UNIVERSITY OF Waterloo

Department of Mechanical and Mechatronics Engineering

MTE 219 Mechanics of Deformation Winter 2022

Term Project

A Report Prepared For:

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March 14, 2022

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1.0 Executive Summary

Design projects offer a chance to apply theory in real life applications, and thus allow engineers to see if the models for the forces are valid. The purpose of this project was to come up with the design of a truss bridge that had the highest possible ratio of **weight supported to weight of the bridge (performance value or PV)**.

Like most engineering projects, the design of the truss bridge project was an iterative process, and there were several versions of the bridge throughout the process and challenges the team encountered. We used facts like wood is stronger under compression to our advantage to come up with an optimum design. The other fact was that weight would be loaded at the center and thus supporting it with a greater number of trusses was key.

To design the bridge, the team did calculations for different truss designs to find a truss that would best. Two tests were conducted to test the capabilities of the bridge and determine the modes of failure and identify problems that were not considered at the initial design. The final design had a performance value of 199.2.

2.0 Introduction

The design problem is to design a 40 cm ± 1 cm bridge by length and achieve a performance value that is at least 100. The performance value ("PV") will be calculated by the maximum applied load that the truss can support divided by the mass of the truss before loaded; both masses are measured in grams.

2.1 Design Constraints and Criteria

The design constraints for this project are as followed:

- Length of bridge is 40.0 ± 1.0 cm wide
- Height of bridge is 10.0 ± 0.5 cm wide (from bottom pin to top pin)
- Supported ends for the bridge must be 8.0 ± 0.5 cm wide
- Joints must be able to freely rotate
- No more than three 4" x 0.125" x 36" balsa wood sheets
- No more than two 0.125" and 36" dowels
- Only two force members are allowed as well as pinned connections

The criteria are as follows:

- All members should break at the same time to achieve an optimal performance value
- Bridge should not use all the material to allow for the design of a few test bridges

Material properties can be found in Table 1 below. These properties are used in the calculations for normal, shear, and bending forces in the Design Analysis section.

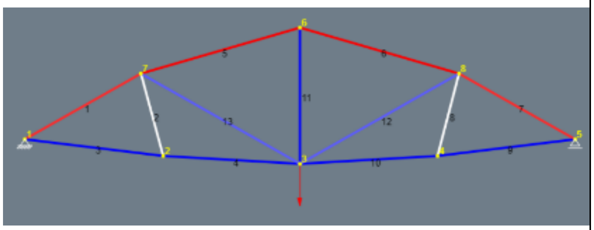
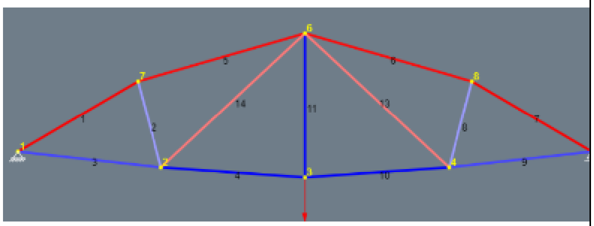
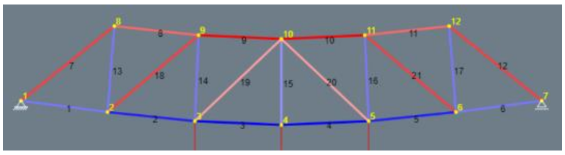
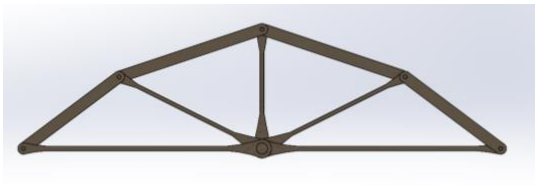
Table 1: Material Properties of Balsa Wood and Wooden Dowels [1]

Properties	Balsa Wood	Dowel
Density, ρ (kg/m^3)	132	650
Elastic Modulus, E (GPa)	3.112	16
Ultimate Normal Force, σ_{ult} (MPa)	31.178	171
Maximum Bending Moment, M_{max} (Nmm)	-	368
Ultimate Shear Force τ_{ult} (MPa)	1.6	23

3.0 Preliminary Designs and Decision Matrices

From the initial concepts, each group member created a design and tested it on an online simulation. The group determined the potential modes of failure for the design. For each mode of failure, calculations were performed for each mode of failure to determine the required member dimensions and pin order to minimize shear. *Table 2* shows the designs that each group member came up with.

