

Visualizing Evolving Trees

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Abstract. Evolving trees arise in many real-life scenarios from computer file systems and dynamic call graphs, to fake news propagation and disease spread. Most layout algorithms for static trees, however, do not work well in an evolving setting (e.g., they are not designed to be stable between time steps). Dynamic graph layout algorithms are better suited to this task, although they often introduce unnecessary edge crossings. With this in mind we propose two methods for visualizing evolving trees that guarantee no edge crossings, while optimizing (1) desired edge length realization, (2) layout compactness, and (3) stability. We evaluate the two new methods, along with four prior approaches (two static and two dynamic), on real-world datasets using quantitative metrics: stress, desired edge length realization, layout compactness, stability, and running time. The new methods are fully functional and available on github.

1 Introduction

Dynamic graph visualization is used in many fields including social networks [27], bibliometric networks [47], software engineering [13], and pandemic modeling [6]; see the survey by Beck *et al.* [9]. Here we focus on a special case, *evolving trees*. In evolving trees the dynamics are captured only by growth (whereas in general dynamic graphs, nodes and edges can also disappear). While this is a significant restriction of the general dynamic graph model, evolving trees are common in many domains including the Tree of Life [35] and the Mathematics Genealogy Graph [34]. An evolving tree can also model disease spread, where nodes correspond to infected individuals and a new node v is added along with an edge to existing node u if u infected v . Visualizing this process can help us see how the infection spreads, the rate of infection, and to identify “super-spreaders.”

There are several methods and tools that can be used to visualize evolving trees [2, 14, 15, 37], however, most of them have limitations that can impact their usability in this domain. Many represent nodes only as points ignoring labels [14, 15], which makes them less useful in real-life applications where it is important to see what each node represents. Others utilize the level-by-level approach to drawing hierarchical graphs [26, 39], which does not capture the underlying graph structure well. Force-directed algorithms tend to better capture the underlying graph structure [30], although they may introduce unnecessary edge crossings. With this in mind, we propose two methods for drawing crossing-free evolving trees that optimize the following desirable properties:

- 36 1. **Desired edge length realization:** The Euclidean distance between two
 37 nodes u and v in the layout should realize the corresponding pre-specified
 38 edge length $l(u, v)$, or be uniform when no additional information is given.
- 39 2. **Layout compactness:** The drawing area should be proportional to the total
 40 area needed for all the labels. A good visualization should have the labeled
 41 graph drawn in a compact way [38]. This prevents the trivial solution of
 42 scaling the layout until all overlaps and crossings are removed, which can
 43 create vast empty spaces in the visualization.
- 44 3. **Stability:** Between time steps, nodes should move as little as possible. This
 45 helps the viewer maintain a mental map of the graph [36]. If the graph moves
 46 around too much, it is difficult to see where new nodes and edges are added
 47 and we lose the context of the new node's relation to the rest of the graph.

48 We propose two force-directed methods that ensure no edge crossings and
 49 optimize desired edge length, compactness and stability. Minimizing edge cross-
 50 ings is important in graph readability [41], and since we work with trees, a layout
 51 without edge crossings is possible and desirable. We use two trees extracted from
 52 Tree of Life [35] and the Mathematics Genealogy [34] projects to demonstrate the
 53 new methods and quantitatively evaluate their performance, measuring desired
 54 edge length realization, compactness, stability, stress, crossings, and running
 55 time. We also evaluate the performance of four earlier methods, showing the two
 56 proposed methods perform well overall; see Fig. 1.

59 2 Related Work

60 Dynamic graph drawing has a long history [10, 45]. It can be divided into two
 61 broad categories: offline and online. In the easier offline setting we assume that
 62 all the data about the dynamics is known in advance. Algorithms for offline
 63 dynamic visualization use different approaches including combining all time-
 64 slice instances into a single supergraph [17–19], connecting the same node in
 65 consecutive time-slices and optimizing them simultaneously [21–23], providing
 66 animation [4], and showing small multiples type visualization [3]. *DynNoSlice* by
 67 Simonetto *et al.* [42] is one of the most recent approaches for this setting and is
 68 different from the prior methods as it does not rely on discrete time-slices.

69 Online dynamic graph drawing deals with the harder problem – when we do
 70 not know in advance what changes will occur. One can optimize the current view,
 71 given what has happened in the past, but cannot look into the future, as the
 72 information is not available. Cohen *et al.* [14, 15] and Workman *et al.* [49] describe
 73 algorithms for dynamic drawing of trees that place nodes that are equidistant
 74 from the root on the same level (same y coordinate). These algorithms do not
 75 take edge lengths into consideration and the hierarchical nature of the layout
 76 can lead to exponential differences between the shortest and longest edges.

77 *DynaDAG* is an online graph drawing method for drawing dynamic directed
 78 acyclic graph as hierarchies [39]. This method moves nodes between adjacent
 79 ranks based on the median sort. It was not specifically developed for trees and
 80 may introduce crossings; see Fig. 2a. Other approaches for online dynamic graph

