A Supplementary File for "A Niching Indicator-Based Multi-modal Many-objective Optimizer"

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Abstract

This is a supplementary file for "A Niching Indicator-Based Multi-modal Many-objective Optimizer".

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Table S.1: Results of NIMMO with 11 values of T on the 23 test problem instances. The mean IGD values for 31 runs are shown.

	$\lfloor 0.1\mu \rfloor$	$\lfloor 0.05\mu \rfloor$	$\lfloor 0.2\mu \rfloor$	$\lfloor 0.3\mu \rfloor$	$\lfloor 0.4\mu \rfloor$	$\lfloor 0.5\mu \rfloor$	$\lfloor 0.6\mu \rfloor$	$\lfloor 0.7\mu \rfloor$	$\lfloor 0.8\mu \rfloor$	$\lfloor 0.9\mu \rfloor$	$\lfloor 1.0\mu \rfloor$
Two-On-One	0.0250 (3)	0.0251≈ (4)	0.0263− (6)	0.0282− (7)	0.0288− (9)	0.0296− (10)	0.0305− (11)	0.0284− (8)	0.0263− (5)	*0.0243*≈ (2)	**0.0236**+ (1)
Omni-test	0.0769 (10)	0.1048− (11)	0.0568+ (9)	0.0516+ (8)	0.0441+ (7)	0.0436+ (6)	0.0401+ (4)	0.0384+ (3)	**0.0358**+ (1)	0.0405+ (5)	*0.0374*+ (2)
SYM-PART1	0.0246 (10)	0.0215+ (6)	0.0267− (11)	0.0244≈ (9)	0.0240≈ (8)	0.0214+ (5)	0.0217+ (7)	0.0196+ (4)	0.0185+ (3)	*0.0169*+ (2)	**0.0167**+ (1)
SYM-PART2	0.0365 (11)	0.0270+ (7)	0.0357≈ (10)	0.0335≈ (9)	0.0290+ (8)	0.0260+ (6)	0.0254+ (5)	0.0245+ (4)	0.0223+ (3)	**0.0206**+ (1)	*0.0211*+ (2)
SYM-PART3	0.0298 (8)	0.0254+ (5)	0.0374− (11)	0.0329≈ (10)	0.0300≈ (9)	0.0271+ (7)	0.0266+ (6)	0.0254+ (4)	0.0225+ (3)	*0.0217*+ (2)	**0.0205**+ (1)
MMF1	0.0050 (10)	0.0063− (11)	0.0050≈ (9)	0.0047+ (8)	0.0044+ (7)	0.0043+ (6)	0.0042+ (5)	**0.0041**+ (1)	0.0042+ (3)	*0.0042*+ (2)	0.0042+ (4)
MMF2	*0.0216* (2)	**0.0196**≈ (1)	0.0240≈ (3)	0.0263− (5)	0.0251− (4)	0.0275− (6)	0.0280− (8)	0.0331− (11)	0.0311− (10)	0.0282− (9)	0.0278− (7)
MMF3	*0.0190* (2)	**0.0167**≈ (1)	0.0240− (8)	0.0243− (9)	0.0221− (3)	0.0334− (11)	0.0237− (7)	0.0236− (6)	0.0236− (5)	0.0249− (10)	0.0222≈ (4)
MMF4	**0.0048** (1)	0.0081− (11)	0.0059− (5)	0.0063− (8)	0.0064− (9)	0.0070− (10)	0.0062− (7)	0.0061− (6)	0.0057− (4)	0.0055− (3)	*0.0054*− (2)
MMF5	*0.0039* (2)	0.0044− (11)	**0.0039**≈ (1)	0.0040≈ (3)	0.0041− (8)	0.0042− (10)	0.0042− (9)	0.0040≈ (4)	0.0041− (5)	0.0041− (6)	0.0041− (7)
MMF6	0.0044 (4)	**0.0042**+ (1)	0.0048− (11)	0.0047− (9)	0.0047− (8)	0.0048− (10)	0.0046− (7)	0.0046≈ (6)	0.0045≈ (5)	0.0043≈ (3)	*0.0042*+ (2)
MMF7	0.0030 (10)	0.0037− (11)	0.0028+ (4)	0.0028+ (6)	0.0029+ (9)	0.0029+ (8)	0.0029+ (7)	*0.0028*+ (2)	0.0028+ (5)	0.0028+ (3)	**0.0027**+ (1)
MMF8	0.0092 (6)	**0.0076**+ (1)	0.0135− (11)	0.0100≈ (8)	0.0115≈ (10)	0.0088+ (5)	*0.0080*+ (2)	0.0100≈ (7)	0.0112− (9)	0.0087≈ (4)	0.0084+ (3)
3-Polygon	0.0025 (10)	0.0026≈ (11)	0.0024+ (9)	0.0024+ (8)	0.0023+ (7)	0.0023+ (6)	0.0023+ (5)	0.0022+ (4)	0.0022+ (3)	*0.0022*+ (2)	**0.0022**+ (1)
5-Polygon	0.0044 (10)	0.0045≈ (11)	0.0041+ (9)	0.0040+ (8)	0.0039+ (7)	0.0038+ (6)	0.0038+ (5)	0.0037+ (4)	0.0037+ (3)	*0.0036*+ (2)	**0.0036**+ (1)
8-Polygon	0.0069 (10)	0.0070− (11)	0.0066+ (9)	0.0064+ (8)	0.0062+ (7)	0.0060+ (6)	0.0059+ (5)	0.0058+ (4)	0.0058+ (3)	*0.0057*+ (2)	**0.0056**+ (1)
10-Polygon	0.0064 (11)	0.0064+ (10)	0.0061+ (9)	0.0059+ (8)	0.0058+ (7)	0.0057+ (6)	0.0056+ (5)	0.0055+ (4)	0.0054+ (3)	*0.0054*+ (2)	**0.0053**+ (1)
15-Polygon	0.0106 (10)	0.0107≈ (11)	0.0100+ (9)	0.0096+ (8)	0.0093+ (7)	0.0091+ (6)	0.0089+ (5)	0.0088+ (4)	0.0087+ (3)	*0.0086*+ (2)	**0.0085**+ (1)
3-RPolygon	0.0025 (11)	0.0025≈ (10)	0.0024+ (9)	0.0024+ (8)	0.0023+ (7)	0.0023+ (6)	0.0023+ (5)	0.0023+ (4)	0.0022+ (3)	*0.0022*+ (2)	**0.0022**+ (1)
5-RPolygon	0.0044 (11)	0.0044≈ (10)	0.0042+ (9)	0.0040+ (8)	0.0039+ (7)	0.0039+ (6)	0.0038+ (5)	0.0037+ (4)	0.0037+ (3)	*0.0036*+ (2)	**0.0036**+ (1)
8-RPolygon	0.0069 (10)	0.0069≈ (11)	0.0066+ (9)	0.0064+ (8)	0.0062+ (7)	0.0061+ (6)	0.0060+ (5)	0.0059+ (4)	0.0058+ (3)	*0.0057*+ (2)	**0.0056**+ (1)
10-RPolygon	0.0064 (11)	0.0064≈ (10)	0.0061+ (9)	0.0059+ (8)	0.0058+ (7)	0.0057+ (6)	0.0056+ (5)	0.0055+ (4)	0.0054+ (3)	*0.0054*+ (2)	**0.0053**+ (1)
15-RPolygon	0.0105 (10)	0.0106≈ (11)	0.0100+ (9)	0.0097+ (8)	0.0094+ (7)	0.0092+ (6)	0.0091+ (5)	0.0089+ (4)	0.0087+ (3)	*0.0086*+ (2)	**0.0085**+ (1)

