Solution

First, we will write the equation in slope-intercept form to find the slope.

$$\begin{aligned}5x+3y&=1\\3y&=-5x+1\\y&=-\tfrac{5}{3}x+\tfrac{1}{3}\end{aligned}$$

The slope is $m=-\frac{5}{3}$. The y-intercept is $\frac{1}{3}$, but that really does not enter into our problem, as the only thing we need for two lines to be parallel is the same slope. The one exception is that if the y-intercepts are the same, then the two lines are the same line. The next step is to use this slope and the given point with the point-slope formula.

$$\begin{aligned}y-5&=-\tfrac{5}{3}(x-3)\\y-5&=-\tfrac{5}{3}x+5\\y&=-\tfrac{5}{3}x+10\end{aligned}$$

The equation of the line is $y=-\frac{5}{3}x+10$. See Figure 7.

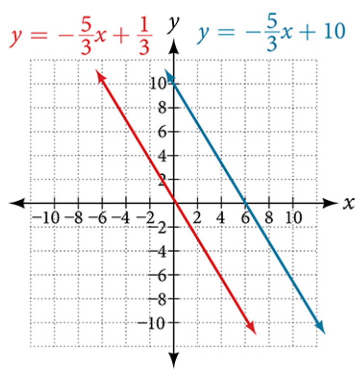

Figure 7

TRY IT #12 Find the equation of the line parallel to $5x=7+y$ and passing through the point $(-1,-2)$.

EXAMPLE 16

Finding the Equation of a Line Perpendicular to a Given Line Passing Through a Given Point

Find the equation of the line perpendicular to $5x-3y+4=0$ and passing through the point $(-4,1)$.

Solution

The first step is to write the equation in slope-intercept form.

$$\begin{aligned}5x-3y+4&=0\\-3y&=-5x-4\\y&=\tfrac{5}{3}x+\tfrac{4}{3}\end{aligned}$$

We see that the slope is $m=\frac{5}{3}$. This means that the slope of the line perpendicular to the given line is the negative reciprocal, or $-\frac{3}{5}$. Next, we use the point-slope formula with this new slope and the given point.

$$\begin{aligned}y-1&=-\tfrac{3}{5}(x-(-4))\\y-1&=-\tfrac{3}{5}x-\tfrac{12}{5}\\y&=-\tfrac{3}{5}x-\tfrac{12}{5}+\tfrac{5}{5}\\y&=-\tfrac{3}{5}x-\tfrac{7}{5}\end{aligned}$$

Solution

ⓐ To evaluate $f(2)$, locate the point on the curve where $x = 2$, then read the y-coordinate of that point. The point has coordinates $(2, 1)$, so $f(2) = 1$. See Figure 5.

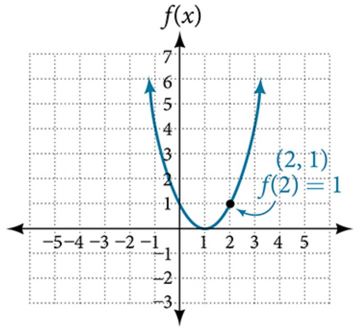

Figure 5

ⓑ To solve $f(x) = 4$, we find the output value 4 on the vertical axis. Moving horizontally along the line $y = 4$, we locate two points of the curve with output value 4: $(-1, 4)$ and $(3, 4)$. These points represent the two solutions to $f(x) = 4$: -1 or 3. This means $f(-1) = 4$ and $f(3) = 4$, or when the input is -1 or 3, the output is 4. See Figure 6.

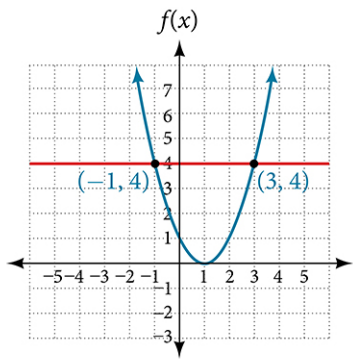

Figure 6

TRY IT #8 Using Figure 4, solve $f(x) = 1$.

Determining Whether a Function is One-to-One

Some functions have a given output value that corresponds to two or more input values. For example, in the stock chart shown in the figure at the beginning of this chapter, the stock price was $1000 on five different dates, meaning that there were five different input values that all resulted in the same output value of $1000.

However, some functions have only one input value for each output value, as well as having only one output for each input. We call these functions one-to-one functions. As an example, consider a school that uses only letter grades and decimal equivalents, as listed in Table 12.

