Regime-Aware Risk and Portfolio Allocation in the S&P 500 (H1 2025)

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***Abstract*—This project analyzes daily trade data for S&P 500 companies during the first half of 2025 to study how risk and return vary across time and sectors. We construct an equal weight pseudo index from individual stock returns and apply time series diagnostics, GARCH volatility modeling, and a two state Markov switching model to identify high and low volatility regimes. Sector tail risk is measured using historical Value at Risk (VaR) and Expected Shortfall (ES). We compare equal weight, inverse volatility, and minimum variance portfolios using a shrinkage covariance estimator. The results show clear volatility clustering and distinct regimes, with high volatility periods aligned with drawdowns. Portfolios that respond to risk achieve shallower drawdowns and better risk adjusted performance, which illustrates how a regime based allocation can support more robust investment decisions.**

I. Introduction

Equity market volatility is not constant. Calm periods are often interrupted by episodes of stress that can cause concentrated portfolio losses. Standard allocations that ignore time varying risk may be suboptimal when volatility regimes differ sharply. This project addresses three questions for January through June 2025:

1) How strong are volatility regimes in the S&P 500?
2) How does tail risk differ across sectors?
3) Can simple allocation rules based on risk, such as inverse volatility and minimum variance portfolios, improve risk adjusted performance relative to equal weight portfolios?

Understanding volatility regimes and sector tail risk is important for position sizing, risk budgeting, and capital allocation. GARCH type models and Markov switching regimes are standard tools for capturing volatility clustering and regime changes [1], [2]. Combining these with portfolio optimization and covariance shrinkage [3] gives a practical example of how statistically informed risk models can support portfolio construction in noisy, finite sample settings.

II. Data Description and Preprocessing

A. Dataset

We use the Kaggle dataset "S&P 500 Stocks Trade Data for First 6 Months of 2025," which contains daily OHLCV fields (open, high, low, close, adjusted close, volume) for S&P 500 constituents from early January through June 30, 2025. Each row is a ticker day observation. Some days such as holidays are missing uniformly, but the panel is approximately balanced over this horizon.

B. Preprocessing Steps

We pivot the data into a wide date by ticker matrix of adjusted close prices and compute log returns

$$r_{t,i} = \log P_{t,i} - \log P_{t-1,i}, \tag{1}$$

for stock i on day t. The first observation per ticker is dropped, which yields a balanced panel of daily returns. An equal weight pseudo index is the cross sectional average of ticker returns each day.

Additional preprocessing includes alignment of trading days and removal of rows with incomplete cross sectional coverage, winsorization or removal of obvious data errors such as extreme returns beyond realistic market moves, and mapping each ticker to a sector using a public S&P 500 constituents file, with "Unknown" assigned when the mapping is unavailable. The result is a clean panel of daily log returns with sector labels.

III. Methods

A. Time Series Diagnostics

We analyze the equal weight index returns using a histogram with an overlaid Normal density and a QQ plot to assess deviations from normality. Autocorrelation functions of returns and squared returns reveal serial dependence and volatility clustering. As is typical for equity indexes, raw return autocorrelation is weak, while squared returns show strong positive autocorrelation, which motivates conditional heteroskedasticity models.

B. GARCH(1,1) with Student-t Innovations

We fit a GARCH(1,1) model with Student-t innovations using the `arch` Python library [4]:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \tag{2}$$
$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{3}$$

where z_t follows a standardized Student-t distribution. The conditional variance depends on one lag of the squared shock and one lag of previous variance. The Student-t distribution accommodates heavy tailed shocks better than Gaussian innovations. We examine the persistence $\alpha_1 + \beta_1$ and the conditional volatility series $\hat{\sigma}_t$.

C. Markov Switching Volatility Regimes

To identify discrete volatility regimes, we estimate a two state Markov switching model [1] using `statsmodels` [5]. Let $S_t \in \{1, 2\}$ denote the latent regime:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t \mid S_t = s \sim \mathcal{N}(0, \sigma_s^2). \tag{4}$$

The regime evolves as a first order Markov chain with transition probabilities $P_{ij} = \Pr(S_t = j \mid S_{t-1} = i)$. Maximum likelihood yields regime specific variances and transition probabilities. Smoothed probabilities $\Pr(S_t = \text{high vol} \mid \text{data})$ define a time series of regime labels, which we overlay on the pseudo index level to compare portfolio performance across calm and turbulent periods.

D. Sector Tail Risk: VaR and ES

For sector level risk, daily sector returns are cross sectional averages of constituent returns. For sector k, we compute historical 95% Value at Risk and Expected Shortfall:

$$\mathrm{VaR}_{0.95}^k = q_{0.05}(r^k), \tag{5}$$
$$\mathrm{ES}_{0.95}^k = \mathbb{E}[r^k \mid r^k \le q_{0.05}(r^k)], \tag{6}$$

where $q_{0.05}(r^k)$ is the 5th percentile of sector k's return distribution. The empirical breach rate, defined as the fraction of days with $r_t^k \le \widehat{\mathrm{VaR}}_{0.95}^k$, serves as a simple VaR backtest.

E. Portfolio Construction

Using the ticker by day returns matrix, we form three daily rebalanced long only portfolios:

- **Equal weight (EW):** $w_i = 1/N$ for N stocks.
- **Inverse volatility (IV):** $w_i \propto 1/\hat{\sigma}_i$, based on each stock's return standard deviation.
- **Minimum variance (MV):** weights that minimize $w^\top \Sigma w$ subject to $w^\top \mathbf{1} = 1$, $w \ge 0$, where Σ is a Ledoit Wolf shrinkage covariance estimate [3].

For each portfolio, we compute daily returns, annualized return and volatility, a Sharpe ratio that assumes a zero risk free rate, and maximum drawdown. Metrics are also computed conditional on high and low volatility regimes.

IV. Results and Visualization

A. Diagnostics and Volatility Clustering

Fig. 1 displays the equal weight index histogram, QQ plot, and autocorrelation functions of returns and squared returns. The empirical distribution has heavier tails than a Normal fit, and the autocorrelation of squared returns is significantly positive over multiple lags. These patterns confirm volatility clustering and support the use of conditional volatility models.

B. GARCH Results

The GARCH(1,1) fit yields persistent conditional variance with $\alpha_1 + \beta_1$ close to but below one, which indicates high volatility persistence. The Student-t degrees of freedom parameter is finite, which captures heavy tails. Fig. 2 plots returns alongside GARCH conditional volatility, and volatility spikes line up with visually turbulent periods.

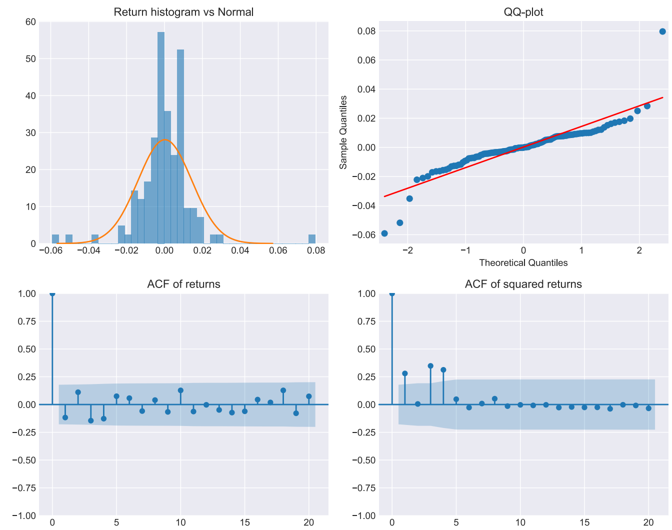

Fig. 1. Equal weight index diagnostics: histogram versus Normal fit (top left), Q Q plot (top right), autocorrelation of returns (bottom left), and autocorrelation of squared returns (bottom right). Heavy tails and positive autocorrelation in squared returns are evident.

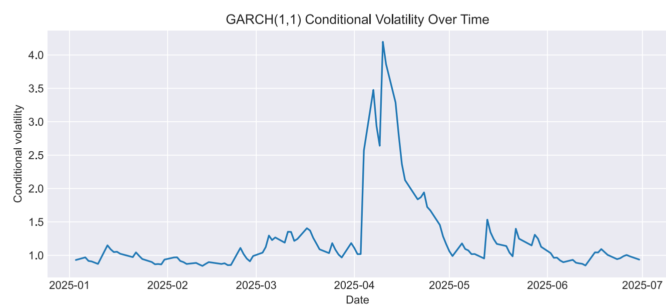

Fig. 2. Student-t GARCH(1,1) conditional volatility (shaded) for the equal weight index. Spikes correspond to periods of elevated market stress.

C. Markov Switching Regimes

Fig. 3 overlays the pseudo index level with shaded high volatility regimes and shows smoothed regime probabilities. The two regimes differ substantially in variance, and high volatility windows coincide with drawdowns. This shows that a discrete regime framework can capture meaningful shifts in market risk.

D. Sector VaR and ES

Fig. 4 reports historical 95% VaR and ES by sector. Over H1 2025, the aggregate sector has a VaR between 1% and 2% daily loss and an ES around 3% in the worst 5% of days, with breach frequencies close to the nominal 5%. More cyclical sectors show more negative VaR and ES, which indicates larger expected losses on extreme down days.

E. Portfolio Performance

Table I summarizes performance metrics for the three portfolios. Inverse volatility delivers higher annualized return and lower volatility than equal weight, which improves the return

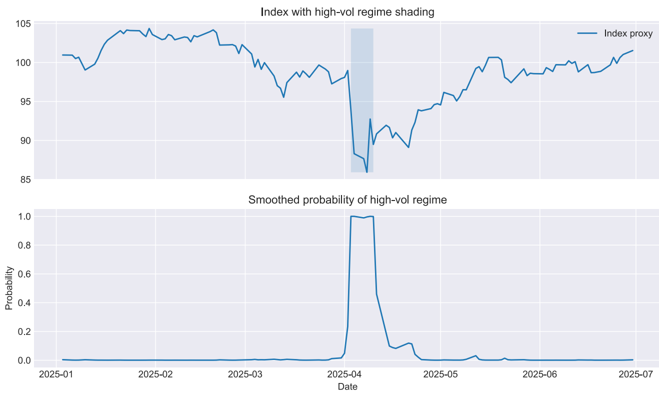

Fig. 3. Pseudo index level with high volatility regimes shaded (top) and smoothed probability of the high volatility regime (bottom). Regime switches align with drawdown periods.

