26. **TAKS REASONING** Which quadrant of the coordinate plane contains no solutions of the system of inequalities?

$$y \le -|x - 3| + 2$$
$$4x - 5y \le 20$$

Ⓐ Quadrant I Ⓑ Quadrant II Ⓒ Quadrant III Ⓓ Quadrant IV

27. **TAKS REASONING** Write a system of two linear inequalities that has $(2, -1)$ as a solution.

ABSOLUTE VALUE SYSTEMS **Graph the system of inequalities.**

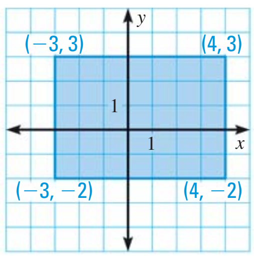
28. $y < |x|$
$y > -|x|$

29. $y \le |x - 2|$
$y \ge |x| - 2$

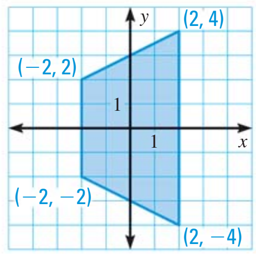
30. $y \le -|x - 3| + 2$
$y > |x - 3| - 1$

CHALLENGE **Write a system of linear inequalities for the shaded region.**

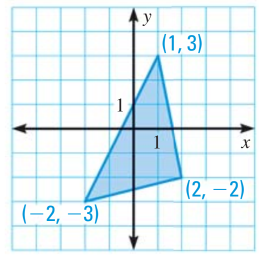
31.



32.



33.



PROBLEM SOLVING

EXAMPLE 4
on p. 170
for Exs. 34–39

34. **SUMMER JOBS** You can work at most 20 hours next week. You need to earn at least \$92 to cover your weekly expenses. Your dog-walking job pays \$7.50 per hour and your job as a car wash attendant pays \$6 per hour. Write a system of linear inequalities to model the situation.

TEXAS @HomeTutor for problem solving help at classzone.com

35. **VIDEO GAME SALE** An online media store is having a sale, as described in the ad shown. Use the information in the ad to write and graph a system of inequalities for the regular video game prices and possible sale prices. Then use the graph to estimate the range of possible sale prices for games that are regularly priced at \$20.

TEXAS @HomeTutor for problem solving help at classzone.com

36. **TAKS REASONING** A book on the care of tropical fish states that the pH level of the water should be between 8.0 and 8.3 pH units and the temperature of the water should be between 76°F and 80°F. Let x be the pH level and y be the temperature. Write and graph a system of inequalities that describes the proper pH level and temperature of the water. *Compare* this graph to the graph you would obtain if the temperatures were given in degrees Celsius.

40. **SUMMER OLYMPICS** The top three countries in the final medal standings for the 2004 Summer Olympics were the United States, China, and Russia. Each gold medal is worth 3 points, each silver medal is worth 2 points, and each bronze medal is worth 1 point. Organize the information using matrices. How many points did each country score?

Medals Won

	Gold	Silver	Bronze
USA	35	39	29
China	32	17	14
Russia	27	27	38

41. **TAKS REASONING** Matrix S gives the numbers of three types of cars sold in February by two car dealers, dealer A and dealer B. Matrix P gives the profit for each type of car sold. Which matrix is defined, SP or PS? Find this matrix and explain what its elements represent.

Matrix *S*

$$\begin{array}{r} \\ \text{Compact} \\ \text{Mid-size} \\ \text{Full-size} \end{array} \begin{array}{c} \quad\text{A} \quad \text{B} \\ \begin{bmatrix} 21 & 16 \\ 40 & 33 \\ 15 & 19 \end{bmatrix} \end{array}$$

Matrix *P*

$$\begin{array}{rc} & \begin{array}{ccc} \text{Compact} & \text{Mid-size} & \text{Full-size} \end{array} \\ \text{Profit} & \begin{bmatrix} \$650 & \$825 & \$1050 \end{bmatrix} \end{array}$$

42. **GRADING** Your overall grade in math class is a weighted average of three components: homework, quizzes, and tests. Homework counts for 20% of your grade, quizzes count for 30%, and tests count for 50%. The spreadsheet below shows the grades on homework, quizzes, and tests for five students. Organize the information using a matrix, then multiply the matrix by a matrix of weights to find each student's overall grade.

