Network Thresholds and Multiple Equilibria in the Diffusion of Content-based Platforms. *

Ryo Suzuki †

1 Introduction

This paper provides a dynamic economic model which explains adoption process of an online platform. We examine a effect of specific network externality on users' behaviors and find that a slight difference in the initial condition may cause huge difference in a equilibrium. Our model suggests there exist multiplicity equilibria in a social network platform. We identify the scale up condition called a critical value, and we also prove that if the initial quality of the platform exceed the critical value, the number of adopted population reach a greater equilibrium. Moreover, we develop a social networking platform service with Facebook Graph API in order to compare a simulation from the model to a real data. As a result, the simulation based on our model can predict better than the previous models. Finally, we discuss the open questions and the direction for the future research.

1.1 The problem discussed in the paper

In this paper, we work on the following research question: why a social network platform grows so rapidly while the others usually fail to attract potential users. It is commonly said that the diffusion theory can answer this kind of question. Certainly, it is possible to explain so-called successful "S-shaped curve" with the model provided in the previous papers. However, the problem is that the existing models cannot explain a failure of scaling up in the same framework. In related research, they usually deal with the diffusion process of innovations such as a new product and an agricultural technology. We hypothesize that when we consider the diffusion mechanism of an online social network platform, we have to take into account two kinds of network effects: user based network effects and content based network effects, and we think the latter one plays an important role in an online platform. Therefore, we attempt to solve this problem by developing the model that is able to suggest the multiple equilibria.

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†University of Tokyo Graduate School of Economics, Email: 1253852881@mail.ecc.u-tokyo.ac.jp

Our main goal is predicting the actual adoption curve more accurately with the dynamic model and initial parameters.

1.2 The approach and results

The novelty of the approach and results is twofold. First, we consider not only effects of the amount of users but also the effects of the amount of contents in an online service as the factor of network effects. And we find that the latter effect is a crucial factor of determining the equilibrium. As a result, we show that a slight difference in the initial condition may result in the huge different in equilibria. This multiplicity of equilibria suggests that unlike the previous model, our model is able to predict not only a path of traditional S-shaped curve but also a path of failure in scaling up.

Second, we test our model with the time series user data from an social network application available on iPhone. We develop and distribute the application via App Store and gather the anonymous user data in 5 months. The simulation results show that our model can give a better prediction of the actual user data than the previous one.

1.3 The related works

The diffusion theory have been studied in the field of sociology [10], marketing [1], [6], [13] and economics [12], [4], [5], [7]. In these related works, [15] propose the simple dynamic model that investigate the diffusion process from the viewpoint of economics, and explain the time series data of agricultural innovation presented in [11]. [8] and [9] study the economics of a platform as a "two-sided markets". The incentive of knowledge sharing in virtual communities have been investigated in [2] and [3]. [14] also studies a critical mass from the aspect of the applied physics.

2 The Model

Consider the infinite many users on $[0, 1]$ and they will be adopted by a social network service. Let $m_t \in [0, 1]$ be the population of adopted users at time t. We assume that a user's decision whether he or she adopts or not depends on the proportion of adopted users. For each user i, we define $r_i \geq 0$ as a threshold for user i, in other words, a minimum proportion that user i get adopted. For instance, $r_i = 0.5$ means that user i get adopted if at least half of all users have been adopted. ($r_i > 1$ means that the user will be never adopted.) Let $f(r)$ and $F(r)$ be the distribution function and the cumulative distribution function of thresholds respectively. This determines the dynamics of adopted population.

Next, let us consider a quality of the social network service. We assume the quality depends on contents provided by adopted users. Let q_t, c, and β be the amount of the quality, the amount of contents provided per capita, and depressing rate of quality. The quality of social network service which affects the users' behaviors. Hence, we think the threshold function $F(\cdot)$ is a conditional function of q_t.

The dynamics of the adopted population and the quality of the service is given by the following dynamic equations.

$$\dot{m_t} = \lambda(F(m_t|q_t) - m_t)$$
$$\dot{q_t} = cm_t - \beta q_t$$

where $\lambda \in (0, 1)$ represents a speed of adoption.