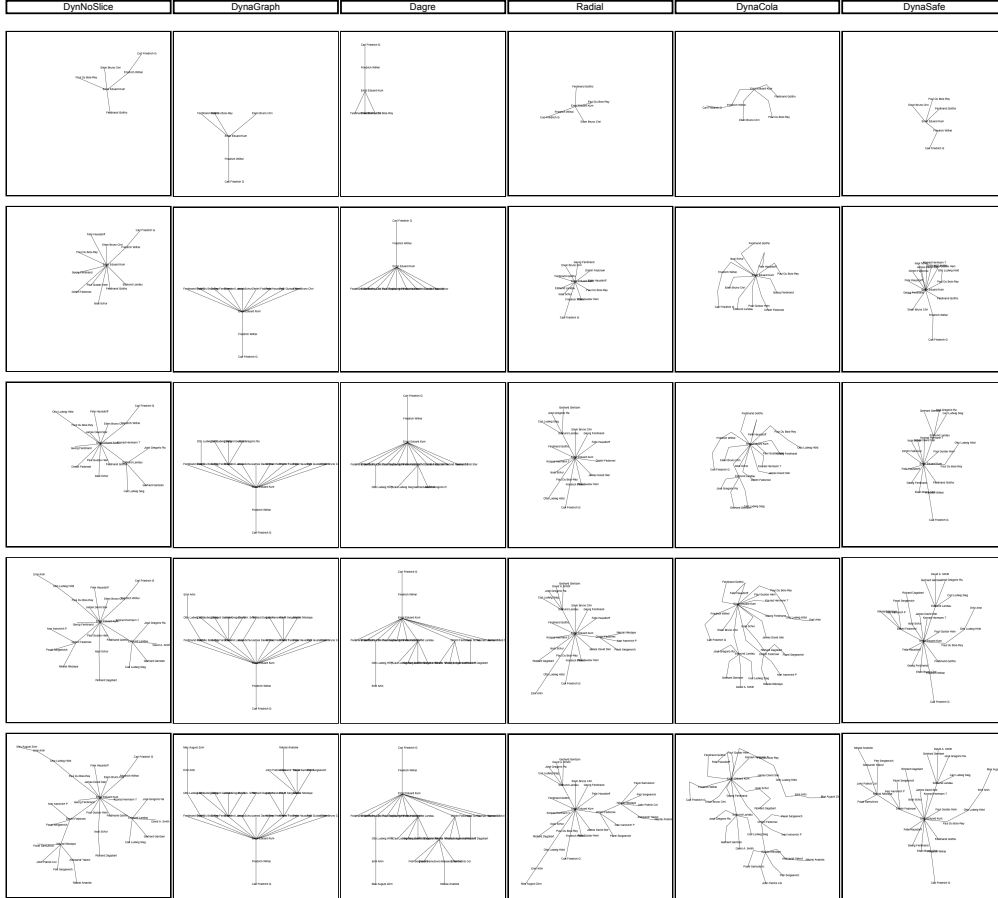


Fig. 1. The layouts obtained by DynNoSlice, DynaGraph, Dagre, Radial, DynaCola and DynaSafe of the same evolving math genealogy tree; each row adds six new nodes.

drawing have maintained the horizontal and vertical position of nodes [36], used node aging methods [28], and adapted multilevel approaches [16] (using FM³ [29]). Online approaches have also been implemented on the GPU [24]. However, these methods do not guarantee crossing free layouts for trees and do not take into account desired edge lengths; see Fig. 2a.

Dagre is a multi-phase algorithm for drawing directed graphs based on [26]. The initial phase finds an optimal rank assignment using the network simplex algorithm. Then it sets the node order within ranks by an iterative heuristic incorporating a weight function and local transpositions to reduce crossings (via the barycenter heuristic) [31]. However, since *Dagre* draws graphs in a hierarchical structure, the edge lengths may vary arbitrarily; see Fig. 2b.

The *radial* layout implemented in yFiles [48] displays each biconnected component in a circular fashion. Radial layouts were introduced by Kar *et al.* [32]

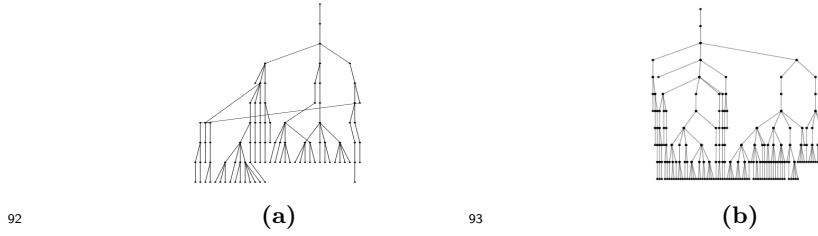


Fig. 2. (a) A DynaDAG layout of a tree with 100 nodes that introduces some edge crossings. (b) A Dagre layout of a tree with 300 nodes, with large edge length variability. Both trees are extracted from the math genealogy dataset.

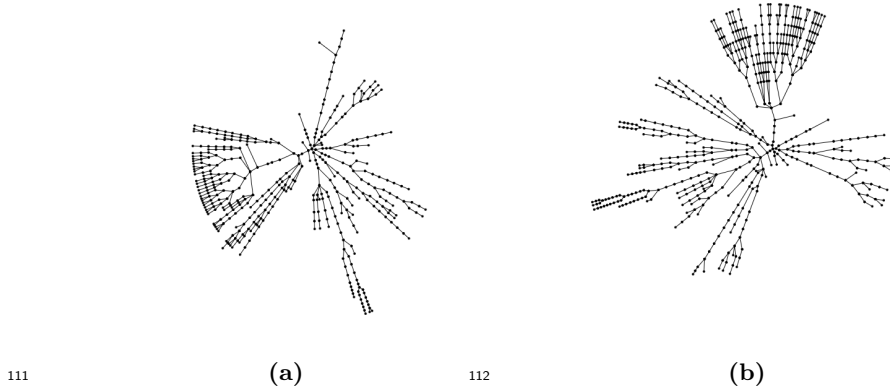


Fig. 3. A radial layout of a 400-node evolving tree of life (a) and the layout after adding 100 nodes (b). As most of the growth occurred on the right side, a lot of movement was needed to accommodate it.

for static graphs. Dougrusoz *et al.* [20] described an interactive tool for dynamic graph visualization based on the radial layout. Six and Tollis [44] adapted the radial idea to circular drawings of static biconnected graphs and experimentally showed that their layout has fewer edge crossings. Kaufmann *et al.* [33] extended this model to handle dynamic graphs, providing the basis of the yFiles *radial* implementation. Pavlo *et al.* [40] adapted the idea to make the radial layout computation parallelizable. Bachmaier [5] further improved the radial layout algorithm for static graphs by adapting the hierarchical approach [46] to minimize edge crossings. Radial layout methods have more freedom than the traditional level-by-level tree layout methods. Nevertheless, they are still constrained and can result in unstable visualization for dynamic graphs in general and evolving trees in particular; see Fig. 3.

Force-directed algorithms [17–19, 42] underlie many static and dynamic graph visualization methods. Unlike hierarchical and radial approaches, force-directed algorithms place nodes at arbitrary positions, and so tend to generate more

compact layouts that better realize desired edge lengths. By adjusting forces appropriately, one can also generate stable layouts by this approach. To the best of our knowledge, there are no force-directed methods for evolving trees.

3 Algorithms for Visualizing Evolving Trees

Here, we describe two force-directed algorithms for evolving tree visualization, *DynaCola* and *DynaSafe*, that realize desired edge lengths without creating crossings, and optimize compactness and stability. *DynaCola* avoids edge crossings by creating and maintaining a “collision region” for each edge. While collision detection/prevention is usually applied to nodes, by carefully applying it to the edges we can prevent all edge crossings. *DynaSafe* prevents edge crossings with a “safe” coordinate update at every step of the algorithm. Before updating a coordinate, it first checks whether the update will introduce a crossing and then limits the update magnitude to avoid the crossing.

3.1 DynaCola

DynaCola stands for Dynamic Collision, as the algorithm uses the collision forces to prevent edge crossings. This is a force-directed algorithm, augmented with edge-regions used to prevent crossings; see Algorithm 1. Recall that we are gradually growing a tree, one node at a time, while maintaining a crossing-free layout and optimizing desirable properties (desired edge lengths, compactness, stability). The *DynaCola* force-directed algorithm relies on the following forces and is implemented in d3.js [11].

