DTL Written Test 2018 - Intern

September 10, 2018

Declaration:
1. Please DO NOT disclose this written test to any third party.
2. Please use only pen and paper for working out the questions. You may only use computers to key in the steps/answers for submission.
3. Please write down the detailed steps for getting the final answer. Simply writing down the final answer without steps will get 0 marks.

1. (10p) Re-order the numbers 1 to 16 into $a_1, \ldots, a_{16}$, such that $a_i + a_{i+1}$ is a square number for $i = 1, \ldots, 15$. Write all the possible answers.

2. (15p) In the future, you will have 6 exams in 18 days. Now they are still not scheduled. Assumption:

   (a) Exams can be in morning or afternoon.

   (b) There is no overlapping of exams.

   (c) The arrangement of exams is totally random.

   What's the probability that the 6 exams will be scheduled in 4 days?

3. (15p) Find all primes p such that $2^p + p^2$ is also prime. Prove there are no others.

4. (20p) Person A and B keep tossing a fair coin. A stops until he gets 2 consecutive heads; B stops until he gets 3 consecutive heads. What's the probability that number of tosses by A is less than the number of tosses by B.

5. (20p) Given 3 weightless planks with length 1m. For each plank, we stick gold bar of 2kg and 3kg to either end (the gold bar is very small comparing to the plank). We now put 3 planks on the edge of the table and each of them should be stable (can not fall down). How distant could the farthest end reach outside the table? For example, for one plank, the farthest distance is 3/5m.

6. (20p) Let $P(n)$ denote the number of partitions of n. For example, $P(4) = 5$, because $4 = 4$; $4 = 1 + 3$; $4 = 2 + 2$; $4 = 1 + 1 + 2$; $4 = 1 + 1 + 1 + 1$. Find all positive intergers n, that satisfy $P(n) + P(n + 4) = P(n + 2) + P(n + 3)$.