EXERCISES FOR SECTION 5-3

5-34. Determine the value of c such that the function $f(x, y) = cxy$ for $0 < x < 3$ and $0 < y < 3$ satisfies the properties of a joint probability density function.

5-35. Continuation of Exercise 5-34. Determine the following:

5-36. Continuation of Exercise 5-34. Determine the following:

(a) Marginal probability distribution of the random variable X
(b) Conditional probability distribution of Y given that $X = 1.5$
(c) $E(Y|X) = 1.5)$
(d) $P(Y < 2|X = 1.5)$
(e) Conditional probability distribution of X given that $Y = 2$

5-37. Determine the value of c that makes the function $f(x, y) = c(x + y)$ a joint probability density function over the range $0 < x < 3$ and $x < y < x + 2$.

5-38. Continuation of Exercise 5-37. Determine the following:

(a) $P(X < 1, Y < 2)$ (b) $P(1 < X < 2)$
(c) $P(Y > 1)$ (d) $P(X < 2, Y < 2)$
(e) $E(X)$

5-39. Continuation of Exercise 5-37. Determine the following:

(a) Marginal probability distribution of X
(b) Conditional probability distribution of Y given that $X = 1$
(c) $E(Y|X = 1)$
(d) $P(Y > 2|X = 1)$
(e) Conditional probability distribution of X given that $Y = 2$

5-40. Determine the value of c that makes the function $f(x, y) = cxy$ a joint probability density function over the range $0 < x < 3$ and $0 < y < x$.

5-41. Continuation of Exercise 5-40. Determine the following:

(a) $P(X < 1, Y < 2)$ (b) $P(1 < X < 2)$
(c) $P(Y > 1)$ (d) $P(X < 2, Y < 2)$
(e) $E(X)$ (f) $E(Y)$

5-42. Continuation of Exercise 5-40. Determine the following:

(a) Marginal probability distribution of X
(b) Conditional probability distribution of Y given $X = 1$
(c) $E(Y|X = 1)$
(d) $P(Y > 2|X = 1)$
(e) Conditional probability distribution of X given $Y = 2$

5-43. Determine the value of c that makes the function $f(x, y) = ce^{-2x-3y}$ a joint probability density function over the range $0 < x$ and $0 < y < x$.

5-44. Continuation of Exercise 5-43. Determine the following:

(a) $P(X < 1, Y < 2)$ (b) $P(1 < X < 2)$
(c) $P(Y > 3)$ (d) $P(X < 2, Y < 2)$
(e) $E(X)$ (f) $E(Y)$

5-45. Continuation of Exercise 5-43. Determine the following:

(a) Marginal probability distribution of X
(b) Conditional probability distribution of Y given $X = 1$
(c) $E(Y|X = 1)$
(d) $P(Y < 2|X = 1)$
(e) Conditional probability distribution of X given $Y = 2$

5-46. Determine the value of c that makes the function $f(x, y) = ce^{-2x-3y}$ a joint probability density function over the range $0 < x$ and $x < y$.

5-47. Continuation of Exercise 5-46. Determine the following:

(a) $P(X < 1, Y < 2)$ (b) $P(1 < X < 2)$
(c) $P(Y > 3)$ (d) $P(X < 2, Y < 2)$
(e) $E(X)$ (f) $E(Y)$

5-48. Continuation of Exercise 5-46. Determine the following:

(a) Marginal probability distribution of X
(b) Conditional probability distribution of Y given $X = 1$
(c) $E(Y|X = 1)$
(d) $P(Y < 2|X = 1)$
(e) Conditional probability distribution of X given $Y = 2$

5-49. Two methods of measuring surface smoothness are used to evaluate a paper product. The measurements are recorded as deviations from the nominal surface smoothness in coded units. The joint probability distribution of the two measurements is a uniform distribution over the region $0 < x < 4$, $0 < y$, and $x - 1 < y < x + 1$. That is, $f_{XY}(x, y) = c$ for x and y in the region. Determine the value for c such that $f_{XY}(x, y)$ is a joint probability density function.

5-50. Continuation of Exercise 5-49. Determine the following:

(a) $P(X < 0.5, Y < 0.5)$ (b) $P(X < 0.5)$
(c) $E(X)$ (d) $E(Y)$

5-51. Continuation of Exercise 5-49. Determine the following:

(a) Marginal probability distribution of X
(b) Conditional probability distribution of Y given $X = 1$
(c) $E(Y|X = 1)$
(d) $P(Y < 0.5|X = 1)$

5-52. The time between surface finish problems in a galvanizing process is exponentially distributed with a mean of 40 hours. A single plant operates three galvanizing lines that are assumed to operate independently.

(a) What is the probability that none of the lines experiences a surface finish problem in 40 hours of operation?
(b) What is the probability that all three lines experience a surface finish problem between 20 and 40 hours of operation?
(c) Why is the joint probability density function not needed to answer the previous questions?

5-53. A popular clothing manufacturer receives Internet orders via two different routing systems. The time between orders for each routing system in a typical day is known to be exponentially distributed with a mean of 3.2 minutes. Both systems operate independently.

(a) What is the probability that no orders will be received in a 5 minute period? In a 10 minute period?
(b) What is the probability that both systems receive two orders between 10 and 15 minutes after the site is officially open for business?

(c) Why is the joint probability distribution not needed to answer the previous questions?

5-54. The conditional probability distribution of Y given $X = x$ is $f_{Y|x}(y) = xe^{-xy}$ for $y > 0$ and the marginal probability distribution of X is a continuous uniform distribution over 0 to 10.

(a) Graph $f_{Y|X}(y) = xe^{-xy}$ for $y > 0$ for several values of x.
Determine
(b) $P(Y < 2|X = 2)$ (c) $E(Y|X = 2)$
(d) $E(Y|X = x)$ (e) $f_{XY}(x, y)$
(f) $f_Y(y)$