Monetary policy topics

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NBER Heterogeneous-Agent Macro Workshop, 2025

This session

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- **Now:** more advanced topics on monetary policy, using the toolkit!
  1. Cyclical income risk
  2. Maturity structure
  3. Nominal assets
  4. Investment

Cyclical income risk

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- Recall canonical model: household takes $n_{it} = N_t$ as given and solves

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$$c_{it} + a_{it} \leq (1 + r_t^p) a_{it-1} + (1 - \tau_t) \frac{W_t}{P_t} n_{it} e_{it}$$

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- This is restrictive: hours n_{it} don't move uniformly across people over the cycle!
- Relaxing this assumption will make a difference, but also uncover a big puzzle

Introducing unequal rationing

- Simple way to relax is to assume that labor is rationed as a function of e_{it} per

$$n_{it} = N_t \frac{\left(e_{it}\right)^{\zeta \log N_t}}{\mathbb{E}\left[e_i^{1+\zeta \log N_t}\right]} \equiv N_t \Gamma\left(e_{it}, N_t\right)$$

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$$\mathrm{sd}\left(\log y_{it}\right) = \left(1 + \zeta \log N_t\right) \mathrm{sd}\left(\log e_i\right)$$

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- $\zeta > 0$: procyclical inequality+income risk, $\zeta < 0$ countercyclical, $\zeta = 0$ acyclical
- Matters because: 1) current shocks redistribute between different MPCs, and 2) future shocks change perceived income risk

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  - $Y_t = \mathscr{C}_t\left(r_0^p, \{r_s\}, \{Z_s\}, \{Y_s\}\right) + G_{ss}$
  - $Z_t = \dfrac{Y_t - T_t}{\mu}$
  - $1 + r_0^p = (1 + r_{ss})\omega + \dfrac{1}{A_{ss}} \sum_{s=0}^{\infty} \left(\prod_{u=0}^{s} \dfrac{1}{1 + r_u} \right) \left(1 - \dfrac{1}{\mu}\right) (Y_t - T_t)$
  - $T_t = T_{ss} + \dfrac{r_t - r_{ss}}{1 + r_t} B_{ss}$

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  - $Y_t = \mathscr{C}_t\left(r_0^p, \{r_s\}, \{Z_s\}, \{Y_s\}\right) + G_{ss}$ $\longrightarrow$ Additional Jacobian for effect of countercyclical risk!

    Once we substitute everything, adds an additional term to IKC
  - $Z_t = \dfrac{Y_t - T_t}{\mu}$
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SSJ implementation

- Add a “hetinput” to the household block:

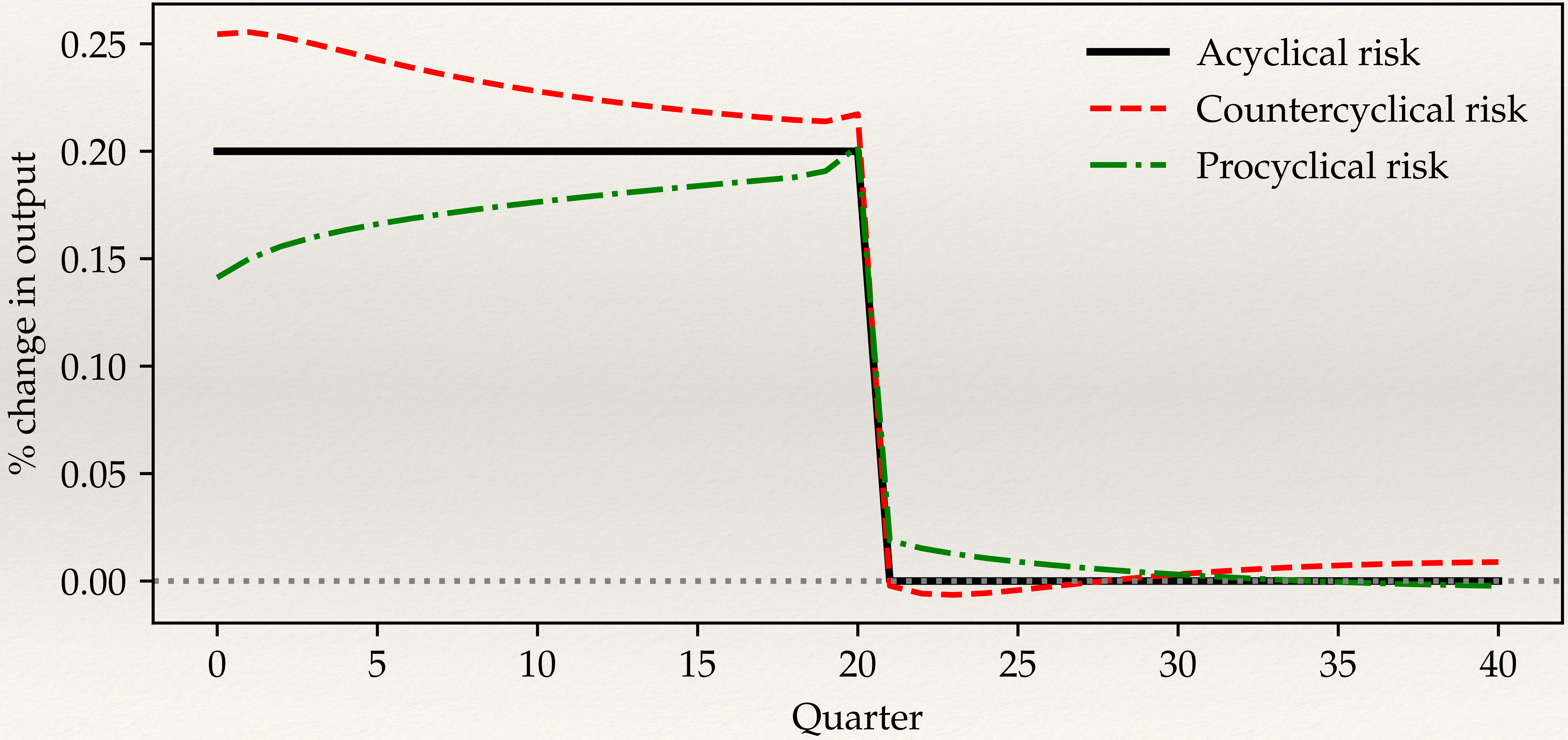
```
def income_cyclical(Z, N, e_grid, zeta, pi_pdf):
    # Auclert-Rognlie 2020 incidence function for labor income, with cyclicality parameter zeta
    # in default case with zeta = 0, this is just gamma / N = 1 and irrelevant
    gamma_N = e_grid ** (zeta * np.log(N)) / np.vdot(e_grid ** (1 + zeta * np.log(N)), pi_pdf)

    # net after-tax income
    y = Z * e_grid * gamma_N

    y = y.reshape(-1, 11)                        # reshape to beta*e grid
    y = y.ravel()                                 # flatten back
    return y

hh_cyclical = hh_raw.add_hetinputs([make_grids, income_cyclical])
```

Result: countercylical risk amplifies FG puzzle!