	A	B	C	D
1	Name	Homework	Quizzes	Test
2	Jean	82	88	86
3	Ted	92	88	90
4	Pat	82	73	81
5	Al	74	75	78
6	Matt	88	92	90

43. **MULTI-STEP PROBLEM** Residents of a certain suburb commute to a nearby city either by driving or by using public transportation. Each year, 20% of those who drive switch to public transportation, and 5% of those who use public transportation switch to driving.

a. The information above can be represented by the *transition matrix*

$$T = \begin{bmatrix} 1 - p & q \\ p & 1 - q \end{bmatrix}$$

where p is the percent of commuters who switch from driving to public transportation and q is the percent of commuters who switch from public transportation to driving. (Both p and q are expressed as decimals.) Write a transition matrix for the given situation.

b. Suppose 5000 commuters drive and 8000 commuters take public transportation. Let M_0 be the following matrix:

$$M_0 = \begin{bmatrix} 5000 \\ 8000 \end{bmatrix}$$

Find $M_1 = TM_0$. What does this matrix represent?

c. Find $M_2 = TM_1$, $M_3 = TM_2$, and $M_4 = TM_3$. What do these matrices represent?

26. **ERROR ANALYSIS** A student tried to explain how the graphs of $y = -2\sqrt[3]{x}$ and $y = -2\sqrt[3]{x+1} - 3$ are related. *Describe* and correct the error.

> The graph of $y = -2\sqrt[3]{x+1} - 3$ is the graph of $y = -2\sqrt[3]{x}$ translated right 1 unit and down 3 units.

27. **MULTIPLE CHOICE** If the graph of $y = 3\sqrt[3]{x}$ is shifted left 2 units, what is the equation of the translated graph?

Ⓐ $y = 3\sqrt[3]{x-2}$ Ⓑ $y = 3\sqrt[3]{x} - 2$ Ⓒ $y = 3\sqrt[3]{x+2}$ Ⓓ $y = 3\sqrt[3]{x} + 2$

REASONING Find the domain and range of the function without graphing. *Explain* how you found your answers.

28. $y = \sqrt{x+5}$

29. $y = \sqrt{x-12}$

30. $y = \frac{1}{3}\sqrt{x} - 4$

31. $y = \frac{1}{2}\sqrt[3]{x+7}$

32. $g(x) = \sqrt[3]{x+7}$

33. $f(x) = \frac{1}{4}\sqrt{x-3} + 6$

34. **CHALLENGE** Graph $y = \sqrt[4]{x}$, $y = \sqrt[5]{x}$, $y = \sqrt[6]{x}$, and $y = \sqrt[7]{x}$ on a graphing calculator. Make generalizations about the graph of $y = \sqrt[n]{x}$ when n is even and when n is odd.

PROBLEM SOLVING

EXAMPLE 3 on p. 447 for Exs. 35–36

35. **INDIRECT MEASUREMENT** The distance d (in miles) that a pilot can see to the horizon can be modeled by $d = 1.22\sqrt{a}$ where a is the plane's altitude (in feet above sea level). Graph the model on a graphing calculator. Then determine at what altitude the pilot can see 8 miles.

TEXAS @HomeTutor for problem solving help at classzone.com

36. **PENDULUMS** Use the model $T = 1.11\sqrt{\ell}$ for the period of a pendulum from Example 3 on page 447.

a. Find the period of a pendulum with a length of 2 feet.

b. Find the length of a pendulum with a period of 2 seconds.

