

1. a) Compute the divided difference table for the tabulated function  $f(x) = 3 \cdot 2^x$ . b) Write down the Newton polynomials  $P_3(x)$  and  $P_4(x)$ . c) Evaluate the Newton polynomials at point  $x = 1.5$  and compare them to function value  $f(1.5)$ .

$k$	$x_k$	$f(x_k)$	1 <sup>st</sup> dd	2 <sup>nd</sup> dd	3 <sup>rd</sup> dd	4 <sup>th</sup> dd
0	-1	1.5				
1	0	3.0	1.5			
2	1	6.0	3.0	0.75		
3	2	12.0	6.0	1.5	0.125	
4	3	24.0	12.0	3.0	0.5	0.0625

(I) 1<sup>st</sup> dd

$$f(x_0, x_1) = \frac{3.0 - 1.5}{0 - (-1)} = 1.5$$

$$f(x_1, x_2) = \frac{6.0 - 3.0}{1 - 0} = 3.0$$

$$f(x_2, x_3) = \frac{12.0 - 6.0}{2 - 1} = 6.0$$

$$f(x_3, x_4) = \frac{24.0 - 12.0}{3 - 2} = 12.0$$

(II) 2<sup>nd</sup> dd

$$f(x_0, x_1, x_2) = \frac{3.0 - 1.5}{1 - (-1)} = 0.75$$

$$f(x_1, x_2, x_3) = \frac{6.0 - 3.0}{2 - 0} = 1.5$$

$$f(x_2, x_3, x_4) = \frac{12.0 - 6.0}{3 - 1} = 3.0$$

(III) 3<sup>rd</sup> dd

$$f(x_0, x_1, x_2, x_3) = \frac{1.5 - 0.75}{2 - (-1)} = 0.125$$

$$f(x_1, x_2, x_3, x_4) = \frac{3.0 - 1.5}{3 - 0} = 0.5$$

(IV) 4<sup>th</sup> dd

$$\begin{matrix} 0.75 \\ 4 \end{matrix}$$

$$\begin{matrix} 0.0625 \\ \overline{0.125} \\ 2 \\ \hline 0.25 \\ \hline 0.25 \end{matrix}$$

$$\begin{aligned} f(x_0, \dots, x_4) &= \frac{0.15 - 0.15}{3 - (-1)} \\ &= 0.0625 \end{aligned}$$

$$a_0 = 1.5, a_1 = 1.5, a_2 = 0.75, a_3 = 0.25, a_4 = 0.0625$$

$$P_3(x) = 1.5 + 1.5(x+1) + 0.75(x+1)x + 0.125(x+1)x(x-1)$$

$$P_4(x) = P_3(x) + 0.0625(x+1)x(x-1)(x-2)$$

$$\chi = 115 \text{ or } 87$$

$$P_3(1.5) = 1.5 + 1.5 \cdot 2.5 + 0.75 \cdot 2.5 \cdot 1.5 + 0.125 \cdot 2.5 \cdot 1.5 \cdot 0.5 \\ = 8.53125$$

$$P_4(1.5) = P_3(1.5) + 0.65 \cdot 2.5 \cdot 1.5 \cdot 0.5 + (-0.5) \\ = 8.47265625$$

$$f(1.5) = 3 \cdot 2^{1.5} \\ = 8.48528131423 \dots$$

$$P_3(1.5) - f(1.5) = 0.0459686257614 \dots$$

$$P_4(1.5) - f(1.5) = -0.0126251242385 \dots$$

2. Find the least-squares line  $y = Ax + B$  for the data given in table below. Draw a graph of a given data and the least-squares line.

$k$	$x_k$	$y_k$
1	-2	1.4
2	-1	1.7
3	0	2.7
4	1	3.4
5	2	3.8

3, 1  
8, 2  
b, 1  
1, 5

$N=5$

$\mathcal{L}$

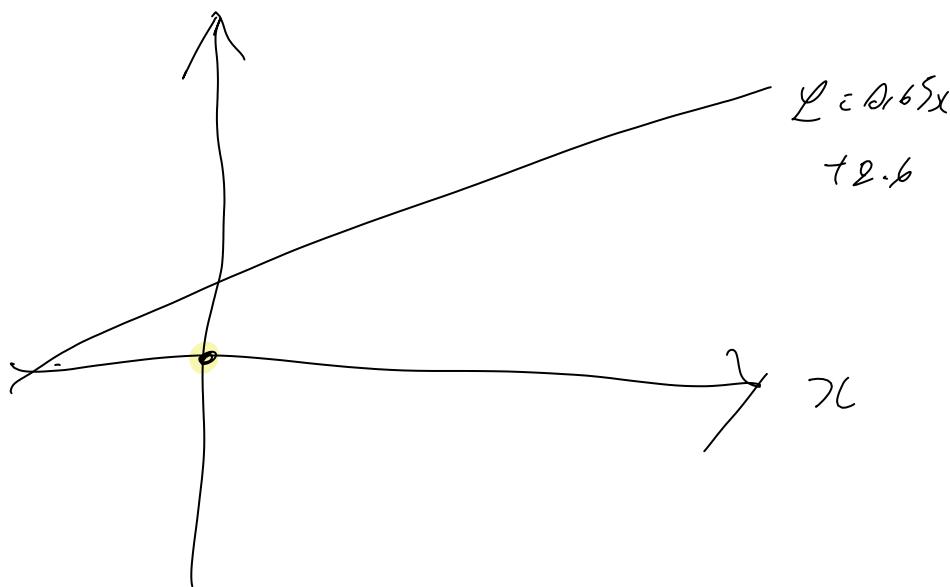
$$\begin{cases} A \sum_{k=1}^N x_k^2 + B \sum_{k=1}^N x_k = \sum_{k=1}^N x_k y_k \\ A \sum_{k=1}^N x_k + B N = \sum_{k=1}^N y_k \end{cases}$$

$$\sum_{k=1}^5 x_k = 0$$

$$\sum_{k=1}^5 y_k = 13.0$$

$$\sum_{k=1}^5 x_k^2 = 10$$

$$\sum_{k=1}^5 x_k y_k = -2.8 - 1.7 + 3.4 + 9.6 \\ = 6.5$$



$$\begin{cases} 10A = 6.5 \Rightarrow A = 0.65 \\ 5B = 13.0 \Rightarrow B = 2.6 \end{cases}$$

$$y = 0.65x + 2.6$$