Ca}, {}_{28}^{78}\text{Ni}, {}_{50}^{132}\text{Sn}$
- 66.** 0.6996 MeV
- 67.** 0.013 mG
- 68.** 3.64
- 69.** 8 alpha, 6 beta
- 70.** ${}_{90}^{232}\text{Th}$
- 71.** 4.8 fm
- 72.** 4.2 MeV
- 73.** 1.67×10^{17} Bq, 1.61×10^{17} Bq
- 74.** 5.49 MeV
- 75.** 883 kg
- 76.** 24.2%
- 77.** 5×10^{15} J
- 78.** 8
- 79.** 1.782×10^4 m/s
- 80.** ${}_{36}^{92}\text{Kr}$
- 81.** 0.999639 c
- 82.** 1.73×10^{-19} m
- 83.** 1.16 fm
- 84.** 2.28 T
- 85.** 8.0 GeV
- 86.** 1.87 T
- 87.** **a)** 0
b) 1
c) 0
d) 0
- 89.** $\overline{udd}, \overline{uud}$
- 90.** Os
- 91.** \overline{ud}
- 92.** 8×10^{-24} s
- 96.** $p^2 = -2.165 \times 10^{-13} \text{ N}^2 \cdot \text{s}^2$
- 98.** -1
- 99.** 1

Glossary

Additive colour theory—the combination of red, green, and blue that results in a neutral (white) colour

Angular acceleration—the change in angular velocity of an object over a period of time

Antiparticle—an elementary particle having the same mass as a given particle but the opposite charge

Antiquark—the antiparticle of the quark

Apogee—the point in a celestial body's orbit where it is farthest from Earth

Atoms—small particles that make up all matter

Balanced forces—equal forces acting in opposite directions, canceling each other out

Barycentre—the centre of mass in any system of celestial objects moving under mutual gravity

Beams—the main horizontal supports of buildings

Beta decay—Radioactive disintegration with the emission of an electron or positron accompanied by an antineutrino or neutrino

Binding energy—the energy required to break a nucleus into its smaller component particles

Black hole—a region of spacetime from which matter and energy cannot escape; a star or galactic nucleus that has collapsed in on itself to the point where its escape velocity exceeds the speed of light

Black-body radiation—the characteristic radiation re-radiated by an object or system that absorbs all radiation incident upon it

Black-body radiator—a body or surface that can absorb all the radiation that falls on it and re-radiate at a characteristic spectrum

Bohr radius—the mean distance of an electron from the nucleus in the ground state of the hydrogen atom

Bone marrow—a soft fatty substance in the cavities of bones, of major importance in blood cell formation

Bound system—a system in which work must be done to separate the constituents

Breed—create by means of nuclear reaction

Brewster's angle—a special angle of incidence at which 100% polarization can occur

Buckminsterfullerene (C_{60})—an extremely unstable form of carbon whose molecule consists of 60 carbon atoms

Bulk modulus (B)—the ratio of the change in pressure applied on a body to the corresponding fractional change in volume that this pressure produces

Buttress—a structure built against a wall or building to strengthen and support it

Cathode rays—beams of electrons emitted from the cathode of a vacuum tube

Cathode-ray tube—a vacuum tube in which cathode rays produce a luminous image on a fluorescent screen, such as a television screen or computer monitor

Centre of mass—a single point at which the entire mass of a body is considered to be concentrated for the purpose of analyzing its motion

Centripetal force—the force required to give the centripetal acceleration that moves a body along a curved path

Chain reaction—a reaction during which the number of subsequent fission reactions increases at a geometric rate

Cherenkov radiation—Radiation emitted by a massive particle that is moving faster than light in the medium through which it is travelling

Chromatic aberration—the failure of different wavelengths of electromagnetic radiation to come to the same focus after refraction

Closed system—an isolated system having no interaction with an environment

Coefficient of friction—the ratio of two forces, the frictional force and the normal force

Coefficient of kinetic friction (μ_k)—the ratio between the force of friction and the normal force when the object is moving

Coefficient of static friction (μ_s)—the ratio between the force of friction and the normal force when the object is at rest; $\mu_s > \mu_k$

Colour charge—the charge carried by gluons that bind quarks together in hadrons by way of the strong nuclear force. There are three colour charges and three corresponding anti-colour (complementary colour) charges. Quarks constantly change their colour charge as they exchange gluons with other quarks.

Colour force field—the force field created when two or more quarks close to each other rapidly exchange gluons that bind the quarks together

Compton effect—the increase in wavelength of x-rays after collision with electrons, providing evidence for the wave-particle duality of light

Contact—touch

Continuous spectrum—electromagnetic radiation at all wavelengths

Coulomb (C)—the SI unit of electric charge

Critical mass—the minimum mass of nuclear material needed for a self-sustaining chain reaction to take place

Dark matter—hypothetical non-luminous material in space, not detected, but predicted by many cosmological theories

Diffraction grating—a large number of closely spaced parallel slits

Displacement—the net travel of an object as measured from its starting point to its end point in a straight line, with direction

Dose equivalent—the product of absorbed dose and the quality and distribution factors compensating for variations in biological effectiveness of different types of radiation

Dynamic equilibrium—see Uniform motion

Elastic modulus (E)—see Young's modulus

Elastic potential energy (E_g)—energy stored in elastic materials as the result of their stretching or compressing

Electric field—the space around a single charge or an array of charges in which electric forces act

Electric potential energy—energy stored when static electric charges are held at a certain distance apart

Electromotive force (EMF)—the electric potential difference (voltage) between two points where no external current flows

Electroweak force—the combined interaction of the electromagnetic and weak interactions

Energy well—a region where an object has a low energy relative to surrounding regions; extra energy is needed to remove an object from such a region

Equilibrium—a condition of balance in which opposing forces equal each other

Equipotential lines—lines along which the potential (electric field strength) is equal at all points

Escape trajectory—a parabolic path where an object has just enough energy to depart a system

Ether—a medium formerly assumed to permeate space and fill the gaps between particles of matter, and conduct light, electric waves, etc.

Event horizon—the boundary of a black hole where the force of gravity is so strong that light cannot escape it

Field map—a set of lines that represent the shape of a magnetic, electric, or gravitational field around a body

Flavour—type of quark (i.e., *up*, *down*, *strange*, etc.)

Flow-through capacitor—a capacitor through which water flows

Fluorescent—exhibiting the radiation produced from certain substances as a result of incident radiation of a shorter wavelength, such as x-rays, ultraviolet light, etc.

Flux—the rate of flow of mass, volume, or energy per unit cross-section normal to the direction of flow

Footprint—the support base of a structure

Force—any cause that produces, changes, or stops the motion of an object

Friction—a force produced from contact between two surfaces

Gimbal—a device, usually composed of rings and pivots, for keeping objects, especially instruments such as a compass, horizontal aboard a ship or aircraft

Glancing collision—a collision during which the objects involved are deflected in more than one dimension

Grand unified theory (GUT)—a theory that would show the interdependence of the electromagnet, weak, and strong forces

Gravitational potential energy (E_g)—the energy stored in an object as the result of its vertical position (i.e., height) due to gravitational attraction

Gravitational well—a region of lower gravitational potential energy relative to some other region

Gravity—the force that attracts a body toward the centre of any body having mass

Group velocity—the linear velocity of a wave

Half-life—the amount of time required for half the number of unstable nuclei in an isotope to decay

Hydrogen bonds—weak intermolecular forces that attract and bond the positive and negative poles of water molecules

Hydroxyapatite—crystals containing calcium that provide strength to bone tissue

Induce—to cause a change without contact

Induction—the process by which electrical or magnetic properties are transferred from one circuit or object to another without direct contact

Inelastic—pertaining to a spring that has been stretched past its elastic limit

Inelastic collision—a collision during which there is an overall loss of translational kinetic energy

Instantaneous velocity—velocity of an object at a specific time

Intensity—the amount or degree of strength of heat, light, or sound per unit area or volume

Ionized—converted to an ion (charged particle) by having (an) electron(s) removed

Isotopes—atoms of the same element type that have different numbers of neutrons

Kinematics—a sub-branch of mechanics dealing with motion only, without regard to any underlying causes

Kinetic friction—the force that acts in a direction opposite to that of the object's motion

Leptons—a class of fundamental particles that consists of the electron, muon, tauon, and three types of neutrino

Leyden jar—a device for collecting and storing electric charge

Line spectrum—a set of wavelengths at which the excited atoms or molecules in the source emit electromagnetic radiation consisting of characteristic emission lines

Linear momentum—the product of a body's mass and velocity

Linearly polarized—see Plane polarized

Lodestone—iron oxide (magnetite) that is naturally magnetic

Longitudinal wave—a wave where particles of the medium vibrate parallel to the direction of wave motion

Macroscopic—visible by the naked eye

Magnetic domain—the effect produced when dipoles of a magnet line up

Magnetron—an electronic tube that oscillates microwaves

Mechanics—the study of motions and forces

Members—constituent parts of a complex structure

Metabolize—to build up food into living matter and use living matter so that it is broken down into simpler substances or waste matter, giving off energy