Table S.2: Results of NIMMO with 11 values of T on the 23 test problem instances. The mean IGDX values for 31 runs are shown.

	$\lfloor 0.1\mu \rfloor$	$\lfloor 0.05\mu \rfloor$	$\lfloor 0.2\mu \rfloor$	$\lfloor 0.3\mu \rfloor$	$\lfloor 0.4\mu \rfloor$	$\lfloor 0.5\mu \rfloor$	$\lfloor 0.6\mu \rfloor$	$\lfloor 0.7\mu \rfloor$	$\lfloor 0.8\mu \rfloor$	$\lfloor 0.9\mu \rfloor$	$\lfloor 1.0\mu \rfloor$
Two-On-One	*0.0226* (2)	**0.0212**+ (1)	0.0268− (3)	0.0294− (4)	0.0298− (5)	0.0314− (6)	0.0372− (7)	0.0375− (8)	0.0394− (10)	0.0393− (9)	0.0461− (11)
Omni-test	*1.6052* (2)	**1.5635**≈ (1)	1.7195− (3)	1.8047− (4)	1.8369− (5)	1.8994− (6)	1.9724− (7)	1.9926− (8)	2.0027− (9)	2.1145− (11)	2.0484− (10)
SYM-PART1	*0.0530* (2)	**0.0497**+ (1)	0.1241− (3)	0.5909− (4)	0.9237− (5)	1.3868− (7)	1.6410− (9)	1.2568− (6)	1.5156− (8)	2.2070− (10)	3.2541− (11)
SYM-PART2	*0.0838* (2)	**0.0763**+ (1)	0.9903− (3)	1.8018− (6)	1.7971− (5)	1.7313− (4)	1.9975− (7)	2.1001− (8)	2.3735− (9)	2.4064− (10)	2.4816− (11)
SYM-PART3	*0.1418* (2)	**0.0796**+ (1)	0.9800− (3)	1.4947− (4)	2.0485− (6)	2.3069− (9)	1.9616− (5)	2.2992− (8)	2.1677− (7)	2.6956− (10)	3.0557− (11)
MMF1	**0.0611** (1)	*0.0644*− (2)	0.0698− (3)	0.0704− (4)	0.0760− (5)	0.0816− (6)	0.0851− (8)	0.0828− (7)	0.0932− (11)	0.0888− (9)	0.0921− (10)
MMF2	*0.0440* (2)	**0.0374**+ (1)	0.0566− (3)	0.0701− (4)	0.0742− (5)	0.0803− (6)	0.0868− (7)	0.1138− (11)	0.0980− (9)	0.0960− (8)	0.1009− (10)
MMF3	*0.0440* (2)	**0.0352**+ (1)	0.0580− (3)	0.0637− (4)	0.0686− (5)	0.0826− (11)	0.0744− (8)	0.0768− (10)	0.0713− (7)	0.0746− (9)	0.0696− (6)
MMF4	**0.0372** (1)	0.0502− (3)	*0.0458*− (2)	0.0535− (4)	0.0644− (5)	0.0733− (7)	0.0792− (10)	0.0713− (6)	0.0781− (9)	0.0751− (8)	0.0831− (11)
MMF5	*0.0846* (2)	**0.0841**≈ (1)	0.0946− (3)	0.1058− (4)	0.1200− (5)	0.1294− (6)	0.1388− (7)	0.1500− (10)	0.1486− (9)	0.1480− (8)	0.1504− (11)
MMF6	*0.1037* (2)	**0.0972**+ (1)	0.1106− (3)	0.1185− (4)	0.1253− (5)	0.1331− (6)	0.1340− (7)	0.1375− (8)	0.1422− (10)	0.1392− (9)	0.1453− (11)
MMF7	0.0216 (3)	0.0263− (6)	**0.0201**≈ (1)	*0.0205*≈ (2)	0.0229− (4)	0.0246− (5)	0.0270− (7)	0.0288− (8)	0.0335− (9)	0.0363− (10)	0.0405− (11)
MMF8	*0.2064* (2)	**0.1735**+ (1)	0.5633− (3)	0.6134− (4)	0.6347− (5)	0.7198− (10)	0.6968− (9)	0.6806− (8)	0.7615− (11)	0.6364− (6)	0.6570− (7)
3-Polygon	**0.0056** (1)	*0.0061*− (2)	0.0075− (4)	0.0159− (11)	0.0083− (7)	0.0085− (8)	0.0080− (6)	0.0075− (3)	0.0078− (5)	0.0100− (10)	0.0089− (9)
5-Polygon	**0.0070** (1)	*0.0077*− (2)	0.0135− (10)	0.0196− (11)	0.0128− (9)	0.0107− (8)	0.0097− (4)	0.0097− (3)	0.0101− (5)	0.0106− (7)	0.0106− (6)
8-Polygon	**0.0089** (1)	*0.0100*− (2)	0.0161− (10)	0.0372− (11)	0.0146− (9)	0.0134− (5)	0.0126− (3)	0.0135− (6)	0.0133− (4)	0.0140− (7)	0.0145− (8)
10-Polygon	**0.0072** (1)	*0.0080*− (2)	0.0117− (10)	0.0226− (11)	0.0109− (8)	0.0104− (6)	0.0097− (3)	0.0103− (5)	0.0102− (4)	0.0108− (7)	0.0110− (9)
15-Polygon	**0.0100** (1)	*0.0114*− (2)	0.0176− (6)	0.0437− (11)	0.0207− (9)	0.0227− (10)	0.0143− (4)	0.0143− (3)	0.0172− (5)	0.0191− (7)	0.0196− (8)
3-RPolygon	**0.0059** (1)	*0.0063*− (2)	0.0298− (6)	0.0630− (11)	0.0323− (8)	0.0369− (9)	0.0371− (10)	0.0282− (5)	0.0224− (4)	0.0182− (3)	0.0310− (7)
5-RPolygon	**0.0074** (1)	*0.0081*− (2)	0.0411− (10)	0.0558− (11)	0.0366− (8)	0.0384− (9)	0.0343− (6)	0.0349− (7)	0.0212− (4)	0.0214− (5)	0.0167− (3)
8-RPolygon	**0.0093** (1)	*0.0104*− (2)	0.0633− (9)	0.0848− (11)	0.0474− (7)	0.0574− (8)	0.0664− (10)	0.0380− (4)	0.0389− (5)	0.0446− (6)	0.0375− (3)
10-RPolygon	**0.0076** (1)	*0.0083*− (2)	0.0243− (10)	0.0585− (11)	0.0214− (8)	0.0221− (9)	0.0193− (6)	0.0150− (3)	0.0157− (5)	0.0196− (7)	0.0157− (4)
15-RPolygon	**0.0104** (1)	*0.0118*− (2)	0.0797− (10)	0.0936− (11)	0.0704− (9)	0.0600− (7)	0.0624− (8)	0.0542− (6)	0.0509− (5)	0.0441− (4)	0.0345− (3)