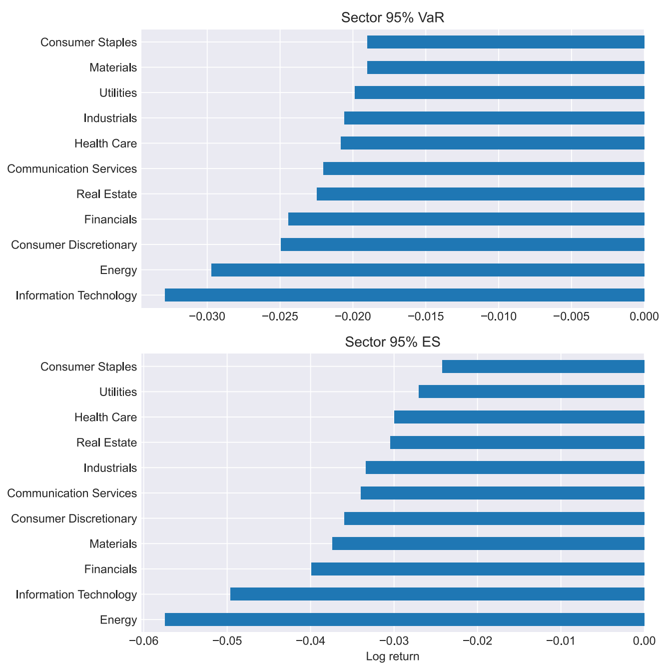

Fig. 4. Historical 95% VaR and ES by sector. Cyclical sectors show greater tail risk than defensive sectors.

to risk tradeoff. The minimum variance portfolio is the most defensive, with the lowest volatility and intermediate returns. Both IV and MV have smaller maximum drawdowns than EW.

Fig. 5 plots cumulative returns and drawdowns for the three strategies with regime shading. The risk sensitive portfolios keep better downside protection when volatility is elevated, which supports the value of allocations that respond to risk during stress periods.

V. Implications and Conclusion

A. Key Findings

The analysis shows that S&P 500 risk over H1 2025 is highly time varying, with distinct high and low volatility regimes. Heavy tails and volatility clustering break simple i.i.d.

TABLE I
PORTFOLIO METRICS (JAN–JUN 2025)

Portfolio	Ann. Ret.	Ann. Vol.	Sharpe	Max DD
EW	1.2%	22.5%	0.05	−17.7%
IV	3.5%	19.9%	0.17	−14.5%
MV	0.9%	18.7%	0.05	−15.6%

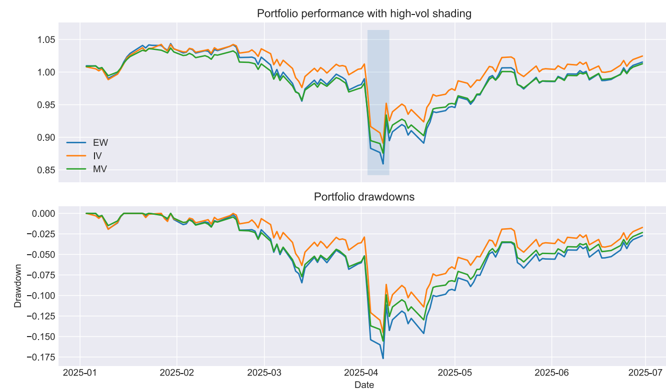

Fig. 5. Cumulative returns (top) and drawdowns (bottom) for EW, IV, and MV portfolios. Shaded regions indicate high volatility regimes. IV and MV show smaller drawdowns during turbulent periods.

assumptions and support the use of conditional volatility and regime models. From a portfolio perspective, inverse volatility and minimum variance strategies with covariance shrinkage reach more favorable risk adjusted performance and smaller drawdowns than a naive equal weight allocation, especially when volatility is high. Sector level VaR and ES show that tail risk is concentrated in certain sectors, which can guide risk budgeting and sector tilts.

B. Practical Implications

For practitioners, these results suggest that including volatility regime information and covariance shrinkage in portfolio construction can improve robustness without excessively complex optimization. Simple rules such as inverse volatility weighting already give meaningful drawdown reduction. Monitoring regime probabilities can also inform dynamic position sizing and hedging decisions.

C. Limitations and Future Work

Limitations include the short six month horizon of our dataset, construction of an equal weight index rather than a cap weighted index, omission of transaction costs and turnover constraints, and reliance on a two state regime specification. Future work could extend the horizon, add macro predictors to regime probabilities, explore alternative volatility models such as EGARCH or GJR GARCH, and implement explicit regime based allocation rules that scale exposure based on inferred risk states.

Appendix: Code

A complete Python implementation is provided in the accompanying Jupyter notebook, which loads the Kaggle dataset, performs preprocessing, estimates GARCH and Markov switching models using the `arch` [4] and `statsmodels` [5] libraries, computes sector VaR and ES, and constructs the three portfolios.

References

[1] J. D. Hamilton, “A new approach to the economic analysis of nonstationary time series and the business cycle,” *Econometrica*, vol. 57, no. 2, pp. 357–384, 1989.

[2] T. Bollerslev, “Generalized autoregressive conditional heteroskedasticity,” *J. Econometrics*, vol. 31, no. 3, pp. 307–327, 1986.

[3] O. Ledoit and M. Wolf, “Honey, I shrunk the sample covariance matrix,” *J. Portfolio Manage.*, vol. 30, no. 4, pp. 110–119, 2004.

[4] K. Sheppard, “arch: Autoregressive conditional heteroskedasticity models in Python,” https://arch.readthedocs.io/, accessed Dec. 2025.

[5] S. Seabold and J. Perktold, “Statsmodels: Econometric and statistical modeling with Python,” in *Proc. 9th Python Sci. Conf.*, 2010.

225-project

December 11, 2025

1 ECE 225A Project: Regime Aware Risk in the S&P 500

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Abstract: This notebook investigates whether modeling volatility regimes improves risk assessment and portfolio allocation for the S&P 500 during the first half of 2025. We fit a GARCH(1,1) model with Student-t innovations, apply a two state Markov switching model to identify high and low volatility regimes, compute sector level tail risk using Value at Risk (VaR) and Expected Shortfall (ES), and compare equal weight, inverse volatility, and minimum variance portfolios.

1.1 1. Data Description

- **Source:** Kaggle dataset "S&P 500 Stocks Trade Data for First 6 Months of 2025"
- **Contents:** Daily open, close, and volume for 503 S&P 500 constituents
- **Time period:** January 2 to June 30, 2025 (122 trading days)
- **Identifiers:** `company_name` and `ticker` for each stock
- **Supplementary data:** Sector mappings from GitHub `datasets/s-and-p-500-companies`

1.1.1 Question and Motivation

Can regime aware modeling improve risk assessment and portfolio allocation?

Significance: - Financial markets exhibit volatility with distinct calm and turbulent periods - Traditional risk measures such as unconditional VaR may underestimate risk during transitions - Portfolio strategies that adapt to volatility can improve risk adjusted returns - Sector level tail risk varies across sectors, suggesting diversification benefits

We investigate whether identifying high and low volatility regimes via Markov switching leads to: 1. Better understanding of tail risk (VaR and ES) across sectors 2. Improved portfolio allocation (comparing equal weight, inverse volatility, and minimum variance portfolios) 3. More accurate risk assessment during regime transitions

1.2 2. Preprocessing and Log Returns

This section loads the raw CSV data, reshapes it from wide to long format, and computes daily log returns for each stock. Log returns are additive over time and approximate normality better than simple returns.

The data loader checks for the Kaggle input path first. If unavailable, it falls back to `sp500_2025_h1.csv` in the repository root.

- Source: Kaggle dataset "S&P 500 stocks trade data for first 6 months of 2025" (daily open/close/volume per constituent)
- **Structure:** 503 tickers × daily columns (e.g., `02-01-2025_opening`, `02-01-2025_closing`, `02-01-2025_volume`) plus `company_name` and `ticker` identifiers
- **Time span:** 2025-01-03 to 2025-06-30 (121 trading days after differencing)
- **Sector mappings:** GitHub `datasets/s-and-p-500-companies` constituents file

1.2.1 Reshaping and Return Calculation

The code below reshapes the wide format data into a tidy panel (one row per ticker and date), then computes log returns as $r_t = \ln(P_t/P_{t-1})$.

[42]:
```
%pip install -q numpy pandas matplotlib statsmodels scikit-learn scipy arch
```

```
[notice] A new release of pip is
available: 24.3.1 -> 25.3
[notice] To update, run:
python3.11 -m pip install --upgrade pip
Note: you may need to restart the kernel to use updated packages.
```

[43]:
```
import os
import random
from pathlib import Path
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import statsmodels.api as sm
from statsmodels.graphics.tsaplots import plot_acf
from sklearn.covariance import LedoitWolf
from scipy.stats import norm
from arch import arch_model

plt.style.use("seaborn-v0_8-darkgrid")

random.seed(0)
np.random.seed(0)

kaggle_path = Path("/kaggle/input/
 ↪s-and-p-500-stocks-trade-data-for-first-6-month-2025/sp500_2025_h1.csv")
local_path = Path("sp500_2025_h1.csv")

if kaggle_path.exists():
    data_path = kaggle_path
elif local_path.exists():
    data_path = local_path
```

```python
else:
    raise FileNotFoundError("Could not find sp500_2025_h1.csv in Kaggle input␣
↪or repo root.")

df = pd.read_csv(data_path)

print("Shape:", df.shape)
print(df.head())
```

```
Shape: (503, 368)
     company_name ticker  02-01-2025_opening  02-01-2025_closing  \
0          Nvidia   NVDA             136.000             138.310
1       Microsoft   MSFT             425.530             418.580
2      Apple Inc.   AAPL             248.657             243.582
3          Amazon   AMZN             222.030             220.220
4  Meta Platforms   META             589.720             599.240

   02-01-2025_volume  03-01-2025_opening  03-01-2025_closing  \
0          198247166             140.010             144.470
1           16896469             421.080             423.350
2           55802016             243.093             243.093
3           33956579             222.505             224.190
4           12682269             604.760             604.630

   03-01-2025_volume  06-01-2025_opening  06-01-2025_closing  …  \
0          229322478             148.590             149.430  …
1           16662943             428.000             427.850  …
2           40288361             244.042             244.731  …
3           27515606             226.780             227.610  …
4           11436784             611.825             630.200  …

   25-06-2025_volume  26-06-2025_opening  26-06-2025_closing  \
0          269146471             155.975              155.02
1           17495099             492.980              497.45
2           39525730             201.430              201.00
3           31755698             213.120              217.12
4            9320436             714.355              726.09

   26-06-2025_volume  27-06-2025_opening  27-06-2025_closing  \
0          198145746             156.040              157.75
1           21578853             497.550              495.94
2           50799121             201.890              201.08
3           50480814             219.920              223.30
4           13964793             726.515              733.63

   27-06-2025_volume  30-06-2025_opening  30-06-2025_closing  \
0          263234539              158.40              157.99
1           34539236              497.04              497.41
```

```
2           73188571              202.01              205.17
3          119217138              223.52              219.39
4           18775735              744.55              738.09

   30-06-2025_volume
0          194580316
1           28368991
2           91912816
3           58887780
4           15402105

[5 rows x 368 columns]
```