Letter grade	Grade point average
A	4.0
B	3.0
C	2.0

Table 12

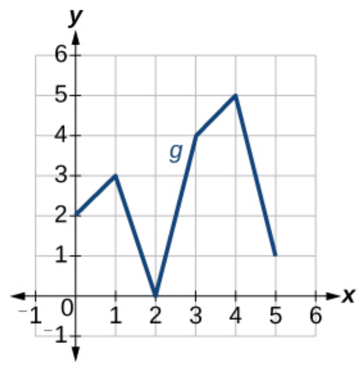

Figure 5

42. $f\,(g(3))$

43. $f\,(g(1))$

44. $g\,(f(1))$

45. $g\,(f(0))$

46. $f\,(f(5))$

47. $f\,(f(4))$

48. $g\,(g(2))$

49. $g\,(g(0))$

For the following exercises, use graphs of $f(x)$, shown in [Figure 6](#), $g(x)$, shown in [Figure 7](#), and $h(x)$, shown in [Figure 8](#), to evaluate the expressions.

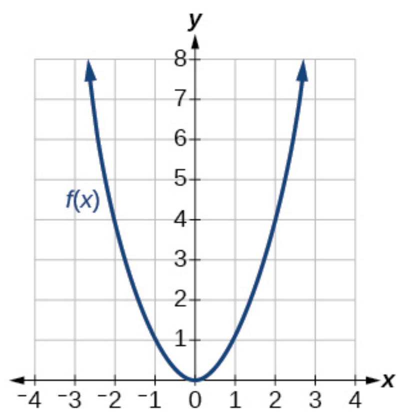

Figure 6

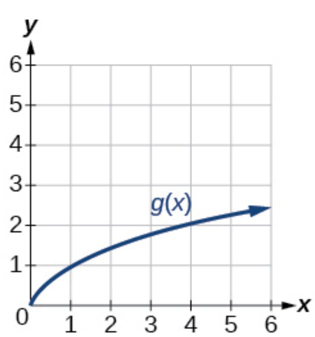

Figure 7

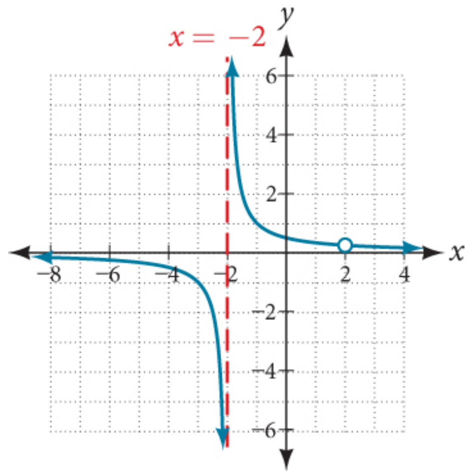

Figure 11

The graph of this function will have the vertical asymptote at $x = -2$, but at $x = 2$ the graph will have a hole.

TRY IT #5 Find the vertical asymptotes and removable discontinuities of the graph of $f(x) = \frac{x^2-25}{x^3-6x^2+5x}$.

Identifying Horizontal Asymptotes of Rational Functions

While vertical asymptotes describe the behavior of a graph as the *output* gets very large or very small, horizontal asymptotes help describe the behavior of a graph as the *input* gets very large or very small. Recall that a polynomial's end behavior will mirror that of the leading term. Likewise, a rational function's end behavior will mirror that of the ratio of the function that is the ratio of the leading terms.

There are three distinct outcomes when checking for horizontal asymptotes:

Case 1: If the degree of the denominator > degree of the numerator, there is a horizontal asymptote at $y = 0$.

$$\text{Example: } f(x) = \frac{4x+2}{x^2+4x-5}$$

In this case, the end behavior is $f(x) \approx \frac{4x}{x^2} = \frac{4}{x}$. This tells us that, as the inputs increase or decrease without bound, this function will behave similarly to the function $g(x) = \frac{4}{x}$, and the outputs will approach zero, resulting in a horizontal asymptote at $y = 0$. See Figure 12. Note that this graph crosses the horizontal asymptote.