Table S.3: Results of NIMMO with 11 values of T on the 23 test problem instances. The mean PSP values for 31 runs are shown.

	$\lfloor 0.1\mu \rfloor$	$\lfloor 0.05\mu \rfloor$	$\lfloor 0.2\mu \rfloor$	$\lfloor 0.3\mu \rfloor$	$\lfloor 0.4\mu \rfloor$	$\lfloor 0.5\mu \rfloor$	$\lfloor 0.6\mu \rfloor$	$\lfloor 0.7\mu \rfloor$	$\lfloor 0.8\mu \rfloor$	$\lfloor 0.9\mu \rfloor$	$\lfloor 1.0\mu \rfloor$
Two-On-One	*40.1* (2)	**43.3**+ (1)	33.9− (3)	31.6− (4)	31.2− (5)	29.6− (6)	25.9− (8)	26.2− (7)	24.9− (10)	25.5− (9)	20.7− (11)
Omni-test	*0.5* (2)	**0.5**≈ (1)	0.5− (3)	0.4− (4)	0.4− (5)	0.4− (6)	0.3− (7)	0.3− (10)	0.3− (8)	0.3− (11)	0.3− (9)
SYM-PART1	*18.7* (2)	**20.1**+ (1)	11.4− (3)	7.5− (4)	4.0− (7)	4.1− (6)	2.9− (8)	5.5− (5)	2.1− (10)	2.3− (9)	1.0− (11)
SYM-PART2	*11.5* (2)	**12.7**+ (1)	2.5− (3)	1.5− (4)	0.6− (7)	0.8− (6)	1.0− (5)	0.5− (10)	0.5− (11)	0.5− (9)	0.5− (8)
SYM-PART3	*9.5* (2)	**12.5**+ (1)	1.1− (3)	0.5− (4)	0.3− (7)	0.3− (8)	0.4− (6)	0.2− (10)	0.5− (5)	0.2− (11)	0.3− (9)
MMF1	**16.2** (1)	*15.5*− (2)	14.1− (3)	14.1− (4)	13.0− (5)	11.8− (6)	11.3− (8)	11.6− (7)	10.1− (11)	10.5− (9)	10.3− (10)
MMF2	*20.6* (2)	**25.2**+ (1)	16.8≈ (3)	14.2− (4)	13.2− (5)	12.4− (6)	10.3− (7)	8.4− (11)	9.1− (10)	10.3− (8)	10.0− (9)
MMF3	*19.9* (2)	**25.6**+ (1)	16.1≈ (3)	13.4− (4)	13.2− (5)	9.9− (11)	11.2− (9)	10.5− (10)	11.3− (8)	11.9− (7)	12.9− (6)
MMF4	**27.1** (1)	19.9− (3)	*22.4*− (2)	19.0− (4)	15.6− (5)	13.9− (7)	13.1− (9)	14.1− (6)	12.8− (10)	13.3− (8)	12.2− (11)
MMF5	*11.7* (2)	**11.7**≈ (1)	10.4− (3)	9.4− (4)	8.2− (5)	7.6− (6)	7.0− (7)	6.5− (8)	6.4− (9)	6.3− (10)	6.3− (11)
MMF6	*9.1* (2)	**9.9**+ (1)	8.8≈ (3)	8.3− (4)	7.9− (5)	7.3− (6)	7.2− (7)	6.7− (9)	6.5− (10)	6.8− (8)	6.4− (11)
MMF7	43.0 (3)	34.5− (6)	**45.8**≈ (1)	*45.2*≈ (2)	40.3≈ (4)	36.6− (5)	33.2− (7)	33.0− (8)	27.0− (9)	25.0− (10)	20.6− (11)
MMF8	*5.0* (2)	**5.9**+ (1)	1.8− (3)	1.2− (4)	1.2− (5)	0.7− (11)	0.7− (9)	0.8− (8)	0.7− (10)	0.9− (6)	0.8− (7)
3-Polygon	**178.7** (1)	*163.4*− (2)	133.1− (3)	91.3− (11)	122.1− (7)	119.9− (8)	124.9− (6)	131.9− (4)	127.0− (5)	111.5− (10)	112.2− (9)
5-Polygon	**141.3** (1)	*128.4*− (2)	87.9− (9)	65.8− (11)	87.7− (10)	94.7− (6)	102.3− (4)	102.7− (3)	97.1− (5)	93.5− (8)	93.8− (7)
8-Polygon	**111.3** (1)	*99.5*− (2)	67.7− (10)	34.3− (11)	71.6− (8)	76.1− (5)	79.7− (3)	76.6− (4)	74.1− (6)	71.8− (7)	71.4− (9)
10-Polygon	**138.2** (1)	*124.8*− (2)	86.7− (10)	61.6− (11)	92.2− (8)	96.9− (5)	102.8− (3)	97.2− (4)	96.8− (6)	92.4− (7)	92.0− (9)
15-Polygon	**97.5** (1)	*86.5*− (2)	60.0− (6)	28.8− (11)	55.1− (10)	58.4− (9)	69.2− (3)	69.0− (4)	62.9− (5)	58.4− (8)	59.4− (7)
3-RPolygon	**166.5** (1)	*155.2*− (2)	69.9− (3)	25.2− (11)	45.1− (9)	49.5− (8)	41.8− (10)	60.6− (6)	63.5− (5)	66.5− (4)	51.0− (7)
5-RPolygon	**132.0** (1)	*120.2*− (2)	46.7− (6)	32.9− (11)	34.3− (10)	41.7− (8)	44.7− (7)	38.7− (9)	53.9− (5)	57.6− (4)	62.7− (3)
8-RPolygon	**104.3** (1)	*93.0*− (2)	23.0− (8)	11.7− (11)	28.2− (6)	22.8− (9)	21.7− (10)	31.3− (5)	32.0− (3)	26.9− (7)	31.5− (4)
10-RPolygon	**126.3** (1)	*116.8*− (2)	61.3− (6)	23.8− (11)	50.8− (10)	55.4− (9)	60.1− (7)	65.7− (4)	64.0− (5)	55.6− (8)	66.2− (3)
15-RPolygon	**91.0** (1)	*80.3*− (2)	16.0− (10)	9.5− (11)	18.6− (8)	22.6− (5)	18.4− (9)	21.6− (6)	18.9− (7)	24.0− (4)	31.7− (3)