The following code performs the wide to long transformation and computes log returns:

```
[44]: id_cols = ["company_name", "ticker"]
all_cols = df.columns.tolist()

# opening/closing/volume columns
open_cols = [c for c in all_cols if c.endswith("_opening")]
close_cols = [c for c in all_cols if c.endswith("_closing")]
volume_cols = [c for c in all_cols if c.endswith("_volume")]

print(f"# opening columns: {len(open_cols)}")
print(f"# closing columns: {len(close_cols)}")
print(f"# volume columns:  {len(volume_cols)}")

# daily close prices into long format
df_close = df.melt(
    id_vars=id_cols,
    value_vars=close_cols,
    var_name="date_col",
    value_name="close"
)
df_close["date_str"] = df_close["date_col"].str.replace("_closing", "",␣
 ↪regex=False)
df_close = df_close.drop(columns=["date_col"])

# daily opening prices into long format
df_open = df.melt(
    id_vars=id_cols,
    value_vars=open_cols,
    var_name="date_col",
    value_name="open"
)
df_open["date_str"] = df_open["date_col"].str.replace("_opening", "",␣
 ↪regex=False)
df_open = df_open.drop(columns=["date_col"])
```

```
if len(volume_cols) > 0:
    df_vol = df.melt(
        id_vars=id_cols,
        value_vars=volume_cols,
        var_name="date_col",
        value_name="volume"
    )
    df_vol["date_str"] = df_vol["date_col"].str.replace("_volume", "",␣
↪regex=False)
    df_vol = df_vol.drop(columns=["date_col"])
else:
    df_vol = None

# merge close and open prices & compute log returns
df_long = df_close.merge(
    df_open,
    on = id_cols + ["date_str"],
    how = "left",
    suffixes = ("_close", "_open")
)
if df_vol is not None:
    df_long = df_long.merge(
        df_vol[id_cols + ["date_str", "volume"]],
        on = id_cols + ["date_str"],
        how = "left"
    )

df_long["date"] = pd.to_datetime(df_long["date_str"], format = "%d-%m-%Y")
df_long = df_long.sort_values(["ticker", "date"])

df_long["log_return"] = (
    df_long.groupby("ticker")["close"].transform(lambda x: np.log(x).diff())
)

# drop rows with missing log returns
df_long = df_long.dropna(subset = ["log_return"])

print(df_long.head())
print("Unique tickers:", df_long["ticker"].nunique())
print("Date range:", df_long["date"].min(), "→", df_long["date"].max())
```

```
# opening columns: 122
# closing columns: 122
# volume columns:  122
              company_name ticker   close    date_str     open   volume  \
753   Agilent Technologies      A  135.69  03-01-2025  133.525  1246919
1256  Agilent Technologies      A  136.43  06-01-2025  135.340  1047034
```

```
1759  Agilent Technologies      A  137.41  07-01-2025  135.980  1056693
2262  Agilent Technologies      A  137.00  08-01-2025  137.220  1684573
2765  Agilent Technologies      A  137.47  10-01-2025  135.195  1369875

           date  log_return
753  2025-01-03    0.016796
1256 2025-01-06    0.005439
1759 2025-01-07    0.007157
2262 2025-01-08   -0.002988
2765 2025-01-10    0.003425
Unique tickers: 503
Date range: 2025-01-03 00:00:00 → 2025-06-30 00:00:00
```

1.3 3. Time Series Diagnostics

Before modeling, we examine the statistical properties of returns to justify our modeling choices. We construct an equal weight pseudo index and inspect:

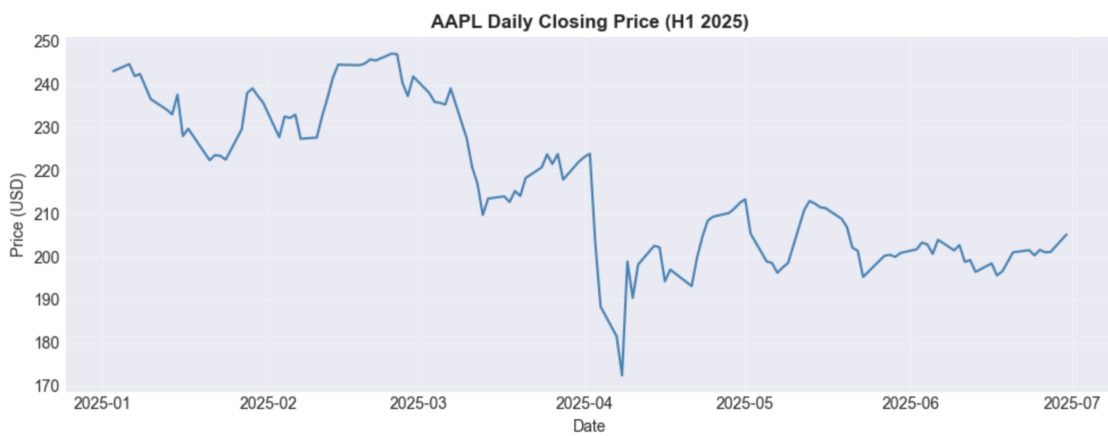
1. **Distribution shape:** Histogram and QQ plot to check for departures from normality (fat tails)
2. **Autocorrelation of returns:** ACF to detect predictable patterns in returns
3. **Autocorrelation of squared returns:** ACF to detect volatility clustering (conditional heteroskedasticity)

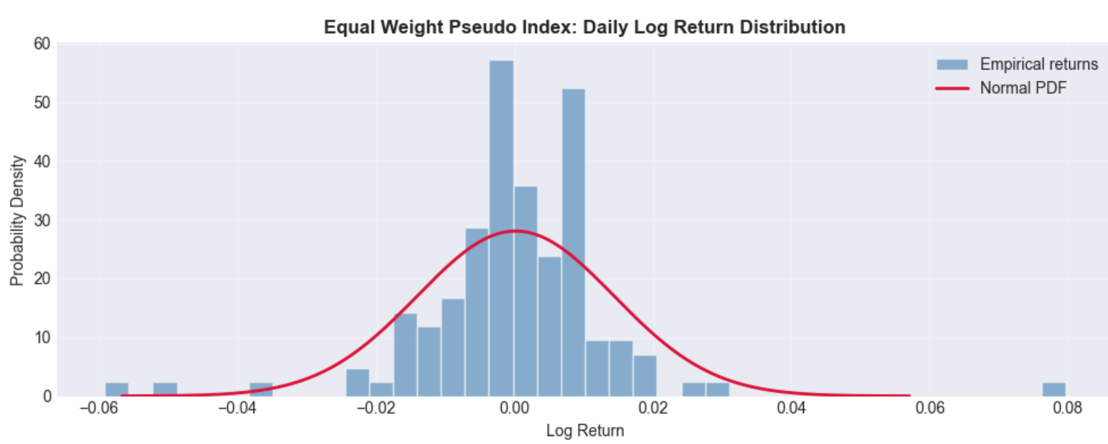
[45]:
```python
# equal weight pseudo index: average log return across all tickers each day
index_returns = (
    df_long.groupby("date")["log_return"]
    .mean()
    .sort_index()
)
index_returns.name = "sp500_eqw"

# sample stock for price path visualization
tickers = df_long["ticker"].unique()
sample_ticker = "AAPL" if "AAPL" in tickers else tickers[0]
print("Sample ticker:", sample_ticker)

sample_df = (
    df_long[df_long["ticker"] == sample_ticker]
    .sort_values("date")
    .set_index("date")
)

# figure 1: sample stock price path
plt.figure(figsize=(10, 4))
plt.plot(sample_df.index, sample_df["close"], color="steelblue", linewidth=1.5)
plt.title(f"{sample_ticker} Daily Closing Price (H1 2025)", fontsize=12,
 ↪fontweight="bold")
```

