Nested grouping symbols are when you have grouping symbols inside other grouping symbols.

When you see nested grouping symbols, you always start from the inside and work outwards.

Example 2

Evaluate $\{5 - [11 - (7 - 2)]\} + 34$.

Solution

Start from the inside and work outwards:

$$\begin{aligned}\{5 - [11 - \mathbf{(7 - 2)}]\} + 34 &= \{5 - \mathbf{[11 - 5]}\} + 34 \\ &= \mathbf{\{5 - 6\}} + 34 \\ &= -1 + 34 \\ &= \mathbf{33}\end{aligned}$$

Guided Practice

Evaluate the following:

11. $8 + [10 + (6 - 9) + 7]$
12. $9 - \{[(-4) + 10] + 7\}$
13. $[(13 - 12) + 6] - (4 - 2)$
14. $14 - \{8 + [5 - (-2)]\} - 6$
15. $13 + [10 - (4 + 5)] - (11 + 8)$
16. $10 + \{[7 - (-2)] - (3 - 1)\} + (-14)$

There are Other Rules About What to Evaluate First

This order of operations is used by all mathematicians, so that every mathematician in the world evaluates expressions in the same way.

1. First calculate expressions within **grouping symbols** — working from the innermost grouping symbols to the outermost.
2. Then calculate expressions involving **exponents**.
3. Next do all **multiplication** and **division**, working from left to right. Multiplication and division have equal priority, so do them in the order they appear from left to right.
4. Lastly, do any **addition** or **subtraction**, again from left to right. Addition and subtraction have the same priority, so do them in the order they appear from left to right too.

Check it out

See Topic 1.3.1 for more about exponents.

Check it out

When you're simplifying expressions within grouping symbols, follow steps 2–4 (see Example 3).

Example 1

Solve $x + 9 = 16$.

Solution

You want x on its own, but here x has 9 added to it.
So subtract 9 from both sides to get x on its own.

$(x + 9) - 9 = 16 - 9$
$x + (9 - 9) = 16 - 9$
$x + 0 = 16 - 9$
$x = 16 - 9$
$\mathbf{x = 7}$

In Example 1, $x = 7$ is the root of the equation. If x takes the value 7, then the equation is **satisfied**.

If x takes **any other value**, then the equation is **not satisfied**.
For example, if $x = 6$, then the left-hand side has the value $6 + 9 = 15$, which does not equal the right-hand side, 16.

When you're actually solving equations, you won't need to go through all the stages each time — but it's really important that you understand the theory of the properties of equality.

- If you have a "**+ 9**" that you don't want, you can get rid of it by just **subtracting 9** from both sides.
- If you have a "**– 9**" that you want to get rid of, you can just **add 9** to both sides.

In other words, you just need to use the **inverse operations**.

Example 2

Solve $x + 10 = 12$.

Solution

$x + 10 = 12$
$x = 12 - 10$ **Subtract 10 from both sides**
$\mathbf{x = 2}$

Example 3

Solve $x - 7 = 8$.

Solution

$x - 7 = 8$
$x = 8 + 7$ **Add 7 to both sides**
$\mathbf{x = 15}$

Independent Practice

1. Three consecutive integers have a sum of 90. Find the numbers.

2. Find the four consecutive integers whose sum is 318.

3. Find three consecutive integers such that the difference between three times the largest and two times the smallest integer is 30.

4. Find three consecutive integers such that the sum of the first two integers is equal to three times the highest integer.

5. Two numbers have a sum of 65. Four times the smaller number is equal to 10 more than the larger number. Find the numbers.

6. Four consecutive even integers have a sum of 140.
What are the integers?

7. Find three consecutive even integers such that six more than three times the smallest integer is 54.

8. Find three consecutive odd integers whose sum is 273.

9. Find four consecutive odd integers such that 12 more than four times the smallest integer is 144.

10. Find three consecutive even integers such that six more than twice the first number is 94.

11. Find three consecutive even integers such that the product of 16 and the third integer is the same as the product of 20 and the second integer.

12. A 36-foot pole is cut into two parts such that the longer part is 11 feet longer than 4 times the shorter part. How long is each piece of the pole?

13. Find three consecutive odd integers such that four times the largest is one more than nine times the smallest integer.

14. Ten thousand people attended a three-day outdoor music festival. If there were 800 more girls than boys, and 1999 fewer adults than boys, how many people of each group attended the festival?

Round Up

Consecutive integer tasks are a strange application of math equations — but they appear a lot in Algebra I. Always make sure you've answered the question — you've always got to remember that your solution isn't complete until you've stated what the integers actually are.

