We will apply this method as we subtract the fractions in Example 1.82.

EXAMPLE 1.82

Subtract: $\frac{7}{15} - \frac{19}{24}$.

Solution

Do the fractions have a common denominator? No, so we need to find the LCD.

	$\frac{7}{15} - \frac{19}{24}$
Find the LCD.	$\begin{array}{r} 15 = \qquad\qquad 3 \cdot 5 \\ 24 = 2 \cdot 2 \cdot 2 \cdot 3 \quad \\ \hline \text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \\ \text{LCD} = 120 \qquad\qquad \end{array}$
Notice, 15 is “missing” three factors of 2 and 24 is “missing” the 5 from the factors of the LCD. So we multiply 8 in the first fraction and 5 in the second fraction to get the LCD.	
Rewrite as equivalent fractions with the LCD.	$\frac{7 \cdot 8}{15 \cdot 8} - \frac{19 \cdot 5}{24 \cdot 5}$
Simplify.	$\frac{56}{120} - \frac{95}{120}$
Subtract.	$-\frac{39}{120}$
Check to see if the answer can be simplified.	$-\frac{13 \cdot 3}{40 \cdot 3}$
Both 39 and 120 have a factor of 3.	
Simplify.	$-\frac{13}{40}$

Do not simplify the equivalent fractions! If you do, you’ll get back to the original fractions and lose the common denominator!

TRY IT : : 1.163 Subtract: $\frac{13}{24} - \frac{17}{32}$.

TRY IT : : 1.164 Subtract: $\frac{21}{32} - \frac{9}{28}$.

In the next example, one of the fractions has a variable in its numerator. Notice that we do the same steps as when both numerators are numbers.

EXAMPLE 1.83

Add: $\frac{3}{5} + \frac{x}{8}$.

Solution

The fractions have different denominators.

	$\frac{3}{5} + \frac{x}{8}$

    - **Addition:** If a, b, c are real numbers, then $(a+b)+c = a+(b+c)$.
    - **Multiplication:** If a, b, c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
    When adding or multiplying, changing the *grouping* gives the same result.
- **Distributive Property:** If a, b, c are real numbers, then
    - $a(b+c) = ab + ac$
    - $(b+c)a = ba + ca$
    - $a(b-c) = ab - ac$
    - $(b-c)a = ba - ca$
- **Identity Property**
    - **of Addition:** For any real number a: $a + 0 = a \quad 0 + a = a$
    0 is the **additive identity**
    - **of Multiplication:** For any real number a: $a \cdot 1 = a \quad 1 \cdot a = a$
    $\mathbf{1}$ is the **multiplicative identity**
- **Inverse Property**
    - **of Addition:** For any real number a, $a + (-a) = 0$. A number and its *opposite* add to zero. $-a$ is the **additive inverse** of a.
    - **of Multiplication:** For any real number a, $(a \neq 0)$ $a \cdot \frac{1}{a} = 1$. A number and its *reciprocal* multiply to one.
    $\frac{1}{a}$ is the **multiplicative inverse** of a.
- **Properties of Zero**
    - For any real number a,
    $a \cdot 0 = 0 \quad 0 \cdot a = 0$ – The product of any real number and 0 is 0.
    - $\frac{0}{a} = 0$ for $a \neq 0$ – Zero divided by any real number except zero is zero.
    - $\frac{a}{0}$ is undefined – Division by zero is undefined.

1.10 Systems of Measurement

- **Metric System of Measurement**
    - **Length**

$$\begin{array}{lcl}
1 \text{ kilometer (km)} & = & 1{,}000 \text{ m} \\
1 \text{ hectometer (hm)} & = & 100 \text{ m} \\
1 \text{ dekameter (dam)} & = & 10 \text{ m} \\
1 \text{ meter (m)} & = & 1 \text{ m} \\
1 \text{ decimeter (dm)} & = & 0.1 \text{ m} \\
1 \text{ centimeter (cm)} & = & 0.01 \text{ m} \\
1 \text{ millimeter (mm)} & = & 0.001 \text{ m} \\
1 \text{ meter} & = & 100 \text{ centimeters} \\
1 \text{ meter} & = & 1{,}000 \text{ millimeters}
\end{array}$$

    - **Mass**

Solution

	$\underbrace{144}\ \underbrace{\text{is}}\ \underbrace{\text{what percent}}\ \underbrace{\text{of}}\ \underbrace{96?}$
Translate into algebra. Let $p =$ the percent.	$144 \quad = \quad p \quad \cdot \quad 96$
Multiply.	$144 = 96p$
Divide by 96 and simplify.	$1.5 = p$
Convert to percent.	$150\% = p$
	$144 \text{ is } 150\% \text{ of } 96$

Note that we are asked to find percent, so we must have our final result in percent form.