Metric—a decimal system of weights and measures based on the metre, litre, and kilogram

Microscopic—visible only by looking through a microscope

Moderate—to slow down

Modulus—a constant indicating the relation between the physical effect and the force producing that effect

Moment of inertia (I)—the sum of all the products formed by multiplying the magnitude of each element of mass by the square of its distance from the axis

Monochromatic—having one wavelength or frequency of light

Neutral equilibrium—an object's state when any disruptive force acts horizontally but the vertical height of the centre of mass remains unchanged

Neutron star—a compact stellar object that is supported against collapse under self-gravity by the pressure of the neutrons of which it is primarily composed; formed as the end product of the evolution of stars of mass greater than 4–10 solar masses

Newton's law of universal gravitation—two bodies attract each other with equal and opposite forces; the magnitude of this force is proportional to the product of the two masses and to the inverse square of the distance between their centres

Normal force (F_n)—the reaction force pressing back on the object exerting an action force; perpendicular to the surface on which the action force acts

Nucleons—the particles that make up an atom's nucleus (i.e., protons and neutrons)

Null result—the result obtained regardless of the way an experiment is done

Open system—an entity with a boundary that is not closed

Parallel-axis theorem—if the moment of inertia of a body of mass M about an axis through its centre of mass is I_0 , then the moment of inertia about a parallel axis a distance l from it is $(I_0 + Ml^2)$

Pauli exclusion principle—no two identical particles in a system, such as electrons in an atom, can have an identical set of quantum numbers

Perigee—the point in a celestial body's orbit where it is nearest Earth

Periodic wave—a wave occurring at regular intervals

Phase—the relationship of position and time between two points on a wave

Phase shift—the relative position of the wave compared to a standard representation

Phase velocity—the speed of propagation of a pure sine wave

Phosphors—synthetic fluorescent or phosphorescent substances used in cathode ray tubes that emit light when subjected to radiation

Plane polarized—light that has had one of the components of its electric field absorbed, so its electric field oscillates in one plane only

Point of insertion—the end of a muscle that is attached to the part of the bone that moves when a muscle contracts

Polaroid—a material that polarizes light that passes through it (i.e., removes a component of its electric field)

Positron—the antiparticle of an electron

Posts—the main vertical supports of buildings

Post-stressed—having the reinforcement in a concrete beam pulled after the concrete has been placed

Potential energy—the stored energy of position of an object

Precessing—the slow movement of the axis of a spinning body around another axis

Preferential direction of transmission—a characteristic of a Polaroid that causes it to absorb one component of light's electric field, allowing only one component to pass through

Pre-stressed—having the reinforcement in a concrete beam pulled before the concrete has been placed

Principle of complementarity—a given system cannot exhibit both wave-like and particle-like behaviour at the same time

Probability distribution—a mathematical function that describes the probabilities of possible events in a sample space

Propagate—transmit, as in a wave through a medium

Proton-proton cycle—a sequence of fusion reactions within a star that leads to the creation of helium and energy

Pulsar—see Neutron star

Quality factor—a number by which the absorbed dose is multiplied to reflect the relative biological effectiveness of radiation. The result is the dose equivalent.

Quanta—discrete quantities of energy proportional to the frequency of radiation they represent; the smallest amount of energy capable of existing independently

Quantum electrodynamics—the study of the properties of electromagnetic radiation and the way in which it interacts with charged matter in terms of quantum mechanics

Quark—a particle that is the fundamental constituent of hadrons and that interacts via the strong force, which is mediated by gluons

Radioactive decay—the continuous disintegration of the nuclei of unstable elements

Recoil velocity—the speed at which a device or object moves backwards after firing a projectile

Rectilinear propagation of light—light travelling in a straight line

Relativistic length—the length measured in a reference frame in which the observed object is moving at a speed close to the speed of light

Residual force—the strong force that protons and neutrons exert on each other due to the colour charge of the quarks that comprise them

Resolving power—a measure of the ability of a lens or optical system to form separate and distinct images of two objects with small angular separation

Rest mass—the mass of a stationary object

Restoring force—a force that acts in an equal and opposite way to another force in order to restore a displaced system to the equilibrium position

Retrograde motion—the apparent backward motion of a celestial body

Rotational inertia—the inertia of an object rotating on an axis that does not pass through its centre of mass

Scalar—a quantity specified by a value (magnitude) only and no direction

Sedimentation—deposition of material in the form of a sediment, as a geological process, or as a liquid in a tank, centrifuge, etc.

Shape—the pattern, strength, and direction of a field (electric, magnetic, gravitational)

Shear modulus (*G*)—the strength factor for a material under shear stress, expressed by the relationship of the shear force applied to it to the change in position produced by this force

Sieverts (Sv)—the derived SI unit of dose equivalent, defined as the absorbed dose of ionizing radiation multiplied by internationally agreed-upon dimensionless weights

Simple harmonic motion—a form of periodic motion in which a point or body oscillates along a line about a central point in such a way that it ranges an equal distance on either side of the central point that is always proportional to its distance from it

Sinusoidally—in a way that maintains the same sine-wave phase

Snell's law—the ratio of the sines of the angles of incidence and refraction of a wave is constant when it passes between two given media

Spacetime—a geometry that includes the three dimensions of space and a fourth dimension of time where an event is identified by a point in a four-dimensional continuum

Spin quantum number—a number that describes an elementary particle's spin direction

Stable equilibrium—the state of an object when the vertical line from the centre of mass remains inside the area of the base of the body

Static equilibrium—an object's state of no motion when all the forces acting on it are balanced

Static friction—the force that tends to prevent a stationary object from starting to move

Stress—the force per unit area on a body that tends to cause it to deform; a measure of the internal forces in a body between particles of the material of which it consists as they resist separation, compression, or sliding in response to externally applied forces

System—an object or group of objects considered as a separate entity for the purpose of restricted study

Tangential acceleration—the tangential, linear acceleration of a point on a rotating object at a distance *r* from the axis of rotation

Tangential velocity—the linear velocity of a point on a rotating rigid object at a distance *r* from the axis of rotation

Tendons—cords or strands of fibrous tissue that connect bones to muscles, thereby giving one mobility

Tensile forces—forces that pull

Terminal velocity (*v_t*)—velocity reached when the upward frictional force on a falling object balances the downward force of gravity

Test charge—a small charge used to check for the presence of an electric field

Test compass—a small compass used to check for the presence of a magnetic field

Test magnet—a small magnet used to check for the presence of a magnetic field

Test mass—a small mass used to check for the presence of a gravitational field

Thrust—the force exerted by a high-speed jet of gas, etc., ejected to the rear of a vehicle, producing forward motion

Time dilation—the change in the rate time passes as an object approaches the speed of light

Total mechanical energy—the sum of an object's kinetic and potential energies

Total moment of inertia—see Rotational inertia

Transuranic elements—elements having a higher atomic number than uranium

Transverse wave—a wave the direction of which is perpendicular to the direction of vibration of the particles of the medium

Travelling wave—a wave in which the medium moves in the direction of propagation

Truss—a metal or wooden structural framework consisting of rafters, posts, and struts, supporting a roof or bridge, etc.

Twin paradox—a paradox resulting from the special theory of relativity; if one of a pair of twins remains on Earth while the other twin makes a journey to a distant star at close to the speed of light and subsequently returns to Earth, the twins will have aged differently. The twin remaining on Earth will have aged more than the twin who travelled to a star

Unified field theory—a theory that unifies all field theories; that is, the fundamental forces of nature (the weak force, the strong force, gravity, and electromagnetism)

Uniform motion—motion at a constant speed in a straight line

Uniform velocity—See Uniform motion

Unstable equilibrium—an object's state when a disruption moves the vertical line from the centre of the mass outside of the base

UV catastrophe—a shortcoming of the Rayleigh-Jeans law, which attempted to describe the radiancy of a black body at various frequencies of the electromagnetic spectrum

Vector—a quantity that is specified by both a magnitude and a direction

Viscosity—the property of a fluid that tends to prevent it from flowing; the frictional resistance of a fluid to the motion of its molecules

Wave-particle duality—the principle of quantum mechanics that implies that light sometimes acts like a wave and sometimes like a particle, depending on the experiment you are performing

Weight—the gravitational pull on an object toward Earth's centre

Young's modulus (*E*)—an inverse constant of the ratio of the longitudinal stress applied to a body to the strain produced; indicates how much the length of an object will change when subjected to a certain force

Index

For entries with multiple page references, the page with the entry's **definition** is given in bold font.

