QUANTUM MECHANICAL EFFECTS OF LEDS AND LASERS

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ABSTRACT

Three experiments were carried out to examine quantum mechanical effects. Light emitting diode threshold potentials were used to find an experimental Planck's constant value of $(6.1\pm.2)\times10^{-34}$ m^2 kg s^{-1}, with an error of 7.94% from the accepted value. Photos of a single slit diffraction pattern from a red laser were analyzed for its intensity profile. The shape was found to have a coefficient of correlation of 0.744 with theory, indicating a moderate fit. Conversely, the intensity distribution had a mean S/N ratio of $(0.3\pm.8)$, indicating significant error. A light box and diffraction grating were set up to produce a white light spectrum, from which a black body curve and experimental temperature of (4810 ± 20) K for the light bulb filament were found.

I INTRODUCTION

On quantum mechanical scales, electromagnetic (EM) waves such as light define the transfer of energy. These particles then form atoms and molecules, and consequently, matter. Thus, an understanding of EM waves aids in the comprehension of interactions between all substances.

II THEORY

The energy of an EM wave is given by:

$$E = hf = \frac{hc}{\lambda} \qquad [1]$$

(HRW, 2011)

Where:

E = Energy of the wave (J)
h = Planck's constant (6.62607004E-34 m^2 kg s^{-1})
f = Frequency of the wave (Hz)
c = Speed of light in a vacuum (m s^{-1})
λ = Wavelength (m)

Wien's displacement law relates black body curve peak to its temperature by:

$$\lambda_{max} = \frac{b}{T} \qquad [2]$$

(HRW, 2011)

Where:

λ_{max} = Wavelength of peak (m)
b = Displacement constant (2.898×10^{-3} m K)
T = Temperature of black body (K)

The potential of a discharging capacitor in an RC circuit with a diode is given by:

$$V(t) = V_{th} + (V_s - V_{th})e^{-\frac{t}{\tau}} \qquad [3]$$

(Sunnu et al., 2012)

Where:

$V(t)$ = Potential across capacitor (V)
V_{th} = Threshold potential (V)
V_S = Initial source potential (V)
t = Time (s)
τ = RC time constant (s)

The energy gap of an LED is:

$$E_{light} = eV_{th} \qquad [4]$$

(Kansas State University, 2010)

Where:

E_{light} = Energy of light from the diode (J)
e = Elementary charge (C)

The de Broglie equation states:

$$p = \frac{h}{\lambda} \quad [5]$$

(HRW, 2011)

Where:

p = Energy gap of the diode (kg m s^{-1})

The minima created by single slit diffraction, as well as the angles created by a diffraction grating, are given by:

$$\theta = arcsin(m\lambda a^{-1}) \quad [6]$$

(HRW, 2011)

Where:

θ = Deflection angle (rad)
m = Minima number
a = Width of slit (m)

The intensity of light passing through a single slit is given by:

$$I = I_m \left(\frac{\sin(\pi a y \lambda^{-1} d^{-1})}{\pi a y \lambda^{-1} d^{-1}}\right)^2 \quad [7]$$

(HRW, 2011)

Where:

I = Intensity (candela)
I_m = Maximum intensity (candela)
y = Position along projection surface (m)
d = Distance from slit to projection surface (m)

Planck's Law provides a black body's spectral radiance of light from the light wavelength and black body temperature:

$$B = 2hc^2\lambda^{-5}\left(e^{\frac{hc}{\lambda kT}} - 1\right)^{-1} \quad [8]$$

(Gregerson, 2016)

Where:

B = Spectral radiance (W sr^{-1} m^{-2} Hz^{-1})

III Method

A 1 mF capacitor, 1 kΩ resistor, 3.0 V battery, and eight light emitting diodes (LEDs) of various colours were acquired pursuant to van Bemmel (2019). An ML 820 Helium Neon Laser was used as a laser source and used in conjunction with a slit of width (87.864±.005) µm. An Alden Industries Ltd Model 401 1003 Lightbulb encased in a ray box was utilized as a light source and was used with a diffraction grating. A Nikon D3300 camera was used for imaging purposes, which was then adjusted using Adobe Lightroom's Lens Correction.