- A force f_E for each edge, to realize the desired edge length. The strength of this force is proportional to the difference between the edge distance in the layout and the desired edge length.
- A general repulsive force f_R defined for all pairs of nodes and implemented with the Barnes-Hut quad-tree data structure [7]. This helps realize the global structure of the underlying tree.
- A collision force f_C for each edge, described in details below. This force prevents edge crossings.
- A gravitational force f_G that attracts all nodes to the center of mass. This force draws the nodes closer together and improves compactness.

To ensure that no edges cross during an update, we define a collision region around each edge: if any edge/node moves too close to another edge, it will be pushed away. To create a collision region for an edge $e = (u, v)$, we can create collision circles with diameter equal to the length of e for both u and v . Then every point of e will be either inside the collision region of u or v . However, the sum of all collision regions for all nodes will be unnecessarily large and the layout will not be compact. With the help of subdivision nodes along the edges, we can reduce the sum of all collision regions. Let $e = (u, v)$ be an edge in the graph. We

158 use a set of subdivision nodes $V_s = \{v_1, v_2, \dots, v_k\}$ and replace the edge (u, v)
 159 by a set of edges $E_s = \{(u, v_1), (v_1, v_2), \dots, (v_k, v)\}$. We assign the desired edge
 160 length of an edge in E_s equal to $l(u, v)/|E_s|$, where $l(u, v)$ is the desired edge
 161 length. In general, the number of subdivision nodes per edge should be a small
 162 constant n_s (by default $n_s = 1$), since the complexity of the algorithm increases
 163 as n_s increases. Also, note that n_s determines the number of bends per edge (no
 164 bends when $n_s = 0$, one bend when $n_s = 1$, and so on).

165 When a new node is added to the tree, a new edge also is added, with one
 166 of its endpoints already placed. To place the new node, we randomly sample a
 167 set of 100 nearby points at a distance equal to the desired edge length, trying to
 168 find a crossing-free position. If we cannot find such a suitable point, we gradually
 169 reduce the distance and repeat the search until we find a crossing free position.
 170 Once the new node has been placed, we subdivide its adjacent edge as described
 171 above.

Algorithm 1: DynaCola(Current tree layout Γ_T , new edge e , $nIters$,
 Number of subdivision nodes n_s)

Result: A crossing-free layout, maintaining the desired properties

```

  Let  $u, v = e$ ;
  Let  $u \in T$ ;
  Find a crossing free position for  $v$  by random sampling;
  Reduce the edge length of  $e$  if necessary;
  Add  $n_s$  subdivision nodes for  $e$ ;
  Add the new edges to  $T$ ;
  for each node  $w$  of  $T$  do
     $\Delta w = 0$ ;
  for  $i \in \{1, 2, \dots, nIters\}$  do
    Compute the collision circle radius;
    for each edge  $e'$  of  $T$  do
      Let  $u', v' = e'$ ;
       $\Delta u', \Delta v' += f_E(e')$ ;
      for each node  $w$  of  $T$  do
         $\Delta w += F_R(w)$ ;
         $\Delta w += F_C(w)$ ;
         $\Delta w += F_G(w)$ ;
    for each node  $w$  of  $T$  do
       $X_w += \Delta w$ ;

```

172 3.2 DynaSafe

173 DynaSafe stands for Dynamic Safety, as the algorithm prioritizes safe moves and
 174 will not make a move if it introduces an edge crossing; see Algorithm 2. DynaSafe
 175 is also a force-directed algorithm, however, it differs from DynaCola as it draws

176 straight-line edges (rather than edges with bends). The algorithm utilizes the
 177 following forces and is implemented in d3.js [11]

- 178 – A force f_E on the edges to realize the desired edge length. The strength of
 179 the force is proportional to the difference between the edge distance in the
 180 layout and the desired edge length.
- 181 – A stress-minimizing force f_S on every pair of nodes not connected by an edge,
 182 used to improve global structure. The desired distance is the shortest-path
 183 distance between the pair, and the magnitude of the force is proportional to
 184 the difference between the realized and desired distance.
- 185 – A repulsive force f_R between nodes, which somewhat counter-intuitively,
 186 improves the compactness of the layout. The force has an elliptical contour
 187 since we are optimizing for labeled nodes, where the value of the x coordinate
 188 must be farther away from other labels than the value of the y coordinate.
- 189 – A gravitational force f_G that attracts all nodes to the center of mass. This
 190 force draws the nodes closer together and improves compactness.

191 DynaSafe prevents edge crossings from occurring at any time by updating the
 192 coordinates safely: if the proposed new coordinate of a node introduces crossings,
 193 we gradually reduce the magnitude of the movement until the crossing is avoided.
 194 To place the new node, we randomly sample a set of 100 nearby points to find
 195 a crossing-free position for its adjacent edge. If we cannot find a crossing free
 196 position using the sample points, we continuously reduce the edge length until
 197 we find a crossing free position. Once the node is added, an iteration of force-
 198 directed algorithm optimizes the layout (again without introducing crossings).

199 By the nature of force-directed algorithms, after one phase of force compu-
 200 tations each node has a proposed new position. Before moving any node to its
 201 proposed new position, we check that the move is “safe,” i.e., it does not intro-
 202 duce a crossing. If the movement of a node introduces any crossings, then the
 203 magnitude of the move is set to $p\%$ of the original movement. This is repeated
 204 (if needed) at most q times and if the crossing is still unavoidable, then the node
 205 is not moved in this phase. By default $p = 0.8$ and $q = 12$.

206 4 Experimental Evaluation

207 We evaluate DynaCola and DynaSafe, along with 4 earlier methods: DynNoSlice,
 208 DynaGraph, Dagre, and Radial. We use two evolving trees to visually compare
 209 the results, as well quantitatively evaluate the desired properties.

210 4.1 Datasets

211 We use two real-world datasets to extract evolving trees for our experiments.

212 **The Tree of Life:** captures the evolutionary progression of life on Earth [35].
 213 The underlying data is a tree structure with a natural time component. As a new
 214 species evolves, a new node in the tree is added. The edges give the parent-child

Algorithm 2: DynaSafe(Current tree layout Γ_T , new edge e , $nIters$)**Result:** A crossing-free layout, maintaining the desired properties

```

Let  $u, v = e$ ;
Let  $u \in T$ ;
Find a crossing free position for  $v$  by random sampling;
Reduce the edge length of  $e$  if necessary;
Add the new edge to  $T$ ;
for each node  $w$  of  $T$  do
     $\Delta w = 0$ ;
for  $i \in \{1, 2, \dots, nIters\}$  do
    for each edge  $e'$  of  $T$  do
        Let  $u', v' = e'$ ;
         $\Delta u', \Delta v' += f_E(e')$ ;
    for each node  $w$  of  $T$  do
         $\Delta w += F_R(w)$ ;
         $\Delta w += F_G(w)$ ;
    for each node pair  $w, x$  do
         $\Delta w, \Delta x += F_S(w, x)$ ;
for each node  $w$  of  $T$  do
    Reduce  $\Delta w$  by  $p\%$  for at most  $q$  times;
    Update  $X_w$  by  $\Delta w$  if no crossing occurs;

```

215 relation of the nodes, where the parent is the original species, and the child is
 216 the new species. We use a subset of this graph with 500 nodes. The maximum
 217 node degree of this tree is 5, and the radius is 24.