Symbols and numbers

3-D movies, 515

A

Absorbed dose, 703, 704 *table*
Absorption spectra, 506
Acceleration, **9**, 10
analysis in a yo-yo, 353
centripetal, **100**, 325
due to gravity, **20**
graphical derivation of, 27
linear vs. angular, 323, 326
tangential, 324
Achromatism, 515
Action-reaction forces, **39**–42
Additive colour theory, 731
Affleck, Ian, 735
Air bags, 267
Air wedges, 552
Al-hazen, Ali, 482
All-terrain vehicles, 158, 159
Alpha decay, 691, 696 *illus.*
Alpha particles, 690–693
Ampere, A, 455
Ampère, André Marie, 453
Ampère's law, 453
Amplitude modulation, 495
Amplitude of a wave, 488
Analyzer of a Polaroid, 510
Angle of magnetic inclination
(*also* Dip angle), 450
Angular acceleration, 323
Angular displacement, 318, 319
Angular momentum, 347–350, 610
of a gyroscope, 354, 355
Angular motion conventions, 319 *illus.*
Angular velocity, **322**
Angular work, **339**–341
Anisotropic crystals, 512, 513
Annihilation, 723, 732–734
Anode, 413
Antibaryons, 724 *table*
Antigravity, 285
Antimatter, 720
Antineutrino, 694
Antiparticle, 694, 722
Antiquark, 723–725 *illus.*
 colours of, 731 *table*
Apocynthion, 301
Arc length, 318, 319

Arch, 171
Archimedes, 2
Aristotle, 2, 186, 482
Artificial gravity, 327
Atom, 725 *illus*
 Bohr-Rutherford model, 372
 electrical charge, 373–375
 nuclear structure of, 686
Atomic bombs, 707–709
Atomic mass number, 686
Atomic number, 686
Average speed, **8**
Average velocity, **8**, 26
Avogadro's number, 706

B

Bacon, Francis, 186
Balanced forces, 33
 and centre of mass, 130–132
 problem solving, 85, 86
Balmer series, 608, 609
Balmer, Johann, 608
Banked curves, 106
Bartholinus, Erasmus, 512
Barycentre, 300
Baryons, 723–725
Beam splitter, 544
Beams, 170
Becquerel, Bq, 703, 704 *table*
Bessemer, Henry, 171
Beta decay, 693–696, 733
Beta emission, 695
Beta particles, 690, 693–696
Binding energy, 297, 298
 of nucleons, 688, 689
Biot's law, 452, 453
Birefringence, 513, 517, 518
Black hole, 716
Black-body radiation, 595, 596
Black-body radiator, 499
Bohr atom, 608–614
Bohr radius, 611
Bohr, Niels, 608 *illus.*
Bohr's principle of complementarity, 614
Bohr-Rutherford model of the atom, 372 *illus.*, 721 *illus.*
Bose, Nath, 727
Bosons, 727, 728
Bound system, 301
Bragg, W.L., 572
Bragg's law, **572**
Brahe, Tycho, 2
Breeding in fusion reactions, 715

Brewster's angle, 511, 512
Bright filament lamp, 506 *illus.*
Brockhouse, Bertram, 735
Brooks, Harriet, 735
Bubble chamber, 722 *illus.*
Bucherer, H., 652
Bulk modulus, 165 *table*
Buttress, 171

C

Calandria, 711
Calcite crystals, 512, 513
CANDU reactor, 711
Capacitance, **418**
Capacitors, 418, 419
Carbon dating, 698–700
Cartesian coordinate system, 79
Cathode, 413
Cathode rays, 413
 and motor principle, 462
Cathode-ray tube, 413
Cavendish, Henry, 48, 49, 377
Cavendish's torsion balance, 377
Centre of mass (*also* Centre of gravity), 128, **129**, 130
 and linear momentum, 211, 212
 and parallel-axis theorem, 337
Centrifugation, 107–109
Centrifuge, **107**–109
Centripetal acceleration, **100**, 325
Centripetal force, **103**–110, 295
Centripetal magnetic force, 461, 462
Chadwick, James, 701
Chain reaction, 708
Change in potential energy (ΔE_p), 289
Charge (Q), 372
 equation for, 380
 of an elementary particle, 415–417
 of capacitors, 418, 419
Charge distribution, 388
Charge-to-mass ratio, 463–465
Charging capacitor, 418
Chromatic aberration, 515
Circle, equation of, 492
Circularly polarized light, 516
Classical physics, 2
Closed (isolated) system, 199, **230**, 231 *illus.*
Cockcroft, John, 701
Coefficient of friction, **44**
Coefficient of kinetic friction, 45
Coefficient of static friction, 45
 on an inclined plane, 91

Coherence, 537
 and holography, 546
 Collision
 graphical representations of, 264

D
 Dalton, John, 721 *illus.*
 Damped simple harmonic motion, 308, 309
 Dark matter, 734
 Daughter nucleus, 691
 Davisson, Clinton J., 608
 de Broglie wavelength, 718
 de Broglie, Louis, 606 *illus.*, 610
 de Broglie's equation, 606
 Decay, 732–734
 Decay series, 702
 and the food chain, 704, 705
 Degrees, converting from radians, 319, 321
 Demagnetization, 438 *table*
 Democritus, 371, 721 *illus.*
 Derived unit, 9
 Descartes, René, 482
 Destructive interference, 534, 536
 in single-slit diffraction, 556–561
 Determinacy, 616
 Deuterium, 687
 in fusion reaction, 713, 715
 Diamagnetism, 443
 Dichroism, 508
 Diffraction, 553–562
 applications of, 569–572
 single vs. double slit patterns, 562 *illus.*
 Diffraction grating, 505, 563–568
 Diffraction-grating equation, 564, 565
 Dimensional unit analysis, for work, 237
 Dipole, 392
 in magnetism, 437
 Direction
 convention for field strength, 394
 conventions for current flow, 442
 conventions for rotation, 136
 conventions for torque, 140
 defining, 7
 of centripetal motion, 101
 of magnetic fields, 441–444
 right-hand rule for torque, 135 *illus.*
 Discharging capacitor, 418
 Dispersion, 505, 571
 Displacement, 5, 6, 7
 graphical derivation of, 28
 Distance, 5, 6, 7
 linear vs. angular, 325, 326 *illus.*
 Domain theory, 437, 438
 Dose, 703
 Dose equivalent, 703, 704 *table*
 Double-slit equations (see Young's three double-slit equations)

E
 Drag, 112
 Dynamic equilibrium, 128
 Dynamics, 5, 32, 33

E
 Eccentricity, 301
 Eiffel tower, 171
 Einstein, Albert, 483, 501, 598 *illus.*, 634 *illus.*, 683 *illus.*
 Elastic collision, 260, 264 *table*
 equation for one-dimensional cases, 260–263
 Elastic modulus (*see* Young's modulus)
 Elastic object, 250
 Elastic potential energy (E_e), 251–253
 Elasticity, 159, 160
 Electric bell, 445 *table*
 Electric charges
 transfer of, 373–376
 vector nature of forces between, 384–386
 Electric dipole, 390
 Electric double-layer capacitors, 419
 Electric field, 388
 of a transverse wave, 487
 polarization of, 507
 Electric field configurations, 391 *table*
 Electric field lines, rules for drawing, 390
 Electric field strength, of a parallel-plate apparatus, 414, 415
 Electric force, vs. charge position, 406
 Electric potential, around a point charge, 409–411
 Electric potential energy (E_e), 400–403
 vs. charge separation, 408 *illus.*
 Electrical emission lines, safety of, 451, 452
 Electromagnet, 443
 Electromagnetic fields, safety of, 451, 452
 Electromagnetic induction, 466–471
 Electromagnetic spectrum, 495 *illus.*, 497 *illus.*
 Electromagnetic strength, factors determining, 444 *table*
 Electromagnetic waves, 485
 generation of, 499
 properties of, 494 *illus.*, 495
 waves, self-propagation of, 471 *illus.*
 sources and uses, 498 *table*
 Electromagnetism, 441–445
 Electromagnets, applications of, 445 *table*
 Electromotive force, 467
 in elementary particles, 728

Electron, 372
 acceleration, 639
 affinity, **375**
 capture, 695, 696 *illus.*
 charge of, 380
 charge-to-mass ratio, 463
 circular motion of, 653
 conservation of energy of, 609
 conservation of momentum of, 610
 determining the mass of, 462–464
 dilated, 653
 emission in beta decay, 693,
 696 *illus.*
 energy vs. light intensity, 599
 energy, E_n , 612, 613
 in magnetic field, 656, 657
 mass of, 408
 oscillators, 504 *illus.*
 volt, eV, 403, 595, 670
 Electronic water purification device, 419
 Electroscope, 372, 373 *illus.*
 Electrostatic force, 371
 vs. gravitational force, 285,
 387 *illus.*
 Electrostatic series, 375 *table*
 Electroweak force, 733
 Elementary charge, 415–417
 Elementary particles, 720–726
 fundamental forces of, 734 *illus.*
 Elliptical orbit, 299 *illus.*
 total energy of, 301, 302
 Emission spectra (*see* Line spectra)
 Empedocles, 720
 Energy
 analysis in a yo-yo, 352
 and gravity, 285–294
 conservation of, 344–346
 fusion vs. fission sources, 717 *table*
 history of, 186, 187
 levels, 610, 613, 614 *illus.*
 relativistic, 664–667
 transfer and escape speed, 294 *illus.*
 transfer in systems, 230–231
 Energy triangle of special relativity, 671
 Energy well, 285
 Equilibrium
 and stability, 155–158
 in a spring, 249
 types of, 155 *table*
 Escape speed, **292**–294
 Escape trajectory, 302
 Ether, 637
 Euclid, 2
 Extensive properties, 404
 Extraordinary (e) ray, 512, 513

