Numerical Analysis Exercise 4. Student name Ryoji Hagihara Student ID 51310061

1. Solve the linear equation system $\mathbf{Ax} = \mathbf{b}$ with

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix} \qquad \mathbf{b} = \begin{Bmatrix} 1.234 \\ 2.234 \\ 3.334 \\ 4.444 \end{Bmatrix}$$

using Gaussian elimination procedure. Show the matrix and the right-hand side after each elimination step.

拡大係数行列

$$(A|b) = \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 1.234 \\ 2 & 2 & 3 & 4 & 2.234 \\ 3 & 3 & 3 & 4 & 3.334 \\ 4 & 4 & 4 & 4 & 4.444 \end{array}\right) \begin{matrix} \cdots ① \\ \cdots ② \\ \cdots ③ \\ \cdots ④ \end{matrix}$$

② − ①×2 , ③ − ①×3, ④ − ①×4 を行う

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 1.234 \\ 0 & -2 & -3 & -4 & -0.234 \\ 0 & -3 & -6 & -8 & -0.368 \\ 0 & -4 & -8 & -12 & -0.492 \end{array}\right) \begin{matrix} \cdots ① \\ \cdots ②' \\ \cdots ③' \\ \cdots ④' \end{matrix}$$

③′ − $\frac{3}{2}$×②′ , ④′ − 2×②′ を行う

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 1.234 \\ 0 & -2 & -3 & -4 & -0.234 \\ 0 & 0 & -\frac{3}{2} & -2 & -0.017 \\ 0 & 0 & -2 & -4 & -0.024 \end{array}\right) \begin{matrix} \cdots ① \\ \cdots ②' \\ \cdots ③'' \\ \cdots ④'' \end{matrix}$$

④″ − $\frac{4}{3}$③″ を行う.

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 1.234 \\ 0 & -2 & -3 & -4 & -0.234 \\ 0 & 0 & -\frac{3}{2} & -2 & -0.017 \\ 0 & 0 & 0 & -\frac{4}{3} & -0.001333 \end{array}\right)$$

$-0.024 + \frac{0.068}{3}$

$$-\frac{4}{3}x_4 = -0.024 + \frac{0.068}{3}$$

0.006

$$x_4 = 0.018 - 0.017$$

$$x_4 = 0.001$$

$$-\frac{3}{2}x_3 - 2\cdot 0.001 = -0.017$$

$$-3x_3 - 0.004 = -0.034$$

$$-3x_3 = -0.03$$

$$x_3 = 0.01$$

$$-2x_2 - 3\cdot 0.01 - 4\cdot 0.001 = -0.234$$

$$-2x_2 - 0.034 = -0.234$$

$$-2x_2 = -0.2$$

$$x_2 = 0.1$$

$$x_1 = 1.0$$

$$x = \begin{pmatrix} 1.0 \\ 0.1 \\ 0.01 \\ 0.001 \end{pmatrix}$$

下三角行列を0にする.

$Ax = b$

0 2 3 4

2. Solve the ill-conditioned system $\mathbf{Ax} = \mathbf{b}$ with the following coefficient matrix and three right-hand sides

$$\mathbf{A} = \begin{bmatrix} 1.01 & 0.99 \\ 0.99 & 1.01 \end{bmatrix} \quad \mathbf{b}_1 = \begin{Bmatrix} 2.00 \\ 2.00 \end{Bmatrix} \quad \mathbf{b}_2 = \begin{Bmatrix} 2.02 \\ 1.98 \end{Bmatrix} \quad \mathbf{b}_3 = \begin{Bmatrix} 1.98 \\ 2.02 \end{Bmatrix}$$

Compare solutions.

(I) $Ax = b_1$

$$\begin{cases} 1.01 x_1 + 0.99 x_2 = 2.00 \\ 0.99 x_1 + 1.01 x_2 = 2.00 \end{cases}$$

$\therefore x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(II) $Ax = b_2$

$$\begin{cases} 1.01 x_1 + 0.99 x_2 = 2.02 \cdots ① \\ 0.99 x_1 + 1.01 x_2 = 1.98 \cdots ② \end{cases}$$

① + ②, ① − ②

$$\begin{cases} 2.00 x_1 + 2.00 x_2 = 4.00 \\ 0.02 x_1 - 0.02 x_2 = 0.04 \end{cases}$$

$$\begin{cases} x_1 + x_2 = 2.00 \\ x_1 - x_2 = 2.00 \end{cases}$$

$\therefore x = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(III) $Ax = b_3$

$$\begin{cases} 1.01 x_1 + 0.99 x_2 = 1.98 \cdots ① \\ 0.99 x_1 + 1.01 x_2 = 2.02 \cdots ② \end{cases}$$

① + ②, ① − ②

$$\begin{cases} 2.00 x_1 + 2.00 x_2 = 4.00 \\ 0.02 x_1 - 0.02 x_2 = -0.04 \end{cases}$$

$$\begin{cases} x_1 + x_2 = 2 \\ -x_1 + x_2 = 2 \end{cases}$$

$\therefore x = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

b が 1% ずれた
だけで解が異なる.

単位行列化

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 1.234 \\ 0 & -2 & -3 & -4 & -0.234 \\ 0 & 0 & -\frac{3}{2} & -2 & -0.017 \\ 0 & 0 & 0 & -\frac{4}{3} & -0.001333\cdots \end{array}\right) \begin{array}{l} -ⓐ \\ -ⓑ \\ -ⓒ \\ -ⓓ \end{array}$$

ⓐ + ⓑ

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & -2 & -3 & -4 & -0.234 \\ 0 & 0 & -\frac{3}{2} & -2 & -0.017 \\ 0 & 0 & 0 & -\frac{4}{3} & -0.0013 \end{array}\right) \begin{array}{l} \cdots ⓐ' \\ \cdots ⓑ \\ \cdots ⓒ \\ \cdots ⓓ \end{array}$$

1.234

$-0.0234 + \frac{0.068}{3}$

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \end{array}$$

$0.072 + 0.068$

$$\begin{array}{r} 0.072 \\ -0.068 \\ \hline 0.004 \end{array}$$

ⓒ × 2 , ⓓ × 3 ÷ (−4)

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & -2 & -3 & -4 & -0.234 \\ 0 & 0 & -3 & -4 & -0.034 \\ 0 & 0 & 0 & 1 & 0.001 \end{array}\right) \begin{array}{l} -ⓐ' \\ -ⓑ \\ -ⓒ \\ -ⓓ' \end{array}$$

ⓑ − ⓒ′ , ⓒ′ + 4ⓓ

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 & -0.2 \\ 0 & 0 & -3 & 0 & -0.03 \\ 0 & 0 & 0 & 1 & 0.001 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0.1 \\ 0 & 0 & 1 & 0 & 0.01 \\ 0 & 0 & 0 & 1 & 0.001 \end{array}\right)$$

$$x = \begin{pmatrix} 1.0 \\ 0.1 \\ 0.01 \\ 0.001 \end{pmatrix}$$