218 **The Mathematics Genealogy:** shows advisor-advisee relationships in the
 219 world of mathematics, stretching back to the middle ages [34]. The dataset in-
 220 cludes the thesis titles, students, advisors, dates, and number of descendants. The
 221 total number of nodes is around 260,000 and is continuously updated. While this
 222 data is not quite a tree (or even connected, or planar), we extract a subset to
 223 create a tree with 500 nodes. The maximum node degree of this tree is 5 and
 224 the radius is 14.

225 4.2 Evaluation Metrics

226 We use standard metrics for each of our desired properties: desired edge length
 227 preservation, compactness, and stability. Additionally, we compute the stress of
 228 the drawing and the number of crossings. This gives a total of five quantitative
 229 measures. For each of these measures we define a loss function as follows:

230 **Desired Edge Length (DEL):** To measure how close the realized edge lengths
 231 are to the desired edge lengths, we find the mean squared error between these
 232 two values. Given the desired edge lengths $\{l_{ij} : (i, j) \in E\}$, and coordinates of
 233 the nodes X in the computed layout, we evaluate with the following formula:

$$\text{Desired edge length loss} = \sqrt{\frac{1}{|E|} \sum_{(i,j) \in E} \left(\frac{\|X_i - X_j\| - l_{ij}}{l_{ij}} \right)^2} \quad (1)$$

234 This measures the root mean square of the relative error as in [1], producing
 235 a non-negative number, with 0 corresponding to a perfect realization.

236 **Compactness:** To measure the compactness of each layout, we use the ratio
 237 between the drawing area and the sum of the areas for all labels [8]. We assume
 238 that a label is at most 16 characters, as we abbreviate longer labels. The sum
 239 of the areas for all labels gives the minimum possible area needed to draw all
 240 labels without overlaps (ignoring any space needed for edges). The area of the
 241 actual drawing is given by the smallest bounding rectangle, once the drawing has
 242 been scaled up until there are no overlapping labels. Once we have this scaled
 243 drawing, we find the positions of the nodes with the largest and smallest x and
 244 y values ($X_{max,0}$, $X_{min,0}$, $X_{max,1}$ and $X_{min,1}$). Using these values we calculate
 245 the area of the bounding rectangle.

$$\text{Compactness loss} = \frac{(X_{max,0} - X_{min,0})(X_{max,1} - X_{min,1})}{\sum_{v \in V} \text{label_area}(v)} \quad (2)$$

246 This formula produces a non-negative number; the ideal value for this mea-
 247 sure is 0 and corresponds to a perfect space utilization.

248 **Stability:** To measure stability, we consider how much each of the nodes moved
 249 after adding a new node. We then sum the movements of all nodes over all
 250 time steps. Since different algorithms use different amounts of drawing areas, we
 251 divide the value by the drawing area to normalize the results. This measure is
 252 similar to that used in DynNoSlice [43], but since DynNoSlice does not use time
 253 slices, it is closer to the measure found in [12]:

$$\text{Stability loss} = \frac{\sum_{v \in V} \sum_{t=1}^{T-1} \|X_v(t+1) - X_v(t)\|}{(X_{max,0} - X_{min,0})(X_{max,1} - X_{min,1})} \quad (3)$$

254 Here, T is the maximum time (500 in our two datasets). This formula pro-
 255 duces a non-negative number; the ideal value is 0 and corresponds to a perfectly
 256 stable layout (no movement of any already placed nodes).

257 **Stress:** This measure evaluates the global quality of the layout, looking at
 258 the differences between the realized distance between any pair of nodes and the
 259 actual distance between them. This measure is used in a variety of graph drawing
 260 algorithms [12, 25, 43]:

$$\text{Stress loss} = \frac{\left(\sum_{i \neq j} (D_{i,j} - \|X_i - X_j\|)^2 \right)^{1/2}}{\sum_{i \neq j} \|X_i - X_j\|} \quad (4)$$

261 This formula produces a non-negative number; the ideal value is 0 and corre-
 262 sponds to a perfect embedding (that captures all graph distances by the realized
 263 Euclidean distances).

264 **Edge Crossings:** Finally, we measure the number of edge crossings in each of
 265 the outputs. Note that our algorithms DynaSafe and DynaCola enforce “no edge
 266 crossings” as a hard constraint. However, DynNoSlice and Dynagraph do not
 267 have such a constraint and so can and indeed do, introduce crossings. Therefore
 268 we include the number of edge crossings for a complete comparison.

269 4.3 Experimental Setup

270 We compare these algorithms to four previous algorithms: DynNoSlice, Dyna-
 271 Graph, Dagre, and Radial. We note that while Dagre and Radial are not specif-
 272 ically designed for dynamic graphs, they can be modified very naturally for this
 273 purpose. We can simply pass in the trees at every timestep and redraw.

274 We consider the simplest case for the desired edge length by using a uniform
 275 length of 100 for all edges. This is a necessary parameter for our algorithms
 276 DynaCola and DynaSafe, but only needed in the other four algorithms in order
 277 to compute the desired edge length measure. To be able to compare our methods
 278 to the other four (that do not take desired edge length into account), we set the
 279 desired edge length equal to the average edge obtained in the layout. We then
 280 normalize these values for a fair comparison.