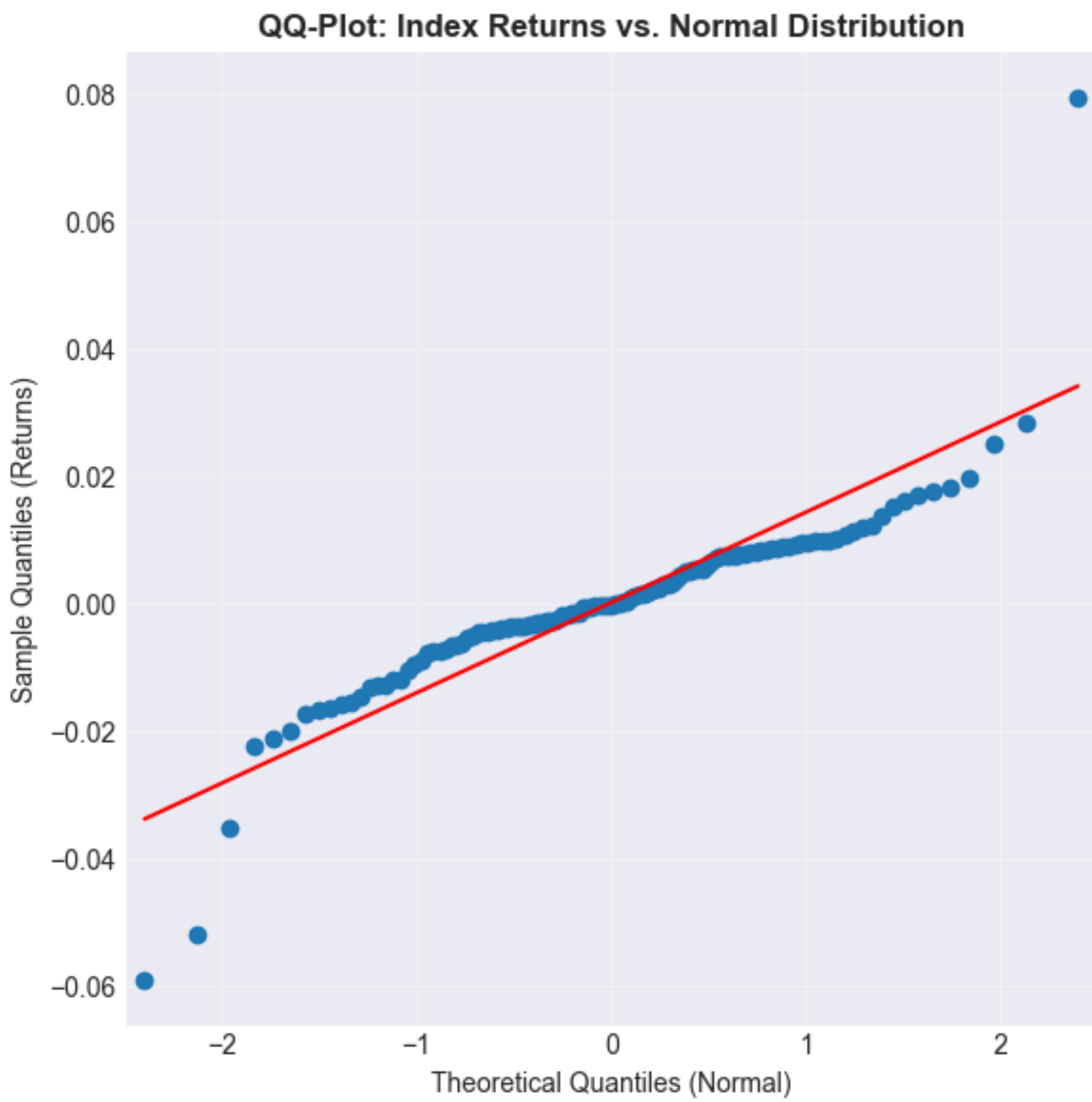
```python
plt.xlabel("Date")
plt.ylabel("Price (USD)")
plt.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

r = index_returns.dropna()
mu = r.mean()
sigma = r.std()

# figure 2: return distribution vs Normal (distribution has fatter tails,␣
↪excess kurtosis)
plt.figure(figsize=(10, 4))
plt.hist(r, bins=40, density=True, alpha=0.6, color="steelblue",␣
↪edgecolor="white", label="Empirical returns")
x = np.linspace(mu - 4 * sigma, mu + 4 * sigma, 200)
plt.plot(x, norm.pdf(x, mu, sigma), lw=2, color="crimson", label="Normal PDF")
plt.title("Equal Weight Pseudo Index: Daily Log Return Distribution",␣
↪fontsize=12, fontweight="bold")
plt.xlabel("Log Return")
plt.ylabel("Probability Density")
plt.legend(loc="upper right")
plt.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

# figure 3: QQ plot (points deviate in both tails, confirming fat tails)
fig = sm.qqplot(r, line="s")
fig.set_size_inches(6, 6)
ax = fig.axes[0]
ax.set_title("QQ-Plot: Index Returns vs. Normal Distribution", fontsize=12,␣
↪fontweight="bold")
ax.set_xlabel("Theoretical Quantiles (Normal)")
ax.set_ylabel("Sample Quantiles (Returns)")
ax.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

# figure 4: ACF of returns (little autocorrelation, mostly within confidence␣
↪bands)
fig, ax = plt.subplots(figsize=(10, 4))
plot_acf(r, lags=20, ax=ax)
ax.set_title("Autocorrelation Function (ACF) of Daily Returns", fontsize=12,␣
↪fontweight="bold")
ax.set_xlabel("Lag (Days)")
ax.set_ylabel("Autocorrelation")
```

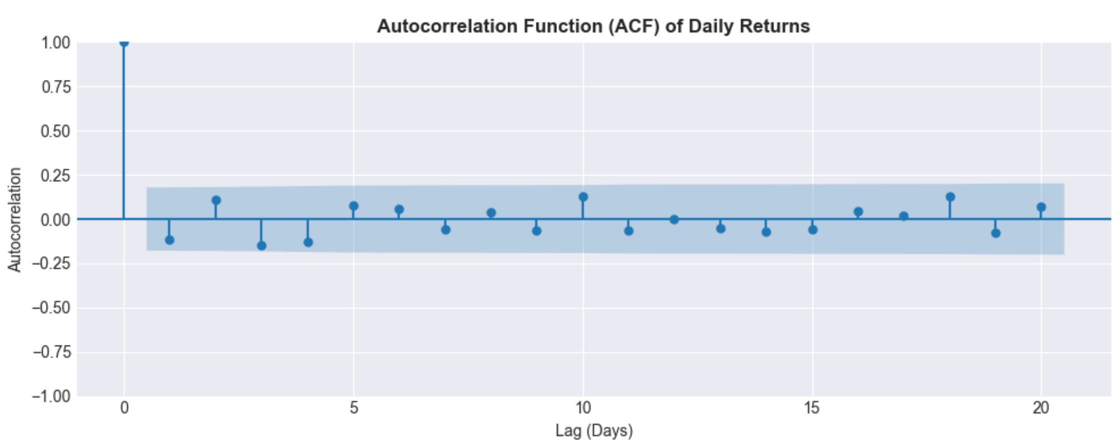
```
plt.tight_layout()
plt.show()

# figure 5: ACF of squared returns (significant autocorrelation, evidence of␣
↪volatility clustering)
fig, ax = plt.subplots(figsize=(10, 4))
plot_acf(r ** 2, lags=20, ax=ax)
ax.set_title("ACF of Squared Returns (Volatility Clustering)", fontsize=12,␣
↪fontweight="bold")
ax.set_xlabel("Lag (Days)")
ax.set_ylabel("Autocorrelation")
plt.tight_layout()
plt.show()
```

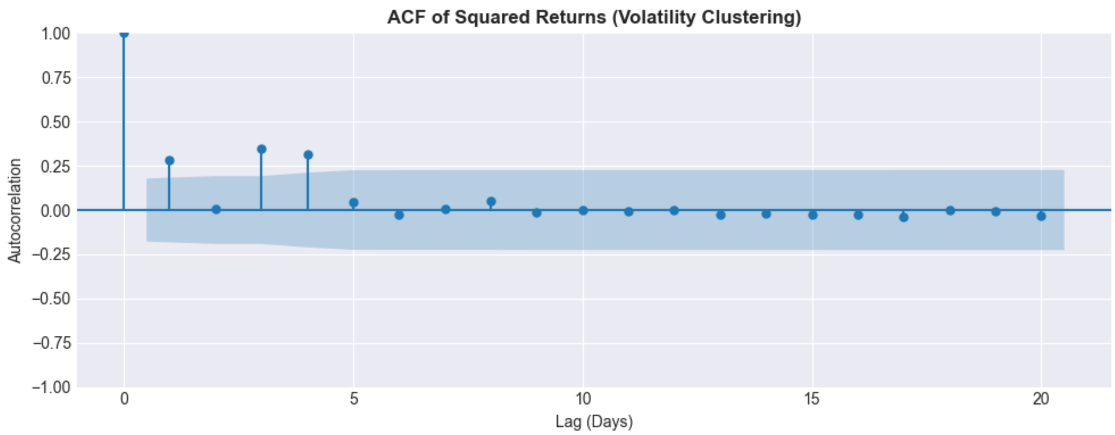
```
Sample ticker: AAPL
```






QQ-Plot: Index Returns vs. Normal Distribution
Sample Quantiles (Returns)
Theoretical Quantiles (Normal)
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
−2
−1
0
1
2


Autocorrelation Function (ACF) of Daily Returns
Autocorrelation
Lag (Days)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
5
10
15
20



1.4 4. Volatility Modeling (GARCH)

Given the evidence of volatility clustering from Section 3, we fit a GARCH(1,1) model with Student-t innovations to capture time varying conditional volatility. The GARCH(1,1) specification is:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

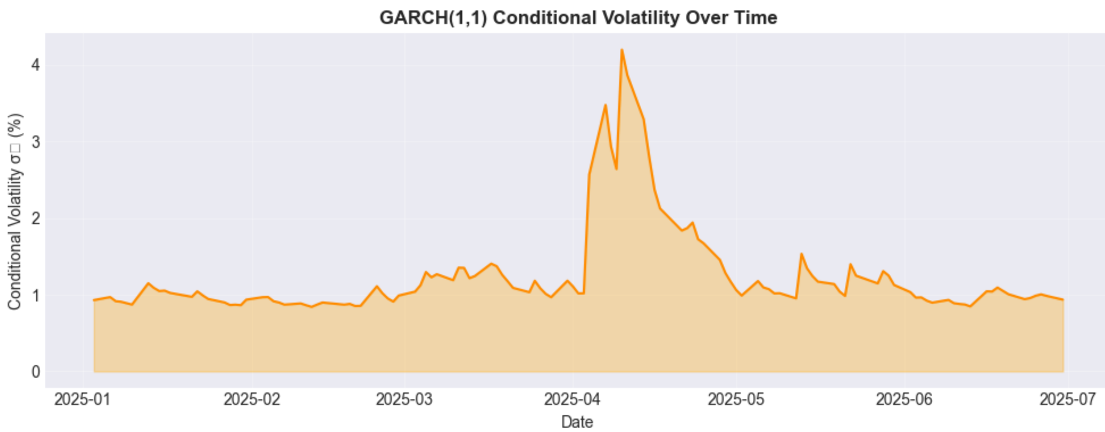
where $\alpha + \beta < 1$ ensures stationarity. We use Student-t innovations to accommodate the fat tails observed in the QQ plot.

[46]:
```
# scale to percentage points for typical GARCH parameter magnitudes
index_returns_pct = r * 100

# Student-t GARCH(1,1) for fat tails and volatility clustering
am = arch_model(index_returns_pct, vol="Garch", p=1, q=1, dist="t")
res_garch = am.fit(update_freq=5, disp="off")

print(res_garch.summary())

# extract conditional volatility and align index with returns
cond_vol = res_garch.conditional_volatility
if isinstance(cond_vol, pd.Series):
    cond_vol = cond_vol.reindex(r.index)
else:
    cond_vol = pd.Series(cond_vol, index=r.index, name="cond_vol")

# figure 6: GARCH conditional volatility (spikes during turbulent periods, mean␣
↪reverts during calm)
plt.figure(figsize=(10, 4))
```

```python
plt.plot(cond_vol.index, cond_vol.values, color="darkorange", linewidth=1.5)
plt.fill_between(cond_vol.index, 0, cond_vol.values, alpha=0.3, color="orange")
plt.title("GARCH(1,1) Conditional Volatility Over Time", fontsize=12,␣
 ↪fontweight="bold")
plt.xlabel("Date")
plt.ylabel("Conditional Volatility   (%)")
plt.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()
```

```
                    Constant Mean - GARCH Model Results
================================================================================
====
Dep. Variable:                  sp500_eqw   R-squared:
0.000
Mean Model:                 Constant Mean   Adj. R-squared:
0.000
Vol Model:                          GARCH   Log-Likelihood:
-179.855
Distribution:      Standardized Student's t   AIC:
369.711
Method:                Maximum Likelihood   BIC:
383.690
                                            No. Observations:
121
Date:                    Thu, Dec 11 2025   Df Residuals:
120
Time:                            21:44:04   Df Model:
1
                                 Mean Model
===========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
---------------------------------------------------------------------------
mu             0.0936  7.816e-02      1.198      0.231 [-5.955e-02,  0.247]
                              Volatility Model
===========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
---------------------------------------------------------------------------
omega          0.1933      0.126      1.530      0.126 [-5.426e-02,  0.441]
alpha[1]       0.2045      0.123      1.667  9.544e-02 [-3.590e-02,  0.445]
beta[1]        0.6845      0.117      5.834  5.408e-09    [  0.455,  0.914]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             4.0522      1.388      2.919  3.506e-03 [  1.332,  6.773]
========================================================================
```

```
Covariance estimator: robust

/var/folders/nz/c2k7vtjn0fj1yg5s18_dn6hc0000gn/T/ipykernel_55596/259260619.py:25
: UserWarning: Glyph 8348 (\N{LATIN SUBSCRIPT SMALL LETTER T}) missing from
font(s) Arial.
  plt.tight_layout()
/Users/ryanluo/Library/Python/3.11/lib/python/site-
packages/IPython/core/pylabtools.py:170: UserWarning: Glyph 8348 (\N{LATIN
SUBSCRIPT SMALL LETTER T}) missing from font(s) Arial.
  fig.canvas.print_figure(bytes_io, **kw)
```