Table 2: Technical Drawings and Freehand Sketches of 4 Truss Designs

Design 1	Design 2
	
Design 3	Design 4
	

Our group used a decision matrix with several criteria to compare and decide the best bridge design. The first criteria the group decided was the number of members. It was decided to rank 1 to the truss with the greatest number of members more would mean more material meaning more weight, as well as more complexity when assembling. The next criteria were the number of joints where rank 1 went to the member with the most joints. More joints mean more pins are needed and more material is used. After, buckling prevention was considered used zero-force members to approximate it. It was either ranked 1 or 4, with 1 having no zero force members, and 4 having them. Furthermore, tensile forces were considered with the lowest tensile force member being ranked 1. Finally, the highest compressive force got a rank of 4 and the other members were ranked in descending order.

Table 3 depicts the decision matrix used. The weighting of each criterion can be seen in the second row, and the rankings can be seen below them: 1 being the best in that category, and 4 being the worst. Designs 1-3 were made using [2].

Table 3: Decision matrix for optimal truss design

	Highest Tensile	Highest Compression	Buckling prevention?	Number of members	Number of joints	Total Points
Weight	3	2	1	5	3	
Truss 1	4	4	1	2	2	37
Truss 2	3	1	1	3	4	39
Truss 3	2	2	4	1	1	22
Truss 4	1	3	1	4	4	42

4.0 Design Analysis and Optimization

After selecting a truss design, the next part would be finding the desired dimensions for the different members of the truss. The forces in each member relative to the mass of the load *m* were first determined using static analysis and the method of sections. The forces relative to the mass *m* of the load are as follows. For reference, the member numbering is shown in Figure 1.

Table 4: Force in truss members

Members	Force (N)	Members	Force (N)
1,2	3.27*m*	7,9	1.83*m*
3,6	-4.09*m*	8	3.27*m*
4,5	-5.17*m*		

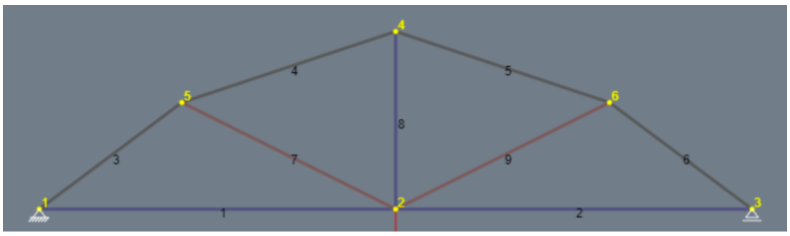
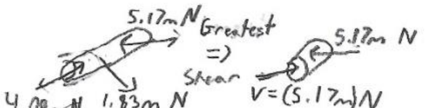


Figure 1: Truss member numbering

A major challenge was the pin in the middle. If left unsupported, it would be the first part of the truss to break; it would not matter how strong the truss design was if the dowel acting as a pin broke. To allow the pin to support the acting load, it was decided to make the pin larger by using more material and gluing it together. Sample calculations for the six modes of failure are shown below in Figure 2 and Figure 3. The maximum allowable mass was determined using the stress and force properties of each member or pin. Pin shear calculations were done according to [3].

Failure due to Pin Shear
Ex. Pin 5



Area $= \pi r^2 = \pi (1.5875)^2 = 7.92\ mm^2$

$\tau = \frac{V}{A} = \frac{(5.17m)\ N}{7.92\ mm^2} = (0.653m)\ MPa$

$\tau_{max} = \tau_{ult} = 23\ MPa = (0.653m_{all})\ MPa$ (Dowel)

$\Rightarrow m_{All} = 35.2\ kg$ for this mode of failure

Failure due to Member Tearing
Ex. Member 7

$t = 3.175\ mm$ $P = (1.83m)\ N$
$b = 3.175\ mm$

$\tau = \frac{P}{2bt} = \frac{(1.83m)\ N}{2(3.175mm)(3.175mm)} = (0.0908m)\ MPa$

$\tau_{max} = \tau_{ult} = 1.6\ MPa = (0.0908m_{All})\ MPa$ (Balsa wood) $\Rightarrow m_{All} = 17.6\ kg$ for this mode of failure