For the first experiment, an LED was connected in a circuit, as shown:

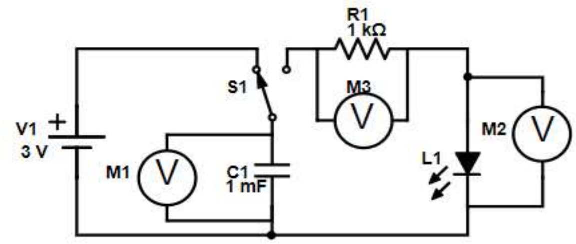

Fig 1. LED Circuit. The circuit that was constructed to measure the properties of the light emitted by eight unique LEDs. Upon full charge of C1, the switch S1 flips and allows C1 to discharge into the LED.

The capacitor was first charged by connecting it to the battery. The capacitor was then quickly connected to the other half of the circuit and allowed to discharge through the diode. Vernier differential voltage probes, set to take measurements 50 times per second were used to find the potential over time as shown in Fig 1. Probes within 10mV of each other were used. Three runs were taken for each diode, and the process was repeated for all eight LEDs.

For the second experiment, the laser was aimed through the single slit to form a diffraction pattern on sheets of white paper on a wall with a metre stick attached. The camera was fixed with a tripod and calibrated by having two pictures taken from the same position. One with the laser on, one with it off. From the metre stick scaling's, it was found that 1 pixel corresponds to $(1.488\pm.007)\times10^{-4}$ m. Fifteen photos of the pattern were taken and straightened digitally.

For the third experiment, the light box was shined through a small opening into a diffraction grating. This created a white light spectrum on a sheet of white paper taped to the wall. The camera was calibrated by taking images before and after the light box was turned on, with a ruler attached to the background. It was found that each pixel has a physical length of (3.161±.007)E-5 m. Fifteen photos were taken and straightened digitally, and each was analyzed in a MATLAB software for intensity.

IV Data

Experiment 1 used the following parts:

Index	Component	Unit	Measured Value
1	Battery	V	4.38±.01
2	Resistor	Ω	980±1
3	Capacitor	mF	1.0±.2

Fig 2. Circuit Components. The components of the first circuit were measured using a multimeter or in previous experiments. These values define properties such as the source potential and the time constant of the circuit.

The following distances were measured for experiment 3 such that trigonometry could be used to find the angles of deflection.

Index	Dimension	Value (cm)
1	d (Grating to Projection)	23.7±.2
2	l (Horizontal from Grating to Violet Edge of Gradient)	4.7±.4
3	w (width of pattern)	4.8±.3

Fig 3. Experiment 3 Dimensions. The distances between various components of experiment 3. This allowed for basic trigonometry to be used with conjunction with [6] to find the angles of refraction, grating width and wavelengths.

15 photos were taken each for the single slit diffraction pattern and white spectrum, both in raw image format. Lens correction was applied to minimize image distortions.

Fig 4. Single Slit Diffraction Pattern. A laser was shone through a single slit onto a white paper background.

Fig 5. White Spectrum Image. A light box was shone through a diffraction grating to produce a spectrum against a white surface with a ruler attached for scale.

V Analysis

The curves for each diode of potential over time were exponentially regressed to the form given by [3]. This process was repeated for all eight LEDs, and the resulting threshold potential values are shown:

Index	Diode Wavelength (±5 nm)	Threshold Potential (V)
1	420	2.87±.05
2	470	2.54±.07
3	565	2.2±.1
4	575	1.80±.07
5	589	1.76±.06
6	606	1.8±.1
7	610	1.8±.1
8	623	1.77±.08

Fig 6. Diode Threshold Potentials. The wavelengths of the diodes in the first experiment are shown with their measured threshold potentials. The former was found from the diode datasheets, while the latter was found experimentally.

