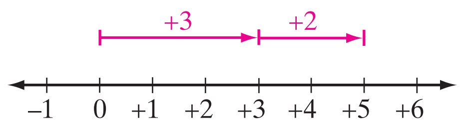
Solution Start at 0 and move 3 units to the right to $+3$; then move 2 more units to the right, arriving at $+5$.



Calculator Solution ENTER: 3 [+] 2 [ENTER]

DISPLAY:

```
3+2
                5
```

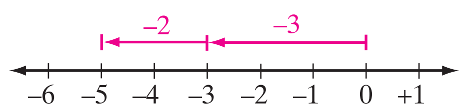
Answer 5

The sum of two positive integers is the same as the sum of two whole numbers. The sum $+5$ is a number whose absolute value is the sum of the absolute values of $+3$ and $+2$ and whose sign is the same as the sign of $+3$ and $+2$.

EXAMPLE 2

Add -3 and -2.

Solution Start at 0 and move 3 units to the left to -3: then move 2 more units to the left, arriving at -5.



Calculator Solution ENTER: [(-)] 3 [+] [(-)] 2 [ENTER]

DISPLAY:

```
-3+-2
               -5
```

Answer -5

The sum -5 is a number whose absolute value is the sum of the absolute values of -3 and -2 and whose sign is the same as the sign of -3 and -2.

Examples 1 and 2 illustrate that the sum of two numbers with the same sign is a number whose absolute value is the sum of the absolute values of the numbers and whose sign is the sign of the numbers.

Procedure

To add two numbers that have the same sign:

1. Find the sum of the absolute values.
2. Give the sum the common sign.

EXERCISES

Writing About Mathematics

1. Explain why the sum of a and 4 can be written as $a + 4$ or as $4 + a$.
2. Explain why 3 less than a can be written as $a - 3$ but not as $3 - a$.

Developing Skills

In 3–20, use mathematical symbols to translate the verbal phrases into algebraic language.

3. y plus 8
4. 4 minus r
5. 7 times x
6. x times 7
7. x divided by 10
8. 10 divided by x
9. c decreased by 6
10. one-tenth of w
11. the product of x and y
12. 5 less than d
13. 8 divided by y
14. y multiplied by 10
15. t more than w
16. one-third of z
17. twice the difference of p and q
18. a number that exceeds m by 4
19. 5 times x, increased by 2
20. 10 decreased by twice a

In 21–30, using the letter n to represent "number," write each verbal phrase in algebraic language.

21. a number increased by 2
22. 20 more than a number
23. 8 increased by a number
24. a number decreased by 6
25. 2 less than a number
26. 3 times a number
27. three-fourths of a number
28. 4 times a number, increased by 3
29. 3 less than twice a number
30. 10 times a number, decreased by 2

In 31–34, use the given variable(s) to write an algebraic expression for each verbal phrase.

31. the number of baseball cards, if b cards are added to a collection of 100 cards
32. Hector's height, if he was h inches tall before he grew 2 inches
33. the total cost of n envelopes that cost \$0.39 each
34. the cost of one pen, if 12 pens cost d dollars

3-2 TRANSLATING VERBAL PHRASES INTO SYMBOLS

A knowledge of arithmetic is important in algebra. Since the variables represent numbers that are familiar to you, it will be helpful to solve each problem by first using a simpler related problem; that is, relate similar arithmetic problems to the given algebraic one.

a. Find the next number in the list.

b. Write a rule or explain how the next number is determined.

c. What is the 25th number in the list?

27. Each of the numbers given below is different from the others, that is, it belongs to a set of numbers to which the others do not belong. Explain why each is different.

3 6 9 35

28. Two oranges cost as much as five bananas. One orange costs the same as a banana and an apple. How many apples cost the same as three bananas?

Exploration

STEP 1. Write a three-digit multiple of 11 by multiplying any whole number from 10 to 90 by 11. Add the digits in the hundreds and the ones places. If the sum is greater than or equal to 11, subtract 11. Compare this result to the digit in the tens place. Repeat the procedure for other three-digit multiples of 11.

