Numerical Analysis. Exercise 10

Partial Differential Equations

Task. Solve the Poisson equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 2 = \nabla^2 u + 2 = 0$$

on a square region with its edge lengths 1 by 1. Boundary condition is **u = 0** on edges (boundary of the squares).
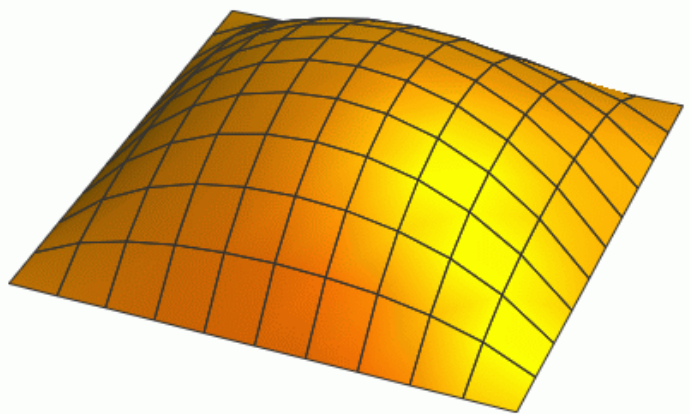
Use

a) Liebman method;

b) SOR method with overrelaxation factor 1.6.

For both methods use grid **n** by **n** cells with stepsizes **h = 0.1** and **h = 0.05** and error tolerance **eps = 1.0e-10**.

Create computer code, which performs computations a and b.

Output of your code should contain for each method and for each stepsize:
n = number of subdivisions in each direction; **h** = stepsize;
iter = number of iterations;
function values along line y = 0.5

(Graphical illustration of solution for h=0.1 is given below:)

The results may be as follows:

```
d^2u/dx^2 + d^2u/dy^2 + 2 = 0, edge length=1


Liebman method
n = 10   h = 0.100
iter = 210
x = 0.00    u =  0.00000
x = 0.10    u =  0.05766

x = 0.20    u =  0.09863
```

```
x = 0.30    u =   0.12577
x = 0.40    u =   0.14120
x = 0.50    u =   0.14620
x = 0.60    u =   0.14120
x = 0.70    u =   0.12577
x = 0.80    u =   0.09863
x = 0.90    u =   0.05766
x = 1.00    u =   0.00000

SOR method, overrelaxation factor = 1.6
n = 10    h = 0.100
iter = 51
x = 0.00    u =   0.00000
x = 0.10    u =   0.05766
x = 0.20    u =   0.09863
x = 0.30    u =   0.12577
x = 0.40    u =   0.14120
x = 0.50    u =   0.14620
x = 0.60    u =   0.14120
x = 0.70    u =   0.12577
x = 0.80    u =   0.09863
x = 0.90    u =   0.05766
x = 1.00    u =   0.00000

Liebman method
n = 20    h = 0.050
iter = 787
x = 0.00    u =   0.00000
x = 0.05    u =   0.03128
x = 0.10    u =   0.05798
x = 0.15    u =   0.08049
x = 0.20    u =   0.09920
x = 0.25    u =   0.11444
x = 0.30    u =   0.12651
x = 0.35    u =   0.13564
x = 0.40    u =   0.14203
x = 0.45    u =   0.14580
x = 0.50    u =   0.14705
x = 0.55    u =   0.14580
x = 0.60    u =   0.14203
x = 0.65    u =   0.13564
x = 0.70    u =   0.12651
x = 0.75    u =   0.11444
x = 0.80    u =   0.09920
x = 0.85    u =   0.08049
x = 0.90    u =   0.05798
x = 0.95    u =   0.03128
x = 1.00    u =   0.00000

SOR method, overrelaxation factor = 1.6
n = 20    h = 0.050
iter = 195
x = 0.00    u =   0.00000
x = 0.05    u =   0.03128
x = 0.10    u =   0.05798
x = 0.15    u =   0.08049
x = 0.20    u =   0.09920
x = 0.25    u =   0.11444
x = 0.30    u =   0.12651
x = 0.35    u =   0.13564
x = 0.40    u =   0.14203
x = 0.45    u =   0.14580
x = 0.50    u =   0.14705
x = 0.55    u =   0.14580
x = 0.60    u =   0.14203
x = 0.65    u =   0.13564
x = 0.70    u =   0.12651
x = 0.75    u =   0.11444
x = 0.80    u =   0.09920
x = 0.85    u =   0.08049
x = 0.90    u =   0.05798
x = 0.95    u =   0.03128
x = 1.00    u =   0.00000
```

Show source code and demonstrate results to your teaching assistant.