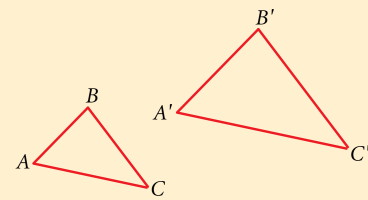
In the Activity below, you can see a connection between the concepts of geometric similarity, proportion, and direct variation.

CONNECTION
GEOMETRY

Activity
Exploring Similarity and Direct Variation

You will need: a calculator

Recall from geometry that *similar* figures have the same shape. This means that the corresponding angles of similar polygons are congruent, and their corresponding sides are proportional.



1. Copy and complete the table below to compare the lengths of the sides in $\triangle A'B'C'$ with the corresponding lengths in $\triangle ABC$.

Length in $\triangle ABC$	Length in $\triangle A'B'C'$	Ratio of $\triangle A'B'C'$ to $\triangle ABC$
$AB = 16$	$A'B' = 24$	$\frac{A'B'}{AB} = ?$
$BC = 20$	$B'C' = 30$	$\frac{B'C'}{BC} = ?$
$AC = 24$	$A'C' = 36$	$\frac{A'C'}{AC} = ?$

CHECKPOINT ✔ **2.** Do your calculations in the third column indicate a direct-variation relationship between the lengths of the sides of $\triangle A'B'C'$ and those of $\triangle ABC$? Explain your response.

It is said that if y varies directly as x, then y is *proportional* to x.

A **proportion** is a statement that two *ratios* are equal. A ratio is the comparison of two quantities by division. A proportion of the form $\frac{a}{b} = \frac{c}{d}$ can be rearranged as follows:

$$\frac{a}{b} = \frac{c}{d}$$
$$\frac{a}{b} \cdot bd = \frac{c}{d} \cdot bd$$
$$ad = bc$$

The result is called the *Cross-Product Property of Proportions*.

Cross-Product Property of Proportions

For $b \neq 0$ and $d \neq 0$:
If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

In a proportion of the form $\frac{a}{b} = \frac{c}{d}$, a and d are the *extremes* and b and c are the *means*. By the Cross-Product Property, the product of the extremes equals the product of the means.

EXAMPLE 2 **Find the inverse of each relation. State whether the relation is a function. State whether the inverse is a function.**

a. $\{(1, 2), (2, 4), (3, 6), (4, 8)\}$ **b.** $\{(1, 5), (1, 6), (3, 6), (4, 9)\}$

SOLUTION

a. relation: $\{(1, 2), (2, 4), (3, 6), (4, 8)\}$
inverse: $\{(2, 1), (4, 2), (6, 3), (8, 4)\}$

The given relation is a function because each domain value is paired with exactly one range value. The inverse is also a function because each domain value is paired with exactly one range value.

b. relation: $\{(1, 5), (1, 6), (3, 6), (4, 9)\}$
inverse: $\{(6, 1), (5, 1), (6, 3), (9, 4)\}$

The given relation is not a function because the domain value 1 is paired with two range values, 5 and 6. The inverse is not a function because the domain value 6 is paired with two range values, 1 and 3.

Example 3 shows you how to find the inverse of a function by interchanging x and y and then solving for y.

EXAMPLE 3 **Find an equation for the inverse of $y = 3x - 2$.**

SOLUTION

In $y = 3x - 2$, interchange x and y. Then solve for y.

$$\begin{aligned} x &= 3y - 2 \\ x + 2 &= 3y \\ \frac{x+2}{3} &= y \\ y &= \frac{1}{3}x + \frac{2}{3} \end{aligned}$$

TRY THIS Find an equation for the inverse of $y = 4x - 5$.

In the Activity below, you can explore the relationship between the graph of a function and the graph of its inverse.

Activity
Exploring Functions and Their Inverses

TECHNOLOGY
GRAPHICS CALCULATOR
Keystroke Guide, page 151

You will need: a graphics calculator

1. Graph $y = 2x - 1$, its inverse, and $y = x$ in a square viewing window. Use the inverse feature of the calculator. How do the graphs of these functions relate to one another? Consider symmetry in your response.
2. Repeat Step 1 for each function listed at right.
3. Write a generalization about the relationship between the graph of a function and the graph of its inverse. **CHECKPOINT ✔**

Function
$y = 3x - 2$
$y = 3x + 2$
$y = -2x + 5$
$y = x^2$

EXAMPLE 1 **Refer to the lawn-care problem described at the beginning of the lesson.**

a. Use the row-reduction method to solve the system.

b. Find the hourly wages for Maya, Amit, and Nina. Then find the total amount that each partner earns for this job.