TEXAS @HomeTutor for problem solving help at classzone.com

37. **SHORT RESPONSE** The speed v (in meters per second) of sound waves in air depends on the temperature K (in kelvins) and can be modeled by:

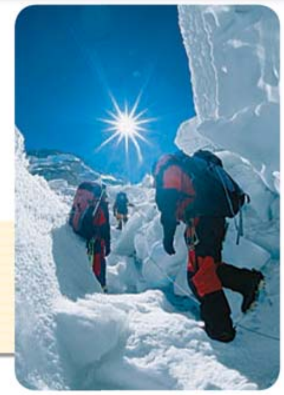
$$v = 331.5\sqrt{\frac{K}{273.15}},\ K \geq 0$$

a. Kelvin temperature K is related to Celsius temperature C by the formula $K = 273.15 + C$. Write an equation that gives the speed v of sound waves in air as a function of the temperature C in degrees Celsius.

b. What are a reasonable domain and range for the function from part (a)?

8.3 Graph General Rational Functions

TEKS 2A.10.A, 2A.10.B, 2A.10.C, 2A.10.F



Before You graphed rational functions involving linear polynomials.

Now You will graph rational functions with higher-degree polynomials.

Why? So you can solve problems about altitude, as in Ex. 35.

Key Vocabulary
- **end behavior,** *p. 339*
- **asymptote,** *p. 478*
- **rational function,** *p. 558*

KEY CONCEPT — *For Your Notebook*

Graphs of Rational Functions

Let $p(x)$ and $q(x)$ be polynomials with no common factors other than ± 1. The graph of the following rational function has the characteristics listed below.

$$f(x) = \frac{p(x)}{q(x)} = \frac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}$$

1. The x-intercepts of the graph of f are the real zeros of $p(x)$.
2. The graph of f has a vertical asymptote at each real zero of $q(x)$.
3. The graph of f has at most one horizontal asymptote, which is determined by the degrees m and n of $p(x)$ and $q(x)$.

$m < n$	The line $y = 0$ is a horizontal asymptote.
$m = n$	The line $y = \frac{a_m}{b_n}$ is a horizontal asymptote.
$m > n$	The graph has no horizontal asymptote. The graph's end behavior is the same as the graph of $y = \frac{a_m}{b_n}x^{m-n}$.

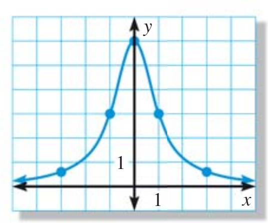
EXAMPLE 1 Graph a rational function ($m < n$)

Graph $y = \dfrac{6}{x^2 + 1}$. State the domain and range.

Solution

The numerator has no zeros, so there is no x-intercept. The denominator has no real zeros, so there is no vertical asymptote.

The degree of the numerator, 0, is less than the degree of the denominator, 2. So, the line $y = 0$ (the x-axis) is a horizontal asymptote.



The graph passes through the points $(-3, 0.6)$, $(-1, 3)$, $(0, 6)$, $(1, 3)$, and $(3, 0.6)$. The domain is all real numbers, and the range is $0 < y \le 6$.

EXAMPLE 2 Graph a rational function ($m = n$)

Graph $y = \dfrac{2x^2}{x^2 - 9}$.

REVIEW ZEROS OF FUNCTIONS

For help with finding zeros of functions, see p. 252.

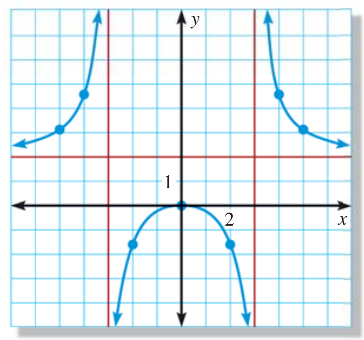
Solution

The zero of the numerator $2x^2$ is 0, so 0 is an x-intercept. The zeros of the denominator $x^2 - 9$ are ± 3, so $x = 3$ and $x = -3$ are vertical asymptotes.