What is happening?

- Can show, in the "Zero-Liquidity" limit of the model where $A_{ss} \to 0$, that

$$c_t = \delta \cdot \mathbb{E}_t \left[c_{t+1} \right] - \frac{1}{\sigma} \cdot \text{const} \cdot \left(r_t^{ante} - \bar{r} \right)$$

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  - Empirically, eg, work by Guvenen&co and Storesletten&co
- Bilbiie's "catch-22": countercylical risk plausible at micro level, not macro!

Indirect ways to make income risk cyclical

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- Then income risk is procyclical! This is McKay, Nakamura, Steinsson

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- Asset pricing condition:

$$1 + r_t = \frac{1 + \delta q_{t+1}}{q_t}$$

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- Redefine $a_{it} = q_t \lambda_{it}$. Using asset pricing condition, we have

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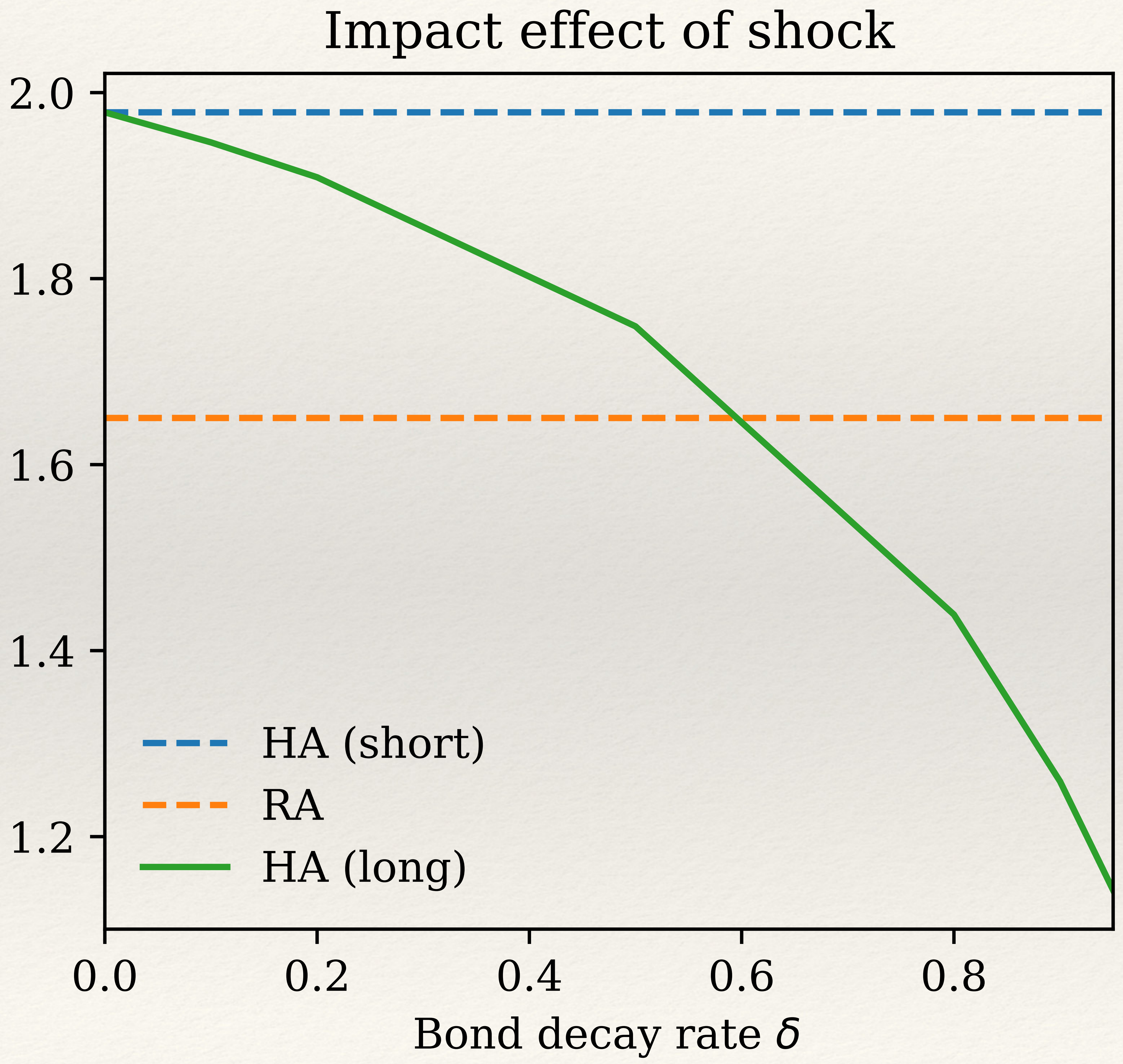
- As in stock market example, use standard model + valuation equation for r_0^p
- Lower ex-ante $r_t \rightarrow$ higher bond price $q_0 \rightarrow$ higher r_0^p (capital gain)

Output effect of a monetary policy shock



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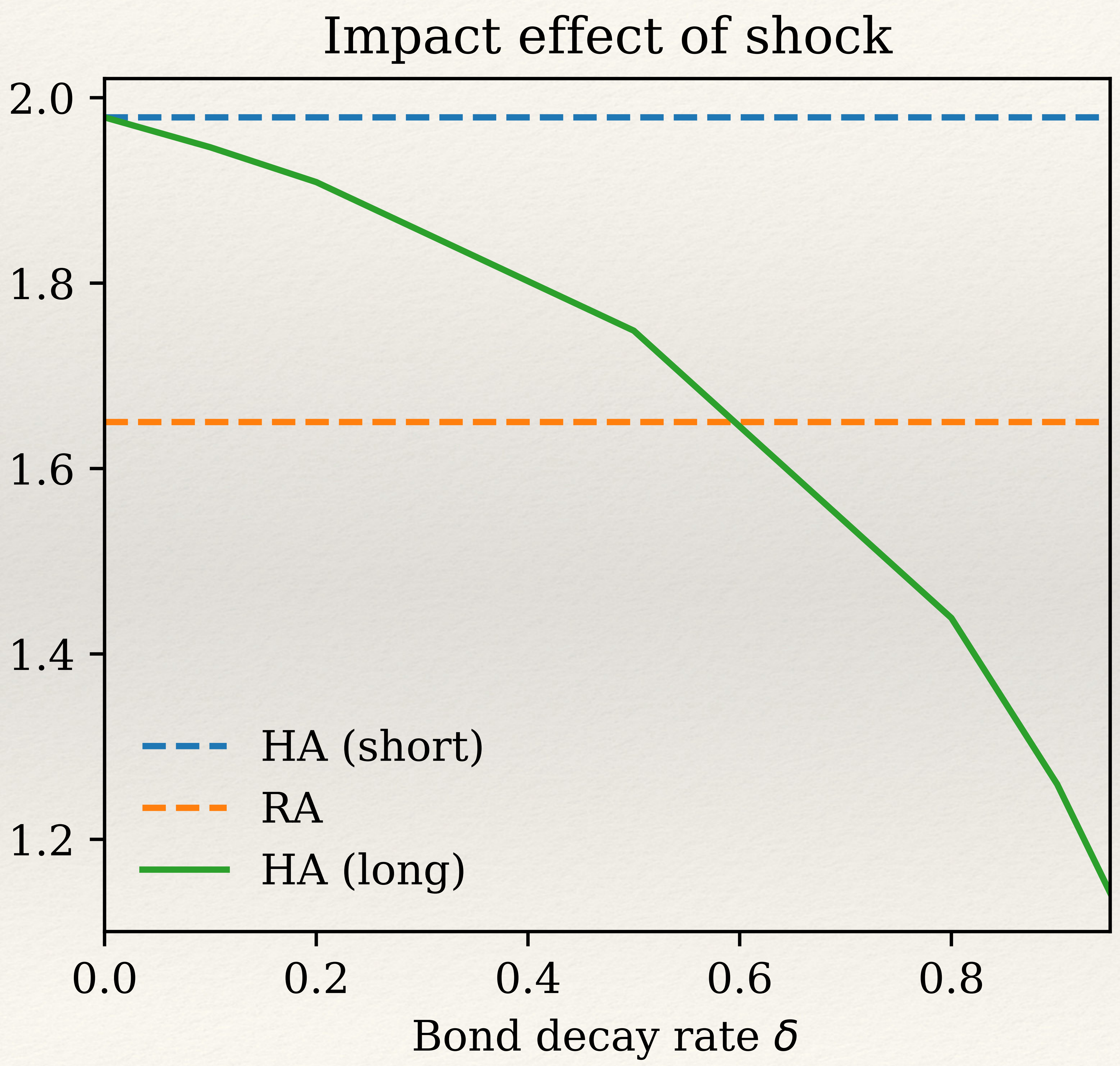