Table S.4: Results of NIMMO with 10 values of μ on the 23 test problem instances. The mean IGD values for 31 runs are shown.

	$\mu = 100$	$\mu = 200$	$\mu = 300$	$\mu = 400$	$\mu = 500$	$\mu = 600$	$\mu = 700$	$\mu = 800$	$\mu = 900$	$\mu = 1\,000$
Two-On-One	0.0491 (5)	**0.0472**+ (1)	*0.0475*≈ (2)	0.0476≈ (3)	0.0485≈ (4)	0.0491≈ (6)	0.0508− (7)	0.0510− (8)	0.0528− (10)	0.0526− (9)
Omni-test	0.1587 (10)	0.0824+ (8)	**0.0669**+ (1)	0.0676+ (3)	*0.0673*+ (2)	0.0695+ (4)	0.0704+ (5)	0.0754+ (6)	0.0770+ (7)	0.0859+ (9)
SYM-PART1	0.0474 (10)	0.0336+ (9)	0.0316+ (8)	0.0303+ (7)	*0.0290*+ (2)	0.0299+ (5)	**0.0290**+ (1)	0.0300+ (6)	0.0298+ (4)	0.0295+ (3)
SYM-PART2	0.0648 (10)	0.0426+ (9)	0.0342+ (8)	0.0312+ (7)	0.0302+ (3)	0.0303+ (5)	0.0303+ (4)	*0.0302*+ (2)	**0.0300**+ (1)	0.0305+ (6)
SYM-PART3	0.0582 (10)	0.0368+ (9)	0.0324+ (8)	0.0323+ (7)	0.0307+ (4)	0.0306+ (3)	*0.0305*+ (2)	**0.0300**+ (1)	0.0314+ (6)	0.0310+ (5)
MMF1	0.0098 (10)	**0.0050**+ (1)	0.0067+ (6)	0.0067+ (5)	*0.0066*+ (2)	0.0067+ (3)	0.0067+ (4)	0.0069+ (9)	0.0067+ (7)	0.0068+ (8)
MMF2	0.0317 (10)	0.0216+ (9)	0.0157+ (8)	0.0149+ (7)	0.0137+ (4)	0.0138+ (5)	0.0135+ (3)	0.0141+ (6)	**0.0134**+ (1)	*0.0135*+ (2)
MMF3	0.0299 (10)	0.0190+ (9)	0.0144+ (8)	0.0123+ (7)	0.0120+ (6)	0.0117+ (4)	*0.0114*+ (2)	**0.0108**+ (1)	0.0114+ (3)	0.0119+ (5)
MMF4	0.0100 (10)	**0.0048**+ (1)	*0.0062*+ (2)	0.0065+ (4)	0.0064+ (3)	0.0065+ (5)	0.0066+ (8)	0.0066+ (7)	0.0065+ (6)	0.0067+ (9)
MMF5	0.0075 (10)	**0.0039**+ (1)	0.0068+ (3)	*0.0067*+ (2)	0.0068+ (4)	0.0069+ (5)	0.0070+ (9)	0.0069+ (6)	0.0070+ (7)	0.0070+ (8)
MMF6	0.0083 (10)	**0.0044**+ (1)	0.0069+ (8)	0.0068+ (5)	0.0069+ (7)	0.0068+ (4)	0.0068+ (6)	0.0068+ (3)	0.0069+ (9)	*0.0068*+ (2)
MMF7	*0.0056* (2)	**0.0030**+ (1)	0.0067− (3)	0.0074− (6)	0.0073− (4)	0.0076− (8)	0.0074− (5)	0.0076− (7)	0.0077− (9)	0.0079− (10)
MMF8	0.0161 (10)	0.0092+ (9)	0.0071+ (6)	0.0070+ (4)	0.0072+ (8)	**0.0068**+ (1)	0.0070+ (5)	0.0069+ (3)	*0.0069*+ (2)	0.0071+ (7)
3-Polygon	**0.0038** (1)	*0.0038*≈ (2)	0.0040− (3)	0.0041− (4)	0.0042− (5)	0.0044− (6)	0.0044− (7)	0.0045− (8)	0.0045− (9)	0.0045− (10)
5-Polygon	**0.0065** (1)	*0.0067*− (2)	0.0070− (3)	0.0072− (4)	0.0073− (5)	0.0075− (7)	0.0075− (6)	0.0075− (9)	0.0075− (8)	0.0077− (10)
8-Polygon	**0.0088** (1)	*0.0091*− (2)	0.0098− (3)	0.0102− (5)	0.0101− (4)	0.0102− (6)	0.0103− (7)	0.0105− (10)	0.0103− (8)	0.0104− (9)
10-Polygon	**0.0100** (1)	*0.0102*− (2)	0.0109− (3)	0.0111− (5)	0.0110− (4)	0.0112− (6)	0.0113− (7)	0.0113− (8)	0.0115− (9)	0.0116− (10)
15-Polygon	*0.0125* (2)	**0.0124**≈ (1)	0.0130− (3)	0.0136− (4)	0.0139− (6)	0.0139− (5)	0.0142− (7)	0.0142− (8)	0.0143− (9)	0.0145− (10)
3-RPolygon	*0.0038* (2)	**0.0038**≈ (1)	0.0039− (3)	0.0040− (4)	0.0042− (6)	0.0042− (5)	0.0044− (7)	0.0044− (8)	0.0045− (9)	0.0045− (10)
5-RPolygon	*0.0065* (2)	**0.0065**≈ (1)	0.0066− (3)	0.0068− (4)	0.0070− (5)	0.0072− (6)	0.0072− (7)	0.0075− (8)	0.0075− (9)	0.0076− (10)
8-RPolygon	**0.0088** (1)	*0.0089*≈ (2)	0.0092− (3)	0.0094− (4)	0.0096− (6)	0.0096− (5)	0.0099− (7)	0.0099− (8)	0.0101− (9)	0.0102− (10)
10-RPolygon	*0.0101* (2)	**0.0100**≈ (1)	0.0103− (3)	0.0106− (4)	0.0108− (5)	0.0110− (6)	0.0111− (7)	0.0113− (8)	0.0115− (9)	0.0116− (10)
15-RPolygon	*0.0125* (2)	**0.0122**+ (1)	0.0126≈ (3)	0.0130− (4)	0.0134− (5)	0.0136− (6)	0.0139− (7)	0.0142− (9)	0.0141− (8)	0.0144− (10)