F
 Faraday, Michael, 466, 483
 Faraday's law of electromagnetic induction, 466, 467
 Ferromagnetic materials, 437
 Feynman diagrams, 729, 730
 Feynman, Richard, 729, 730
 Field, **388**–393
 Field map, 388
 drawing, 439 *illus.*
 Field shapes, electric vs. gravitational vs. magnetic, 393 *illus.*
 Field strength, **394**–399
 around a current-carrying conductor, 452–454
 Coulomb's law vs. Newton's gravitational law, 399 *illus.*
 equations for various conductor configurations, 454 *table*
 Field theory, 494
 First ionization energy, 374 *illus.*, **375**
 Fission reactors, 710–712
 Fission, 707–709, 712 *illus.*
 Fizeau, Armand Hippolyte, 483
 Flat of a CD, 575
 Flavour change in particle decay, 733
 Flight data recorders, 232, 233
 Flow-through capacitor, 419
 Fluorescent lamp, 506 *illus.*
 Flux, 596
 Footprint, for stability, 156
 Force, **32**
 analysis in a yo-yo, 352, 353
 at a distance, 388, 436
 field, 389
 gravitational vs. electrical, 395 *illus.*
 points on the human body,
 148 *table*, 149 *table*
 Foucault, Jean Bernard Leon, 483
 Frame of reference, **35**, 634
 and relative motion, 70, 71
 Franklin, Benjamin, 372
 Fraunhofer diffraction, 555
 Fraunhofer lines, 506
 Free fall, 19–23
 Free-body diagrams, 33
 Frequency modulation (FM), 495
 Frequency of a wave, 488
 Frequency of rotation, 102
 Fresnel diffraction, 555
 Fresnel, Augustin, 483, 553
 Friction, 44–47
 and tires, 52
 in transfer of charge, 374, 375
 calculation of force, 37
 Fusion, 712–715
 Fusion reactors, 713–715

G
 Galilei, Galileo, 2, 3, 19 *illus.*, 186
 Galileo's guinea and feather demonstration, 19, 20 *illus.*
 Gamma decay, 695, 696 *illus.*
 Gamma ray wavelength, 495 *illus.*,
 497 *illus.*, 498 *table*
 Gamma rays, 690
 Geiger counter, 703 *illus.*
 Geiger, Hans, 412
 Gell-Mann, M., 723
 Geosynchronous Earth orbit (GEO)
 (also Geostationary orbit), 109, 110
 Germer, L.H., 608
 Gilbert, Sir William, 440, 441
 Gimbal, 355
 Glancing collision, 203
 Glashow, S, 733
 Gluon, 727 *table*, 731
 and colour theory, 732 *table*
 Gradians, 320
 Grand unified theory, 734
 Graphs
 acceleration-time analysis, 28
 of linear motion, 24–31
 position-time analysis, 24, 25
 velocity-time analysis, 27–31
 Grating spectroscope, 569
 Gravitational constant, 244
 Gravitational potential energy (E_g),
243–248
 Gravitational well, 285
 Graviton, 734
 Gravity, 20, **33**, **48**
 and Coulomb's laws, 382
 and energy, 285–294
 and field strength, 394
 and magnetism, 390
 artificial, 327
 Gravity spot (*see* Centre of mass)
 Grays, Gy, 703, 704 *table*
 Gregory, James, 482
 Grimaldi, Francesco, 482, 553
 Group velocity of a wave, 501
 Gyroscopes, 354, 355
 Gyrostabilizers, 355

H
 Hadrons, 723, 724, 725 *illus.*
 and colour theory, 732 *table*
 Half-life, 697, 698
 Han, Moo-Young, 731
 Heavy elements, creating, 715–717
 Heavy water, 710 *illus.*, 711
 Heisenberg, Werner, 617 *illus.*
 Heisenberg's uncertainty principle,
619–620

- Helium, nuclear fusion of, 716
 Helmets, 267
 Hero of Alexandria, 482
 Hertz, Heinrich, 495
 Hertz, Hz, 488
 Higgs boson, 734
 Holograms, 546, 547
 Hooke, Robert, 187, 250, 482
 Hooke's law, 159, 160, **250**
 and acceleration of mass on a spring, 304
 and simple harmonic motion, 305, 491
 Horizontal plane
 and Newton's laws in two dimension, 87
 centripetal force in, 104
 Human body
 and power, 259
 and static equilibrium, 148–153
 centre of mass of, 147
 force and pivot points in, 148 *table*, 149 *table*
 stress and strain on, 169, 170
 Huygens, Christian, 3, 483, 555
 Huygens' principle, 555
 Huygens' wavelets, 555, 556
 Hydrogen bonds, 499, 500 *illus.*
 Hydrogen, isotopes of, 687
- I**
- I-beams, 171
 Impulse, **191**–197
 Inclined plane, 89–92
 Induction, 375, 376 *table*
 Inelastic collision, **260**, 266 *table*
 Inelastic object, 250
 Inertial frame of reference, **35**
 and special relativity, 634–636
 Infrared wavelength, 495 *illus.*, 497 *illus.*, 498 *table*
 Instantaneous acceleration, graphical derivation of, 27
 Instantaneous velocity, **8**
 Insulator, 375
 Intensity, 510
 Intensive properties, 404
 Interference, 534–537, **553**
 in a thin film, 548–552
 of light, 537–543
 Interferometers, 544, 545, 639
 Intermolecular forces, microwave effects on, 500 *illus.*
 International Space Station, 310, 311
 International Thermonuclear Experimental Reactor (ITER), 713, 714
 Ionization energy, 614
- Ionizing ability, 691
 Isolated system, **230**
 Isotopes, 465, 687
 decay series, 702
 half-lives of, 697 *table*
 Israel, Werner, 735
- J**
- Jannsen, Hans, 482
 Jannsen, Zacharias, 482
 Jeans, James, 596
 Joliet, Pierre, 695
- K**
- Kepler's laws of planetary motion, 298–300
 Kepler, Johannes, 2, 3, 298, 482
 Kepler's third law for large masses, 300
 Kinematics, **5**
 Kinematics equations
 applied for uniform linear acceleration, 10–19
 derivations of, 10, 11, 12
 Kinetic energy (E_k), 239, **240**, 241
 and gravity, 290, 291
 and momentum, 241, 242
 linear vs. rotational, 343 *illus.*
 rotational, 342, 343
 Kinetic friction, 45
- L**
- Land, Edwin, 508
 Large Hadron Collider (LHC), 674, 720
 Laser, 546, 547
 Laser light in CD players, 574, 575
 Law of conservation of energy, **199**, **253**, 254
 and movement of charged particles, 404–413
 Law of conservation of linear momentum, **199**
 Law of electric charges, 372
 Law of inertia (*see* Newton's first law of motion)
 Law of magnetic forces, 437
 Lawrence, E.O., 652, 653
 Leibniz, Gottfried, 187
 Length contraction, 643–645
 Lenz, Heinrich, 467
 Lenz's law, 467–469
 Leptons, 721, 722 *table*
 decay, 733
 Leyden jar, 419
 Lifting electromagnets, 445 *table*
 Light, 487
 and thin-film interference, 548–552
 classical wave theory of, 593
- diffraction of, 553–562
 dispersion of, 505
 interference of, 537–543
 polarization, 507–513
 quantum theory, 594–598
 rectilinear propagation of, 553
 scattering of, 519–521
 speed of, 495, 497
 the photoelectric effect, 598–603
 wavelengths of visible region, 506
 wave–particle duality of, 614
- Light year, ca, 648
 Line spectra (*also* Emission spectra), 506, 568
 Linear accelerators (Linacs), 668–672
 Linear momentum, **189**, 190
 and centre of mass, 211, 212
 and impulse, 190–197
 conservation in one dimension, 199–202
 conservation in two dimensions, 203–210
 Linear motion
 algebraic description of, 10–19
 graphical analysis of, 24–31
 Linear polarization (*see* Plane polarization)
 Lippershey, Hans, 482
 Liquid crystal displays (LCDs), 516, 517
 Lithium
 atomic model of, 686 *illus.*
 in nuclear fusion, 715
 Lodestone, 436
 Longitudinal waves, 486
 Long-wave radio wavelength, 497 *illus.*
 Lord Rayleigh, 561, 596
- M**
- Mach number, 496
 Macroscopic waves, 554
 Magnetic domains, 437
 Magnetic field
 electrons moving in, 656, 657
 in current-carrying conductors, 444 *illus.*
 in solenoids, 443, 444
 lines, 392 *illus.*
 maps, 438–440
 of a transverse wave, 487
 Magnetic flux, 439
 Magnetic forces, 436
 law of, 437
 on conductors and charges, 447–459
 on moving charges, 457–459
 Magnetic induction, 438 *table*
 Magnetic permeability, 445

