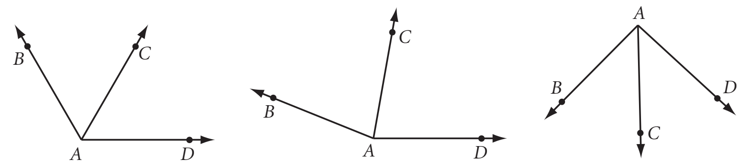
The next example shows how to use both kinds of reasoning: inductive reasoning to discover the property and deductive reasoning to explain why it works.

EXAMPLE B

In each diagram, $\overrightarrow{AC}$ bisects obtuse angle BAD. Classify $\angle BAD$, $\angle DAC$, and $\angle CAB$ as *acute*, *right*, or *obtuse*. Then complete the conjecture.



$m\angle BAD = 120°$ $m\angle BAD = 158°$ $m\angle BAD = 92°$

Conjecture: If an obtuse angle is bisected, then the two newly formed congruent angles are _?_.

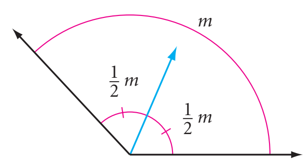
Justify your answers with a deductive argument.

▶ Solution

In each diagram, $\angle BAD$ is obtuse because $m\angle BAD$ is greater than 90°. In each diagram, the angles formed by the bisector are acute because their measures—60°, 79°, and 46°—are less than 90°. So one possible conjecture is

Conjecture: If an obtuse angle is bisected, then the two newly formed congruent angles are *acute*.

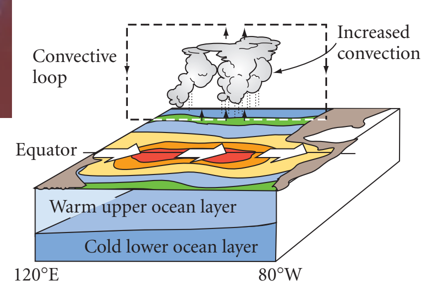
Why? According to our definition of an angle, every angle measure is less than 180°. So, using algebra, if m is the measure of an obtuse angle, then $m < 180°$. When you bisect an angle, the two newly formed angles each measure half of the original angle, or $\frac{1}{2}m$. If $m < 180°$, then $\frac{1}{2}m < \frac{1}{2}(180)$, so $\frac{1}{2}m < 90°$. The two angles are each less than 90°, so they are acute.



Science CONNECTION

Here is an example of inductive reasoning, supported by deductive reasoning. El Niño is the warming of water in the tropical Pacific Ocean, which produces unusual weather conditions and storms worldwide. For centuries, farmers living in the Andes Mountains of South America have observed the stars in the Pleiades constellation to predict El Niño conditions. If the Pleiades look dim in June, they predict an El Niño year. What is the connection? Scientists have recently found that in an El Niño year, increased evaporation from the ocean produces high-altitude clouds that are invisible to the eye, but create a haze that makes stars more difficult to see. Therefore, the pattern that the Andean farmers knew about for centuries is now supported by a scientific explanation. To find out more about this story, go to www.keymath.com/DG .

El Niño Conditions



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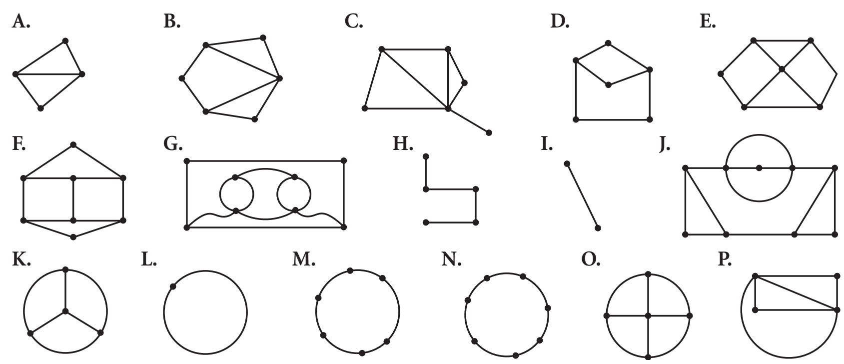
Step 1 | Try these networks and see which ones can be traveled and which are impossible to travel.

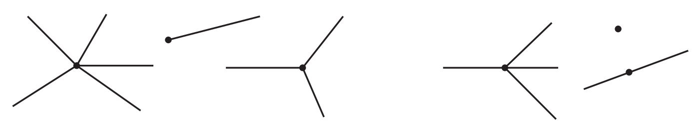


Which networks were impossible to travel? Are they impossible or just difficult? How can you be sure? As you do the next few steps, see if you can find the reason why some networks are impossible to travel.