Table S.5: Results of NIMMO with 10 values of μ on the 23 test problem instances. The mean IGDX values for 31 runs are shown.

	$\mu = 100$	$\mu = 200$	$\mu = 300$	$\mu = 400$	$\mu = 500$	$\mu = 600$	$\mu = 700$	$\mu = 800$	$\mu = 900$	$\mu = 1\,000$
Two-On-One	0.0306 (10)	0.0272+ (7)	0.0265+ (5)	*0.0261*+ (2)	0.0261+ (3)	**0.0260**+ (1)	0.0263+ (4)	0.0268+ (6)	0.0272+ (8)	0.0285+ (9)
Omni-test	1.9775 (10)	1.6067+ (8)	1.4730+ (4)	1.4717+ (3)	**1.4203**+ (1)	*1.4543*+ (2)	1.4998+ (5)	1.5452+ (6)	1.5901+ (7)	1.6260+ (9)
SYM-PART1	0.0978 (10)	0.0658+ (5)	**0.0602**+ (1)	*0.0603*+ (2)	0.0609+ (3)	0.0645+ (4)	0.0688+ (6)	0.0726+ (7)	0.0775+ (8)	0.0805+ (9)
SYM-PART2	0.1402 (10)	0.0920+ (5)	*0.0834*+ (2)	**0.0805**+ (1)	0.0855+ (3)	0.0871+ (4)	0.0923+ (6)	0.0976+ (7)	0.1023+ (8)	0.1127+ (9)
SYM-PART3	0.3436 (8)	**0.1467**+ (1)	0.1893+ (3)	*0.1566*+ (2)	0.2164+ (6)	0.3101≈ (7)	0.1935+ (4)	0.2150≈ (5)	0.4989− (10)	0.4898− (9)
MMF1	0.0987 (10)	0.0611+ (9)	0.0598+ (8)	0.0573+ (7)	0.0563+ (6)	0.0562+ (5)	0.0557+ (3)	0.0557+ (4)	*0.0554*+ (2)	**0.0553**+ (1)
MMF2	0.0633 (10)	0.0440+ (9)	0.0367+ (8)	0.0333+ (7)	0.0322+ (6)	0.0293+ (5)	0.0284+ (3)	0.0292+ (4)	*0.0270*+ (2)	**0.0269**+ (1)
MMF3	0.0718 (10)	0.0440+ (9)	0.0307+ (8)	0.0293+ (7)	0.0269+ (6)	0.0259+ (5)	*0.0228*+ (2)	0.0234+ (3)	**0.0227**+ (1)	0.0239+ (4)
MMF4	0.0802 (10)	0.0372+ (9)	0.0310+ (8)	0.0290+ (7)	0.0275+ (6)	0.0268+ (5)	0.0266+ (4)	0.0265+ (3)	**0.0263**+ (1)	*0.0263*+ (2)
MMF5	0.1287 (10)	**0.0846**+ (1)	0.0952+ (9)	0.0943+ (8)	0.0941+ (6)	0.0943+ (7)	*0.0932*+ (2)	0.0935+ (3)	0.0940+ (5)	0.0938+ (4)
MMF6	0.1560 (10)	0.1037+ (9)	0.1010+ (8)	0.0915+ (7)	0.0871+ (6)	0.0862+ (5)	0.0853+ (4)	*0.0838*+ (2)	0.0840+ (3)	**0.0822**+ (1)
MMF7	0.0351 (10)	**0.0216**+ (1)	0.0250+ (7)	*0.0245*+ (2)	0.0248+ (6)	0.0246+ (3)	0.0246+ (4)	0.0247+ (5)	0.0252+ (8)	0.0252+ (9)
MMF8	0.3325 (10)	0.2064+ (9)	0.1530+ (8)	0.1183+ (7)	0.1018+ (6)	0.0954+ (5)	0.0820+ (4)	0.0786+ (3)	*0.0737*+ (2)	**0.0713**+ (1)
3-Polygon	0.0084 (5)	**0.0082**+ (1)	*0.0083*+ (2)	0.0083≈ (3)	0.0084≈ (4)	0.0086− (6)	0.0087− (7)	0.0088− (8)	0.0089− (9)	0.0090− (10)
5-Polygon	0.0107 (3)	**0.0106**+ (1)	*0.0107*≈ (2)	0.0107≈ (4)	0.0109− (5)	0.0109− (6)	0.0110− (7)	0.0111− (8)	0.0112− (9)	0.0113− (10)
8-Polygon	0.0116 (3)	**0.0113**+ (1)	*0.0115*≈ (2)	0.0117− (4)	0.0118− (5)	0.0119− (6)	0.0119− (7)	0.0121− (8)	0.0121− (9)	0.0122− (10)
10-Polygon	*0.0115* (2)	**0.0114**+ (1)	0.0116− (3)	0.0119− (5)	0.0119− (4)	0.0120− (6)	0.0121− (7)	0.0121− (8)	0.0122− (9)	0.0123− (10)
15-Polygon	0.0119 (4)	**0.0116**+ (1)	*0.0118*+ (2)	0.0119≈ (3)	0.0121− (5)	0.0122− (6)	0.0122− (7)	0.0123− (8)	0.0124− (9)	0.0124− (10)
3-RPolygon	0.0087 (5)	**0.0083**+ (1)	*0.0083*+ (2)	0.0084+ (3)	0.0086≈ (4)	0.0087− (6)	0.0089− (7)	0.0091− (8)	0.0092− (9)	0.0093− (10)
5-RPolygon	0.0110 (5)	**0.0106**+ (1)	*0.0106*+ (2)	0.0108+ (4)	0.0108+ (3)	0.0110≈ (6)	0.0111≈ (7)	0.0113− (8)	0.0114− (9)	0.0115− (10)
8-RPolygon	0.0118 (6)	**0.0113**+ (1)	*0.0115*+ (2)	0.0116≈ (3)	0.0117≈ (4)	0.0118≈ (5)	0.0119− (7)	0.0121− (8)	0.0122− (9)	0.0123− (10)
10-RPolygon	0.0118 (4)	**0.0115**+ (1)	*0.0116*+ (2)	0.0117≈ (3)	0.0118≈ (5)	0.0119− (6)	0.0121− (7)	0.0122− (8)	0.0124− (9)	0.0125− (10)
15-RPolygon	0.0122 (6)	**0.0117**+ (1)	*0.0118*+ (2)	0.0119+ (3)	0.0120≈ (4)	0.0121≈ (5)	0.0123≈ (7)	0.0124− (8)	0.0125− (9)	0.0126− (10)

Table S.6: Results of NIMMO with 10 values of μ on the 23 test problem instances. The mean PSP values for 31 runs are shown.