Independent Practice

Solve:

1. $|2x| = 84$

2. $|3z| = -9$

3. $|x + 8| = 24$

4. $|-x + 4| = 3$

5. $|2x + 8| = |4 - 3x|$

6. $|7x - 4| = |3x + 9|$

7. $|0.1x - 0.3| = |0.3x + 4.1|$

8. $-2|x - 20| = -8$

9. $\left|\frac{1}{3}x + \frac{1}{8}\right| = \left|\frac{1}{12}x + \frac{1}{4}\right|$

10. $\left|-\frac{x}{4}\right| = 12$

11. $\left|\frac{1}{2}x\right| + 1 = 5$

12. $\frac{4}{3}|2x + 1| = 8x$

13. $\left|\frac{x-8}{0.1}\right| = \left|\frac{10(x-4)}{3}\right|$

14. $\left|\frac{x-4}{7}\right| = \left|\frac{2x+8}{14}\right|$

In Exercises 15–18 you will need to form an absolute value equation and solve it to find the unknown.

15. If $(x + 4)$ is $3x$ from 0, what are the possible values of x?

16. If $(4x - 5)$ is $(2x + 1)$ from 0, what are the possible values of x?

17. If $(3w + 2)$ and $(w - 4)$ are the same distance away from 0, what are the possible values of w?

18. If $(4x - 5 + x)$ and $(7 + 5x + 2)$ are the same distance from 0, what are the possible values of x?

19. Given that $|3x - 5| = |2x + 6|$, find the two possible values of $b^2 - 2bx + x^2$ if $b = -3$.

If $a = 2$, $b = 4$, and $c = 6$ then solve each absolute value equation for x:

20. $|ax - b| = |x + c|$

21. $|ax + b| + c = a - bx$

22. $\frac{1}{a}|cx + ab| = |ax - c|$

Round Up

Equations with absolute values on both sides look really difficult at first. Make sure you understand how Example 3 shows that you still only get two distinct equations when there are two absolute values.

Topic 4.3.2

Point-Slope Formula

California Standards:

7.0: Students verify that a point lies on a line, given an equation of the line. **Students are able to derive linear equations by using the point-slope formula.**

What it means for you:

You'll learn about the point-slope formula and use it to find the equation of a line.

Key words:

- slope
- point-slope formula

Check it out:

The point-slope formula is just the one from Topic 4.3.1 rearranged. (x, y) is any point on the line, so (x_1, y_1) and (x, y) are the two points on the line.

The point-slope formula is a really useful way of calculating the equation of a straight line.

Use the Formula to Find the Equation of a Straight Line

If you know the **slope** of the line and a **point** on the line, you can use the **point-slope formula** to find the equation of the line.

The point-slope formula for finding the equation of a line is:

$$y - y_1 = m(x - x_1)$$

where m is the slope and (x_1, y_1) is a point on the line.

You substitute the ***x*-coordinate** of a point on the line for x_1 and the y-coordinate of the same point for y_1. Watch out though — x and y are variables and they stay as letters in the equation of the line.

Example 1

Find the equation of the line through (–4, 6) that has a slope of –3.

Solution

$(x_1, y_1) = (-4, 6)$ and $m = -3$

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 6 = -3[x - (-4)]$

$\Rightarrow y - 6 = -3(x + 4)$

$\Rightarrow y - 6 = -3x - 12$

$\Rightarrow \mathbf{y + 3x = -6}$

Guided Practice

Write the equation of the line that passes through the given point and has the given slope.

1. Point (–2, –3), slope = –1

2. Point (3, –5), slope = 2

3. Point (–7, –2), slope = –5

4. Point (4, –3), slope = $\frac{2}{3}$

5. Point (2, 6), slope = $-\frac{3}{4}$

6. Point (–2, –3), slope = $\frac{5}{8}$

7. Point (–5, –3), slope = $-\frac{6}{7}$

8. Point $\left(-\frac{2}{3}, \frac{1}{4}\right)$, slope = $\frac{2}{5}$

Topic 4.5.2

Borders of Regions

California Standards:

6.0: Students graph a linear equation and compute the x- and y-intercepts (e.g., graph $2x + 6y = 4$). **They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).**

What it means for you:

You'll learn how to show the different types of inequality on a graph.

Key words:

- inequality
- strict inequality
- border
- region

Don't forget:

Remember, ≤ means "less than or equal to," ≥ means "greater than or equal to."

In Topic 4.5.1 you were dealing with regions defined by strict inequalities — the ones involving a < or > sign. This Topic shows you how to graph inequalities involving ≤ and ≥ signs too.

Regions Can Have Different Types of Borders

The region defined by a **strict** inequality **doesn't** include points on the **border line**, and you draw the border line as a **dashed** line.

For example, the region defined by $y > -x + 3$ doesn't include any points on the line $y = -x + 3$.

