

Numerical Analysis. Exercise 08

Task 1. Numerical Differentiation

For function $f(x) = \sin(x)$ calculate the derivative at points 0, 0.1PI, 0.2PI, 0.3PI, 0.4PI, 0.5PI

- a) using forward difference formula with $h = 0.025\pi$;
- b) using central difference formula with $h = 0.025\pi$.

Create computer code, which performs computations a-b. The output of your code should contain the following for two methods:

Method

x, Derivative value, Error

Here *Derivative value* is the value obtained by numerical differentiation, *Error* is the difference between your numerical value and the exact value.

Task 2. Numerical Integration

Calculate the following integral using different methods:

$$I = \int_0^1 \frac{dx}{x^2 + 1} = \arctan(1) = \frac{\pi}{4}$$

- a) Use trapezoidal rule with $n = 24$ (n is the number of subintervals for integration).
- b) Use Simpson's 1/3 rule with $n = 24$.

Create computer code, which performs computations. The output of your code should contain the following for all methods:

Method

Integral value, Error, Relative error (%)

Here *Integral value* is the value obtained by numerical integration, *Error* is the difference between your numerical value and the exact value.

The results may be as follows:

Numerical differentiation

```
Forward difference
x = 0.0 PI Deriv = 0.998972 Error = -0.001028
x = 0.1 PI Deriv = 0.937950 Error = -0.013106
x = 0.2 PI Deriv = 0.785115 Error = -0.023902
x = 0.3 PI Deriv = 0.555427 Error = -0.032358
x = 0.4 PI Deriv = 0.271371 Error = -0.037646
x = 0.5 PI Deriv = -0.039250 Error = -0.039250

Central difference
x = 0.0 PI Deriv = 0.998972 Error = -0.001028
x = 0.1 PI Deriv = 0.950079 Error = -0.000977
x = 0.2 PI Deriv = 0.808186 Error = -0.000831
x = 0.3 PI Deriv = 0.587181 Error = -0.000604
x = 0.4 PI Deriv = 0.308699 Error = -0.000318
x = 0.5 PI Deriv = 0.000000 Error = -0.000000

Numerical integration
I = Integral(1/(x^2+1)), limits: [0,1]

Trapezoidal rule, nsub = 24
I = 0.785325825437 Error = -7.2338e-05 R = -9.21e-03 %

Simpson 1/3 rule, nsub = 24
I = 0.785398163346 Error = -5.1911e-11 R = -6.61e-09 %
```

Show source code and demonstrate results to your teaching assistant.