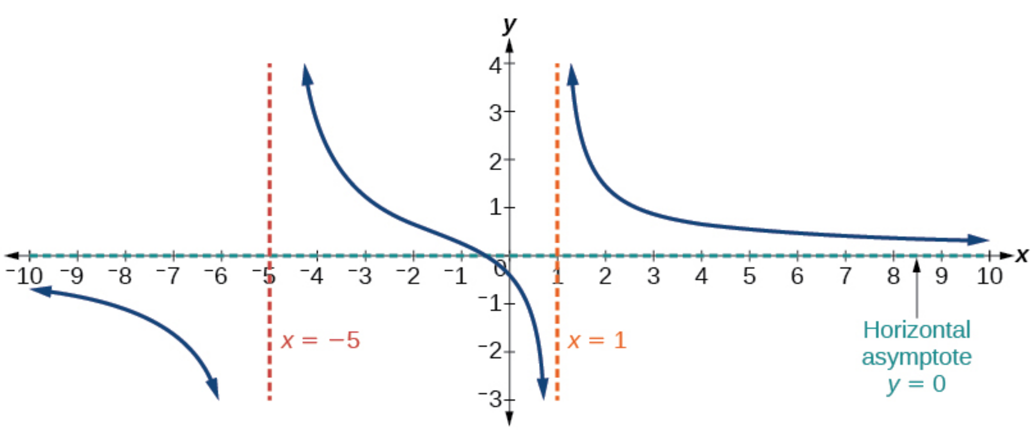

Figure 12 Horizontal asymptote $y = 0$ when $f(x) = \frac{p(x)}{q(x)},\ q(x) \neq 0$ where degree of $p <$ degree of q.

Case 2: If the degree of the denominator < degree of the numerator by one, we get a slant asymptote.

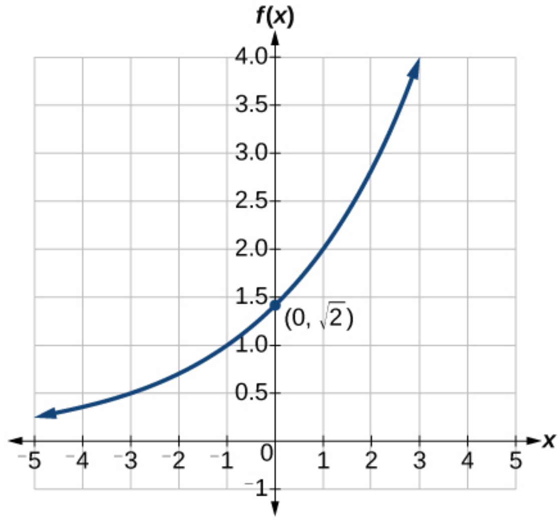

Figure 6

HOW TO

Given two points on the curve of an exponential function, use a graphing calculator to find the equation.

1. Press **[STAT].**
2. Clear any existing entries in columns **L1** or **L2.**
3. In **L1**, enter the *x*-coordinates given.
4. In **L2**, enter the corresponding *y*-coordinates.
5. Press **[STAT]** again. Cursor right to **CALC**, scroll down to **ExpReg (Exponential Regression)**, and press **[ENTER].**
6. The screen displays the values of *a* and *b* in the exponential equation $y = a \cdot b^x$.

EXAMPLE 7

Using a Graphing Calculator to Find an Exponential Function

Use a graphing calculator to find the exponential equation that includes the points $(2, 24.8)$ and $(5, 198.4)$.

Solution

Follow the guidelines above. First press **[STAT]**, **[EDIT]**, **[1: Edit…],** and clear the lists **L1** and **L2**. Next, in the **L1** column, enter the *x*-coordinates, 2 and 5. Do the same in the **L2** column for the *y*-coordinates, 24.8 and 198.4.

Now press **[STAT]**, **[CALC]**, **[0: ExpReg]** and press **[ENTER]**. The values $a = 6.2$ and $b = 2$ will be displayed. The exponential equation is $y = 6.2 \cdot 2^x$.

TRY IT #7 Use a graphing calculator to find the exponential equation that includes the points (3, 75.98) and (6, 481.07).

Applying the Compound-Interest Formula

Savings instruments in which earnings are continually reinvested, such as mutual funds and retirement accounts, use **compound interest**. The term *compounding* refers to interest earned not only on the original value, but on the accumulated value of the account.

The **annual percentage rate (APR)** of an account, also called the **nominal rate**, is the yearly interest rate earned by an investment account. The term *nominal* is used when the compounding occurs a number of times other than once per year. In fact, when interest is compounded more than once a year, the effective interest rate ends up being *greater* than the nominal rate! This is a powerful tool for investing.

TRY IT #5 Given that $\sin\ \alpha = -\frac{4}{5}$ and α lies in quadrant IV, find the exact value of $\cos\ \left(\frac{\alpha}{2}\right)$.