1.5 5. Regime Switching (Markov Switching Model)

We fit a two state Markov switching model to identify distinct volatility regimes:

- **Low volatility regime:** Calm market conditions with smaller daily moves
- **High volatility regime:** Turbulent periods with large swings

The model estimates regime specific means and variances, plus transition probabilities $P(S_t = j|S_{t-1} = i)$. We extract smoothed regime probabilities to classify each day and visualize how regimes align with market movements.

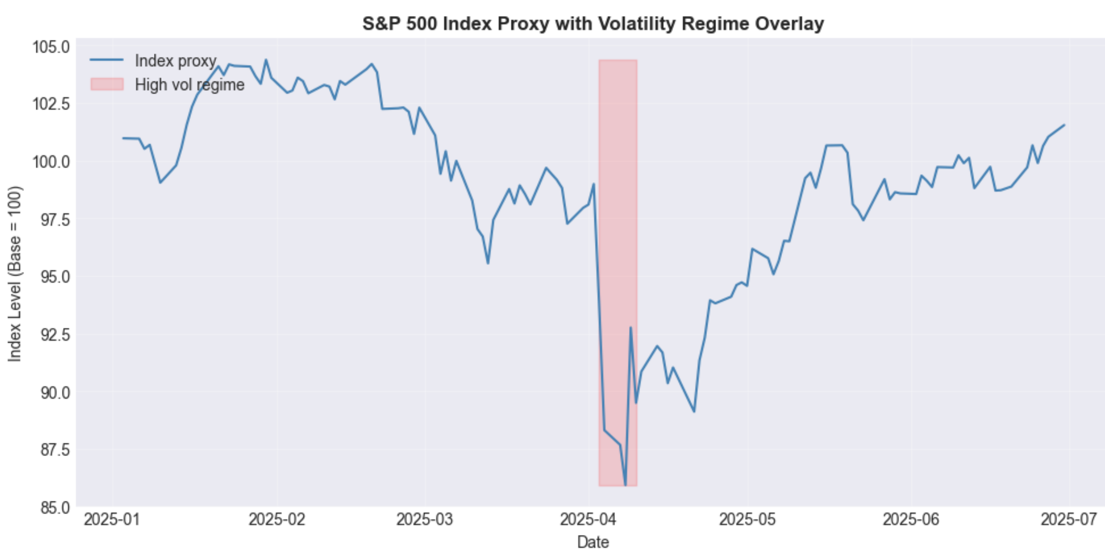
[47]:
```python
from statsmodels.tsa.regime_switching.markov_regression import MarkovRegression

# two state Markov switching on index returns (switching variance)
y = r.values
mod_ms = MarkovRegression(
    y,
    k_regimes=2,
    trend="c",
    switching_variance=True
)
res_ms = mod_ms.fit(em_iter=50, search_reps=20)
print(res_ms.summary())
```

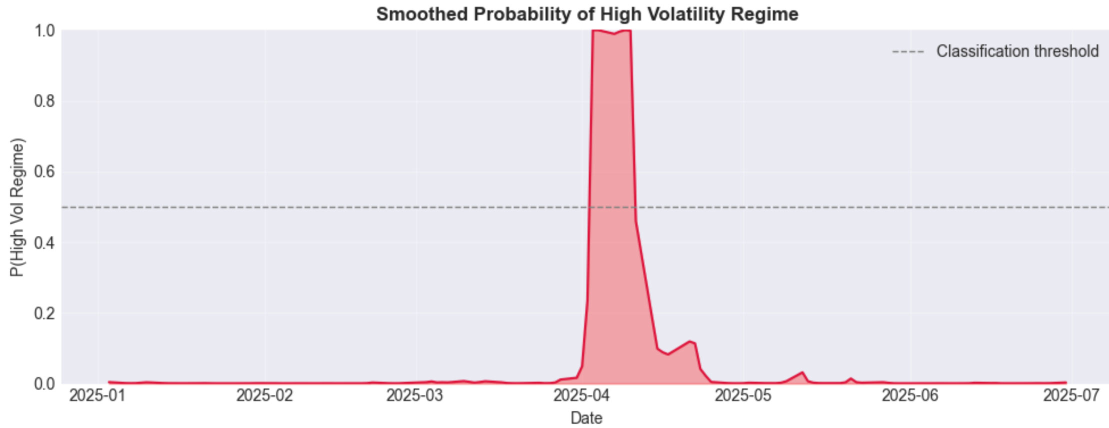
```python
# identify high volatility regime by computing empirical variance using smoothed probabilities
prob_raw = res_ms.smoothed_marginal_probabilities
if hasattr(prob_raw, "values"):
    prob_0 = pd.Series(prob_raw[0].values, index=r.index)
    prob_1 = pd.Series(prob_raw[1].values, index=r.index)
else:
    prob_0 = pd.Series(prob_raw[:, 0], index=r.index)
    prob_1 = pd.Series(prob_raw[:, 1], index=r.index)

# get regime means from params
params = res_ms.params
if isinstance(params, pd.Series):
    const_keys = [k for k in params.index if "const" in str(k).lower()]
    mu_0 = params[const_keys[0]] if len(const_keys) > 0 else 0
    mu_1 = params[const_keys[1]] if len(const_keys) > 1 else 0
else:
    # compute means empirically if params is array
    mu_0 = (r.values * prob_0.values).sum() / prob_0.sum()
    mu_1 = (r.values * prob_1.values).sum() / prob_1.sum()

# compute weighted variance for each regime: E[(r - )²] weighted by regime probabilities
sigma2_0 = ((r.values - mu_0)**2 * prob_0.values).sum() / prob_0.sum()
sigma2_1 = ((r.values - mu_1)**2 * prob_1.values).sum() / prob_1.sum()

high_vol_regime_idx = 1 if sigma2_1 > sigma2_0 else 0
print(f"\nRegime 0 sigma²: {sigma2_0:.6f}, Regime 1 sigma²: {sigma2_1:.6f}")
print(f"High volatility regime: {high_vol_regime_idx}")

# smoothed probability of the high volatility regime
prob_raw = res_ms.smoothed_marginal_probabilities
if hasattr(prob_raw, "values"):
    prob_high = pd.Series(prob_raw[high_vol_regime_idx].values, index=r.index, name="prob_high")
else:
    prob_high = pd.Series(prob_raw[:, high_vol_regime_idx], index=r.index, name="prob_high")
regime_series = (prob_high > 0.5).astype(int)
regime_series.name = "high_vol_regime"

index_level = (1 + r).cumprod() * 100

# figure 7: index level with regime overlay (red shading marks high volatility periods)
plt.figure(figsize=(10, 5))
```

```python
plt.plot(index_level.index, index_level.values, label="Index proxy",
    color="steelblue", linewidth=1.5)
high_mask = regime_series == 1
plt.fill_between(
    index_level.index,
    index_level.min(),
    index_level.max(),
    where=high_mask,
    color="red",
    alpha=0.15,
    label="High vol regime"
)
plt.title("S&P 500 Index Proxy with Volatility Regime Overlay", fontsize=12,
    fontweight="bold")
plt.xlabel("Date")
plt.ylabel("Index Level (Base = 100)")
plt.legend(loc="upper left")
plt.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

# figure 8: smoothed regime probabilities (near 1 = high vol, near 0 = low vol)
plt.figure(figsize=(10, 4))
plt.plot(prob_high.index, prob_high.values, color="crimson", linewidth=1.5)
plt.fill_between(prob_high.index, 0, prob_high.values, alpha=0.3, color="red")
plt.axhline(y=0.5, color="gray", linestyle="--", linewidth=1,
    label="Classification threshold")
plt.title("Smoothed Probability of High Volatility Regime", fontsize=12,
    fontweight="bold")
plt.xlabel("Date")
plt.ylabel("P(High Vol Regime)")
plt.ylim(0, 1)
plt.legend(loc="upper right")
plt.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()
```

```
                        Markov Switching Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  121
Model:               MarkovRegression   Log Likelihood                 377.421
Date:                Thu, 11 Dec 2025   AIC                           -742.841
Time:                        21:44:04   BIC                           -726.067
Sample:                             0   HQIC                          -736.028
                                - 121
Covariance Type:               approx
                             Regime 0 parameters
```

```
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0010      0.001      1.100      0.272      -0.001       0.003
sigma2      8.449e-05   1.18e-05      7.144      0.000    6.13e-05       0.000
                             Regime 1 parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0105      0.016     -0.656      0.512      -0.042       0.021
sigma2         0.0018      0.001      1.735      0.083      -0.000       0.004
                         Regime transition parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
p[0->0]        0.9896      0.011     92.333      0.000       0.969       1.011
p[1->0]        0.1648      0.152      1.085      0.278      -0.133       0.463
==============================================================================

Warnings:
[1] Covariance matrix calculated using numerical (complex-step) differentiation.

Regime 0 sigma²: 0.000084, Regime 1 sigma²: 0.001775
High volatility regime: 1
```





1.6 6. Sector Risk: Value at Risk and Expected Shortfall

We assess tail risk at the sector level using two standard risk measures:

- **Value at Risk (VaR):** The α-quantile loss; at 95% confidence, the loss exceeded only 5% of the time
- **Expected Shortfall (ES):** The average loss conditional on exceeding VaR (also called CVaR)

We map tickers to GICS sectors using an external reference file, compute equal weight sector returns, estimate historical VaR and ES, and backtest by checking breach ratios.