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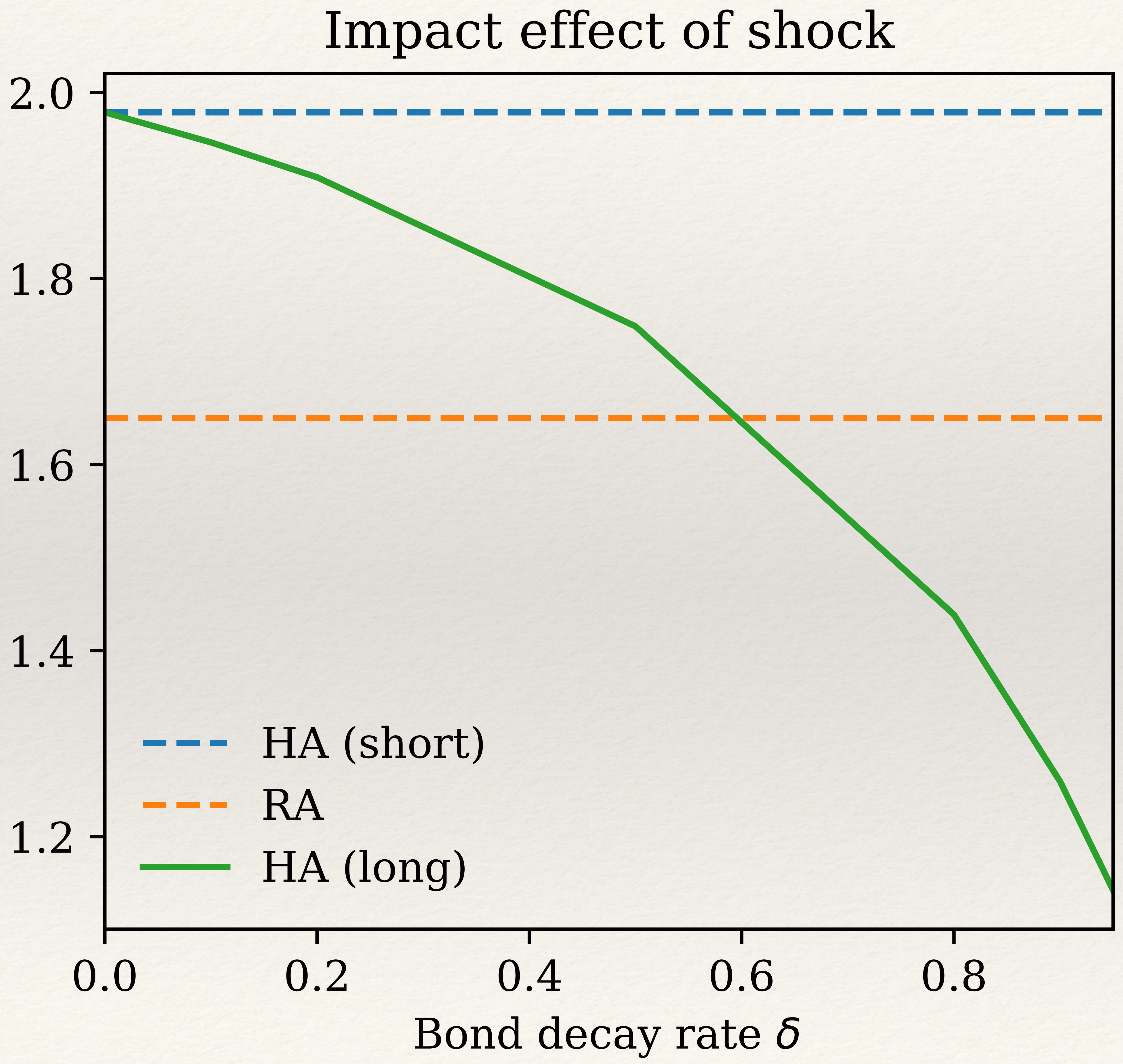