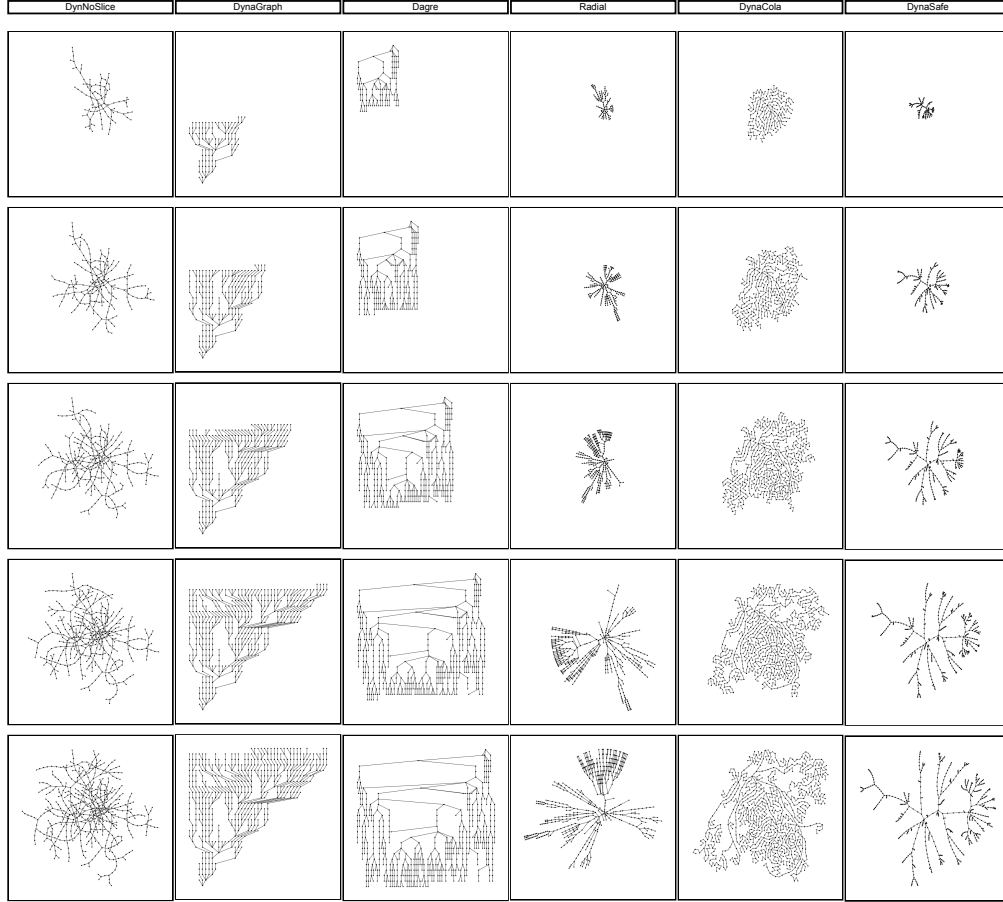
281 The performance of DynNoSlice depends heavily on two parameters, τ and
 282 δ that must be tuned. With the help of the authors, we found $\tau = 16$ and $\delta = 4$
 283 worked well for our 500-node trees. The performance of DynaCola depends on
 284 the number of subdivision nodes n_s ; we use $n_s = 1$ for the experiments. For
 285 the radial layout algorithm, we have used the default settings in the yFiles [48]
 286 implementation. The other algorithms are also used with their default settings.

287 4.4 Results

288 Both the visual and quantitative results indicate that the two new methods
 289 perform well overall; see Fig. 4.

291 **Desired Edge Lengths:** The quantitative results are shown in Table 5 and
 292 indicate that DynaCola does best. DynNoSlice is second best and DynaSafe is
 293 third. Dynagraph has the worst performance – not surprising given that it is a
 294 hierarchical layout, which is forced to use some very long edges near the root.

296 **Compactness:** The quantitative results are shown in Table 6 and indicate that
 297 DynNoSlice outperforms the rest of the algorithms. DynaCola is second best and
 298 DynaSafe is third. Here, Dagre has the worst performance.



290

Fig. 4. The layouts of the six methods for the tree of life dataset.

Nodes	DynNoSlice	Dynagraph	Dagre	Radial	DynaCola	DynaSafe
100 MG	85.60	192.07	219.20	153.53	124.90	147.80
200 MG	87.94	196.29	224.54	162.80	130.87	153.31
300 MG	95.24	201.41	225.86	169.34	137.00	159.87
400 MG	98.89	206.48	227.74	175.07	145.47	169.68
500 MG	106.82	208.39	236.94	192.53	149.43	174.93
100 TOL	96.46	196.79	223.43	179.05	147.82	160.58
200 TOL	100.24	214.29	231.38	183.06	154.10	169.38
300 TOL	110.56	216.16	239.98	190.82	157.19	170.04
400 TOL	119.98	233.85	255.76	203.92	167.52	194.28
500 TOL	126.85	235.72	272.82	214.09	173.94	196.03

299

Fig. 6. Compactness of math genealogy tree (MG) and the tree of life (TOL).

Nodes	DynNoSlice	Dynagraph	Dagre	Radial	DynaCola	DynaSafe
100 MG	0.37691	1.95933	0.682568	0.653853	0.282521	0.589865
200 MG	0.36552	1.95179	0.679827	0.640628	0.270322	0.575430
300 MG	0.35007	1.94213	0.666440	0.63058	0.253877	0.564747
400 MG	0.34402	1.93822	0.646479	0.619203	0.243184	0.553141
500 MG	0.33377	1.91979	0.639766	0.592694	0.237139	0.548756
100 TOL	0.21675	1.28710	0.448483	0.448205	0.158071	0.411747
200 TOL	0.21972	1.37271	0.494261	0.460161	0.166190	0.443935
300 TOL	0.23986	1.40404	0.510473	0.481034	0.176748	0.453332
400 TOL	0.25597	1.45660	0.553543	0.510352	0.183856	0.470334
500 TOL	0.26652	1.52650	0.581648	0.530249	0.189373	0.485759

295 **Fig. 5.** Desired edge lengths of math genealogy tree (MG) and tree of life (TOL).

300 **Stability:** The quantitative results are shown in Table 7 and indicate that Dyna-
301 Cola does best. DynaGraph is second, and DynaSafe is third. The radial layout
302 performs worse in this metric because it rotates the subtrees as more edges are
303 added.

Nodes	DynNoSlice	Dynagraph	Dagre	Radial	DynaCola	DynaSafe
100 MG	0.001584	0.001393	0.0016502	0.001998	0.001348	0.001459
200 MG	0.000752	0.000447	0.0012497	0.001839	0.000264	0.000410
300 MG	0.000577	0.000227	0.0010083	0.001450	0.000225	0.000295
400 MG	0.000249	0.000190	0.0009504	0.001203	0.000164	0.000216
500 MG	0.000037	0.000014	0.0007591	0.001047	0.000011	0.000019
100 TOL	0.000437	0.000139	0.001241	0.003609	0.000105	0.000163
200 TOL	0.000323	0.000125	0.001196	0.003408	0.000097	0.000146
300 TOL	0.000263	0.000099	0.001163	0.003174	0.000072	0.000106
400 TOL	0.000235	0.000073	0.001136	0.002490	0.000064	0.000101
500 TOL	0.000199	0.000071	0.000834	0.001941	0.000052	0.000101

304 **Fig. 7.** Stability of math genealogy tree (MG) and tree of life (TOL).

305 **Stress:** The quantitative results are shown in Table 8. In general, DynaSafe
306 does much better on this measure than any of the rest of them. DynNoSlice is
307 second and DynaCola third. DynaGraph and Dagre perform the worst in this
308 metric due to the limitations inherent in the hierarchical layout. Note that the
309 stress is normalized, so the numbers are comparable.