Step 2 | Draw the River Pregel and the two islands shown on the first page of this exploration. Draw an eighth bridge so that you can travel over all the bridges exactly once if you start at point *C* and end at point *B*.

Step 3 | Draw the River Pregel and the two islands. Can you draw an eighth bridge so that you can travel over all the bridges exactly once, starting and finishing at the same point? How many solutions can you find?

Step 4 | Euler realized that it is the points of intersection that determine whether a network can be traveled. Each point of intersection is either "odd" or "even."



Odd points　　　Even points

Did you find any networks that have only one odd point? Can you draw one? Try it. How about three odd points? Or five odd points? Can you create a network that has an odd number of odd points? Explain why or why not.

Step 5 | How does the number of even points and odd points affect whether a network can be traveled?

Conjecture

A network can be traveled if __?__.

LESSON

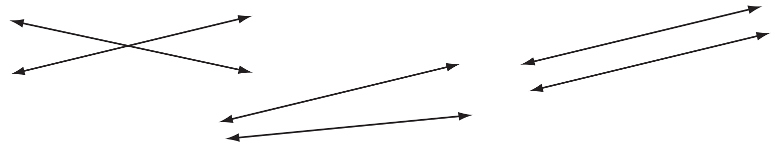
3.5

Constructing Parallel Lines

When you stop to think, don't forget to start up again.

ANONYMOUS

Parallel lines are lines that lie in the same plane and do not intersect.



The lines in the first pair shown above intersect. They are clearly not parallel. The lines in the second pair do not meet as drawn. However, if they were extended, they would intersect. Therefore, they are not parallel. The lines in the third pair appear to be parallel, but if you extend them far enough in both directions, can you be sure they won't meet? There are many ways to be sure that the lines are parallel.

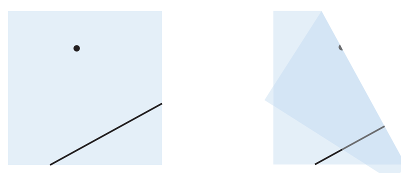
Investigation

Constructing Parallel Lines by Folding

You will need

- patty paper

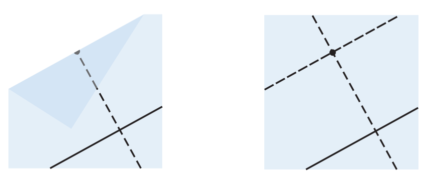
How would you check whether two lines are parallel? One way is to draw a transversal and compare corresponding angles. You can also use this idea to *construct* a pair of parallel lines.



Step 1 Step 2

Step 1 Draw a line and a point on patty paper as shown.

Step 2 Fold the paper to construct a perpendicular so that the crease runs through the point as shown. Describe the four newly formed angles.



Step 3 Step 4

Step 3 Through the point, make another fold that is perpendicular to the first crease.

Step 4 Match the pairs of corresponding angles created by the folds. Are they all congruent? Why? What conclusion can you make about the lines?

CHAPTER

4

Discovering and Proving Triangle Properties

Is it possible to make a representation of recognizable figures that has no background?

M. C. ESCHER

Symmetry Drawing E103, M. C. Escher, 1959

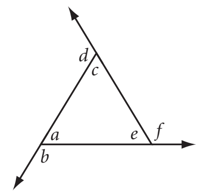
OBJECTIVES

In this chapter you will

- learn why triangles are so useful in structures
- discover relationships between the sides and angles of triangles
- learn about the conditions that guarantee that two triangles are congruent

EXERCISES

1. Complete this flowchart proof of the Exterior Angle Sum Conjecture for a triangle.



Flowchart Proof

1 $a + b = 180°$
$\underline{?}$

2 $c + d = 180°$
$\underline{?}$

3 $e + f = 180°$
$\underline{?}$

4 $a + b + c + d + e + f = \underline{?}\,°$
Addition property of equality

5 $a + c + e = \underline{?}\,°$
$\underline{?}$

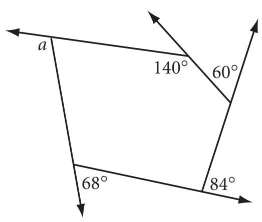
6 $b + d + f = \underline{?}\,°$
Subtraction property of equality

2. What is the sum of the measures of the exterior angles of a decagon?

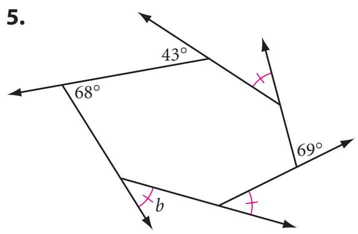
3. What is the measure of an exterior angle of an equiangular pentagon? An equiangular hexagon?