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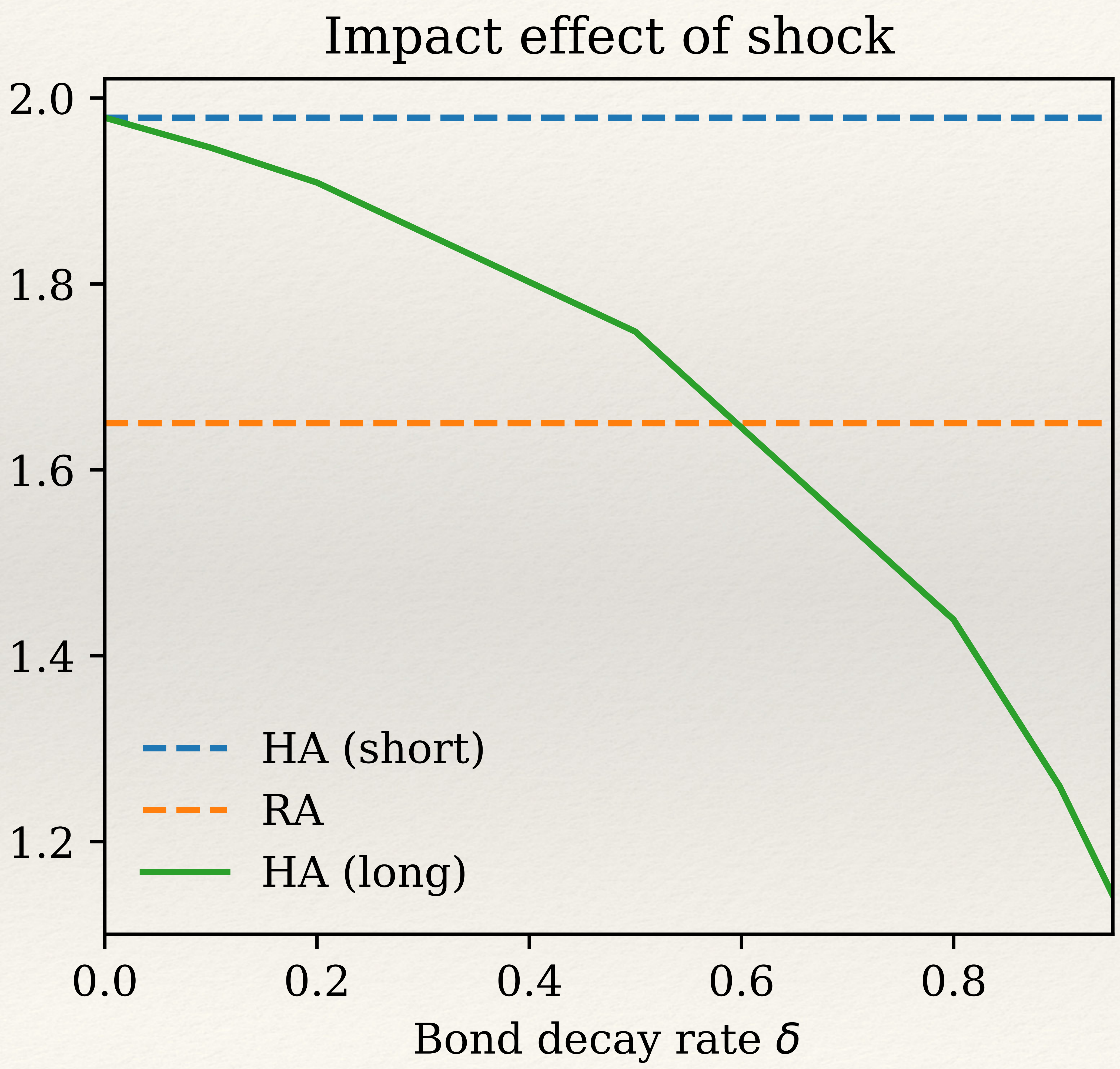

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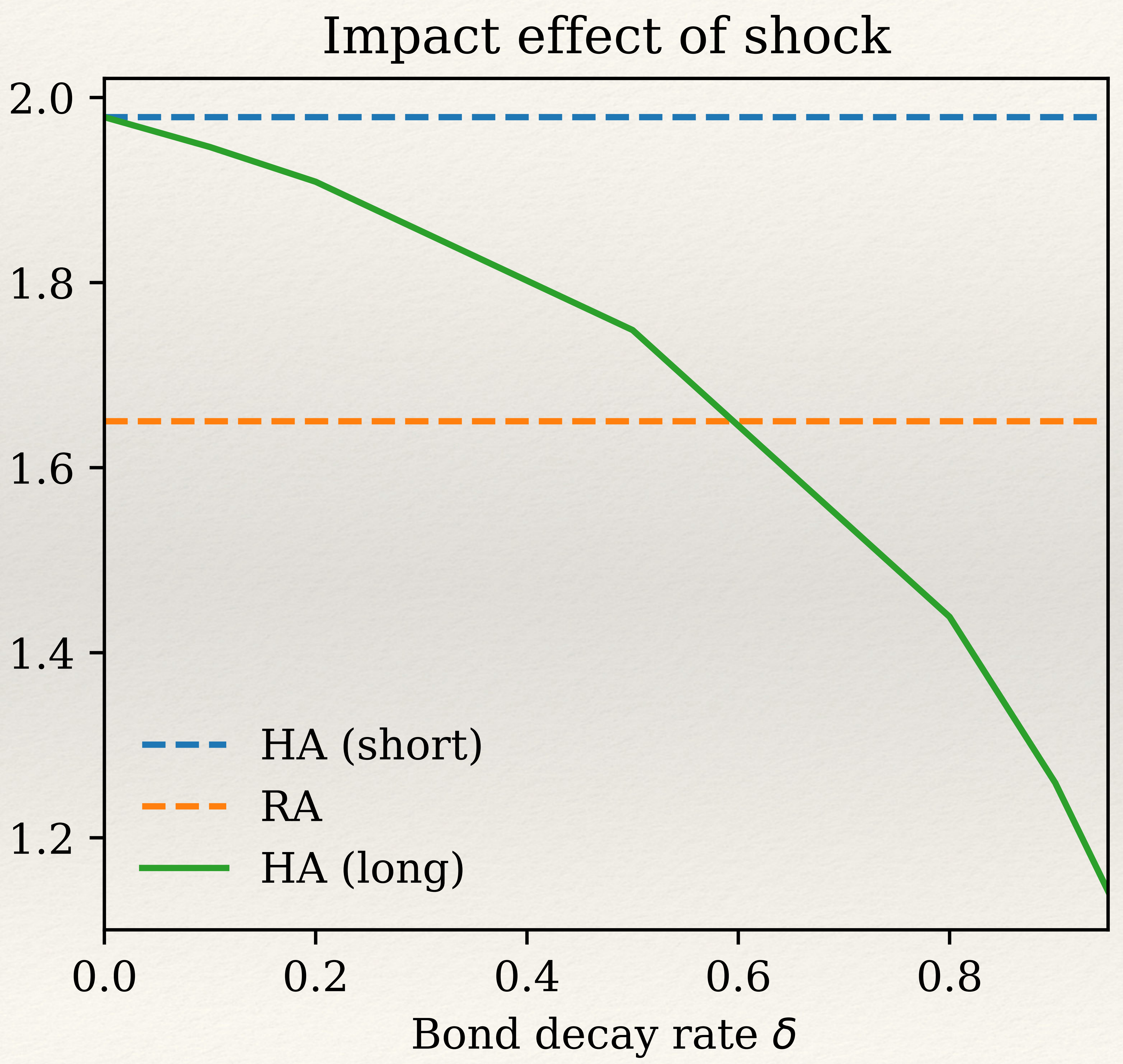

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  - true in data too! (Calza et al)

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- Fisher equation:

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Rewriting the problem

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- Even with r_t rule, inflation now matters for aggregate demand due to nominal revaluation!