Nodes	DynNoSlice	Dynagraph	Dagre	Radial	DynaCola	DynaSafe
100 MG	113.10	150.59	230.75	125.37	76.51	49.44
200 MG	161.45	186.17	250.98	172.05	151.74	68.42
300 MG	179.48	264.90	286.77	205.98	184.47	94.05
400 MG	186.68	292.66	286.83	262.09	227.98	107.56
500 MG	249.61	393.11	396.72	314.93	291.39	109.00
100 TOL	136.52	210.06	263.98	192.64	128.45	59.77
200 TOL	165.75	262.24	325.65	243.59	201.76	65.10
300 TOL	181.62	305.15	369.57	287.93	220.63	81.81
400 TOL	254.20	328.59	398.11	317.28	306.89	93.99
500 TOL	285.19	400.81	461.43	374.02	351.23	119.77

Fig. 8. Stress scores of math genealogy tree (MG) and tree of life (TOL).

Edge Crossings: The quantitative results are shown in Table 9. There are four winners here – the four algorithms that prevent any edge crossings: DynaCola, DynaSafe, Radial, and Dagre. DynNoSlice and Dynagraph do introduce some crossings.

Nodes	DynNoSlice	Dynagraph	Dagre	Radial	DynaCola	DynaSafe
100 MG	43	8	0	0	0	0
200 MG	82	11	0	0	0	0
300 MG	168	11	0	0	0	0
400 MG	217	13	0	0	0	0
500 MG	277	13	0	0	0	0
100 TOL	21	0	0	0	0	0
200 TOL	67	0	0	0	0	0
300 TOL	106	0	0	0	0	0
400 TOL	176	0	0	0	0	0
500 TOL	231	0	0	0	0	0

Fig. 9. Edge crossings of math genealogy tree (MG) and tree of life (TOL).

Running time: The Radial layout has the lowest running time, taking 34.93 seconds and 28.03 seconds, respectively, to draw the 500-node math genealogy tree and tree of life. On the other end, DynNoSlice is the slowest algorithm, taking more than 6 hours to draw the 500-node trees. Our two new methods are not as fast as the Radial algorithm and not as slow as DynNoSlice, taking about 5 minutes on the 500-node trees; see Fig. 10.

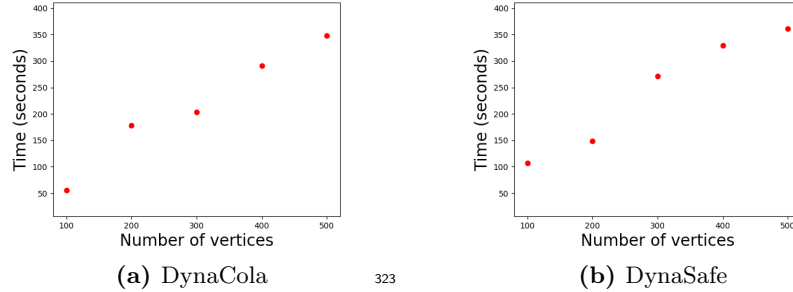


Fig. 10. Running time w.r.t. number of nodes in the math genealogy tree.

5 Discussion, Limitations, Conclusions and Future Work

While there are many algorithms and tools for drawing static trees, there are only few that can handle dynamic trees well. Among those, there are even fewer that take edge labels into account while also preventing edge crossings. With this in mind, we described two methods for visualizing evolving trees. Fully functional prototypes and videos showing them in action are available online: <https://rynggray.github.io/dynamic-trees/index.html>. Source code and all experimental data can be found on github: https://github.com/abureyanahmed/evolving_tree.

The two new methods, DynaCola and DynaSafe perform well overall. However, we only used two 500-node trees in the evaluation. We used 5 metrics to quantitatively evaluate the new methods against 4 earlier methods, but it is likely that other earlier methods can be adapted to this setting and might perform better than the ones we used. Further, we did not evaluate how the algorithms behave as the size of the trees grows. Our algorithms have not been optimized for speed and will likely struggle with larger instances.

The two new methods are designed for evolving trees, where growth is the only type of change. A natural question is whether these algorithms can be generalized to the more difficult problem of online dynamic tree visualization.

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References

1. Ahmed, R., Luca, F.D., Devkota, S., Kobourov, S., Li, M.: Graph drawing via gradient descent, $(gd)^2$. In: 28th International Symposium on Graph Drawing and Network Visualization (GD). pp. 3–17. Springer (2020)

- 352 2. Archambault, D., Purchase, H., Pinaud, B.: Animation, small multiples, and the
353 effect of mental map preservation in dynamic graphs. *IEEE Transactions on Visu-*
354 *alization and Computer Graphics* **17**(4), 539–552 (2010)
- 355 3. Bach, B., Henry-Riche, N., Dwyer, T., Madhyastha, T., Fekete, J.D., Grabowski,
356 T.: Small multiples: Piling time to explore temporal patterns in dynamic networks.
357 *Computer Graphics Forum* **34**(3), 31–40 (2015)
- 358 4. Bach, B., Pietriga, E., Fekete, J.D.: Graphdiaries: Animated transitions and tem-
359 poral navigation for dynamic networks. *IEEE Transactions on Visualization and*
360 *Computer Graphics* **20**(5), 740–754 (2013)
- 361 5. Bachmaier, C.: A radial adaptation of the sugiyama framework for visualizing hier-
362 archical information. *IEEE Transactions on Visualization and Computer Graphics*
363 **13**(3), 583–594 (2007)
- 364 6. Balcan, D., Gonçalves, B., Hu, H., Ramasco, J.J., Colizza, V., Vespignani, A.:
365 Modeling the spatial spread of infectious diseases: The global epidemic and mobility
366 computational model. *Journal of Computational Science* **1**(3), 132–145 (2010)
- 367 7. Barnes, J., Hut, P.: A hierarchical $O(n \log n)$ force-calculation algorithm. *Nature*
368 **324**(6096), 446–449 (1986)
- 369 8. Barth, L., Kobourov, S.G., Pupyrev, S.: Experimental comparison of semantic word
370 clouds. In: *Experimental Algorithms*. pp. 247–258. Springer (2014)
- 371 9. Beck, F., Burch, M., Diehl, S., Weiskopf, D.: The state of the art in visualizing
372 dynamic graphs. In: *16th Eurographics Conference on Visualization, (EuroVis)*.
373 *Eurographics Association* (2014)
- 374 10. Beck, F., Burch, M., Diehl, S., Weiskopf, D.: A taxonomy and survey of dynamic
375 graph visualization. In: *Computer Graphics Forum*. vol. 36, pp. 133–159. Wiley
376 *Online Library* (2017)
- 377 11. Bostock, M., Ogievetsky, V., Heer, J.: D³ data-driven documents. *IEEE Transac-*
378 *tions on Visualization and Computer Graphics* **17**(12), 2301–2309 (2011)
- 379 12. Brandes, U., Mader, M.: A quantitative comparison of stress-minimization ap-
380 proaches for offline dynamic graph drawing. In: *19th International Symposium on*
381 *Graph Drawing (GD)*. pp. 99–110. Springer (2011)
- 382 13. Burch, M., Müller, C., Reina, G., Schmauder, H., Greis, M., Weiskopf, D.: Visu-
383 alizing dynamic call graphs. In: *Vision, Modeling, and Visualization (VMV)*. pp.
384 207–214 (2012)
- 385 14. Cohen, R.F., Di Battista, G., Tamassia, R., Tollis, I.G.: Dynamic graph drawings:
386 Trees, series-parallel digraphs, and planar st-digraphs. *SIAM Journal on Comput-*
387 *ing* **24**(5), 970–1001 (1995)
- 388 15. Cohen, R.F., Di Battista, G., Tamassia, R., Tollis, I.G., Bertolazzi, P.: A framework
389 for dynamic graph drawing. In: *Proceedings of the eighth annual symposium on*
390 *Computational geometry*. pp. 261–270 (1992)
- 391 16. Crnovrsanin, T., Chu, J., Ma, K.L.: An incremental layout method for visualizing
392 online dynamic graphs. In: *23rd International Symposium on Graph Drawing (GD)*.
393 pp. 16–29. Springer (2015)
- 394 17. Diehl, S., Görg, C.: Graphs, they are changing. *10th International Symposium on*
395 *Graph Drawing (GD)* pp. 23–31 (2002)
- 396 18. Diehl, S., Görg, C., Kerren, A.: Foresighted graph layout. Technical Report, Uni-
397 versity of Saarland (2000)
- 398 19. Diehl, S., Görg, C., Kerren, A.: Preserving the mental map using foresighted layout.
399 *Proceedings of Joint Eurographics - IEEE TCVG Symposium on Visualization*
400 *(VisSym)* (2001)