STEP 2. Write a three-digit number that is not a multiple of 11 by adding any counting number less than 11 to a multiple of 11 used in step 1. Add the digits in the hundreds and the ones places. If the sum is greater than or equal to 11, subtract 11. Compare this result to the digit in the tens place. Repeat the procedure with another number.

STEP 3. Based on steps 1 and 2, can you suggest a way of determining whether or not a three-digit number is divisible by 11?

STEP 4. Write a four-digit multiple of 11 by multiplying any whole number from 91 to 909 by 11. Add the digits in the hundreds and ones places. Add the digits in the thousands and tens places. If one sum is greater than or equal to 11, subtract 11. Compare these results. Repeat the procedure for another four-digit multiple of 11.

STEP 5. Write a four-digit number that is not a multiple of 11 by adding any counting number less than 11 to a multiple of 11 used in step 4. Add the digits in the hundreds place and ones place. Add the digits in the thousands place and tens place. If one sum is greater than or equal to 11, subtract 11. Compare these results. Repeat the procedure starting with another number.

STEP 6. Based on steps 4 and 5, can you suggest a way of determining whether or not a four-digit number is divisible by 11?

STEP 7. Write a rule for determining whether or not any whole number is divisible by 11.

$$\overbrace{a : \underbrace{b = c}_{\text{means}} : d}^{}\quad\text{extremes}\qquad\text{or}\qquad \frac{a}{b} = \frac{c}{d}$$

$$a : \overbrace{b = c}^{\text{means}} : d \;\;(\text{extremes: } a, d) \qquad \text{or} \qquad \overset{\text{extreme}}{\underset{\text{mean}}{\frac{a}{b}}} = \overset{\text{mean}}{\underset{\text{extreme}}{\frac{c}{d}}}$$

In the proportion, $4 : 20 = 1 : 5$, the product of the means, 20(1), is equal to the product of the extremes, 4(5).

In the proportion, $\frac{5}{15} = \frac{10}{30}$, the product of the means, 15(10), is equal to the product of the extremes, 5(30).

In any proportion $\frac{a}{b} = \frac{c}{d}$, we can show that the product of the means is equal to the product of the extremes, $ad = bc$. Since $\frac{a}{b} = \frac{c}{d}$ is an equation, we can multiply both members by bd, the least common denominator of the fractions in the equation.

$$\frac{a}{b} = \frac{c}{d}$$
$$bd\left(\frac{a}{b}\right) = bd\left(\frac{c}{d}\right)$$
$$\cancel{b}d\left(\frac{a}{\cancel{b}}\right) = b\cancel{d}\left(\frac{c}{\cancel{d}}\right)$$
$$d \cdot a = b \cdot c$$
$$ad = bc$$

Therefore, we have shown that the following statement is always true:

► **In a proportion, the product of the means is equal to the product of the extremes.**

Notice that the end result, $ad = bc$, is the result of multiplying the terms that are cross-wise from each other:

$$\frac{a}{b} \times \frac{c}{d}$$

This is called **cross-multiplying**, which we have just shown to be valid.

If the product of two cross-wise terms is called a **cross product**, then the following is also true:

► **In a proportion, the cross products are equal.**

If a, b, c, and d are nonzero numbers and $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. There are three other proportions using a, b, c and d for which $ad = bc$.

$$\frac{a}{c} = \frac{b}{d} \qquad \frac{d}{b} = \frac{c}{a} \qquad \frac{d}{c} = \frac{b}{a}$$

For example, we know that $\frac{6}{4} = \frac{15}{10}$ is a proportion because $6(10) = 4(15)$. Therefore, each of the following is also a proportion.

$$\frac{6}{15} = \frac{4}{10} \qquad \frac{10}{4} = \frac{15}{6} \qquad \frac{10}{15} = \frac{4}{6}$$

EXERCISES

Writing About Mathematics

1. A right prism has bases that are regular hexagons. The measure of each of the six sides of the hexagon is represented by a and the height of the solid by $2a$.

a. How many surfaces make up the solid?

b. Describe the shape of each face

c. Express the dimensions and the area of each face in terms of a.