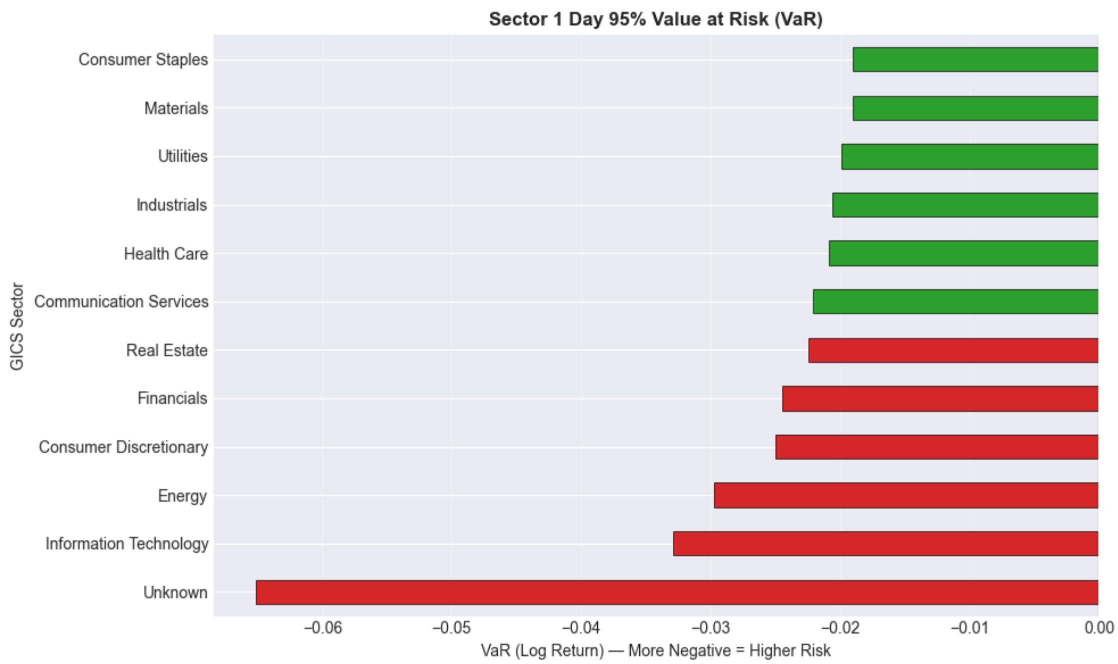
[48]:
```
df_long_sect = df_long.copy()

# map tickers to sectors; fall back to Unknown if mapping unavailable
try:
    sector_url = "https://raw.githubusercontent.com/datasets/
 ↪s-and-p-500-companies/master/data/constituents.csv"
    cons = pd.read_csv(sector_url)
    cons.columns = cons.columns.str.strip().str.lower()
    # detect ticker column
    ticker_col = None
    for c in ["symbol", "ticker"]:
        if c in cons.columns:
            ticker_col = c
            break
    # detect sector column (try multiple common names)
    sector_col = None
    for c in ["sector", "gics sector", "gics_sector", "industry"]:
        if c in cons.columns:
            sector_col = c
            break
    if ticker_col and sector_col:
```

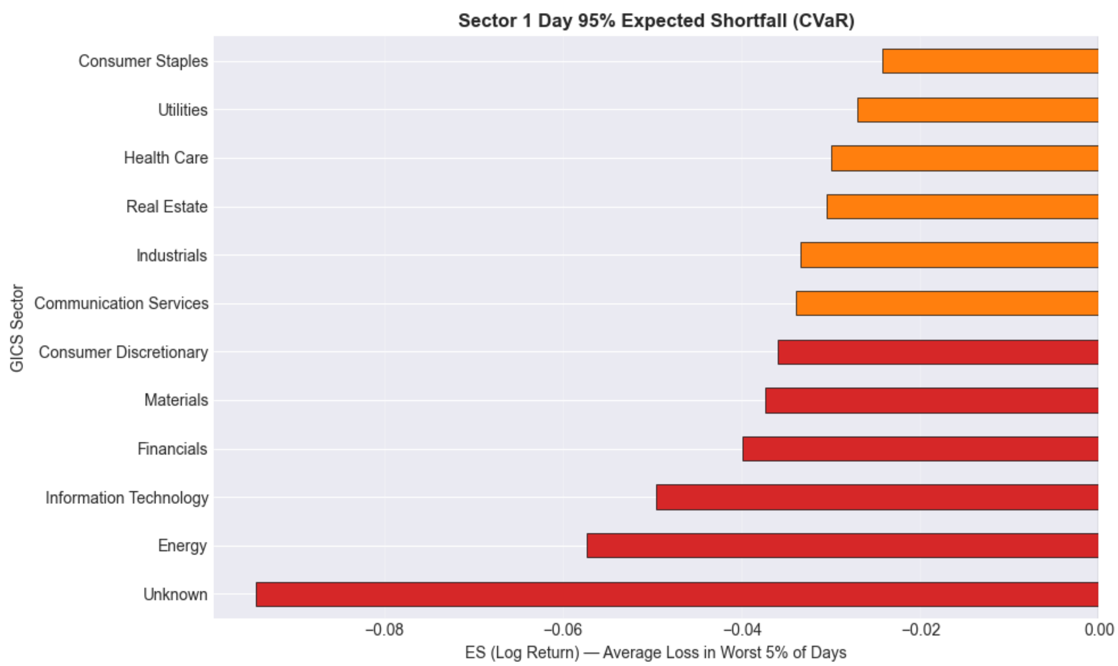
```
        cons = cons.rename(columns = {ticker_col: "ticker", sector_col:␣
↪"sector"})
        cons["ticker"] = cons["ticker"].str.upper()
        sector_map = cons.set_index("ticker")["sector"]
        df_long_sect["sector"] = df_long_sect["ticker"].str.upper().
↪map(sector_map)
        missing = df_long_sect["sector"].isna().sum()
        df_long_sect["sector"] = df_long_sect["sector"].fillna("Unknown")
        msg = f"Loaded sector info from GitHub (cols: {ticker_col},␣
↪{sector_col}); {missing} rows mapped to 'Unknown'."
        print(msg)
    else:
        raise ValueError(f"Could not find ticker/sector columns. Available:␣
↪{list(cons.columns)}")
except Exception as e:
    print("Could not load sector info, using 'Unknown' for all. Error:", e)
    df_long_sect["sector"] = "Unknown"

df_long_sect = df_long_sect.dropna(subset = ["log_return"])

sector_returns = (
    df_long_sect.groupby(["date", "sector"])["log_return"]
    .mean()
    .unstack("sector")
    .sort_index()
)

print("Sector returns shape:", sector_returns.shape)
print("Sectors:", sector_returns.columns.tolist())

alpha = 0.95

def compute_var_es(series, alpha=0.95):
    """
    Compute Value-at-Risk (VaR) and Expected Shortfall (ES) for a return series.

    Convention: For log returns, negative values = losses.
    VaR at 95% confidence: 5th percentile (worst 5% of returns).
    ES: Average return conditional on exceeding VaR (mean of tail losses).

    Returns:
        q: VaR (negative value = loss threshold)
        es: ES (average loss in worst 5% of days)
    """
    series = series.dropna()
    # 5th percentile for 95% VaR
    q = series.quantile(1 - alpha)
```

```
    # ES: mean of returns in the tail
    es = series[series <= q].mean()
    return q, es

var_sector = {}
es_sector = {}

for sector in sector_returns.columns:
    q, es = compute_var_es(sector_returns[sector], alpha = alpha)
    var_sector[sector] = q
    es_sector[sector] = es

var_sector = pd.Series(var_sector)
es_sector = pd.Series(es_sector)

print("Sector VaR (95%):")
print(var_sector.sort_values())
print("\nSector ES (95%):")
print(es_sector.sort_values())

# figure 9: sector Value at Risk (more negative VaR = higher tail risk)
plt.figure(figsize=(10, 6))
colors = ["#d62728" if v < var_sector.median() else "#2ca02c" for v in␣
 ↪var_sector.sort_values()]
var_sector.sort_values().plot(kind="barh", color=colors, edgecolor="black",␣
 ↪linewidth=0.5)
plt.title("Sector 1 Day 95% Value at Risk (VaR)", fontsize=12,␣
 ↪fontweight="bold")
plt.xlabel("VaR (Log Return) - More Negative = Higher Risk")
plt.ylabel("GICS Sector")
plt.axvline(x=0, color="black", linewidth=0.8)
plt.grid(True, axis="x", alpha=0.3)
plt.tight_layout()
plt.show()

# figure 10: sector Expected Shortfall (ES captures average loss in worst 5% of␣
 ↪days)
plt.figure(figsize=(10, 6))
colors = ["#d62728" if v < es_sector.median() else "#ff7f0e" for v in es_sector.
 ↪sort_values()]
es_sector.sort_values().plot(kind="barh", color=colors, edgecolor="black",␣
 ↪linewidth=0.5)
plt.title("Sector 1 Day 95% Expected Shortfall (CVaR)", fontsize=12,␣
 ↪fontweight="bold")
plt.xlabel("ES (Log Return) - Average Loss in Worst 5% of Days")
plt.ylabel("GICS Sector")
```

```python
plt.axvline(x=0, color="black", linewidth=0.8)
plt.grid(True, axis="x", alpha=0.3)
plt.tight_layout()
plt.show()

# VaR backtest: count days where return < VaR (breach ratio should be ~5%)
breach_ratios = (sector_returns.lt(var_sector)).sum() / sector_returns.notna().
↪sum()
print("\nVaR breach ratios (should be ~5% for a perfect 95% VaR):")
print(breach_ratios.sort_values())
```

```
Loaded sector info from GitHub (cols: symbol, gics sector); 484 rows mapped to
'Unknown'.
Sector returns shape: (121, 12)
Sectors: ['Communication Services', 'Consumer Discretionary', 'Consumer
Staples', 'Energy', 'Financials', 'Health Care', 'Industrials', 'Information
Technology', 'Materials', 'Real Estate', 'Unknown', 'Utilities']
Sector VaR (95%):
Unknown                  -0.065108
Information Technology   -0.032883
Energy                   -0.029714
Consumer Discretionary   -0.024959
Financials               -0.024437
Real Estate              -0.022470
Communication Services   -0.022044
Health Care              -0.020824
Industrials              -0.020579
Utilities                -0.019883
Materials                -0.019027
Consumer Staples         -0.019010
dtype: float64

Sector ES (95%):
Unknown                  -0.094387
Energy                   -0.057422
Information Technology   -0.049587
Financials               -0.039918
Materials                -0.037403
Consumer Discretionary   -0.035933
Communication Services   -0.033955
Industrials              -0.033373
Real Estate              -0.030468
Health Care              -0.029960
Utilities                -0.027019
Consumer Staples         -0.024212
dtype: float64
```






```
VaR breach ratios (should be ~5% for a perfect 95% VaR):
sector
Consumer Discretionary    0.049587
```

```
Materials                 0.049587
Real Estate               0.049587
Communication Services    0.057851
Consumer Staples          0.057851
Energy                    0.057851
Financials                0.057851
Health Care               0.057851
Industrials               0.057851
Information Technology    0.057851
Unknown                   0.057851
Utilities                 0.057851
dtype: float64
```

1.7 7. Portfolio Strategies and Performance

We construct and compare three portfolio strategies:

Strategy	Description
Equal Weight (EW)	$w_i = 1/N$ for all assets
Inverse Volatility (IV)	$w_i \propto 1/\sigma_i$, tilts toward lower volatility stocks
Minimum Variance (MV)	$w = \Sigma^{-1}\mathbf{1}/(\mathbf{1}'\Sigma^{-1}\mathbf{1})$, uses Ledoit Wolf shrinkage covariance

We evaluate annualized return, volatility, Sharpe ratio, and maximum drawdown. Finally, we split performance by volatility regime to see whether regime aware analysis provides additional insight.