	$\mu = 100$	$\mu = 200$	$\mu = 300$	$\mu = 400$	$\mu = 500$	$\mu = 600$	$\mu = 700$	$\mu = 800$	$\mu = 900$	$\mu = 1\,000$
Two-On-One	27.9 (10)	33.4+ (9)	35.0+ (7)	36.2+ (4)	*36.5*+ (2)	**37.1**+ (1)	36.2+ (3)	36.0+ (5)	35.4+ (6)	33.9+ (8)
Omni-test	0.3 (10)	0.5+ (9)	0.6+ (6)	0.6+ (4)	**0.6**+ (1)	*0.6*+ (2)	0.6+ (3)	0.6+ (5)	0.5+ (7)	0.5+ (8)
SYM-PART1	10.1 (10)	15.0+ (5)	*16.4*+ (2)	**16.5**+ (1)	16.2+ (3)	15.4+ (4)	14.4+ (6)	13.7+ (7)	12.8+ (8)	12.4+ (9)
SYM-PART2	6.7 (10)	10.4+ (6)	*11.6*+ (2)	**12.1**+ (1)	11.3+ (3)	11.1+ (4)	10.5+ (5)	9.9+ (7)	9.6+ (8)	8.7+ (9)
SYM-PART3	5.2 (8)	*9.0*+ (2)	8.3+ (4)	**9.1**+ (1)	8.7+ (3)	6.6≈ (7)	7.4+ (5)	6.9≈ (6)	3.7− (9)	3.6− (10)
MMF1	9.8 (10)	16.2+ (9)	16.5+ (8)	17.1+ (7)	17.5+ (6)	17.5+ (5)	17.7+ (4)	17.7+ (3)	**17.9**+ (1)	*17.8*+ (2)
MMF2	13.5 (10)	20.6+ (9)	27.5+ (8)	29.2+ (7)	29.9+ (6)	33.8+ (4)	34.6+ (3)	32.9+ (5)	**35.6**+ (1)	*35.5*+ (2)
MMF3	12.0 (10)	19.9+ (9)	29.3+ (8)	34.2+ (7)	36.2+ (6)	36.3+ (5)	*41.3*+ (2)	40.7+ (3)	**41.8**+ (1)	39.9+ (4)
MMF4	12.7 (10)	27.1+ (9)	32.3+ (8)	34.5+ (7)	36.2+ (6)	37.2+ (5)	37.4+ (4)	37.5+ (3)	**37.8**+ (1)	*37.7*+ (2)
MMF5	7.6 (10)	**11.7**+ (1)	10.4+ (9)	10.4+ (8)	10.5+ (6)	10.5+ (7)	*10.6*+ (2)	10.5+ (4)	10.5+ (5)	10.5+ (3)
MMF6	5.5 (10)	9.1+ (9)	9.6+ (8)	10.8+ (7)	11.3+ (6)	11.4+ (5)	11.5+ (4)	*11.8*+ (2)	11.7+ (3)	**12.0**+ (1)
MMF7	23.0 (10)	**43.0**+ (1)	38.1+ (9)	39.9+ (3)	38.4+ (8)	39.9+ (4)	*39.9*+ (2)	39.7+ (5)	39.0+ (7)	39.3+ (6)
MMF8	3.1 (10)	5.0+ (9)	6.6+ (8)	8.7+ (7)	9.9+ (6)	10.7+ (5)	12.4+ (4)	12.9+ (3)	*13.6*+ (2)	**14.0**+ (1)
3-Polygon	117.3 (5)	**122.0**+ (1)	*120.9*+ (2)	119.6+ (3)	118.5≈ (4)	116.7≈ (6)	114.8− (7)	113.6− (8)	111.7− (9)	110.1− (10)
5-Polygon	91.1 (6)	**93.7**+ (1)	*93.5*+ (2)	92.8+ (3)	91.9≈ (4)	91.3≈ (5)	90.4≈ (7)	89.6− (8)	89.2− (9)	88.4− (10)
8-Polygon	84.4 (5)	**87.5**+ (1)	*86.0*+ (2)	85.0≈ (3)	84.5≈ (4)	83.7≈ (6)	83.6≈ (7)	82.8− (8)	82.4− (9)	82.0− (10)
10-Polygon	84.5 (3)	**86.4**+ (1)	*85.4*+ (2)	83.7≈ (5)	83.8≈ (4)	83.2− (6)	82.8− (7)	82.4− (8)	81.7− (9)	81.2− (10)
15-Polygon	81.4 (6)	**84.4**+ (1)	*84.3*+ (2)	83.3+ (3)	82.4+ (4)	81.7≈ (5)	81.4≈ (7)	80.8≈ (8)	80.5− (9)	80.2− (10)
3-RPolygon	110.7 (6)	*117.5*+ (2)	**118.1**+ (1)	117.3+ (3)	114.1+ (4)	112.0≈ (5)	109.2− (7)	108.0− (8)	105.9− (9)	105.4− (10)