Bearing Failure
Ex. Member 7 (Pin 5)

$t = 3.175\ mm$ $P = (1.83m)\ N$
$d = 3.175\ mm$

$\sigma_b = \frac{P}{\text{Contact Area}} = \frac{P}{td} = \frac{(1.83m)\ N}{(3.175mm)(3.175mm)} = 0.182m\ MPa$

$\sigma_{max} = \sigma_{ult} = 171\ MPa = 0.182m_{All}\ MPa$ (Dowel) $\Rightarrow m_{All} = 940\ kg$ for this mode of Failure

Failure due to Member in Tension - Rupture
Ex. Member 7

Smallest Cross-sectional Area $= tw_{min} = 3.175\ mm \cdot 3.175\ mm = 10.08\ mm^2$

$\sigma_{max} = \frac{P}{A_{min}} = \frac{(1.83m)N}{(10.08)mm^2} = 0.181m\ MPa$

$\sigma_{ult} = 31.178\ MPa = 0.181m_{All}\ MPa \Rightarrow m_{All} = 172\ kg$ (Balsa Wood) For this mode of Failure

Figure 2: Calculations for pin shear, member tearing and bearing failure

Failure due to Member in Compression - Buckling

Ex. Member 4 (length 126.49)

$I_{min} = \frac{1}{12} bh^3 = \frac{1}{12}(9.525)(9.525)^3 = 685.93\ mm^4$

$l_e = 126.49\ mm$

$E = 3.112\ GPa$ (balsa wood)

$$P_{cr} = \frac{\pi^2 E I_{min}}{l_e^2} = \frac{\pi^2 (3.112\ GPa)(685.93\ mm^4)}{(126.49\ mm)^2} = 1316.76\ N$$

Force in member 4: $5.17 m_{All}\ N = P_{cr} = 1316.76\ N$

$\Rightarrow m_{All} = 255$ kg for this mode of failure

Failure due to Bending in Pin

Using pin 2, $c = r$

$I = \frac{\pi}{4} c^4$ $\quad l = 6.32\ mm$

$P = (9.81 m)\ N$ ↳ Hanger diameter for loading

$c = 4.7625\ mm$

$$(\sigma_{All})_{Max} = \sigma_{Ult\ (Dowel)} = 171 = \frac{Mc}{I} = \frac{Pl}{\pi c^3} = \frac{(9.81 m_{All})\ N \cdot 6.32\ mm}{\pi (4.7625\ mm)^3} = 0.1827 m$$

$\Rightarrow m_{All} = 436$ kg for this mode of failure

Figure 3: Calculations for rupture, buckling and bending of pin

4.1 Design of Members

Table 5: Summary Table of the Optimizing Team's Design

Criteria	Rationale and Design
Members should have the necessary thickness before failure	Goal is for all members to break at the same time. If there was a weak member, it would be the first to break regardless of the other members. Members taking on more normal force will need to be thicker compared to members taking on less loads.
Member thickness should not be too thin	If members are too thin, they may experience buckling.

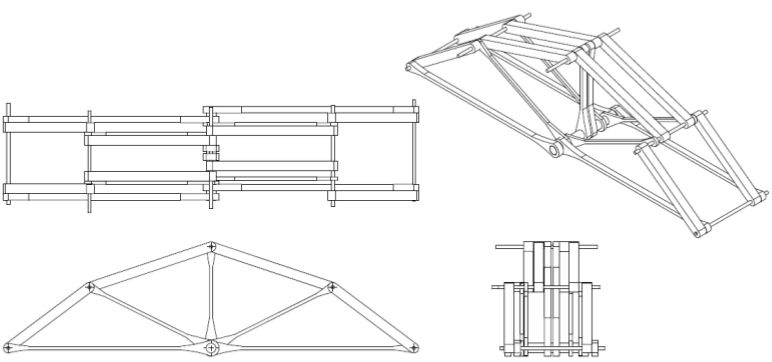


Figure 4: Different Views of the Final 3D Truss Design

The bridge and the different views can be seen in Figure 4. The bridge is symmetrical and uses the same members on both sides of the dowel. Note that in Figure 4 the middle dowel is simplified to a large one-piece dowel.