In Exercises 4–9, use your new conjectures to calculate the measure of each lettered angle.

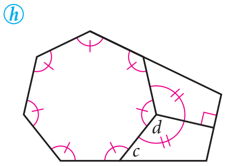
4.



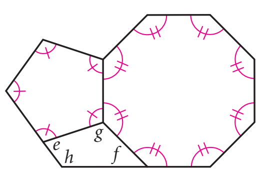
5.



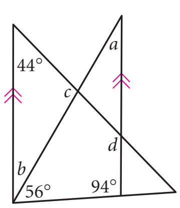
6. ⓗ



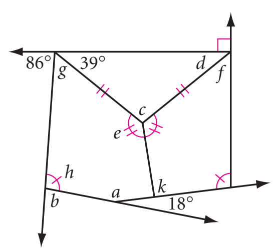
7.



8.



9.



10. How many sides does a regular polygon have if each exterior angle measures 24°? ⓗ

11. How many sides does a polygon have if the sum of its interior angle measures is 7380°?

12. Is there a maximum number of obtuse exterior angles that any polygon can have? If so, what is the maximum? If not, why not? Is there a minimum number of acute interior angles that any polygon must have? If so, what is the minimum? If not, why not? ⓗ

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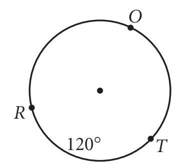
EXAMPLE C

If the length of $\overset{\frown}{ROT}$ is 116π meters, what is the radius of the circle?



▸ Solution

$m\overset{\frown}{ROT} = 240°$, so $\overset{\frown}{ROT}$ is $\frac{240}{360}$, or $\frac{2}{3}$ of the circumference.

$116\pi = \frac{2}{3}C$	Apply the Arc Length Conjecture.
$116\pi = \frac{2}{3}(2\pi r)$	Substitute $2\pi r$ for C.
$348\pi = 4\pi r$	Multiply both sides by 3.
$87 = r$	Divide both sides by 4π.

The radius is 87 m.

EXERCISES

You will need

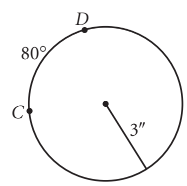
A calculator for Exercises **9–14**

Construction tools for Exercise **16**

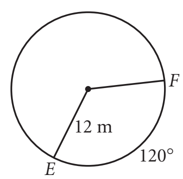
Geometry software for Exercise **16**

For Exercises 1–8, state your answers in terms of π.

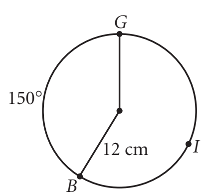
1. Length of $\overset{\frown}{CD}$ is $\underline{\ ?\ }$.



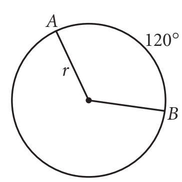
2. Length of $\overset{\frown}{EF}$ is $\underline{\ ?\ }$.



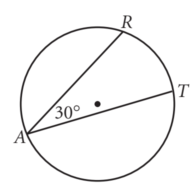
3. Length of $\overset{\frown}{BIG}$ is $\underline{\ ?\ }$. ⓗ



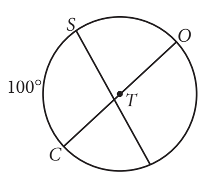
4. Length of $\overset{\frown}{AB}$ is 6π m. The radius is $\underline{\ ?\ }$.



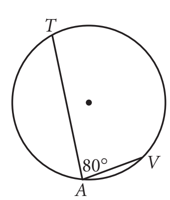
5. The radius is 18 ft. Length of $\overset{\frown}{RT}$ is $\underline{\ ?\ }$.



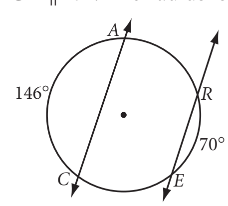
6. The radius is 9 m. Length of $\overset{\frown}{SO}$ is $\underline{\ ?\ }$.