These values were then used to find the threshold energies, using [1] and [4]:

$$E_{light} = eV_{th} = hf \qquad [9]$$

This relation was then regressed using one point for each LED, solving for the energy as shown and comparing with the measured frequency, found from the wavelength as seen in [1]. The slope of the regression is the desired Planck's constant.

$$E(f) = (6.1 \pm .2) \times 10^{-34} f \qquad [10]$$

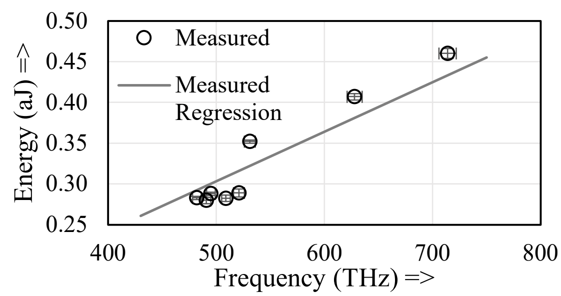

Fig 7. LED Frequency vs. Energy. The frequency of each LED was plotted against its energy by [4] and the trend was linearly regressed by [1] to produce [10] with a slope equal to Planck's constant. The fit has an R^2 value of 0.99.

The resulting value for Planck's constant is (6.1±.2)×10^{-34} m^2 kg s^{-1}, which differs from the accepted value of 6.626×10^{-34} J s and has a resulting error of 7.94%.

It is believed that the discrepancy is partially due to differences between the advertised and actual wavelength. Moreover, the "turn-off" potential of the LED is likely to be higher than the regressed threshold potential, as the LED visibly appears to turn off prior to the end of the regression that the threshold is based on. As a result, regressed values would be lower than accepted, thus leading to lower values of Planck's constant.

For the single-slit experiment, software was created in MATLAB to find the relative luminance for each column of pixels.

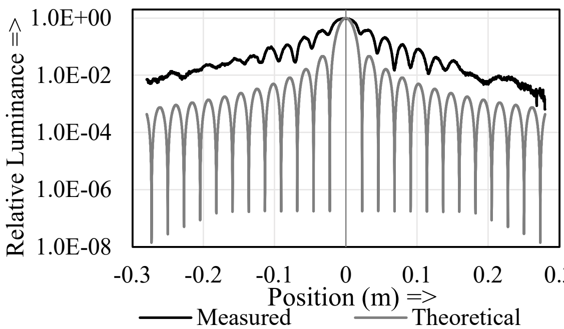

Fig 8. Relative Luminance of Single Slit Diffraction. The relative luminance of the single slit pattern was plotted against position. It is that the measured values are consistently higher than the theoretical. Uncertainties not shown for clarity.

From Fig 8, it is noted that although the positions of maxima and minima from the experiment correlate strongly to theoretical values from [6] and [7], they differ greatly in terms of intensity. Quantitatively, the R^2 between the theoretical and measured relative luminance is 0.744, indicative of a moderate correlation as both share maxima and minima at the same horizontal positions. This supports [6], which gives the positions of minima. Conversely, the mean error for intensity values at each data point yields a mean signal-to-noise ratio of (0.3±.8), showing the error is approximately three times the signal, and thus, poor correlation.

This discrepancy in measured and theoretical values is attributed to the lowering of peak intensity in practice. While the central peak of the theoretical regression has the same intensity as the source as it is comprised of unaltered photons passing through the slit, this likely does not happen in practice due to imperfection in slit and laser construction. As a result, this may substantially lower the central luminance of the measured values, and thus, increase the relative luminance of surrounding regions.

For the black body experiment, the wavelengths of the resulting rainbow after the white light was shone through the diffraction grating were found using a MATLAB program with [6] and plotted against the relative luminance for each pixel column. Relative luminance was then plotted against light wavelength, along with the theoretical values as given by [8]. The literature temperature value was taken to be approximately (2700±5) K.

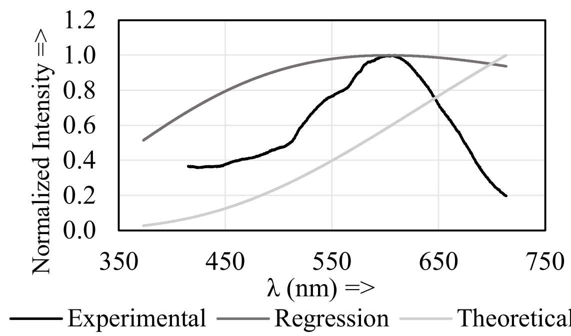

Fig 9. Black Body Curves. The experimental black body curves of the light box compared to the regressed value based on Wein's Law. The horizontal uncertainty for each value is 20nm, while the vertical uncertainties vary but are not shown for clarity.