TRY IT : : 3.27 Translate and solve:

110 is what percent of 88?

TRY IT : : 3.28 Translate and solve:

126 is what percent of 72?

Solve Applications of Percent

Many applications of percent—such as tips, sales tax, discounts, and interest—occur in our daily lives. To solve these applications we'll translate to a basic percent equation, just like those we solved in previous examples. Once we translate the sentence into a percent equation, we know how to solve it.

We will restate the problem solving strategy we used earlier for easy reference.

HOW TO : : USE A PROBLEM-SOLVING STRATEGY TO SOLVE AN APPLICATION.

Step 1. **Read** the problem. Make sure all the words and ideas are understood.

Step 2. **Identify** what we are looking for.

Step 3. **Name** what we are looking for. Choose a variable to represent that quantity.

Step 4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation.

Step 5. **Solve** the equation using good algebra techniques.

Step 6. **Check** the answer in the problem and make sure it makes sense.

Step 7. **Answer** the question with a complete sentence.

Now that we have the strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications will involve everyday situations, you can rely on your own experience.

EXAMPLE 3.15

Dezohn and his girlfriend enjoyed a nice dinner at a restaurant and his bill was $68.50. He wants to leave an 18% tip. If the tip will be 18% of the total bill, how much tip should he leave?

express train	60 mph (4 hours) = 240 miles
local train	48 mph (5 hours) = 240 miles ✓

Table 3.11

Step 7. Answer the question with a complete sentence.

- The speed of the local train is 48 mph and the speed of the express train is 60 mph.

TRY IT : : 3.95

Wayne and Dennis like to ride the bike path from Riverside Park to the beach. Dennis's speed is seven miles per hour faster than Wayne's speed, so it takes Wayne 2 hours to ride to the beach while it takes Dennis 1.5 hours for the ride. Find the speed of both bikers.

TRY IT : : 3.96

Jeromy can drive from his house in Cleveland to his college in Chicago in 4.5 hours. It takes his mother 6 hours to make the same drive. Jeromy drives 20 miles per hour faster than his mother. Find Jeromy's speed and his mother's speed.

In Example 3.48, the last example, we had two trains traveling the same distance. The diagram and the chart helped us write the equation we solved. Let's see how this works in another case.

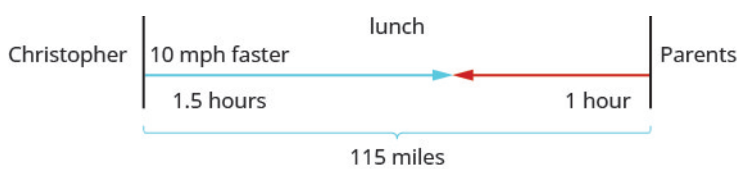
EXAMPLE 3.49

Christopher and his parents live 115 miles apart. They met at a restaurant between their homes to celebrate his mother's birthday. Christopher drove 1.5 hours while his parents drove 1 hour to get to the restaurant. Christopher's average speed was 10 miles per hour faster than his parents' average speed. What were the average speeds of Christopher and of his parents as they drove to the restaurant?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.

- Draw a diagram to illustrate what it happening. Below shows a sketch of what is happening in the example.



- Create a table to organize the information.
- Label the columns rate, time, distance.
- List the two scenarios.
- Write in the information you know.

	Rate (mph)	• Time (hrs)	= Distance (miles)
Christopher		1.5	
Parents		1	
			115

Step 2. Identify what we are looking for.

- We are asked to find the average speeds of Christopher and his parents.

Step 3. Name what we are looking for. Choose a variable to represent that quantity.