7. Length of $\overset{\frown}{TV}$ is 12π in. The diameter is $\underline{\ ?\ }$.



8. Length of $\overset{\frown}{AR}$ is 40π cm. $\overline{CA} \parallel \overline{RE}$. The radius is $\underline{\ ?\ }$. ⓗ



Review

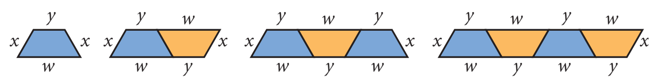
In Exercises 15 and 16, graph the two lines, then find the area bounded by the x-axis, the y-axis, and both lines.

15. $y = \frac{1}{2}x + 5, y = -2x + 10$ ⓗ

16. $y = -\frac{1}{3}x + 6, y = -\frac{4}{3}x + 12$

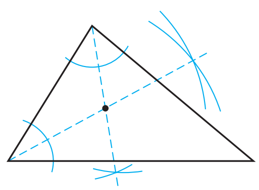
17. ***Technology*** Construct a triangle and its three medians. Compare the areas of the six small triangles that the three medians formed. Make a conjecture, and support it with a convincing argument.

18. If the pattern continues, write an expression for the perimeter of the nth figure in the picture pattern.

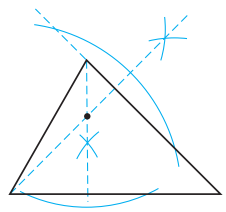


19. Identify the point of concurrency from the construction marks.

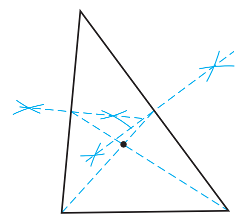
a.



b.



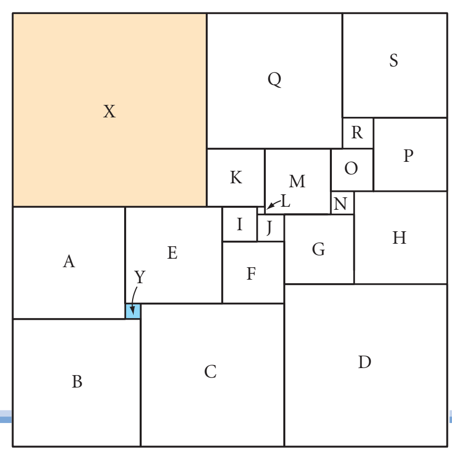
c.



IMPROVING YOUR VISUAL THINKING SKILLS

The Squared Square Puzzle

The square shown is called a "squared square." A square 112 units on a side is divided into 21 squares. The area of square X is 50^2, or 2500, and the area of square Y is 4^2, or 16. Find the area of each of the other squares.



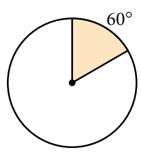
EXERCISES

You will need

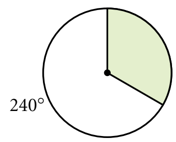
Construction tools for Exercise 16

In Exercises 1–8, find the area of the shaded region. The radius of each circle is r. If two circles are shown, r is the radius of the smaller circle and R is the radius of the larger circle.

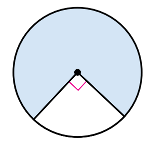
1. $r = 6$ cm



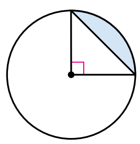
2. $r = 8$ cm



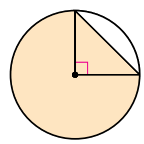
3. $r = 16$ cm



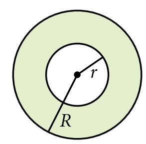
4. $r = 2$ cm



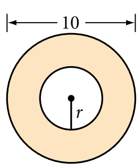
5. $r = 8$ cm



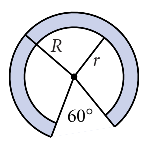
6. $R = 7$ cm
$r = 4$ cm ⓗ



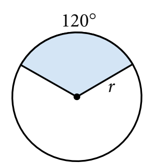
7. $r = 2$ cm



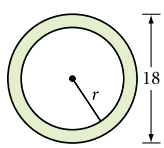
8. $R = 12$ cm
$r = 9$ cm



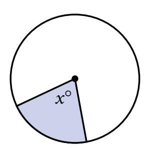
9. The shaded area is 12π cm^2. Find r.



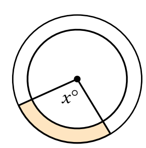
10. The shaded area is 32π cm^2. Find r.



11. The shaded area is 120π cm^2, and the radius is 24 cm. Find x.



12. The shaded area is 10π cm^2. The radius of the large circle is 10 cm, and the radius of the small circle is 8 cm. Find x. ⓗ



13. Suppose the pizza slice in the photo at the beginning of this lesson is a sector with a 36° arc, and the pizza has a radius of 20 ft. If one can of tomato sauce will cover 3 ft^2 of pizza, how many cans would you need to cover this slice?