- Magnetic Resonance Imaging (MRI), 472
- Magnetism and gravity, 390
- Magnetohydrodynamics, 460
- Magnetron, 499
- Magnitude of centripetal motion, 101
- Maiman, T.H., 483
- Malus' law, 509, 510
- Maric, Mileva, 646 *illus.*
- Marsden, Ernest, 412
- Mass, **33**
- defect, 688
 - difference, 688
 - dilation, 652–658
 - equivalence, 605
 - of atomic particles, 408
 - of electrons and protons, 462–464
- Mass spectrometer, 464, 465
- Mass–energy equivalence, 662–668
- Matter waves, 485, 606–608
- Maximum lines, 536–542
- in single-slit diffraction, 557–561
- Maxwell, James Clark, 469, 483, 495
- Maxwell's equation of electromagnetism, 469, 470
- Mechanical energy, 248 *illus.*
- Mechanical waves, 485
- Mechanics, 2, 3, **5**
- Medium-wave radio wavelength, 497 *illus.*
- Members, 171
- Mendeleev, Dmitri, 720
- Mesons, 724, 725 *illus.*
- Metre, standard length of, 547
- Metric system, 3 (*see also* SI units)
- prefixes of, 7 *table*
- Metric unit, 6, 7
- Michelson, Albert A., 497, 544, 547, 638, 639
- Michelson-Morley null result, 639
- Microscopic waves, 554
- Microwave oven, 499 *illus.*
- Microwave safety, 522, 523
- Micro-wavelength, 495 *illus.*, 497 *illus.*, 498 *table*
- Millikan, Robert A., 380, 415
- Millikan's elementary charge calculations, 416 *table*
- Millikan's oil-drop apparatus, 415 *illus.*
- Minimum lines (*also* Nodes), 536–542
- in single-slit diffraction, 556–561
- Moderation of fission, 708, 710
- Modulus, 165
- values for various substances, 166 *table*
- Molecules, 725 *illus.*
- Moment of force (*see* Torque)
- Moment of inertia, **332**–338
- Momentum (*see also* Linear momentum), 190
- and kinetic energy, 241, 242
 - conservation of, 202 *illus.*
 - history of, 186, 187
 - linear vs. angular, 348 *illus.*, 350 *illus.*
 - of photons, 603–606
 - relativistic, 663, 664
- Monopole, 390
- Morley, E.W., 638, 639
- Motion
- angular equations of, 327, 328, 331
 - linear vs. angular, 329 *illus.*
 - states of, 35 *illus.*
 - uniform, **9**
- Motor principle, 447, **448**
- applying, 460–466
- Muon, 641
- N**
- Nambu, Yoichiro, 731
- Natural resonance frequency, 520
- Negative force and electric charges, 383
- Negative time, 82
- Net force, 36
- and static equilibrium, 130
- Neutral equilibrium, **155** *table*
- Neutrino, 654, 694, 722, 725, 726
- Neutron cycle, 712 *illus.*
- Neutron star (*also* Pulsar), 716
- Neutrons, 372, 686
- mass of, 408
- Newton, Sir Isaac, 2, 3, 186, 187, 189, 469, 482, 483
- Newton spring scale, 394 *illus.*
- Newton's first law of motion (*also* Law of inertia), **34**, 35, 42, **128**
- rotational equivalent, 336 *illus.*
- Newton's law of universal gravitation, **48**–**51**
- vs. Coulomb's law, 382 *illus.*
- Newton's laws in two dimension, 85–88
- Newton's second law of motion, **36**–**38**, 42, 192 *illus.*
- rotational equivalent, 336 *illus.*
- Newton's third law, **39**–**42**
- and simple harmonic motion, 491
- Nodal lines (*see* Minimum of waves)
- Non-inertial frame of reference, **35**
- Non-isolated system, **230**
- Non-perpendicular vectors, problem solving, 74–77
- Non-reflective coatings, 553
- Normal force, **44**–**47**
- Nuclear activity, measure of, 703, 704 *table*
- Nuclear binding energy, 688
- Nuclear force, 690
- Nuclear stability, 690, 691
- Nucleic acids, microwave effects on, 522, 523
- Nucleons
- binding energy of, 688, 689
 - probing of, 718, 719
- Nucleus, 686
- O**
- Objects
- moments of inertia of, 333 *table*, 334 *table*
 - physical effects as speed approaches c, 668 *illus.*
- Oersted, Hans Christian, 441
- Oersted's principle, 441
- vs. Faraday's principles, 467 *illus.*
- Ommatidia, 518
- Open system, 231 *illus.*
- Optic axis, 513
- Optical activity, 518, 519
- Orbital elements, 301 *illus.*
- Orbital period, 301
- Orbital shapes, 302 *illus.*
- Orbital speed, equation for, 296
- Orbits, 295–302
- Order numbers, 537
- Ordinary (o) ray, 512, 513
- Overdamping, 308
- Ozone layer, 499
- P**
- Paradoxes, 647–649
- Parallel-axis theorem, 337
- Paramagnetism, 442
- Pardies, Ignace, 483
- Partial polarization, 511, 512
- Particle acceleration, 668–672, 674, 718
- Path difference, 538
- effect on thin-film interference, 548, 549
- Pauli exclusion principle, 730
- Pendulums, 306, 307
- Pericynthion, 301
- Period of a wave, 488
- Period of rotation, 102
- Periodic waves, 486
- Permanent magnetism, 438 *table*
- Perpendicular vectors, 74 *illus.*
- Phase, 486
- lag, 504
 - shift, 490, 535, 536

- Phase velocity of a wave, 501
 Phosphors, 506 *illus.*
 Photoelastic analysis, 517
 Photoelectric effect, 598–603
 Photon wavelength, 613
 Photons, 595, 599
 - and momentum, 603–606
 - energy of, 600, 601, 603
 - position uncertainly in diffraction, 617–619
 - probability distribution, 616
 Pi meson, 724, 725 *illus.*
 Piezoelectric crystals, 624
 Pions, 654, 655
 Pit, of a CD, 575
 Pivot point, 135
 - on the human body, 148 *table*, 149 *table*
 Planck, Max, 595
 Planck's black-body equation, 596
 Planck's constant, in eVs, 600
 Planck's equation, 595
 Plane polarization (*also* Linear polarization), 507
 Planetary motion, 298–300
 Plasma gas, 714
 Plato, 186
 Point charges, 377
 - electric potential around, 409–411
 - field lines around, 392 *illus.*
 - force–distance relationship between, 407
 Points of insertion, for tensile forces, 148
 Poisson, Simon, 553
 Polarization, 507–513
 - applications of, 514–519
 - in insect eyes, 518
 Polarized light microscopy, 518
 Polarizer, of a Polaroid, 510
 Polarizing filters, 514
 Polaroid, 508, 509 *illus.*
 Position, 6
 Positive force, 383
 Positron, 723
 - emission in beta decay, 695, 696 *illus.*
 Positron emission tomography (PET), 736, 737
 Posts, 170
 Post-stressed concrete, 171
 Potential (*also* Electric potential), 401–403
 Potential energy (E_p), 249
 - and gravity, 287, 288
 - between point charges, 407
 - change in, 244
 gravitational vs. electrostatic, 400 *illus.*
 - vs. change in potential energy (ΔE_p), 289, 290
 Power, 255–258
 - and the human body, 259
 Precessing, 472
 Pressure, 161, 165 *table*
 Pre-stressed concrete, 171
 Principle of superposition, 534
 Probability waves, 616, 616
 Projectile motion, 78–84
 Projectiles, elliptical path of, 302, 303
 Proper length, 643
 Proper time, 641
 Proton, 372, 686
 - mass of, 408
 Proton-proton cycle, 716
 Ptolemy, 2
 Pulsar (*see* Neutron star)
 Pythagoras, 2, 482
 Pythagoras' theorem, 64
- Q**
 Quadratic equation, 14
 Quality factor, 703
 Quanta, 595
 Quantum chromodynamics, 730, 731
 Quantum electrodynamics, 729
 Quantum theory, 593–598
 Quantum tunnelling, 622, 623
 Quarks, 723–725
 - colour charge of, 731, 732
 - decay, 733
- R**
 Radar, 516
 Radian measure, 318–321, 490
 Radiation detection, 703, 704
 Radio wavelength, 495, 498
 Radioactive dating, 698–700
 Radioactive decay curve, 698
 Radioactive emissions, 691 *table*
 Radioactivity, 690
 Range, 74
 - of projectiles, 302
 Range equation, 83, 112
 Rarefaction, 486 *illus.*, 487 *illus.*
 Rayleigh criterion, 561
 Rayleigh-Jeans law, 596
 Re-bars, 171
 Recoilless rifle, 43
 Rectilinear propagation of light, 553
 Reflection
 - and polarization, 511, 512
 - in a thin film, 548
 Reflection grating, 563
- Refraction, 500–506
 - of optical medium, 504, 505
 Refractive index, 501, 502 *table*
 - effect on thin-film interference, 549
 Relative motion, 70–77, 634
 Relativistic effects, 658
 Relativistic energy, 664–667
 Relativistic length, 643
 - equation for, 644
 Relativistic momentum, 663, 664
 Relativistic time, 641
 Relativistic velocity addition, 660
 Relays, 445 *table*
 Residual force, 730
 Resolution, 561, 562
 - by spectrometry, 569, 570
 Resolving power, 570, 571 *illus.*
 Rest energy, 665
 Rest mass, m_0 , 653
 Retrograde motion, 298 *illus.*
 Reverse magnetization, 438
 Right-hand rule #1, 442 *illus.*
 Right-hand rule #2
 - and Lenz's law, 467–469
 - for conventional current flow, 444 *illus.*
 Right-hand rule #3
 - and magnetohydrodynamics
 - propulsion, 460 *illus.*
 - for convention current flow, 448
 - and force direction of a moving charge, 458, 459
 Right-hand rule for torque, 135
 Romer, Olaf, 483
 Rotation direction conventions, 136
 Rotational energy, 339–341
 Rotational equilibrium, 137
 Rotational inertia, 337
 Rotational kinetic energy, 342, 343
 Rudolf, Heinrich, 483
 Rutherford, Ernest, 372, 412, 721 *illus.*
 Rutherford's gold-foil experiment, 412
- S**
 Salam, A, 733
 Satellite orbits, 297
 Satellites, 109, 110
 Scalar, 6
 Scanning tunnelling microscopy, 624, 625
 Scattering, 519–521
 Schrödinger, Erwin, 721 *illus.*
 Scientific method, 2
 Seat belts, 267
 Secondary waves, 504
 Sedimentation, 107
 Semimajor axis, 301

