定量的マクロ経済学 a 後半課題問 1

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1.
異質的な個人を含むモデルとその均衡を定義する。

【Households】
Households solve following dynamic optimization problem.

$$\max_{\{c_{it}\},\{a_{it+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}) \; s.t.$$
$$c_{it} + a_{it+1} = (1 + r - r\tau_k)a_{it} + wh_{it} + T$$
$$a_{it+1} \geq -\underline{B}, a_{i0} \; given$$

【Firms】
All the firms have following same production function.

$$Y = F(K, H)$$

Profit is represented as below.

$$\max_{K,H} F(K, H) - (r + \delta)K - wH$$

【Markets】
Wage is w, rent is r. Goods' price is normalized 1 in steady state.

【Government】
Government imposes capital income tax with rate τ_k and rebate it as lump-sum transfer T.

【Stationary competitive equilibrium】
A stationary CE with policy is a list of functions $V(a,h), g_a(a,h), K, H, r, w, \mu(a,h), T \; s.t.$
1.(Household optimization)
Taking r and w as given, $V(a,h)$ solves

$$V(a,h) = \max_{a'} u\big((1 + r - r\tau_k)a + wh + T - a'\big) + \beta \sum_{h'} V(a', h')\pi(h'|h) \text{ s.t.}$$

$$-\underline{B} \leq a' \leq (1 + r - \tau_k r)a + wh + T$$

and $g_a(a,h)$ is an optimal decision rule.

2.(Firm optimization)

Taking r and w as given, K and H solve firms problem

$$\max_{k,h} F(k,h) - (r+\delta)k - wh \text{ s.t. } k \geq 0, h \geq 0.$$

3.(Government)

$$\tau_k r K = T$$

4.(Market clearing)

(1) Labor $H = \sum_h h\pi^*(h)$

(2) Assets $K = \sum_a \sum_h g_a(a,h)\mu(a,h)$

(3) Goods $F(K,H) = \sum_a \sum_h \big((1+r-\tau_k r)a + wh + T - g_a(a,h)\big)\mu(a,h) + \delta K$

5.(Aggregate law of motion)

Distribution of agents over states μ is stationary

$$\mu(a',h') = \sum_a \sum_h \mathbf{1}\{a: g_a(a,h) \in a'\}\pi(h'|h)\mu(a,h).$$