APPLICATION
SMALL BUSINESS

SOLUTION

System

$$\begin{cases} m + a + n = 21 \\ 2m + a = 23 \\ a + 3n = 25 \end{cases}$$

Augmented Matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 21 \\ 2 & 1 & 0 & 23 \\ 0 & 1 & 3 & 25 \end{array}\right]$$

a. Perform row operations.

- Inspect column 1.

The first row begins with 1, but the 2 in the second row needs to become 0.

$-2R_1 + R_2 \rightarrow R_2$

Replace row 2 with the sum of row 2 and –2 times row 1.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 21 \\ 0 & -1 & -2 & -19 \\ 0 & 1 & 3 & 25 \end{array}\right]$$

- Inspect column 2.

Row 1:
Change the entry to 0.

$R_2 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & -1 & -2 & -19 \\ 0 & 1 & 3 & 25 \end{array}\right]$$

Row 2:
Change the entry to 1.

$-1R_2 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 19 \\ 0 & 1 & 3 & 25 \end{array}\right]$$

Row 3:
Change the entry to 0.

$-1R_2 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 19 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

- Inspect column 3.

Row 1:
Change the entry to 0.

$R_3 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 2 & 19 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

Row 2:
Change the entry to 0.

$-2R_3 + R_2 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

The matrix is now in reduced row-echelon form.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 6 \end{array}\right] \qquad \begin{array}{l} m = 8 \\ a = 7 \\ n = 6 \end{array}$$

b. Maya receives \$8 an hour; since she works 3 hours, she will earn \$24. Amit receives \$7 an hour and works 3 hours, so he will earn \$21. Nina receives \$6 an hour and works 4 hours, so she will earn \$24.

CHECKPOINT ✔ Check the solution to Example 1 by using substitution.

5.8 Solving Quadratic Inequalities

Why *You can solve many real-world problems, such as those involving business profits related to revenue and cost, by solving quadratic inequalities.*

Objectives

- Write, solve, and graph a quadratic inequality in one variable.
- Write, solve, and graph a quadratic inequality in two variables.

APPLICATION
SMALL BUSINESS

Katie makes and sells T-shirts. A consultant found that her monthly costs, C, are related to the selling price, p, of the shirts by the function $C(p) = 75p + 2500$. The revenue, R, from the sale of the shirts is represented by $R(p) = -25p^2 + 700p$. Her profit, P, is the difference between the revenue and the costs each month.

$$\begin{aligned} P(p) &= R(p) - C(p) \\ &= -25p^2 + 700p - (75p + 2500) \\ &= -25p^2 + 625p - 2500 \end{aligned}$$

For what range of prices can Katie sell the shirts in order to make a profit? That is, for what values of p will $-25p^2 + 625p - 2500 > 0$? *You will answer this question in Example 2.*

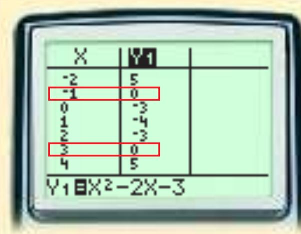
One-Variable Quadratic Inequalities

TECHNOLOGY
GRAPHICS CALCULATOR
Keystroke Guide, page 351

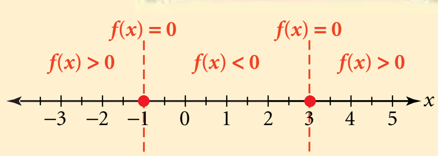
Activity Exploring Quadratic Inequalities

You will need: a graphics calculator

The display at right shows the values of $f(x) = x^2 - 2x - 3$ for integer values of x between -2 and 4 inclusive.



The table suggests the following three cases:
- When $x = -1$ or $x = 3$, $f(x) = 0$.
- When $x < -1$ or $x > 3$, $f(x) > 0$.
- When $-1 < x < 3$, $f(x) < 0$.



LESSON 6.5

EXAMPLE 1 Page 387

Graph $y = 10 \log x$, and evaluate y for $x = 300$.

Use viewing window [−50, 350] by [−10, 50].

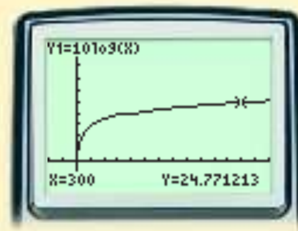
Graph the function:

[Y=] 10 [LOG] [X,T,Θ,n] [)] [GRAPH]

TI-82: [(]

Find the point on the graph where $x = 300$:

[2nd] [TRACE] (CALC) [1: value] [ENTER] (X=) 300 [ENTER]



EXAMPLE 3 Page 387

Solve $5^x = 62$ by graphing.