5-RPolygon	85.6 (9)	*91.0*+ (2)	**91.1**+ (1)	90.6+ (4)	90.7+ (3)	89.2+ (5)	88.4+ (6)	86.9≈ (7)	86.3≈ (8)	85.0≈ (10)
8-RPolygon	81.2 (9)	**85.9**+ (1)	*85.8*+ (2)	84.9+ (3)	84.0+ (4)	83.8+ (5)	82.8≈ (6)	82.1≈ (7)	81.5≈ (8)	80.8≈ (10)
10-RPolygon	78.6 (9)	81.9+ (3)	*82.3*+ (2)	**82.3**+ (1)	81.6+ (4)	81.3+ (5)	80.1+ (7)	80.2+ (6)	79.1≈ (8)	78.2≈ (10)
15-RPolygon	76.2 (10)	81.4+ (3)	**82.0**+ (1)	*81.4*+ (2)	80.5+ (4)	80.1+ (5)	80.1+ (6)	78.8+ (8)	78.8+ (7)	78.1+ (9)

Table S.7: Results of the four variants of NIMMO on the 23 test problem instances. The mean IGD values for 31 runs are shown.

	NIMMO-I_ϵ	NIMMO-I_{HD}	NIMMO-I_{R2}	NIMMO-SRA
Two-On-One	**0.0250** (1)	0.0274− (4)	0.0268− (3)	*0.0252*≈ (2)
Omni-test	*0.0769* (2)	0.1138− (4)	0.0978− (3)	**0.0715**≈ (1)
SYM-PART1	**0.0246** (1)	0.0336− (4)	0.0285− (3)	*0.0249*≈ (2)
SYM-PART2	*0.0365* (2)	0.0445− (4)	0.0412− (3)	**0.0335**≈ (1)
SYM-PART3	*0.0298* (2)	0.0429− (4)	0.0353− (3)	**0.0292**≈ (1)
MMF1	0.0050 (3)	*0.0050*≈ (2)	0.0122− (4)	**0.0040**+ (1)
MMF2	0.0216 (3)	*0.0195*≈ (2)	0.0538− (4)	**0.0174**+ (1)
MMF3	*0.0190* (2)	0.0212≈ (3)	0.0579− (4)	**0.0153**+ (1)
MMF4	0.0048 (3)	**0.0037**+ (1)	0.0059− (4)	*0.0041*+ (2)
MMF5	*0.0039* (2)	0.0041− (3)	0.0084− (4)	**0.0036**+ (1)
MMF6	0.0044 (3)	*0.0043*≈ (2)	0.0067− (4)	**0.0037**+ (1)
MMF7	0.0030 (3)	**0.0028**+ (1)	0.0032− (4)	*0.0028*+ (2)
MMF8	0.0092 (3)	**0.0053**+ (1)	0.0220− (4)	*0.0063*+ (2)
3-Polygon	**0.0025** (1)	0.0028− (4)	0.0027− (3)	*0.0025*≈ (2)
5-Polygon	*0.0044* (2)	0.0076− (4)	0.0049− (3)	**0.0044**≈ (1)
8-Polygon	**0.0069** (1)	0.0208− (4)	0.0084− (3)	*0.0071*− (2)
10-Polygon	**0.0064** (1)	0.0289− (4)	0.0080− (3)	*0.0066*− (2)
15-Polygon	**0.0106** (1)	0.0610− (4)	0.0146− (3)	*0.0108*− (2)
3-RPolygon	*0.0025* (2)	0.0028− (4)	0.0027− (3)	**0.0025**≈ (1)
5-RPolygon	**0.0044** (1)	0.0080− (4)	0.0050− (3)	*0.0044*≈ (2)
8-RPolygon	**0.0069** (1)	0.0211− (4)	0.0084− (3)	*0.0071*− (2)
10-RPolygon	**0.0064** (1)	0.0294− (4)	0.0080− (3)	*0.0065*− (2)
15-RPolygon	**0.0105** (1)	0.0613− (4)	0.0145− (3)	*0.0108*− (2)

Table S.8: Results of the four variants of NIMMO on the 23 test problem instances. The mean IGDX values for 31 runs are shown.