[49]:
```
# wide returns matrix; drop days with missing values for consistent asset␣
↪universe
returns_wide = (
    df_long.pivot(index="date", columns="ticker", values="log_return")
    .sort_index()
)
returns_wide = returns_wide.dropna(axis=0, how="any")

print("Returns matrix shape:", returns_wide.shape)

tickers_clean = returns_wide.columns
n_assets = len(tickers_clean)

# portfolio 1: equal weight (EW)
w_eq = np.repeat(1.0 / n_assets, n_assets)

# portfolio 2: inverse volatility (IV)
vols = returns_wide.std()
w_inv = 1.0 / vols
w_inv = w_inv / w_inv.sum()  # Normalize to sum to 1
```

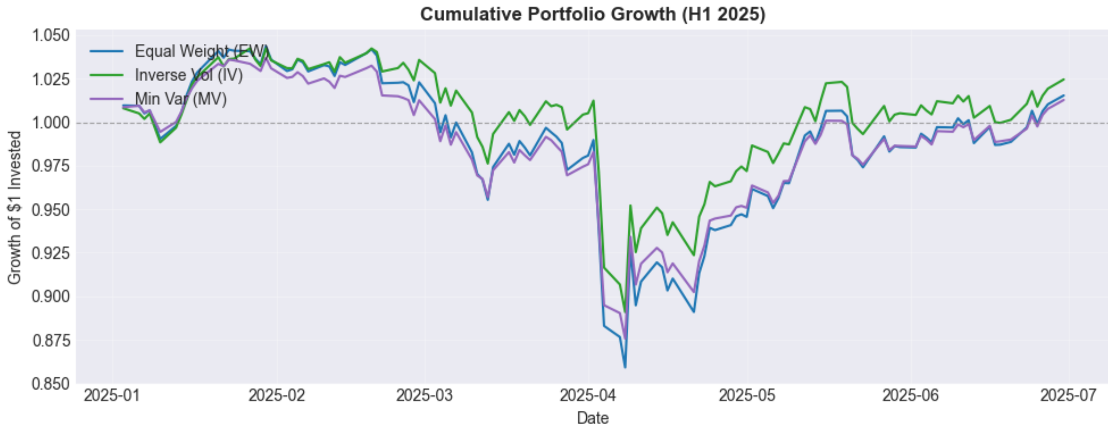
```
# portfolio 3: minimum variance (MV) with Ledoit Wolf shrinkage covariance
lw = LedoitWolf().fit(returns_wide.values)
Sigma = lw.covariance_

# verify covariance matrix is positive definite
eigenvals = np.linalg.eigvals(Sigma)
if np.any(eigenvals <= 0):
    print(f"Warning: Covariance matrix not positive definite. Min eigenvalue:␣
 ↪{eigenvals.min():.2e}")
    Sigma = Sigma + 1e-6 * np.eye(n_assets)

# minimum variance weights: w = Σ ¹·1 / (1'·Σ ¹·1)
ones = np.ones(n_assets)
inv_Sigma = np.linalg.inv(Sigma)
w_mv = inv_Sigma.dot(ones)
w_mv = np.clip(w_mv, 0, None)  # long only
w_mv = w_mv / w_mv.sum()

# verify weights sum to 1
print("Equal weight weights sum:", w_eq.sum())
print("Inverse vol weights sum:", w_inv.sum())
print("Min var weights sum:", w_mv.sum())
assert np.allclose([w_eq.sum(), w_inv.sum(), w_mv.sum()], 1.0), "Weights must␣
 ↪sum to 1"

# compute portfolio returns: r_portfolio = Σ w_i · r_i
port_eq = pd.Series(
    returns_wide.values.dot(w_eq),
    index=returns_wide.index,
    name="EW"
)
port_inv = pd.Series(
    returns_wide.values.dot(w_inv.values),
    index=returns_wide.index,
    name="IV"
)
port_mv = pd.Series(
    returns_wide.values.dot(w_mv),
    index=returns_wide.index,
    name="MV"
)


def perf_stats(r):
    r = r.dropna()
    ann_ret = r.mean() * 252
    ann_vol = r.std() * np.sqrt(252)
```

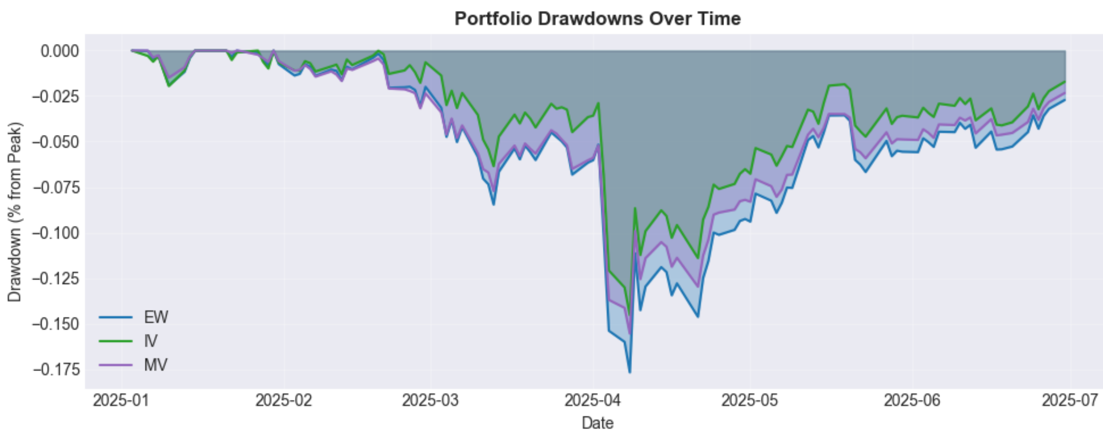
```python
    sharpe = ann_ret / ann_vol if ann_vol > 0 else np.nan
    cum = (1 + r).cumprod()
    peak = cum.cummax()
    dd = cum / peak - 1
    max_dd = dd.min()
    return {
        "ann_return": ann_ret,
        "ann_vol": ann_vol,
        "sharpe": sharpe,
        "max_drawdown": max_dd
    }

# performance summary
for series in [port_eq, port_inv, port_mv]:
    print("\n=== Portfolio:", series.name, "===")
    stats = perf_stats(series)
    for k, v in stats.items():
        print(f"{k:>15}: {v:.4f}")

# cumulative growth
cum_eq = (1 + port_eq).cumprod()
cum_inv = (1 + port_inv).cumprod()
cum_mv = (1 + port_mv).cumprod()

# figure 11: cumulative portfolio growth
plt.figure(figsize=(10, 4))
plt.plot(cum_eq.index, cum_eq, label="Equal Weight (EW)", linewidth=1.5,
 ↪color="#1f77b4")
plt.plot(cum_inv.index, cum_inv, label="Inverse Vol (IV)", linewidth=1.5,
 ↪color="#2ca02c")
plt.plot(cum_mv.index, cum_mv, label="Min Var (MV)", linewidth=1.5,
 ↪color="#9467bd")
plt.axhline(y=1.0, color="gray", linestyle="--", linewidth=0.8, alpha=0.7)
plt.title("Cumulative Portfolio Growth (H1 2025)", fontsize=12,
 ↪fontweight="bold")
plt.xlabel("Date")
plt.ylabel("Growth of $1 Invested")
plt.legend(loc="upper left")
plt.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()


def drawdown_series(r):
    r = r.dropna()
    cum = (1 + r).cumprod()
    peak = cum.cummax()
```

```python
    dd = cum / peak - 1
    return dd


dd_eq = drawdown_series(port_eq)
dd_inv = drawdown_series(port_inv)
dd_mv = drawdown_series(port_mv)

# figure 12: portfolio drawdowns
plt.figure(figsize=(10, 4))
plt.fill_between(dd_eq.index, dd_eq, 0, alpha=0.3, color="#1f77b4")
plt.fill_between(dd_inv.index, dd_inv, 0, alpha=0.3, color="#2ca02c")
plt.fill_between(dd_mv.index, dd_mv, 0, alpha=0.3, color="#9467bd")
plt.plot(dd_eq.index, dd_eq, label="EW", linewidth=1.5, color="#1f77b4")
plt.plot(dd_inv.index, dd_inv, label="IV", linewidth=1.5, color="#2ca02c")
plt.plot(dd_mv.index, dd_mv, label="MV", linewidth=1.5, color="#9467bd")
plt.title("Portfolio Drawdowns Over Time", fontsize=12, fontweight="bold")
plt.xlabel("Date")
plt.ylabel("Drawdown (% from Peak)")
plt.legend(loc="lower left")
plt.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()

# performance by volatility regime
regime_aligned = regime_series.reindex(port_eq.index).ffill().bfill()

for lbl, name in [(0, "Low vol regime"), (1, "High vol regime")]:
    mask = regime_aligned == lbl
    print(f"\n===== {name} =====")
    for series in [port_eq, port_inv, port_mv]:
        stats = perf_stats(series[mask])
        print(
            f"Portfolio {series.name}: "
            f"ann_return={stats['ann_return']:.4f}, "
            f"ann_vol={stats['ann_vol']:.4f}, "
            f"sharpe={stats['sharpe']:.4f}, "
            f"max_dd={stats['max_drawdown']:.4f}"
        )

# figure 13: portfolio growth with regime overlay
plt.figure(figsize=(10, 4))
plt.plot(cum_eq.index, cum_eq, label="EW", linewidth=1.5, color="#1f77b4")
plt.plot(cum_inv.index, cum_inv, label="IV", linewidth=1.5, color="#2ca02c")
plt.plot(cum_mv.index, cum_mv, label="MV", linewidth=1.5, color="#9467bd")

high_mask_port = regime_aligned == 1
```