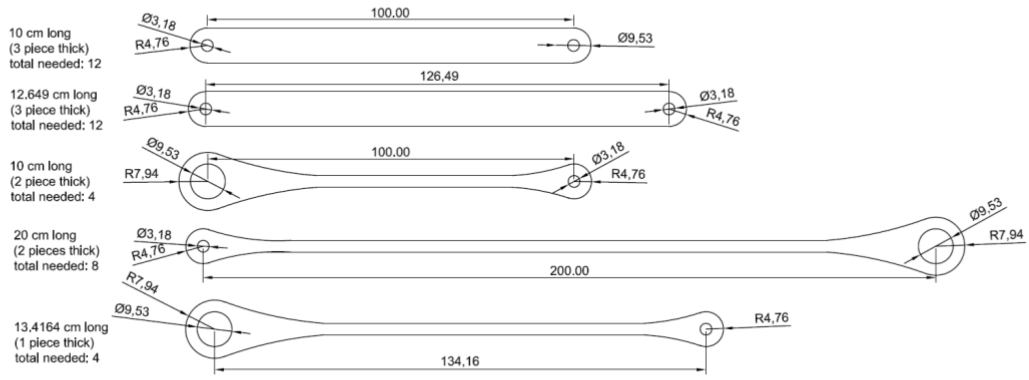


Figure 5: Detailed Drawing with Measurements of each Distinct Member

The five members with their dimensions in millimeters can be seen in Figure 5. They were printed on a laser printer and the specified number of copies is also observed in Figure 2. Since the balsa wood was 3.175 mm thick, the team decided to cut multiple members (two to three) copies to make it two or three pieces thick. It would also reduce buckling as the pieces would have a larger cross-sectional area making the ratio between the length and thickness being greater.

The dimensions and number of each optimized member and dowel were used with their respective densities to calculate the mass of the total bridge. Note that glue was considered negligible. The theoretical mass was calculated as 22.98 g. The theoretical performance value for the optimized design was 765.9.

5.0 Construction, Testing, and Refinement

The group decide to conduct two tests to find any potential flaws with the design and determine where and when the bridge would fail before creating a final design for the competition day. There was a limited number of materials, and our original design used a full sheet of balsa wood meaning two tests and one final bridge design could be done.

It was important to reduce the bending moment in the center of the bridge while ensuring the ends met the 8.0 ± 0.5 cm; we had to put long member on outside. We made the dowels from 8 cm long to 10 cm to ensure there was enough space. The group took photos to document the order required.

Table 6: Observations from Tests for Bridge Design

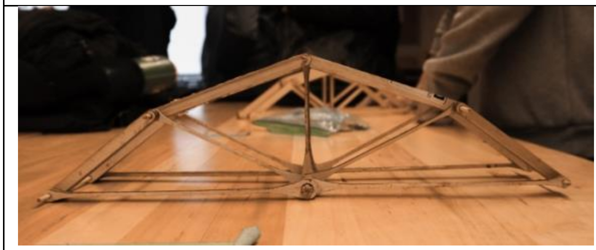
Images of the Broken Members from the Test Bridge	Physical Observations and Other Notes
	Date: March 4, 2022, 3:20 pm Mass: 24 g Applied Load before Fracture: 3.0 kg Mode of Failure: Rupture Causes: Ends not glued (treat as single rather than double), concentrated stress at end due to absence of fillet
	Date: March 7, 2022, 3:35 pm Mass of Truss: 26 g Applied Load before Fracture: 8.5 kg Mode of Failure: Rupture, Normal Stress on Multiple Members Causes: Normal stress and strain on members Design Comments: Design did what was desired–multiple modes of failures occurred at once. Dowels were fine and did not bend

In Table 5, a summary table of observations and images from the tests are shown. After the first test we implemented the design changes: 1) add fillets to reduce stress concentration, 2) glue at the circular areas, the design was significantly better and multiple members broke at the same time for various failure reasons; the design was good. As well, it achieved a PV of 326 far exceeding the required 100. No further changes were made on the CAD design, and the bridge was re-printed on the laser cutter for the final competition.

It is of note that our group decided to use wood glue for the final design over hot glue to reduce the mass and have a stronger adhesive bond between the two- and three-member thick balsa wood members. The final bridge failed with a PV value of 199.2 which happens to be under our previous value of 326. The mass was 25 g – a gram less than the previous iteration.