The numerator and denominator have the same degree, so the horizontal asymptote is $y = \dfrac{a_m}{b_n} = \dfrac{2}{1} = 2$.

Plot points between and beyond the vertical asymptotes.

	x	y
To the left of $x = -3$	−5	3.1
	−4	4.6
Between $x = -3$ and $x = 3$	−2	−1.6
	0	0
	2	−1.6
To the right of $x = 3$	4	4.6
	5	3.1



EXAMPLE 3 Graph a rational function ($m > n$)

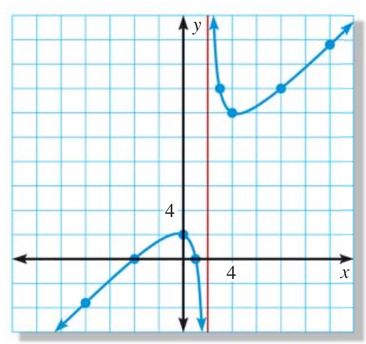
Graph $y = \dfrac{x^2 + 3x - 4}{x - 2}$.

Solution

The numerator factors as $(x + 4)(x - 1)$, so the x-intercepts are -4 and 1. The zero of the denominator $x - 2$ is 2, so $x = 2$ is a vertical asymptote.

The degree of the numerator, 2, is greater than the degree of the denominator, 1, so the graph has no horizontal asymptote. The graph has the same end behavior as the graph of $y = x^{2-1} = x$. Plot points on each side of the vertical asymptote.

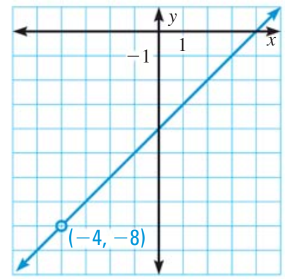
	x	y
To the left of $x = 2$	−8	−3.6
	−4	0
	0	2
	1	0
To the right of $x = 2$	3	14
	4	12
	8	14
	12	17.6



POINT DISCONTINUITY **In Exercises 44–46, use the following information.**

The graph of a rational function can have a hole in it, called a *point discontinuity*, where the function is undefined. An example is shown below.

$$y = \frac{x^2 - 16}{x + 4} = \frac{\cancel{(x+4)}(x-4)}{\cancel{x+4}} = x - 4$$



The graph of $y = \frac{x^2 - 16}{x + 4}$ is the same as the graph of $y = x - 4$ except that there is a hole at $(-4, -8)$ because the rational function is not defined when $x = -4$.

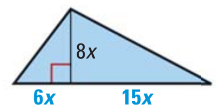
Graph the rational function. Use an open circle for a point discontinuity.

44. $y = \frac{x^2 + 10x + 21}{x + 3}$

45. $y = \frac{x^2 - 36}{x - 6}$

46. $y = \frac{2x^2 - x - 10}{x + 2}$

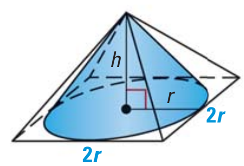
47. **CHALLENGE** Find the ratio of the perimeter to the area of the triangle shown at the right.



PROBLEM SOLVING

EXAMPLE 2 on p. 574 for Exs. 48, 50–52

48. **GEOMETRY** Find the ratio of the volume of the square pyramid to the volume of the inscribed cone. Write your answer in simplified form.