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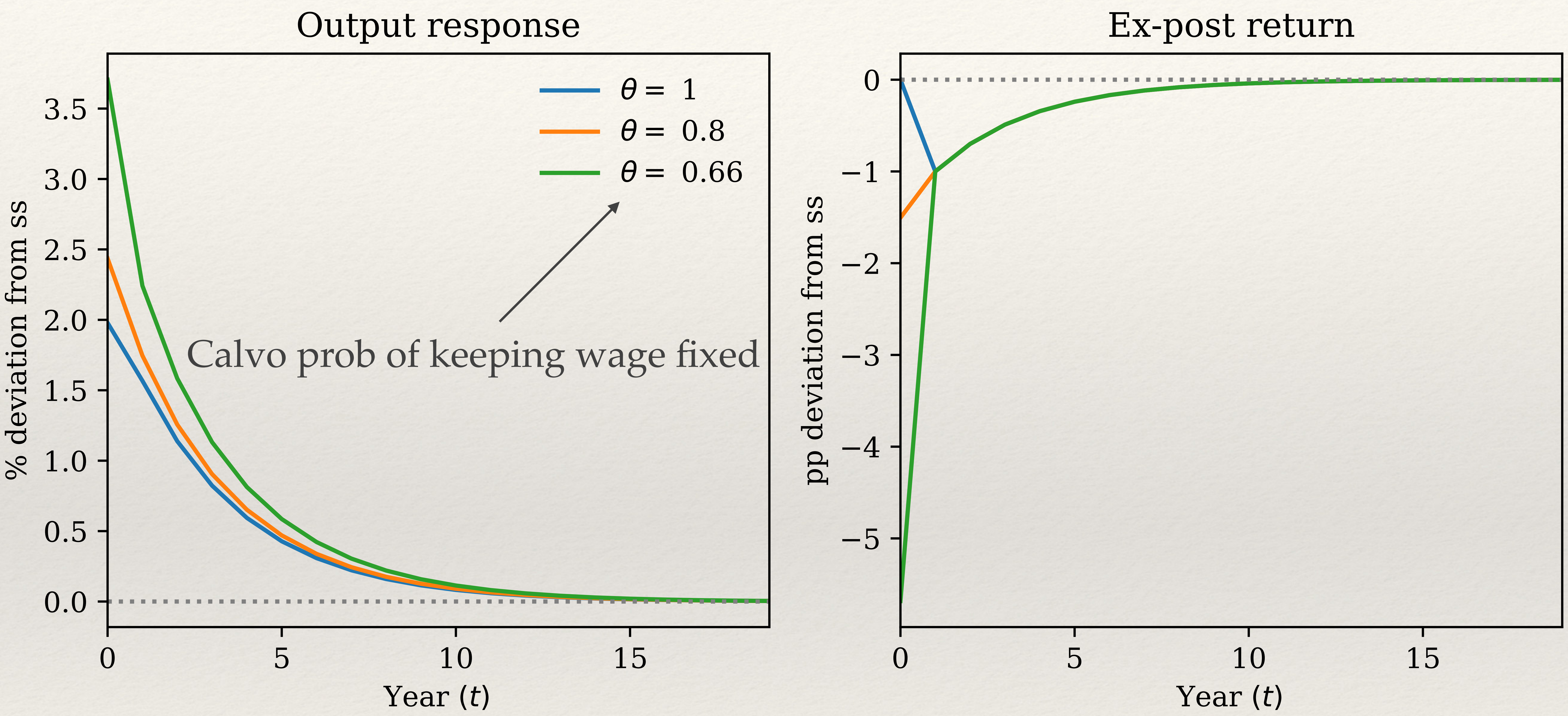

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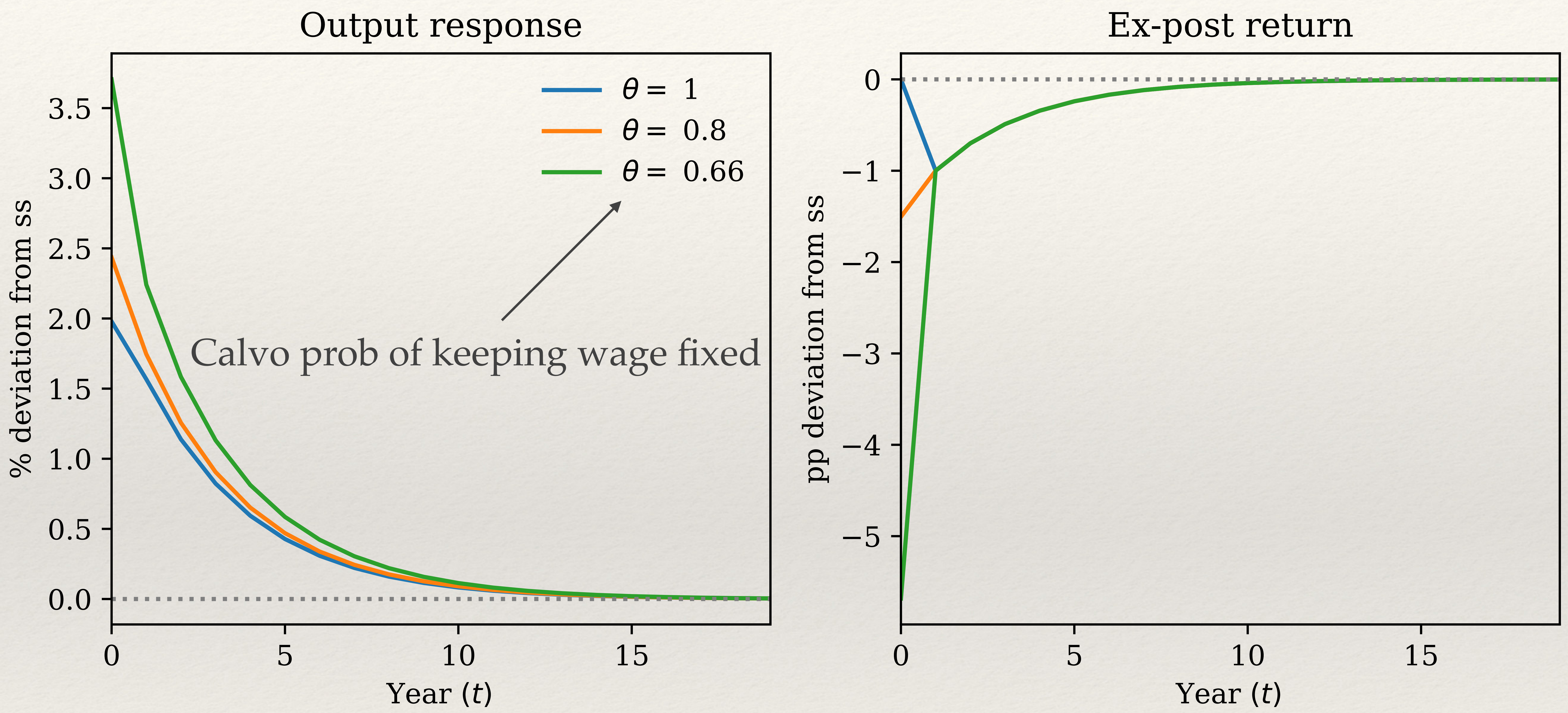