And given these differential equations, the diffusion path $\{m_t, q_t\}_{t=0}^{\infty}$ is determined uniquely when the initial condition (m_0, q_0) is chosen. In the following, we assume that if $q' > q$ holds, then q' has the first order stochastic dominance over q.

Definition 1. *The equilibrium (m^*, q^*) is defined as $m^* = \lim_{t\to\infty} m_t$ and $q^* = \lim_{t\to\infty} q_t$*

Definition 2. *Critical value of quality is defined as the level of quality $\bar{q}$ such that if $q_0 \in [0, \bar{q})$ then $\lim_{t\to\infty} m_t = 0$ and if $q_0 \in (\bar{q}, \infty)$ then $\lim_{t\to\infty} m_t = m^*$ $(m^* > 0)$*

Proposition 1. *Given c, β, $F(\cdot, q)$ and two differential equations. If $F(0|0) = 0$ is satisfied. And if there exists $\hat{q} \in [0, \infty)$ such that $(m_0, q_0) = (0, \hat{q})$ and $\lim_{t\to\infty}(m_t, q_t) = (m^*, q^*) \gg (0, 0)$. Then there exists a critical value of quality $\bar{q}$.*

Proposition 2. *If $c/\beta > a$ and $\lambda(c/\beta - a)^2 \geq ca$ hold, then there exists a critical value of quality $\bar{q} \in [a, c/\beta]$.*

Proposition 3. *If the following condition is satisfied, then there exists $(m^*, q^*) \neq (0, 0)$ such that $\dot{m} = \lambda(F(m^*, q^*) - m^*) = 0$ and $\dot{q} = cm^* - \beta q^* = 0$ are hold.*

$$\frac{c}{\beta} \geq \min_{(\bar{m}, \bar{q}) \in A} \left[\frac{1 - \frac{\partial F(\bar{m}|\bar{q})}{\partial m}}{\frac{\partial F(\bar{m}|\bar{q})}{\partial q}} \right]$$

where $A = \{(\bar{m}, \bar{q}) \in [0, 1] \times [0, \infty) \mid F(\bar{m}|\bar{q}) - \frac{\partial F(\bar{m}|\bar{q})}{\partial m}\bar{m} - \frac{\partial F(\bar{m}|\bar{q})}{\partial q}\bar{q} = 0\}$.

3 The Data

We design iOS application by ourselves and collect data from users' activities on our iOS application with Facebook Graph API [1]. We started from June 21st to November 21st and gather data in 5 months. We define "active at time t" as user who click like button and post data to server at time t.

To compare actual data of the application to our model we simulate the model in the following settings. In the simulation we assume $F(\cdot|q)$ as normal

[1] https://developers.facebook.com

distribution $N(0.1/q, 0.2)$ and $c = 1.0$, $\beta = 0.1$. So the dynamics is given by this.

$$m_{t+1} = 0.6m_t + 0.4 \int_{-\infty}^{m_t} \frac{1}{\sqrt{2\pi}\ 0.2} e^{-\frac{(x-\frac{0.1}{q_t})^2}{0.08}} dx$$
$$q_{t+1} = 1.0m_t + 0.9q_t$$

Simulation result can be seen in Figure.2 when initial condition $(m_0, q_0) = (0, 0.1)$. Left hand side of Figure.2 is dynamics of m_t and right hand side is dynamics of q_t. On the other hand, the actual data from our application also can be seen in Figure. 3. Each graph of Figure.3 represents time series data of active user and quality.

4 Conclusion

We show that there are three factors to determine the result: the quality of initial condition, the provided average contents per capita and rate of diminishing the value of current contents. We suggest that the effect of the amount of content on users' behaviors plays an important role as well, though his kind of network effect has not been fully investigated before.

