

Numerical Analysis. Exercise 08

Task 1. Numerical Differentiation

For function $f(x) = \sin(x)$ calculate the derivative at points 0, 0.1PI, 0.2PI, 0.3PI, 0.4PI, 0.5PI

a) using forward difference formula with $h = 0.025\text{PI}$;

b) using central difference formula with $h = 0.025\text{PI}$.

Create computer code, which performs computations a-b. The output of your code should contain the following for two methods:

Method

x, Derivative value, Error

Here *Derivative value* is the value obtained by numerical differentiation, *Error* is the difference between your numerical value and the exact value.

Task 2. Numerical Integration

Calculate the following integral using different methods:

$$I = \int_0^1 \frac{dx}{x^2 + 1} = \arctan(1) = \frac{\pi}{4}$$

a) Use trapezoidal rule with $n = 24$ (n is the number of subintervals for integration).

b) Use Simpson's 1/3 rule with $n = 24$.

Create computer code, which performs computations. The output of your code should contain the following for all methods:

Method

Integral value, Error, Relative error (%)

Here *Integral value* is the value obtained by numerical integration, *Error* is the difference between your numerical value and the exact value.

The results may be as follows:

Numerical differentiation

Forward difference

x = 0.0	PI	Deriv =	0.998972	Error =	-0.001028
x = 0.1	PI	Deriv =	0.937950	Error =	-0.013106
x = 0.2	PI	Deriv =	0.785115	Error =	-0.023902
x = 0.3	PI	Deriv =	0.555427	Error =	-0.032358
x = 0.4	PI	Deriv =	0.271371	Error =	-0.037646
x = 0.5	PI	Deriv =	-0.039250	Error =	-0.039250

Central difference

x = 0.0	PI	Deriv =	0.998972	Error =	-0.001028
x = 0.1	PI	Deriv =	0.950079	Error =	-0.000977
x = 0.2	PI	Deriv =	0.808186	Error =	-0.000831
x = 0.3	PI	Deriv =	0.587181	Error =	-0.000604
x = 0.4	PI	Deriv =	0.308699	Error =	-0.000318
x = 0.5	PI	Deriv =	0.000000	Error =	-0.000000

Numerical integration

I = Integral(1/(x^2+1)), limits: [0,1]

Trapezoidal rule, nsub = 24

I = 0.785325825437 Error = -7.2338e-05 R = -9.21e-03 %

Simpson 1/3 rule, nsub = 24

I = 0.785398163346 Error = -5.1911e-11 R = -6.61e-09 %

Show source code and demonstrate results to your teaching assistant.