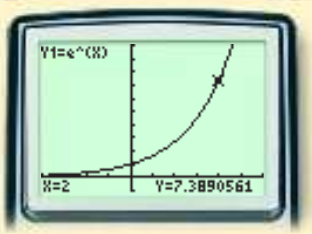
Use viewing window [−1, 5] by [−40, 100].

Graph the related equations, and use a keystroke sequence similar to that in Example 4 of Lesson 6.3 to find any points of intersection.

LESSON 6.6

EXAMPLES 1 and 3 Pages 393 and 394

For part a of Example 1, evaluate $y = e^x$ to the nearest thousandth for $x = 2$.



Use viewing window [−2, 3] by [−1, 10].

Enter the equation: [Y=] [2nd] [LN] (e^x) [X,T,Θ,n] [)] [ENTER]

TI-82: [(]

Create a table of values:
Use a keystroke sequence similar to that in the Activity in Lesson 6.3. Use TblStart = −1 and △Tbl = 0.5.

Find the point on the graph where $x = 2$:
Use a keystroke sequence similar to that in Example 1 of Lesson 6.5.

For parts **b** and **c** of Example 1, use a similar keystroke sequence.

For Example 3, use viewing window [−2, 10] by [−2, 4] and a similar keystroke sequence.

EXAMPLE 4 Page 395

Solve $2 = e^{0.085x}$ by graphing.

Use viewing window [−20, 100] by [−3, 10].

Graph the related equations:

[Y=] 2 [ENTER] (Y2=) [2nd] [LN] (e^x) .085 [X,T,Θ,n] [)] [ENTER]

TI-82: [(]

Find the point of intersection:
Use a keystroke sequence similar to that in Example 4 of Lesson 6.3.

Solve each equation by factoring. ***(LESSON 5.2)***

72. $x^2 + 12x + 11 = 0$

73. $x^2 - 2x - 15 = 0$

74. $x^2 + 14x + 48 = 0$

75. $x^2 + 17x + 72 = 0$

76. Give an example of a perfect-square trinomial and then write it as a binomial squared. ***(LESSON 5.3)***

Use the quadratic formula to solve each equation. ***(LESSONS 5.5 AND 5.6)***

77. $3x^2 + 7x + 2 = 0$

78. $-2x^2 + 4x + 5 = 0$

79. $5x^2 + 2x + 4 = 0$

80. $-6x^2 + 5x - 4 = 0$

internet connect

Portfolio Extension

Go To: **go.hrw.com**
Keyword:
MB1 Polynomials

Look Beyond

81. Sketch the graph of a cubic polynomial that intersects the x-axis at exactly the number of points indicated. Write *impossible* if appropriate.
a. 3 points **b.** 2 points **c.** 1 point **d.** 0 points

PORTFOLIO ACTIVITY

The bottle below has a circular base, a flat bottom, and curved sides. There is no geometric formula for the volume of a solid with this shape. In this activity, you will find a polynomial function that models the volume of a solid with this shape.

1. Obtain a bottle with a circular base, a flat bottom, and curved sides. Measure the diameter of the circular base in centimeters. Calculate the radius and the area of the circular base ($A = \pi r^2$).
2. Pour water into the bottle until the bottle is approximately half full. Place a cap on the bottle, and measure the height, h_1, in centimeters of the water. The volume in milliliters of the part of the bottle containing water, W, can be approximated by $W(r) = \pi r^2 h_1$. Calculate the approximate volume of this part of the bottle.

3. Turn the bottle upside-down, and measure the height, h_2, in centimeters of the air space above the water. The volume of the air space, A, in milliliters can be approximated by $A(r) = \pi r^2 h_2$. Calculate the approximate volume of the air space.

4. The total volume in milliliters of the bottle, V, can be modeled by the function below. Find the total volume of the bottle.

$$\begin{aligned} V(r) &= W(r) + A(r) \\ &= \pi r^2 h_1 + \pi r^2 h_2 \end{aligned}$$

WORKING ON THE CHAPTER PROJECT

You should now be able to complete Activity 1 of the Chapter Project.