2. A regular hexagon can be divided into six equilateral triangles. If the length of a side of an equilateral triangle is a, the height is $\frac{\sqrt{3}}{2}a$. For the rectangular solid described in exercise 1:

a. Express the area of each base in terms of a.

b. Express the surface area in terms of a.

Developing Skills

In 3–9, find the surface area of each rectangular prism or cylinder to the nearest *tenth of a square unit*.

3. Bases are circles with a radius of 18 inches. The height of the cylinder is 48 inches.

4. Bases are squares with sides that measure 27 inches. The height is 12 inches.

5. Bases are rectangles with dimensions of 8 feet by 12 feet. The height is 3 feet.

6. Bases are isosceles trapezoids with parallel sides that measure 15 centimeters and 25 centimeters. The distance between the parallel sides is 12 centimeters and the length of each of the equal sides is 13 centimeters. The height of the prism is 20 centimeters.

7. Bases are isosceles right triangles with legs that measure 5 centimeters. The height is 7 centimeters.

8. Bases are circles with a diameter of 42 millimeters. The height is 3.4 centimeters.

9. Bases are circles each with an area of 314.16 square feet. The height is 15 feet.

Applying Skills

10. Agatha is using scraps of wallpaper to cover a box that is a rectangular solid whose base measures 8 inches by 5 inches and whose height is 3 inches. The box is open at the top. How many square inches of wallpaper does she need to cover the outside of the box?

11. Agatha wants to make a cardboard lid for the box described in exercise 10. Her lid will be a rectangular solid that is open at the top, with a base that is slightly larger than that of the box. She makes the base of the lid 8.1 inches by 5.1 inches with a height of 2.0 inches. To the nearest tenth of a square inch, how much wallpaper does she need to cover the outside of the lid?

To understand volume, count the cubes in the diagram on the previous page. There are 3 layers, each containing 8 cubes, for a total of 24 cubes. Note that 3 corresponds to the height, h, that 8 corresponds to the area of the base, B, and that 24 corresponds to the volume V in cubic units.

A cube that measures 1 foot on each side represents 1 cubic foot. Each face of the cube is 1 square foot. Since each foot can be divided into 12 inches, the area of each face is 12×12 or 144 square inches and the volume of the cube is $12 \times 12 \times 12$ or 1,728 cubic inches.

1 square foot = 144 square inches

1 cubic foot = 1,728 cubic inches

A cube that measures 1 meter on each side represents 1 cubic meter. Each face of the cube is 1 square meter. Since each meter can be divided into 100 centimeters, the area of each face is 100×100 or 10,000 square centimeters and the volume of the cube is $100 \times 100 \times 100$ or 1,000,000 cubic centimeters.

1 square meter = 10,000 square centimeters

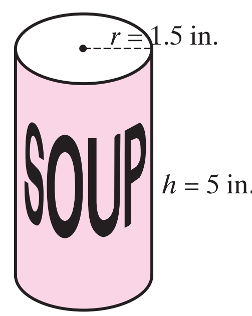
1 cubic meter = 1,000,000 cubic centimeters

EXAMPLE 1

A cylindrical can of soup has a radius of 1.5 inches and a height of 5 inches. Find the volume of this can:

a. in terms of π.

b. to the nearest cubic inch.



Solution This can is a right circular cylinder. Use the formula $V = Bh$. Since the base is a circle whose area equals πr^2, the area B of the base can be replaced by πr^2.

a. $V = Bh = (\pi r^2)h = \pi(1.5)^2(5) = 11.25\pi$

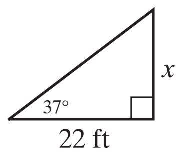
b. When we use a calculator to evaluate 11.25π, the calculator gives 35.34291735 as a rational approximation. This answer rounded to the nearest integer is 35.

Answers **a.** 11.25π cu in. **b.** 35 cu in.

Developing Skills

In 3–10: In each given right triangle, find to the *nearest foot* the length of the side marked x; or find to the *nearest degree* the measure of the angle marked x. Assume that each measure is given to the nearest foot or to the nearest degree.

3.