The mechanism can be illustrated as the followings. The more users the platform has, the more contents platforms can accumulate and the higher the quality will be. Hence, once this positive feedback loop happens, the platforms keep attractive in the future and finally reach the good equilibrium. On the other hand, there is also negative feedback resultant from the network externalities. If the quality is not so high, the speed of reduce its quality is faster than the that of increasing of accumulation. Even though it has some quality at the beginning, once the negative loop occurs platforms cannot increase their users. In summary, the contribution of the paper is that we provide a better tool to understand the a mechanism behind the diffusion process of platforms.

4.1 Open questions and future research

The remaining open question is about the competition between two platforms. It is common that more than two platforms compete for their market share in the reality. The most famous and interesting example is the competition between Facebook and MySpace in early 2000. However, we do not illustrate the situation where multiple platforms exists. Therefore, an important extension for the future research is developing this kind of multiple platforms model.

In addition to the extension for competing, it is also important to consider that microeconomic foundation for the threshold distribution. In the model, we assume the normal distribution as a threshold distribution because of the complexity of calculation. However, we acknowledge that we should consider the microeconomic foundation which represents the shape of threshold distribution in the future. Thus, future research will also involve developing a general theory which explain the relationship between the threshold function and the utility function.

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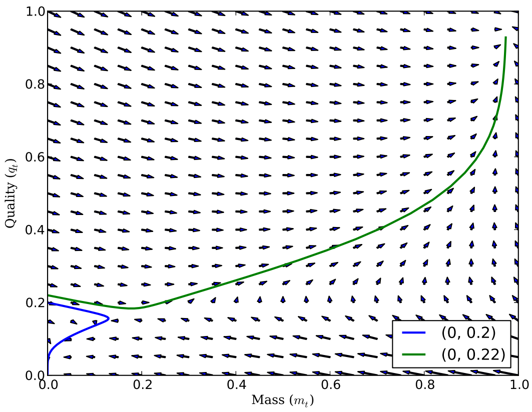

Figure 1: Multiple Equilibria in Phase Diagram: diagram of m_t (horizontal) and q_t (vertical) when normal distribution $N(0.1/q, 0.2)$, $\lambda = 0.4$, $c = 0.1$ and $\beta = 0.1$. Blue line and green line represent path of the initial condition $(m_0, q_0) = (0, 0.2)$ and path of the initial condition $(m_0, q_0) = (0, 0.22)$ respectively.

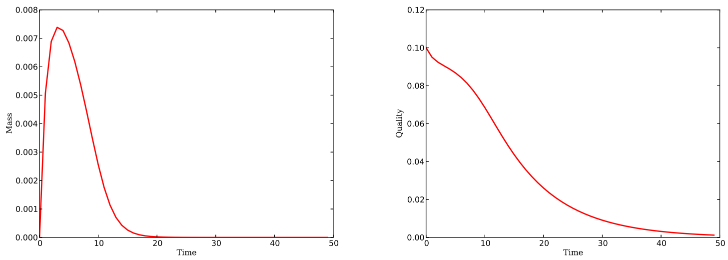

Figure 2: Simulation: dynamics of mass (left) and quality (right) when $N(0.1/q_t, 0.2)$, $\lambda = 0.4$, $c = 1.0$, $\beta = 0.1$, and $q_0 = 0.1$

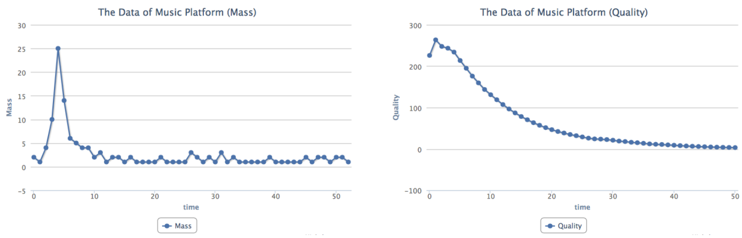

Figure 3: Actual Data: active user data (left) and amount of contents provided users assuming $\beta = 0.1$ (right) in the App from June, 21 to November, 21