Using Combinations and Probability

Recall from Lesson 10.1 that you can find the probability of event A by using the following ratio:

$$P(A) = \frac{\text{number of outcomes in event } A}{\text{number of outcomes in the sample space}}$$

In many situations, you can find and evaluate the numerator and the denominator by applying the formula for combinations.

EXAMPLE 4

APPLICATION
SURVEYS

In a recent survey of 25 voters, 17 favor a new city regulation and 8 oppose it.

Find the probability that in a random sample of 6 respondents from this survey, exactly 2 favor the proposed regulation and 4 oppose it.

Survey of 25 people

17 favor	8 oppose
2 of 17	4 of 8

SOLUTION

1. Find the number of outcomes in the event. Use the Fundamental Counting Principle.

$$_{17}C_2 \times {_8C_4}$$

Choose 2 respondents of 17 respondents who favor.

Choose 4 respondents of 8 respondents who oppose.

2. Find the number of outcomes in the sample space.

$_{25}C_6$ — *Choose 6 people from the 25 respondents.*

3. Find the probability.

$$\frac{\text{number of outcomes in event A}}{\text{number of outcomes in the sample space}} = \frac{_{17}C_2 \times {_8C_4}}{_{25}C_6} \approx 0.05$$

TECHNOLOGY
GRAPHICS CALCULATOR
Keystroke Guide, page 686

Thus, the probability of selecting exactly 2 respondents who favor the proposed regulation and 4 who oppose it in a randomly selected group of 6 respondents is about 0.05, or 5%.

TRY THIS Find the probability that in a random sample of 10 respondents from the above survey, all 10 favor the proposed regulation.

CRITICAL THINKING Sets A and B are two nonoverlapping sets. Set A contains a distinct objects and set B contains b distinct objects. You wish to choose x objects randomly from both set A and set B. Find the probability of choosing r objects from set A and s objects from set B. Are there any restrictions on r and s?

Set A a objects	Set B b objects
r	s

CONNECTION
TRANSFORMATIONS

The function $y = a \sin b(t - c)$, where t is in radians and y represents relative air pressure, can represent a particular sound as follows:

- The amplitude of the graph, a, represents the relative *intensity* of the changes in air pressure. A higher intensity results in a louder sound.
- The frequency of the sound wave, measured in units called hertz (Hz), or cycles per second, determines the pitch of the sound. The value of b in the function is equal to the frequency multiplied by 2π.
- The phase shift, c, represents the change in the position of the sound wave over time.
- The period of the sound wave is the reciprocal of the frequency.

EXAMPLE 3

APPLICATION
ACOUSTICS

A particular sound has a frequency of 55 hertz and an amplitude of 3.

a. Write a transformed sine function to represent this sound.

b. Write a new function that represents a phase shift of $\frac{1}{2}$ of a period to the right of the function from part **a**. Then use a graphics calculator to graph at least one period of both functions on the same coordinate plane.

Denyce Graves singing in the opera Carmen

SOLUTION

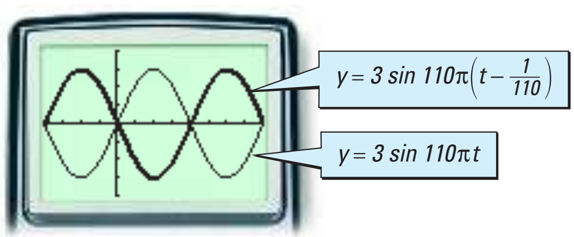
a. The parent function is $y = \sin t$. Write the transformed function in the form $y = a \sin b(t - c)$.

The amplitude is 3.	$a = 3$
The frequency is related to b.	$b = 2\pi \times 55 = 110\pi$
There is no phase shift.	$c = 0$

Thus, the transformed function is $y = 3 \sin 110\pi t$.

b. The period is the reciprocal of the frequency: $\frac{1}{55}$. One-half of the period is $\frac{1}{2} \times \frac{1}{55} = \frac{1}{110}$. Thus, a phase shift of $\frac{1}{110}$ radian to the right of the function $y = 3 \sin 110\pi t$ is given by $y = 3 \sin 110\pi\left(t - \frac{1}{110}\right)$. Graph both functions in radian mode.

TECHNOLOGY
GRAPHICS CALCULATOR
Keystroke Guide, page 883



TRY THIS

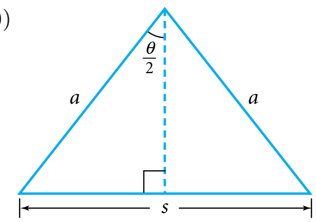
Write the function for a sound with a frequency of 120 hertz, an amplitude of 1.5, and a phase shift of $\frac{1}{3}$ of a period to the left. Graph at least one period of the function along with its parent function.