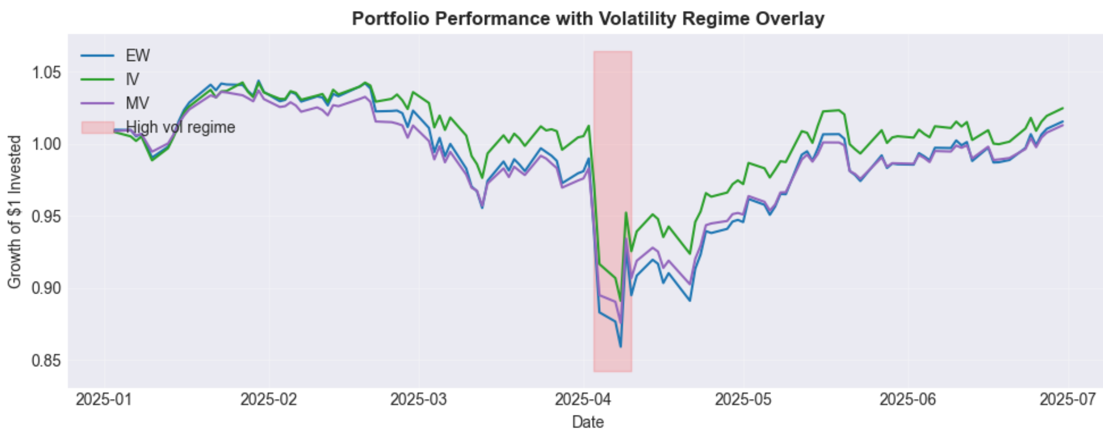
```
plt.fill_between(
    cum_eq.index,
    min(cum_eq.min(), cum_inv.min(), cum_mv.min()) * 0.98,
    max(cum_eq.max(), cum_inv.max(), cum_mv.max()) * 1.02,
    where=high_mask_port,
    color="red",
    alpha=0.15,
    label="High vol regime"
)

plt.title("Portfolio Performance with Volatility Regime Overlay", fontsize=12,␣
 ↪fontweight="bold")
plt.xlabel("Date")
plt.ylabel("Growth of $1 Invested")
plt.legend(loc="upper left")
plt.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()
```

```
Returns matrix shape: (121, 503)
Equal weight weights sum: 1.0
Inverse vol weights sum: 1.0000000000000002
Min var weights sum: 1.0

=== Portfolio: EW ===
     ann_return: 0.0569
        ann_vol: 0.2256
         sharpe: 0.2523
   max_drawdown: -0.1768

=== Portfolio: IV ===
     ann_return: 0.0703
        ann_vol: 0.1995
         sharpe: 0.3523
   max_drawdown: -0.1453

=== Portfolio: MV ===
     ann_return: 0.0437
        ann_vol: 0.1871
         sharpe: 0.2338
   max_drawdown: -0.1555
```






```
===== Low vol regime =====
Portfolio EW: ann_return=0.2653, ann_vol=0.1480, sharpe=1.7926, max_dd=-0.0846
Portfolio IV: ann_return=0.2591, ann_vol=0.1336, sharpe=1.9389, max_dd=-0.0635
Portfolio MV: ann_return=0.2120, ann_vol=0.1222, sharpe=1.7353, max_dd=-0.0772

===== High vol regime =====
Portfolio EW: ann_return=-3.9374, ann_vol=0.8012, sharpe=-4.9142, max_dd=-0.0845
Portfolio IV: ann_return=-3.5490, ann_vol=0.6945, sharpe=-5.1098, max_dd=-0.0827
Portfolio MV: ann_return=-3.1817, ann_vol=0.6692, sharpe=-4.7546, max_dd=-0.0700
```



1.8 8. Discussion and Insights

1.8.1 Key Findings

1. **Non-normality:** The QQ plot shows clear fat tails, and the squared return ACF confirms volatility clustering. This supports using GARCH and Markov switching models.

2. **GARCH captures persistence:** The fitted GARCH(1,1) has $\alpha + \beta \approx 0.89$, indicating persistent volatility. Conditional volatility spikes align with the high volatility regime from the Markov model.

3. **Regime switching is informative:** The two state model cleanly separates calm and turbulent periods. Regime probabilities could support dynamic risk management, for example reducing exposure when $P(\text{high vol}) > 0.5$.

4. **Sector tail risk varies:** VaR and ES estimates differ across sectors. Information Technology and Energy show higher tail risk than Consumer Staples and Utilities.

5. **Portfolio performance:** Inverse volatility weighting delivers the highest Sharpe ratio and shallowest drawdowns. Minimum variance is the most conservative. Regime conditioned analysis shows all strategies struggle during high volatility periods.

1.8.2 Limitations

- **Short sample:** Only 6 months of data (121 trading days) limits statistical power
- **Equal weight proxy:** Our pseudo index ignores market cap weighting
- **No transaction costs:** Real implementation would incur rebalancing costs
- **Sector coverage:** Some tickers could not be mapped to sectors
- **In sample only:** We did not perform out of sample backtesting

1.8.3 References

- Kaggle dataset: S&P 500 Stocks Trade Data for First 6 Months of 2025
- Sector mappings: GitHub `datasets/s-and-p-500-companies`
- GARCH modeling: Bollerslev (1986), `arch` Python package

- Markov switching: Hamilton (1989), `statsmodels` implementation

[50]:
```python
import os
os.makedirs("figures", exist_ok=True)

# diagnostics plot
fig, axes = plt.subplots(2, 2, figsize=(10, 8))

ax = axes[0, 0]
ax.hist(r, bins=40, density=True, alpha=0.6)
x = np.linspace(mu - 4 * sigma, mu + 4 * sigma, 200)
ax.plot(x, norm.pdf(x, mu, sigma))
ax.set_title("Return histogram vs Normal")

sm.qqplot(r, line="s", ax=axes[0, 1])
axes[0, 1].set_title("QQ-plot")

plot_acf(r, lags=20, ax=axes[1, 0])
axes[1, 0].set_title("ACF of returns")

plot_acf(r ** 2, lags=20, ax=axes[1, 1])
axes[1, 1].set_title("ACF of squared returns")

plt.tight_layout()
fig.savefig("figures/returns_diagnostics.pdf", bbox_inches="tight")
plt.close(fig)

# garch plot
fig, ax = plt.subplots(figsize=(10, 4))
ax.plot(cond_vol.index, cond_vol.values)
ax.set_title("GARCH(1,1) Conditional Volatility Over Time")
ax.set_xlabel("Date")
ax.set_ylabel("Conditional volatility")
fig.savefig("figures/garch_volatility.pdf", bbox_inches="tight")
plt.close(fig)

# regime overlay plot
fig, axes = plt.subplots(2, 1, figsize=(10, 6), sharex=True)

ax = axes[0]
ax.plot(index_level.index, index_level.values, label="Index proxy")
high_mask = regime_series == 1
ax.fill_between(
    index_level.index,
    index_level.min(),
    index_level.max(),
    where=high_mask,
```

```
    alpha=0.15
)
ax.set_title("Index with high-vol regime shading")
ax.legend()

ax2 = axes[1]
ax2.plot(prob_high.index, prob_high.values)
ax2.set_title("Smoothed probability of high-vol regime")
ax2.set_xlabel("Date")
ax2.set_ylabel("Probability")

plt.tight_layout()
fig.savefig("figures/regime_probabilities.pdf", bbox_inches="tight")
plt.close(fig)

# sector var/es plot
fig, axes = plt.subplots(2, 1, figsize=(8, 8))

var_sector.sort_values().plot(kind="barh", ax=axes[0])
axes[0].set_title("Sector 95% VaR")

es_sector.sort_values().plot(kind="barh", ax=axes[1])
axes[1].set_title("Sector 95% ES")
axes[1].set_xlabel("Log return")

plt.tight_layout()
fig.savefig("figures/sector_var_es.pdf", bbox_inches="tight")
plt.close(fig)

# portfolio performance plot
fig, axes = plt.subplots(2, 1, figsize=(10, 6), sharex=True)

ax = axes[0]
ax.plot(cum_eq.index, cum_eq, label="EW")
ax.plot(cum_inv.index, cum_inv, label="IV")
ax.plot(cum_mv.index, cum_mv, label="MV")
high_mask_port = regime_aligned == 1
ax.fill_between(
    cum_eq.index,
    min(cum_eq.min(), cum_inv.min(), cum_mv.min()) * 0.98,
    max(cum_eq.max(), cum_inv.max(), cum_mv.max()) * 1.02,
    where=high_mask_port,
    alpha=0.15
)
ax.set_title("Portfolio performance with high-vol shading")
ax.legend()
```

```
ax2 = axes[1]
ax2.plot(dd_eq.index, dd_eq, label="EW")
ax2.plot(dd_inv.index, dd_inv, label="IV")
ax2.plot(dd_mv.index, dd_mv, label="MV")
ax2.set_title("Portfolio drawdowns")
ax2.set_xlabel("Date")
ax2.set_ylabel("Drawdown")

plt.tight_layout()
fig.savefig("figures/portfolio_performance.pdf", bbox_inches="tight")
plt.close(fig)
```

[51]:
```
rets_eq = cum_eq.pct_change().fillna(0)
rets_inv = cum_inv.pct_change().fillna(0)
rets_mv = cum_mv.pct_change().fillna(0)

import pandas as pd
import numpy as np

def portfolio_stats(r):
    ann_ret = (1 + r).prod()**(252 / len(r)) - 1
    ann_vol = r.std() * np.sqrt(252)
    sharpe = ann_ret / ann_vol
    cum = (1 + r).cumprod()
    peak = cum.cummax()
    dd = cum / peak - 1
    max_dd = dd.min()
    return ann_ret, ann_vol, sharpe, max_dd

stats = {}
for name, r in [("EW", rets_eq), ("IV", rets_inv), ("MV", rets_mv)]:
    stats[name] = portfolio_stats(r)

metrics = pd.DataFrame(
    stats,
    index=["ann_ret", "ann_vol", "sharpe", "max_dd"]
).T

# Pretty print as percentages with 1 decimal place
display(metrics.apply(
    lambda col: col if col.name == "sharpe" else 100 * col
))
```

	ann_ret	ann_vol	sharpe	max_dd
EW	1.187874	22.517831	0.052753	-17.681064
IV	3.454386	19.919113	0.173421	-14.533446
MV	0.860698	18.671438	0.046097	-15.551305