



Exercise 6. Student name _____ Student ID _____

1. a) Compute the divided difference table for the tabulated function $f(x) = 3 \cdot 2^x$. b) Write down the Newton polynomials $P_3(x)$ and $P_4(x)$. c) Evaluate the Newton polynomials at point $x = 1.5$ and compare them to function value $f(1.5)$.

| k | x_k | $f(x_k)$ | 1 st dd | 2 nd dd | 3 rd dd | 4 th dd |
|---|-------|----------|--------------------|--------------------|--------------------|--------------------|
| 0 | -1 | 1.5 | | | | |
| 1 | 0 | 3.0 | 1.5 | | | |
| 2 | 1 | 6.0 | 3.0 | 0.75 | | |
| 3 | 2 | 12.0 | 6.0 | 1.5 | 0.25 | |
| 4 | 3 | 24.0 | 12.0 | 3.0 | 0.5 | 0.0625 |

(I) 1st dd

$$f(x_0, x_1) = \frac{3.0 - 1.5}{0 - (-1)} = 1.5$$

$$f(x_1, x_2) = \frac{6.0 - 3.0}{1 - 0} = 3.0$$

$$f(x_2, x_3) = \frac{12.0 - 6.0}{2 - 1} = 6.0$$

$$f(x_3, x_4) = \frac{24.0 - 12.0}{3 - 2} = 12.0$$

(II) 2nd dd

$$f(x_0, x_1, x_2) = \frac{3.0 - 1.5}{1 - (-1)} = 0.75$$

$$f(x_1, x_2, x_3) = \frac{6.0 - 3.0}{2 - 0} = 1.5$$

$$f(x_2, x_3, x_4) = \frac{12 - 6}{3 - 1} = 3.0$$

(III) 3rd dd

$$f(x_0, x_1, x_2, x_3) = \frac{1.5 - 0.75}{2 - (-1)} = 0.25$$

$$f(x_1, x_2, x_3, x_4) = \frac{3.0 - 1.5}{3 - 0} = 0.5$$

(IV) 4th dd

$$f(x_0, \dots, x_4) = \frac{0.5 - 0.25}{3 - (-1)} = 0.0625$$

$$\begin{array}{r} 0.25 \\ 4 \end{array}$$

$$\begin{array}{r} 0.0625 \\ \times 4 \\ \hline 0.25 \\ 24 \\ \hline 24 \\ \hline 0 \end{array}$$

$$a_0 = 1.5, a_1 = 1.5, a_2 = 0.75, a_3 = 0.25, a_4 = 0.0625$$

$$P_3(x) = 1.5 + 1.5(x+1) + 0.75(x+1)x + 0.25(x+1)x(x-1)$$

$$P_4(x) = P_3(x) + 0.0625(x+1)x(x-1)(x-2)$$

$$x = 1.5 \text{ の } x^k$$

$$P_3(1.5) = 1.5 + 1.5 \cdot 2.5 + 0.175 \cdot 2.5 \cdot 1.5 + 0.125 \cdot 2.5 \cdot 1.5 \cdot 0.5$$

$$= 8.53125$$

$$P_4(1.5) = P_3(1.5) + 0.65 \cdot 2.5 \cdot 1.5 \cdot 0.5 \cdot (-0.5)$$

$$= 8.47265625$$

$$f(1.5) = 3 \cdot 2^{1.5}$$

$$= 8.48528139423 \dots$$

$$P_3(1.5) - f(1.5) = 0.0459686257614 \dots$$

$$P_4(1.5) - f(1.5) = -0.0126251242385 \dots$$

2. Find the least-squares line $y = Ax + B$ for the data given in table below. Draw a graph of a given data and the least-squares line.

| k | x_k | y_k |
|-----|-------|-------|
| 1 | -2 | 1.4 |
| 2 | -1 | 1.7 |
| 3 | 0 | 2.7 |
| 4 | 1 | 3.4 |
| 5 | 2 | 3.8 |

3.1 2.2 N=5
6.1 3.5

$$\begin{cases} A \sum_{k=1}^N x_k^2 + B \sum_{k=1}^N x_k = \sum_{k=1}^N x_k y_k \\ A \sum_{k=1}^N x_k + B N = \sum_{k=1}^N y_k \end{cases}$$

$$\sum_{k=1}^5 x_k = 0$$

$$\sum_{k=1}^5 y_k = 13.0$$

$$\sum_{k=1}^5 x_k^2 = 10$$

$$\sum_{k=1}^5 x_k y_k = -2.8 - 1.7 + 2.7 + 3.4 + 7.6 = 6.5$$

$$\begin{cases} 10 A = 6.5 \Rightarrow A = 0.65 \\ 5 B = 13.0 \Rightarrow B = 2.6 \end{cases}$$

$$y = 0.65x + 2.6$$