- 401 20. Doğrusöz, U., Madden, B., Madden, P.: Circular layout in the graph layout toolkit.
402 In: 4th International Symposium on Graph Drawing (GD). pp. 92–100. Springer
403 (1996)
- 404 21. Erten, C., Harding, P.J., Kobourov, S.G., Wampler, K., Yee, G.: Graphael: Graph
405 animations with evolving layouts. In: 11th International Symposium on Graph
406 Drawing (GD). pp. 98–110. Springer (2003)
- 407 22. Erten, C., Kobourov, S.G., Le, V., Navabi, A.: Simultaneous graph drawing: Layout
408 algorithms and visualization schemes. In: 11th International Symposium on Graph
409 Drawing (GD). pp. 437–449. Springer (2003)
- 410 23. Forrester, D., Kobourov, S.G., Navabi, A., Wampler, K., Yee, G.V.: graphael: A
411 system for generalized force-directed layouts. In: 12th International Symposium on
412 Graph Drawing (GD). pp. 454–464. Springer (2004)
- 413 24. Frishman, Y., Tal, A.: Online dynamic graph drawing. *IEEE Transactions on Vi-*
414 *ualization and Computer Graphics* **14**(4), 727–740 (2008)
- 415 25. Gansner, E.R., Koren, Y., North, S.: Graph drawing by stress majorization. In:
416 12th International Symposium on Graph Drawing (GD). pp. 239–250. Springer
417 (2004)
- 418 26. Gansner, E.R., Koutsofios, E., North, S.C., Vo, K.P.: A technique for drawing
419 directed graphs. *IEEE Transactions on Software Engineering* **19**(3), 214–230 (1993)
- 420 27. Gilbert, F., Simonetto, P., Zaidi, F., Jourdan, F., Bourqui, R.: Communities and
421 hierarchical structures in dynamic social networks: analysis and visualization. *Soc-*
422 *ial Network Analysis and Mining* **1**(2), 83–95 (2011)
- 423 28. Gorochowski, T.E., di Bernardo, M., Grierson, C.S.: Using aging to visually un-
424 cover evolutionary processes on networks. *IEEE Transactions on Visualization and*
425 *Computer Graphics* **18**(8), 1343–1352 (2011)
- 426 29. Hachul, S., Jünger, M.: Drawing large graphs with a potential-field-based multilevel
427 algorithm. In: 12th International Symposium on Graph Drawing (GD). pp. 285–
428 295. Springer (2004)
- 429 30. Hu, Y., Koren, Y.: Extending the spring-electrical model to overcome warping
430 effects. In: 2009 IEEE Pacific Visualization Symposium. pp. 129–136. IEEE (2009)
- 431 31. Jünger, M., Mutzel, P.: 2-layer straightline crossing minimization: Performance of
432 exact and heuristic algorithms. In: *Graph Algorithms and Applications I*, pp. 3–27.
433 World Scientific (2002)
- 434 32. Kar, G., Madden, B., Gilbert, R.: Heuristic layout algorithms for network man-
435 agement presentation services. *IEEE Network* **2**(6), 29–36 (1988)
- 436 33. Kaufmann, M., Wiese, R.: Maintaining the mental map for circular drawings. In:
437 10th International Symposium on Graph Drawing (GD). pp. 12–22. Springer (2002)
- 438 34. Keller, M.T.: Math genealogy project, [https://genealogy.math.ndsu.nodak.](https://genealogy.math.ndsu.nodak.edu/)
439 [edu/](https://genealogy.math.ndsu.nodak.edu/)
- 440 35. Maddison, D., Schulz, K., Lenards, A., Maddison, W.: Tree of life web project,
441 <http://tolweb.org/tree/>
- 442 36. Misue, K., Eades, P., Lai, W., Sugiyama, K.: Layout adjustment and the mental
443 map. *Journal of Visual Languages & Computing* **6**(2), 183–210 (1995)
- 444 37. Moen, S.: Drawing dynamic trees. *IEEE Software* **7**(4), 21–28 (1990)
- 445 38. Nguyen, Q.H.: INKA: an ink-based model of graph visualization. *CoRR*
446 **abs/1801.07008** (2018)
- 447 39. North, S.C.: Incremental layout in dynadag. In: 3rd International Symposium on
448 Graph Drawing (GD). pp. 409–418. Springer (1995)
- 449 40. Pavlo, A., Homan, C., Schull, J.: A parent-centered radial layout algorithm for
450 interactive graph visualization and animation. arXiv preprint [cs/0606007](https://arxiv.org/abs/cs/0606007) (2006)