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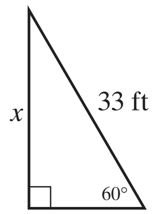

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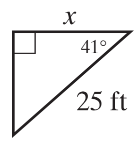

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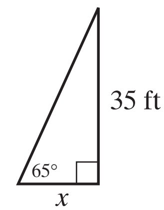

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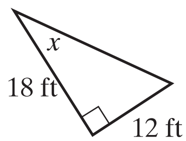

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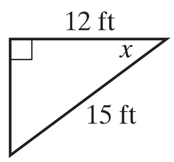

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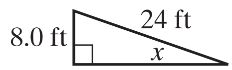

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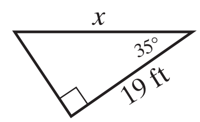


11. In $\triangle ABC$, $m\angle A = 42$, $AB = 14$, and $\overline{BD}$ is the altitude to $\overline{AC}$. Find BD to the *nearest tenth*.

12. In $\triangle ABC$, $\overline{AC} \cong \overline{BC}$, $m\angle A = 50$, and $AB = 30$. Find to the *nearest tenth* the length of the altitude from vertex C.

13. The legs of a right triangle measure 84 and 13. Find to the *nearest degree* the measure of the smallest angle of this triangle.

14. The length of hypotenuse $\overline{AB}$ of right triangle ABC is twice the length of leg $\overline{BC}$. Find the number of degrees in $\angle B$.

15. The longer side of a rectangle measures 10, and a diagonal makes an angle of 27° with this side. Find to the *nearest integer* the length of the shorter side.

16. In rectangle $ABCD$, diagonal $\overline{AC}$ measures 11 and side $\overline{AB}$ measures 7. Find to the *nearest degree* the measure of $\angle CAB$.

17. In right triangle ABC, $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$, $AB = 25$, and $AC = 20$. Find lengths AD, DB, and CD to the *nearest integer* and the measure of $\angle B$ to the *nearest degree*.

18. The lengths of the diagonals of a rhombus are 10 and 24.

a. Find the perimeter of the rhombus.

b. Find to the *nearest degree* the measure of the angle that the longer diagonal makes with a side of the rhombus.

19. The altitude to the hypotenuse of a right triangle ABC divides the hypotenuse into segments whose measures are 9 and 4. The measure of the altitude is 6. Find to the *nearest degree* the measure of the smaller acute angle of $\triangle ABC$.

7. The product of 3.40×10^{-3} and 8.50×10^{2} equals

(1) 2.89×10^{0} (2) 2.89×10^{1} (3) 2.89×10^{2} (4) 2.89×10^{-1}

8. The measure of $\angle A$ is 12° less than twice the measure of its complement. What is the measure of $\angle A$?

(1) 51° (2) 34° (3) 39° (4) 56°

9. The y-intercept of the graph of $2x + 3y = 6$ is

(1) 6 (2) 2 (3) 3 (4) -3

10. When $2a^2 - 5a$ is subtracted from $5a^2 + 1$, the difference is

(1) $-3a^2 - 5a - 1$
(2) $3a^2 - 5a + 1$
(3) $3a^2 + 6a$
(4) $3a^2 + 5a + 1$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

11. A straight line has a slope of -3 and contains the point $(0, 6)$. What are the coordinates of the point at which the line intersects the x-axis?

12. Last year the Edwards family spent \$6,200 for food. This year, the cost of food for the family was \$6,355. What was the percent of increase in the cost of food for the Edwards family?

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

13. Dana paid \$3.30 for 2 muffins and a cup of coffee. At the same prices, Damion paid \$5.35 for 3 muffins and 2 cups of coffee. What is the cost of a muffin and of a cup of coffee?

14. In trapezoid $ABCD$, $\overline{BC} \perp \overline{AB}$ and $\overline{BC} \perp \overline{DC}$. If $AB = 54.8$ feet, $DC = 37.2$ feet, and $BC = 15.8$ feet, find to the *nearest degree* the measure of $\angle A$.

11-1 FACTORS AND FACTORING

When two numbers are multiplied, the result is called their **product**. The numbers that are multiplied are *factors* of the product. Since $3(5) = 15$, the numbers 3 and 5 are factors of 15.