Half-Angle Identities

APPLICATION
ARCHITECTURE

Refer to the isosceles triangle described at the beginning of the lesson. A half-angle identity for the sine function can be found by solving the two expressions for s that relate $\sin \frac{\theta}{2}$ and $\cos \theta$.

$$2a \sin \frac{\theta}{2} = a\sqrt{2 - 2\cos\theta} \quad (0^\circ < \theta < 180^\circ \text{ and } a > 0)$$
$$\sin \frac{\theta}{2} = \frac{\sqrt{2 - 2\cos\theta}}{2}$$
$$\sin \frac{\theta}{2} = \sqrt{\frac{2 - 2\cos\theta}{4}}$$
$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos\theta}{2}}$$



The *half-angle identities* for the sine and cosine of any angle are given below.

Half-Angle Identities

$$\sin \frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}} \qquad \cos \frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

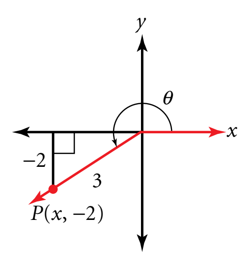
Choose + or − depending on the sign of the value for $\sin \frac{\theta}{2}$ or $\cos \frac{\theta}{2}$.

EXAMPLE 3 **Given $180^\circ \le \theta \le 270^\circ$ and $\sin \theta = -\frac{2}{3}$, find the exact value of $\cos \frac{\theta}{2}$.**

SOLUTION

PROBLEM SOLVING

Draw a diagram and find the exact value of $\cos \theta$.



$$x^2 + y^2 = r^2$$
$$x^2 + (-2)^2 = 3^2$$
$$x = \pm\sqrt{3^2 - (-2)^2}$$
$$x = -\sqrt{5} \qquad \textit{x is negative in Quadrant III.}$$

Thus, $\cos \theta = \frac{x}{r} = \frac{-\sqrt{5}}{3}$.

Use the half-angle identity for cosine. If $180^\circ \le \theta \le 270^\circ$, then $\frac{180^\circ}{2} \le \frac{\theta}{2} \le \frac{270^\circ}{2}$, or $90^\circ \le \theta \le 135^\circ$. Therefore, the sign of the value for $\cos \frac{\theta}{2}$ will be negative.

$$\cos \frac{\theta}{2} = -\sqrt{\frac{1 + \cos\theta}{2}}$$
$$= -\sqrt{\frac{1 + \left(\frac{-\sqrt{5}}{3}\right)}{2}}$$
$$= -\sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{5}}{3}\right)}$$
$$= -\sqrt{\frac{1}{2} - \frac{\sqrt{5}}{6}}$$

TRY THIS Given $90^\circ \le \theta \le 180^\circ$ and $\cos \theta = -\frac{1}{3}$, find the exact value of $\sin \frac{\theta}{2}$.

LESSON 14.6

Activity

Page 923

Graph $y = \sin x$ and $y = 1$ on the same screen.

Use viewing window [0, 360] by [–3, 3] in degree mode.

Y= SIN X,T,θ,n ENTER (Y2=) 1 GRAPH

For Step 2 use viewing window [0, 720] by [–3, 3].

EXAMPLE 1

Page 923

Graph $y = \cos x$ and $y = \sqrt{3} - \cos x$ on the same screen, and find any points of intersection.

Use viewing window [0, 360] by [–2, 3] in degree mode.

Graph the functions:

Y= COS X,T,θ,n ENTER (Y2=) 2nd $\sqrt{\ }$ x^2 3) –

COS X,T,θ,n GRAPH

Find any points of intersection:

2nd CALC TRACE 5:intersect (First curve?) ENTER

(Second curve?) ENTER (Guess?) ENTER

EXAMPLE 3

Page 924

Graph $y = 2\cos^2 x$ and $y = \sin x + 1$ on the same screen, and find any points of intersection.

Use viewing window [0, 360] by [–0.5, 2.5] in degree mode.

Graph the functions:

Y= 2 (COS X,T,θ,n)) x^2 ENTER (Y2=) SIN X,T,θ,n

⇑ TI-82: (⇑ TI-82: (

) + 1 GRAPH

Find any points of intersection:

2nd CALC TRACE 5:intersect (First curve?) ENTER

(Second curve?) ENTER (Guess?) ENTER

The calculator may return an error message when looking for the intersection point (270°, 0).