- 451 41. Purchase, H.: Which aesthetic has the greatest effect on human understanding?
452 In: 5th International Symposium on Graph Drawing (GD). pp. 248–261. Springer
453 (1997)
- 454 42. Simonetto, P., Archambault, D., Kobourov, S.: Event-based dynamic graph vi-
455 sualisation. *IEEE Transactions on Visualization and Computer Graphics* **26**(7),
456 2373–2386 (2018)
- 457 43. Simonetto, P., Archambault, D., Kobourov, S.G.: Drawing dynamic graphs without
458 timeslices. *CoRR* **abs/1709.00372** (2017)
- 459 44. Six, J.M., Tollis, I.G.: A framework for circular drawings of networks. In: 7th
460 International Symposium on Graph Drawing (GD). pp. 107–116. Springer (1999)
- 461 45. Skambath, M., Tantau, T.: Offline drawing of dynamic trees: Algorithmics and
462 document integration. *CoRR* **abs/1608.08385** (2016)
- 463 46. Sugiyama, K., Tagawa, S., Toda, M.: Methods for visual understanding of hierar-
464 chical system structures. *IEEE Transactions on Systems, Man, and Cybernetics*
465 **11**(2), 109–125 (1981)
- 466 47. Van Eck, N.J., Waltman, L.: Visualizing bibliometric networks. In: *Measuring*
467 *Scholarly Impact*, pp. 285–320. Springer (2014)
- 468 48. Wiese, R., Eiglsperger, M., Kaufmann, M.: yfiles visualization and automatic lay-
469 out of graphs. In: *Graph Drawing Software*, pp. 173–191. Springer (2004)
- 470 49. Workman, D., Bernard, M., Pothoven, S.: An incremental editor for dynamic hier-
471 archical drawing of trees. In: *International Conference on Computational Science*.
472 pp. 986–995. Springer (2004)

Appendix

Fig. ?? shows a small evolving tree with labels, as obtained by the six algorithms: DynNoSlice, DynaGraph, Dagre, Radial, DynaCola and DynaSafe. Fig. 11 shows the same layouts without the labels, allowing us to see the structure a bit better.

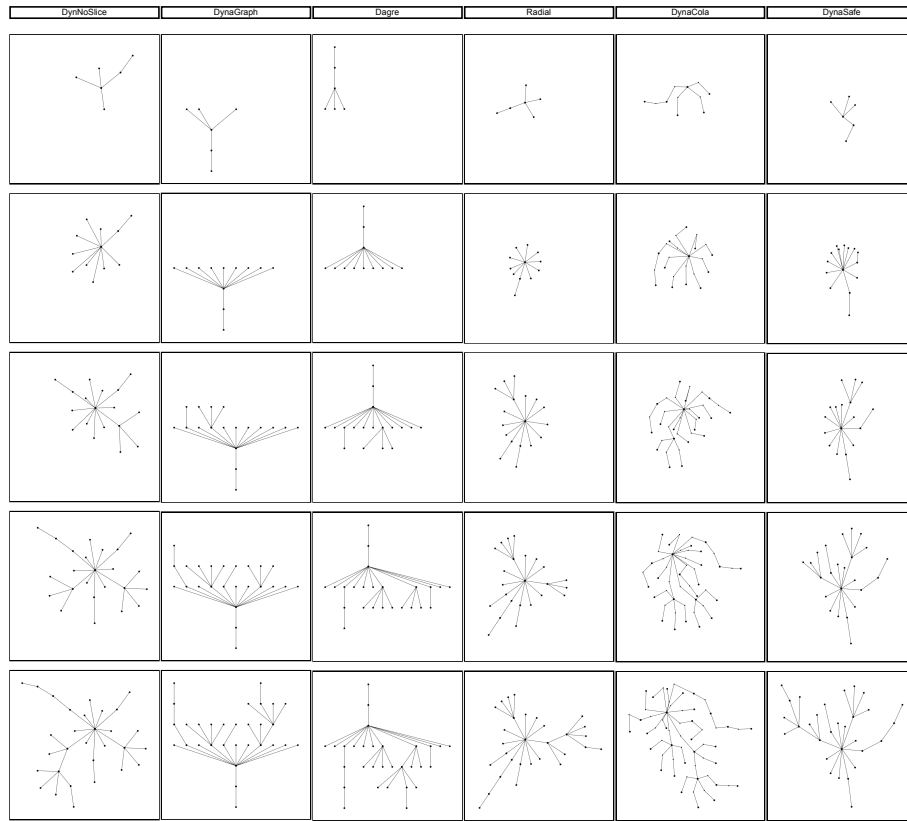
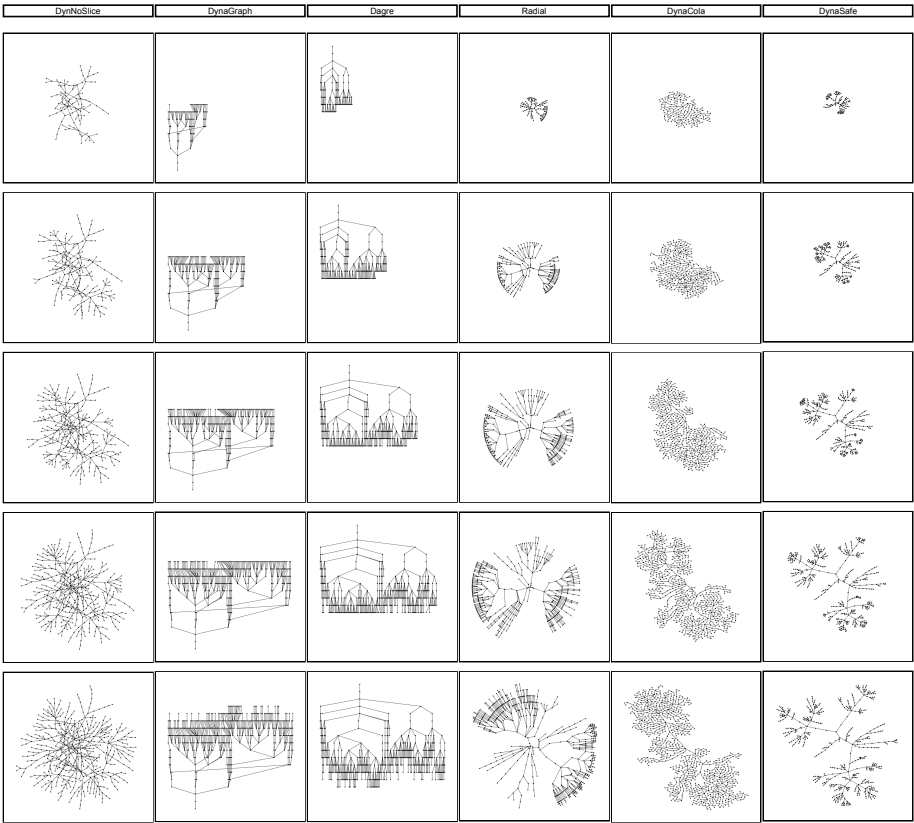


Fig. 11. The layouts of different algorithms of the same evolving trees at different steps.

The graph below shows a comparison of the different algorithms on the math genealogy graph. This shows the timesteps with 100,200,300,400, and 500 nodes.



479 **Fig. 12.** The layouts of different algorithms of the same evolving math genealogy
480 trees at different steps.