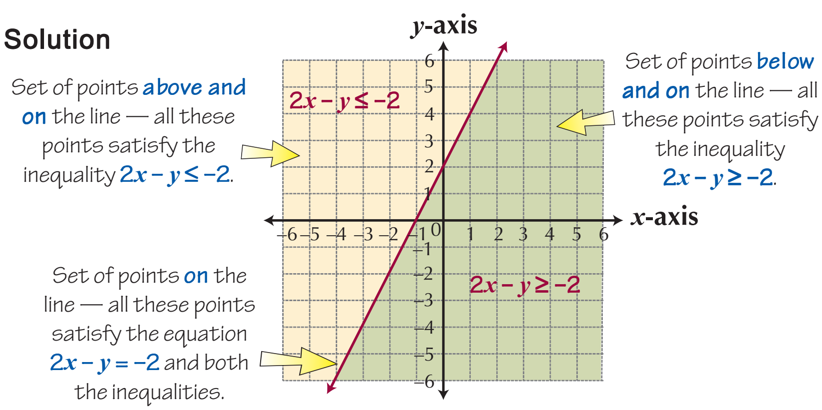
Regions defined by inequalities involving a **≤** or **≥** sign **do** include points on the **border line**. In this case, you draw the border line as a **solid** line.

For example, the region defined by $y \geq -x + 3$ includes all the points on the line $y = -x + 3$.

Example 1

Graph $2x - y = -2$ and show the three regions of the plane that include all the points on this line.

Solution



Topic 5.3.2

Systems of Equations — Integer Problems

California Standards:

9.0: Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

15.0: Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

What it means for you:

You'll solve integer problems involving systems of linear equations.

Key words:

- system of linear equations
- substitution method

In this Topic you'll solve systems of linear equations to figure out solutions to problems involving integers.

Using Systems of Equations to Solve Integer Problems

Example 1

The sum of two integers is 53. The larger number is 7 less than three times the smaller one. Find the numbers.

Solution

First form a system of two equations.

Let x = smaller number

y = larger number

The sum of the two integers is 53, so $x + y = 53$.
The larger is 7 less than 3 times the smaller one, so $y = 3x - 7$.
So the system of equations is $\boldsymbol{x + y = 53}$ and $\boldsymbol{y = 3x - 7}$.

Now solve the system of equations. The variable y in the second equation is already expressed in terms of x, so it makes sense to use the substitution method.

Substitute $3x - 7$ for y in the first equation.

$$\begin{aligned} x + y &= 53 \\ x + (3x - 7) &= 53 \\ 4x - 7 &= 53 \\ 4x &= 60 \\ \boldsymbol{x} &\boldsymbol{= 15} \end{aligned}$$

Now, substitute 15 for x in the equation $y = 3x - 7$.

$$\begin{aligned} y &= 3x - 7 \\ y &= 3(15) - 7 \\ y &= 45 - 7 \\ \boldsymbol{y} &\boldsymbol{= 38} \end{aligned}$$

That means that the integers are 15 and 38.

The solution should work in both equations:

$x + y = 53 \Rightarrow 15 + 38 = 53 \Rightarrow 53 = 53$ — True

$y = 3x - 7 \Rightarrow 38 = 3(15) - 7 \Rightarrow 38 = 38$ — True

A useful check is to make sure that the answer matches the information given in the question:

- The sum of the two integers is $15 + 38 = 53$. This matches the question.
- Three times the smaller integer is $15 \times 3 = 45$. The larger integer is $45 - 38 = 7$ less than this.

Example 8

Simplify $(x - 2)(2x^2 - 3x + 4)$.

Solution

$$\begin{array}{rrrr}
 & 2x^2 & -\,3x & +\,4 \\
\times & & x & -\,2 \\
\hline
 & -4x^2 & +\;\;6x & -\,8 \\
2x^3 & -\,3x^2 & +\;\;4x & \\
\hline
\mathbf{2x^3} & \mathbf{-\,7x^2} & \mathbf{+\,10x} & \mathbf{-\,8}
\end{array}$$

$-2(2x^2 - 3x + 4)$

$x(2x^2 - 3x + 4)$

Guided Practice

Use the stacking method to multiply these polynomials:

21. $(3x + y)(x + 2y)$

22. $(4x + 5y)(2x + 3y)$

23. $(3x^2 + 2x + 3)(3x - 4)$

24. $(4x^2 - 5x + 6)(4x + 5)$

25. $(a + b)^2$

26. $(a - b)^2$

27. $(a - b)(a + b)$

28. $(a - b)(a^2 + ab + b^2)$

29. $(a + b)(a^2 - ab + b^2)$

30. $(a^2 - b^2)(a^2 + b^2)$

Independent Practice

Expand and simplify each product, using the distributive method. Show all your work.

1. $(2x + 8)(x - 4)$

2. $(x^2 + 3)(x - 2)$

3. $(x - 3)(2 - x)$

4. $(2x + 7)(3x + 5)$

5. $(3x - 8)(x^2 - 4x + 2)$

6. $(2x - 4y)(3x - 3y + 4)$

Use the stack method to multiply. Show all your work.