6.0 Conclusions

Table 7: Final Construction of the Bridge, Broken Members after Failure

Final Truss Assembly on the Competition Day	Broken Members after Failure
	

The project exceeded the required performance value of 100 by two folds. The project's criteria and constraints were also met. There are some design considerations that would be worthwhile to consider for a future design. In the final bridge design, the failure was a result of member tearing due to shear on two members shown in Table 7. No other members broke in the truss which is different from the second bridge iteration. It is theorized that this was due to some misalignment issues which were a result of using multiple layers of members and wood glue and last-minute cracks that were introduced in the members during the assembly process. Furthermore, when completing a final design, no changes should be made as untested problems can occur. The project was a fascinating look at engineering design and apply the engineering principles from the MTE219: Materials of Deformation class. Good planning, testing, and iterating were all instrumental to bring this project to life.

7.0 References

[1] Univeristy of Waterloo, "MTE219_W22_MaterialData_Solution," February 2022. [Online]. Available: https://learn.uwaterloo.ca/d2l/le/content/783116/viewContent/4253245/View. [Accessed 1 March 2022].

[2] Valdivia, "2D-Truss Analysis," [Online]. Available: https://valdivia.staff.jade-hs.de/fachwerk_en.html. [Accessed 1 March 2022].

[3] M. Vable, "Mechanics of Materials," Michigan Technoogical University, [Online]. Available: https://madhuvable.org/wp-content/uploads/2016/04/Entire%20Book%202018.pdf. [Accessed 1 March 2022].

7.0 Appendix

Appendix 1: Calculated Values for Different Forces Experiences

Failure due to Member in Tension - Rupture

Members	Smallest Cross-sectional Area (mm^2)	Normal Stress (MPa)	Allowable Mass (kg)
1,2	20.16	0.162m	192
7,9	10.08	0.181m	**172**
8	20.16	0.162m	192

Failure due to Member in Compression - Buckling

Members	Minimum Moment of Inertia (mm^4)	Length (mm)	Calculated Critical Buckling Load (N)	Allowable Mass (kg)
3, 6	685.93	100	2106.77	515
4, 5	685.93	126.49	1316.76	**255**

Failure due to Pin Bending

Loading Force (N)	Length (mm)	Radius (mm)	Bending Stress	Allowable Mass (kg)
9.81m	6.32	4.7625	0.1827m	**936**

Failure due to Pin Shear

Pins	Pin Area (mm^2)	Greatest Shear Force (N)	Shear Stress (MPa)	Allowable Mass (kg)
1, 3	7.92	3.27m	0.413m	55.7
5, 6	7.92	5.17m	0.653m	**35.2**
4	7.92	5.17m	0.653m	**35.2**
2	71.26	4.26m	0.0598m	384.7

Failure due to Member Tearing

Members	Thickness (mm)	Joint Depth (mm)	Shear Stress (MPa)	Allowable Mass (kg)
1,2	6.350	3.175	0.0811*m*	19.7
7,9	3.175	3.175	0.0908*m*	**17.6**
8	6.350	3.175	0.0811*m*	19.7

Bearing Failure

Members	Thickness (mm)	Bearing Diameter (mm)	Bearing Stress (MPa)	Allowable Mass (kg)
1,2 (Pins 1,3)	6.350	3.175	0.162*m*	1056
1,2 (Pin 2)	6.350	9.525	0.0541*m*	3161
3,6	9.525	3.175	0.135*m*	1267
4,5	9.525	3.175	0.171*m*	1000
7,9 (Pins 5,6)	3.175	3.175	0.182*m*	**940**
7,9 (Pin 2)	3.175	9.525	0.0605*m*	2826
8 (Pin 4)	6.350	3.175	0.162*m*	1056
8 (Pin 2)	6.350	9.525	0.0541*m*	3161

Maximum Allowable Mass: 17.6 kg

Mass calculation for Members

Member	Thickness (mm)	Area (mm^2)	Mass per Member (g)
1,2	6.350	891.1434	0.7470
3,6	9.525	1007.92211	1.267
4,5	9.525	1260.2384	1.584
7,9	3.175	683.0678	0.2863
8	6.350	574.5971	0.4816

Mass calculation for Pins

Pin	Height (mm)	Area (mm^2)	Mass per Pin (g)
1,3,4,5,6	100	7.92	0.4633
2	100	71.26	4.169

Total Mass: 22.98 g

Theoretical PV: 765.9