■ A computer or graphing calculator finds the regression line by the method of **least squares**, which is to minimize the sum of the squares of the vertical distances between the data points and the line. The details are explained in Section 14.7.

called *linear regression*. If we use a graphing calculator, we enter the data from Table 1 into the data editor and choose the linear regression command. (With Maple we use the fit[leastsquare] command in the stats package; with Mathematica we use the Fit command.) The machine gives the slope and y-intercept of the regression line as

$$m = 1.55192 \qquad b = -2734.55$$

So our least squares model for the CO_2 level is

$$\boxed{2} \qquad C = 1.55192t - 2734.55$$

In Figure 6 we graph the regression line as well as the data points. Comparing with Figure 5, we see that it gives a better fit than our previous linear model.

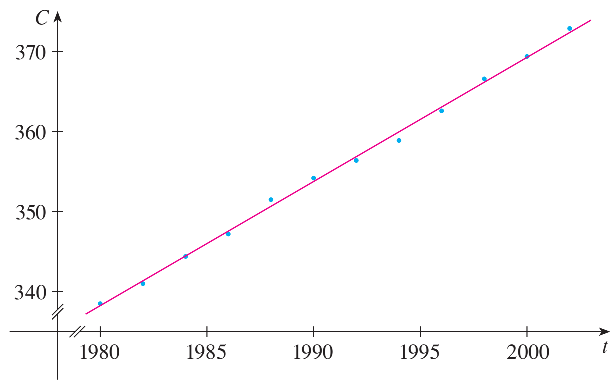

FIGURE 6
The regression line

■

V EXAMPLE 3 Use the linear model given by Equation 2 to estimate the average CO_2 level for 1987 and to predict the level for the year 2010. According to this model, when will the CO_2 level exceed 400 parts per million?

SOLUTION Using Equation 2 with $t = 1987$, we estimate that the average CO_2 level in 1987 was

$$C(1987) = (1.55192)(1987) - 2734.55 \approx 349.12$$

This is an example of *interpolation* because we have estimated a value *between* observed values. (In fact, the Mauna Loa Observatory reported that the average CO_2 level in 1987 was 348.93 ppm, so our estimate is quite accurate.)

With $t = 2010$, we get

$$C(2010) = (1.55192)(2010) - 2734.55 \approx 384.81$$

So we predict that the average CO_2 level in the year 2010 will be 384.8 ppm. This is an example of *extrapolation* because we have predicted a value *outside* the region of observations. Consequently, we are far less certain about the accuracy of our prediction.

Using Equation 2, we see that the CO_2 level exceeds 400 ppm when

$$1.55192t - 2734.55 > 400$$

Solving this inequality, we get

$$t > \frac{3134.55}{1.55192} \approx 2019.79$$