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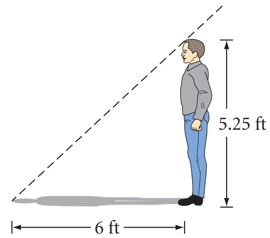
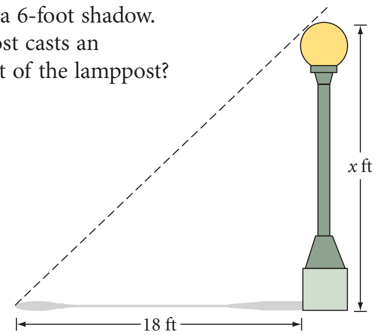
EXAMPLE A person 5 feet 3 inches tall casts a 6-foot shadow. At the same time of day, a lamppost casts an 18-foot shadow. What is the height of the lamppost?





▶ Solution The light rays that create the shadows hit the ground at congruent angles. Assuming both the person and the lamppost are perpendicular to the ground, you have similar triangles by the AA Similarity Conjecture. Solve a proportion that relates corresponding lengths.

$$\frac{5.25}{6} = \frac{x}{18}$$

$$18 \cdot \frac{5.25}{6} = x$$

$$15.75 = x$$

The height of the lamppost is 15 feet 9 inches.

EXERCISES

You will need

Construction tools for Exercise 14

Geometry software for Exercises 17 and 18

1. A flagpole 4 meters tall casts a 6-meter shadow. At the same time of day, a nearby building casts a 24-meter shadow. How tall is the building?

2. Five-foot-tall Melody casts an 84-inch shadow. How tall is her friend if, at the same time of day, his shadow is 1 foot shorter than hers?

3. A 10 m rope from the top of a flagpole reaches to the end of the flagpole's 6 m shadow. How tall is the nearby football goalpost if, at the same moment, it has a shadow of 4 m? ⓗ

4. Private eye Samantha Diamond places a mirror on the ground between herself and an apartment building and stands so that when she looks into the mirror, she sees into a window. The mirror's crosshairs are 1.22 meters from her feet and 7.32 meters from the base of the building. Sam's eye is 1.82 meters above the ground. How high is the window?

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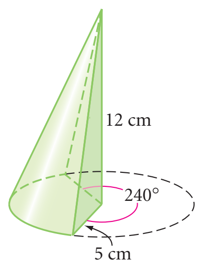
53. A 32-foot telephone pole casts a 12-foot shadow at the same time a boy nearby casts a 1.75-foot shadow. How tall is the boy?

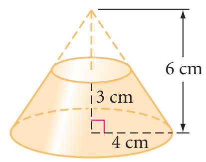
A. 4 ft 8 in. **B.** 4 ft 6 in. **C.** 5 ft 8 in. **D.** 6 ft

Exercises 54–56 are portions of cones. Find the volume of each solid.

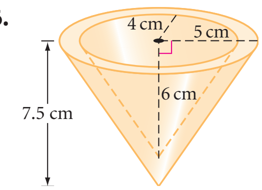
54.



55.



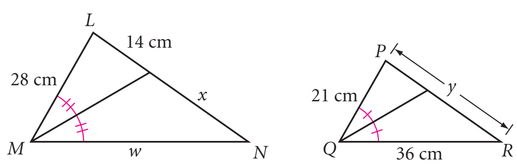
56.



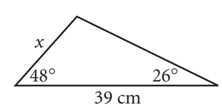
57. Each person at a family reunion hugs everyone else exactly once. There were 528 hugs. How many people were at the reunion?

58. Triangle TRI with vertices $T(-7, 0)$, $R(-5, 3)$, and $I(-1, 0)$ is translated by the rule $(x, y) \rightarrow (x + 2, y - 1)$. Then its image is translated by the rule $(x, y) \rightarrow (x - 1, y - 2)$. What single translation is equivalent to the composition of these two translations?

59. $\triangle LMN \sim \triangle PQR$. Find w, x, and y.



60. Find x.



61. The diameter of a circle has endpoints $(5, -2)$ and $(5, 4)$. Find the equation of the circle.

62. Explain why a regular pentagon cannot create a monohedral tessellation.

63. Archaeologist Ertha Diggs uses a clinometer to find the height of an ancient temple. She views the top of the temple with a 37° angle of elevation. She is standing 130 meters from the center of the temple's base, and her eye is 1.5 meters above the ground. How tall is the temple?

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