- Complete the chart.

PRACTICE TEST

425. Four-fifths of the people on a hike are children. If there are 12 children, what is the total number of people on the hike?

426. One number is three more than twice another. Their sum is -63. Find the numbers.

427. The sum of two consecutive odd integers is -96. Find the numbers.

428. Marla's breakfast was 525 calories. This was 35% of her total calories for the day. How many calories did she have that day?

429. Humberto's hourly pay increased from \$16.25 to \$17.55. Find the percent increase.

430. Melinda deposited \$5,985 in a bank account with an interest rate of 1.9%. How much interest was earned in 2 years?

431. Dotty bought a freezer on sale for \$486.50. The original price of the freezer was \$695. Find ⓐ the amount of discount and ⓑ the discount rate.

432. Bonita has \$2.95 in dimes and quarters in her pocket. If she has five more dimes than quarters, how many of each coin does she have?

433. At a concert, \$1,600 in tickets were sold. Adult tickets were \$9 each and children's tickets were \$4 each. If the number of adult tickets was 30 less than twice the number of children's tickets, how many of each kind were sold?

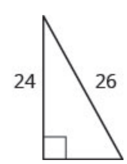
434. Kim is making eight gallons of punch from fruit juice and soda. The fruit juice costs \$6.04 per gallon and the soda costs \$4.28 per gallon. How much fruit juice and how much soda should she use so that the punch costs \$5.71 per gallon?

435. The measure of one angle of a triangle is twice the measure of the smallest angle. The measure of the third angle is 14 more than the measure of the smallest angle. Find the measures of all three angles.

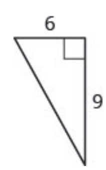
436. What is the height of a triangle with area 277.2 square inches and base 44 inches?

In the following exercises, use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.

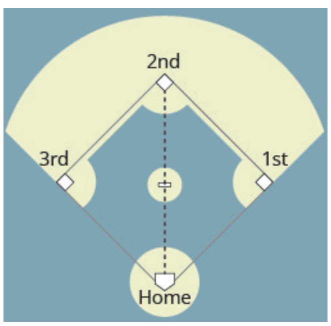
437.



438.



439. A baseball diamond is really a square with sides of 90 feet. How far is it from home plate to second base, as shown?



440. The length of a rectangle is two feet more than five times the width. The perimeter is 40 feet. Find the dimensions of the rectangle.

441. Two planes leave Dallas at the same time. One heads east at a speed of 428 miles per hour. The other plane heads west at a speed of 382 miles per hour. How many hours will it take them to be 2,025 miles apart?

442. Leon drove from his house in Cincinnati to his sister's house in Cleveland, a distance of 252 miles. It took him $4\frac{1}{2}$ hours. For the first half hour he had heavy traffic, and the rest of the time his speed was five miles per hour less than twice his speed in heavy traffic. What was his speed in heavy traffic?

5.4

Solve Applications with Systems of Equations

Learning Objectives

By the end of this section, you will be able to:

- Translate to a system of equations
- Solve direct translation applications
- Solve geometry applications
- Solve uniform motion applications

BE PREPARED : : 5.10

Before you get started, take this readiness quiz.

The sum of twice a number and nine is 31. Find the number.
If you missed this problem, review **Example 3.4**.

BE PREPARED : : 5.11

Twins Jon and Ron together earned \$96,000 last year. Ron earned \$8,000 more than three times what Jon earned. How much did each of the twins earn?
If you missed this problem, review **Example 3.11**.

BE PREPARED : : 5.12

Alessio rides his bike $3\frac{1}{2}$ hours at a rate of 10 miles per hour. How far did he ride?

If you missed this problem, review **Example 2.58**.

Previously in this chapter we solved several applications with systems of linear equations. In this section, we'll look at some specific types of applications that relate two quantities. We'll translate the words into linear equations, decide which is the most convenient method to use, and then solve them.

We will use our Problem Solving Strategy for Systems of Linear Equations.

HOW TO : : USE A PROBLEM SOLVING STRATEGY FOR SYSTEMS OF LINEAR EQUATIONS.