The maximum relative intensity is seen to be at λ = (600±20) nm. By [2], the experimental temperature is then (4810±20) K, which slightly by approximately 6% lower than the experimental regression which yields T = (5130±10) K and is significantly different from the stated temperature of (2700±5) K. This can be attributed to the manner in which RGB values are perceived and calculated by a camera. With regard to intensity: green and red contribute more to the brightness of a pixel than blue does. (Cook 2009) As a result, the intensity of the range between the two colors may appear higher despite calibrations designed to correct for camera imperfections. This could then lead to an artificial peak at around λ = 600 nm, which is observed by experiment.

To derive the Uncertainty Principle, the uncertainty of the photon's location is the slit width. By [5]:

$$p_y = p \sin\theta = \frac{h}{\lambda}\sin\theta \qquad [11]$$

With m=1 in [6], which was supported by the second experiment, we have:

$$sin(\theta) = \frac{\lambda}{a} \qquad [12]$$

Therefore, from [11], the uncertainty of momentum in the y direction is given by:

$$\sigma_{p_y} = p_y = \frac{h}{\lambda}\cdot\frac{\lambda}{a} = \frac{h}{a} \qquad [13]$$

By letting the width of the slit be equal to the y-position uncertainty, we have:

$$\sigma_y = d \qquad [14]$$

$$\sigma_y \sigma_{p_y} = h \qquad [15]$$

Allowing ħ to be the reduced Planck's constant and equal to $\frac{h}{2\pi}$ gives:

$$\sigma_y \sigma_{p_y} \geq \frac{1}{2}\,\bar{h} \qquad [16]$$

The final equation is known as Heisenberg's Uncertainty Principle. It states that for any particle, the accuracy to which the position can be known is inversely proportional to the accuracy to which the momentum can be known. It thus places a strict limit on the accuracy of observations, especially at the quantum level.

VI Sources of Error

LEDs do not emit a single wavelength, but rather a small interval of them, which was not given in the operating data for the selected diodes. Although the typical uncertainty value was used in calculation, it is possible for this range to be larger than expected or skewed, thus reducing the accuracy or precision of the experiment.

While efforts were made to ensure a dark room while taking images for Experiments 2 and 3, there was still some amount of light shining onto the surface from distant sources, which may have unevenly affected the pixels.

VII Conclusion

In this experiment, light-emitting diodes, single slit diffraction patterns, and white light spectrums were tested. The LED experiment returned a Planck's constant value of $(6.1\pm.2)\times10^{-34}$ $m^2\,kg\,s^{-1}$, returning an experimental error of 7.94%. The signal-to-noise of the measured versus theoretical relative luminance of the single slit experiment was (0.3±.8), likely due to imperfections in slit constructed that lowered the maximum intensity. However, measured values correlated with theory for the position of peaks, with an R^2 value of 0.744 between the two datasets. The temperature of a lightbulb was found based on its blackbody radiation spectrum. Wein's Law was used to find the experimental temperature of the bulb at (4810±20), as opposed to the (2700±5) K that was sourced. Finally, the Uncertainty Principle was derived using equations supported by the single slit experiment.

VIII Sources

van Bemmel, H., "AP Physics C Laboratory Manual", http://www.hmvb.org/apc1920lm.pdf, 2019

Cook, D., "The Algorithms for Converting Colour to Grayscale", https://www.johndcook.com/blog/2009/08/24/algorithms-convert-color-grayscale/, 2009

Gregerson, E., "Planck's Radiation Law", https://www.britannica.com/science/blackbody, 2016

Halliday, D., Resnick, R., Walker, J. (HRW), Fundamentals of Physics 9e, Wiley, 2011

Kansas State University, "Determining Planck's Constant with LEDs", https://web.phys.ksu.edu/vqm/tutorials/planck/, 2010

Nave, R., "Single Slit Diffraction Intensity", http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinint.html

Sunnu, J., Siriya, S., Thamaphat, K., Oopathump, C., Bharmanee, P., Limsuwan, P., "Experimental Set for Measuring the Planck's Constant using LED", https://pdfs.semanticscholar.org/6b77/2821f3d2590ce3bbabe36493bb8f90af7db5.pdf, 2012