TEXAS @HomeTutor for problem solving help at classzone.com

49. **ENTERTAINMENT** From 1992 to 2002, the gross ticket sales S (in millions of dollars) to Broadway shows and the total attendance A (in millions) at the shows can be modeled by

$$S = \frac{-6420t + 292{,}000}{6.02t^2 - 125t + 1000} \quad \text{and} \quad A = \frac{-407t + 7220}{5.92t^2 - 131t + 1000}$$

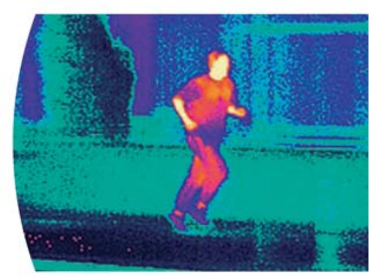
where t is the number of years since 1992. Write a model for the *average* dollar amount a person paid per ticket as a function of the year. What was the average amount a person paid per ticket in 1999?

TEXAS @HomeTutor for problem solving help at classzone.com

50. **TAKS REASONING** Almost all of the energy generated by a long-distance runner is released in the form of heat. For a runner with height H and speed V, the rate h_g of heat generated and the rate h_r of heat released can be modeled by $h_g = k_1H^3V^2$ and $h_r = k_2H^2$ where k_1 and k_2 are constants.

a. Write the ratio of heat generated to heat released. Simplify the expression.

b. When the ratio of heat generated to heat released equals 1, how is speed related to height? Does a taller or shorter runner have the advantage? *Explain.*



Thermogram of runner

8 TAKS PREPARATION

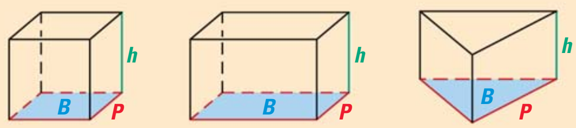
TEXAS

TAKS Obj. 8
TEKS G.8.D

REVIEWING SURFACE AREA AND VOLUME

To solve math problems involving surface area and volume of solids, you can use the following formulas.

Prisms

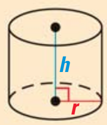


Cube   Rectangular Prism   Triangular Prism

Surface Area $= 2B + Ph$

Volume $= Bh$

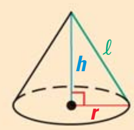
Cylinder



Surface Area $= 2\pi r^2 + 2\pi rh$

Volume $= \pi r^2 h$

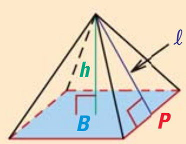
Cone



Surface Area $= \pi r^2 + \pi r\ell$

Volume $= \frac{1}{3}\pi r^2 h$

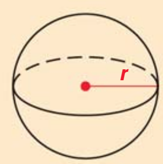
Pyramid



Surface Area $= B + \frac{1}{2}P\ell$

Volume $= \frac{1}{3}Bh$

Sphere



Surface Area $= 4\pi r^2$

Volume $= \frac{4}{3}\pi r^3$

EXAMPLE

Find the volume of a desktop computer with the dimensions shown.

Solution

STEP 1 **Find** the area of the base.

$B = 18 \cdot 60$

$= 1080$

STEP 2 **Calculate** the volume of the computer.

Volume $= Bh$	**Formula for the volume of a prism**
$= (1080)(45)$	**Substitute 1080 for B and 45 for h.**
$= 48{,}600$	**Simplify.**

▶ The volume of the computer is 48,600 cubic centimeters.

EXAMPLE 2 Use an exponential model

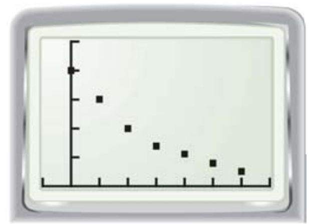
COOLING RATES You are storing leftover chili in a freezer. The table shows the chili's temperature y (in degrees Fahrenheit) after x minutes in the freezer. Use a graphing calculator to find a model for the data.

x	0	10	20	30	40	50	60
y	100	75	50	35	28	20	15

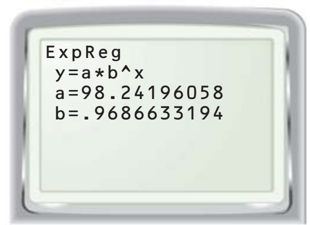
ANOTHER WAY
For an extension of the problem in Example 2, turn to page 781 for the **Problem Solving Workshop**.