Step 1. **Read** the problem. Make sure all the words and ideas are understood.
Step 2. **Identify** what we are looking for.
Step 3. **Name** what we are looking for. Choose variables to represent those quantities.
Step 4. **Translate** into a system of equations.
Step 5. **Solve** the system of equations using good algebra techniques.
Step 6. **Check** the answer in the problem and make sure it makes sense.
Step 7. **Answer** the question with a complete sentence.

Translate to a System of Equations

Many of the problems we solved in earlier applications related two quantities. Here are two of the examples from the chapter on Math Models.

- The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.
- A married couple together earns \$110,000 a year. The wife earns \$16,000 less than twice what her husband earns. What does the husband earn?

In that chapter we translated each situation into one equation using only one variable. Sometimes it was a bit of a challenge figuring out how to name the two quantities, wasn't it?

Let's see how we can translate these two problems into a system of equations with two variables. We'll focus on Steps 1

Solution

	$(b+3)(2b^2-5b+8)$
Distribute.	$b(2b^2-5b+8)+3(2b^2-5b+8)$
Multiply.	$2b^3-5b^2+8b+6b^2-15b+24$
Combine like terms.	$2b^3+b^2-7b+24$

> **TRY IT : : 6.89** Multiply using the Distributive Property: $(y-3)(y^2-5y+2)$.

> **TRY IT : : 6.90** Multiply using the Distributive Property: $(x+4)(2x^2-3x+5)$.

Now let's do this same multiplication using the Vertical Method.

EXAMPLE 6.46

Multiply using the Vertical Method: $(b+3)(2b^2-5b+8)$.

Solution

It is easier to put the polynomial with fewer terms on the bottom because we get fewer partial products this way.

	$2b^2-5b+8$
	$\times \quad b+3$
Multiply $(2b^2-5b+8)$ by 3.	$6b^2-15b+24$
	$2b^3-5b^2+8b$
Multiply $(2b^2-5b+8)$ by b.	$2b^3+b^2-7b+24$
Add like terms.	

> **TRY IT : : 6.91** Multiply using the Vertical Method: $(y-3)(y^2-5y+2)$.

> **TRY IT : : 6.92** Multiply using the Vertical Method: $(x+4)(2x^2-3x+5)$.

We have now seen two methods you can use to multiply a trinomial by a binomial. After you practice each method, you'll probably find you prefer one way over the other. We list both methods are listed here, for easy reference.

Multiplying a Trinomial by a Binomial

To multiply a trinomial by a binomial, use the:

- Distributive Property
- Vertical Method

	$z^2 + 4z - 5$
Factors will be two binomials with first terms z.	$(z\quad)(z\quad)$
Use $-1,\ 5$ as the last terms of the binomials.	$(z-1)(z+5)$

Check.

$$(z-1)(z+5)$$
$$z^2 + 5z - 1z - 5$$
$$z^2 + 4z - 5\ ✓$$

> **TRY IT : : 7.41** Factor: $h^2 + 4h - 12$.

> **TRY IT : : 7.42** Factor: $k^2 + k - 20$.

Let's make a minor change to the last trinomial and see what effect it has on the factors.

EXAMPLE 7.22

Factor: $z^2 - 4z - 5$.

Solution

This time, we need factors of -5 that add to -4.

Factors of -5	Sum of factors
$1,\ -5$	$1 + (-5) = -4^*$
$-1,\ 5$	$-1 + 5 = 4$

	$z^2 - 4z - 5$
Factors will be two binomials with first terms z.	$(z\quad)(z\quad)$
Use $1,\ -5$ as the last terms of the binomials.	$(z+1)(z-5)$

Check.

$$(z+1)(z-5)$$
$$z^2 - 5z + 1z - 5$$
$$z^2 - 4z - 5\ ✓$$

Notice that the factors of $z^2 - 4z - 5$ are very similar to the factors of $z^2 + 4z - 5$. It is very important to make sure you choose the factor pair that results in the correct sign of the middle term.

> **TRY IT : : 7.43** Factor: $x^2 - 4x - 12$.

> **TRY IT : : 7.44** Factor: $y^2 - y - 20$.

EXAMPLE 7.23

Factor: $q^2 - 2q - 15$.

HOW TO : : FACTOR POLYNOMIALS.

