

DTL Written Test 2018 - Intern

October 22, 2018

Declaration:

1. Please DO NOT disclose this written test to any third party.
2. Please use only pen and paper for working out the questions. You may only use computers to key in the steps/answers for submission.
3. Please write down the detailed steps for getting the final answer. Simply writing down the final answer without steps will get 0 marks.

1. (10p) Re-order the numbers 1 to 16 into a_1, \dots, a_{16} , such that $a_i + a_{i+1}$ is a square number for $i = 1, \dots, 15$. Write all the possible answers.
2. (15p) In the future, you will have 6 exams in 18 days. Now they are still not scheduled. Assumption:
 - (a) Exams can be in morning or afternoon.
 - (b) There is no overlapping of exams.
 - (c) The arrangement of exams is totally random.

What's the probability that the 6 exams will be scheduled in 4 days?

3. (15p) Find all primes p such that $2^p + p^2$ is also prime. Prove there are no others.
4. (15p) Two sequences a_n and b_n ($n \geq 1$) match the following requirements:
 - (1) All the elements are positive integers, and there are both monotonically increasing (i.e. for any i , $a_i < a_{i+1}$, $b_i < b_{i+1}$),
 - (2) $a_9 = b_9 < 2018$,
 - (3) $b_{i+1} = 2 * b_i$,
 - (4) $a_{i+2} = a_{i+1} + a_i$.

Please find out all the possible sequences.

5. (20p) If the citizens of a country stand in a circle (citizen number > 2). Everyone has some coins. If the government finds a person has more coins than the sum of his two neighbors, it will take one coin from the rich person and distribute two coins to his two neighbors (each gets one). The government will randomly select one rich person if there are more than one rich persons. Do you think there is a probability the government need to spend infinite coins?
6. (25p) Given 3 weightless planks with length 1m. For each plank, we stick gold bar of 2kg and 3kg to either end (the gold bar is very small comparing to the plank). We now put 3 planks on the edge of the table and each of them should be stable (can not fall down). How distant could the farthest end reach outside the table? For example, for one plank, the farthest distance is $3/5$ m.