EXAMPLE 9

Finding the Measurement of a Half Angle

Now, we will return to the problem posed at the beginning of the section. A bicycle ramp is constructed for high-level competition with an angle of θ formed by the ramp and the ground. Another ramp is to be constructed half as steep for novice competition. If $\tan\ \theta = \frac{5}{3}$ for higher-level competition, what is the measurement of the angle for novice competition?

Solution

Since the angle for novice competition measures half the steepness of the angle for the high level competition, and $\tan\ \theta = \frac{5}{3}$ for high competition, we can find $\cos\ \theta$ from the right triangle and the Pythagorean theorem so that we can use the half-angle identities. See Figure 4.

$$\begin{aligned} 3^2 + 5^2 &= 34 \\ c &= \sqrt{34} \end{aligned}$$

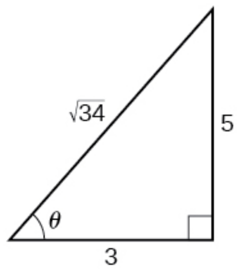

Figure 4

We see that $\cos\ \theta = \frac{3}{\sqrt{34}} = \frac{3\sqrt{34}}{34}$. We can use the half-angle formula for tangent: $\tan\ \frac{\theta}{2} = \sqrt{\frac{1-\cos\ \theta}{1+\cos\ \theta}}$. Since $\tan\ \theta$ is in the first quadrant, so is $\tan\ \frac{\theta}{2}$.

$$\begin{aligned} \tan\ \frac{\theta}{2} &= \sqrt{\frac{1-\frac{3\sqrt{34}}{34}}{1+\frac{3\sqrt{34}}{34}}} \\ &= \sqrt{\frac{\frac{34-3\sqrt{34}}{34}}{\frac{34+3\sqrt{34}}{34}}} \\ &= \sqrt{\frac{34-3\sqrt{34}}{34+3\sqrt{34}}} \\ &\approx 0.57 \end{aligned}$$

We can take the inverse tangent to find the angle: $\tan^{-1}(0.57) \approx 29.7°$. So the angle of the ramp for novice competition is $\approx 29.7°$.

MEDIA

Access these online resources for additional instruction and practice with double-angle, half-angle, and reduction formulas.

Double-Angle Identities (http://openstax.org/l/doubleangiden)
Half-Angle Identities (http://openstax.org/l/halfangleident)

76. A yield sign measures 30 inches on all three sides. What is the area of the sign?

77. Naomi bought a dining table whose top is in the shape of a triangle. Find the area of the table top if two of the sides measure 4 feet and 4.5 feet, and the smaller angles measure 32° and 42°, as shown in Figure 35.

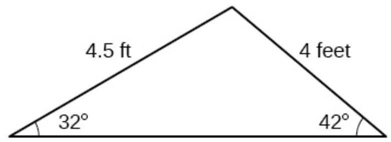

Figure 35

10.2 Non-right Triangles: Law of Cosines

Learning Objectives

In this section, you will:

- Use the Law of Cosines to solve oblique triangles.
- Solve applied problems using the Law of Cosines.
- Use Heron's formula to find the area of a triangle.

Suppose a boat leaves port, travels 10 miles, turns 20 degrees, and travels another 8 miles as shown in Figure 1. How far from port is the boat?

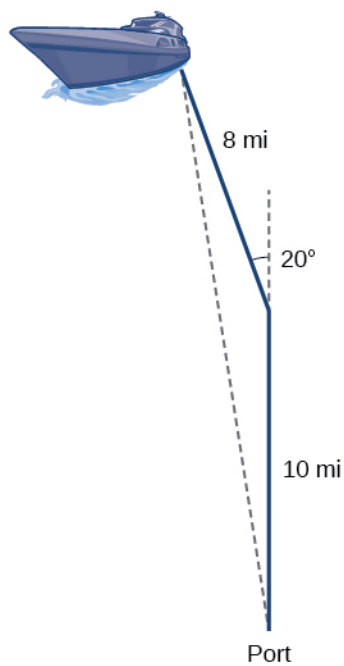

Figure 1

Unfortunately, while the Law of Sines enables us to address many non-right triangle cases, it does not help us with triangles where the known angle is between two known sides, a SAS (side-angle-side) triangle, or when all three sides are known, but no angles are known, a SSS (side-side-side) triangle. In this section, we will investigate another tool for solving oblique triangles described by these last two cases.