Step 1. Is there a greatest common factor?

- Factor it out.

Step 2. Is the polynomial a binomial, trinomial, or are there more than three terms?

- If it is a binomial:
  Is it a sum?
  - Of squares? Sums of squares do not factor.
  - Of cubes? Use the sum of cubes pattern.

  Is it a difference?
  - Of squares? Factor as the product of conjugates.
  - Of cubes? Use the difference of cubes pattern.
- If it is a trinomial:
  Is it of the form $x^2 + bx + c$? Undo FOIL.
  Is it of the form $ax^2 + bx + c$?
  - If a and c are squares, check if it fits the trinomial square pattern.
  - Use the trial and error or "ac" method.
- If it has more than three terms:
  Use the grouping method.

Step 3. Check.

- Is it factored completely?
- Do the factors multiply back to the original polynomial?

Remember, a polynomial is completely factored if, other than monomials, its factors are prime!

EXAMPLE 7.59

Factor completely: $4x^5 + 12x^4$.

Solution

Is there a GCF?	Yes, $4x^4$.	$4x^5 + 12x^4$
	Factor out the GCF.	$4x^4(x+3)$
In the parentheses, is it a binomial, a trinomial, or are there more than three terms?	Binomial.	
Is it a sum?		Yes.
Of squares? Of cubes?		No.
Check.		
Is the expression factored completely?		Yes.
Multiply.		

$$4x^4(x+3)$$
$$4x^4 \cdot x + 4x^4 \cdot 3$$
$$4x^5 + 12x^4 \checkmark$$

TRY IT : : 7.117 Factor completely: $3a^4 + 18a^3$.

Solution

	$\frac{2a}{2ab+b^2}+\frac{3a}{4a^2-b^2}$
Do the expressions have a common denominator? No. Rewrite each expression with the LCD.	
Find the LCD. $\begin{array}{rl} 2ab+b^2 &= b(2a+b) \\ 4a^2-b^2 &= \quad (2a+b)(2a-b) \\ \hline \text{LCD} &= b(2a+b)(2a-b) \end{array}$	
Rewrite each rational expression as an equivalent rational expression with the LCD.	$\frac{2a(2a-b)}{b(2a+b)(2a-b)}+\frac{3a\cdot b}{(2a+b)(2a-b)\cdot b}$
Simplify the numerators.	$\frac{4a^2-2ab}{b(2a+b)(2a-b)}+\frac{3ab}{b(2a+b)(2a-b)}$
Add the rational expressions.	$\frac{4a^2-2ab+3ab}{b(2a+b)(2a-b)}$
Simplify the numerator.	$\frac{4a^2+ab}{b(2a+b)(2a-b)}$
Factor the numerator.	$\frac{a(4a+b)}{b(2a+b)(2a-b)}$
There are no factors common to the numerator and denominator. The fraction cannot be simplified.	

TRY IT : : 8.85 Add: $\frac{5x}{xy-y^2}+\frac{2x}{x^2-y^2}$.

TRY IT : : 8.86 Add: $\frac{7}{2m+6}+\frac{4}{m^2+4m+3}$.

Avoid the temptation to simplify too soon! In the example above, we must leave the first rational expression as $\frac{2a(2a-b)}{b(2a+b)(2a-b)}$ to be able to add it to $\frac{3a\cdot b}{(2a+b)(2a-b)\cdot b}$. Simplify only after you have combined the numerators.

EXAMPLE 8.44

Add: $\frac{8}{x^2-2x-3}+\frac{3x}{x^2+4x+3}$.

Solution

	$\frac{8}{x^2-2x-3}+\frac{3x}{x^2+4x+3}$
Do the expressions have a common denominator? No. Rewrite each expression with the LCD.	
Find the LCD. $\begin{array}{rl} x^2-2x-3 &= (x+1)(x-3) \\ x^2+4x+3 &= (x+1)\qquad\quad (x+3) \\ \hline \text{LCD} &= (x+1)(x-3)(x+3) \end{array}$	
Rewrite each rational expression as an equivalent fraction with the LCD.	$\frac{8(x+3)}{(x+1)(x-3)(x+3)}+\frac{3x(x-3)}{(x+1)(x+3)(x-3)}$