Numerical Analysis **Exercise 5.** Student name ______________________ Student ID _____________

1. Use Gaussian elimination to find the inverse of matrix $\mathbf{A}$.
   Check your answer by computing product $\mathbf{A}\mathbf{A}^{-1}$.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 3 & 2 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

拡大行列

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} -① \\ -② \\ -③ \\ -④ \end{array}$$

②' = ① + ②

$$\Rightarrow \left(\begin{array}{cccc|cccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} -① \\ -②' \\ -③ \\ -④ \end{array}$$

①' = ① − 2②', ③' = ③ − 3②', ④' = ④ − ②'

$$\Rightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & -1 & 0 & -1 & -2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -3 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right) \begin{array}{l} -①' \\ -②' \\ -③' \\ -④' \end{array}$$

①'' = ①' + ③', ②'' = ②' − ③'

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 2 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 3 & -3 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right) \begin{array}{l} -①'' \\ -②'' \\ -③' \\ -④' \end{array}$$

①''' = ①'' + ④'', ②''' = ②'' − ④'', ③'' = ③' + ④'

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 1 & 0 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 2 & 2 & -1 & 1 \\ -1 & 1 & 0 & 1 \end{pmatrix}$$

$$AA^{-1} = \begin{pmatrix} 1 & 2 & 1 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 3 & 2 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 2 & 2 & -1 & 1 \\ -1 & 1 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. Find the eigenvalues and eigenvectors of the matrix $\mathbf{A}$ using Jacobi diagonalization. Determine the eigenvalues from the characteristic equation and compare with the eigenvalues found by Jacobi method.

$$\tan 2\alpha = \frac{2a_{12}}{a_{11} - a_{22}} \qquad \mathbf{A} = \begin{bmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$$

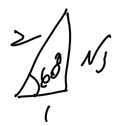
(I) Jacobi

$p = 1, q = 2$

$$\tan 2\alpha = \frac{2a_{12}}{a_{11} - a_{22}} = \frac{2\cdot\sqrt{3}}{2} = \sqrt{3}$$

$\sqrt{3}$を0にする

$a_{12} = a_{21} = \sqrt{3}$



$\frac{\pi}{3}$

$-\frac{\pi}{4} < \alpha < \frac{\pi}{4}$

$\tan 2\alpha = \sqrt{3}$

$2\alpha = \frac{\pi}{3}$

$\alpha = \frac{\pi}{6}$

Nは直交行列

$$N = \begin{pmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

$$\bar{A} = N^T A N = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 2\sqrt{3} & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}$$

固有値：4, 0

固有値4に対する固有ベクトル $\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$

固有値0に対する固有ベクトル $\begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$

(II) characteristic equation

$$\begin{vmatrix} 3-\lambda & \sqrt{3} \\ \sqrt{3} & 1-\lambda \end{vmatrix} = (3-\lambda)(1-\lambda) - 3$$

$$= -4\lambda + \lambda^2$$

$$= \lambda(\lambda - 4)$$

$\lambda = 0, 4$