- Shawlow, A.L., 483
 Shear modulus, 165 *table*
 Shear strength, 169 *table*
 Shear stress, **162** *table*
 Shock absorbers, 267, 309
 Short-wave radio wavelength, 497 *illus.*
 SI units (Système International d'Unités), 3, 6
 - for acceleration, 9
 - for circular motion, 325
 - for electric field strength in a parallel-plate capacitor, 415
 - for electric potential, 401
 - for energy, 335
 - for force, 32
 - for mass, 33
 - for power, 255
 - for pressure, 161
 - for stress, 161
 - for torque, 136, 335
 - for work, 136, 233
 Sieverts, Sv, 703, 704 *table*
 Simple harmonic motion, 303–307, 486, 491
 - damped, 308, 309
 - equations of, 490, 493
 - in two dimensions, 492
 Simultaneity, 646, 647
 Sine law, 67, 206, **488** *illus.*, 490, 491, 492
 Single-slit diffraction, 554–562
 Single-slit equations, 555–561
 Snell, Willebrord, 482
 Snells' law, **502**–504
 Snow mobiles, 214, 215
 Sodium lamp, 570 *illus.*
 Solenoids, 443, 444
 Sound, 487
 Sound waves, 554
 Spacetime interval, 650
 Spacetime invariance, 649–651
 Special relativity
 - Einstein's first postulate, 634–636
 - Einstein's second postulate, 637–639
 - energy triangle of, 671 *illus.*
 - summary of, 673 *illus.*
 Spectra, **506**, 569
 - of hydrogen gas, 609 *illus.*
 Spectroscope, 506
 Speed
 - average, **8**
 - linear vs. angular, 323, 326
 - of electromagnetic waves, 494, 495
 - relation to length, 643
 - tangential vs. angular, 325
 Speed of light, c , 495, 497
 Spin quantum number, 722
 Spine structure, 170 *illus.*
 Spring constant, 160, 251
 Springs
 - solving energy of, 252, 253
 - simple harmonic motion of, 303–307
 - total energy of system, 305
 Square of the spacetime interval, 650
 Stability, **155**
 - and equilibrium, 155–158
 Standard model, 721
 Stanford Linear Accelerator Center, 669, 674, 719 *illus.*
 Static equilibrium, 128
 - and centre of mass, 130–132, 145, 146
 - balancing forces and torque, 139–145
 - conditions for, 139 *table*
 - of human body, 148–153
 Static friction, 45
 Statics, **128**
 Stopping potential, V_{stop} , 599
 Strain, **163**
 - in construction, 170, 171
 - parameters of, 164 *illus.*
 Strength of building materials, 169 *table*
 Stress, **161**–170
 - building collapse from, 172
 - in construction, 170, 171
 String-and-pulley, 93–98
 Sub-critical mass, in fission, 709 *illus.*
 Sudbury neutrino observatory (SNO), 725, 726
 Sun, electromagnetic waves from, 498, 499
 Supercrest, 534 *illus.*
 Supernova, 716
 Supertrough, 534 *illus.*
 Systems, 199, 230–232
- T**
- Tangential acceleration, 323
 Tangential velocity, 324
 Taylor, Richard, 735
 Telsa, Nikola, 449
 Temporary magnetism, 438 *table*
 Tensile forces, 148
 Tensile strength, 169 *table*
 Tensile stress, **162**, 165 *table*
 Tension force
 - and centripetal force, 105, 106
 - of tendons, 169
 Test charge, 388
 Test magnet/mass, 389
 Thales of Miletus, 372
- Thermal neutrons, 707
 Thin-film interference, 548–552
 Thomas, J.J., 462
 Thompson, Benjamin, 187
 Thomson, George, 608
 Thomson, J.J., 721 *illus.*
 Thomson, William (Lord Kelvin), 187
 Thrust, **201**
 Tides, 350, 351
 Time dilation, 640–643
 Tires, 52
 Torque (*also* Moment of force), **134**–136
 - analysis in a yo-yo, 353
 - and moment of inertia, 335, 336
 - direction conventions, 140
 - problem solving, 145 *illus.*
 Total energy, 405 *illus.*, 665
 - of an elliptical orbit, 301, 302
 Total moment of inertia, 337, 338
 Townes, C.H., 483
 Translation equilibrium, 132
 Transmission grating, 563
 Transmutation
 - artificial, 700–702
 - of nuclear particles, 690
 Transuranic, 702
 Transverse waves, 486
 Travelling waves, 486
 Tritium, 687
 - in fusion reaction, 713, 715
 TRIUMF cyclotron, 674, 720
 Trough, 534 *illus.*
 Truss, 171
 Twin paradox, 648
- U**
- Ultracentrifuge, 108
 Ultraviolet wavelength, 495 *illus.*, 497 *illus.*, 498 *table*
 Unbalanced forced, 33
 Underdamping, 308
 Unified atomic mass units, 687
 Unified field theory, 471
 Uniform circular motion, **98**–102
 Uniform motion, **9**
 Uniform velocity, **9**
 Unit analysis, 7, 194
 - for elastic potential energy, 252
 - for kinetic energy, 240
 - for moment of inertia, 336
 - for work, 234
 Unit conversion, 7
 - of kg to MeV/ c^2 , 670
 Unit
 - for electric current, 454, 455
 - for electrical power, 257
 - for magnetic field strength, 449

- Universal gravitation constant, G , 48
 Universal gravitation equation, 285
 Universal wave equation, **495**
 Unruh, William, 735
 Unstable equilibrium, **155** *table*
 Uranium, in fission, 707, 708
 UV catastrophe, 596
- V**
 Van de Graaff generator, 668
 Van Musschenbroek, Pieter, 419
 Vector, **6**
 arrow, 8
 direction, 64 *illus.*
 Vector addition, 64–68
 by component method, 207 *illus.*
 Vector subtraction, 69
 Vectors in two dimensions, 64–70
 Velocity
 addition at speeds close to c , 659–661
 average, **8**
 graphical derivation, 24, 25
 instantaneous, **8**
 tangential, 324
 uniform, **9**
 Vertical plane
- and Newton's laws in two dimension, 86
 centripetal force in, 104
 Very-high-frequency (VHF) radio wavelength, 497 *illus.*
 Viscosity, 45
 Visible wavelength, 495 *illus.*, 497 *illus.*, 498 *table*
 Volta, Alessandro, 401
 von Fraunhofer, Joseph, 483
- W**
 Wallis, John, 186
 Walton, Ernest, 701
 Water waves, 486 *illus.*, 554
 Wave propagation, 486
 Wave theory of light, 593, 594
 Wavefronts, 502 *illus.*, 503
 Wavelength, 488
 Wave-particle duality, 614
 Waves, **485**
 Weight, **33**
 Weinberg, S, 733
 Whimshurst machine, 376
 Wien's law, 596
 Wobble, 300
 Work, **233**–238
- and rotational energy, 339–341
 by gravity, 286–288
 determining graphically, 237, 238
 dimensional unit analysis, 237
 moving a charge between plates, 406, 407
 of a charge in an electric field, 401
 Work function, W_0 , 600, 601, 603
 Work–energy theorem, **240**, 241
- X**
 X-ray diffraction, 571, 572
 X-ray wavelength, 495 *illus.*, 497 *illus.*, 498 *table*
- Y**
 Young, Thomas, 165, 483, 537
 Young's double-slit experiment, 537, 538 *illus.*
 Young's modulus (*also* Elastic modulus), **164**, 165
 Young's three double-slit equations, 538–543
 Yukawa, Hideki, 494
- Z**
 Zweig, G., 723

Photograph Credits

Every effort has been made to find and to acknowledge correctly the sources of the material reproduced in this book. The publisher welcomes any information that will enable it to rectify, in subsequent editions, any errors or omissions.