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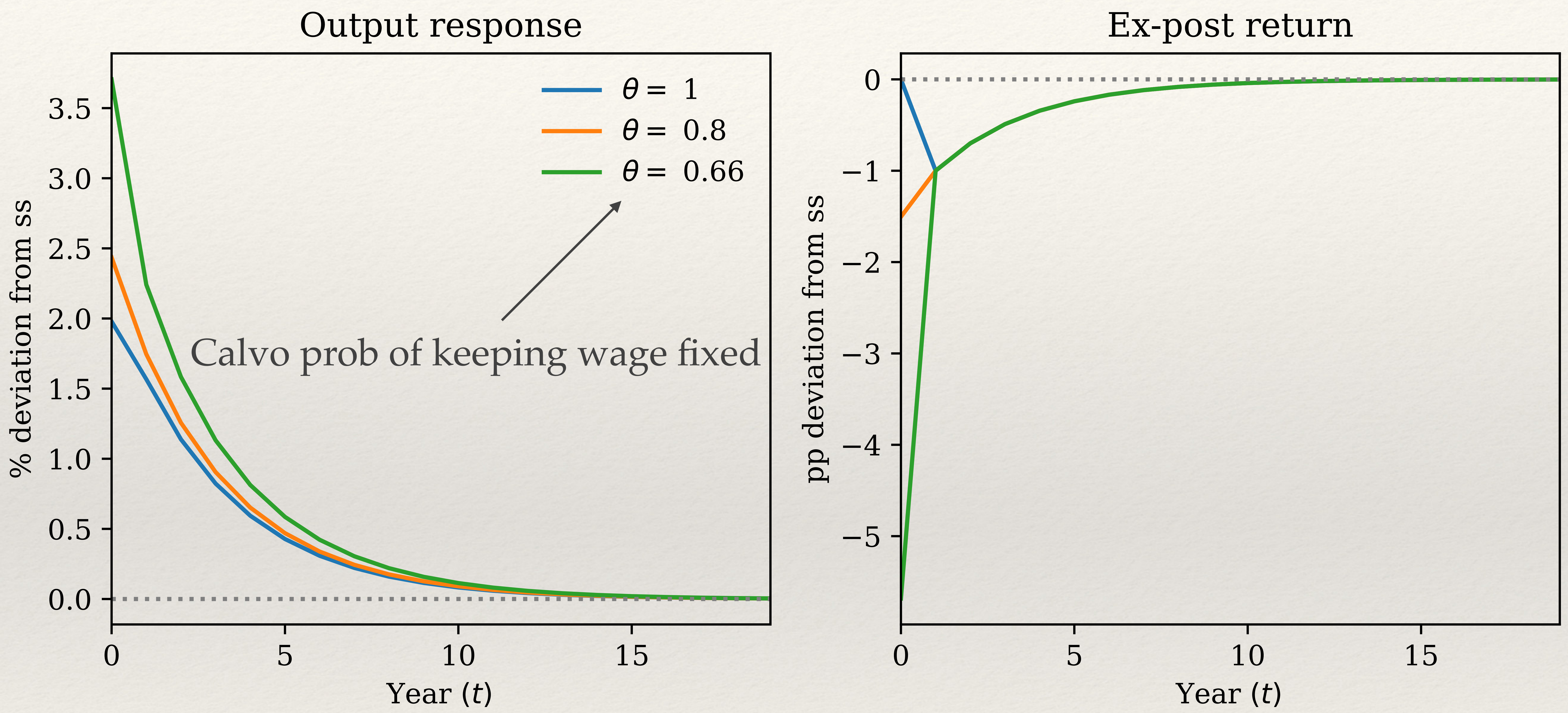

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- Those agents have higher MPCs, this boosts demand (effect bigger with steeper P.C.)

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$$C_t + I_t = Y_t = XK_{t-1}^{\alpha}N_t^{1-\alpha}$$

- Obvious: output is affected differently now since investment responds
- Not so obvious: does **consumption** respond differently?
- Not true in RANK, since there we have the Euler equation:

$$C_t^{-\sigma} = \beta(1 + r_t)C_{t+1}^{-\sigma}$$

  - Consumption C_t only a function of r_t, independent of I_t (or anything else)

Model setup

- Now final goods firm rents capital and labor, flexible prices

$$w_t = X(1-\alpha) K_{t-1}^{\alpha} N_t^{-\alpha} \qquad r_t^K = X\alpha K_{t-1}^{\alpha-1} N_t^{1-\alpha}$$

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- Capital firm owns K_t and and rents it out, invests s.t. quadratic costs, dividend

$$D_t = r_t^K K_{t-1} - I_t - \frac{\Psi}{2}\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right)^2 K_{t-1} \qquad I_t = K_t - (1-\delta)K_{t-1}$$

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Why do we need adjustment costs with sticky prices? Consider the effect of a dr_t shock without them. Then:

$$\frac{dK_t}{K} = -\frac{1}{1-\alpha}\frac{1}{r+\delta}dr_t \qquad \Rightarrow \qquad \frac{dI_0}{I} = -\frac{1}{1-\alpha}\frac{1}{r+\delta}\frac{1}{\delta}dr_0$$

With $\delta = 4\,\%$, $r = 1\,\%$, $\alpha = 0.3$, get semielasticity of investment of -715!!

Model setup continued

- Q theory equations:

$$\frac{I_t}{K_{t-1}} - \delta = \frac{1}{\Psi}\left(Q_t - 1\right)$$

$$p_t = Q_t K_t = \frac{p_{t+1} + D_{t+1}}{1 + r_t}$$

Model setup continued

- Q theory equations:

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- Assume mutual funds owns 100% shares
  - Now our connection to household block is $1 + r_0^p = \dfrac{p_0 + d_0}{p_{ss}}$