Solution

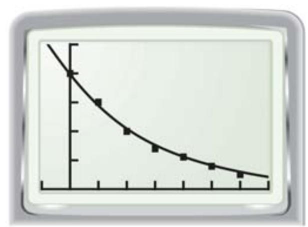
STEP 1 **Make** a scatter plot. The points fall rapidly at first and then begin to level off. This suggests an exponential decay model.



STEP 2 **Use** the exponential regression feature to find an equation of the model.



STEP 3 **Graph** the model along with the data to verify that the model fits the data well.



▸ A model for the data is $y = 98.2(0.969)^x$.

Animated Algebra at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

Use a graphing calculator to find a model for the data. Then graph the model and the data in the same coordinate plane.

1.

x	10	20	30	40	50	60	70	80
y	23.1	28.9	34.9	43.7	53.2	66.5	80.8	99.3

2.

x	0	1	2	3	4	5	6	7
y	33	41	52	68	80	89	102	118

MIXED REVIEW FOR TEKS

TAKS PRACTICE
classzone.com

Lessons 11.3–11.5

MULTIPLE CHOICE

1. **BIOLOGY** A biologist caught, measured, weighed, and then released eight Maine landlocked salmon. The table shows each fish's length x (in inches) and weight y (in pounds). Which equation is the best model for the data? Use a graphing calculator to find the answer. *TEKS 2A.1.B*

x	10.3	15.2	16.2	16.4
y	0.4	1.0	1.3	1.3
x	17.5	18.1	22	23.6
y	1.7	2.0	3.5	4.2

Ⓐ $y = 0.0688(1.20)^x$

Ⓑ $y = 0.525x^2 - 0.502x + 3.18$

Ⓒ $y = 0.332x - 3.84$

Ⓓ $y = 0.000321x^{3.01}$

2. **SHOPPING SURVEY** In a survey of 1022 people who shop online, 73% said that they do so because of the convenience. What is the approximate margin of error for the survey? *TEKS a.1*

Ⓕ $\pm 2.9\%$

Ⓖ $\pm 3.1\%$

Ⓗ $\pm 3.7\%$

Ⓙ $\pm 7.3\%$

3. **SHOE SIZE** The table shows the shoe size of a certain boy at different ages (in years). What is the most reasonable prediction for the boy's shoe size at age 17? Use a quadratic model obtained from a graphing calculator to find the answer. *TEKS 2A.1.B*

Age	6	7	8	10
Shoe size	5	6	7	9
Age	12	14	15	16
Shoe size	10	11	11	12

Ⓐ 10 Ⓑ 11

Ⓒ 12 Ⓓ 13

4. **SPORTS SURVEY** A local sports TV station wants to determine the average number of hours per week people in the viewing area watch sporting events on television. The station surveys people at a nearby sports stadium. Which type of sample is described? *TEKS a.6*

Ⓕ Self-selected Ⓖ Systematic

Ⓗ Convenience Ⓙ Random

5. **SUPERMARKET SURVEY** A survey shows that the time spent by shoppers in a certain supermarket is normally distributed with a mean of 45 minutes and a standard deviation of 12 minutes. What is the approximate probability that a randomly chosen shopper spends between 45 and 69 minutes in the supermarket? *TEKS a.1*

Ⓐ 0.475 Ⓑ 0.4985

Ⓒ 0.95 Ⓓ 0.997

6. **STUDENT SURVEY** A survey claims that 15% of high school students prefer having gym class during the last period of the day. The survey reports a margin of error of $\pm 5\%$. About how many students were surveyed? *TEKS a.1*

Ⓕ 20 Ⓖ 44

Ⓗ 133 Ⓙ 400

GRIDDED ANSWER

7. **TREE HEIGHT** At a tree nursery, the heights of scotch pine trees are normally distributed with a mean of 200 centimeters and a standard deviation of 20 centimeters. Find the percent of scotch pine trees that have a height of at least 220 centimeters. Round your answer to the nearest whole number. *TEKS a.1*