Using the Law of Cosines to Solve Oblique Triangles

The tool we need to solve the problem of the boat's distance from the port is the **Law of Cosines**, which defines the relationship among angle measurements and side lengths in oblique triangles. Three formulas make up the Law of Cosines. At first glance, the formulas may appear complicated because they include many variables. However, once the

Algebraic

For the following exercises, find the absolute value of the given complex number.

6. $5 + 3i$

7. $-7 + i$

8. $-3 - 3i$

9. $\sqrt{2} - 6i$

10. $2i$

11. $2.2 - 3.1i$

For the following exercises, write the complex number in polar form.

12. $2 + 2i$

13. $8 - 4i$

14. $-\frac{1}{2} - \frac{1}{2} i$

15. $\sqrt{3} + i$

16. $3i$

For the following exercises, convert the complex number from polar to rectangular form.

17. $z = 7 \operatorname{cis}\left(\frac{\pi}{6}\right)$

18. $z = 2 \operatorname{cis}\left(\frac{\pi}{3}\right)$

19. $z = 4 \operatorname{cis}\left(\frac{7\pi}{6}\right)$

20. $z = 7 \operatorname{cis}(25°)$

21. $z = 3 \operatorname{cis}(240°)$

22. $z = \sqrt{2} \operatorname{cis}(100°)$

For the following exercises, find $z_1 z_2$ in polar form.

23. $z_1 = 2\sqrt{3} \operatorname{cis}(116°);\quad z_2 = 2 \operatorname{cis}(82°)$

24. $z_1 = \sqrt{2} \operatorname{cis}(205°);\quad z_2 = 2\sqrt{2} \operatorname{cis}(118°)$

25. $z_1 = 3 \operatorname{cis}(120°);\quad z_2 = \frac{1}{4} \operatorname{cis}(60°)$

26. $z_1 = 3 \operatorname{cis}\left(\frac{\pi}{4}\right);\quad z_2 = 5 \operatorname{cis}\left(\frac{\pi}{6}\right)$

27. $z_1 = \sqrt{5} \operatorname{cis}\left(\frac{5\pi}{8}\right);\quad z_2 = \sqrt{15} \operatorname{cis}\left(\frac{\pi}{12}\right)$

28. $z_1 = 4 \operatorname{cis}\left(\frac{\pi}{2}\right);\quad z_2 = 2 \operatorname{cis}\left(\frac{\pi}{4}\right)$

For the following exercises, find $\frac{z_1}{z_2}$ in polar form.

29. $z_1 = 21 \operatorname{cis}(135°);\quad z_2 = 3 \operatorname{cis}(65°)$

30. $z_1 = \sqrt{2} \operatorname{cis}(90°);\quad z_2 = 2 \operatorname{cis}(60°)$

31. $z_1 = 15 \operatorname{cis}(120°);\quad z_2 = 3 \operatorname{cis}(40°)$

32. $z_1 = 6 \operatorname{cis}\left(\frac{\pi}{3}\right);\quad z_2 = 2 \operatorname{cis}\left(\frac{\pi}{4}\right)$

33. $z_1 = 5\sqrt{2} \operatorname{cis}(\pi);\quad z_2 = \sqrt{2} \operatorname{cis}\left(\frac{2\pi}{3}\right)$

34. $z_1 = 2 \operatorname{cis}\left(\frac{3\pi}{5}\right);\quad z_2 = 3 \operatorname{cis}\left(\frac{\pi}{4}\right)$

For the following exercises, find the powers of each complex number in polar form.

35. Find z^3 when $z = 5 \operatorname{cis}(45°)$.

36. Find z^4 when $z = 2 \operatorname{cis}(70°)$.

37. Find z^2 when $z = 3 \operatorname{cis}(120°)$.

38. Find z^2 when $z = 4 \operatorname{cis}\left(\frac{\pi}{4}\right)$.

39. Find z^4 when $z = \operatorname{cis}\left(\frac{3\pi}{16}\right)$.

40. Find z^3 when $z = 3 \operatorname{cis}\left(\frac{5\pi}{3}\right)$.

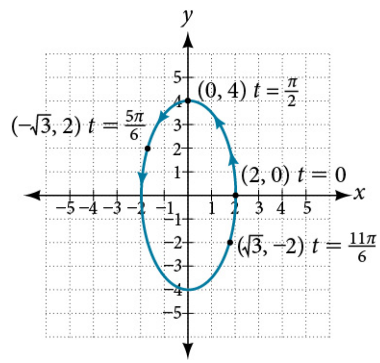

Figure 3

By the symmetry shown in the values of x and y, we see that the parametric equations represent an ellipse. The ellipse is mapped in a counterclockwise direction as shown by the arrows indicating increasing t values.