7. $(x^2 - 4)(x + 3)$

8. $(x - y)(3x^2 + xy + y^2)$

9. $(4x^2 - 5x)(1 + 2x - 3x^2)$

10. $(x + 4)(3x^2 - 2x + 5)$

Use these formulas to find each of the products in Exercises 11–16.

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

11. $(x + 2)^2$

12. $(3x - 1)(3x + 1)$

13. $(2x - 3)^2$

14. $(4x + y)^2$

15. $(5x + 3c)(5x - 3c)$

16. $(8c + 3)^2$

Round Up

You've seen the distributive property lots of times already, but it's easy to make calculation errors when you're multiplying long polynomials — so be careful and check your work.

Topic 6.3.2

Polynomials and Negative Powers

California Standards:
2.0: Students understand and use such operations as taking the opposite**, finding the reciprocal,** taking a root, and raising to a fractional power**. They understand and use the rules of exponents.**
10.0: Students add, subtract, multiply, and **divide monomials and polynomials.** Students solve multistep problems, including word problems, by using these techniques.

What it means for you:
You'll learn how to find the multiplicative inverse of a polynomial, and how to use negative exponents.

Key words:
- polynomial
- monomial
- reciprocal
- exponent

Before you divide one polynomial by another polynomial, you need to know how to write the multiplicative inverse of a polynomial.

Finding the Reciprocal of a Polynomial

The **reciprocal** of a polynomial is its **multiplicative inverse**.

To find the **reciprocal of Polynomial A**, you **divide the number 1 by Polynomial A**.

Example 1

Find the reciprocal of each of these polynomials:
a) $2x - 1$ b) $-5x^2 + 3x - 1$

Solution

a) $\dfrac{1}{2x-1}$ b) $\dfrac{1}{-5x^2+3x-1}$

By the **definition of division**, to **divide by a polynomial you need to multiply by the reciprocal** (the **inverse** under multiplication) of that polynomial.

Guided Practice

Find the multiplicative inverse of each of these expressions.

1. ab **2.** a^2 **3.** $2x + 4b$

4. $3x + 1$ **5.** $8x^3 - 16x^2 + 4$ **6.** $\dfrac{1}{-2x^2+8x-9}$

7. $2x^2y^4$ **8.** $\dfrac{1}{3a^2b^4c^8}$ **9.** $\dfrac{1}{-3x^2y^8c^4}$

Topic
6.4.1

Section 6.4
Special Products of Two Binomials

California Standards:
2.0: Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They **understand and use the rules of exponents.**
10.0: Students add, subtract, **multiply,** and divide **monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.**
11.0 Students apply basic factoring techniques to second and simple third-**degree polynomials. These techniques include finding a common factor for all terms in a polynomial**, **recognizing the difference of two squares, and recognizing perfect squares of binomials.**

What it means for you:
You'll learn how to use special cases of binomial multiplication to save time in calculations.

Key words:
- binomial
- difference of two squares

Check it out:
$(a + b)^2$ expands to give a perfect square trinomial — see Topic 6.8.2.

This Topic is all about special cases of binomial multiplication. Knowing how to expand these special products will save you time when you're dealing with binomials later in Algebra I.

Remember These Three Special Binomial Products

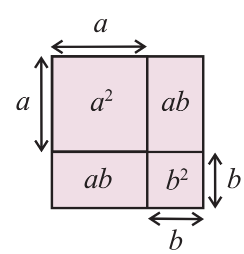
Given any real numbers a and b, then:

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$
$$(a + b)(a - b) = a^2 - b^2$$

When You Expand $(a + b)^2$, You Always Get an ab-Term

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a^2 + ab + ba + b^2 \quad \textbf{Using the distributive property}\\ &= \mathbf{a^2 + 2ab + b^2}\end{aligned}$$

You can relate this equation to the area of a square:



$(a + b)^2$ is the same as the area of this large square — add the areas of the two smaller squares, a^2 and b^2, and the two rectangles, $2 \times ab$.

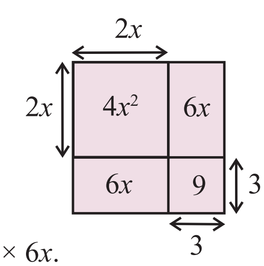
Example 1

Expand and simplify $(2x + 3)^2$.

Solution

Put the expression in the form $(a + b)(a + b)$:

$$\begin{aligned}(2x + 3)^2 &= (2x + 3)(2x + 3)\\ &= 4x^2 + 6x + 6x + 9\\ &= \mathbf{4x^2 + 12x + 9}\end{aligned}$$



$(2x + 3)^2$ is the same as the area of the large square — add the areas of the two smaller squares, $4x^2$ and 9, and the two rectangles, $2 \times 6x$.