EXAMPLE 3
on p. 956
for Exs. 21–29

SIMPLIFYING EXPRESSIONS **Rewrite the expression without double angles or half angles, given that $0 < \theta < \frac{\pi}{2}$. Then simplify the expression.**

21. $\dfrac{\cos 2\theta}{1 - 2\sin^2\theta}$

22. $\dfrac{\sin 2\theta}{2\cos\theta}$

23. $(1 - \tan\theta)\tan 2\theta$

24. $\dfrac{\cos 2\theta}{\sin\theta - \cos\theta}$

25. $\dfrac{-\tan\frac{\theta}{2}}{\csc\theta}$

26. $2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

27. **TAKS REASONING** Which expression is equivalent to $\cot\theta + \tan\theta$?

Ⓐ $\csc 2\theta$ Ⓑ $2\csc 2\theta$ Ⓒ $\sec 2\theta$ Ⓓ $2\sec 2\theta$

ERROR ANALYSIS ***Describe*** **and correct the error in simplifying the expression.**

28.

$$\frac{\cos 2x}{\cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x}$$
$$= \frac{1}{\cos^2 x}$$
$$= \sec^2 x$$

29.

$$\sin 22.5° = \sin \frac{1}{2}(45°)$$
$$= 2\sin 45° \cos 45°$$
$$= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= 1$$

EXAMPLE 5
on p. 958
for Exs. 30–35

VERIFYING IDENTITIES **Verify the identity.**

30. $2\cos^2\theta = 1 + \cos 2\theta$

31. $\sin 3\theta = \sin\theta\,(4\cos^2\theta - 1)$

32. $\frac{1}{2}\sin\frac{2x}{3} = \sin\frac{x}{3}\cos\frac{x}{3}$

33. $2\sin^2 x\tan\frac{x}{2} = 2\sin x - \sin 2x$

34. $-\dfrac{\cos 2\theta}{\sin\theta} = 2\sin\theta - \csc\theta$

35. $\cos 4\theta = \cos^4\theta - 6\sin^2\theta\cos^2\theta + \sin^4\theta$

EXAMPLE 6
on p. 958
for Exs. 36–41

SOLVING EQUATIONS **Solve the equation for $0 \le x < 2\pi$.**

36. $\sin\frac{x}{2} = 1$

37. $2\cos\frac{x}{2} + 1 = 0$

38. $\tan x - \tan 2x = 0$

39. $\tan\frac{x}{2} = \dfrac{2 - \sqrt{2}}{2\sin x}$

40. $\cos 2x = -2\cos^2 x$

41. $2\sin 2x\sin x = 3\cos x$

EXAMPLE 7
on p. 958
for Exs. 42–47

FINDING GENERAL SOLUTIONS **Find the general solution of the equation.**

42. $\cos\frac{x}{2} = 1$

43. $\tan\frac{x}{2} = \sin x$

44. $\sin 2x = \sin x$

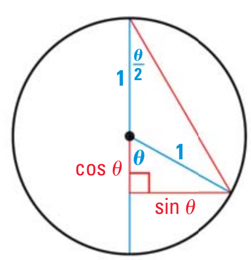
45. $\cos 2x + \cos x = 0$

46. $\cos\frac{x}{2} + \sin x = 0$

47. $\sin\frac{x}{2} + \cos x = 0$

48. **REASONING** Show that the three double-angle formulas for cosine are equivalent.

49. **CHALLENGE** Use the diagram shown at the right to derive the formulas for $\sin\frac{\theta}{2}$, $\cos\frac{\theta}{2}$, and $\tan\frac{\theta}{2}$ when θ is an acute angle.



○ = WORKED-OUT SOLUTIONS
= TAKS PRACTICE AND REASONING