Analysis

We have seen that parametric equations can be graphed by plotting points. However, a graphing calculator will save some time and reveal nuances in a graph that may be too tedious to discover using only hand calculations.

Make sure to change the mode on the calculator to parametric (PAR). To confirm, the $Y =$ window should show

$$X_{1T} =$$
$$Y_{1T} =$$

instead of $Y_1 =$.

TRY IT #2 Graph the parametric equations: $x = 5\cos\ t$, $y = 3\sin\ t$.

EXAMPLE 3

Graphing Parametric Equations and Rectangular Form Together

Graph the parametric equations $x = 5\cos\ t$ and $y = 2\sin\ t$. First, construct the graph using data points generated from the parametric form. Then graph the rectangular form of the equation. Compare the two graphs.

Solution

Construct a table of values like that in Table 3.

t	$x = 5\cos\ t$	$y = 2\sin\ t$
0	$x = 5\cos(0) = 5$	$y = 2\sin(0) = 0$
1	$x = 5\cos(1) \approx 2.7$	$y = 2\sin(1) \approx 1.7$
2	$x = 5\cos(2) \approx -2.1$	$y = 2\sin(2) \approx 1.8$
3	$x = 5\cos(3) \approx -4.95$	$y = 2\sin(3) \approx 0.28$
4	$x = 5\cos(4) \approx -3.3$	$y = 2\sin(4) \approx -1.5$
5	$x = 5\cos(5) \approx 1.4$	$y = 2\sin(5) \approx -1.9$
-1	$x = 5\cos(-1) \approx 2.7$	$y = 2\sin(-1) \approx -1.7$

Table 3

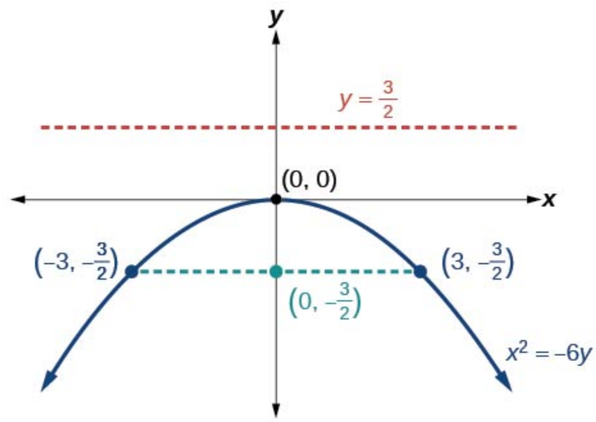

Figure 8

TRY IT #2 Graph $x^2 = 8y$. Identify and label the focus, directrix, and endpoints of the latus rectum.

Writing Equations of Parabolas in Standard Form

In the previous examples, we used the standard form equation of a parabola to calculate the locations of its key features. We can also use the calculations in reverse to write an equation for a parabola when given its key features.

HOW TO

Given its focus and directrix, write the equation for a parabola in standard form.

1. Determine whether the axis of symmetry is the *x*- or *y*-axis.
   a. If the given coordinates of the focus have the form $(p, 0)$, then the axis of symmetry is the *x*-axis. Use the standard form $y^2 = 4px$.
   b. If the given coordinates of the focus have the form $(0, p)$, then the axis of symmetry is the *y*-axis. Use the standard form $x^2 = 4py$.
2. Multiply $4p$.
3. Substitute the value from Step 2 into the equation determined in Step 1.

EXAMPLE 3

Writing the Equation of a Parabola in Standard Form Given its Focus and Directrix

What is the equation for the parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $x = \frac{1}{2}$?

Solution

The focus has the form $(p, 0)$, so the equation will have the form $y^2 = 4px$.

- Multiplying $4p$, we have $4p = 4\left(-\frac{1}{2}\right) = -2$.
- Substituting for $4p$, we have $y^2 = 4px = -2x$.

Therefore, the equation for the parabola is $y^2 = -2x$.

TRY IT #3 What is the equation for the parabola with focus $\left(0, \frac{7}{2}\right)$ and directrix $y = -\frac{7}{2}$?

Graphing Parabolas with Vertices Not at the Origin

Like other graphs we've worked with, the graph of a parabola can be translated. If a parabola is translated h units horizontally and k units vertically, the vertex will be (h, k). This translation results in the standard form of the equation