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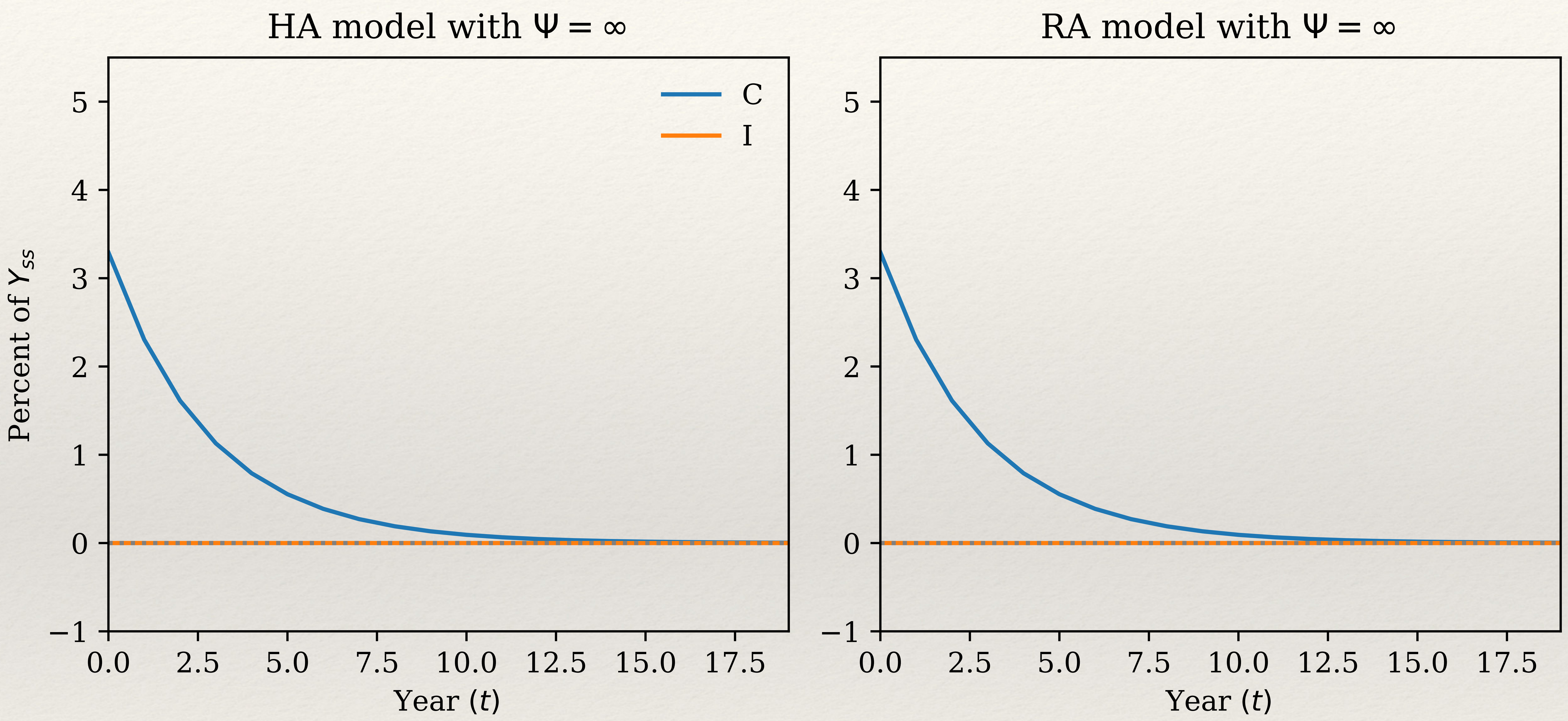
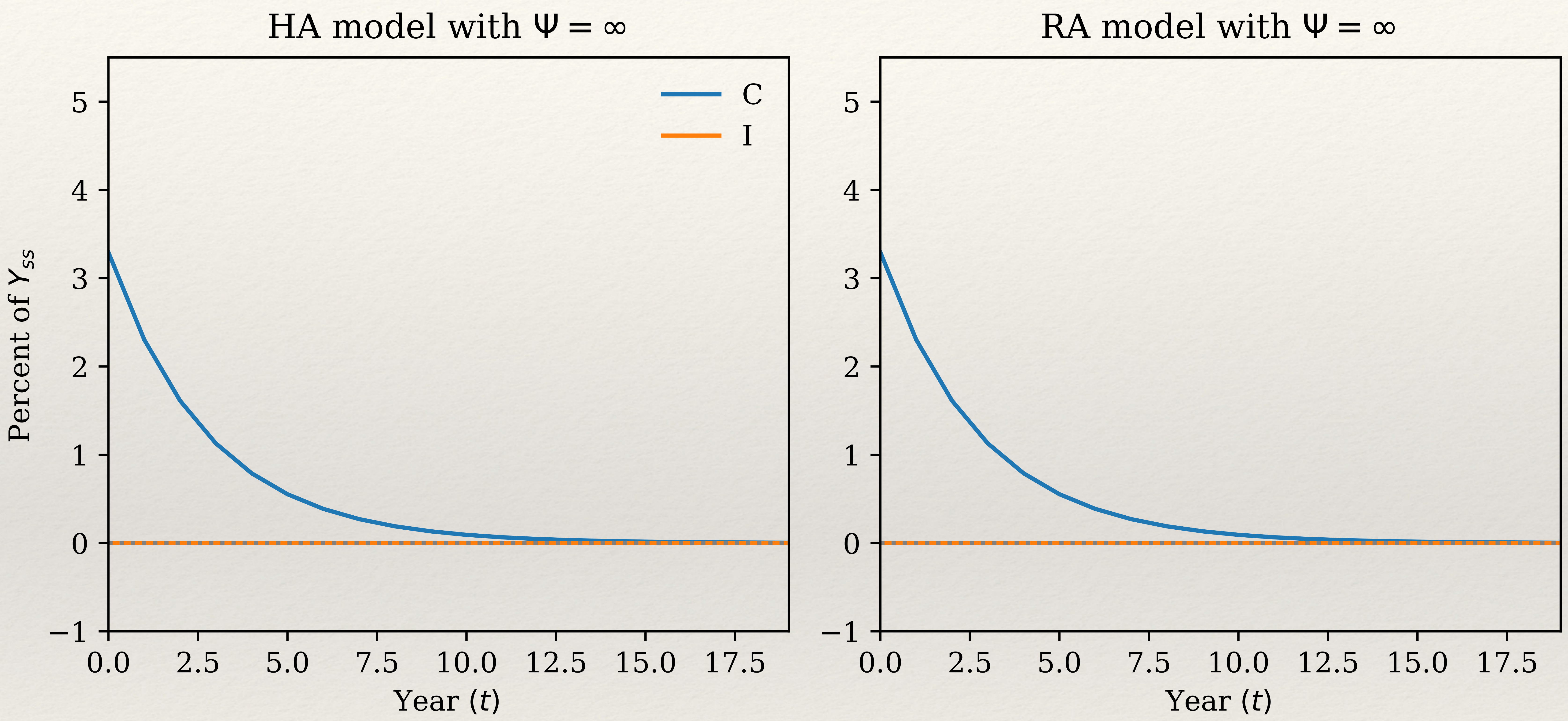
- Assume mutual funds owns 100% shares
  - Now our connection to household block is $1 + r_0^p = \dfrac{p_0 + d_0}{p_{ss}}$
- Asset market clearing $A_t = p_t$