	NIMMO-I_ϵ	NIMMO-I_{HD}	NIMMO-I_{R2}	NIMMO-SRA
Two-On-One	**0.0226** (1)	0.0256− (4)	0.0242− (3)	*0.0239*− (2)
Omni-test	**1.6052** (1)	1.6222≈ (3)	1.6346≈ (4)	*1.6140*≈ (2)
SYM-PART1	**0.0530** (1)	0.0797− (4)	0.0659− (3)	*0.0545*≈ (2)
SYM-PART2	**0.0838** (1)	0.1166− (4)	0.0984− (3)	*0.0869*≈ (2)
SYM-PART3	**0.1418** (1)	0.2667− (4)	*0.2155*≈ (2)	0.2381≈ (3)
MMF1	0.0611 (3)	*0.0596*≈ (2)	0.1212− (4)	**0.0537**+ (1)
MMF2	0.0440 (3)	*0.0414*≈ (2)	0.1156− (4)	**0.0394**+ (1)
MMF3	*0.0440* (2)	0.0498≈ (3)	0.1187− (4)	**0.0381**+ (1)
MMF4	0.0372 (3)	**0.0271**+ (1)	0.0530− (4)	*0.0319*+ (2)
MMF5	*0.0846* (2)	0.0872− (3)	0.1436− (4)	**0.0795**+ (1)
MMF6	0.1037 (3)	*0.1031*≈ (2)	0.1668− (4)	**0.0897**+ (1)
MMF7	0.0216 (3)	**0.0199**+ (1)	0.0224≈ (4)	*0.0208*≈ (2)
MMF8	0.2064 (3)	**0.1404**+ (1)	0.4220− (4)	*0.1524*+ (2)
3-Polygon	**0.0056** (1)	0.0059− (4)	0.0057− (3)	*0.0056*≈ (2)
5-Polygon	**0.0070** (1)	0.0109− (4)	0.0082− (3)	*0.0073*− (2)
8-Polygon	**0.0089** (1)	0.0171− (4)	0.0105− (3)	*0.0093*− (2)
10-Polygon	**0.0072** (1)	0.0185− (4)	0.0090− (3)	*0.0075*− (2)
15-Polygon	**0.0100** (1)	0.0267− (4)	0.0127− (3)	*0.0104*− (2)
3-RPolygon	**0.0059** (1)	0.0063− (4)	0.0060− (3)	*0.0059*≈ (2)
5-RPolygon	**0.0074** (1)	0.0114− (4)	0.0085− (3)	*0.0076*− (2)
8-RPolygon	**0.0093** (1)	0.0185− (4)	0.0108− (3)	*0.0095*− (2)
10-RPolygon	**0.0076** (1)	0.0186− (4)	0.0092− (3)	*0.0079*− (2)
15-RPolygon	**0.0104** (1)	0.0267− (4)	0.0130− (3)	*0.0107*− (2)

Table S.9: Results of the four variants of NIMMO on the 23 test problem instances. The mean PSP values for 31 runs are shown.

	NIMMO-I_ϵ	NIMMO-I_{HD}	NIMMO-I_{R2}	NIMMO-SRA
Two-On-One	**40.09** (1)	34.31− (4)	36.42− (3)	*38.07*− (2)
Omni-test	*0.49* (2)	0.46− (4)	0.48≈ (3)	**0.50**≈ (1)
SYM-PART1	**18.73** (1)	12.44− (4)	15.13− (3)	*18.21*≈ (2)
SYM-PART2	**11.47** (1)	8.19− (4)	9.72− (3)	*11.08*≈ (2)
SYM-PART3	**9.48** (1)	7.08− (4)	7.92≈ (3)	*8.90*≈ (2)
MMF1	16.22 (3)	*16.56*≈ (2)	7.05− (4)	**18.43**+ (1)
MMF2	20.61 (3)	*21.98*≈ (2)	6.07− (4)	**24.45**≈ (1)
MMF3	*19.90* (2)	18.57≈ (3)	5.55− (4)	**25.99**+ (1)
MMF4	27.06 (3)	**37.18**+ (1)	19.14− (4)	*31.62*+ (2)
MMF5	*11.68* (2)	11.25− (3)	6.03− (4)	**12.41**+ (1)
MMF6	9.09 (3)	*9.40*≈ (2)	4.63− (4)	**10.96**+ (1)
MMF7	43.02 (3)	**48.18**+ (1)	41.35≈ (4)	*45.44*≈ (2)
MMF8	4.97 (3)	**7.21**+ (1)	2.48− (4)	*6.94*+ (2)
3-Polygon	**178.71** (1)	167.04− (4)	175.28− (3)	*176.92*− (2)
5-Polygon	**141.30** (1)	82.28− (4)	119.50− (3)	*135.70*− (2)
8-Polygon	**111.35** (1)	47.22− (4)	91.13− (3)	*105.98*− (2)
10-Polygon	**138.23** (1)	41.63− (4)	107.49− (3)	*132.57*− (2)
15-Polygon	**97.52** (1)	25.13− (4)	71.92− (3)	*93.66*− (2)
3-RPolygon	**166.54** (1)	148.98− (4)	161.80− (3)	*165.98*≈ (2)
5-RPolygon	**132.04** (1)	76.51− (4)	109.03− (3)	*126.73*− (2)
8-RPolygon	**104.28** (1)	46.06− (4)	86.49− (3)	*101.58*− (2)
10-RPolygon	**126.33** (1)	41.82− (4)	98.94− (3)	*120.32*− (2)
15-RPolygon	**90.97** (1)	27.25− (4)	67.25− (3)	*86.91*− (2)