COVER PAGE: Getty Images

UNIT A: Opener: Gilbert Lundt/Corbis/Magma; Timeline photos: Isaac Newton: Original Artwork by John Vanderbank (in preparation for painting), courtesy AIP Emilio Segre Visual Archives, Lande Collection; Lunar vehicle: Courtesy NASA; Hockey image: Karl Weatherly/Corbis/Magma; CHAPTER 1: Opener: EyeWire Inc.; Fig.1.1: Images/Firstlight.ca; Fig.1.2: Wothe/Firstlight.ca; Fig.1.4: Ken Straaten:Firstlight.ca; Fig.1.5: EyeWire Inc.; Fig.1.6: Alan Marsh/Firstlight.ca; Fig.1.7: Duomo/Corbis/Magma; Fig.1.9: Rubberball/EyeWire Inc.; Fig.1.10: Ford of Canada Archive; Fig.1.13a: Corbis/Magma; Fig.1.13b: Istituto e Museo di Storia della Scienza – Photographic Department; Fig.1.14: Courtesy NASA; Fig.1.16: Courtesy CN Tower; Fig.1.31: Tim Wright/Corbis/Magma; Fig.1.34: PORSCHE, CARRERA and the Porsche crest are registered trademarks, 911 and the distinctive shapes of PORSCHE automobiles are trademarks of Dr. Ing. h.c.F. Porsche AG. Photograph of the PORSCHE automobile used with permission of Porsche Cars North America; Fig.1.40: Yokohama/Firstlight.ca; Fig.1.41: Jeremy Davey, ThrustSSC Team; Fig.1.47: Courtesy Tara Wildlife Inc.; STSE.1.1: Michelin North America (Canada) Inc.; Fig.1.59: Photodisc/EyeWire Inc.; Fig.1.60: Corbis/Magma; Fig.1.63a: EyeWire Inc.; Fig.1.66: Nathan J. Torkington; CHAPTER 2: Opener: ©Jonathan Selkowitz; Fig.2.6: Corbis/Magma; Fig.2.11: Ranger Boats; Fig.2.15a: Rubberball/EyeWire Inc.; Fig.2.17: Corbis/Magma; Fig.2.23: Photodisc/EyeWire Inc.; Fig.2.26: MaXx Images/Indexstock; Fig.2.30: Photodisc/EyeWire Inc.; Fig.2.31: EyeWire Inc.; Fig.2.33a: Bettman/Corbis/Magma; Fig.2.43: Corbis/Magma; Fig.2.46: Baine Stanley/Firstlight.ca; Fig.2.47: Darwin Wiggett/Firstlight.ca; Fig.2.51a and b: Erik Konopka; Fig.2.52a: Corbis/Magma; Fig.2.52b: Telesat Canada; Fig.2.54: Courtesy NASA; Fig.STSE.2.1: Karl Weatherly/Corbis/Magma; Fig.Lab.2.3: Corbis/Magma; Fig.Lab.2.4: Ken Denton; CHAPTER 3: Opener: Kevin Schafer/Corbis/Magma; Fig.3.1: MSCUA, University of Washington Libraries, Farquharson 12; Fig.3.3: David Nunuk/Firstlight.ca; Fig.3.7a: Brian Heimbecker; Fig.3.12a: Courtesy Karen and Jeff D'elia; Fig.3.21a: Parks Canada, Trent-Severn Waterway; Fig.3.22a: Kevin Fleming/Corbis/Magma; Fig.3.28b: Ken Redmond Photography; Fig.3.28c: Bettmann/Corbis/Magma; Fig.3.36: MaXx Images/Indexstock; Fig.3.42: It Stock/Firstlight.ca; Fig.3.44: Brian Heimbecker; Fig.3.48a and b: Mr. Masataka Mori, a director of the Japan Karate Association and chief instructor of JKA Shotokan Karate-Do International; Fig.3.48c: TIP-IT® is a trademark owned by Mattel, Inc., used with permission. Copyright 2002 Mattel Inc., all rights reserved; Fig.3.48d: Jenga® is a registered trademark of Pokonobe Associates and is used with its per-

mission. © 2002 Pokonobe Associates. All rights reserved.; Fig.3.48e: Yann Arthus-Bertrand/Corbis/Magma; Fig.3.48f: Photo by Kathis Wilson/Jacko von Tem, BH, BST, TT, CGC; Fig.3.51a: Joseph Sohm; ChromoSohm Inc./Corbis/Magma; Fig.3.51b: Patrick Ward/Corbis/Magma; Fig.3.58a: EyeWire Inc.; Fig.3.59 (left): EyeWire Inc.; Fig.3.59 (right): Irwin Publishing Ltd.; Fig.3.64b: Kevin Schafer/Corbis/Magma; Fig.3.64d: Martin Jones/Corbis/Magma; Fig.3.64e: Corbis/Magma; Fig.3.64f: Brian Heimbecker; Fig.STSE.3.1: AP/Wide World Photos; Fig.STSE.3.2: Staff of Level3.com/DHD Multimedia Gallery; Fig.3.65: Brian Heimbecker; UNIT B: Opener: Bryan Mumford, copyright 2002; Timeline photos: Snowboarder: Duomo/Corbis/Magma; Football Tackle: Reuters NewsMedia Inc./Corbis/Magma; Pool table image: David Katzenstein/Corbis/Magma; CHAPTER 4: Opener: Patrick Kovarik/Corbis/Magma; Fig.4.7a: Flagworld.com; Fig.4.8a: Chuck Savage/Firstlight.ca; Fig.4.8b: H.T. Kaiser/Firstlight.ca; Fig.4.8c: Mendola/Bill Vann/Firstlight.ca; Fig.4.9c: Comstock Images/Russ Kinne; Fig.4.16: Reuters NewMedia Inc./Corbis/Magma; Fig.4.25: Images/Firstlight.ca; Fig.STSE.4.1: Alan Marsh/Firstlight.ca; Fig.STSE.4.2: ©ZAP with permission ZAPWORLD.com 2002; Fig.STSE.4.3: Bob Krist/Corbis/Magma; CHAPTER 5: Opener: Courtesy NASA; Fig.5.4: AFP/Corbis/Magma; Fig.5.22a: Irwin Publishing Ltd.; Fig.5.22b: Courtesy Sony Corp.; Fig.5.23: Gunter Marx Photography/Corbis/Magma; Fig.5.26: Wally McNamee/Corbis/Magma; Fig.5.39a: Duome/Corbis/Magma; Fig.5.39b: Joe Marquette/Corbis/Magma; Fig.5.39c: Corbis/Magma; STSE.5.1: © 2002 Nintendo/Creatures inc. GAME FREAK inc. All Rights Reserved.; STSE.5.3: <http://www.ameritech.net/users/paulcarlisle/trebuchet.html>; CHAPTER 6: Opener: Courtesy of SOHO/EIT consortium. SOHO is a project of international cooperation between ESA and NASA; Fig.6.5: Courtesy NASA; Fig.6.6: Courtesy NASA; Fig.6.7: Courtesy NASA; Fig.6.8: Courtesy NASA; Fig.6.11: Courtesy NASA; Fig.6.14: Courtesy NASA; Fig.6.19: Bettmann/Corbis/Magma; Fig.6.20: Smithsonian Institution; Fig.6.27b: Irwin Publishing Ltd.; Fig.6.28: Eric Stern; Fig.STSE.6.1: Courtesy NASA; CHAPTER 7: Opener: Firstlight.ca; Fig.7.1a: MaXx Images/Indexstock; Fig.7.1b: Wally McNamee/Corbis/Magma; Fig.7.1c: Atlan Jean Louis/Corbis Sygma/Magma; Fig.7.11: Bohemian Nomad Picturemakers/Corbis/Magma; Fig.7.13: Courtesy NASA; Fig.7.16: Copyright 2001 Bentley Systems, Inc. Used with permission; Fig.7.26: Philip Harvey/Corbis/Magma; Fig.7.43a: Estock Photography/Everett Johnson; Fig.7.44a and b: Corbis/Magma; Fig.7.46: Firstlight.ca; Fig.STSE.7.1a: John Sokolowski; Fig.STSE.7.1b: Alain Nogues/Corbis Sygma/Magma; Fig.STSE.7.2a: Reproduced with the permission of Stoddart Publishing; Fig.STSE.7.3a: Courtesy of the Canada Science and Technology Museum, Ottawa; UNIT C Opener: Michael Deyoung/firstlight.ca; Timeline photos: Cliff diver: Scott Spiker/Firstlight.ca; Microchip: Charles O'Rear/Corbis/Magma; Northern Lights: Bryan Alexander/firstlight.ca; CHAPTER 8: Opener:

Ontario Science Scentre; Fig.8.1: Irwin Publishing Ltd.; Fig.8.5: Olivier McKay/Firstlight.ca; Fig.8.7a: Ontario Science Centre; Fig.8.9: Courtesy of Physics Department, University of Texas at Dallas; Fig.8.25d: Phil Degginger/Getty Images; Fig.8.46: Courtesy Jerry DiMarco/Montana State University; Fig.8.47: Courtesy Sony Corp.; Fig.8.49: Courtesy of the Archives, California Institute of Technology; Fig.STSE.8.5: Sabrex of Texas, Inc.; Fig.8.57: MaXx Images/Indexstock; CHAPTER 9: Opener: Bettmann/Corbis/Magma; Fig.9.5: MaXx Images/Indexstock; Fig.9.8a and b: Jerry DiMarco/Montana State University; Fig.9.10a and b: Jerry DiMarco/Montana State University; Fig.9.11a: Jim Lyons/Jim MacLachlan; Fig.9.14: Jerry DiMarco/Montana State University; Fig.9.15a: Jim Lyons/Jim MacLachlan; Fig.9.15b: Science Software Systems; Fig.9.18b: ©Wayne Decker/Fundamental Photos NYC; Fig.9.19b: With permission of Goodheart-Willcox, publisher. *Applied Electricity and Electronics*, by Clair A. Baynes; Fig.20.b: www.umei.com/fire-protection-accessories/fire-alarm-bells/fire-alarm-bell-1-10.htm; Fig.9.36: Joel W. Rogers/Corbis/Magma; Fig.9.37: This material used with permission of Johnson & Wiley Sons, Inc.; Fig.9.40a: Papilio/Corbis/Magma; Fig.9.42a: Courtesy of Pacific Northwest National Laboratory; Fig.STSE.9.1: Photo by Paul Barrette, courtesy of Canada Diagnostic Centres; Fig.STSE.9.3: Firstlight.ca; Fig.lab.9.1: Brian Heimbecker; UNIT D: Opener: © 1997 Michael Dalton, Fundamental Photographs, NYC; Timeline photo: eye and laser: Image Bank/Mel Digiocomo. Reproduced with the permission of Stoddart Publishing; CHAPTER 10: Opener: Darwin Wiggett/Firstlight.ca; Fig.10.1a: John Gillmour/Firstlight.ca; Fig.10.1b: Tekinga Microphones, Sweden; Fig.10.1c: Corbis/Magma; Fig.10.3: Steve Short/Firstlight.ca; Fig.10.7: US Geological Survey; photo obtained from the National Geophysical Data Centre, Boulder, CO; Fig.10.32: Jerry DiMarco/Montana State University; Fig.10.33a and b: reproduced with the permission of Stoddart Publishing; Fig.10.37a: Roger Tidman/Corbis/Magma; Fig.10.37b: Courtesy Jason Robertson; Fig.10.39a and b: Erik Konopka; Fig.10.44: figure 8.21, p. 233 from OPTICS by Eugene Hecht and Alfred Zajac. Copyright © 1974 by Addison-Wesley Publishing Co. Inc. Reprinted by permission of Pearson Education, Inc.; Fig.10.46: Irwin Publishing Ltd.; Fig.10.48: Ed Bock/Corbis/Magma; Fig.10.49: Corbis/Magma; Fig.10.52: Peter Aprahamian/Sharples Stress Engineers LTD/Science Photo Library; Fig.10.53: Dr. Jeremy Burgess/Science Photo Library; Fig.10.57a: Corbis/Magma; Fig.10.57b: MaXx Images/Indexstock; Fig.10.57c: MaXx Images/Indexstock; Fig.10.57d: Darwin Wiggett/Firstlight.ca; Fig.STSE.10.1: Corbis/Magma; CHAPTER 11: Opener: Corbis/Magma; Fig.11.1a: ©1990 Richard Megna, Fundamental Photographs, NYC; Fig.11.3a and b: reproduced with the permission of Stoddart Publishing; Fig.11.8a: from *Matter and Energy*, MacLachlan/MacNeill/Bell/Spencer; Fig.11.17: © Longman Group Limited 1972; Fig.11.23: reproduced with the permission of Stoddart Publishing; Fig.11.24: Photograph/Hologram by Stephen W. Michael; Fig.11.26: hologram-NEED; Fig.11.27: Image

Bank/Mel Digiocomo. Reproduced with the permission of Stoddart Publishing; Fig.11.29: reproduced with the permission of Stoddart Publishing; Fig.11.33a: Science Software Systems, Inc.; Fig.11.35: Flagworld.com; Fig.11.37: Cagnet, Fancon & Thrierr, *Atlas of Optical Phenomena*, copyright 1962, Springer-Verlag OHG, Berlin; Fig.11.39: From *The Nature of Light and Sound*, Holt, Rinehart and Winston of Canada, 1974; Fig.11.42: John Gillmour/Firstlight.ca; Fig.11.52a: Truax/The Image Finders®, Fig.11.52b: Courtesy NASA; Fig.11.55: Corbis/Magma; Fig.11.58b and c: reproduced with the permission of Stoddart Publishing; Fig.11.60a: Courtesy of Ajax Scientific Ltd.; Fig.11.61: reproduced with the permission of Stoddart Publishing; Fig.11.63: reproduced with the permission of Stoddart Publishing; Fig.11.66: Andrew Syred/Science Photo Library; Fig.11.70: © Longman Group Limited 1972; Fig.STSE.11.1a: Dr. Jeremy Burgess/Science Photo Library; Fig.STSE.11.3: www.howstuffworks.com/cd2.htm; Fig.lab.11.3a and b: © Longman Group Limited 1972; UNIT E: Opener: Courtesy of Sandia National Laboratories; CHAPTER 12: Opener: Dr. David Wexler, Coloured by Dr. Jeremy Burgess/Science Photo Library; Fig.12.3: A. Farnsworth/Firstlight.ca; Fig.12.4: MaXx Images/Indexstock; Fig.12.5: Steve Lawrence/Firstlight.ca; Fig.12.6: Bettmann/Corbis/Magma; Fig.12.7: reproduced with the permission of Stoddart Publishing; Fig.12.9: Bettmann/Corbis/Magma; Fig.12.15a: Corbis/Magma; Fig.12.17: Bettmann/Corbis/Magma; Fig.12.20: www.godunov.com/Bucky/fullerene.html; Fig.12.21a: Bettmann/Corbis/Magma; Fig.12.25: Reprinted with permission from *American Journal of Physics* 57(2). Copyright 1989, American Association of Physics Teachers; Fig.12.29: Bettmann/Corbis/Magma; Fig.STSE.12.1: L. Medard/Eurelios/Science Photo Library; Fig.STSE.12.2: Lawrence Livermore Laboratory/Science Photo Library; Fig.STSE.12.4: E. Graugnard, T. Lee and R. Reifenberger, Purdue University; Department of Physics; Chapter 13: Opener: Courtesy of R. Williams, the HDG Team (ST scl) and NASA; Fig.13.1: Corbis/Magma; Fig.13.5: MC Escher's "Waterfall" © 2002 Cordon Art BV – Baarn – Holland, all rights reserved; Fig.13.8b: Bettman/Corbis/Magma; Fig.13.15: Archiv Daniel Spoerri, Swiss National Library, Berne; Fig.13.36: Courtesy Stanford Linear Accelerator Centre; Fig.13.39: Courtesy Radionics; Fig.13.42: Wally McNamee/Corbis/Magma; Fig.13.45: CERN; Fig.STSE.13.2: TRIUMF-ISAC, Dr. C. Kost; Fig.STSE.13.3: CERN; Fig.STSE.13.4: copyright 1990 Richard Megna, Fundamental Photos, NYC; Fig.STSE.13.5: Kevin M. Dunn; CHAPTER 14: Opener: Lawrence Berkely National Library; Fig.14.10: Copyright Parks Canada, A. Cornellier; Fig.14.13b: Roger Ressmeyer/Corbis/Magma; Fig.14.14: Brian Milne/Firstlight.ca; Fig.14.18: Copyright Wardrop Engineering Inc., 2001; Fig.14.19: Reprinted with permission from "Russian Nuclear Weapons Museum," *Physics Today* 49(11), November 1996, p. 31 © 1996, American Institute of Physics; Fig.14.20: Princeton Plasma Physics Laboratory; Fig.14.21: Courtesy NASA; Fig.14.21: Courtesy US Department of Energy; Fig.14.23: Courtesy Ontario Power Generation; Fig.14.24: Courtesy Atomic Energy of Canada Ltd.; Fig.14.28: Copyright

© Wardrop Engineering Inc., 2001 (www.wardrop.com); Fig.14.30: Courtesy Princeton Plasma Physics Laboratory; Fig.14.31: Courtesy NASA; Fig.14.32: Courtesy Stanford Linear Accelerator Centre; Fig.14.34: Courtesy Stanford Linear Accelerator Centre; Fig.14.35: David Parker/Science Photo Library; Fig.14.37a: Courtesy Fermi National Accelerator Laboratory; Fig.14.37b: CERN photo; Fig.14.54: C.Powell, P. Fowler and D. Perkins/Science Photo Library; Fig.14.55: C.Powell, P. Fowler and D. Perkins/Science Photo Library; STSE.14.1 (left): PET Gamma detectors; STSE.14.1 (right): PET Machine; STSE.14.3: National Cancer Institute/Science Photo Library; APPENDICES: Loren Santow/Getty Images

Some textual material in the Appendices initially appeared in J. Cutnell and K. Johnson, eds., *Physics*, 5th ed. Published by John Wiley and Sons, Inc, 2001. This material is used by permission of John Wiley and Sons Inc.