Effect of monetary shock: inelastic investment

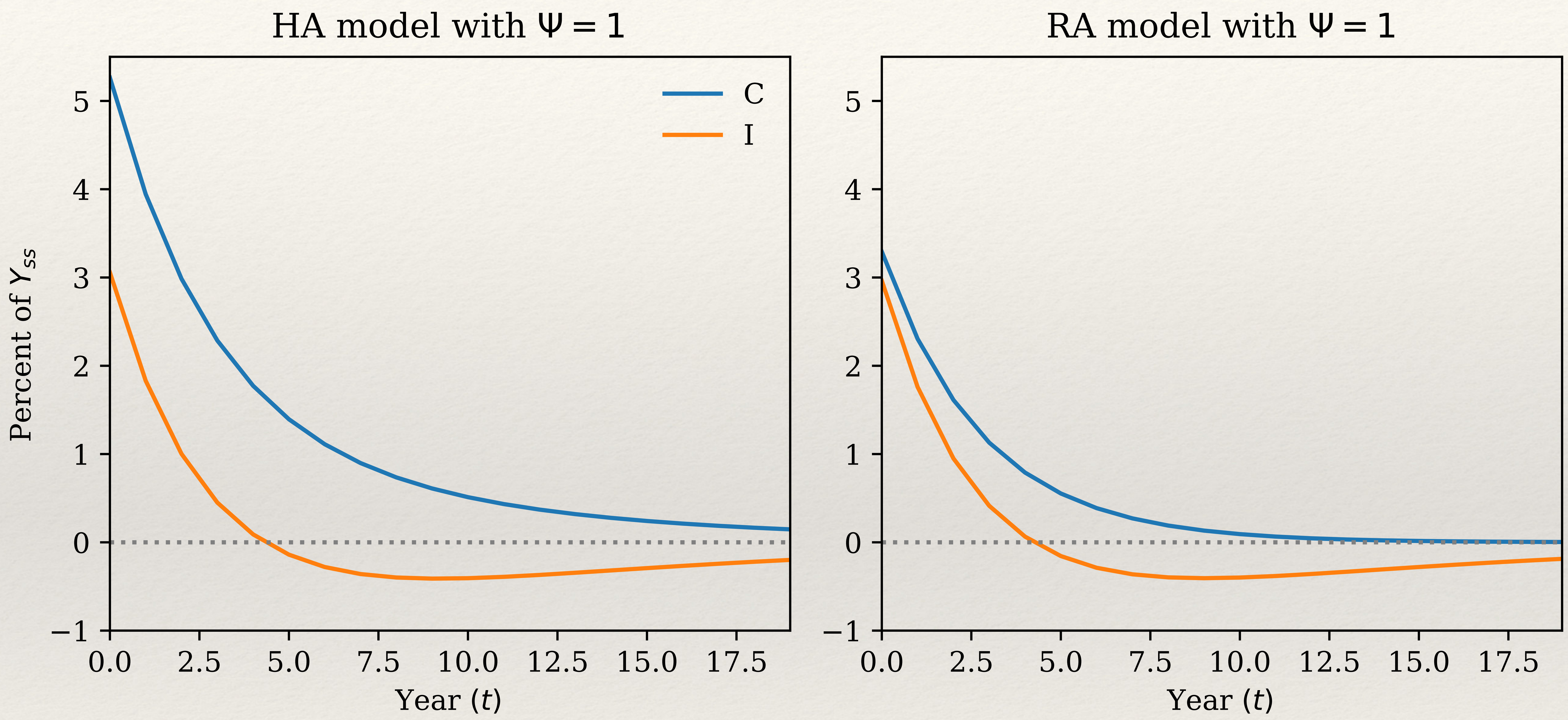


Effect of monetary shock: inelastic investment

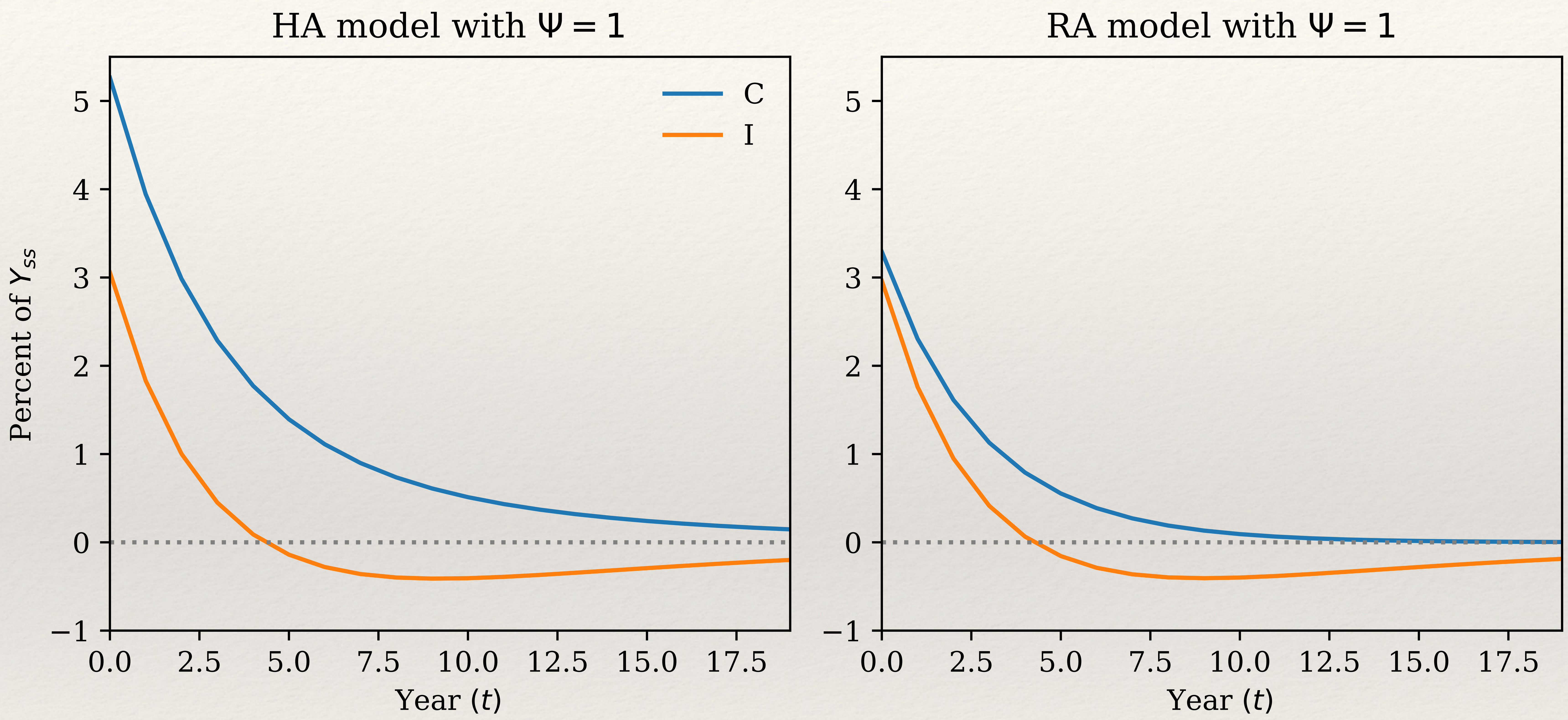


- With inelastic investment $\Psi = \infty, \delta = 0$, but capital $\alpha = 0$, just as in stock market case: equivalence between RA and HA (Werning result)

Effect of monetary shock: elastic investment



Effect of monetary shock: elastic investment



- With elastic investment, consumption gets amplified!
- Why? Aggregate demand propagation $I \to Y \to C$ (Auclert, Rognlie, Straub)

Bottom line: what does investment bring to HANK?

- **Complementarity** between investment and consumption:

Consumption response	No Investment	Investment
RA	Euler equation	same
HA	same (Werning)	**Amplification!**

Summary

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- HANK substantially enriches the analysis of monetary policy
- Key points:
  - Countercyclical income risk has large amplification effects
  - Maturity structure important due to capital gains-induced redistribution
  - Nominal positions relevant due to inflation-induced redistribution
  - Complementarity between investment and high MPCs