Factoring a number is the process of finding those numbers whose product is the given number. Usually, when we factor, we are finding the factors of an integer and we find only those factors that are integers. We call this **factoring over the set of integers**.

Factors of a product can be found by using division. Over the set of integers, if the divisor and the quotient are both integers, then they are factors of the dividend. For example, $35 \div 5 = 7$. Thus, $35 = 5(7)$, and 5 and 7 are factors of 35.

Every positive integer that is the product of two positive integers is also the product of the opposites of those integers.

$$+21 = (+3)(+7) \qquad\qquad +21 = (-3)(-7)$$

Every negative integer that is the product of a positive integer and a negative integer is also the product of the opposites of those integers.

$$-21 = (+3)(-7) \qquad\qquad -21 = (-3)(+7)$$

Usually, when we factor a positive integer, we write only the positive integral factors.

Two factors of any number are 1 and the number itself. To find other integral factors, if they exist, we use division, as stated above. We let the number being factored be the dividend, and we divide this number in turn by the whole numbers 2, 3, 4, and so on. If the quotient is an integer, then both the divisor and the quotient are factors of the dividend.

For example, use a calculator to find the integral factors of 126. We will use integers as divisors and look for quotients that are integers. Pairs of factors of 126 are listed to the right of the quotients.

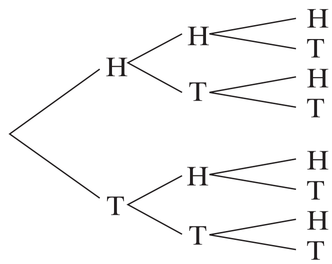
Quotients	*Pairs of Factors of 126*	*Quotients*	*Pairs of Factors of 126*
$126 \div 1 = 126$	$1 \cdot 126$	$126 \div 7 = 18$	$7 \cdot 18$
$126 \div 2 = 63$	$2 \cdot 63$	$126 \div 8 = 15.75$	—
$126 \div 3 = 42$	$3 \cdot 42$	$126 \div 9 = 14$	$9 \cdot 14$
$126 \div 4 = 31.5$	—	$126 \div 10 = 12.6$	—
$126 \div 5 = 25.2$	—	$126 \div 11 = 11.\overline{45}$	—
$126 \div 6 = 21$	$6 \cdot 21$	$126 \div 12 = 10.5$	—

When the quotient is smaller than the divisor (here, $10.5 < 12$), we have found all possible positive integral factors.

The factors of 126 are 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, and 126.

Recall that a *prime number* is an integer greater than 1 that has no positive integral factors other than itself and 1. The first seven prime numbers are 2, 3, 5, 7, 11, 13, and 17. Integers greater than 1 that are not prime are called *composite numbers*.

Three Tosses of a Coin



Tree diagram

(H, H, H)
(H, H, T)
(H, T, H)
(H, T, T)

(T, H, H)
(T, H, T)
(T, T, H)
(T, T, T)

List of ordered triples

three-dimensional. Although such a graph can be drawn, it is too difficult at this time. We can conclude that:

1. Tree diagrams, or lists of ordered elements, are effective ways to indicate any compound event of two or more activities.
2. Graphs should be limited to ordered pairs, or to events consisting of exactly two activities.

EXAMPLE 1

The school cafeteria offers four types of salads, three types of beverages, and five types of desserts. If a lunch consists of one salad, one beverage, and one dessert, how many possible lunches can be chosen?

Solution By the counting principle, we multiply the number of possibilities for each choice:

$$4 \times 3 \times 5 = 12 \times 5 = 60 \text{ possible lunches } \textit{Answer}$$

Independent Events

The probability of rolling 5 on one toss of a die is $\frac{1}{6}$. What is the probability of rolling a pair of 5's when two dice are tossed?

When we roll two dice, the number obtained on the second die is in no way determined by of the result obtained on the first die. When we toss two coins, the face that shows on the second coin is in no way determined by the face that shows on the first coin.

When the result of one activity in no way influences the result of a second activity, the results of these activities are called **independent events**. In cases where two events are independent, we may extend the counting principle to find the probability that both independent events occur at the same time.